

# Computer algebra independent integration tests

1-Algebraic-functions/1.1-Binomial-products/1.1.3-General/1.1.3.3-a+b-x^n-p-c+d-x^n-q

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June 28, 2021

Compiled on June 28, 2021 at 11:15pm

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3.208	$\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)^2} dx$	887
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3.233	$\int \left(a+\frac{b}{x}\right)^{3/2} \left(c+\frac{d}{x}\right) dx$	982
3.234	$\int \left(a+\frac{b}{x}\right)^{3/2} dx$	986
3.235	$\int \frac{\left(a+\frac{b}{x}\right)^{3/2}}{c+\frac{d}{x}} dx$	990

3.236	$\int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^2} dx$	994
3.237	$\int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^3} dx$	999
3.238	$\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 dx$	1006
3.239	$\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 dx$	1014
3.240	$\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right) dx$	1019
3.241	$\int \left(a + \frac{b}{x}\right)^{5/2} dx$	1023
3.242	$\int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{c + \frac{d}{x}} dx$	1027
3.243	$\int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^2} dx$	1032
3.244	$\int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^3} dx$	1037
3.245	$\int \frac{\left(c + \frac{d}{x}\right)^3}{\sqrt{a + \frac{b}{x}}} dx$	1043
3.246	$\int \frac{\left(c + \frac{d}{x}\right)^2}{\sqrt{a + \frac{b}{x}}} dx$	1047
3.247	$\int \frac{c + \frac{d}{x}}{\sqrt{a + \frac{b}{x}}} dx$	1051
3.248	$\int \frac{1}{\sqrt{a + \frac{b}{x}}} dx$	1054
3.249	$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} dx$	1057
3.250	$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} dx$	1061
3.251	$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3} dx$	1067
3.252	$\int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{3/2}} dx$	1074
3.253	$\int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{3/2}} dx$	1078
3.254	$\int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{3/2}} dx$	1082
3.255	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2}} dx$	1086
3.256	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} dx$	1089
3.257	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} dx$	1095
3.258	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3} dx$	1103

3.259	$\int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{5/2}} dx$	1113
3.260	$\int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{5/2}} dx$	1118
3.261	$\int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{5/2}} dx$	1122
3.262	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2}} dx$	1127
3.263	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)} dx$	1131
3.264	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2} dx$	1139
3.265	$\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3} dx$	1149
3.266	$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$	1158
3.267	$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$	1164
3.268	$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx$	1168
3.269	$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$	1172
3.270	$\int \frac{a + \frac{b}{x^2}}{c + \frac{d}{x^2}} dx$	1175
3.271	$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$	1178
3.272	$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$	1182
3.273	$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$	1186
3.274	$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$	1190
3.275	$\int \frac{a + \frac{b}{x^3}}{c + \frac{d}{x^3}} dx$	1193
3.276	$\int \frac{a + b\sqrt{x}}{c + d\sqrt{x}} dx$	1197
3.277	$\int \frac{-1 + \sqrt[3]{x}}{1 + \sqrt[3]{x}} dx$	1200
3.278	$\int \frac{1 + x^{2/3}}{-1 + x^{2/3}} dx$	1202
3.279	$\int \frac{-16 + x^{3/4}}{16 + x^{3/4}} dx$	1205
3.280	$\int \frac{1 + \frac{1}{\sqrt[3]{x}}}{-1 + \frac{1}{\sqrt[3]{x}}} dx$	1209
3.281	$\int (a - bx^n)^{3/2} (a + bx^n)^{3/2} dx$	1212
3.282	$\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$	1215
3.283	$\int (a - bx^n)^p (a + bx^n)^p dx$	1218
3.284	$\int (a + bx^n) (c + dx^n)^4 dx$	1220
3.285	$\int (a + bx^n) (c + dx^n)^3 dx$	1224

3.286	$\int (a + bx^n)(c + dx^n)^2 dx$	1227
3.287	$\int (a + bx^n)(c + dx^n) dx$	1230
3.288	$\int \frac{a+bx^n}{c+dx^n} dx$	1232
3.289	$\int \frac{a+bx^n}{(c+dx^n)^2} dx$	1234
3.290	$\int \frac{a+bx^n}{(c+dx^n)^3} dx$	1237
3.291	$\int \frac{a+bx^n}{(c+dx^n)^4} dx$	1241
3.292	$\int (a + bx^n)^2 (d + ex^n)^3 dx$	1244
3.293	$\int (a + bx^n)^2 (d + ex^n)^2 dx$	1247
3.294	$\int (a + bx^n)^2 (c + dx^n) dx$	1250
3.295	$\int \frac{(a+bx^n)^2}{c+dx^n} dx$	1253
3.296	$\int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx$	1256
3.297	$\int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx$	1259
3.298	$\int \frac{(c+dx^n)^4}{a+bx^n} dx$	1262
3.299	$\int \frac{(c+dx^n)^3}{a+bx^n} dx$	1266
3.300	$\int \frac{(c+dx^n)^2}{a+bx^n} dx$	1269
3.301	$\int \frac{c+dx^n}{a+bx^n} dx$	1272
3.302	$\int \frac{1}{(a+bx^n)(c+dx^n)} dx$	1274
3.303	$\int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$	1276
3.304	$\int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$	1279
3.305	$\int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$	1282
3.306	$\int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx$	1285
3.307	$\int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$	1288
3.308	$\int \frac{c+dx^n}{(a+bx^n)^2} dx$	1291
3.309	$\int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$	1294
3.310	$\int \frac{1}{(a+bx^n)^2(c+dx^n)^2} dx$	1297
3.311	$\int \frac{1}{(a+bx^n)^2(c+dx^n)^3} dx$	1300
3.312	$\int (a + bx^n)^p (c + dx^n)^q dx$	1304
3.313	$\int (a + bx^n)^p (c + dx^n)^3 dx$	1306
3.314	$\int (a + bx^n)^p (c + dx^n)^2 dx$	1310
3.315	$\int (a + bx^n)^p (c + dx^n) dx$	1313
3.316	$\int (a + bx^n)^p dx$	1316
3.317	$\int \frac{(a+bx^n)^p}{c+dx^n} dx$	1318
3.318	$\int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$	1320
3.319	$\int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$	1323
3.320	$\int (a + bx^n)^p (c + dx^n)^{-1-\frac{1}{n}-p} dx$	1326
3.321	$\int (a + bx^n)^3 (c + dx^n)^{-4-\frac{1}{n}} dx$	1328
3.322	$\int (a + bx^n)^2 (c + dx^n)^{-3-\frac{1}{n}} dx$	1331
3.323	$\int (a + bx^n) (c + dx^n)^{-2-\frac{1}{n}} dx$	1334

3.324	$\int (c + dx^n)^{-1-\frac{1}{n}} dx$	1337
3.325	$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx$	1339
3.326	$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$	1341
3.327	$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx$	1343
3.328	$\int (a + bx^n)^p (c + dx^n)^{-2-\frac{1}{n}-p} dx$	1345
3.329	$\int (a + bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c + dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx$	1348
3.330	$\int (a + bx^n)^2 (c + dx^n)^{-4-\frac{1}{n}} dx$	1351
3.331	$\int (a + bx^n) (c + dx^n)^{-3-\frac{1}{n}} dx$	1354
3.332	$\int (c + dx^n)^{-2-\frac{1}{n}} dx$	1357
3.333	$\int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx$	1359
3.334	$\int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx$	1362
3.335	$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx$	1365
3.336	$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$	1368
3.337	$\int x^5 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1371
3.338	$\int x^3 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1374
3.339	$\int x \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1377
3.340	$\int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x} dx$	1380
3.341	$\int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^3} dx$	1383
3.342	$\int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^5} dx$	1387
3.343	$\int x^4 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1391
3.344	$\int x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1396
3.345	$\int \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$	1400
3.346	$\int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^2} dx$	1404
3.347	$\int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^4} dx$	1407
3.348	$\int \frac{x^4(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$	1411
3.349	$\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$	1415
3.350	$\int \frac{x^2(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$	1418
3.351	$\int \frac{x(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$	1421
3.352	$\int \frac{a+bx^2}{\sqrt{-1+cx} \sqrt{1+cx}} dx$	1424
3.353	$\int \frac{a+bx^2}{x \sqrt{-1+cx} \sqrt{1+cx}} dx$	1427
3.354	$\int \frac{a+bx^2}{x^2 \sqrt{-1+cx} \sqrt{1+cx}} dx$	1430
3.355	$\int \frac{a+bx^2}{x^3 \sqrt{-1+cx} \sqrt{1+cx}} dx$	1433
3.356	$\int \frac{a+bx^2}{x^4 \sqrt{-1+cx} \sqrt{1+cx}} dx$	1436
3.357	$\int \frac{a+bx^2}{x^5 \sqrt{-1+cx} \sqrt{1+cx}} dx$	1439

3.358	$\int \frac{x^4(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$	1443
3.359	$\int \frac{x^3(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$	1448
3.360	$\int \frac{x^2(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$	1451
3.361	$\int \frac{x(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$	1455
3.362	$\int \frac{a+bx^2}{\sqrt{-c+dx} \sqrt{c+dx}} dx$	1458
3.363	$\int \frac{a+bx^2}{x\sqrt{-c+dx} \sqrt{c+dx}} dx$	1461
3.364	$\int \frac{a+bx^2}{x^2\sqrt{-c+dx} \sqrt{c+dx}} dx$	1464
3.365	$\int \frac{a+bx^2}{x^3\sqrt{-c+dx} \sqrt{c+dx}} dx$	1467
3.366	$\int \frac{a+bx^2}{x^4\sqrt{-c+dx} \sqrt{c+dx}} dx$	1470
3.367	$\int \frac{a+bx^2}{x^5\sqrt{-c+dx} \sqrt{c+dx}} dx$	1473
3.368	$\int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1477
3.369	$\int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1481
3.370	$\int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1484
3.371	$\int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1488
3.372	$\int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1491
3.373	$\int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1494
3.374	$\int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1497
3.375	$\int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1500
3.376	$\int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1503
3.377	$\int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}} dx$	1506
3.378	$\int \frac{1+c^2x^2}{x\sqrt{-1+cx} \sqrt{1+cx}} dx$	1510
3.379	$\int \frac{x \sqrt{\frac{2b^2c+a^2d}{b^2c+a^2d}} (c+dx^2)}{\sqrt{-a+bx} \sqrt{a+bx}} dx$	1513
3.380	$\int \frac{1}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}} \sqrt{1+x}} dx$	1516
3.381	$\int \frac{1}{\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}} \sqrt{a^2+b^2x}} dx$	1519
3.382	$\int (a-bx^n)^p (a+bx^n)^p (c+dx^{2n})^q dx$	1522
3.383	$\int (a-bx^n)^p (a+bx^n)^p (a^2+b^2x^{2n})^p dx$	1525
3.384	$\int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)} dx$	1528
3.385	$\int (a-bx^{n/2})^p (a+bx^{n/2})^p \left( \frac{a^2d(1+p)}{b^2\left(1+\frac{-1-2n-np}{n}\right)} + dx^n \right)^{\frac{-1-2n-np}{n}} dx$	1531

<b>4</b>	<b>Listing of Grading functions</b>	<b>1535</b>
4.0.1	Mathematica and Rubi grading function	1535
4.0.2	Maple grading function	1537
4.0.3	Sympy grading function	1540
4.0.4	SageMath grading function	1542

# Chapter 1

## Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [ 385 ]. This is test number [ 26 ].

### 1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

### 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric $2F1$  functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 ( 385 )	% 0.00 ( 0 )
Mathematica	% 99.48 ( 383 )	% 0.52 ( 2 )
Maple	% 51.17 ( 197 )	% 48.83 ( 188 )
Maxima	% 43.38 ( 167 )	% 56.62 ( 218 )
Fricas	% 55.58 ( 214 )	% 44.42 ( 171 )
Sympy	% 37.40 ( 144 )	% 62.60 ( 241 )
Giac	% 34.81 ( 134 )	% 65.19 ( 251 )
Mupad	% 44.16 ( 170 )	% 55.84 ( 215 )

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ol>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Table 1.2: Description of grading applied to integration result

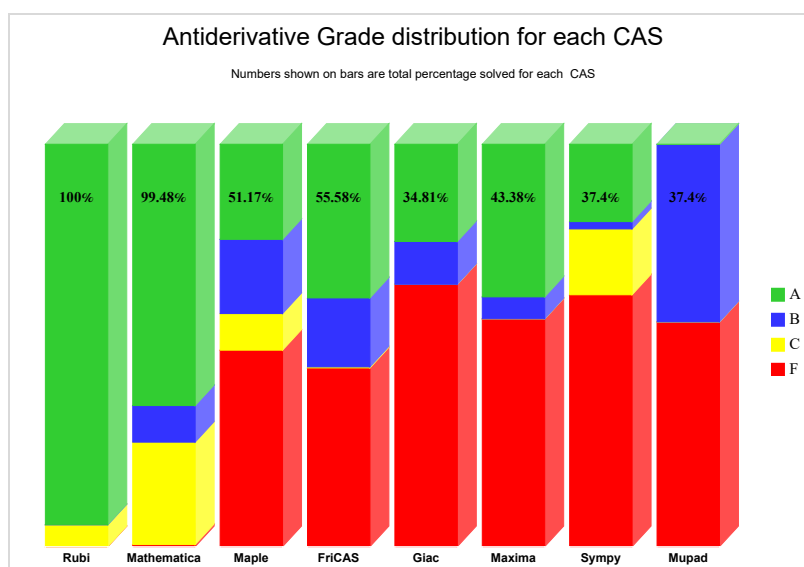
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.



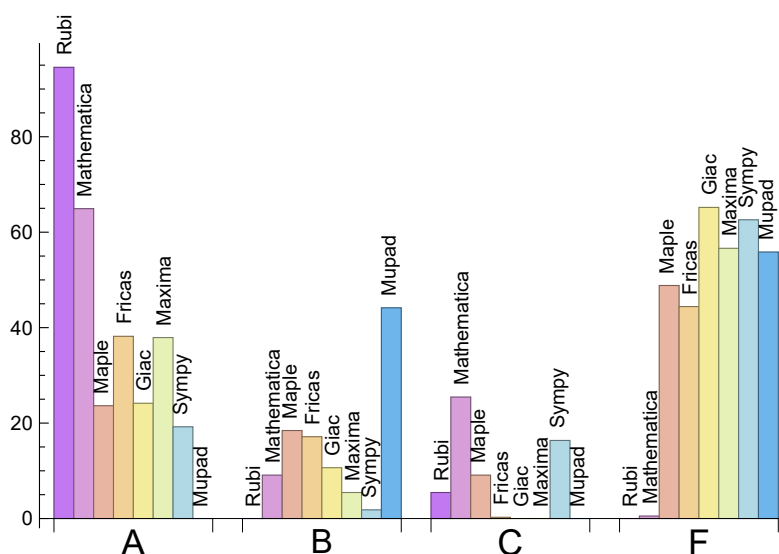
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.55	0.00	5.45	0.00
Mathematica	64.94	9.09	25.45	0.52
Maple	23.64	18.44	9.09	48.83
Maxima	37.92	5.45	0.00	56.62
Fricas	38.18	17.14	0.26	44.42
Sympy	19.22	1.82	16.36	62.60
Giac	24.16	10.65	0.00	65.19
Mupad	0.00	44.16	0.00	55.84

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure F.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned F(-1).

The third is due to an exception generated. Assigned F(-2). This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	50.00 %	50.00 %	0.00 %
Maple	188	99.47 %	0.53 %	0.00 %
Maxima	218	100.00 %	0.00 %	0.00 %
Fricas	171	56.14 %	42.11 %	1.75 %
Sympy	241	47.72 %	46.47 %	5.81 %
Giac	251	89.64 %	0.00 %	10.36 %
Mupad	215	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

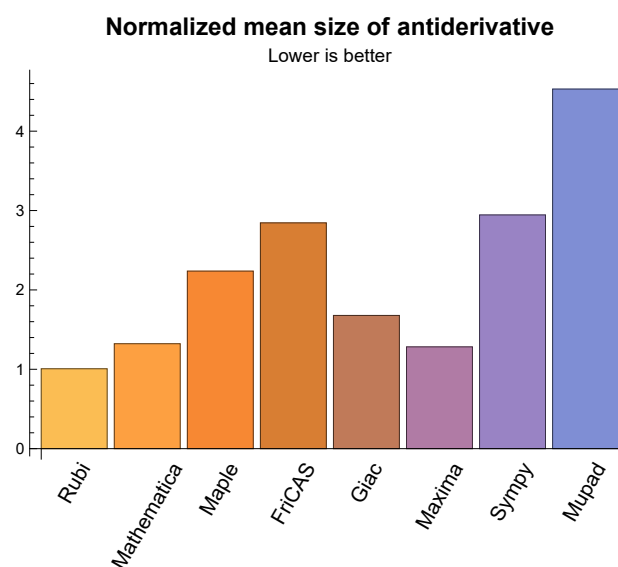
## 1.3 Performance

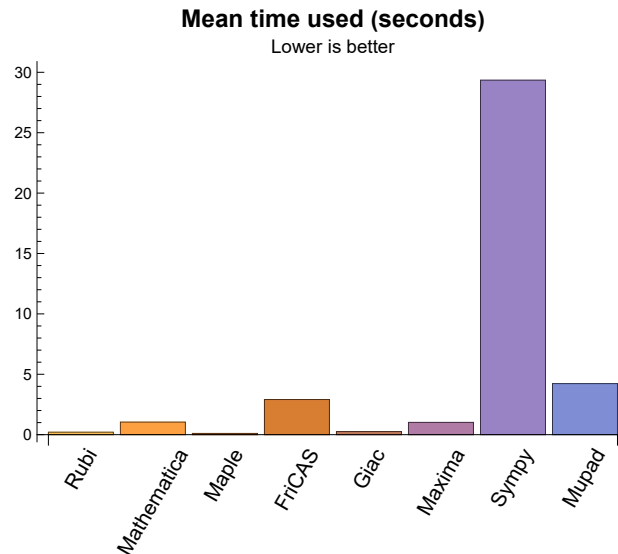
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.21	169.10	1.01	118.00	1.00
Mathematica	1.04	185.35	1.32	129.00	0.96
Maple	0.10	409.23	2.24	198.00	1.34
Maxima	1.02	185.69	1.28	144.00	1.14
Fricas	2.91	554.92	2.84	232.00	2.11
Sympy	29.35	314.96	2.94	153.50	1.20
Giac	0.26	261.01	1.68	200.00	1.36
Mupad	4.23	1166.19	4.53	148.50	1.22

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





## 1.4 list of integrals that has no closed form antiderivative

{}

## 1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

## 1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 115, 116}

**Mathematica** {34, 35, 36, 37, 38, 39, 75, 79, 81, 83, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 136, 137, 141, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 218, 221, 222, 269, 274, 307, 312, 317, 318, 319, 320, 328, 334, 335, 352, 384}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Mupad** Verification phase not implemented yet.

## 1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.9 Important notes about some of the results

### 1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

## 1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

## 1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at [https://ask.sagemath.org/question/57123/could-we-have-a-leaf\\_count-function-in-base-sagemath/](https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/)

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

## 1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

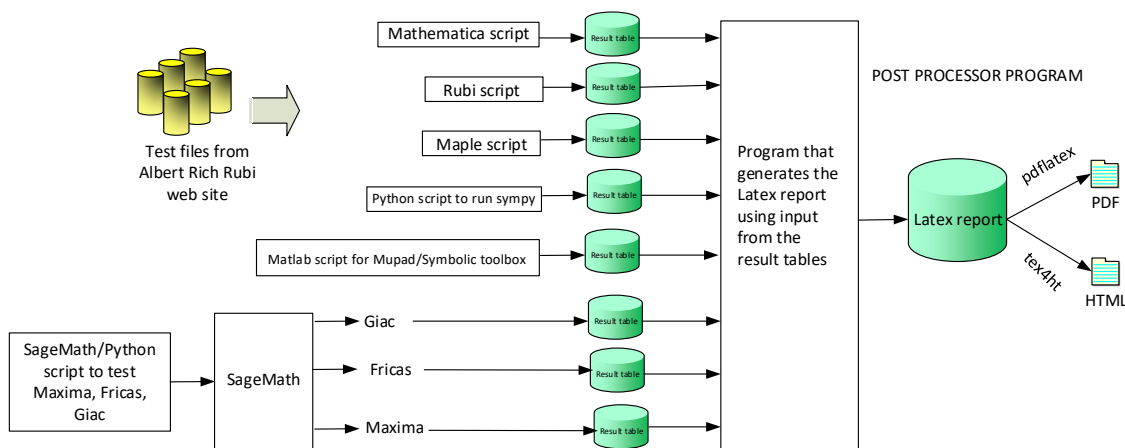
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives  $\sin(x)^2/2$

## 1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"  
*The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.  
*The following fields present only in Rubi Tables*
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system





# Chapter 2

## detailed summary tables of results

### 2.1 List of integrals sorted by grade for each CAS

#### 2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 90, 93, 94, 95, 96, 97, 100, 101, 104, 105, 106, 107, 108, 112, 113, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

B grade: { }

C grade: { 34, 35, 36, 37, 38, 39, 86, 87, 88, 91, 92, 98, 99, 102, 103, 109, 110, 111, 114, 115, 116 }

F grade: { }

#### 2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 30, 31, 32, 33, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 89, 92, 103, 116, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 138, 139, 140, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 195, 198, 216, 217, 219, 220, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 259, 266, 267, 268, 270, 272, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 313, 314, 315, 316, 320, 321, 322, 324, 325, 326, 327, 329, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 353, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 378, 380, 381, 383, 385 }

B grade: { 93, 94, 95, 96, 97, 104, 105, 106, 107, 108, 117, 118, 119, 120, 121, 133, 136, 137, 142,

143, 144, 218, 221, 222, 269, 312, 317, 318, 319, 328, 334, 335, 352, 354, 384 }

C grade: { 27, 29, 34, 35, 36, 37, 38, 39, 56, 57, 59, 86, 87, 88, 90, 91, 98, 99, 100, 101, 102, 109, 110, 111, 112, 113, 114, 115, 141, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 197, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 223, 252, 254, 255, 256, 257, 258, 260, 261, 262, 263, 264, 265, 271, 273, 279, 298, 299, 300, 307, 323, 330, 331, 332, 333, 375, 377, 379 }

F grade: { 336, 382 }

### 2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 17, 18, 19, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 159, 163, 164, 165, 170, 171, 172, 191, 192, 224, 248, 270, 271, 272, 273, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 349, 351, 353, 355, 356, 357, 359, 361, 366, 369, 371, 374, 376, 378, 379 }

B grade: { 11, 12, 14, 15, 16, 20, 21, 22, 23, 157, 158, 160, 161, 162, 166, 167, 168, 169, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 324, 340, 341, 342, 363, 365, 367, 373, 375, 377 }

C grade: { 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 216, 343, 344, 345, 346, 347, 348, 350, 352, 354, 358, 360, 362, 364, 368, 370, 372 }

F grade: { 13, 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

### 2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 29, 30, 31, 44, 45, 59, 60, 61, 62, 63, 74, 75, 76, 77, 78, 85, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 231, 232, 233, 234, 238, 239, 240, 241, 245, 248, 252, 253, 255, 259, 260, 261, 262, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379 }

B grade: { 26, 27, 28, 32, 33, 40, 41, 42, 43, 46, 47, 56, 57, 58, 70, 71, 72, 73, 246, 247, 254 }

C grade: { }

F grade: { 34, 35, 36, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 256, 257, 258, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 380, 381, 382, 383, 384, 385 }

## 2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 14, 15, 16, 17, 18, 19, 24, 25, 30, 31, 32, 33, 40, 41, 44, 45, 46, 47, 56, 57, 58, 60, 61, 62, 63, 70, 71, 72, 75, 76, 77, 78, 85, 145, 146, 147, 148, 149, 153, 154, 155, 156, 224, 225, 226, 227, 228, 229, 231, 232, 233, 234, 235, 236, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 259, 260, 261, 262, 266, 267, 268, 270, 275, 276, 277, 278, 279, 280, 287, 322, 323, 324, 329, 330, 331, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 373, 374, 375, 376, 377, 378, 379, 381, 385 }

B grade: { 7, 12, 13, 20, 21, 22, 23, 26, 27, 28, 29, 36, 42, 43, 59, 73, 74, 86, 87, 88, 98, 99, 110, 111, 150, 151, 152, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 191, 192, 193, 194, 205, 206, 216, 230, 237, 251, 256, 257, 258, 263, 264, 265, 284, 285, 286, 292, 293, 294, 321, 372 }

C grade: { 380 }

F grade: { 34, 35, 37, 38, 39, 48, 49, 50, 51, 52, 53, 54, 55, 64, 65, 66, 67, 68, 69, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 93, 94, 95, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 218, 219, 220, 221, 222, 223, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 325, 326, 327, 328, 333, 334, 335, 336, 382, 383, 384 }

## 2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 20, 21, 22, 23, 24, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 224, 225, 226, 227, 231, 232, 233, 238, 239, 240, 241, 245, 246, 247, 248, 255, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 293, 294, 324 }

B grade: { 30, 60, 234, 254, 261, 262, 270 }

C grade: { 27, 28, 29, 40, 41, 48, 49, 50, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 79, 80, 81, 82, 135, 140, 141, 219, 220, 288, 289, 290, 295, 298, 299, 300, 301, 308, 313, 314, 315, 316, 349, 351, 352, 353, 354, 355, 356, 359, 361, 362, 363, 364, 365, 366, 369, 371, 372, 373, 374, 378 }

F grade: { 19, 25, 26, 31, 32, 33, 34, 35, 36, 37, 38, 39, 42, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 61, 62, 63, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 142, 143, 144, 145, 164, 165, 166, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 221, 222, 223, 228, 229, 230, 235, 236, 237, 242, 243, 244, 249, 250, 251, 252, 253, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 271, 272, 273, 274, 281, 282, 283, 291, 292, 296, 297, 302, 303, 304, 305, 306, 307, 309, 310, 311, 312, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 350, 357, 358, 360, 367, 368, 370, 375, 376, 377, 379, 380, 381, 382, 383, 384, 385 }

## 2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 162, 163, 164, 165, 168, 169, 170, 171, 245, 246, 247, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 263, 265, 270, 275, 276, 277, 278, 279, 280, 340, 341, 346, 348, 349, 350, 351, 352, 353, 354, 358, 359, 360, 361, 362, 363, 364, 368, 370, 378 }

B grade: { 160, 161, 166, 167, 172, 227, 230, 237, 244, 248, 257, 264, 284, 285, 286, 287, 292, 293, 294, 337, 338, 339, 342, 343, 344, 345, 347, 355, 356, 357, 365, 366, 367, 369, 371, 372, 373, 374, 375,

376, 377 }

C grade: { }

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## 2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 30, 31, 32, 33, 44, 45, 46, 47, 60, 61, 62, 63, 75, 76, 77, 78, 85, 141, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 270, 275, 276, 277, 278, 279, 280, 284, 285, 286, 287, 292, 293, 294, 316, 324, 332, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 369, 371, 374, 376, 378, 379 }

C grade: { }

F grade: { 27, 28, 29, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 268, 269, 271, 272, 273, 274, 281, 282, 283, 288, 289, 290, 291, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 333, 334, 335, 336, 368, 370, 372, 373, 375, 377, 380, 381, 382, 383, 384, 385 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	97	104	97	87
normalized size	1	1.00	1.00	1.03	1.02	1.03	1.11	1.03	0.93
time (sec)	N/A	0.072	0.049	0.038	0.475	0.363	0.125	0.173	0.051
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	74	80	74	66
normalized size	1	1.00	1.00	1.04	1.00	1.06	1.14	1.06	0.94
time (sec)	N/A	0.043	0.015	0.044	0.562	0.364	0.079	0.148	0.034
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	51	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96
time (sec)	N/A	0.028	0.008	0.044	0.481	0.359	0.074	0.189	0.048
Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.013	0.005	0.040	0.536	0.353	0.066	0.149	0.037
Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	128	195	128	369	71	133	123
normalized size	1	1.00	0.89	1.35	0.89	2.56	0.49	0.92	0.85
time (sec)	N/A	0.094	0.123	0.048	1.277	0.445	0.417	0.202	1.383

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	158	537	97	160	143
normalized size	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.082	0.096	0.052	1.157	0.447	0.578	0.191	1.398
Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	175	249	192	743	133	180	173
normalized size	1	1.00	0.89	1.26	0.97	3.77	0.68	0.91	0.88
time (sec)	N/A	0.106	0.131	0.059	1.434	0.460	0.777	0.191	1.396
Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	132	139	132	116
normalized size	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95
time (sec)	N/A	0.071	0.019	0.050	0.547	0.379	0.091	0.159	1.199
Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	91	90	91	75
normalized size	1	1.00	1.00	1.06	1.00	1.11	1.10	1.11	0.91
time (sec)	N/A	0.046	0.012	0.037	0.707	0.370	0.084	0.208	0.042
Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	51	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.02	1.00	0.96
time (sec)	N/A	0.029	0.007	0.039	0.574	0.356	0.072	0.159	0.045
Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	189	505	156	211	152
normalized size	1	1.00	0.97	1.93	1.09	2.92	0.90	1.22	0.88
time (sec)	N/A	0.128	0.142	0.046	1.230	0.482	0.679	0.194	1.386

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	210	367	226	771	189	233	191
normalized size	1	1.00	1.03	1.81	1.11	3.80	0.93	1.15	0.94
time (sec)	N/A	0.244	0.221	0.062	1.282	0.463	1.135	0.190	1.412
Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F(-1)	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	234	0	267	1067	233	264	249
normalized size	1	1.00	0.91	0.00	1.03	4.14	0.90	1.02	0.97
time (sec)	N/A	0.233	0.276	180.000	1.321	0.449	1.623	0.197	1.428
Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	253	661	364	873	371	391	250
normalized size	1	1.00	1.00	2.62	1.44	3.46	1.47	1.55	0.99
time (sec)	N/A	0.191	0.127	0.047	1.189	0.466	1.310	0.201	1.430
Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	203	486	273	700	257	296	192
normalized size	1	1.00	0.98	2.34	1.31	3.37	1.24	1.42	0.92
time (sec)	N/A	0.148	0.106	0.046	1.319	0.444	1.003	0.185	1.402
Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	167	334	190	507	156	211	152
normalized size	1	1.00	0.97	1.93	1.10	2.93	0.90	1.22	0.88
time (sec)	N/A	0.124	0.149	0.043	1.321	0.452	0.690	0.324	1.375
Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	128	390	71	133	123
normalized size	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.078	0.075	0.046	1.075	0.434	0.436	0.191	1.379

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	224	222	293	254	447	278	1364
normalized size	1	1.00	0.78	0.77	1.02	0.88	1.55	0.97	4.74
time (sec)	N/A	0.147	0.107	0.050	1.235	0.497	79.722	0.266	7.705
Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	336	406	489	432	0	443	2589
normalized size	1	1.00	0.97	1.17	1.41	1.25	0.00	1.28	7.48
time (sec)	N/A	0.270	0.235	0.054	1.298	7.437	0.000	0.203	16.807
Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	313	905	509	1619	546	529	416
normalized size	1	1.00	0.98	2.83	1.59	5.06	1.71	1.65	1.30
time (sec)	N/A	0.298	0.303	0.060	1.346	0.470	12.429	0.190	0.390
Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	260	708	397	1316	405	412	302
normalized size	1	1.00	0.97	2.65	1.49	4.93	1.52	1.54	1.13
time (sec)	N/A	0.226	0.238	0.057	1.156	0.487	8.537	0.190	1.493
Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	227	529	306	1027	291	319	240
normalized size	1	1.00	0.97	2.26	1.31	4.39	1.24	1.36	1.03
time (sec)	N/A	0.220	0.156	0.053	1.219	0.458	4.330	0.200	0.296
Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	205	367	220	768	189	227	191
normalized size	1	1.00	1.01	1.81	1.08	3.78	0.93	1.12	0.94
time (sec)	N/A	0.231	0.198	0.053	1.395	0.467	2.556	0.213	1.467



Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	145	221	158	537	97	160	143
normalized size	1	1.00	0.86	1.31	0.93	3.18	0.57	0.95	0.85
time (sec)	N/A	0.084	0.091	0.054	1.439	0.465	1.418	0.171	1.428
Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	337	406	489	440	0	443	2492
normalized size	1	1.00	0.97	1.17	1.41	1.27	0.00	1.28	7.20
time (sec)	N/A	0.255	0.195	0.056	1.290	7.375	0.000	0.201	15.930
Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	381	606	784	897	0	664	3637
normalized size	1	1.00	0.91	1.45	1.87	2.14	0.00	1.58	8.68
time (sec)	N/A	0.493	0.630	0.062	1.257	85.305	0.000	0.222	24.310
Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	62	0	322	399	80	0	-1
normalized size	1	1.00	0.55	0.00	2.88	3.56	0.71	0.00	-0.01
time (sec)	N/A	0.032	0.106	0.429	1.219	0.436	4.984	0.000	0.000
Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	134	0	244	363	76	0	-1
normalized size	1	1.00	1.47	0.00	2.68	3.99	0.84	0.00	-0.01
time (sec)	N/A	0.020	0.084	0.366	1.216	0.444	5.525	0.000	0.000
Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	62	0	130	372	70	0	-1
normalized size	1	1.00	0.73	0.00	1.53	4.38	0.82	0.00	-0.01
time (sec)	N/A	0.013	0.037	0.387	1.254	0.443	16.137	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	28	25	50	44	190	0	27
normalized size	1	1.00	0.60	0.53	1.06	0.94	4.04	0.00	0.57
time (sec)	N/A	0.009	0.015	0.046	0.523	0.420	91.681	0.000	1.348
Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	40	37	85	69	0	0	44
normalized size	1	1.00	0.73	0.67	1.55	1.25	0.00	0.00	0.80
time (sec)	N/A	0.013	0.021	0.043	0.624	0.417	0.000	0.000	1.425
Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	51	48	119	91	0	0	58
normalized size	1	1.00	0.69	0.65	1.61	1.23	0.00	0.00	0.78
time (sec)	N/A	0.019	0.022	0.048	0.584	0.431	0.000	0.000	1.391
Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	62	59	153	113	0	0	73
normalized size	1	1.00	0.67	0.63	1.65	1.22	0.00	0.00	0.78
time (sec)	N/A	0.026	0.026	0.046	0.538	0.431	0.000	0.000	1.371
Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	483	56	232	0	0	0	0	0	-1
normalized size	1	0.12	0.48	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.384	0.607	0.000	0.000	0.000	0.000	0.000
Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	464	55	217	0	0	0	0	0	-1
normalized size	1	0.12	0.47	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.116	0.609	0.000	34.203	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	398	58	151	0	0	644	0	0	-1
normalized size	1	0.15	0.38	0.00	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.026	0.134	0.698	0.000	29.485	0.000	0.000	0.000
Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	58	153	0	0	0	0	0	-1
normalized size	1	0.13	0.34	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.133	0.584	0.000	0.000	0.000	0.000	0.000
Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	473	58	213	0	0	0	0	0	-1
normalized size	1	0.12	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.029	0.139	0.681	0.000	0.000	0.000	0.000	0.000
Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	492	58	240	0	0	0	0	0	-1
normalized size	1	0.12	0.49	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.176	0.600	0.000	0.000	0.000	0.000	0.000
Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	151	0	552	421	126	0	-1
normalized size	1	1.00	1.09	0.00	3.97	3.03	0.91	0.00	-0.01
time (sec)	N/A	0.057	0.137	0.375	1.527	0.881	9.174	0.000	0.000
Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	141	0	436	399	121	0	-1
normalized size	1	1.00	1.18	0.00	3.63	3.32	1.01	0.00	-0.01
time (sec)	N/A	0.042	0.078	0.384	1.396	0.815	7.599	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	137	0	296	412	0	0	-1
normalized size	1	1.00	1.21	0.00	2.62	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.082	0.532	1.373	0.843	0.000	0.000	0.000
Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	131	0	180	521	0	0	-1
normalized size	1	1.00	1.19	0.00	1.64	4.74	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.063	0.533	1.354	0.912	0.000	0.000	0.000
Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	40	37	105	67	0	0	44
normalized size	1	1.00	0.53	0.49	1.38	0.88	0.00	0.00	0.58
time (sec)	N/A	0.021	0.015	0.042	0.498	0.861	0.000	0.000	1.432
Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	51	48	155	91	0	0	56
normalized size	1	1.00	0.49	0.46	1.48	0.87	0.00	0.00	0.53
time (sec)	N/A	0.035	0.032	0.050	0.585	0.908	0.000	0.000	1.389
Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	62	59	206	113	0	0	71
normalized size	1	1.00	0.63	0.60	2.10	1.15	0.00	0.00	0.72
time (sec)	N/A	0.035	0.035	0.046	0.574	0.914	0.000	0.000	1.436
Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	73	70	257	135	0	0	86
normalized size	1	1.00	0.62	0.60	2.20	1.15	0.00	0.00	0.74
time (sec)	N/A	0.044	0.040	0.049	0.510	0.990	0.000	0.000	1.427

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	97	0	0	0	168	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	1.79	0.00	-0.01
time (sec)	N/A	0.034	0.049	0.407	0.000	0.789	8.409	0.000	0.000
Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	85	0	0	0	126	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	1.34	0.00	-0.01
time (sec)	N/A	0.034	0.046	0.382	0.000	0.646	5.592	0.000	0.000
Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	121	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	1.29	0.00	-0.01
time (sec)	N/A	0.034	0.047	0.368	0.000	0.631	5.758	0.000	0.000
Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	62	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.045	0.556	0.000	0.637	0.000	0.000	0.000
Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	70	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.069	0.510	0.000	0.698	0.000	0.000	0.000
Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	85	0	0	0	0	0	-1
normalized size	1	1.00	1.10	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.118	0.535	0.000	0.773	0.000	0.000	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	95	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.126	0.566	0.000	0.646	0.000	0.000	0.000
Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	106	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.034	0.141	0.553	0.000	0.693	0.000	0.000	0.000
Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	75	0	406	482	170	0	-1
normalized size	1	1.00	0.43	0.00	2.33	2.77	0.98	0.00	-0.01
time (sec)	N/A	0.059	0.068	0.374	1.678	0.714	10.424	0.000	0.000
Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	72	0	322	424	82	0	-1
normalized size	1	1.00	0.51	0.00	2.28	3.01	0.58	0.00	-0.01
time (sec)	N/A	0.045	0.095	0.375	1.437	0.872	5.347	0.000	0.000
Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	141	0	244	362	78	0	-1
normalized size	1	1.00	1.27	0.00	2.20	3.26	0.70	0.00	-0.01
time (sec)	N/A	0.030	0.154	0.395	1.198	0.653	4.439	0.000	0.000
Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	61	0	134	488	71	0	-1
normalized size	1	1.00	0.62	0.00	1.35	4.93	0.72	0.00	-0.01
time (sec)	N/A	0.024	0.052	0.380	1.224	0.686	12.828	0.000	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	37	34	51	54	190	0	33
normalized size	1	1.00	0.79	0.72	1.09	1.15	4.04	0.00	0.70
time (sec)	N/A	0.010	0.036	0.049	0.470	0.828	82.052	0.000	1.370
Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	59	57	86	87	0	0	87
normalized size	1	1.00	0.65	0.63	0.95	0.96	0.00	0.00	0.96
time (sec)	N/A	0.028	0.039	0.045	0.608	0.724	0.000	0.000	1.424
Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	80	81	120	121	0	0	105
normalized size	1	1.00	0.66	0.67	0.99	1.00	0.00	0.00	0.87
time (sec)	N/A	0.035	0.034	0.042	0.499	0.579	0.000	0.000	1.460
Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	100	105	154	155	0	0	132
normalized size	1	1.00	0.66	0.70	1.02	1.03	0.00	0.00	0.87
time (sec)	N/A	0.047	0.062	0.046	0.657	0.671	0.000	0.000	1.453
Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	0	0	0	265	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.00	3.12	0.00	-0.01
time (sec)	N/A	0.023	0.052	0.420	0.000	0.665	10.988	0.000	0.000
Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	75	0	0	0	170	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	2.05	0.00	-0.01
time (sec)	N/A	0.022	0.143	0.385	0.000	0.706	7.402	0.000	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	72	0	0	0	82	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.021	0.138	0.403	0.000	0.938	4.966	0.000	0.000
Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	73	0	0	0	78	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.022	0.069	0.374	0.000	0.691	5.072	0.000	0.000
Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	66	0	0	0	78	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.84	0.00	-0.01
time (sec)	N/A	0.025	0.047	0.390	0.000	0.702	16.363	0.000	0.000
Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	75	0	0	0	78	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.83	0.00	-0.01
time (sec)	N/A	0.028	0.044	0.400	0.000	0.661	117.701	0.000	0.000
Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	238	0	672	717	270	0	-1
normalized size	1	1.00	0.91	0.00	2.56	2.74	1.03	0.00	-0.00
time (sec)	N/A	0.165	5.190	0.385	1.294	0.854	13.131	0.000	0.000
Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	203	0	552	634	131	0	-1
normalized size	1	1.00	0.93	0.00	2.52	2.89	0.60	0.00	-0.00
time (sec)	N/A	0.166	5.171	0.385	1.479	0.632	7.275	0.000	0.000



Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	172	0	436	554	126	0	-1
normalized size	1	1.00	0.98	0.00	2.49	3.17	0.72	0.00	-0.01
time (sec)	N/A	0.096	5.150	0.381	1.275	0.732	6.437	0.000	0.000
Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	168	0	301	652	0	0	-1
normalized size	1	1.00	1.06	0.00	1.89	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.102	5.163	0.559	1.137	0.583	0.000	0.000	0.000
Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	180	0	190	719	0	0	-1
normalized size	1	1.00	1.18	0.00	1.25	4.73	0.00	0.00	-0.01
time (sec)	N/A	0.068	5.247	0.550	1.193	0.736	0.000	0.000	0.000
Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	126	76	109	103	0	0	148
normalized size	1	1.00	1.62	0.97	1.40	1.32	0.00	0.00	1.90
time (sec)	N/A	0.021	0.097	0.049	0.508	0.617	0.000	0.000	1.427
Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	106	115	159	152	0	0	176
normalized size	1	1.00	0.61	0.66	0.91	0.87	0.00	0.00	1.01
time (sec)	N/A	0.073	5.106	0.046	0.712	0.726	0.000	0.000	1.450
Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	138	156	210	200	0	0	217
normalized size	1	1.00	0.65	0.74	1.00	0.95	0.00	0.00	1.03
time (sec)	N/A	0.127	5.176	0.047	0.503	0.645	0.000	0.000	1.430

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	169	197	261	246	0	0	257
normalized size	1	1.00	0.67	0.78	1.03	0.97	0.00	0.00	1.02
time (sec)	N/A	0.207	5.153	0.049	0.643	0.830	0.000	0.000	1.481
Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	177	0	0	0	418	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.00	3.10	0.00	-0.01
time (sec)	N/A	0.067	3.587	0.416	0.000	0.667	12.604	0.000	0.000
Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	171	0	0	0	270	0	-1
normalized size	1	1.00	1.29	0.00	0.00	0.00	2.03	0.00	-0.01
time (sec)	N/A	0.077	5.189	0.373	0.000	0.669	7.059	0.000	0.000
Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	179	0	0	0	131	0	-1
normalized size	1	1.00	1.37	0.00	0.00	0.00	1.00	0.00	-0.01
time (sec)	N/A	0.062	3.997	0.388	0.000	0.851	3.938	0.000	0.000
Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	104	0	0	0	126	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.95	0.00	-0.01
time (sec)	N/A	0.075	5.135	0.388	0.000	0.691	4.271	0.000	0.000
Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	0	0	0	0	0	-1
normalized size	1	1.00	1.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	3.763	0.605	0.000	0.679	0.000	0.000	0.000

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	128	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	5.162	0.604	0.000	0.614	0.000	0.000	0.000
Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	120	134	182	166	0	0	271
normalized size	1	1.00	1.10	1.23	1.67	1.52	0.00	0.00	2.49
time (sec)	N/A	0.036	0.043	0.053	0.626	0.667	0.000	0.000	1.556
Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	62	655	0	0	643	0	0	-1
normalized size	1	0.19	1.98	0.00	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.232	0.701	0.000	15.642	0.000	0.000	0.000
Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	60	443	0	0	535	0	0	-1
normalized size	1	0.22	1.62	0.00	0.00	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.711	0.559	0.000	1.746	0.000	0.000	0.000
Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	59	161	0	0	469	0	0	-1
normalized size	1	0.25	0.69	0.00	0.00	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.027	0.224	0.570	0.000	1.015	0.000	0.000	0.000
Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	207	168	0	0	0	0	0	-1
normalized size	1	1.40	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.151	0.526	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	238	256	0	0	0	0	0	-1
normalized size	1	1.33	1.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.193	0.801	0.574	0.000	0.000	0.000	0.000	0.000
Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	621	621	0	0	0	0	0	-1
normalized size	1	2.75	2.75	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.580	2.673	0.558	0.000	0.000	0.000	0.000	0.000
Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	1172	277	0	0	0	0	0	-1
normalized size	1	4.19	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	6.642	5.746	0.605	0.000	0.000	0.000	0.000	0.000
Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	346	0	0	0	0	0	-1
normalized size	1	1.00	5.77	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.611	0.612	0.000	0.000	0.000	0.000	0.000
Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	160	0	0	0	0	0	-1
normalized size	1	1.00	2.71	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.166	0.587	0.000	0.000	0.000	0.000	0.000
Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	161	0	0	0	0	0	-1
normalized size	1	1.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.039	0.613	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	332	0	0	0	0	0	-1
normalized size	1	1.00	5.35	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.279	0.643	0.000	0.000	0.000	0.000	0.000
Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	429	0	0	0	0	0	-1
normalized size	1	1.00	6.92	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.867	0.629	0.000	0.000	0.000	0.000	0.000
Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	351	62	698	0	0	819	0	0	-1
normalized size	1	0.18	1.99	0.00	0.00	2.33	0.00	0.00	-0.00
time (sec)	N/A	0.029	1.083	0.398	0.000	12.135	0.000	0.000	0.000
Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	301	60	450	0	0	631	0	0	-1
normalized size	1	0.20	1.50	0.00	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.028	0.643	0.582	0.000	1.335	0.000	0.000	0.000
Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	241	78	0	0	0	0	0	-1
normalized size	1	1.32	0.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.041	0.538	0.000	0.000	0.000	0.000	0.000
Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	276	99	0	0	0	0	0	-1
normalized size	1	1.27	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.204	0.145	0.555	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	625	625	0	0	0	0	0	-1
normalized size	1	2.39	2.39	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.934	2.032	0.600	0.000	0.000	0.000	0.000	0.000
Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	324	1214	288	0	0	0	0	0	-1
normalized size	1	3.75	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	5.692	5.944	0.594	0.000	0.000	0.000	0.000	0.000
Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	341	0	0	0	0	0	-1
normalized size	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.323	0.513	0.000	0.000	0.000	0.000	0.000
Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	232	0	0	0	0	0	-1
normalized size	1	1.00	3.93	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.211	0.572	0.000	0.000	0.000	0.000	0.000
Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	393	0	0	0	0	0	-1
normalized size	1	1.00	6.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.268	0.573	0.000	0.000	0.000	0.000	0.000
Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	386	0	0	0	0	0	-1
normalized size	1	1.00	6.23	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.575	0.583	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	550	0	0	0	0	0	-1
normalized size	1	1.00	8.87	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.986	0.599	0.000	0.000	0.000	0.000	0.000
Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	541	62	1171	0	0	0	0	0	-1
normalized size	1	0.11	2.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.027	2.341	0.395	0.000	0.000	0.000	0.000	0.000
Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	458	62	908	0	0	1246	0	0	-1
normalized size	1	0.14	1.98	0.00	0.00	2.72	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.838	0.412	0.000	79.231	0.000	0.000	0.000
Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	B	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	391	62	651	0	0	954	0	0	-1
normalized size	1	0.16	1.66	0.00	0.00	2.44	0.00	0.00	-0.00
time (sec)	N/A	0.028	1.164	0.607	0.000	8.886	0.000	0.000	0.000
Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	276	79	0	0	0	0	0	-1
normalized size	1	1.27	0.36	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	0.033	0.572	0.000	0.000	0.000	0.000	0.000
Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	326	153	0	0	0	0	0	-1
normalized size	1	1.22	0.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.243	0.219	0.546	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	307	167	168	0	0	0	0	0	-1
normalized size	1	0.54	0.55	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	0.380	0.579	0.000	0.000	0.000	0.000	0.000
Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	377	428	428	0	0	0	0	0	-1
normalized size	1	1.14	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.742	3.084	0.592	0.000	0.000	0.000	0.000	0.000
Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	C	A	F	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	463	1990	337	0	0	0	0	0	-1
normalized size	1	4.30	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	8.662	5.878	0.559	0.000	0.000	0.000	0.000	0.000
Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	316	0	0	0	0	0	-1
normalized size	1	1.00	5.27	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.519	0.568	0.000	0.000	0.000	0.000	0.000
Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	431	0	0	0	0	0	-1
normalized size	1	1.00	7.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	0.617	0.560	0.000	0.000	0.000	0.000	0.000
Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	442	0	0	0	0	0	-1
normalized size	1	1.00	7.49	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.772	0.590	0.000	0.000	0.000	0.000	0.000



Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	531	0	0	0	0	0	-1
normalized size	1	1.00	8.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.095	0.585	0.000	0.000	0.000	0.000	0.000
Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	515	0	0	0	0	0	-1
normalized size	1	1.00	8.31	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	1.616	0.585	0.000	0.000	0.000	0.000	0.000
Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.043	0.505	0.000	77.528	0.000	0.000	0.000
Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	90	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.031	0.537	0.000	3.007	0.000	0.000	0.000
Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.020	0.552	0.000	47.913	0.000	0.000	0.000
Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	89	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.025	0.535	0.000	2.249	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.062	0.538	0.000	51.161	0.000	0.000	0.000
Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	86	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.017	0.022	0.526	0.000	3.995	0.000	0.000	0.000
Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	89	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.016	0.042	0.561	0.000	64.290	0.000	0.000	0.000
Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.050	0.552	0.000	2.820	0.000	0.000	0.000
Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	89	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.037	0.559	0.000	64.256	0.000	0.000	0.000
Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	0.046	0.496	0.000	2.847	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	90	0	0	0	0	0	-1
normalized size	1	1.00	0.59	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.036	0.549	0.000	66.283	0.000	0.000	0.000
Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
normalized size	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.432	0.556	0.000	1.537	0.000	0.000	0.000
Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	176	106	0	0	0	0	0	-1
normalized size	1	1.05	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.079	0.529	0.000	1.160	0.000	0.000	0.000
Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	85	90	0	0	0	75	0	-1
normalized size	1	1.01	1.07	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.039	0.037	0.396	0.000	0.989	82.864	0.000	0.000
Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.204	0.563	0.000	1.161	0.000	0.000	0.000
Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.258	0.580	0.000	0.882	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	137	0	0	0	0	0	-1
normalized size	1	1.00	0.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	5.077	0.577	0.000	1.324	0.000	0.000	0.000
Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	0	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	5.043	0.540	0.000	1.226	0.000	0.000	0.000
Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
normalized size	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.042	0.046	0.390	0.000	0.759	100.310	0.000	0.000
Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	196	0	0	0	34	0	41
normalized size	1	1.00	4.45	0.00	0.00	0.00	0.77	0.00	0.93
time (sec)	N/A	0.010	0.198	0.361	0.000	1.264	15.563	0.000	1.346
Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.250	0.566	0.000	0.839	0.000	0.000	0.000
Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.025	0.257	0.525	0.000	1.202	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.413	0.557	0.000	1.192	0.000	0.000	0.000
Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	71	0	91	0	0	131
normalized size	1	1.00	0.98	1.34	0.00	1.72	0.00	0.00	2.47
time (sec)	N/A	0.019	0.038	0.041	0.000	1.323	0.000	0.000	1.905
Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	94	97	96	98	107	98	88
normalized size	1	1.00	1.00	1.03	1.02	1.04	1.14	1.04	0.94
time (sec)	N/A	0.068	0.024	0.040	0.651	0.897	0.090	0.156	1.303
Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	73	70	74	76	74	66
normalized size	1	1.00	1.00	1.04	1.00	1.06	1.09	1.06	0.94
time (sec)	N/A	0.049	0.017	0.039	0.647	0.962	0.084	0.150	1.242
Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	53	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96
time (sec)	N/A	0.029	0.012	0.036	0.605	0.773	0.078	0.161	0.048
Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	26	26	26	25
normalized size	1	1.00	1.00	0.89	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.014	0.009	0.045	0.493	0.839	0.065	0.150	0.036

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	212	639	87	245	720
normalized size	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.151	0.179	0.048	1.278	1.318	0.659	0.165	1.480
Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	236	711	112	266	740
normalized size	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.147	0.202	0.053	1.158	1.186	0.849	0.169	1.520
Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	243	314	271	787	151	286	762
normalized size	1	1.00	0.89	1.15	0.99	2.88	0.55	1.05	2.79
time (sec)	N/A	0.174	0.223	0.054	1.211	1.277	1.031	0.194	1.581
Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	154	163	158	173	185	173	146
normalized size	1	1.00	1.00	1.06	1.03	1.12	1.20	1.12	0.95
time (sec)	N/A	0.114	0.033	0.041	0.546	0.606	0.117	0.150	0.067
Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	122	125	124	132	139	132	116
normalized size	1	1.00	1.00	1.02	1.02	1.08	1.14	1.08	0.95
time (sec)	N/A	0.077	0.023	0.035	0.544	0.732	0.098	0.151	1.297
Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	87	82	91	97	91	75
normalized size	1	1.00	1.00	1.06	1.00	1.11	1.18	1.11	0.91
time (sec)	N/A	0.049	0.017	0.041	0.695	0.995	0.090	0.166	0.046

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	50	49	48	50	53	50	48
normalized size	1	1.00	1.00	0.98	0.96	1.00	1.06	1.00	0.96
time (sec)	N/A	0.030	0.008	0.038	0.478	0.809	0.080	0.149	0.045
Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	286	1239	187	353	1081
normalized size	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.194	0.131	0.048	1.285	1.126	1.119	0.198	1.485
Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	298	475	319	1335	219	376	1254
normalized size	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.366	0.216	0.056	1.104	1.267	1.979	0.174	1.536
Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	319	499	361	1411	264	407	1401
normalized size	1	1.00	0.91	1.43	1.03	4.04	0.76	1.17	4.01
time (sec)	N/A	0.266	0.227	0.058	1.220	1.398	5.850	0.212	1.660
Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	322	837	489	2477	435	617	1822
normalized size	1	1.00	0.97	2.52	1.47	7.46	1.31	1.86	5.49
time (sec)	N/A	0.267	0.224	0.047	1.437	1.305	3.585	0.484	1.515
Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	271	627	385	1855	303	481	1433
normalized size	1	1.00	0.94	2.18	1.34	6.44	1.05	1.67	4.98
time (sec)	N/A	0.223	0.166	0.046	1.210	1.157	1.696	0.173	1.488

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	231	436	287	1240	187	353	1081
normalized size	1	1.00	0.91	1.72	1.13	4.90	0.74	1.40	4.27
time (sec)	N/A	0.190	0.132	0.046	1.093	1.117	1.084	0.177	1.466
Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	196	266	212	639	87	245	720
normalized size	1	1.00	0.88	1.19	0.95	2.87	0.39	1.10	3.23
time (sec)	N/A	0.138	0.151	0.048	1.126	1.315	0.610	0.165	0.221
Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	340	320	365	1356	0	437	6153
normalized size	1	1.00	0.76	0.71	0.81	3.02	0.00	0.97	13.70
time (sec)	N/A	0.268	0.143	0.056	1.434	1.361	0.000	0.209	2.755
Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	498	550	481	3299	0	667	21975
normalized size	1	1.00	0.97	1.07	0.94	6.43	0.00	1.30	42.84
time (sec)	N/A	0.420	0.335	0.056	1.566	46.363	0.000	0.195	4.004
Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	391	1118	644	3222	0	798	2490
normalized size	1	1.00	0.96	2.75	1.58	7.92	0.00	1.96	6.12
time (sec)	N/A	0.396	0.453	0.060	1.446	1.008	0.000	0.179	1.707
Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	341	885	521	2580	471	642	2043
normalized size	1	1.00	0.96	2.48	1.46	7.23	1.32	1.80	5.72
time (sec)	N/A	0.367	0.349	0.062	1.454	1.040	47.538	0.174	0.302



Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	301	669	405	1938	337	496	1616
normalized size	1	1.00	0.95	2.11	1.28	6.11	1.06	1.56	5.10
time (sec)	N/A	0.317	0.254	0.055	1.309	0.897	6.989	0.179	1.530
Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	297	475	319	1335	219	376	1254
normalized size	1	1.00	1.02	1.63	1.10	4.59	0.75	1.29	4.31
time (sec)	N/A	0.377	0.227	0.058	1.261	1.085	2.174	0.168	0.298
Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	212	295	236	711	112	266	740
normalized size	1	1.00	0.87	1.20	0.96	2.90	0.46	1.09	3.02
time (sec)	N/A	0.153	0.172	0.053	1.348	0.878	0.965	0.166	1.533
Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	499	550	470	3299	0	667	21975
normalized size	1	1.00	0.97	1.07	0.92	6.43	0.00	1.30	42.84
time (sec)	N/A	0.428	0.347	0.058	1.457	36.676	0.000	0.217	3.817
Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	596	596	629	784	670	0	0	967	37266
normalized size	1	1.00	1.06	1.32	1.12	0.00	0.00	1.62	62.53
time (sec)	N/A	0.739	6.192	0.066	1.292	0.000	0.000	0.218	5.617
Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	290	408	0	0	0	0	-1
normalized size	1	1.00	0.90	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.383	0.820	0.310	0.000	0.000	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	341	311	0	0	0	0	-1
normalized size	1	1.00	1.23	1.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.262	0.461	0.293	0.000	0.000	0.000	0.000	0.000
Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	155	259	0	0	0	0	-1
normalized size	1	1.00	0.65	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.163	0.209	0.314	0.000	0.000	0.000	0.000	0.000
Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	156	183	0	0	0	0	-1
normalized size	1	1.00	0.96	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.101	0.309	0.000	0.000	0.000	0.000	0.000
Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	381	301	0	0	0	0	-1
normalized size	1	1.00	1.36	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.222	0.316	0.362	0.000	0.000	0.000	0.000	0.000
Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	422	361	0	0	0	0	-1
normalized size	1	1.00	1.26	1.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.400	0.853	0.320	0.000	0.000	0.000	0.000	0.000
Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	926	926	346	322	0	0	0	0	-1
normalized size	1	1.00	0.37	0.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.655	0.518	0.477	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	881	881	161	273	0	0	0	0	-1
normalized size	1	1.00	0.18	0.31	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.880	0.186	0.327	0.000	0.000	0.000	0.000	0.000
Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	742	742	161	191	0	0	0	0	-1
normalized size	1	1.00	0.22	0.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.633	0.032	0.332	0.000	0.000	0.000	0.000	0.000
Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	913	913	331	313	0	0	0	0	-1
normalized size	1	1.00	0.36	0.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.132	0.285	0.286	0.000	0.000	0.000	0.000	0.000
Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	976	976	429	371	0	0	0	0	-1
normalized size	1	1.00	0.44	0.38	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.683	0.992	0.337	0.000	0.000	0.000	0.000	0.000
Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	477	540	0	0	0	0	-1
normalized size	1	1.00	1.12	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.918	0.292	0.000	0.000	0.000	0.000	0.000
Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	396	412	0	0	0	0	-1
normalized size	1	1.00	1.08	1.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.405	0.669	0.340	0.000	0.000	0.000	0.000	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	342	329	0	0	0	0	-1
normalized size	1	1.00	1.11	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	0.436	0.339	0.000	0.000	0.000	0.000	0.000
Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	233	294	0	0	0	0	-1
normalized size	1	1.00	0.84	1.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.215	0.130	0.306	0.000	0.000	0.000	0.000	0.000
Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	386	322	0	0	0	0	-1
normalized size	1	1.00	1.25	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.247	0.280	0.336	0.000	0.000	0.000	0.000	0.000
Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	374	375	0	0	0	0	-1
normalized size	1	1.00	1.03	1.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.404	0.515	0.351	0.000	0.000	0.000	0.000	0.000
Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	439	439	382	484	0	0	0	0	-1
normalized size	1	1.00	0.87	1.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.536	0.880	0.349	0.000	0.000	0.000	0.000	0.000
Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	155	103	0	315	0	0	-1
normalized size	1	1.00	1.50	1.00	0.00	3.06	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.171	0.208	0.000	2.745	0.000	0.000	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	155	158	0	339	0	0	-1
normalized size	1	1.00	1.34	1.36	0.00	2.92	0.00	0.00	-0.01
time (sec)	N/A	0.023	0.167	0.221	0.000	1.917	0.000	0.000	0.000
Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	364	0	0	2381	0	0	-1
normalized size	1	1.00	1.73	0.00	0.00	11.28	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.629	0.587	0.000	9.033	0.000	0.000	0.000
Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	161	0	0	844	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	4.88	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.180	0.575	0.000	1.061	0.000	0.000	0.000
Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	84	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.045	0.566	0.000	0.000	0.000	0.000	0.000
Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	256	0	0	0	0	0	-1
normalized size	1	1.00	1.91	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.577	0.598	0.000	0.000	0.000	0.000	0.000
Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	621	0	0	0	0	0	-1
normalized size	1	1.00	3.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	2.368	0.603	0.000	0.000	0.000	0.000	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	231	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	5.439	0.615	0.000	0.000	0.000	0.000	0.000
Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	294	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.338	0.698	0.557	0.000	0.000	0.000	0.000	0.000
Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	346	0	0	0	0	0	-1
normalized size	1	1.00	1.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	0.321	0.572	0.000	0.000	0.000	0.000	0.000
Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	0	0	0	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.166	0.569	0.000	0.000	0.000	0.000	0.000
Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	161	0	0	0	0	0	-1
normalized size	1	1.00	0.62	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	0.039	0.550	0.000	0.000	0.000	0.000	0.000
Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	332	0	0	0	0	0	-1
normalized size	1	1.00	1.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.273	0.592	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	430	0	0	0	0	0	-1
normalized size	1	1.00	1.20	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.401	0.773	0.624	0.000	0.000	0.000	0.000	0.000
Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	560	0	0	3308	0	0	-1
normalized size	1	1.00	2.00	0.00	0.00	11.81	0.00	0.00	-0.00
time (sec)	N/A	0.359	0.956	0.430	0.000	45.476	0.000	0.000	0.000
Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	358	0	0	1667	0	0	-1
normalized size	1	1.00	1.56	0.00	0.00	7.25	0.00	0.00	-0.00
time (sec)	N/A	0.175	0.638	0.646	0.000	3.263	0.000	0.000	0.000
Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	78	0	0	0	0	0	-1
normalized size	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.060	0.608	0.000	0.000	0.000	0.000	0.000
Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	99	0	0	0	0	0	-1
normalized size	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.140	0.595	0.000	0.000	0.000	0.000	0.000
Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	625	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	2.265	0.596	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	1216	0	0	0	0	0	-1
normalized size	1	1.00	4.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	5.005	0.609	0.000	0.000	0.000	0.000	0.000
Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	392	0	0	0	0	0	-1
normalized size	1	1.00	1.11	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.348	0.591	0.410	0.000	0.000	0.000	0.000	0.000
Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	341	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.490	0.589	0.000	0.000	0.000	0.000	0.000
Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	233	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.203	0.314	0.657	0.000	0.000	0.000	0.000	0.000
Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	337	0	0	0	0	0	-1
normalized size	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	0.344	0.675	0.000	0.000	0.000	0.000	0.000
Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	387	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.406	0.579	0.661	0.000	0.000	0.000	0.000	0.000



Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	211	0	208	0	0	-1
normalized size	1	1.00	0.83	3.98	0.00	3.92	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.019	3.245	0.000	16.992	0.000	0.000	0.000
Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	48	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.022	0.025	0.619	0.000	0.000	0.000	0.000	0.000
Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	172	0	0	0	0	0	-1
normalized size	1	1.00	2.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	0.263	0.553	0.000	1.815	0.000	0.000	0.000
Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	106	0	0	0	119	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	0.68	0.00	-0.01
time (sec)	N/A	0.133	0.056	0.532	0.000	1.767	166.026	0.000	0.000
Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	85	90	0	0	0	75	0	-1
normalized size	1	0.91	0.97	0.00	0.00	0.00	0.81	0.00	-0.01
time (sec)	N/A	0.041	0.033	0.376	0.000	1.453	61.010	0.000	0.000
Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.217	0.560	0.000	1.307	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	162	0	0	0	0	0	-1
normalized size	1	1.00	2.84	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.028	0.242	0.565	0.000	1.191	0.000	0.000	0.000
Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	545	545	48	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.085	0.020	0.566	0.000	0.000	0.000	0.000	0.000
Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	118	248	164	306	454	0	173
normalized size	1	1.00	0.83	1.73	1.15	2.14	3.17	0.00	1.21
time (sec)	N/A	0.123	0.190	0.063	1.453	1.193	57.222	0.000	2.593
Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	84	191	126	208	121	0	99
normalized size	1	1.00	0.85	1.93	1.27	2.10	1.22	0.00	1.00
time (sec)	N/A	0.067	0.158	0.059	1.191	1.008	36.578	0.000	1.901
Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	52	163	106	128	87	0	92
normalized size	1	1.00	0.70	2.20	1.43	1.73	1.18	0.00	1.24
time (sec)	N/A	0.048	0.042	0.057	1.371	1.496	41.308	0.000	1.962
Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	74	50	99	42	64	58
normalized size	1	1.00	1.00	1.90	1.28	2.54	1.08	1.64	1.49
time (sec)	N/A	0.019	0.073	0.049	1.201	1.081	2.189	0.200	0.076

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	100	287	0	482	0	0	149
normalized size	1	1.00	0.96	2.76	0.00	4.63	0.00	0.00	1.43
time (sec)	N/A	0.111	0.244	0.108	0.000	0.970	0.000	0.000	1.631
Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	122	943	0	801	0	0	1195
normalized size	1	1.00	0.83	6.41	0.00	5.45	0.00	0.00	8.13
time (sec)	N/A	0.209	0.386	0.066	0.000	1.357	0.000	0.000	2.263
Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	330	1972	0	1749	0	820	1895
normalized size	1	1.00	1.55	9.26	0.00	8.21	0.00	3.85	8.90
time (sec)	N/A	0.340	0.823	0.068	0.000	2.251	0.000	0.412	3.729
Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	159	353	190	380	1817	0	327
normalized size	1	1.00	0.97	2.15	1.16	2.32	11.08	0.00	1.99
time (sec)	N/A	0.142	0.213	0.060	1.488	1.240	123.483	0.000	3.878
Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	106	260	152	268	534	0	197
normalized size	1	1.00	0.84	2.06	1.21	2.13	4.24	0.00	1.56
time (sec)	N/A	0.080	0.260	0.060	1.160	1.087	94.065	0.000	2.579
Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	205	132	164	163	0	81
normalized size	1	1.00	0.73	2.05	1.32	1.64	1.63	0.00	0.81
time (sec)	N/A	0.063	0.091	0.057	1.326	1.027	56.146	0.000	2.512

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	46	100	63	100	92	0	34
normalized size	1	1.00	0.85	1.85	1.17	1.85	1.70	0.00	0.63
time (sec)	N/A	0.025	0.025	0.052	1.202	0.881	2.692	0.000	1.501
Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	528	0	519	0	0	556
normalized size	1	1.00	0.96	4.98	0.00	4.90	0.00	0.00	5.25
time (sec)	N/A	0.128	0.236	0.061	0.000	1.081	0.000	0.000	1.684
Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	143	834	0	769	0	0	448
normalized size	1	1.00	0.92	5.35	0.00	4.93	0.00	0.00	2.87
time (sec)	N/A	0.222	0.387	0.062	0.000	0.954	0.000	0.000	2.165
Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	168	1817	0	1765	0	727	1664
normalized size	1	1.00	0.80	8.69	0.00	8.44	0.00	3.48	7.96
time (sec)	N/A	0.345	0.604	0.067	0.000	1.227	0.000	0.611	3.472
Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	201	457	219	494	5513	0	487
normalized size	1	1.00	1.02	2.31	1.11	2.49	27.84	0.00	2.46
time (sec)	N/A	0.158	0.255	0.067	1.207	0.852	158.704	0.000	6.049
Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	121	336	181	350	1841	0	271
normalized size	1	1.00	0.80	2.21	1.19	2.30	12.11	0.00	1.78
time (sec)	N/A	0.102	0.158	0.060	1.251	1.019	112.031	0.000	3.791

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	94	253	161	222	520	0	99
normalized size	1	1.00	0.75	2.02	1.29	1.78	4.16	0.00	0.79
time (sec)	N/A	0.077	0.112	0.060	1.505	0.964	82.773	0.000	3.480
Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	120	78	139	99	0	34
normalized size	1	1.00	0.90	1.69	1.10	1.96	1.39	0.00	0.48
time (sec)	N/A	0.034	0.062	0.058	1.249	0.940	4.364	0.000	1.630
Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	116	859	0	659	0	0	1427
normalized size	1	1.00	0.87	6.41	0.00	4.92	0.00	0.00	10.65
time (sec)	N/A	0.221	0.270	0.056	0.000	1.306	0.000	0.000	2.156
Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	145	1323	0	1001	0	0	1153
normalized size	1	1.00	0.87	7.97	0.00	6.03	0.00	0.00	6.95
time (sec)	N/A	0.234	0.430	0.063	0.000	1.225	0.000	0.000	2.311
Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	191	1638	0	1445	0	945	1476
normalized size	1	1.00	0.81	6.91	0.00	6.10	0.00	3.99	6.23
time (sec)	N/A	0.373	0.849	0.071	0.000	1.164	0.000	0.489	3.439
Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	95	535	166	233	386	158	107
normalized size	1	1.00	0.75	4.25	1.32	1.85	3.06	1.25	0.85
time (sec)	N/A	0.090	0.147	0.063	1.327	0.981	89.999	0.204	1.727

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	66	348	129	158	114	99	63
normalized size	1	1.00	0.90	4.77	1.77	2.16	1.56	1.36	0.86
time (sec)	N/A	0.054	0.087	0.063	1.244	0.931	83.772	0.183	1.621
Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	173	109	115	82	78	88
normalized size	1	1.00	1.04	3.39	2.14	2.25	1.61	1.53	1.73
time (sec)	N/A	0.033	0.039	0.059	1.298	0.982	60.252	0.197	1.977
Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	71	67	98	44	71	66
normalized size	1	1.00	1.00	1.65	1.56	2.28	1.02	1.65	1.53
time (sec)	N/A	0.020	0.018	0.046	1.208	0.927	3.088	0.239	1.441
Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	104	228	0	542	0	134	1183
normalized size	1	1.00	0.96	2.11	0.00	5.02	0.00	1.24	10.95
time (sec)	N/A	0.096	0.246	0.064	0.000	0.991	0.000	0.181	1.982
Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	150	1135	0	1163	0	300	3813
normalized size	1	1.00	0.87	6.60	0.00	6.76	0.00	1.74	22.17
time (sec)	N/A	0.218	0.775	0.069	0.000	1.065	0.000	0.220	3.536
Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	216	2269	0	2307	0	352	2890
normalized size	1	1.00	0.86	9.08	0.00	9.23	0.00	1.41	11.56
time (sec)	N/A	0.400	1.746	0.069	0.000	1.937	0.000	0.288	5.479

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	92	969	200	336	0	222	172
normalized size	1	1.00	0.70	7.34	1.52	2.55	0.00	1.68	1.30
time (sec)	N/A	0.101	0.075	0.067	1.259	0.977	0.000	0.209	1.908
Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	81	789	164	272	0	160	120
normalized size	1	0.96	0.86	8.39	1.74	2.89	0.00	1.70	1.28
time (sec)	N/A	0.079	0.103	0.064	1.157	0.893	0.000	0.226	1.834
Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	48	387	144	210	224	127	71
normalized size	1	1.00	0.63	5.09	1.89	2.76	2.95	1.67	0.93
time (sec)	N/A	0.050	0.024	0.056	1.374	1.052	81.344	0.196	2.438
Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	36	198	85	156	71	86	34
normalized size	1	1.02	0.60	3.30	1.42	2.60	1.18	1.43	0.57
time (sec)	N/A	0.031	0.013	0.058	1.183	0.773	4.794	0.165	1.868
Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	106	962	0	1075	0	200	3000
normalized size	1	1.00	0.72	6.54	0.00	7.31	0.00	1.36	20.41
time (sec)	N/A	0.194	0.072	0.074	0.000	1.166	0.000	0.206	2.684
Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	164	3119	0	2321	0	424	4274
normalized size	1	1.00	0.73	13.92	0.00	10.36	0.00	1.89	19.08
time (sec)	N/A	0.323	0.145	0.073	0.000	2.002	0.000	0.256	6.202

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	239	5158	0	4093	0	516	8936
normalized size	1	1.00	0.75	16.12	0.00	12.79	0.00	1.61	27.92
time (sec)	N/A	0.525	0.321	0.079	0.000	5.028	0.000	0.309	9.488
Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	145	1150	228	483	0	203	194
normalized size	1	1.00	1.01	8.04	1.59	3.38	0.00	1.42	1.36
time (sec)	N/A	0.151	0.430	0.061	1.341	0.984	0.000	0.256	2.050
Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	118	97	588	190	407	0	163	144
normalized size	1	0.97	0.80	4.82	1.56	3.34	0.00	1.34	1.18
time (sec)	N/A	0.095	0.082	0.065	1.308	1.011	0.000	0.205	2.219
Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	60	541	170	331	1479	145	87
normalized size	1	1.00	0.58	5.25	1.65	3.21	14.36	1.41	0.84
time (sec)	N/A	0.065	0.035	0.064	1.251	0.921	155.819	0.220	2.910
Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	82	38	271	101	225	774	98	34
normalized size	1	1.04	0.48	3.43	1.28	2.85	9.80	1.24	0.43
time (sec)	N/A	0.038	0.025	0.068	1.291	0.592	7.925	0.239	1.722
Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	118	1767	0	1990	0	247	5387
normalized size	1	1.00	0.59	8.79	0.00	9.90	0.00	1.23	26.80
time (sec)	N/A	0.315	0.084	0.071	0.000	5.603	0.000	0.219	4.622



Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	178	4644	0	3887	0	576	5789
normalized size	1	1.00	0.62	16.18	0.00	13.54	0.00	2.01	20.17
time (sec)	N/A	0.448	0.189	0.080	0.000	5.602	0.000	0.326	8.729
Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	409	409	239	7300	0	6171	0	523	4284
normalized size	1	1.00	0.58	17.85	0.00	15.09	0.00	1.28	10.47
time (sec)	N/A	0.702	0.387	0.079	0.000	14.874	0.000	0.285	8.227
Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	167	253	0	890	0	0	4674
normalized size	1	1.00	1.36	2.06	0.00	7.24	0.00	0.00	38.00
time (sec)	N/A	0.094	1.401	0.096	0.000	2.164	0.000	0.000	22.224
Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	81	155	0	247	0	0	478
normalized size	1	1.00	1.00	1.91	0.00	3.05	0.00	0.00	5.90
time (sec)	N/A	0.052	0.068	0.096	0.000	0.931	0.000	0.000	6.583
Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	87	280	0	319	0	0	-1
normalized size	1	1.00	0.71	2.30	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.106	0.080	0.000	1.299	0.000	0.000	0.000
Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	206	0	0	0	0	0	-1
normalized size	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.367	0.297	0.000	1.037	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	40	45	33	98	82	33	32
normalized size	1	1.00	1.03	1.15	0.85	2.51	2.10	0.85	0.82
time (sec)	N/A	0.020	0.032	0.046	1.305	0.884	0.330	0.153	0.069
Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	205	277	0	0	0	0	-1
normalized size	1	1.00	0.88	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	0.378	0.142	0.000	0.882	0.000	0.000	0.000
Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	86	94	0	0	0	0	-1
normalized size	1	1.00	0.37	0.41	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.205	0.074	0.069	0.000	1.006	0.000	0.000	0.000
Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	191	185	0	0	0	0	-1
normalized size	1	1.00	0.73	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.280	0.328	0.172	0.000	0.799	0.000	0.000	0.000
Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	104	0	0	0	0	0	-1
normalized size	1	1.00	1.32	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.149	0.225	0.000	1.328	0.000	0.000	0.000
Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	129	195	128	390	71	133	123
normalized size	1	1.00	0.89	1.34	0.88	2.69	0.49	0.92	0.85
time (sec)	N/A	0.106	0.102	0.050	1.294	0.663	0.454	0.222	0.274

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	41	59	47	48	82	49	49
normalized size	1	1.00	0.84	1.20	0.96	0.98	1.67	1.00	1.00
time (sec)	N/A	0.049	0.072	0.052	0.638	0.974	0.299	0.195	0.074
Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	21	20	20	24	20	20
normalized size	1	1.00	1.00	0.81	0.77	0.77	0.92	0.77	0.77
time (sec)	N/A	0.016	0.013	0.040	0.550	0.913	0.196	0.205	0.034
Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	23	23	27	24	13
normalized size	1	1.00	1.00	1.41	1.35	1.35	1.59	1.41	0.76
time (sec)	N/A	0.015	0.006	0.048	0.564	0.690	0.266	0.182	1.458
Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	22	66	71	71	102	71	90
normalized size	1	1.00	0.21	0.63	0.68	0.68	0.98	0.68	0.87
time (sec)	N/A	0.086	0.007	0.044	1.135	0.823	5.721	0.166	1.497
Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	23	22	22	26	23	22
normalized size	1	1.00	1.00	0.77	0.73	0.73	0.87	0.77	0.73
time (sec)	N/A	0.019	0.031	0.042	0.558	0.593	0.184	0.152	0.041
Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.040	0.767	0.000	0.000	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.027	0.018	0.594	0.000	0.000	0.000	0.000	0.000
Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	72	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.029	0.043	0.928	0.000	0.981	0.000	0.000	0.000
Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	138	186	527	2744	740	131
normalized size	1	1.00	0.83	1.05	1.41	3.99	20.79	5.61	0.99
time (sec)	N/A	0.111	0.284	0.053	0.614	0.682	3.673	0.216	1.638
Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	90	104	140	319	1540	450	99
normalized size	1	1.00	0.91	1.05	1.41	3.22	15.56	4.55	1.00
time (sec)	N/A	0.073	0.128	0.055	0.503	1.010	3.346	0.193	1.557
Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	94	175	726	232	71
normalized size	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.045	0.102	0.054	0.440	0.979	1.956	0.188	1.535
Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	37	43	48	69	236	83	38
normalized size	1	1.00	0.92	1.08	1.20	1.72	5.90	2.08	0.95
time (sec)	N/A	0.021	0.075	0.045	0.510	0.979	0.650	0.169	1.483

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	40	0	0	0	73	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	1.70	0.00	-0.02
time (sec)	N/A	0.017	0.014	0.632	0.000	0.667	3.333	0.000	0.000
Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	56	0	0	0	592	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	8.11	0.00	-0.01
time (sec)	N/A	0.031	0.046	0.587	0.000	0.956	8.694	0.000	0.000
Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	3706	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	47.51	0.00	-0.01
time (sec)	N/A	0.031	0.056	0.609	0.000	0.956	64.993	0.000	0.000
Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	58	0	0	0	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	0.065	0.585	0.000	0.836	0.000	0.000	0.000
Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	149	164	242	667	0	947	157
normalized size	1	1.00	0.94	1.04	1.53	4.22	0.00	5.99	0.99
time (sec)	N/A	0.130	0.211	0.057	0.622	1.002	0.000	0.243	1.705
Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	105	117	168	370	1765	539	108
normalized size	1	1.00	0.94	1.04	1.50	3.30	15.76	4.81	0.96
time (sec)	N/A	0.082	0.197	0.055	0.604	0.862	77.474	0.225	1.566

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	74	94	175	726	232	71
normalized size	1	1.00	1.00	1.06	1.34	2.50	10.37	3.31	1.01
time (sec)	N/A	0.047	0.108	0.049	0.633	1.082	2.737	0.272	1.531
Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	170	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.094	0.049	0.602	0.000	1.078	6.831	0.000	0.000
Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	95	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.155	0.585	0.000	0.966	0.000	0.000	0.000
Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	133	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.160	0.192	0.614	0.000	0.945	0.000	0.000	0.000
Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	133	0	0	0	369	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	1.19	0.00	-0.00
time (sec)	N/A	0.501	4.125	0.654	0.000	0.928	9.725	0.000	0.000
Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	104	0	0	0	269	0	-1
normalized size	1	1.00	0.60	0.00	0.00	0.00	1.55	0.00	-0.01
time (sec)	N/A	0.267	1.588	0.634	0.000	1.023	13.012	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	0	0	0	170	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	2.02	0.00	-0.01
time (sec)	N/A	0.097	0.361	0.604	0.000	0.669	12.793	0.000	0.000
Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	40	0	0	0	73	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	1.74	0.00	-0.02
time (sec)	N/A	0.016	0.011	0.614	0.000	0.730	4.112	0.000	0.000
Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.026	0.063	0.912	0.000	1.007	0.000	0.000	0.000
Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	121	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.217	0.928	0.000	1.233	0.000	0.000	0.000
Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	210	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	0.264	0.902	0.000	0.838	0.000	0.000	0.000
Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	217	0	0	0	0	0	-1
normalized size	1	1.00	0.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.548	5.264	0.617	0.000	1.514	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	167	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	5.379	0.676	0.000	0.863	0.000	0.000	0.000
Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	666	0	0	0	0	0	-1
normalized size	1	1.00	5.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	2.580	0.608	0.000	0.901	0.000	0.000	0.000
Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	56	0	0	0	592	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	8.22	0.00	-0.01
time (sec)	N/A	0.031	0.044	0.590	0.000	0.645	8.769	0.000	0.000
Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	108	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.142	0.201	0.910	0.000	0.987	0.000	0.000	0.000
Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	147	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.285	0.913	0.000	1.048	0.000	0.000	0.000
Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	233	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.438	0.927	0.000	0.990	0.000	0.000	0.000



Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	190	0	0	0	0	0	-1
normalized size	1	1.00	2.35	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.566	0.889	0.000	0.775	0.000	0.000	0.000
Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	401	168	0	0	0	199	0	-1
normalized size	1	1.00	0.42	0.00	0.00	0.00	0.50	0.00	-0.00
time (sec)	N/A	0.578	5.318	0.742	0.000	1.114	93.229	0.000	0.000
Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	197	140	0	0	0	143	0	-1
normalized size	1	0.98	0.69	0.00	0.00	0.00	0.71	0.00	-0.00
time (sec)	N/A	0.269	5.202	0.720	0.000	0.838	42.751	0.000	0.000
Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	89	94	0	0	0	87	0	-1
normalized size	1	0.91	0.96	0.00	0.00	0.00	0.89	0.00	-0.01
time (sec)	N/A	0.047	0.047	0.604	0.000	0.690	6.154	0.000	0.000
Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	0	0	0	37	0	47
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.80	0.00	1.02
time (sec)	N/A	0.011	0.005	0.036	0.000	0.705	1.902	0.000	2.299
Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.309	0.914	0.000	1.071	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.030	0.342	0.900	0.000	0.901	0.000	0.000	0.000
Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	180	0	0	0	0	0	-1
normalized size	1	1.00	3.05	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.029	0.651	0.930	0.000	0.998	0.000	0.000	0.000
Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	94	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.088	0.910	0.000	1.095	0.000	0.000	0.000
Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	218	0	0	478	0	0	-1
normalized size	1	1.00	1.22	0.00	0.00	2.69	0.00	0.00	-0.01
time (sec)	N/A	0.086	0.142	0.828	0.000	0.803	0.000	0.000	0.000
Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	113	0	0	231	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	1.99	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.083	0.797	0.000	1.232	0.000	0.000	0.000
Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	82	0	0	85	0	0	-1
normalized size	1	1.00	1.41	0.00	0.00	1.47	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.137	0.598	0.000	0.832	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	53	0	31	211	0	75
normalized size	1	1.00	1.00	2.94	0.00	1.72	11.72	0.00	4.17
time (sec)	N/A	0.003	0.036	0.073	0.000	1.005	33.049	0.000	1.756
Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.014	0.979	0.000	0.806	0.000	0.000	0.000
Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.014	0.015	0.925	0.000	1.159	0.000	0.000	0.000
Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	0	0	0	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.015	0.013	0.947	0.000	1.286	0.000	0.000	0.000
Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	193	179	1414	0	0	0	0	0	-1
normalized size	1	0.93	7.33	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	51.213	0.921	0.000	1.136	0.000	0.000	0.000
Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	55	0	0	108	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	1.89	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.069	0.904	0.000	1.270	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	136	0	0	400	0	0	-1
normalized size	1	1.00	0.42	0.00	0.00	1.22	0.00	0.00	-0.00
time (sec)	N/A	0.184	0.463	0.766	0.000	0.966	0.000	0.000	0.000
Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	94	0	0	173	0	0	-1
normalized size	1	1.00	0.74	0.00	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.170	0.632	0.000	1.347	0.000	0.000	0.000
Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	0	0	68	0	0	64
normalized size	1	1.00	1.10	0.00	0.00	1.36	0.00	0.00	1.28
time (sec)	N/A	0.012	0.029	0.605	0.000	1.408	0.000	0.000	1.763
Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	153	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.031	6.942	0.836	0.000	1.056	0.000	0.000	0.000
Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	1070	0	0	0	0	0	-1
normalized size	1	1.00	8.43	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	63.260	1.038	0.000	1.237	0.000	0.000	0.000
Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	1241	0	0	0	0	0	-1
normalized size	1	1.00	9.47	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	44.863	1.131	0.000	1.493	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F(-1)	F	F	F	F(-2)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.055	180.013	0.952	0.000	1.372	0.000	0.000	0.000
Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	164	110	92	178	114	0	621	152
normalized size	1	1.08	0.72	0.61	1.17	0.75	0.00	4.09	1.00
time (sec)	N/A	0.118	0.104	0.043	0.500	1.340	0.000	0.760	1.761
Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	118	88	68	124	90	0	495	118
normalized size	1	1.08	0.81	0.62	1.14	0.83	0.00	4.54	1.08
time (sec)	N/A	0.087	0.060	0.046	0.498	0.990	0.000	0.748	1.736
Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	62	44	70	66	0	361	83
normalized size	1	1.07	0.93	0.66	1.04	0.99	0.00	5.39	1.24
time (sec)	N/A	0.044	0.046	0.046	0.619	1.141	0.000	0.371	1.641
Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	85	174	52	80	0	78	248
normalized size	1	1.00	1.06	2.18	0.65	1.00	0.00	0.98	3.10
time (sec)	N/A	0.078	0.243	0.101	1.300	0.941	0.000	0.320	3.598
Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	114	114	182	98	85	0	157	584
normalized size	1	1.19	1.19	1.90	1.02	0.89	0.00	1.64	6.08
time (sec)	N/A	0.084	0.056	0.067	1.462	1.067	0.000	0.421	6.890

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	164	137	226	162	100	0	324	1004
normalized size	1	1.36	1.13	1.87	1.34	0.83	0.00	2.68	8.30
time (sec)	N/A	0.104	0.092	0.066	1.353	1.085	0.000	0.455	15.557
Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	161	298	246	138	0	558	2314
normalized size	1	1.00	0.77	1.43	1.18	0.66	0.00	2.68	11.12
time (sec)	N/A	0.149	0.309	0.095	0.548	1.319	0.000	0.627	39.151
Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	135	240	192	112	0	432	1681
normalized size	1	1.00	0.85	1.51	1.21	0.70	0.00	2.72	10.57
time (sec)	N/A	0.123	0.191	0.067	0.666	1.500	0.000	0.458	42.568
Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	129	182	137	88	0	288	734
normalized size	1	1.00	1.13	1.60	1.20	0.77	0.00	2.53	6.44
time (sec)	N/A	0.047	0.279	0.059	0.543	1.128	0.000	0.350	17.425
Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	101	153	105	83	0	110	243
normalized size	1	1.00	0.97	1.47	1.01	0.80	0.00	1.06	2.34
time (sec)	N/A	0.087	0.089	0.064	1.500	1.216	0.000	0.395	3.486
Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	105	153	75	100	0	171	236
normalized size	1	1.00	1.25	1.82	0.89	1.19	0.00	2.04	2.81
time (sec)	N/A	0.080	0.086	0.066	1.472	1.336	0.000	0.383	3.438

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	117	191	153	96	0	172	1154
normalized size	1	1.00	0.94	1.53	1.22	0.77	0.00	1.38	9.23
time (sec)	N/A	0.085	0.143	0.106	0.554	1.247	0.000	0.231	32.625
Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	57	95	55	216	108	108
normalized size	1	1.00	0.68	0.55	0.92	0.53	2.10	1.05	1.05
time (sec)	N/A	0.075	0.043	0.046	0.633	1.127	62.839	0.223	2.440
Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	98	147	113	77	0	121	720
normalized size	1	1.00	1.13	1.69	1.30	0.89	0.00	1.39	8.28
time (sec)	N/A	0.069	0.105	0.078	0.537	1.010	0.000	0.373	22.496
Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	52	38	54	37	202	59	66
normalized size	1	1.00	0.80	0.58	0.83	0.57	3.11	0.91	1.02
time (sec)	N/A	0.040	0.054	0.044	0.469	1.036	41.910	0.222	2.359
Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	101	103	74	55	182	69	293
normalized size	1	1.00	2.15	2.19	1.57	1.17	3.87	1.47	6.23
time (sec)	N/A	0.020	0.212	0.066	0.497	1.099	45.512	0.176	12.685
Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	66	62	29	48	162	45	77
normalized size	1	1.00	1.43	1.35	0.63	1.04	3.52	0.98	1.67
time (sec)	N/A	0.061	0.033	0.072	1.152	0.983	40.135	0.224	3.865

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	73	77	44	56	148	58	61
normalized size	1	1.00	2.21	2.33	1.33	1.70	4.48	1.76	1.85
time (sec)	N/A	0.057	0.030	0.072	1.177	1.178	35.169	0.216	2.593
Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	84	45	57	141	114	297
normalized size	1	1.00	1.28	1.40	0.75	0.95	2.35	1.90	4.95
time (sec)	N/A	0.063	0.071	0.082	1.226	1.074	63.618	0.213	8.668
Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	37	54	52	146	116	53
normalized size	1	1.00	0.82	0.60	0.87	0.84	2.35	1.87	0.85
time (sec)	N/A	0.062	0.018	0.050	1.233	0.699	61.444	0.206	2.440
Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	102	125	85	78	0	268	650
normalized size	1	1.00	1.03	1.26	0.86	0.79	0.00	2.71	6.57
time (sec)	N/A	0.078	0.102	0.074	1.107	1.310	0.000	0.221	21.455
Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	148	240	196	115	0	203	1682
normalized size	1	1.00	0.90	1.46	1.20	0.70	0.00	1.24	10.26
time (sec)	N/A	0.120	0.124	0.099	0.568	1.240	0.000	0.280	42.656
Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	87	68	124	66	240	124	130
normalized size	1	1.00	0.74	0.58	1.05	0.56	2.03	1.05	1.10
time (sec)	N/A	0.086	0.062	0.046	0.551	1.096	70.843	0.251	2.700



Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	121	182	142	90	0	140	1048
normalized size	1	1.00	1.03	1.54	1.20	0.76	0.00	1.19	8.88
time (sec)	N/A	0.097	0.098	0.075	0.639	0.906	0.000	0.233	25.513
Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	61	43	69	42	223	65	76
normalized size	1	1.00	0.85	0.60	0.96	0.58	3.10	0.90	1.06
time (sec)	N/A	0.046	0.038	0.040	0.645	1.143	44.702	0.198	2.663
Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	119	124	89	63	199	79	417
normalized size	1	1.00	1.75	1.82	1.31	0.93	2.93	1.16	6.13
time (sec)	N/A	0.032	0.349	0.071	0.459	1.175	41.776	0.265	10.800
Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	87	108	37	61	178	55	108
normalized size	1	1.00	1.55	1.93	0.66	1.09	3.18	0.98	1.93
time (sec)	N/A	0.069	0.037	0.077	1.342	0.967	39.158	0.209	3.967
Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	90	97	55	68	165	66	77
normalized size	1	1.00	1.58	1.70	0.96	1.19	2.89	1.16	1.35
time (sec)	N/A	0.075	0.041	0.069	1.257	1.290	36.825	0.217	2.945
Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	102	158	60	73	162	141	457
normalized size	1	1.00	1.34	2.08	0.79	0.96	2.13	1.86	6.01
time (sec)	N/A	0.073	0.102	0.072	1.292	1.271	69.669	0.494	7.500

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	66	49	75	67	170	137	79
normalized size	1	1.00	0.88	0.65	1.00	0.89	2.27	1.83	1.05
time (sec)	N/A	0.069	0.031	0.041	1.319	0.670	70.802	0.244	2.765
Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	144	227	114	100	0	325	1005
normalized size	1	1.00	1.17	1.85	0.93	0.81	0.00	2.64	8.17
time (sec)	N/A	0.095	0.108	0.070	1.208	1.084	0.000	0.271	19.135
Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	119	316	196	190	0	214	-1
normalized size	1	1.00	0.74	1.96	1.22	1.18	0.00	1.33	-0.01
time (sec)	N/A	0.123	0.162	0.087	0.492	1.085	0.000	0.398	0.000
Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	72	68	123	80	226	200	90
normalized size	1	1.00	0.63	0.59	1.07	0.70	1.97	1.74	0.78
time (sec)	N/A	0.094	0.054	0.049	0.464	1.180	177.265	0.435	2.801
Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	90	254	138	159	0	147	-1
normalized size	1	1.00	0.59	1.67	0.91	1.05	0.00	0.97	-0.01
time (sec)	N/A	0.115	0.121	0.077	0.500	0.980	0.000	0.321	0.000
Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	45	43	69	56	201	152	67
normalized size	1	1.00	0.59	0.57	0.91	0.74	2.64	2.00	0.88
time (sec)	N/A	0.054	0.067	0.043	0.576	1.137	136.305	0.280	2.747

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	86	160	76	129	182	113	-1
normalized size	1	1.00	1.37	2.54	1.21	2.05	2.89	1.79	-0.02
time (sec)	N/A	0.032	0.244	0.073	0.548	1.407	112.361	0.280	0.000
Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	84	188	58	101	172	115	-1
normalized size	1	1.00	1.29	2.89	0.89	1.55	2.65	1.77	-0.02
time (sec)	N/A	0.081	0.044	0.076	1.463	1.102	136.445	0.362	0.000
Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	48	71	103	165	219	73
normalized size	1	1.00	0.76	0.72	1.06	1.54	2.46	3.27	1.09
time (sec)	N/A	0.077	0.027	0.050	1.365	0.996	136.133	0.459	2.866
Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	75	315	104	138	0	211	-1
normalized size	1	1.00	0.64	2.69	0.89	1.18	0.00	1.80	-0.01
time (sec)	N/A	0.098	0.032	0.091	1.265	1.065	0.000	0.542	0.000
Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	77	73	125	132	0	242	104
normalized size	1	1.00	0.65	0.61	1.05	1.11	0.00	2.03	0.87
time (sec)	N/A	0.096	0.031	0.046	1.342	0.877	0.000	0.728	2.897
Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	78	387	162	165	0	402	-1
normalized size	1	1.00	0.47	2.33	0.98	0.99	0.00	2.42	-0.01
time (sec)	N/A	0.120	0.033	0.084	1.480	0.689	0.000	0.794	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	56	53	23	39	148	40	72
normalized size	1	1.00	1.40	1.32	0.58	0.98	3.70	1.00	1.80
time (sec)	N/A	0.054	0.026	0.069	1.351	1.238	30.114	0.171	3.652
Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	244	66	79	65	0	0	96
normalized size	1	1.00	4.60	1.25	1.49	1.23	0.00	0.00	1.81
time (sec)	N/A	0.091	0.314	0.053	1.244	1.011	0.000	0.000	3.274
Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	0	0	69	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	1.92	0.00	0.00	-0.03
time (sec)	N/A	0.020	0.020	0.857	0.000	0.915	0.000	0.000	0.000
Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	0	0	50	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.67	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.058	0.877	0.000	1.323	0.000	0.000	0.000
Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.438	1.602	0.000	1.409	0.000	0.000	0.000
Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.049	1.495	0.000	1.308	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	258	0	0	0	0	0	-1
normalized size	1	1.00	3.39	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.429	1.225	0.000	0.998	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	104	103	0	0	180	0	0	-1
normalized size	1	1.08	1.07	0.00	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.394	2.167	0.000	1.244	0.000	0.000	0.000

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [279] had the largest ratio of [.5294]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	1	1.00	17	0.059
2	A	2	1	1.00	17	0.059
3	A	2	1	1.00	17	0.059
4	A	2	1	1.00	15	0.067
5	A	7	7	1.00	17	0.412
6	A	7	7	1.00	17	0.412
7	A	8	8	1.00	17	0.471
8	A	2	1	1.00	19	0.053
9	A	2	1	1.00	19	0.053
10	A	2	1	1.00	17	0.059
11	A	8	7	1.00	19	0.368
12	A	9	8	1.00	19	0.421
13	A	8	8	1.00	19	0.421

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
14	A	8	7	1.00	19	0.368
15	A	8	7	1.00	19	0.368
16	A	8	7	1.00	19	0.368
17	A	7	7	1.00	17	0.412
18	A	13	7	1.00	19	0.368
19	A	14	8	1.00	19	0.421
20	A	9	8	1.00	19	0.421
21	A	9	8	1.00	19	0.421
22	A	9	8	1.00	19	0.421
23	A	9	8	1.00	19	0.421
24	A	7	7	1.00	17	0.412
25	A	14	8	1.00	19	0.421
26	A	15	9	1.00	19	0.474
27	A	3	3	1.00	20	0.150
28	A	2	2	1.00	20	0.100
29	A	2	2	1.00	20	0.100
30	A	2	2	1.00	20	0.100
31	A	3	3	1.00	20	0.150
32	A	4	3	1.00	20	0.150
33	A	5	3	1.00	20	0.150
34	C	2	2	0.12	22	0.091
35	C	2	2	0.12	22	0.091
36	C	2	2	0.15	22	0.091
37	C	2	2	0.13	22	0.091
38	C	2	2	0.12	22	0.091
39	C	2	2	0.12	22	0.091
40	A	4	4	1.00	22	0.182
41	A	3	3	1.00	22	0.136
42	A	3	3	1.00	22	0.136
43	A	3	3	1.00	22	0.136
44	A	3	2	1.00	22	0.091
45	A	4	3	1.00	22	0.136
46	A	5	4	1.00	22	0.182
47	A	6	4	1.00	22	0.182
48	A	4	4	1.00	22	0.182
49	A	4	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
50	A	4	4	1.00	22	0.182
51	A	4	4	1.00	22	0.182
52	A	4	4	1.00	22	0.182
53	A	4	4	1.00	22	0.182
54	A	4	4	1.00	22	0.182
55	A	4	4	1.00	22	0.182
56	A	4	3	1.00	19	0.158
57	A	3	3	1.00	19	0.158
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105
60	A	2	2	1.00	19	0.105
61	A	3	3	1.00	19	0.158
62	A	4	3	1.00	19	0.158
63	A	5	3	1.00	19	0.158
64	A	3	3	1.00	19	0.158
65	A	3	3	1.00	19	0.158
66	A	3	3	1.00	19	0.158
67	A	3	3	1.00	19	0.158
68	A	3	3	1.00	19	0.158
69	A	3	3	1.00	19	0.158
70	A	5	4	1.00	21	0.190
71	A	4	4	1.00	21	0.190
72	A	3	3	1.00	21	0.143
73	A	3	3	1.00	21	0.143
74	A	3	3	1.00	21	0.143
75	A	3	2	1.00	21	0.095
76	A	4	3	1.00	21	0.143
77	A	5	4	1.00	21	0.190
78	A	6	4	1.00	21	0.190
79	A	4	4	1.00	21	0.190
80	A	4	4	1.00	21	0.190
81	A	4	4	1.00	21	0.190
82	A	4	4	1.00	21	0.190
83	A	4	4	1.00	21	0.190
84	A	4	4	1.00	21	0.190
85	A	4	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	C	2	2	0.19	21	0.095
87	C	2	2	0.22	21	0.095
88	C	2	2	0.25	21	0.095
89	A	7	7	1.40	21	0.333
90	A	8	8	1.33	21	0.381
91	C	2	2	2.75	21	0.095
92	C	2	2	4.19	21	0.095
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	C	2	2	0.18	21	0.095
99	C	2	2	0.20	21	0.095
100	A	8	8	1.32	21	0.381
101	A	8	8	1.27	21	0.381
102	C	2	2	2.39	21	0.095
103	C	2	2	3.75	21	0.095
104	A	2	2	1.00	21	0.095
105	A	2	2	1.00	21	0.095
106	A	2	2	1.00	21	0.095
107	A	2	2	1.00	21	0.095
108	A	2	2	1.00	21	0.095
109	C	2	2	0.11	21	0.095
110	C	2	2	0.14	21	0.095
111	C	2	2	0.16	21	0.095
112	A	9	8	1.27	21	0.381
113	A	9	9	1.22	21	0.429
114	C	2	2	0.54	21	0.095
115	C	2	2	1.14	21	0.095
116	C	2	2	4.30	21	0.095
117	A	2	2	1.00	21	0.095
118	A	2	2	1.00	21	0.095
119	A	2	2	1.00	21	0.095
120	A	2	2	1.00	21	0.095
121	A	2	2	1.00	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
122	A	3	2	1.00	23	0.087
123	A	3	2	1.00	23	0.087
124	A	2	2	1.00	23	0.087
125	A	2	2	1.00	23	0.087
126	A	1	1	1.00	23	0.043
127	A	1	1	1.00	23	0.043
128	A	1	1	1.00	23	0.043
129	A	2	2	1.00	23	0.087
130	A	2	2	1.00	23	0.087
131	A	3	2	1.00	23	0.087
132	A	3	2	1.00	23	0.087
133	A	3	2	1.00	19	0.105
134	A	4	4	1.05	19	0.210
135	A	3	3	1.01	17	0.176
136	A	2	2	1.00	19	0.105
137	A	2	2	1.00	19	0.105
138	A	5	5	1.00	19	0.263
139	A	4	4	1.00	19	0.210
140	A	3	3	0.91	17	0.176
141	A	2	2	1.00	9	0.222
142	A	2	2	1.00	19	0.105
143	A	2	2	1.00	19	0.105
144	A	2	2	1.00	19	0.105
145	A	1	1	1.00	50	0.020
146	A	2	1	1.00	17	0.059
147	A	2	1	1.00	17	0.059
148	A	2	1	1.00	17	0.059
149	A	2	1	1.00	15	0.067
150	A	10	7	1.00	17	0.412
151	A	10	7	1.00	17	0.412
152	A	11	8	1.00	17	0.471
153	A	2	1	1.00	19	0.053
154	A	2	1	1.00	19	0.053
155	A	2	1	1.00	19	0.053
156	A	2	1	1.00	17	0.059
157	A	11	7	1.00	19	0.368

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
158	A	12	8	1.00	19	0.421
159	A	11	8	1.00	19	0.421
160	A	11	7	1.00	19	0.368
161	A	11	7	1.00	19	0.368
162	A	11	7	1.00	19	0.368
163	A	10	7	1.00	17	0.412
164	A	19	7	1.00	19	0.368
165	A	20	8	1.00	19	0.421
166	A	12	8	1.00	19	0.421
167	A	12	8	1.00	19	0.421
168	A	12	8	1.00	19	0.421
169	A	12	8	1.00	19	0.421
170	A	10	7	1.00	17	0.412
171	A	20	8	1.00	19	0.421
172	A	21	9	1.00	19	0.474
173	A	10	8	1.00	23	0.348
174	A	9	7	1.00	23	0.304
175	A	8	6	1.00	23	0.261
176	A	5	3	1.00	23	0.130
177	A	9	7	1.00	23	0.304
178	A	10	8	1.00	23	0.348
179	A	10	6	1.00	21	0.286
180	A	9	5	1.00	21	0.238
181	A	7	4	1.00	21	0.190
182	A	10	6	1.00	21	0.286
183	A	11	7	1.00	21	0.333
184	A	11	8	1.00	23	0.348
185	A	10	8	1.00	23	0.348
186	A	9	7	1.00	23	0.304
187	A	9	7	1.00	23	0.304
188	A	9	7	1.00	23	0.304
189	A	10	8	1.00	23	0.348
190	A	11	8	1.00	23	0.348
191	A	4	4	1.00	25	0.160
192	A	1	1	1.00	25	0.040
193	A	10	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
194	A	9	8	1.00	21	0.381
195	A	4	4	1.00	21	0.190
196	A	5	5	1.00	21	0.238
197	A	7	7	1.00	21	0.333
198	A	8	7	1.00	21	0.333
199	A	11	10	1.00	21	0.476
200	A	10	9	1.00	21	0.429
201	A	4	3	1.00	21	0.143
202	A	9	8	1.00	21	0.381
203	A	10	9	1.00	21	0.429
204	A	11	10	1.00	21	0.476
205	A	11	10	1.00	21	0.476
206	A	10	9	1.00	21	0.429
207	A	5	5	1.00	21	0.238
208	A	5	5	1.00	21	0.238
209	A	7	7	1.00	21	0.333
210	A	8	7	1.00	21	0.333
211	A	11	10	1.00	21	0.476
212	A	10	9	1.00	21	0.429
213	A	10	9	1.00	21	0.429
214	A	10	9	1.00	21	0.429
215	A	11	10	1.00	21	0.476
216	A	4	4	1.00	17	0.235
217	A	4	4	1.00	26	0.154
218	A	3	2	1.00	19	0.105
219	A	4	4	1.00	19	0.210
220	A	3	3	0.91	17	0.176
221	A	2	2	1.00	19	0.105
222	A	2	2	1.00	19	0.105
223	A	7	7	1.00	21	0.333
224	A	6	6	1.00	21	0.286
225	A	6	6	1.00	21	0.286
226	A	5	5	1.00	19	0.263
227	A	4	4	1.00	11	0.364
228	A	7	6	1.00	21	0.286
229	A	8	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
230	A	9	7	1.00	21	0.333
231	A	7	7	1.00	21	0.333
232	A	7	6	1.00	21	0.286
233	A	6	5	1.00	19	0.263
234	A	5	5	1.00	11	0.454
235	A	7	6	1.00	21	0.286
236	A	8	7	1.00	21	0.333
237	A	9	7	1.00	21	0.333
238	A	8	7	1.00	21	0.333
239	A	8	6	1.00	21	0.286
240	A	7	5	1.00	19	0.263
241	A	6	5	1.00	11	0.454
242	A	8	7	1.00	21	0.333
243	A	8	7	1.00	21	0.333
244	A	9	8	1.00	21	0.381
245	A	5	5	1.00	21	0.238
246	A	5	5	1.00	21	0.238
247	A	4	4	1.00	19	0.210
248	A	4	4	1.00	11	0.364
249	A	7	6	1.00	21	0.286
250	A	8	7	1.00	21	0.333
251	A	9	7	1.00	21	0.333
252	A	5	5	1.00	21	0.238
253	A	5	5	0.96	21	0.238
254	A	5	5	1.00	19	0.263
255	A	5	4	1.02	11	0.364
256	A	8	7	1.00	21	0.333
257	A	9	8	1.00	21	0.381
258	A	10	8	1.00	21	0.381
259	A	5	5	1.00	21	0.238
260	A	6	6	0.97	21	0.286
261	A	6	5	1.00	19	0.263
262	A	6	4	1.04	11	0.364
263	A	9	7	1.00	21	0.333
264	A	10	8	1.00	21	0.381
265	A	11	8	1.00	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
266	A	8	8	1.00	23	0.348
267	A	4	4	1.00	23	0.174
268	A	5	5	1.00	23	0.217
269	A	3	3	1.00	19	0.158
270	A	3	3	1.00	17	0.176
271	A	6	6	1.00	23	0.261
272	A	7	7	1.00	23	0.304
273	A	7	7	1.00	23	0.304
274	A	4	3	1.00	19	0.158
275	A	8	8	1.00	17	0.471
276	A	3	2	1.00	21	0.095
277	A	3	2	1.00	17	0.118
278	A	4	4	1.00	17	0.235
279	A	9	9	1.00	17	0.529
280	A	4	3	1.00	17	0.176
281	A	3	3	1.00	24	0.125
282	A	3	3	1.00	24	0.125
283	A	3	3	1.00	20	0.150
284	A	2	1	1.00	17	0.059
285	A	2	1	1.00	17	0.059
286	A	2	1	1.00	17	0.059
287	A	2	1	1.00	15	0.067
288	A	2	2	1.00	17	0.118
289	A	2	2	1.00	17	0.118
290	A	2	2	1.00	17	0.118
291	A	2	2	1.00	17	0.118
292	A	2	1	1.00	19	0.053
293	A	2	1	1.00	19	0.053
294	A	2	1	1.00	17	0.059
295	A	3	3	1.00	19	0.158
296	A	3	3	1.00	19	0.158
297	A	3	3	1.00	19	0.158
298	A	5	4	1.00	19	0.210
299	A	4	4	1.00	19	0.210
300	A	3	3	1.00	19	0.158
301	A	2	2	1.00	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
302	A	3	2	1.00	19	0.105
303	A	4	3	1.00	19	0.158
304	A	5	4	1.00	19	0.210
305	A	5	4	1.00	19	0.210
306	A	4	4	1.00	19	0.210
307	A	3	3	1.00	19	0.158
308	A	2	2	1.00	17	0.118
309	A	4	3	1.00	19	0.158
310	A	5	4	1.00	19	0.210
311	A	6	4	1.00	19	0.210
312	A	3	2	1.00	19	0.105
313	A	5	5	1.00	19	0.263
314	A	4	4	0.98	19	0.210
315	A	3	3	0.91	17	0.176
316	A	2	2	1.00	9	0.222
317	A	2	2	1.00	19	0.105
318	A	2	2	1.00	19	0.105
319	A	2	2	1.00	19	0.105
320	A	1	1	1.00	28	0.036
321	A	4	2	1.00	25	0.080
322	A	3	2	1.00	25	0.080
323	A	2	2	1.00	23	0.087
324	A	1	1	1.00	15	0.067
325	A	1	1	1.00	23	0.043
326	A	1	1	1.00	25	0.040
327	A	1	1	1.00	25	0.040
328	A	2	2	0.93	28	0.071
329	A	1	1	1.00	69	0.014
330	A	5	3	1.00	25	0.120
331	A	3	3	1.00	23	0.130
332	A	2	2	1.00	15	0.133
333	A	2	2	1.00	25	0.080
334	A	2	2	1.00	23	0.087
335	A	2	2	1.00	25	0.080
336	A	2	2	1.00	25	0.080
337	A	6	4	1.08	31	0.129

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
338	A	4	4	1.08	31	0.129
339	A	2	2	1.07	29	0.069
340	A	5	5	1.00	31	0.161
341	A	5	5	1.19	31	0.161
342	A	5	4	1.36	31	0.129
343	A	9	8	1.00	31	0.258
344	A	7	7	1.00	31	0.226
345	A	5	5	1.00	28	0.179
346	A	5	5	1.00	31	0.161
347	A	6	6	1.00	31	0.194
348	A	5	5	1.00	29	0.172
349	A	4	4	1.00	29	0.138
350	A	3	3	1.00	29	0.103
351	A	2	2	1.00	27	0.074
352	A	2	2	1.00	26	0.077
353	A	3	3	1.00	29	0.103
354	A	2	2	1.00	29	0.069
355	A	3	3	1.00	29	0.103
356	A	2	2	1.00	29	0.069
357	A	5	5	1.00	29	0.172
358	A	8	7	1.00	31	0.226
359	A	4	4	1.00	31	0.129
360	A	6	6	1.00	31	0.194
361	A	2	2	1.00	29	0.069
362	A	4	4	1.00	28	0.143
363	A	3	3	1.00	31	0.097
364	A	4	4	1.00	31	0.129
365	A	3	3	1.00	31	0.097
366	A	2	2	1.00	31	0.065
367	A	5	5	1.00	31	0.161
368	A	8	8	1.00	31	0.258
369	A	4	4	1.00	31	0.129
370	A	7	7	1.00	31	0.226
371	A	2	2	1.00	29	0.069
372	A	4	4	1.00	28	0.143
373	A	3	3	1.00	31	0.097

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	2	2	1.00	31	0.065
375	A	5	5	1.00	31	0.161
376	A	4	4	1.00	31	0.129
377	A	7	7	1.00	31	0.226
378	A	3	3	1.00	31	0.097
379	A	1	1	1.00	57	0.018
380	A	3	3	1.00	32	0.094
381	A	4	4	1.00	41	0.098
382	A	4	3	1.00	31	0.097
383	A	4	4	1.00	35	0.114
384	A	3	3	1.00	31	0.097
385	A	2	2	1.08	76	0.026



# Chapter 3

## Listing of integrals

### 3.1 $\int (a + bx^3)(c + dx^3)^4 dx$

**Optimal.** Leaf size=94

$$\frac{1}{4}c^3x^4(4ad+bc) + \frac{2}{7}c^2dx^7(3ad+2bc) + \frac{1}{13}d^3x^{13}(ad+4bc) + \frac{1}{5}cd^2x^{10}(2ad+3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

[Out] a\*c^4\*x+1/4\*c^3\*(4\*a\*d+b\*c)\*x^4+2/7\*c^2\*d\*(3\*a\*d+2\*b\*c)\*x^7+1/5\*c\*d^2\*(2\*a\*d+3\*b\*c)\*x^10+1/13\*d^3\*(a\*d+4\*b\*c)\*x^13+1/16\*b\*d^4\*x^16

**Rubi [A]** time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{2}{7}c^2dx^7(3ad+2bc) + \frac{1}{4}c^3x^4(4ad+bc) + \frac{1}{13}d^3x^{13}(ad+4bc) + \frac{1}{5}cd^2x^{10}(2ad+3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^4, x]

[Out] a\*c^4\*x + (c^3\*(b\*c + 4\*a\*d)\*x^4)/4 + (2\*c^2\*d\*(2\*b\*c + 3\*a\*d)\*x^7)/7 + (c\*d^2\*(3\*b\*c + 2\*a\*d)\*x^10)/5 + (d^3\*(4\*b\*c + a\*d)\*x^13)/13 + (b\*d^4\*x^16)/16

### Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)(c + dx^3)^4 dx &= \int (ac^4 + c^3(bc + 4ad)x^3 + 2c^2d(2bc + 3ad)x^6 + 2cd^2(3bc + 2ad)x^9 + d^3(4bc + ad)x^{12} + bd^4x^{15}) dx \\ &= ac^4x + \frac{1}{4}c^3(bc + 4ad)x^4 + \frac{2}{7}c^2d(2bc + 3ad)x^7 + \frac{1}{5}cd^2(3bc + 2ad)x^{10} + \frac{1}{13}d^3(4bc + ad)x^{13} + \frac{1}{16}bd^4x^{16} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 94, normalized size = 1.00

$$\frac{1}{4}c^3x^4(4ad+bc) + \frac{2}{7}c^2dx^7(3ad+2bc) + \frac{1}{13}d^3x^{13}(ad+4bc) + \frac{1}{5}cd^2x^{10}(2ad+3bc) + ac^4x + \frac{1}{16}bd^4x^{16}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^4,x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^4)/4 + (2*c^2*d*(2*b*c + 3*a*d)*x^7)/7 + (c*d^2*(3*b*c + 2*a*d)*x^{10})/5 + (d^3*(4*b*c + a*d)*x^{13})/13 + (b*d^4*x^{16})/16$

**fricas** [A] time = 0.36, size = 97, normalized size = 1.03

$$\frac{1}{16}x^{16}d^4b + \frac{4}{13}x^{13}d^3cb + \frac{1}{13}x^{13}d^4a + \frac{3}{5}x^{10}d^2c^2b + \frac{2}{5}x^{10}d^3ca + \frac{4}{7}x^7dc^3b + \frac{6}{7}x^7d^2c^2a + \frac{1}{4}x^4c^4b + x^4dc^3a + xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^4,x, algorithm="fricas")

[Out]  $1/16*x^{16}*d^4*b + 4/13*x^{13}*d^3*c*b + 1/13*x^{13}*d^4*a + 3/5*x^{10}*d^2*c^2*b + 2/5*x^{10}*d^3*c*a + 4/7*x^7*d*c^3*b + 6/7*x^7*d^2*c^2*a + 1/4*x^4*c^4*b + x^4*d*c^3*a + x*c^4*a$

**giac** [A] time = 0.17, size = 97, normalized size = 1.03

$$\frac{1}{16}bd^4x^{16} + \frac{4}{13}bcd^3x^{13} + \frac{1}{13}ad^4x^{13} + \frac{3}{5}bc^2d^2x^{10} + \frac{2}{5}acd^3x^{10} + \frac{4}{7}bc^3dx^7 + \frac{6}{7}ac^2d^2x^7 + \frac{1}{4}bc^4x^4 + ac^3dx^4 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^4,x, algorithm="giac")

[Out]  $1/16*b*d^4*x^{16} + 4/13*b*c*d^3*x^{13} + 1/13*a*d^4*x^{13} + 3/5*b*c^2*d^2*x^{10} + 2/5*a*c*d^3*x^{10} + 4/7*b*c^3*d*x^7 + 6/7*a*c^2*d^2*x^7 + 1/4*b*c^4*x^4 + a*c^3*d*x^4 + a*c^4*x$

**maple** [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{bd^4x^{16}}{16} + \frac{(ad^4 + 4bcd^3)x^{13}}{13} + \frac{(4acd^3 + 6c^2d^2b)x^{10}}{10} + \frac{(6ac^2d^2 + 4c^3db)x^7}{7} + ac^4x + \frac{(4ac^3d + bc^4)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^4,x)

[Out]  $1/16*b*d^4*x^{16} + 1/13*(a*d^4 + 4*b*c*d^3)*x^{13} + 1/10*(4*a*c*d^3 + 6*b*c^2*d^2)*x^{10} + 1/7*(6*a*c^2*d^2 + 4*b*c^3*d)*x^7 + 1/4*(4*a*c^3*d + b*c^4)*x^4 + a*c^4*x$

**maxima** [A] time = 0.47, size = 96, normalized size = 1.02

$$\frac{1}{16}bd^4x^{16} + \frac{1}{13}(4bcd^3 + ad^4)x^{13} + \frac{1}{5}(3bc^2d^2 + 2acd^3)x^{10} + \frac{2}{7}(2bc^3d + 3ac^2d^2)x^7 + ac^4x + \frac{1}{4}(bc^4 + 4ac^3d)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^4,x, algorithm="maxima")

[Out]  $1/16*b*d^4*x^{16} + 1/13*(4*b*c*d^3 + a*d^4)*x^{13} + 1/5*(3*b*c^2*d^2 + 2*a*c*d^3)*x^{10} + 2/7*(2*b*c^3*d + 3*a*c^2*d^2)*x^7 + a*c^4*x + 1/4*(b*c^4 + 4*a*c^3*d)*x^4$

**mupad** [B] time = 0.05, size = 87, normalized size = 0.93

$$x^4 \left( \frac{bc^4}{4} + adc^3 \right) + x^{13} \left( \frac{ad^4}{13} + \frac{4bcd^3}{13} \right) + \frac{bd^4x^{16}}{16} + ac^4x + \frac{2c^2dx^7(3ad + 2bc)}{7} + \frac{cd^2x^{10}(2ad + 3bc)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)\*(c + d\*x^3)^4,x)

```
[Out] x^4*((b*c^4)/4 + a*c^3*d) + x^13*((a*d^4)/13 + (4*b*c*d^3)/13) + (b*d^4*x^16)/16 + a*c^4*x + (2*c^2*d*x^7*(3*a*d + 2*b*c))/7 + (c*d^2*x^10*(2*a*d + 3*b*c))/5
```

```
sympy [A] time = 0.13, size = 104, normalized size = 1.11
```

$$ac^4x + \frac{bd^4x^{16}}{16} + x^{13} \left( \frac{ad^4}{13} + \frac{4bcd^3}{13} \right) + x^{10} \left( \frac{2acd^3}{5} + \frac{3bc^2d^2}{5} \right) + x^7 \left( \frac{6ac^2d^2}{7} + \frac{4bc^3d}{7} \right) + x^4 \left( ac^3d + \frac{bc^4}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)*(d*x**3+c)**4,x)
```

```
[Out] a*c**4*x + b*d**4*x**16/16 + x**13*(a*d**4/13 + 4*b*c*d**3/13) + x**10*(2*a*c*d**3/5 + 3*b*c**2*d**2/5) + x**7*(6*a*c**2*d**2/7 + 4*b*c**3*d/7) + x**4*(a*c**3*d + b*c**4/4)
```

## 3.2 $\int (a + bx^3)(c + dx^3)^3 dx$

**Optimal.** Leaf size=70

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

[Out]  $a*c^3*x + 1/4*c^2*(3*a*d+b*c)*x^4 + 3/7*c*d*(a*d+b*c)*x^7 + 1/10*d^2*(a*d+3*b*c)*x^{10} + 1/13*b*d^3*x^{13}$

**Rubi [A]** time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^3,x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b*c + a*d)*x^{10})/10 + (b*d^3*x^{13})/13$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)(c + dx^3)^3 dx &= \int (ac^3 + c^2(bc + 3ad)x^3 + 3cd(bc + ad)x^6 + d^2(3bc + ad)x^9 + bd^3x^{12}) dx \\ &= ac^3x + \frac{1}{4}c^2(bc + 3ad)x^4 + \frac{3}{7}cd(bc + ad)x^7 + \frac{1}{10}d^2(3bc + ad)x^{10} + \frac{1}{13}bd^3x^{13} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 70, normalized size = 1.00

$$\frac{1}{4}c^2x^4(3ad + bc) + \frac{1}{10}d^2x^{10}(ad + 3bc) + \frac{3}{7}cdx^7(ad + bc) + ac^3x + \frac{1}{13}bd^3x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^3,x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^4)/4 + (3*c*d*(b*c + a*d)*x^7)/7 + (d^2*(3*b*c + a*d)*x^{10})/10 + (b*d^3*x^{13})/13$

**fricas [A]** time = 0.36, size = 74, normalized size = 1.06

$$\frac{1}{13}x^{13}d^3b + \frac{3}{10}x^{10}d^2cb + \frac{1}{10}x^{10}d^3a + \frac{3}{7}x^7dc^2b + \frac{3}{7}x^7d^2ca + \frac{1}{4}x^4c^3b + \frac{3}{4}x^4dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^3,x, algorithm="fricas")

[Out]  $1/13*x^{13}*d^3*b + 3/10*x^{10}*d^2*c*b + 1/10*x^{10}*d^3*a + 3/7*x^7*d*c^2*b + 3/7*x^7*d^2*c*a + 1/4*x^4*c^3*b + 3/4*x^4*d*c^2*a + x*c^3*a$

**giac** [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{13}bd^3x^{13} + \frac{3}{10}bcd^2x^{10} + \frac{1}{10}ad^3x^{10} + \frac{3}{7}bc^2dx^7 + \frac{3}{7}acd^2x^7 + \frac{1}{4}bc^3x^4 + \frac{3}{4}ac^2dx^4 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^3,x, algorithm="giac")

[Out] 1/13\*b\*d^3\*x^13 + 3/10\*b\*c\*d^2\*x^10 + 1/10\*a\*d^3\*x^10 + 3/7\*b\*c^2\*d\*x^7 + 3/7\*a\*c\*d^2\*x^7 + 1/4\*b\*c^3\*x^4 + 3/4\*a\*c^2\*d\*x^4 + a\*c^3\*x

**maple** [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{bd^3x^{13}}{13} + \frac{(ad^3 + 3bcd^2)x^{10}}{10} + \frac{(3acd^2 + 3bc^2d)x^7}{7} + ac^3x + \frac{(3ac^2d + bc^3)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^3,x)

[Out] 1/13\*b\*d^3\*x^13+1/10\*(a\*d^3+3\*b\*c\*d^2)\*x^10+1/7\*(3\*a\*c\*d^2+3\*b\*c^2\*d)\*x^7+1/4\*(3\*a\*c^2\*d+b\*c^3)\*x^4+a\*c^3\*x

**maxima** [A] time = 0.56, size = 70, normalized size = 1.00

$$\frac{1}{13}bd^3x^{13} + \frac{1}{10}(3bcd^2 + ad^3)x^{10} + \frac{3}{7}(bc^2d + acd^2)x^7 + ac^3x + \frac{1}{4}(bc^3 + 3ac^2d)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^3,x, algorithm="maxima")

[Out] 1/13\*b\*d^3\*x^13 + 1/10\*(3\*b\*c\*d^2 + a\*d^3)\*x^10 + 3/7\*(b\*c^2\*d + a\*c\*d^2)\*x^7 + a\*c^3\*x + 1/4\*(b\*c^3 + 3\*a\*c^2\*d)\*x^4

**mupad** [B] time = 0.03, size = 66, normalized size = 0.94

$$x^4 \left( \frac{bc^3}{4} + \frac{3ad^2c}{4} \right) + x^{10} \left( \frac{ad^3}{10} + \frac{3bcd^2}{10} \right) + \frac{bd^3x^{13}}{13} + ac^3x + \frac{3cdx^7(ad+bc)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)\*(c + d\*x^3)^3,x)

[Out] x^4\*((b\*c^3)/4 + (3\*a\*c^2\*d)/4) + x^10\*((a\*d^3)/10 + (3\*b\*c\*d^2)/10) + (b\*d^3\*x^13)/13 + a\*c^3\*x + (3\*c\*d\*x^7\*(a\*d + b\*c))/7

**sympy** [A] time = 0.08, size = 80, normalized size = 1.14

$$ac^3x + \frac{bd^3x^{13}}{13} + x^{10} \left( \frac{ad^3}{10} + \frac{3bcd^2}{10} \right) + x^7 \left( \frac{3acd^2}{7} + \frac{3bc^2d}{7} \right) + x^4 \left( \frac{3ac^2d}{4} + \frac{bc^3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*3,x)

[Out] a\*c\*\*3\*x + b\*d\*\*3\*x\*\*13/13 + x\*\*10\*(a\*d\*\*3/10 + 3\*b\*c\*d\*\*2/10) + x\*\*7\*(3\*a\*c\*d\*\*2/7 + 3\*b\*c\*\*2\*d/7) + x\*\*4\*(3\*a\*c\*\*2\*d/4 + b\*c\*\*3/4)

### 3.3 $\int (a + bx^3)(c + dx^3)^2 dx$

**Optimal.** Leaf size=50

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

[Out]  $a*c^2*x + 1/4*c*(2*a*d + b*c)*x^4 + 1/7*d*(a*d + 2*b*c)*x^7 + 1/10*b*d^2*x^{10}$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^2,x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^{10})/10$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)(c + dx^3)^2 dx &= \int (ac^2 + c(bc + 2ad)x^3 + d(2bc + ad)x^6 + bd^2x^9) dx \\ &= ac^2x + \frac{1}{4}c(bc + 2ad)x^4 + \frac{1}{7}d(2bc + ad)x^7 + \frac{1}{10}bd^2x^{10} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{7}dx^7(ad + 2bc) + \frac{1}{4}cx^4(2ad + bc) + ac^2x + \frac{1}{10}bd^2x^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^2,x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^4)/4 + (d*(2*b*c + a*d)*x^7)/7 + (b*d^2*x^{10})/10$

**fricas [A]** time = 0.36, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}d^2b + \frac{2}{7}x^7dcb + \frac{1}{7}x^7d^2a + \frac{1}{4}x^4c^2b + \frac{1}{2}x^4dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out]  $1/10*x^{10}*d^2*b + 2/7*x^7*d*c*b + 1/7*x^7*d^2*a + 1/4*x^4*c^2*b + 1/2*x^4*d*c*a + x*c^2*a$

**giac** [A] time = 0.19, size = 50, normalized size = 1.00

$$\frac{1}{10}bd^2x^{10} + \frac{2}{7}bcdx^7 + \frac{1}{7}ad^2x^7 + \frac{1}{4}bc^2x^4 + \frac{1}{2}acdx^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] 1/10\*b\*d^2\*x^10 + 2/7\*b\*c\*d\*x^7 + 1/7\*a\*d^2\*x^7 + 1/4\*b\*c^2\*x^4 + 1/2\*a\*c\*d\*x^4 + a\*c^2\*x

**maple** [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{bd^2x^{10}}{10} + \frac{(ad^2 + 2bcd)x^7}{7} + ac^2x + \frac{(2acd + bc^2)x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^2,x)

[Out] 1/10\*b\*d^2\*x^10+1/7\*(a\*d^2+2\*b\*c\*d)\*x^7+1/4\*(2\*a\*c\*d+b\*c^2)\*x^4+a\*c^2\*x

**maxima** [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{10}bd^2x^{10} + \frac{1}{7}(2bcd + ad^2)x^7 + \frac{1}{4}(bc^2 + 2acd)x^4 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^2,x, algorithm="maxima")

[Out] 1/10\*b\*d^2\*x^10 + 1/7\*(2\*b\*c\*d + a\*d^2)\*x^7 + 1/4\*(b\*c^2 + 2\*a\*c\*d)\*x^4 + a\*c^2\*x

**mupad** [B] time = 0.05, size = 48, normalized size = 0.96

$$x^4 \left( \frac{bc^2}{4} + \frac{adc}{2} \right) + x^7 \left( \frac{ad^2}{7} + \frac{2bcd}{7} \right) + \frac{bd^2x^{10}}{10} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)\*(c + d\*x^3)^2,x)

[Out] x^4\*((b\*c^2)/4 + (a\*c\*d)/2) + x^7\*((a\*d^2)/7 + (2\*b\*c\*d)/7) + (b\*d^2\*x^10)/10 + a\*c^2\*x

**sympy** [A] time = 0.07, size = 51, normalized size = 1.02

$$ac^2x + \frac{bd^2x^{10}}{10} + x^7 \left( \frac{ad^2}{7} + \frac{2bcd}{7} \right) + x^4 \left( \frac{acd}{2} + \frac{bc^2}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c)\*\*2,x)

[Out] a\*c\*\*2\*x + b\*d\*\*2\*x\*\*10/10 + x\*\*7\*(a\*d\*\*2/7 + 2\*b\*c\*d/7) + x\*\*4\*(a\*c\*d/2 + b\*c\*\*2/4)

### 3.4 $\int (a + bx^3)(c + dx^3) dx$

**Optimal.** Leaf size=28

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

[Out] a\*c\*x+1/4\*(a\*d+b\*c)\*x^4+1/7\*b\*d\*x^7

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {373}

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3), x]

[Out] a\*c\*x + ((b\*c + a\*d)\*x^4)/4 + (b\*d\*x^7)/7

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)(c + dx^3) dx &= \int (ac + (bc + ad)x^3 + bdx^6) dx \\ &= acx + \frac{1}{4}(bc + ad)x^4 + \frac{1}{7}bdx^7 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{4}x^4(ad + bc) + acx + \frac{1}{7}bdx^7$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3), x]

[Out] a\*c\*x + ((b\*c + a\*d)\*x^4)/4 + (b\*d\*x^7)/7

**fricas [A]** time = 0.35, size = 26, normalized size = 0.93

$$\frac{1}{7}x^7db + \frac{1}{4}x^4cb + \frac{1}{4}x^4da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c), x, algorithm="fricas")

[Out] 1/7\*x^7\*d\*b + 1/4\*x^4\*c\*b + 1/4\*x^4\*d\*a + x\*c\*a

**giac [A]** time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{7}bdx^7 + \frac{1}{4}bcx^4 + \frac{1}{4}adx^4 + acx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c),x, algorithm="giac")

[Out] 1/7\*b\*d\*x^7 + 1/4\*b\*c\*x^4 + 1/4\*a\*d\*x^4 + a\*c\*x

maple [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bdx^7}{7} + \frac{(ad+bc)x^4}{4} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c),x)

[Out] a\*c\*x+1/4\*(a\*d+b\*c)\*x^4+1/7\*b\*d\*x^7

maxima [A] time = 0.54, size = 24, normalized size = 0.86

$$\frac{1}{7}bdx^7 + \frac{1}{4}(bc+ad)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c),x, algorithm="maxima")

[Out] 1/7\*b\*d\*x^7 + 1/4\*(b\*c + a\*d)\*x^4 + a\*c\*x

mupad [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bdx^7}{7} + \left(\frac{ad}{4} + \frac{bc}{4}\right)x^4 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)\*(c + d\*x^3),x)

[Out] x^4\*((a\*d)/4 + (b\*c)/4) + a\*c\*x + (b\*d\*x^7)/7

sympy [A] time = 0.07, size = 26, normalized size = 0.93

$$acx + \frac{bdx^7}{7} + x^4\left(\frac{ad}{4} + \frac{bc}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*(d\*x\*\*3+c),x)

[Out] a\*c\*x + b\*d\*x\*\*7/7 + x\*\*4\*(a\*d/4 + b\*c/4)

### 3.5 $\int \frac{a+bx^3}{c+dx^3} dx$

**Optimal.** Leaf size=144

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{bx}{d}$$

[Out] b\*x/d-1/3\*(-a\*d+b\*c)\*ln(c^(1/3)+d^(1/3)\*x)/c^(2/3)/d^(4/3)+1/6\*(-a\*d+b\*c)\*ln(c^(2/3)-c^(1/3)\*d^(1/3)\*x+d^(2/3)\*x^2)/c^(2/3)/d^(4/3)+1/3\*(-a\*d+b\*c)\*arctan(1/3\*(c^(1/3)-2\*d^(1/3)\*x)/c^(1/3)\*3^(1/2))/c^(2/3)/d^(4/3)\*3^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {388, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{6c^{2/3}d^{4/3}} - \frac{(bc-ad)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{3c^{2/3}d^{4/3}} + \frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3), x]

[Out] (b\*x)/d + ((b\*c - a\*d)\*ArcTan[(c^(1/3) - 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))])/(Sqrt[3]\*c^(2/3)\*d^(4/3)) - ((b\*c - a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(3\*c^(2/3)\*d^(4/3)) + ((b\*c - a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(6\*c^(2/3)\*d^(4/3))

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Simp}[\frac{d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]]}{b}, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 634

$\text{Int}[\frac{(d_.) + (e_.)*(x_.)}{(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2}, x\_Symbol] \ :> \ \text{Dist}[\frac{2*c*d - b*e}{2*c}, \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{NeQ}[2*c*d - b*e, 0] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0] \ \&\& \ \text{!NiceSqrtQ}[b^2 - 4*a*c]$

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^3}{c + dx^3} dx &= \frac{bx}{d} - \frac{(bc - ad) \int \frac{1}{c+dx^3} dx}{d} \\ &= \frac{bx}{d} - \frac{(bc - ad) \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{2/3}d} - \frac{(bc - ad) \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{2/3}d} \\ &= \frac{bx}{d} - \frac{(bc - ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{4/3}} + \frac{(bc - ad) \int \frac{-\sqrt[3]{c}\sqrt[3]{d} + 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{6c^{2/3}d^{4/3}} - \frac{(bc - ad) \int \frac{1}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{2\sqrt[3]{c}d} \\ &= \frac{bx}{d} - \frac{(bc - ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{4/3}} + \frac{(bc - ad) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{6c^{2/3}d^{4/3}} - \frac{(bc - ad) \text{Subst}\left(\frac{1}{2\sqrt[3]{c}d}, \frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt[3]{c}}\right)}{6c^{2/3}d^{4/3}} \\ &= \frac{bx}{d} + \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{4/3}} - \frac{(bc - ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{4/3}} + \frac{(bc - ad) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{6c^{2/3}d^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 128, normalized size = 0.89

$$\frac{(bc - ad) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2) - 2(bc - ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x) + 2\sqrt{3}(bc - ad) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}}\right) + 6bc^{2/3}}{6c^{2/3}d^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)/(c + d\*x^3), x]

[Out] (6\*b\*c^(2/3)\*d^(1/3)\*x + 2\*Sqrt[3]\*(b\*c - a\*d)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]] - 2\*(b\*c - a\*d)\*Log[c^(1/3) + d^(1/3)\*x] + (b\*c - a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2]/(6\*c^(2/3)\*d^(4/3))

**fricas** [A] time = 0.45, size = 369, normalized size = 2.56

$$\frac{6bc^2dx - 3\sqrt{\frac{1}{3}}(bc^2d - acd^2)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}} \log\left(\frac{2cdx^3 - 3(c^2d)^{\frac{1}{3}}cx - c^2 + 3\sqrt{\frac{1}{3}}\left(2cdx^2 + (c^2d)^{\frac{2}{3}}x - (c^2d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3 + c}\right) + (c^2d)^{\frac{2}{3}}(bc - ad)}{6c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c),x, algorithm="fricas")

[Out] [1/6\*(6\*b\*c^2\*d\*x - 3\*sqrt(1/3)\*(b\*c^2\*d - a\*c\*d^2)\*sqrt(-(c^2\*d)^(1/3)/d)\*log((2\*c\*d\*x^3 - 3\*(c^2\*d)^(1/3)\*c\*x - c^2 + 3\*sqrt(1/3)\*(2\*c\*d\*x^2 + (c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt(-(c^2\*d)^(1/3)/d))/(d\*x^3 + c)) + (c^2\*d)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) - 2\*(c^2\*d)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x + (c^2\*d)^(2/3)))/(c^2\*d^2), 1/6\*(6\*b\*c^2\*d\*x - 6\*sqrt(1/3)\*(b\*c^2\*d - a\*c\*d^2)\*sqrt((c^2\*d)^(1/3)/d)\*arctan(sqrt(1/3)\*(2\*(c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt((c^2\*d)^(1/3)/d)/c^2) + (c^2\*d)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) - 2\*(c^2\*d)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x + (c^2\*d)^(2/3)))/(c^2\*d^2)]

**giac** [A] time = 0.20, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right) + (bc - ad) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right) + \frac{bx}{d} + \frac{(bc - ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3cd}}{3(-cd^2)^{\frac{2}{3}} + 6(-cd^2)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c),x, algorithm="giac")

[Out] 1/3\*sqrt(3)\*(b\*c - a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(-c\*d^2)^(2/3) + 1/6\*(b\*c - a\*d)\*log(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/(-c\*d^2)^(2/3) + b\*x/d + 1/3\*(b\*c - a\*d)\*(-c/d)^(1/3)\*log(abs(x - (-c/d)^(1/3)))/(c\*d)

**maple** [A] time = 0.05, size = 195, normalized size = 1.35

$$\frac{\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right) + a \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) - a \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right) + \sqrt{3} bc \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right) + bc \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d + 3\left(\frac{c}{d}\right)^{\frac{2}{3}}d - 6\left(\frac{c}{d}\right)^{\frac{2}{3}}d + 3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2 + 3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)/(d\*x^3+c),x)

[Out] b\*x/d+1/3/d/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*a-1/3/d^2/(c/d)^(2/3)\*ln(x+(c/d)^(1/3))\*b\*c-1/6/d/(c/d)^(2/3)\*ln(x^2-(c/d)^(1/3)\*x+(c/d)^(2/3))\*a+1/6/d^2/(c/d)^(2/3)

$/d)^{2/3} * \ln(x^2 - (c/d)^{1/3} * x + (c/d)^{2/3}) * b * c + 1/3 * d / (c/d)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(c/d)^{1/3} * x - 1)) * a - 1/3 * d^2 / (c/d)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2/(c/d)^{1/3} * x - 1)) * b * c$

**maxima [A]** time = 1.28, size = 128, normalized size = 0.89

$$\frac{bx}{d} - \frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc - ad) \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc - ad) \log\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c), x, algorithm="maxima")

[Out]  $b*x/d - 1/3*\sqrt{3}*(b*c - a*d)*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{1/3})/(c/d)^{1/3})/(d^2*(c/d)^{2/3}) + 1/6*(b*c - a*d)*\log(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3})/(d^2*(c/d)^{2/3}) - 1/3*(b*c - a*d)*\log(x + (c/d)^{1/3})/(d^2*(c/d)^{2/3})$

**mupad [B]** time = 1.38, size = 123, normalized size = 0.85

$$\frac{bx}{d} + \frac{\ln(d^{1/3}x + c^{1/3})(ad - bc)}{3c^{2/3}d^{4/3}} - \frac{\ln(c^{1/3} - 2d^{1/3}x + \sqrt{3}c^{1/3}1i)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(ad - bc)}{3c^{2/3}d^{4/3}} + \frac{\ln(2d^{1/3}x - c^{1/3} + \sqrt{3}c^{1/3}1i)\left(\frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)(ad - bc)}{3c^{2/3}d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)/(c + d\*x^3), x)

[Out]  $(b*x)/d + (\log(d^{1/3}*x + c^{1/3})*(a*d - b*c))/(3*c^{2/3}*d^{4/3}) - (\log(3^{1/2}*c^{1/3}*1i - 2*d^{1/3}*x + c^{1/3})*((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c))/(3*c^{2/3}*d^{4/3}) + (\log(3^{1/2}*c^{1/3}*1i + 2*d^{1/3}*x - c^{1/3})*((3^{1/2}*1i)/2 - 1/2)*(a*d - b*c))/(3*c^{2/3}*d^{4/3})$

**sympy [A]** time = 0.42, size = 71, normalized size = 0.49

$$\frac{bx}{d} + \text{RootSum}\left(27t^3c^2d^4 - a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3, \left(t \mapsto t \log\left(\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c), x)

[Out]  $b*x/d + \text{RootSum}(27*_t**3*c**2*d**4 - a**3*d**3 + 3*a**2*b*c*d**2 - 3*a*b**2*c**2*d + b**3*c**3, \text{Lambda}(_t, _t*\log(3*_t*c*d/(a*d - b*c) + x)))$

$$3.6 \quad \int \frac{a+bx^3}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=169

$$\frac{(2ad+bc)\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2)}{18c^{5/3}d^{4/3}} + \frac{(2ad+bc)\log(\sqrt[3]{c}+\sqrt[3]{d}x)}{9c^{5/3}d^{4/3}} - \frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} - \frac{x(bc-ad)}{3cd(c+d)}$$

[Out]  $-1/3*(-a*d+b*c)*x/c/d/(d*x^3+c)+1/9*(2*a*d+b*c)*\ln(c^{(1/3)}+d^{(1/3)*x})/c^{(5/3)}/d^{(4/3)}-1/18*(2*a*d+b*c)*\ln(c^{(2/3)}-c^{(1/3)}*d^{(1/3)*x}+d^{(2/3)*x^2})/c^{(5/3)}/d^{(4/3)}-1/9*(2*a*d+b*c)*\arctan(1/3*(c^{(1/3)}-2*d^{(1/3)*x})/c^{(1/3)}*3^{(1/2)})/c^{(5/3)}/d^{(4/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {385, 200, 31, 634, 617, 204, 628}

$$\frac{(2ad+bc)\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2)}{18c^{5/3}d^{4/3}} + \frac{(2ad+bc)\log(\sqrt[3]{c}+\sqrt[3]{d}x)}{9c^{5/3}d^{4/3}} - \frac{(2ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} - \frac{x(bc-ad)}{3cd(c+d)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3)^2, x]

[Out]  $-((b*c - a*d)*x)/(3*c*d*(c + d*x^3)) - ((b*c + 2*a*d)*\text{ArcTan}[(c^{(1/3)} - 2*d^{(1/3)*x})/(\text{Sqrt}[3]*c^{(1/3)})])/(3*\text{Sqrt}[3]*c^{(5/3)}*d^{(4/3)}) + ((b*c + 2*a*d)*\text{Log}[c^{(1/3)} + d^{(1/3)*x}])/(9*c^{(5/3)}*d^{(4/3)}) - ((b*c + 2*a*d)*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)*x} + d^{(2/3)*x^2}])/(18*c^{(5/3)}*d^{(4/3)})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 617**

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^3}{(c + dx^3)^2} dx &= -\frac{(bc - ad)x}{3cd(c + dx^3)} + \frac{(bc + 2ad) \int \frac{1}{c + dx^3} dx}{3cd} \\ &= -\frac{(bc - ad)x}{3cd(c + dx^3)} + \frac{(bc + 2ad) \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{9c^{5/3}d} + \frac{(bc + 2ad) \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{9c^{5/3}d} \\ &= -\frac{(bc - ad)x}{3cd(c + dx^3)} + \frac{(bc + 2ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}d^{4/3}} - \frac{(bc + 2ad) \int \frac{-\sqrt[3]{c}\sqrt[3]{d} + 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{18c^{5/3}d^{4/3}} + \frac{(bc + 2ad) \int \frac{1}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{18c^{5/3}d^{4/3}} \\ &= -\frac{(bc - ad)x}{3cd(c + dx^3)} + \frac{(bc + 2ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}d^{4/3}} - \frac{(bc + 2ad) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{18c^{5/3}d^{4/3}} \\ &= -\frac{(bc - ad)x}{3cd(c + dx^3)} - \frac{(bc + 2ad) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{4/3}} + \frac{(bc + 2ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}d^{4/3}} - \frac{(bc + 2ad) \int \frac{1}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{18c^{5/3}d^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 145, normalized size = 0.86

$$\frac{-(2ad + bc) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2) - \frac{6c^{2/3}\sqrt[3]{d}x(bc - ad)}{c + dx^3} + 2(2ad + bc) \log(\sqrt[3]{c} + \sqrt[3]{d}x) - 2\sqrt{3}(2ad + bc) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{18c^{5/3}d^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^2, x]
```

```
[Out] ((-6*c^(2/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 2*a*d)*A
rcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 2*a*d)*Log[c^(1/3) +
d^(1/3)*x] - (b*c + 2*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/
(18*c^(5/3)*d^(4/3))
```

**fricas** [A] time = 0.45, size = 537, normalized size = 3.18

$$\left[ 3 \sqrt{\frac{1}{3}} (bc^3d + 2ac^2d^2 + (bc^2d^2 + 2acd^3)x^3) \sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}} \log \left( \frac{2cdx^3 - 3(c^2d)^{\frac{1}{3}}cx - c^2 + 3\sqrt{\frac{1}{3}} \left( 2cdx^2 + (c^2d)^{\frac{2}{3}}x - (c^2d)^{\frac{1}{3}}c \right) \sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3 + c} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] [1/18\*(3\*sqrt(1/3)\*(b\*c^3\*d + 2\*a\*c^2\*d^2 + (b\*c^2\*d^2 + 2\*a\*c\*d^3)\*x^3)\*sqrt(-(c^2\*d)^(1/3)/d)\*log((2\*c\*d\*x^3 - 3\*(c^2\*d)^(1/3)\*c\*x - c^2 + 3\*sqrt(1/3)\*(2\*c\*d\*x^2 + (c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt(-(c^2\*d)^(1/3)/d))/(d\*x^3 + c)) - ((b\*c\*d + 2\*a\*d^2)\*x^3 + b\*c^2 + 2\*a\*c\*d)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) + 2\*((b\*c\*d + 2\*a\*d^2)\*x^3 + b\*c^2 + 2\*a\*c\*d)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) - 6\*(b\*c^3\*d - a\*c^2\*d^2)\*x)/(c^3\*d^3\*x^3 + c^4\*d^2), 1/18\*(6\*sqrt(1/3)\*(b\*c^3\*d + 2\*a\*c^2\*d^2 + (b\*c^2\*d^2 + 2\*a\*c\*d^3)\*x^3)\*sqrt((c^2\*d)^(1/3)/d)\*arctan(sqrt(1/3)\*(2\*(c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt((c^2\*d)^(1/3)/d)/c^2) - ((b\*c\*d + 2\*a\*d^2)\*x^3 + b\*c^2 + 2\*a\*c\*d)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) + 2\*((b\*c\*d + 2\*a\*d^2)\*x^3 + b\*c^2 + 2\*a\*c\*d)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) - 6\*(b\*c^3\*d - a\*c^2\*d^2)\*x)/(c^3\*d^3\*x^3 + c^4\*d^2)]

**giac** [A] time = 0.19, size = 160, normalized size = 0.95

$$\frac{\sqrt{3}(bc + 2ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9(-cd^2)^{\frac{2}{3}}c} + \frac{(bc + 2ad) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18(-cd^2)^{\frac{2}{3}}c} + \frac{(bc + 2ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] -1/9\*sqrt(3)\*(b\*c + 2\*a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/(-c/d)^(1/3))/((-c\*d^2)^(2/3)\*c) - 1/18\*(b\*c + 2\*a\*d)\*log(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c\*d^2)^(2/3)\*c) - 1/9\*(b\*c + 2\*a\*d)\*(-c/d)^(1/3)\*log(abs(x - (-c/d)^(1/3)))/(c^2\*d) - 1/3\*(b\*c\*x - a\*d\*x)/((d\*x^3 + c)\*c\*d)

**maple** [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd} + \frac{2a \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd} - \frac{a \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd} + \frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2} + \frac{b \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((b\*x^3+a)/(d\*x^3+c)^2,x)

[Out]  $\frac{1}{3} \frac{(a*d-b*c)}{d*c*x/(d*x^3+c)+2/9/c/d/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*a+1/9/d^{2/3}*\ln(x+(c/d)^{1/3})*b-1/9/c/d/(c/d)^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*a-1/18/d^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*b+2/9/c/d/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a+1/9/d^{2/3}*(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*b}$

**maxima** [A] time = 1.16, size = 158, normalized size = 0.93

$$\frac{(bc-ad)x}{3(cd^2x^3+c^2d)} + \frac{\sqrt{3}(bc+2ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc+2ad)\log\left(x^2-x\left(\frac{c}{d}\right)^{\frac{1}{3}}+\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{18cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(bc+2ad)\log\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9cd^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out]  $-\frac{1}{3} \frac{(b*c - a*d)*x}{(c*d^2*x^3 + c^2*d)} + \frac{1}{9} \frac{\sqrt{3}*(b*c + 2*a*d)*\arctan\left(\frac{1}{3} \frac{\sqrt{3}*(2*x - (c/d)^{1/3})}{(c/d)^{1/3}}\right)}{(c*d^2*(c/d)^{2/3})} - \frac{1}{18} \frac{(b*c + 2*a*d)*\log\left(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3}\right)}{(c*d^2*(c/d)^{2/3})} + \frac{1}{9} \frac{(b*c + 2*a*d)*\log\left(x + (c/d)^{1/3}\right)}{(c*d^2*(c/d)^{2/3})}$

**mupad** [B] time = 1.40, size = 143, normalized size = 0.85

$$\frac{\ln\left(d^{1/3}x + c^{1/3}\right)(2ad + bc)}{9c^{5/3}d^{4/3}} - \frac{\ln\left(c^{1/3} - 2d^{1/3}x + \sqrt{3}c^{1/3}1i\right)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(2ad + bc)}{9c^{5/3}d^{4/3}} + \frac{\ln\left(2d^{1/3}x - c^{1/3} + \sqrt{3}c^{1/3}1i\right)\left(\frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)(2ad + bc)}{9c^{5/3}d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)/(c + d\*x^3)^2,x)

[Out]  $\frac{(\log(d^{1/3}*x + c^{1/3})*(2*a*d + b*c))}{(9*c^{5/3}*d^{4/3})} - \frac{(\log(3^{1/2}*c^{1/3}*1i - 2*d^{1/3}*x + c^{1/3}))*((3^{1/2}*1i)/2 + 1/2)*(2*a*d + b*c)}{(9*c^{5/3}*d^{4/3})} + \frac{(\log(3^{1/2}*c^{1/3}*1i + 2*d^{1/3}*x - c^{1/3}))*((3^{1/2}*1i)/2 - 1/2)*(2*a*d + b*c)}{(9*c^{5/3}*d^{4/3})} + \frac{(x*(a*d - b*c))}{(3*c*d*(c + d*x^3))}$

**sympy** [A] time = 0.58, size = 97, normalized size = 0.57

$$\frac{x(ad - bc)}{3c^2d + 3cd^2x^3} + \text{RootSum}\left(729t^3c^5d^4 - 8a^3d^3 - 12a^2bcd^2 - 6ab^2c^2d - b^3c^3, \left(t \mapsto t \log\left(\frac{9tc^2d}{2ad + bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*2,x)

[Out]  $x*(a*d - b*c)/(3*c**2*d + 3*c*d**2*x**3) + \text{RootSum}(729*_t**3*c**5*d**4 - 8*a**3*d**3 - 12*a**2*b*c*d**2 - 6*a*b**2*c**2*d - b**3*c**3, \text{Lambda}(_t, _t*\log(9*_t*c**2*d/(2*a*d + b*c) + x)))$

$$3.7 \quad \int \frac{a+bx^3}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=197

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d(c+dx^3)}$$

[Out]  $-1/6*(-a*d+b*c)*x/c/d/(d*x^3+c)^2+1/18*(5*a*d+b*c)*x/c^2/d/(d*x^3+c)+1/27*(5*a*d+b*c)*\ln(c^{1/3}+d^{1/3}*x)/c^{8/3}/d^{4/3}-1/54*(5*a*d+b*c)*\ln(c^{2/3}-c^{1/3}*d^{1/3}*x+d^{2/3}*x^2)/c^{8/3}/d^{4/3}-1/27*(5*a*d+b*c)*\arctan(1/3*(c^{1/3}-2*d^{1/3}*x)/c^{1/3}*3^{1/2})/c^{8/3}/d^{4/3}*3^{1/2}$

**Rubi [A]** time = 0.11, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {385, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{(5ad+bc)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)}{54c^{8/3}d^{4/3}}+\frac{(5ad+bc)\log\left(\sqrt[3]{c}+\sqrt[3]{d}x\right)}{27c^{8/3}d^{4/3}}-\frac{(5ad+bc)\tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}}+\frac{x(5ad+bc)}{18c^2d(c+dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)/(c + d\*x^3)^3, x]

[Out]  $-\frac{((b*c - a*d)*x)/(6*c*d*(c + d*x^3)^2) + ((b*c + 5*a*d)*x)/(18*c^2*d*(c + d*x^3)) - ((b*c + 5*a*d)*\text{ArcTan}[(c^{1/3} - 2*d^{1/3}*x)/(\text{Sqrt}[3]*c^{1/3})])}{(9*\text{Sqrt}[3]*c^{8/3}*d^{4/3})} + \frac{((b*c + 5*a*d)*\text{Log}[c^{1/3} + d^{1/3}*x])}{(27*c^{8/3}*d^{4/3})} - \frac{((b*c + 5*a*d)*\text{Log}[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])}{(54*c^{8/3}*d^{4/3})}$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^3}{(c + dx^3)^3} dx &= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad) \int \frac{1}{(c + dx^3)^2} dx}{6cd} \\
&= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad)x}{18c^2d(c + dx^3)} + \frac{(bc + 5ad) \int \frac{1}{c + dx^3} dx}{9c^2d} \\
&= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad)x}{18c^2d(c + dx^3)} + \frac{(bc + 5ad) \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{27c^{8/3}d} + \frac{(bc + 5ad) \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d}}{27c^{8/3}d} \\
&= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad)x}{18c^2d(c + dx^3)} + \frac{(bc + 5ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{27c^{8/3}d^{4/3}} - \frac{(bc + 5ad) \int \frac{-\sqrt[3]{c}\sqrt[3]{d}}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d}}{54c^{8/3}d^{4/3}} \\
&= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad)x}{18c^2d(c + dx^3)} + \frac{(bc + 5ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{27c^{8/3}d^{4/3}} - \frac{(bc + 5ad) \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d)}{54c^{8/3}d^{4/3}} \\
&= -\frac{(bc - ad)x}{6cd(c + dx^3)^2} + \frac{(bc + 5ad)x}{18c^2d(c + dx^3)} - \frac{(bc + 5ad) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{4/3}} + \frac{(bc + 5ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{27c^{8/3}d^{4/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 175, normalized size = 0.89

$$-(5ad + bc) \log\left(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2\right) - \frac{9c^{5/3} \sqrt[3]{d} x(bc-ad)}{(c+dx^3)^2} + \frac{3c^{2/3} \sqrt[3]{d} x(5ad+bc)}{c+dx^3} + 2(5ad + bc) \log\left(\sqrt[3]{c} + \sqrt[3]{d} x\right) - 2\sqrt[3]{c} \sqrt[3]{d} x$$


---


$$54c^{8/3}d^{4/3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^3)/(c + d*x^3)^3,x]
```

```
[Out] ((-9*c^(5/3)*d^(1/3)*(b*c - a*d)*x)/(c + d*x^3)^2 + (3*c^(2/3)*d^(1/3)*(b*c + 5*a*d)*x)/(c + d*x^3) - 2*Sqrt[3]*(b*c + 5*a*d)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]] + 2*(b*c + 5*a*d)*Log[c^(1/3) + d^(1/3)*x] - (b*c + 5*a*d)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2]/(54*c^(8/3)*d^(4/3))
```

**fricas [B]** time = 0.46, size = 743, normalized size = 3.77

$$3(bc^3d^2 + 5ac^2d^3)x^4 + 3\sqrt{\frac{1}{3}}\left((bc^2d^3 + 5acd^4)x^6 + bc^4d + 5ac^3d^2 + 2(bc^3d^2 + 5ac^2d^3)x^3\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}} \log\left(\frac{2cdx^3}{\dots}\right)$$


---

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] [1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 3*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt(-(c^2*d)^(1/3)/d)*log((2*c*d*x^3 - 3*(c^2*d)^(1/3)*c*x - c^2 + 3*sqrt(1/3)*(2*c*d*x^2 + (c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt(-(c^2*d)^(1/3)/d))/(d*x^3 + c)) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x/(c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2), 1/54*(3*(b*c^3*d^2 + 5*a*c^2*d^3)*x^4 + 6*sqrt(1/3)*((b*c^2*d^3 + 5*a*c*d^4)*x^6 + b*c^4*d + 5*a*c^3*d^2 + 2*(b*c^3*d^2 + 5*a*c^2*d^3)*x^3)*sqrt((c^2*d)^(1/3)/d)*arctan(sqrt(1/3)*(2*(c^2*d)^(2/3)*x - (c^2*d)^(1/3)*c)*sqrt((c^2*d)^(1/3)/d)/c^2) - ((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x^2 - (c^2*d)^(2/3)*x + (c^2*d)^(1/3)*c) + 2*((b*c*d^2 + 5*a*d^3)*x^6 + b*c^3 + 5*a*c^2*d + 2*(b*c^2*d + 5*a*c*d^2)*x^3)*(c^2*d)^(2/3)*log(c*d*x + (c^2*d)^(2/3)) - 6*(b*c^4*d - 4*a*c^3*d^2)*x/(c^4*d^4*x^6 + 2*c^5*d^3*x^3 + c^6*d^2)]
```

**giac [A]** time = 0.19, size = 180, normalized size = 0.91

$$\frac{\sqrt{3}(bc + 5ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27(-cd^2)^{\frac{2}{3}}c^2} - \frac{(bc + 5ad) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54(-cd^2)^{\frac{2}{3}}c^2} + \frac{(bc + 5ad)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27c^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c)^3,x, algorithm="giac")

[Out]  $-\frac{1}{27}\sqrt{3}(b*c + 5*a*d)*\arctan\left(\frac{1}{3}\sqrt{3}(2*x + (-c/d)^{1/3})/(-c/d)^{1/3}\right)/((-c*d^2)^{2/3}*c^2) - \frac{1}{54}(b*c + 5*a*d)*\log(x^2 + x*(-c/d)^{1/3} + (-c/d)^{2/3})/((-c*d^2)^{2/3}*c^2) - \frac{1}{27}(b*c + 5*a*d)*(-c/d)^{1/3}*\log(a$   
 $bs(x - (-c/d)^{1/3}))/c^3*d + \frac{1}{18}(b*c*d*x^4 + 5*a*d^2*x^4 - 2*b*c^2*x + 8*a*c*d*x)/((d*x^3 + c)^2*c^2*d)$

**maple [A]** time = 0.06, size = 249, normalized size = 1.26

$$\frac{5\sqrt{3} a \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^2d} + \frac{5a \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^2d} - \frac{5a \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54\left(\frac{c}{d}\right)^{\frac{2}{3}}c^2d} + \frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{27\left(\frac{c}{d}\right)^{\frac{2}{3}}c^2d} + \frac{b \ln}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)/(d\*x^3+c)^3,x)

[Out]  $\frac{1}{18}(5*a*d+b*c)/c^2*x^4 + \frac{1}{9}(4*a*d-b*c)/c/d*x)/(d*x^3+c)^2 + \frac{5}{27}c^2/d/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*a + \frac{1}{27}c/d^2/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*b - \frac{5}{54}c^2/d/(c/d)^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*a - \frac{1}{54}c/d^2/(c/d)^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*b + \frac{5}{27}c^2/d/(c/d)^{2/3}*3^{1/2}*\arctan\left(\frac{1}{3}3^{1/2}*(2/(c/d)^{1/3}*x-1)\right)*a + \frac{1}{27}c/d^2/(c/d)^{2/3}*3^{1/2}*\arctan\left(\frac{1}{3}3^{1/2}*(2/(c/d)^{1/3}*x-1)\right)*b$

**maxima [A]** time = 1.43, size = 192, normalized size = 0.97

$$\frac{(bcd + 5ad^2)x^4 - 2(bc^2 - 4acd)x}{18(c^2d^3x^6 + 2c^3d^2x^3 + c^4d)} + \frac{\sqrt{3}(bc + 5ad) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(bc + 5ad) \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54c^2d^2\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{18}((b*c*d + 5*a*d^2)*x^4 - 2*(b*c^2 - 4*a*c*d)*x)/(c^2*d^3*x^6 + 2*c^3*d^2*x^3 + c^4*d) + \frac{1}{27}\sqrt{3}(b*c + 5*a*d)*\arctan\left(\frac{1}{3}\sqrt{3}(2*x - (c/d)^{1/3})/(c/d)^{1/3}\right)/(c^2*d^2*(c/d)^{2/3}) - \frac{1}{54}(b*c + 5*a*d)*\log(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3})/(c^2*d^2*(c/d)^{2/3}) + \frac{1}{27}(b*c + 5*a*d)*\log(x + (c/d)^{1/3})/(c^2*d^2*(c/d)^{2/3})$

**mupad [B]** time = 1.40, size = 173, normalized size = 0.88

$$\frac{\frac{x^4(5ad+bc)}{18c^2} + \frac{x(4ad-bc)}{9cd}}{c^2 + 2cdx^3 + d^2x^6} + \frac{\ln(d^{1/3}x + c^{1/3})(5ad + bc)}{27c^{8/3}d^{4/3}} - \frac{\ln(c^{1/3} - 2d^{1/3}x + \sqrt{3}c^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(5ad + bc)}{27c^{8/3}d^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)/(c + d\*x^3)^3,x)

[Out]  $\frac{(x^4*(5*a*d + b*c))/(18*c^2) + (x*(4*a*d - b*c))/(9*c*d)}{(c^2 + d^2*x^6 + 2*c*d*x^3) + (\log(d^{1/3}*x + c^{1/3})*(5*a*d + b*c))/(27*c^{8/3}*d^{4/3})} - (\log(3^{1/2}*c^{1/3}*i - 2*d^{1/3}*x + c^{1/3})*((3^{1/2}*i)/2 + 1/2)*$

$$\frac{(5ad + bc)}{(27c^{8/3}d^{4/3})} + (\log(3^{1/2})c^{1/3}1i + 2d^{1/3}x - c^{1/3}) \cdot \left( \frac{3^{1/2}1i}{2} - \frac{1}{2} \right) \cdot \frac{(5ad + bc)}{(27c^{8/3}d^{4/3})}$$

**sympy [A]** time = 0.78, size = 133, normalized size = 0.68

$$\frac{x^4(5ad^2 + bcd) + x(8acd - 2bc^2)}{18c^4d + 36c^3d^2x^3 + 18c^2d^3x^6} + \text{RootSum}\left(19683t^3c^8d^4 - 125a^3d^3 - 75a^2bcd^2 - 15ab^2c^2d - b^3c^3, \left(t \mapsto t \log\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*3,x)

[Out] (x\*\*4\*(5\*a\*d\*\*2 + b\*c\*d) + x\*(8\*a\*c\*d - 2\*b\*c\*\*2))/(18\*c\*\*4\*d + 36\*c\*\*3\*d\*\*2\*x\*\*3 + 18\*c\*\*2\*d\*\*3\*x\*\*6) + RootSum(19683\*\_t\*\*3\*c\*\*8\*d\*\*4 - 125\*a\*\*3\*d\*\*3 - 75\*a\*\*2\*b\*c\*d\*\*2 - 15\*a\*b\*\*2\*c\*\*2\*d - b\*\*3\*c\*\*3, Lambda(\_t, \_t\*log(27\*\_t\*c\*\*3\*d/(5\*a\*d + b\*c) + x)))

### 3.8 $\int (a + bx^3)^2 (c + dx^3)^3 dx$

**Optimal.** Leaf size=122

$$\frac{1}{10}dx^{10}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{7}cx^7(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{4}ac^2x^4(3ad+2bc) + \frac{1}{13}bd^2x^{13}(2ad+3bc)$$

[Out]  $a^2c^3x + \frac{1}{4}ac^2(3ad+2bc)x^4 + \frac{1}{7}c(3a^2d^2+6abcd+b^2c^2)x^7 + \frac{1}{10}d(a^2d^2+6abcd+3b^2c^2)x^{10} + \frac{1}{13}bd^2(2ad+3bc)x^{13} + \frac{1}{16}b^2d^3x^{16}$

**Rubi [A]** time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{1}{10}dx^{10}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{7}cx^7(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{4}ac^2x^4(3ad+2bc) + \frac{1}{13}bd^2x^{13}(2ad+3bc)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3)^3,x]

[Out]  $a^2c^3x + (ac^2(2bc + 3ad)x^4)/4 + (c(b^2c^2 + 6abcd + 3a^2d^2)x^7)/7 + (d(3b^2c^2 + 6abcd + a^2d^2)x^{10})/10 + (bd^2(3bc + 2ad)x^{13})/13 + (b^2d^3x^{16})/16$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)^2 (c + dx^3)^3 dx &= \int (a^2c^3 + ac^2(2bc + 3ad)x^3 + c(b^2c^2 + 6abcd + 3a^2d^2)x^6 + d(3b^2c^2 + 6abcd + 3a^2d^2)x^9 + d^2(3bc + 2ad)x^{12} + d^3x^{15}) dx \\ &= a^2c^3x + \frac{1}{4}ac^2(2bc + 3ad)x^4 + \frac{1}{7}c(b^2c^2 + 6abcd + 3a^2d^2)x^7 + \frac{1}{10}d(3b^2c^2 + 6abcd + 3a^2d^2)x^{10} + \frac{1}{13}d^2(3bc + 2ad)x^{13} + \frac{1}{16}d^3x^{16} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{10}dx^{10}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{7}cx^7(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{4}ac^2x^4(3ad+2bc) + \frac{1}{13}bd^2x^{13}(2ad+3bc)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3)^3,x]

[Out]  $a^2c^3x + (ac^2(2bc + 3ad)x^4)/4 + (c(b^2c^2 + 6abcd + 3a^2d^2)x^7)/7 + (d(3b^2c^2 + 6abcd + a^2d^2)x^{10})/10 + (bd^2(3bc + 2ad)x^{13})/13 + (b^2d^3x^{16})/16$

**fricas [A]** time = 0.38, size = 132, normalized size = 1.08

$$\frac{1}{16}x^{16}d^3b^2 + \frac{3}{13}x^{13}d^2cb^2 + \frac{2}{13}x^{13}d^3ba + \frac{3}{10}x^{10}dc^2b^2 + \frac{3}{5}x^{10}d^2cba + \frac{1}{10}x^{10}d^3a^2 + \frac{1}{7}x^7c^3b^2 + \frac{6}{7}x^7dc^2ba + \frac{3}{7}x^7d^2ca^2 + \frac{1}{2}x^4ac^2(2bc + 3ad) + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^3,x, algorithm="fricas")

[Out]  $1/16*x^{16}*d^3*b^2 + 3/13*x^{13}*d^2*c*b^2 + 2/13*x^{13}*d^3*b*a + 3/10*x^{10}*d*c^2*b^2 + 3/5*x^{10}*d^2*c*b*a + 1/10*x^{10}*d^3*a^2 + 1/7*x^7*c^3*b^2 + 6/7*x^7*d*c^2*b*a + 3/7*x^7*d^2*c*a^2 + 1/2*x^4*c^3*b*a + 3/4*x^4*d*c^2*a^2 + x*c^3*a^2$

**giac** [A] time = 0.16, size = 132, normalized size = 1.08

$$\frac{1}{16} b^2 d^3 x^{16} + \frac{3}{13} b^2 c d^2 x^{13} + \frac{2}{13} a b d^3 x^{13} + \frac{3}{10} b^2 c^2 d x^{10} + \frac{3}{5} a b c d^2 x^{10} + \frac{1}{10} a^2 d^3 x^{10} + \frac{1}{7} b^2 c^3 x^7 + \frac{6}{7} a b c^2 d x^7 + \frac{3}{7} a^2 c d^2 x^7 + \frac{1}{2} x^4 c^3 b a + \frac{3}{4} x^4 d c^2 a^2 + x c^3 a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^3,x, algorithm="giac")

[Out]  $1/16*b^2*d^3*x^{16} + 3/13*b^2*c*d^2*x^{13} + 2/13*a*b*d^3*x^{13} + 3/10*b^2*c^2*d*x^{10} + 3/5*a*b*c*d^2*x^{10} + 1/10*a^2*d^3*x^{10} + 1/7*b^2*c^3*x^7 + 6/7*a*b*c^2*d*x^7 + 3/7*a^2*c*d^2*x^7 + 1/2*a*b*c^3*x^4 + 3/4*a^2*c^2*d*x^4 + a^2*c^3*x$

**maple** [A] time = 0.05, size = 125, normalized size = 1.02

$$\frac{b^2 d^3 x^{16}}{16} + \frac{(2 a b d^3 + 3 b^2 c d^2) x^{13}}{13} + \frac{(a^2 d^3 + 6 a b c d^2 + 3 b^2 c^2 d) x^{10}}{10} + \frac{(3 a^2 c d^2 + 6 a b c^2 d + b^2 c^3) x^7}{7} + a^2 c^3 x + \frac{(3 a^2 c^2 d + 6 a b c^2 d + b^2 c^3) x^7}{7} + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c)^3,x)

[Out]  $1/16*b^2*d^3*x^{16} + 1/13*(2*a*b*d^3 + 3*b^2*c*d^2)*x^{13} + 1/10*(a^2*d^3 + 6*a*b*c*d^2 + 3*b^2*c^2*d)*x^{10} + 1/7*(3*a^2*c*d^2 + 6*a*b*c^2*d + b^2*c^3)*x^7 + 1/4*(3*a^2*c^2*d + 2*a*b*c^3)*x^4 + a^2*c^3*x$

**maxima** [A] time = 0.55, size = 124, normalized size = 1.02

$$\frac{1}{16} b^2 d^3 x^{16} + \frac{1}{13} (3 b^2 c d^2 + 2 a b d^3) x^{13} + \frac{1}{10} (3 b^2 c^2 d + 6 a b c d^2 + a^2 d^3) x^{10} + \frac{1}{7} (b^2 c^3 + 6 a b c^2 d + 3 a^2 c d^2) x^7 + a^2 c^3 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^3,x, algorithm="maxima")

[Out]  $1/16*b^2*d^3*x^{16} + 1/13*(3*b^2*c*d^2 + 2*a*b*d^3)*x^{13} + 1/10*(3*b^2*c^2*d + 6*a*b*c*d^2 + a^2*d^3)*x^{10} + 1/7*(b^2*c^3 + 6*a*b*c^2*d + 3*a^2*c*d^2)*x^7 + a^2*c^3*x + 1/4*(2*a*b*c^3 + 3*a^2*c^2*d)*x^4$

**mupad** [B] time = 1.20, size = 116, normalized size = 0.95

$$x^7 \left( \frac{3 a^2 c d^2}{7} + \frac{6 a b c^2 d}{7} + \frac{b^2 c^3}{7} \right) + x^{10} \left( \frac{a^2 d^3}{10} + \frac{3 a b c d^2}{5} + \frac{3 b^2 c^2 d}{10} \right) + a^2 c^3 x + \frac{b^2 d^3 x^{16}}{16} + \frac{a c^2 x^4 (3 a d + 2 b c)}{4} + \frac{b^2 d^3 x^{16}}{16} + \frac{a c^2 x^4 (3 a d + 2 b c)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^2\*(c + d\*x^3)^3,x)

[Out]  $x^7*((b^2*c^3)/7 + (3*a^2*c*d^2)/7 + (6*a*b*c^2*d)/7) + x^{10}*((a^2*d^3)/10 + (3*b^2*c^2*d)/10 + (3*a*b*c*d^2)/5) + a^2*c^3*x + (b^2*d^3*x^{16})/16 + (a*c^2*x^4*(3*a*d + 2*b*c))/4 + (b*d^2*x^{13}*(2*a*d + 3*b*c))/13$

**sympy** [A] time = 0.09, size = 139, normalized size = 1.14

$$a^2 c^3 x + \frac{b^2 d^3 x^{16}}{16} + x^{13} \left( \frac{2 a b d^3}{13} + \frac{3 b^2 c d^2}{13} \right) + x^{10} \left( \frac{a^2 d^3}{10} + \frac{3 a b c d^2}{5} + \frac{3 b^2 c^2 d}{10} \right) + x^7 \left( \frac{3 a^2 c d^2}{7} + \frac{6 a b c^2 d}{7} + \frac{b^2 c^3}{7} \right) + x^4 \left( \frac{3 a^2 c^2 d}{4} + \frac{2 a b c^3}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((b*x**3+a)**2*(d*x**3+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**16/16 + x**13*(2*a*b*d**3/13 + 3*b**2*c*d**2/13)
+ x**10*(a**2*d**3/10 + 3*a*b*c*d**2/5 + 3*b**2*c**2*d/10) + x**7*(3*a**2*
c*d**2/7 + 6*a*b*c**2*d/7 + b**2*c**3/7) + x**4*(3*a**2*c**2*d/4 + a*b*c**3
/2)
```

### 3.9 $\int (a + bx^3)^2 (c + dx^3)^2 dx$

**Optimal.** Leaf size=82

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

[Out]  $a^2c^2x + \frac{1}{2}acx^4(ad + bc) + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + \frac{1}{13}b^2d^2x^{13}$

**Rubi [A]** time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3)^2,x]

[Out]  $a^2c^2x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b*c + a*d)*x^{10})/5 + (b^2*d^2*x^{13})/13$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)^2 (c + dx^3)^2 dx &= \int (a^2c^2 + 2ac(bc + ad)x^3 + (b^2c^2 + 4abcd + a^2d^2)x^6 + 2bd(bc + ad)x^9 + b^2d^2x^{12}) \\ &= a^2c^2x + \frac{1}{2}ac(bc + ad)x^4 + \frac{1}{7}(b^2c^2 + 4abcd + a^2d^2)x^7 + \frac{1}{5}bd(bc + ad)x^{10} + \frac{1}{13}b^2d^2x^{13} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 82, normalized size = 1.00

$$\frac{1}{7}x^7(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{1}{5}bdx^{10}(ad + bc) + \frac{1}{2}acx^4(ad + bc) + \frac{1}{13}b^2d^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3)^2,x]

[Out]  $a^2c^2x + (a*c*(b*c + a*d)*x^4)/2 + ((b^2*c^2 + 4*a*b*c*d + a^2*d^2)*x^7)/7 + (b*d*(b*c + a*d)*x^{10})/5 + (b^2*d^2*x^{13})/13$

**fricas [A]** time = 0.37, size = 91, normalized size = 1.11

$$\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out]  $\frac{1}{13}x^{13}d^2b^2 + \frac{1}{5}x^{10}dcb^2 + \frac{1}{5}x^{10}d^2ba + \frac{1}{7}x^7c^2b^2 + \frac{4}{7}x^7dcba + \frac{1}{7}x^7d^2a^2 + \frac{1}{2}x^4c^2ba + \frac{1}{2}x^4dca^2 + xc^2a^2$

**giac** [A] time = 0.21, size = 91, normalized size = 1.11

$$\frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} b^2 c d x^{10} + \frac{1}{5} a b d^2 x^{10} + \frac{1}{7} b^2 c^2 x^7 + \frac{4}{7} a b c d x^7 + \frac{1}{7} a^2 d^2 x^7 + \frac{1}{2} a b c^2 x^4 + \frac{1}{2} a^2 c d x^4 + a^2 c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] 1/13\*b^2\*d^2\*x^13 + 1/5\*b^2\*c\*d\*x^10 + 1/5\*a\*b\*d^2\*x^10 + 1/7\*b^2\*c^2\*x^7 + 4/7\*a\*b\*c\*d\*x^7 + 1/7\*a^2\*d^2\*x^7 + 1/2\*a\*b\*c^2\*x^4 + 1/2\*a^2\*c\*d\*x^4 + a^2\*c^2\*x

**maple** [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2 d^2 x^{13}}{13} + \frac{(2 a b d^2 + 2 b^2 c d) x^{10}}{10} + \frac{(a^2 d^2 + 4 a b c d + b^2 c^2) x^7}{7} + a^2 c^2 x + \frac{(2 a^2 c d + 2 a b c^2) x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c)^2,x)

[Out] 1/13\*b^2\*d^2\*x^13+1/10\*(2\*a\*b\*d^2+2\*b^2\*c\*d)\*x^10+1/7\*(a^2\*d^2+4\*a\*b\*c\*d+b^2\*c^2)\*x^7+1/4\*(2\*a^2\*c\*d+2\*a\*b\*c^2)\*x^4+a^2\*c^2\*x

**maxima** [A] time = 0.71, size = 82, normalized size = 1.00

$$\frac{1}{13} b^2 d^2 x^{13} + \frac{1}{5} (b^2 c d + a b d^2) x^{10} + \frac{1}{7} (b^2 c^2 + 4 a b c d + a^2 d^2) x^7 + a^2 c^2 x + \frac{1}{2} (a b c^2 + a^2 c d) x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^2,x, algorithm="maxima")

[Out] 1/13\*b^2\*d^2\*x^13 + 1/5\*(b^2\*c\*d + a\*b\*d^2)\*x^10 + 1/7\*(b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*x^7 + a^2\*c^2\*x + 1/2\*(a\*b\*c^2 + a^2\*c\*d)\*x^4

**mupad** [B] time = 0.04, size = 75, normalized size = 0.91

$$x^7 \left( \frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7} \right) + a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + \frac{a c x^4 (a d + b c)}{2} + \frac{b d x^{10} (a d + b c)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^2\*(c + d\*x^3)^2,x)

[Out] x^7\*((a^2\*d^2)/7 + (b^2\*c^2)/7 + (4\*a\*b\*c\*d)/7) + a^2\*c^2\*x + (b^2\*d^2\*x^13)/13 + (a\*c\*x^4\*(a\*d + b\*c))/2 + (b\*d\*x^10\*(a\*d + b\*c))/5

**sympy** [A] time = 0.08, size = 90, normalized size = 1.10

$$a^2 c^2 x + \frac{b^2 d^2 x^{13}}{13} + x^{10} \left( \frac{a b d^2}{5} + \frac{b^2 c d}{5} \right) + x^7 \left( \frac{a^2 d^2}{7} + \frac{4 a b c d}{7} + \frac{b^2 c^2}{7} \right) + x^4 \left( \frac{a^2 c d}{2} + \frac{a b c^2}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c)\*\*2,x)

[Out] a\*\*2\*c\*\*2\*x + b\*\*2\*d\*\*2\*x\*\*13/13 + x\*\*10\*(a\*b\*d\*\*2/5 + b\*\*2\*c\*d/5) + x\*\*7\*(a\*\*2\*d\*\*2/7 + 4\*a\*b\*c\*d/7 + b\*\*2\*c\*\*2/7) + x\*\*4\*(a\*\*2\*c\*d/2 + a\*b\*c\*\*2/2)

### 3.10 $\int (a + bx^3)^2 (c + dx^3) dx$

**Optimal.** Leaf size=50

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

[Out]  $a^2cx + \frac{1}{4}a*(a*d + 2*b*c)*x^4 + \frac{1}{7}b*(2*a*d + b*c)*x^7 + \frac{1}{10}b^2*d*x^{10}$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3), x]

[Out]  $a^2cx + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^{10})/10$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^3)^2 (c + dx^3) dx &= \int (a^2c + a(2bc + ad)x^3 + b(bc + 2ad)x^6 + b^2dx^9) dx \\ &= a^2cx + \frac{1}{4}a(2bc + ad)x^4 + \frac{1}{7}b(bc + 2ad)x^7 + \frac{1}{10}b^2dx^{10} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 1.00

$$a^2cx + \frac{1}{7}bx^7(2ad + bc) + \frac{1}{4}ax^4(ad + 2bc) + \frac{1}{10}b^2dx^{10}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3), x]

[Out]  $a^2cx + (a*(2*b*c + a*d)*x^4)/4 + (b*(b*c + 2*a*d)*x^7)/7 + (b^2*d*x^{10})/10$

**fricas [A]** time = 0.36, size = 50, normalized size = 1.00

$$\frac{1}{10}x^{10}db^2 + \frac{1}{7}x^7cb^2 + \frac{2}{7}x^7dba + \frac{1}{2}x^4cba + \frac{1}{4}x^4da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c), x, algorithm="fricas")

[Out]  $\frac{1}{10}x^{10}d*b^2 + \frac{1}{7}x^7*c*b^2 + \frac{2}{7}x^7*d*b*a + \frac{1}{2}x^4*c*b*a + \frac{1}{4}x^4*d*a^2 + x*c*a^2$

**giac** [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{10} b^2 dx^{10} + \frac{1}{7} b^2 cx^7 + \frac{2}{7} abdx^7 + \frac{1}{2} abcx^4 + \frac{1}{4} a^2 dx^4 + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c),x, algorithm="giac")

[Out] 1/10\*b^2\*d\*x^10 + 1/7\*b^2\*c\*x^7 + 2/7\*a\*b\*d\*x^7 + 1/2\*a\*b\*c\*x^4 + 1/4\*a^2\*d\*x^4 + a^2\*c\*x

**maple** [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2 d x^{10}}{10} + \frac{(2abd + b^2 c) x^7}{7} + a^2 cx + \frac{(a^2 d + 2abc) x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c),x)

[Out] 1/10\*b^2\*d\*x^10+1/7\*(2\*a\*b\*d+b^2\*c)\*x^7+1/4\*(a^2\*d+2\*a\*b\*c)\*x^4+a^2\*c\*x

**maxima** [A] time = 0.57, size = 48, normalized size = 0.96

$$\frac{1}{10} b^2 dx^{10} + \frac{1}{7} (b^2 c + 2abd) x^7 + \frac{1}{4} (2abc + a^2 d) x^4 + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c),x, algorithm="maxima")

[Out] 1/10\*b^2\*d\*x^10 + 1/7\*(b^2\*c + 2\*a\*b\*d)\*x^7 + 1/4\*(2\*a\*b\*c + a^2\*d)\*x^4 + a^2\*c\*x

**mupad** [B] time = 0.04, size = 48, normalized size = 0.96

$$x^4 \left( \frac{d a^2}{4} + \frac{b c a}{2} \right) + x^7 \left( \frac{c b^2}{7} + \frac{2 a d b}{7} \right) + \frac{b^2 d x^{10}}{10} + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^2\*(c + d\*x^3),x)

[Out] x^4\*((a^2\*d)/4 + (a\*b\*c)/2) + x^7\*((b^2\*c)/7 + (2\*a\*b\*d)/7) + (b^2\*d\*x^10)/10 + a^2\*c\*x

**sympy** [A] time = 0.07, size = 51, normalized size = 1.02

$$a^2 cx + \frac{b^2 dx^{10}}{10} + x^7 \left( \frac{2abd}{7} + \frac{b^2 c}{7} \right) + x^4 \left( \frac{a^2 d}{4} + \frac{abc}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2\*(d\*x\*\*3+c),x)

[Out] a\*\*2\*c\*x + b\*\*2\*d\*x\*\*10/10 + x\*\*7\*(2\*a\*b\*d/7 + b\*\*2\*c/7) + x\*\*4\*(a\*\*2\*d/4 + a\*b\*c/2)

$$3.11 \quad \int \frac{(a+bx^3)^2}{c+dx^3} dx$$

**Optimal.** Leaf size=173

$$\frac{(bc-ad)^2 \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3} d^{7/3}} + \frac{(bc-ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{2/3} d^{7/3}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{bx(bc-2ad)}{d^2}$$

[Out]  $-b*(-2*a*d+b*c)*x/d^2+1/4*b^2*x^4/d+1/3*(-a*d+b*c)^2*\ln(c^{(1/3)}+d^{(1/3)}*x)/c^{(2/3)}/d^{(7/3)}-1/6*(-a*d+b*c)^2*\ln(c^{(2/3)}-c^{(1/3)}*d^{(1/3)}*x+d^{(2/3)}*x^2)/c^{(2/3)}/d^{(7/3)}-1/3*(-a*d+b*c)^2*\arctan(1/3*(c^{(1/3)}-2*d^{(1/3)}*x)/c^{(1/3)}*3^{(1/2)})/c^{(2/3)}/d^{(7/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.13, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)^2 \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3} d^{7/3}} + \frac{(bc-ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{3c^{2/3} d^{7/3}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c}-2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} - \frac{bx(bc-2ad)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3), x]

[Out]  $-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^4)/(4*d) - ((b*c - a*d)^2*ArcTan[(c^{(1/3)} - 2*d^{(1/3)}*x)/(Sqrt[3]*c^{(1/3)})])/(Sqrt[3]*c^{(2/3)}*d^{(7/3)}) + ((b*c - a*d)^2*Log[c^{(1/3)} + d^{(1/3)}*x])/(3*c^{(2/3)}*d^{(7/3)}) - ((b*c - a*d)^2*Log[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])/(6*c^{(2/3)}*d^{(7/3)})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^3)^2}{c + dx^3} dx &= \int \left( -\frac{b(bc - 2ad)}{d^2} + \frac{b^2x^3}{d} + \frac{b^2c^2 - 2abcd + a^2d^2}{d^2(c + dx^3)} \right) dx \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc - ad)^2 \int \frac{1}{c + dx^3} dx}{d^2} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{2/3}d^2} + \frac{(bc - ad)^2 \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{2/3}d^2} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc - ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{7/3}} - \frac{(bc - ad)^2 \int \frac{-\sqrt[3]{c}\sqrt[3]{d} + 2d^{2/3}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{6c^{2/3}d^{7/3}} + \dots \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^4}{4d} + \frac{(bc - ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{7/3}} - \frac{(bc - ad)^2 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{6c^{2/3}d^{7/3}} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^4}{4d} - \frac{(bc - ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}d^{7/3}} + \frac{(bc - ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}d^{7/3}} - \dots \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 167, normalized size = 0.97

$$\frac{-2(bc - ad)^2 \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2) - 12bc^{2/3}\sqrt[3]{d}x(bc - 2ad) + 4(bc - ad)^2 \log(\sqrt[3]{c} + \sqrt[3]{d}x) + 4\sqrt{3}(bc - ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{12c^{2/3}d^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3), x]

[Out] (-12\*b\*c^(2/3)\*d^(1/3)\*(b\*c - 2\*a\*d)\*x + 3\*b^2\*c^(2/3)\*d^(4/3)\*x^4 + 4\*Sqrt[3]\*(b\*c - a\*d)^2\*ArcTan[(-c^(1/3) + 2\*d^(1/3)\*x)/(Sqrt[3]\*c^(1/3))] + 4\*(b\*c - a\*d)^2\*Log[c^(1/3) + d^(1/3)\*x] - 2\*(b\*c - a\*d)^2\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(12\*c^(2/3)\*d^(7/3))

**fricas** [A] time = 0.48, size = 505, normalized size = 2.92

$$\frac{3b^2c^2d^2x^4 + 6\sqrt{\frac{1}{3}}(b^2c^3d - 2abc^2d^2 + a^2cd^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}} \log\left(\frac{2cdx^3 - 3(c^2d)^{\frac{1}{3}}cx - c^2 + 3\sqrt{\frac{1}{3}}\left(2cdx^2 + (c^2d)^{\frac{2}{3}}x - (c^2d)^{\frac{1}{3}}c\right)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}}}{dx^3 + c}}{\right)}{\quad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c),x, algorithm="fricas")

[Out] [1/12\*(3\*b^2\*c^2\*d^2\*x^4 + 6\*sqrt(1/3)\*(b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2 + a^2\*c\*d^3)\*sqrt(-(c^2\*d)^(1/3)/d)\*log((2\*c\*d\*x^3 - 3\*(c^2\*d)^(1/3)\*c\*x - c^2 + 3\*sqrt(1/3)\*(2\*c\*d\*x^2 + (c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt(-(c^2\*d)^(1/3)/d))/(d\*x^3 + c)) - 2\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) + 4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) - 12\*(b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2)\*x/(c^2\*d^3), 1/12\*(3\*b^2\*c^2\*d^2\*x^4 + 12\*sqrt(1/3)\*(b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2 + a^2\*c\*d^3)\*sqrt((c^2\*d)^(1/3)/d)\*arctan(sqrt(1/3)\*(2\*(c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt((c^2\*d)^(1/3)/d)/c^2) - 2\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) + 4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) - 12\*(b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2)\*x/(c^2\*d^3)]

**giac** [A] time = 0.19, size = 211, normalized size = 1.22

$$\frac{\sqrt{3}(b^2c^2 - 2abcd + a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right) (b^2c^2 - 2abcd + a^2d^2) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right) (b^2c^2d^2)}{3(-cd^2)^{\frac{2}{3}}d \quad 6(-cd^2)^{\frac{2}{3}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/((-c/d)^(1/3)))/((-c/d)^(1/3))/((-c\*d^2)^(2/3)\*d) - 1/6\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/((-c\*d^2)^(2/3)\*d) - 1/3\*(b^2\*c^2\*d^2 - 2\*a\*b\*c\*d^3 + a^2\*d^4)\*(-c/d)^(1/3)\*log(abs(x - (-c/d)^(1/3)))/(c\*d^4) + 1/4\*(b^2\*d^3\*x^4 - 4\*b^2\*c\*d^2\*x + 8\*a\*b\*d^3\*x)/d^4

**maple** [B] time = 0.05, size = 334, normalized size = 1.93

$$\frac{b^2x^4}{4d} + \frac{\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d} + \frac{a^2 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d} - \frac{a^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}x + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(\frac{c}{d}\right)^{\frac{2}{3}}d} - \frac{2\sqrt{3}abc \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{c}{d}\right)^{\frac{2}{3}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((b\*x^3+a)^2/(d\*x^3+c), x)

[Out]  $\frac{1}{4}b^2x^4/d + 2*b/d*a*x - b^2/d^2*c*x + 1/3/d/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a^2 - 2/3/d^2/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a*b*c + 1/3/d^3/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*b^2*c^2 - 1/6/d/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*a^2 + 1/3/d^2/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*a*b*c - 1/6/d^3/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*b^2*c^2 + 1/3/d/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a^2 - 2/3/d^2/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a*b*c + 1/3/d^3/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*b^2*c^2$

**maxima** [A] time = 1.23, size = 189, normalized size = 1.09

$$\frac{b^2 dx^4 - 4(b^2 c - 2abd)x}{4d^2} + \frac{\sqrt{3}(b^2 c^2 - 2abcd + a^2 d^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} - \frac{(b^2 c^2 - 2abcd + a^2 d^2) \log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c), x, algorithm="maxima")

[Out]  $\frac{1}{4}*(b^2*d*x^4 - 4*(b^2*c - 2*a*b*d)*x)/d^2 + 1/3*\sqrt{3}*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/(d^3*(c/d)^{(2/3)}) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(d^3*(c/d)^{(2/3)}) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(x + (c/d)^{(1/3)})/(d^3*(c/d)^{(2/3)})$

**mupad** [B] time = 1.39, size = 152, normalized size = 0.88

$$\frac{b^2 x^4}{4d} - x \left( \frac{b^2 c}{d^2} - \frac{2ab}{d} \right) + \frac{\ln(d^{1/3} x + c^{1/3}) (ad - bc)^2}{3c^{2/3} d^{7/3}} + \frac{\ln(2d^{1/3} x - c^{1/3} + \sqrt{3} c^{1/3} i) \left( -\frac{1}{6} + \frac{\sqrt{3} i}{6} \right) (ad - bc)^2}{c^{2/3} d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^2/(c + d\*x^3), x)

[Out]  $\frac{b^2*x^4}{4*d} - x*((b^2*c)/d^2 - (2*a*b)/d) + (\log(d^{(1/3)}*x + c^{(1/3)}))*((a*d - b*c)^2)/(3*c^{(2/3)}*d^{(7/3)}) + (\log(3^{(1/2)}*c^{(1/3)}*1i + 2*d^{(1/3)}*x - c^{(1/3)}))*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^2/(c^{(2/3)}*d^{(7/3)}) - (\log(3^{(1/2)}*c^{(1/3)}*1i - 2*d^{(1/3)}*x + c^{(1/3)}))*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^2/(3*c^{(2/3)}*d^{(7/3)})$

**sympy** [A] time = 0.68, size = 156, normalized size = 0.90

$$\frac{b^2 x^4}{4d} + x \left( \frac{2ab}{d} - \frac{b^2 c}{d^2} \right) + \text{RootSum} \left( 27t^3 c^2 d^7 - a^6 d^6 + 6a^5 b c d^5 - 15a^4 b^2 c^2 d^4 + 20a^3 b^3 c^3 d^3 - 15a^2 b^4 c^4 d^2 + 6ab^5 c^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c), x)

[Out]  $b**2*x**4/(4*d) + x*(2*a*b/d - b**2*c/d**2) + \text{RootSum}(27*_t**3*c**2*d**7 - a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, \text{Lambda}(_t, _t*\log(3*_t*c*d**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x)))$

$$3.12 \quad \int \frac{(a+bx^3)^2}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=203

$$\frac{(bc-ad)(ad+2bc) \log\left(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2\right)}{9c^{5/3} d^{7/3}} - \frac{2(bc-ad)(ad+2bc) \log\left(\sqrt[3]{c} + \sqrt[3]{d} x\right)}{9c^{5/3} d^{7/3}} + \frac{2(bc-ad)(ad+2bc) \arctan\left(\frac{c^{1/3} + d^{1/3} x}{c^{1/3} - d^{1/3} x}\right)}{3\sqrt{3} c^{5/3} d^{7/3}}$$

[Out]  $b^2 x/d^2 + 1/3(-a*d+b*c)^2 x/c/d^2/(d*x^3+c) - 2/9(-a*d+b*c)*(a*d+2*b*c)*\ln(c^{1/3}+d^{1/3}*x)/c^{5/3}/d^{7/3} + 1/9(-a*d+b*c)*(a*d+2*b*c)*\ln(c^{2/3}-c^{1/3}*d^{1/3}*x+d^{2/3}*x^2)/c^{5/3}/d^{7/3} + 2/9(-a*d+b*c)*(a*d+2*b*c)*\arctan(1/3*(c^{1/3}-2*d^{1/3}*x)/c^{1/3}*3^{1/2})/c^{5/3}/d^{7/3}*3^{1/2}$

**Rubi [A]** time = 0.24, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)(ad+2bc) \log\left(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2\right)}{9c^{5/3} d^{7/3}} - \frac{2(bc-ad)(ad+2bc) \log\left(\sqrt[3]{c} + \sqrt[3]{d} x\right)}{9c^{5/3} d^{7/3}} + \frac{2(bc-ad)(ad+2bc) \arctan\left(\frac{c^{1/3} + d^{1/3} x}{c^{1/3} - d^{1/3} x}\right)}{3\sqrt{3} c^{5/3} d^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3)^2, x]

[Out]  $(b^2 x)/d^2 + ((b*c - a*d)^2 x)/(3*c*d^2*(c + d*x^3)) + (2*(b*c - a*d)*(2*b*c + a*d)*\text{ArcTan}[(c^{1/3} - 2*d^{1/3}*x)/(\text{Sqrt}[3]*c^{1/3})])/(3*\text{Sqrt}[3]*c^{5/3}*d^{7/3}) - (2*(b*c - a*d)*(2*b*c + a*d)*\text{Log}[c^{1/3} + d^{1/3}*x])/(9*c^{5/3}*d^{7/3}) + ((b*c - a*d)*(2*b*c + a*d)*\text{Log}[c^{2/3} - c^{1/3}*d^{1/3}*x + d^{2/3}*x^2])/(9*c^{5/3}*d^{7/3})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2}{(c + dx^3)^2} dx &= \int \left( \frac{b^2}{d^2} - \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^3}{d^2(c + dx^3)^2} \right) dx \\
&= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^3}{(c + dx^3)^2} dx}{d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{3cd^2(c + dx^3)} - \frac{(2(bc - ad)(2bc + ad)) \int \frac{1}{c + dx^3} dx}{3cd^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{3cd^2(c + dx^3)} - \frac{(2(bc - ad)(2bc + ad)) \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{9c^{5/3}d^2} - \frac{(2(bc - ad)(2bc + ad)) \int \frac{1}{c^2}}{9c^{5/3}d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{3cd^2(c + dx^3)} - \frac{2(bc - ad)(2bc + ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}d^{7/3}} + \frac{((bc - ad)(2bc + ad)) \int \frac{1}{c^2}}{9c^{5/3}d^{7/3}} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{3cd^2(c + dx^3)} - \frac{2(bc - ad)(2bc + ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}d^{7/3}} + \frac{(bc - ad)(2bc + ad) \log(c)}{9c^{5/3}} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{3cd^2(c + dx^3)} + \frac{2(bc - ad)(2bc + ad) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}d^{7/3}} - \frac{2(bc - ad)(2bc + ad) \log(c)}{9c^{5/3}d^{7/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 210, normalized size = 1.03

$$\frac{2(-a^2d^2 - abcd + 2b^2c^2) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{5/3}} + \frac{2\sqrt{3}(-a^2d^2 - abcd + 2b^2c^2) \tan^{-1}\left(\frac{1 - 2\sqrt[3]{d}x}{\sqrt[3]{c}}\right)}{c^{5/3}} + \frac{(-a^2d^2 - abcd + 2b^2c^2) \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d}x + d^{2/3}x^2)}{c^{5/3}} + \frac{3\sqrt[3]{d}}{c}$$


---


$$9d^{7/3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3)^2,x]

[Out] (9\*b^2\*d^(1/3)\*x + (3\*d^(1/3)\*(b\*c - a\*d)^2\*x)/(c\*(c + d\*x^3)) + (2\*sqrt[3] \* (2\*b^2\*c^2 - a\*b\*c\*d - a^2\*d^2)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/sqrt[3] ])/c^(5/3) - (2\*(2\*b^2\*c^2 - a\*b\*c\*d - a^2\*d^2)\*Log[c^(1/3) + d^(1/3)\*x])/c^(5/3) + ((2\*b^2\*c^2 - a\*b\*c\*d - a^2\*d^2)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/c^(5/3))/(9\*d^(7/3))

**fricas [B]** time = 0.46, size = 771, normalized size = 3.80

$$\left[ \frac{9b^2c^3d^2x^4 - 3\sqrt{\frac{1}{3}}(2b^2c^4d - abc^3d^2 - a^2c^2d^3 + (2b^2c^3d^2 - abc^2d^3 - a^2cd^4)x^3)\sqrt{-\frac{(c^2d)^{\frac{1}{3}}}{d}} \log\left(\frac{2cdx^3 - 3(c^2d)^{\frac{1}{3}}cx - c^2 + (c^2d)^{\frac{1}{3}}}{d}\right)}{9d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] [1/9\*(9\*b^2\*c^3\*d^2\*x^4 - 3\*sqrt(1/3)\*(2\*b^2\*c^4\*d - a\*b\*c^3\*d^2 - a^2\*c^2\*d^3 + (2\*b^2\*c^3\*d^2 - a\*b\*c^2\*d^3 - a^2\*c\*d^4)\*x^3)\*sqrt(-(c^2\*d)^(1/3)/d) \* log((2\*c\*d\*x^3 - 3\*(c^2\*d)^(1/3)\*c\*x - c^2 + 3\*sqrt(1/3)\*(2\*c\*d\*x^2 + (c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt(-(c^2\*d)^(1/3)/d))/(d\*x^3 + c)) + (2\*b^2\*c^3 - a\*b\*c^2\*d - a^2\*c\*d^2 + (2\*b^2\*c^2\*d - a\*b\*c\*d^2 - a^2\*d^3)\*x^3)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) - 2\*(2\*b^2\*c^3 - a\*b\*c^2\*d - a^2\*c\*d^2 + (2\*b^2\*c^2\*d - a\*b\*c\*d^2 - a^2\*d^3)\*x^3)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) + 3\*(4\*b^2\*c^4\*d - 2\*a\*b\*c^3\*d^2 + a^2\*c^2\*d^3)\*x)/(c^3\*d^4\*x^3 + c^4\*d^3), 1/9\*(9\*b^2\*c^3\*d^2\*x^4 - 6\*sqrt(1/3)\*(2\*b^2\*c^4\*d - a\*b\*c^3\*d^2 - a^2\*c^2\*d^3 + (2\*b^2\*c^3\*d^2 - a\*b\*c^2\*d^3 - a^2\*c\*d^4)\*x^3)\*sqrt((c^2\*d)^(1/3)/d)\*arctan(sqrt(1/3)\*(2\*(c^2\*d)^(2/3)\*x - (c^2\*d)^(1/3)\*c)\*sqrt((c^2\*d)^(1/3)/d)/c^2) + (2\*b^2\*c^3 - a\*b\*c^2\*d - a^2\*c\*d^2 + (2\*b^2\*c^2\*d - a\*b\*c\*d^2 - a^2\*d^3)\*x^3)\*(c^2\*d)^(2/3)\*log(c\*d\*x^2 - (c^2\*d)^(2/3)\*x + (c^2\*d)^(1/3)\*c) - 2\*(2\*b^2\*c^3 - a\*b\*c^2\*d - a^2\*c\*d^2 + (2\*b^2\*c^2\*d - a\*b\*c\*d^2 - a^2\*d^3)\*x^3)\*(c^2\*d)^(2/3)\*log(c\*d\*x + (c^2\*d)^(2/3)) + 3\*(4\*b^2\*c^4\*d - 2\*a\*b\*c^3\*d^2 + a^2\*c^2\*d^3)\*x)/(c^3\*d^4\*x^3 + c^4\*d^3)]

**giac [A]** time = 0.19, size = 233, normalized size = 1.15

$$\frac{b^2x}{d^2} + \frac{2\sqrt{3}(2b^2c^2 - abcd - a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9(-cd^2)^{\frac{2}{3}}cd} + \frac{(2b^2c^2 - abcd - a^2d^2) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9(-cd^2)^{\frac{2}{3}}cd} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c)^2,x, algorithm="giac")

[Out]  $b^2x/d^2 + 2/9\sqrt{3}*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/((-c*d^2)^{(2/3)}*c*d) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/((-c*d^2)^{(2/3)}*c*d) + 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/((c^2*d^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((d*x^3 + c)*c*d^2)$

**maple [B]** time = 0.06, size = 367, normalized size = 1.81

$$\frac{a^2x}{3(d^3x^3 + c)c} - \frac{2abx}{3(d^3x^3 + c)d} + \frac{b^2cx}{3(d^3x^3 + c)d^2} + \frac{2\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd} + \frac{2a^2 \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd} - \frac{a^2 \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9\left(\frac{c}{d}\right)^{\frac{2}{3}}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2/(d\*x^3+c)^2,x)

[Out]  $b^2x/d^2 + 1/3/c*x/(d*x^3+c)*a^2 - 2/3/d*x/(d*x^3+c)*a*b + 1/3/d^2*c*x/(d*x^3+c)*b^2 + 2/9/d/c/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a^2 + 2/9/d^2/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a*b - 4/9/d^3*c/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*b^2 - 1/9/d/c/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*a^2 - 1/9/d^2/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*a*b + 2/9/d^3*c/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*b^2 + 2/9/d/c/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a^2 + 2/9/d^2/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a*b - 4/9/d^3*c/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*b^2$

**maxima [A]** time = 1.28, size = 226, normalized size = 1.11

$$\frac{(b^2c^2 - 2abcd + a^2d^2)x}{3(cd^3x^3 + c^2d^2)} + \frac{b^2x}{d^2} - \frac{2\sqrt{3}(2b^2c^2 - abcd - a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}} + \frac{(2b^2c^2 - abcd - a^2d^2) \log\left(\frac{c}{d}\right)}{9cd^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c)^2,x, algorithm="maxima")

[Out]  $1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x/(c*d^3*x^3 + c^2*d^2) + b^2*x/d^2 - 2/9*\sqrt{3}*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/(c*d^3*(c/d)^{(2/3)}) + 1/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(c*d^3*(c/d)^{(2/3)}) - 2/9*(2*b^2*c^2 - a*b*c*d - a^2*d^2)*\log(x + (c/d)^{(1/3)})/(c*d^3*(c/d)^{(2/3)})$

**mupad [B]** time = 1.41, size = 191, normalized size = 0.94

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3c(d^3x^3 + c^2d^2)} + \frac{2 \ln(d^{1/3}x + c^{1/3})(ad - bc)(ad + 2bc)}{9c^{5/3}d^{7/3}} + \frac{2 \ln(2d^{1/3}x - c^{1/3} + \sqrt{3}c^{1/3}i)}{9c^{5/3}d^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^2/(c + d\*x^3)^2,x)

[Out] (b^2\*x)/d^2 + (x\*(a^2\*d^2 + b^2\*c^2 - 2\*a\*b\*c\*d))/(3\*c\*(c\*d^2 + d^3\*x^3)) + (2\*log(d^(1/3)\*x + c^(1/3))\*(a\*d - b\*c)\*(a\*d + 2\*b\*c))/(9\*c^(5/3)\*d^(7/3)) + (2\*log(3^(1/2)\*c^(1/3)\*1i + 2\*d^(1/3)\*x - c^(1/3))\*((3^(1/2)\*1i)/2 - 1/2)\*(a\*d - b\*c)\*(a\*d + 2\*b\*c))/(9\*c^(5/3)\*d^(7/3)) - (2\*log(3^(1/2)\*c^(1/3)\*1i - 2\*d^(1/3)\*x + c^(1/3))\*((3^(1/2)\*1i)/2 + 1/2)\*(a\*d - b\*c)\*(a\*d + 2\*b\*c))/(9\*c^(5/3)\*d^(7/3))

**sympy** [A] time = 1.14, size = 189, normalized size = 0.93

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3c^2d^2 + 3cd^3x^3} + \text{RootSum}\left(729t^3c^5d^7 - 8a^6d^6 - 24a^5bcd^5 + 24a^4b^2c^2d^4 + 88a^3b^3c^3d^3 - 48a^2b^4c^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*2,x)

[Out] b\*\*2\*x/d\*\*2 + x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(3\*c\*\*2\*d\*\*2 + 3\*c\*d\*\*3\*x\*\*3) + RootSum(729\*\_t\*\*3\*c\*\*5\*d\*\*7 - 8\*a\*\*6\*d\*\*6 - 24\*a\*\*5\*b\*c\*d\*\*5 + 24\*a\*\*4\*b\*\*2\*c\*\*2\*d\*\*4 + 88\*a\*\*3\*b\*\*3\*c\*\*3\*d\*\*3 - 48\*a\*\*2\*b\*\*4\*c\*\*4\*d\*\*2 - 96\*a\*b\*\*5\*c\*\*5\*d + 64\*b\*\*6\*c\*\*6, Lambda(\_t, \_t\*log(9\*\_t\*c\*\*2\*d\*\*2/(2\*a\*\*2\*d\*\*2 + 2\*a\*b\*c\*d - 4\*b\*\*2\*c\*\*2) + x)))

$$3.13 \quad \int \frac{(a+bx^3)^2}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=258

$$\frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3}x^2)}{54c^{8/3}d^{7/3}} + \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{27c^{8/3}d^{7/3}} - \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(\sqrt[3]{c} - \sqrt[3]{d} x)}{27c^{8/3}d^{7/3}}$$

[Out]  $-1/6*(-a*d+b*c)*x*(b*x^3+a)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+4*b*c)*x/c^2/d^2/(d*x^3+c)+1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*\ln(c^{(1/3)}+d^{(1/3)*x})/c^{(8/3)}/d^{(7/3)}-1/54*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*\ln(c^{(2/3)}-c^{(1/3)*d^{(1/3)*x}+d^{(2/3)*x^2}})/c^{(8/3)}/d^{(7/3)}-1/27*(5*a^2*d^2+2*a*b*c*d+2*b^2*c^2)*\arctan(1/3*(c^{(1/3)}-2*d^{(1/3)*x})/c^{(1/3)*3^{(1/2)}})/c^{(8/3)}/d^{(7/3)*3^{(1/2)}}$

**Rubi [A]** time = 0.23, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {413, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3}x^2)}{54c^{8/3}d^{7/3}} + \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(\sqrt[3]{c} + \sqrt[3]{d} x)}{27c^{8/3}d^{7/3}} - \frac{(5a^2d^2 + 2abcd + 2b^2c^2) \log(\sqrt[3]{c} - \sqrt[3]{d} x)}{27c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2/(c + d\*x^3)^3, x]

[Out]  $-((b*c - a*d)*x*(a + b*x^3))/(6*c*d*(c + d*x^3)^2) - ((b*c - a*d)*(4*b*c + 5*a*d)*x)/(18*c^2*d^2*(c + d*x^3)) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\text{ArcTan}[c^{(1/3)} - 2*d^{(1/3)*x}]/(\text{Sqrt}[3]*c^{(1/3)}))/ (9*\text{Sqrt}[3]*c^{(8/3)*d^{(7/3)}}) + ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\text{Log}[c^{(1/3)} + d^{(1/3)*x}])/ (27*c^{(8/3)*d^{(7/3)}}) - ((2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\text{Log}[c^{(2/3)} - c^{(1/3)*d^{(1/3)*x} + d^{(2/3)*x^2}}])/ (54*c^{(8/3)*d^{(7/3)}})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_)^3)^-1, x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^-1, x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n +

p, 0])

### Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^2}{(c + dx^3)^3} dx &= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} + \frac{\int \frac{a(bc + 5ad) + 2b(2bc + ad)x^3}{(c + dx^3)^2} dx}{6cd} \\
&= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} - \frac{(bc - ad)(4bc + 5ad)x}{18c^2d^2(c + dx^3)} + \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \int \frac{1}{c + dx^3} dx}{9c^2d^2} \\
&= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} - \frac{(bc - ad)(4bc + 5ad)x}{18c^2d^2(c + dx^3)} + \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{27c^{8/3}d^2} + \dots \\
&= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} - \frac{(bc - ad)(4bc + 5ad)x}{18c^2d^2(c + dx^3)} + \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{27c^{8/3}d^{7/3}} \\
&= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} - \frac{(bc - ad)(4bc + 5ad)x}{18c^2d^2(c + dx^3)} + \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{27c^{8/3}d^{7/3}} \\
&= -\frac{(bc - ad)x(a + bx^3)}{6cd(c + dx^3)^2} - \frac{(bc - ad)(4bc + 5ad)x}{18c^2d^2(c + dx^3)} - \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{9\sqrt{3}c^{8/3}d^{7/3}}
\end{aligned}$$



**Mathematica [A]** time = 0.28, size = 234, normalized size = 0.91

$$2(5a^2d^2 + 2abcd + 2b^2c^2) \log(\sqrt[3]{c} + \sqrt[3]{d}x) - 2\sqrt{3} (5a^2d^2 + 2abcd + 2b^2c^2) \tan^{-1} \left( \frac{1 - \frac{2\sqrt[3]{d}x}{\sqrt[3]{c}}}{\sqrt{3}} \right) - \frac{3c^{2/3} \sqrt[3]{d}x(-a^2d^2(8c + 5d\sqrt[3]{d}x) + b^2c^2(4c + 7d\sqrt[3]{d}x))}{54c^{8/3}d^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2/(c + d\*x^3)^3,x]

[Out]  $((-3c^{2/3}d^{1/3}x(2ab^2cd(2c - dx^3) - a^2d^2(8c + 5d\sqrt[3]{d}x) + b^2c^2(4c + 7d\sqrt[3]{d}x)))/(c + d\sqrt[3]{d}x)^2 - 2\sqrt{3}(2b^2c^2 + 2ab^2cd + 5a^2d^2)\text{ArcTan}[(1 - (2d^{1/3}x)/c^{1/3})/\sqrt{3}] + 2(2b^2c^2 + 2ab^2cd + 5a^2d^2)\text{Log}[c^{1/3} + d^{1/3}x] - (2b^2c^2 + 2ab^2cd + 5a^2d^2)\text{Log}[c^{2/3} - c^{1/3}d^{1/3}x + d^{2/3}x^2])/(54c^{8/3}d^{7/3})$

**fricas [B]** time = 0.45, size = 1067, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c)^3,x, algorithm="fricas")

[Out]  $[-1/54(3(7b^2c^4d^2 - 2ab^2c^3d^3 - 5a^2c^2d^4)x^4 - 3\sqrt{1/3}(2b^2c^5d + 2ab^2c^4d^2 + 5a^2c^3d^3 + (2b^2c^3d^3 + 2ab^2c^2d^4 + 5a^2c^2d^5)x^6 + 2(2b^2c^4d^2 + 2ab^2c^3d^3 + 5a^2c^2d^4)x^3)\sqrt{-(c^2d)^{1/3}/d}\log((2cdx^3 - 3(c^2d)^{1/3}cx - c^2 + 3\sqrt{1/3}(2cdx^2 + (c^2d)^{2/3}x - (c^2d)^{1/3}c)\sqrt{-(c^2d)^{1/3}/d}))/d) + ((2b^2c^2d^2 + 2ab^2c^3d^3 + 5a^2d^4)x^6 + 2b^2c^4 + 2ab^2c^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2ab^2c^2d^2 + 5a^2c^2d^3)x^3)(c^2d)^{2/3}\log(cdx^2 - (c^2d)^{2/3}x + (c^2d)^{1/3}c) - 2((2b^2c^2d^2 + 2ab^2c^3d^3 + 5a^2d^4)x^6 + 2b^2c^4 + 2ab^2c^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2ab^2c^2d^2 + 5a^2c^2d^3)x^3)(c^2d)^{2/3}\log(cdx + (c^2d)^{2/3}) + 12(b^2c^5d + ab^2c^4d^2 - 2a^2c^3d^3)x)/(c^4d^5x^6 + 2c^5d^4x^3 + c^6d^3), -1/54(3(7b^2c^4d^2 - 2ab^2c^3d^3 - 5a^2c^2d^4)x^4 - 6\sqrt{1/3}(2b^2c^5d + 2ab^2c^4d^2 + 5a^2c^3d^3 + (2b^2c^3d^3 + 2ab^2c^2d^4 + 5a^2c^2d^5)x^6 + 2(2b^2c^4d^2 + 2ab^2c^3d^3 + 5a^2c^2d^4)x^3)\sqrt{(c^2d)^{1/3}/d}\arctan(\sqrt{1/3}(2(c^2d)^{2/3}x - (c^2d)^{1/3}c)\sqrt{(c^2d)^{1/3}/d}/c^2) + ((2b^2c^2d^2 + 2ab^2c^3d^3 + 5a^2d^4)x^6 + 2b^2c^4 + 2ab^2c^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2ab^2c^2d^2 + 5a^2c^2d^3)x^3)(c^2d)^{2/3}\log(cdx^2 - (c^2d)^{2/3}x + (c^2d)^{1/3}c) - 2((2b^2c^2d^2 + 2ab^2c^3d^3 + 5a^2d^4)x^6 + 2b^2c^4 + 2ab^2c^3d + 5a^2c^2d^2 + 2(2b^2c^3d + 2ab^2c^2d^2 + 5a^2c^2d^3)x^3)(c^2d)^{2/3}\log(cdx + (c^2d)^{2/3}) + 12(b^2c^5d + ab^2c^4d^2 - 2a^2c^3d^3)x)/(c^4d^5x^6 + 2c^5d^4x^3 + c^6d^3)]$

**giac [A]** time = 0.20, size = 264, normalized size = 1.02

$$\frac{\sqrt{3}(2b^2c^2 + 2abcd + 5a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27(-cd^2)^{\frac{2}{3}}c^2d} - \frac{(2b^2c^2 + 2abcd + 5a^2d^2) \log\left(x^2 + x\left(-\frac{c}{d}\right)^{\frac{1}{3}} + \left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{54(-cd^2)^{\frac{2}{3}}c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2/(d\*x^3+c)^3,x, algorithm="giac")

[Out]  $-1/27*\sqrt{3}*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/((-c*d^2)^{(2/3)}*c^2*d) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/((-c*d^2)^{(2/3)}*c^2*d) - 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/c^3*d^2) - 1/18*(7*b^2*c^2*d*x^4 - 2*a*b*c*d^2*x^4 - 5*a^2*d^3*x^4 + 4*b^2*c^3*x + 4*a*b*c^2*d*x - 8*a^2*c*d^2*x)/((d*x^3 + c)^2*c^2*d^2)$

**maple** [F(-1)] time = 180.00, size = 0, normalized size = 0.00

hanged

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^3+a)^2/(d*x^3+c)^3,x)$

[Out]  $\text{int}((b*x^3+a)^2/(d*x^3+c)^3,x)$

**maxima** [A] time = 1.32, size = 267, normalized size = 1.03

$$\frac{(7b^2c^2d - 2abcd^2 - 5a^2d^3)x^4 + 4(b^2c^3 + abc^2d - 2a^2cd^2)x}{18(c^2d^4x^6 + 2c^3d^3x^3 + c^4d^2)} + \frac{\sqrt{3}(2b^2c^2 + 2abcd + 5a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{27c^2d^3\left(\frac{c}{d}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^3+a)^2/(d*x^3+c)^3,x, \text{algorithm}=\text{"maxima"})$

[Out]  $-1/18*((7*b^2*c^2*d - 2*a*b*c*d^2 - 5*a^2*d^3)*x^4 + 4*(b^2*c^3 + a*b*c^2*d - 2*a^2*c*d^2)*x)/(c^2*d^4*x^6 + 2*c^3*d^3*x^3 + c^4*d^2) + 1/27*\sqrt{3}*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((c^2*d^3*(c/d)^{(2/3)}) - 1/54*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(c^2*d^3*(c/d)^{(2/3)}) + 1/27*(2*b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*\log(x + (c/d)^{(1/3)})/(c^2*d^3*(c/d)^{(2/3)})$

**mupad** [B] time = 1.43, size = 249, normalized size = 0.97

$$\frac{\ln(d^{1/3}x + c^{1/3})(5a^2d^2 + 2abcd + 2b^2c^2)}{27c^{8/3}d^{7/3}} - \frac{2x(-2a^2d^2 + abcd + b^2c^2)}{9cd^2} - \frac{x^4(5a^2d^2 + 2abcd - 7b^2c^2)}{18c^2d} + \frac{\ln(2d^{1/3}x - c^{1/3} + \dots)}{c^2 + 2cdx^3 + d^2x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a + b*x^3)^2/(c + d*x^3)^3,x)$

[Out]  $(\log(d^{(1/3)}*x + c^{(1/3)})*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^{(8/3)}*d^{(7/3)}) - ((2*x*(b^2*c^2 - 2*a^2*d^2 + a*b*c*d))/(9*c*d^2) - (x^4*(5*a^2*d^2 - 7*b^2*c^2 + 2*a*b*c*d))/(18*c^2*d))/(c^2 + d^2*x^6 + 2*c*d*x^3) + (\log(3^{(1/2)}*c^{(1/3)}*i + 2*d^{(1/3)}*x - c^{(1/3)}))*((3^{(1/2)}*i)/2 - 1/2)*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^{(8/3)}*d^{(7/3)}) - (\log(3^{(1/2)}*c^{(1/3)}*i - 2*d^{(1/3)}*x + c^{(1/3)}))*((3^{(1/2)}*i)/2 + 1/2)*(5*a^2*d^2 + 2*b^2*c^2 + 2*a*b*c*d))/(27*c^{(8/3)}*d^{(7/3)})$

**sympy** [A] time = 1.62, size = 233, normalized size = 0.90

$$\frac{x^4(5a^2d^3 + 2abcd^2 - 7b^2c^2d) + x(8a^2cd^2 - 4abc^2d - 4b^2c^3)}{18c^4d^2 + 36c^3d^3x^3 + 18c^2d^4x^6} + \text{RootSum}\left(19683t^3c^8d^7 - 125a^6d^6 - 150a^5bcd^5 - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c)\*\*3,x)

[Out] (x\*\*4\*(5\*a\*\*2\*d\*\*3 + 2\*a\*b\*c\*d\*\*2 - 7\*b\*\*2\*c\*\*2\*d) + x\*(8\*a\*\*2\*c\*d\*\*2 - 4\*a\*b\*c\*\*2\*d - 4\*b\*\*2\*c\*\*3))/(18\*c\*\*4\*d\*\*2 + 36\*c\*\*3\*d\*\*3\*x\*\*3 + 18\*c\*\*2\*d\*\*4\*x\*\*6) + RootSum(19683\*\_t\*\*3\*c\*\*8\*d\*\*7 - 125\*a\*\*6\*d\*\*6 - 150\*a\*\*5\*b\*c\*d\*\*5 - 210\*a\*\*4\*b\*\*2\*c\*\*2\*d\*\*4 - 128\*a\*\*3\*b\*\*3\*c\*\*3\*d\*\*3 - 84\*a\*\*2\*b\*\*4\*c\*\*4\*d\*\*2 - 24\*a\*b\*\*5\*c\*\*5\*d - 8\*b\*\*6\*c\*\*6, Lambda(\_t, \_t\*log(27\*\_t\*c\*\*3\*d\*\*2/(5\*a\*\*2\*d\*\*2 + 2\*a\*b\*c\*d + 2\*b\*\*2\*c\*\*2) + x)))

$$3.14 \quad \int \frac{(c+dx^3)^4}{a+bx^3} dx$$

**Optimal.** Leaf size=252

$$\frac{(bc-ad)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{13/3}} + \frac{(bc-ad)^4 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{13/3}} - \frac{(bc-ad)^4 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{13/3}} + \frac{dx(2bc-ad)}{b^3}$$

[Out] d\*(-a\*d+2\*b\*c)\*(a^2\*d^2-2\*a\*b\*c\*d+2\*b^2\*c^2)\*x/b^4+1/4\*d^2\*(a^2\*d^2-4\*a\*b\*c\*d+6\*b^2\*c^2)\*x^4/b^3+1/7\*d^3\*(-a\*d+4\*b\*c)\*x^7/b^2+1/10\*d^4\*x^10/b+1/3\*(-a\*d+b\*c)^4\*ln(a^(1/3)+b^(1/3)\*x)/a^(2/3)/b^(13/3)-1/6\*(-a\*d+b\*c)^4\*ln(a^(2/3)-a^(1/3)\*b^(1/3)\*x+b^(2/3)\*x^2)/a^(2/3)/b^(13/3)-1/3\*(-a\*d+b\*c)^4\*arctan(1/3\*(a^(1/3)-2\*b^(1/3)\*x)/a^(1/3)\*3^(1/2))/a^(2/3)/b^(13/3)\*3^(1/2)

**Rubi [A]** time = 0.19, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, number of rules / integrand size = 0.368, Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{d^2 x^4 (a^2 d^2 - 4abcd + 6b^2 c^2)}{4b^3} + \frac{dx(2bc - ad)(a^2 d^2 - 2abcd + 2b^2 c^2)}{b^4} - \frac{(bc - ad)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{13/3}} + \frac{dx(2bc - ad)}{b^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^4/(a + b\*x^3), x]

[Out] (d\*(2\*b\*c - a\*d)\*(2\*b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*x)/b^4 + (d^2\*(6\*b^2\*c^2 - 4\*a\*b\*c\*d + a^2\*d^2)\*x^4)/(4\*b^3) + (d^3\*(4\*b\*c - a\*d)\*x^7)/(7\*b^2) + (d^4\*x^10)/(10\*b) - ((b\*c - a\*d)^4\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(13/3)) + ((b\*c - a\*d)^4\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(13/3)) - ((b\*c - a\*d)^4\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(13/3))

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])]; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^4}{a + bx^3} dx &= \int \left( \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^3}{b^3} + \frac{d^3(4bc - ad)x^6}{b^2} \right. \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{4b^3} + \frac{d^3(4bc - ad)x^7}{7b^2} + \dots \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{4b^3} + \frac{d^3(4bc - ad)x^7}{7b^2} + \dots \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{4b^3} + \frac{d^3(4bc - ad)x^7}{7b^2} + \dots \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{4b^3} + \frac{d^3(4bc - ad)x^7}{7b^2} + \dots \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{4b^3} + \frac{d^3(4bc - ad)x^7}{7b^2} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 253, normalized size = 1.00

$$\frac{-\frac{70(bc-ad)^4 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{bx+b^{2/3}x^2})}{a^{2/3}} + \frac{140(bc-ad)^4 \log(\sqrt[3]{a} + \sqrt[3]{bx})}{a^{2/3}} + \frac{140\sqrt{3}(bc-ad)^4 \tan^{-1}\left(\frac{2\sqrt[3]{bx} - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 105b^{4/3}d^2x^4(a^2d^2 - 4a)}{420b^{13/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^4/(a + b*x^3), x]
```

```
[Out] (420*b^(1/3)*d*(4*b^3*c^3 - 6*a*b^2*c^2*d + 4*a^2*b*c*d^2 - a^3*d^3)*x + 10
5*b^(4/3)*d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^4 + 60*b^(7/3)*d^3*(4*b*c
- a*d)*x^7 + 42*b^(10/3)*d^4*x^10 + (140*sqrt[3]*(b*c - a*d)^4*ArcTan[(-a^(
1/3) + 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))]/a^(2/3) + (140*(b*c - a*d)^4*Log[a
^(1/3) + b^(1/3)*x])/a^(2/3) - (70*(b*c - a*d)^4*Log[a^(2/3) - a^(1/3)*b^(1
/3)*x + b^(2/3)*x^2])/a^(2/3))/(420*b^(13/3))
```

**fricas** [A] time = 0.47, size = 873, normalized size = 3.46

$$42 a^2 b^4 d^4 x^{10} + 60 (4 a^2 b^4 c d^3 - a^3 b^3 d^4) x^7 + 105 (6 a^2 b^4 c^2 d^2 - 4 a^3 b^3 c d^3 + a^4 b^2 d^4) x^4 + 210 \sqrt{\frac{1}{3}} (a b^5 c^4 - 4 a^2 b^4 c^3 d + 6 a^3 b^3 c^2 d^2 - 4 a^4 b^2 c d^3 + a^5 b d^4) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^4/(b\*x^3+a),x, algorithm="fricas")

[Out] [1/420\*(42\*a^2\*b^4\*d^4\*x^10 + 60\*(4\*a^2\*b^4\*c\*d^3 - a^3\*b^3\*d^4)\*x^7 + 105\*(6\*a^2\*b^4\*c^2\*d^2 - 4\*a^3\*b^3\*c\*d^3 + a^4\*b^2\*d^4)\*x^4 + 210\*sqrt(1/3)\*(a\*b^5\*c^4 - 4\*a^2\*b^4\*c^3\*d + 6\*a^3\*b^3\*c^2\*d^2 - 4\*a^4\*b^2\*c\*d^3 + a^5\*b\*d^4)\*sqrt(-(a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 - 3\*(a^2\*b)^(1/3)\*a\*x - a^2 + 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt(-(a^2\*b)^(1/3)/b))/(b\*x^3 + a)) - 70\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 140\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 420\*(4\*a^2\*b^4\*c^3\*d - 6\*a^3\*b^3\*c^2\*d^2 + 4\*a^4\*b^2\*c\*d^3 - a^5\*b\*d^4)\*x/(a^2\*b^5), 1/420\*(42\*a^2\*b^4\*d^4\*x^10 + 60\*(4\*a^2\*b^4\*c\*d^3 - a^3\*b^3\*d^4)\*x^7 + 105\*(6\*a^2\*b^4\*c^2\*d^2 - 4\*a^3\*b^3\*c\*d^3 + a^4\*b^2\*d^4)\*x^4 + 420\*sqrt(1/3)\*(a\*b^5\*c^4 - 4\*a^2\*b^4\*c^3\*d + 6\*a^3\*b^3\*c^2\*d^2 - 4\*a^4\*b^2\*c\*d^3 + a^5\*b\*d^4)\*sqrt((a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt((a^2\*b)^(1/3)/b)/a^2) - 70\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 140\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 420\*(4\*a^2\*b^4\*c^3\*d - 6\*a^3\*b^3\*c^2\*d^2 + 4\*a^4\*b^2\*c\*d^3 - a^5\*b\*d^4)\*x/(a^2\*b^5)]

**giac** [A] time = 0.20, size = 391, normalized size = 1.55

$$\frac{\sqrt{3} (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) \arctan\left(\frac{\sqrt{3} \left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3 \left(-ab^2\right)^{\frac{2}{3}} b^3} (b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^4/(b\*x^3+a),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a\*b^2)^(2/3)\*b^3) - 1/6\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a\*b^2)^(2/3)\*b^3) - 1/3\*(b^10\*c^4 - 4\*a\*b^9\*c^3\*d + 6\*a^2\*b^8\*c^2\*d^2 - 4\*a^3\*b^7\*c\*d^3 + a^4\*b^6\*d^4)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a\*b^10) + 1/140\*(14\*b^9\*d^4\*x^10 + 80\*b^9\*c\*d^3\*x^7 - 20\*a\*b^8\*d^4\*x^7 + 210\*b^9\*c^2\*d^2\*x^4 - 140\*a\*b^8\*c\*d^3\*x^4 + 35\*a^2\*b^7\*d^4\*x^4 + 560\*b^9\*c^3\*d\*x - 840\*a\*b^8\*c^2\*d^2\*x + 560\*a^2\*b^7\*c\*d^3\*x - 140\*a^3\*b^6\*d^4\*x)/b^10

**maple [B]** time = 0.05, size = 661, normalized size = 2.62

$$\frac{d^4 x^{10}}{10b} - \frac{a d^4 x^7}{7b^2} + \frac{4c d^3 x^7}{7b} + \frac{a^2 d^4 x^4}{4b^3} - \frac{ac d^3 x^4}{b^2} + \frac{3c^2 d^2 x^4}{2b} + \frac{\sqrt{3} a^4 d^4 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^5} + \frac{a^4 d^4 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^5} - \frac{a^4 d^4}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^4/(b\*x^3+a),x)

[Out] 
$$-4/3/b^4/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*a^3*c*d^3+2/b^3/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*a^2*c^2*d^2-4/3/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*a*c^3*d-1/7*d^4/b^2*x^7*a+4/7*d^3/b*x^7*c-1/6/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^4+1/3/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^4+4*d/b*c^3*x+1/4*d^4/b^3*x^4*a^2+3/2*d^2/b*x^4*c^2-d^4/b^4*a^3*x-4/3/b^4/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*a^3*c*d^3+2/b^3/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*a^2*c^2*d^2-4/3/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*a*c^3*d+2/3/b^4/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a^3*c*d^3-1/b^3/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a^2*c^2*d^2+2/3/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a*c^3*d+1/3/b^5/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*a^4*d^4-6*d^2/b^2*a*c^2*x-d^3/b^2*x^4*a*c+4*d^3/b^3*a^2*c*x+1/3/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^4+1/3/b^5/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*a^4*d^4-1/6/b^5/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a^4*d^4+1/10*d^4*x^10/b$$

**maxima [A]** time = 1.19, size = 364, normalized size = 1.44

$$\frac{14 b^3 d^4 x^{10} + 20 (4 b^3 c d^3 - a b^2 d^4) x^7 + 35 (6 b^3 c^2 d^2 - 4 a b^2 c d^3 + a^2 b d^4) x^4 + 140 (4 b^3 c^3 d - 6 a b^2 c^2 d^2 + 4 a^2 b c d^3 - 4 a^3 b^2 c^2 d^2 + a^4 d^4) x + 140 b^4 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{140 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^4/(b\*x^3+a),x, algorithm="maxima")

[Out] 
$$1/140*(14*b^3*d^4*x^{10} + 20*(4*b^3*c*d^3 - a*b^2*d^4)*x^7 + 35*(6*b^3*c^2*d^2 - 4*a*b^2*c*d^3 + a^2*b*d^4)*x^4 + 140*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - a^3*d^4)*x)/b^4 + 1/3*\sqrt{3}*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)}) - 1/6*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^5*(a/b)^{(2/3)}) + 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*\log(x + (a/b)^{(1/3)})/(b^5*(a/b)^{(2/3)})$$

**mupad [B]** time = 1.43, size = 250, normalized size = 0.99

$$x \left( \frac{4c^3d}{b} - \frac{a \left( \frac{ad^4 - 4cd^3}{b^2} + \frac{6c^2d^2}{b} \right)}{b} \right) - x^7 \left( \frac{ad^4}{7b^2} - \frac{4cd^3}{7b} \right) + x^4 \left( \frac{a \left( \frac{ad^4 - 4cd^3}{b^2} + \frac{3c^2d^2}{2b} \right)}{4b} + \frac{d^4x^{10}}{10b} + \frac{\ln(b^{1/3}x + \dots)}{3a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^4/(a + b*x^3),x)`

[Out]  $x*((4*c^3*d)/b - (a*((a*(a*d^4)/b^2 - (4*c*d^3)/b))/b + (6*c^2*d^2)/b)/b$   
 $- x^7*((a*d^4)/(7*b^2) - (4*c*d^3)/(7*b)) + x^4*((a*((a*d^4)/b^2 - (4*c*d^3)/b))/(4*b) + (3*c^2*d^2)/(2*b) + (d^4*x^10)/(10*b) + (\log(b^{1/3}*x + a^{1/3})*(a*d - b*c)^4)/(3*a^{2/3}*b^{13/3}) + (\log(3^{1/2}*a^{1/3}*1i + 2*b^{1/3}*x - a^{1/3}))*((3^{1/2}*1i)/6 - 1/6)*(a*d - b*c)^4/(a^{2/3}*b^{13/3})$   
 $- (\log(3^{1/2}*a^{1/3}*1i - 2*b^{1/3}*x + a^{1/3}))*((3^{1/2}*1i)/2 + 1/2)*(a*d - b*c)^4/(3*a^{2/3}*b^{13/3})$

**sympy** [A] time = 1.31, size = 371, normalized size = 1.47

$$x^7 \left( -\frac{ad^4}{7b^2} + \frac{4cd^3}{7b} \right) + x^4 \left( \frac{a^2d^4}{4b^3} - \frac{acd^3}{b^2} + \frac{3c^2d^2}{2b} \right) + x \left( -\frac{a^3d^4}{b^4} + \frac{4a^2cd^3}{b^3} - \frac{6ac^2d^2}{b^2} + \frac{4c^3d}{b} \right) + \text{RootSum} \left( 27t^3a^2b^{13} - a^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**4/(b*x**3+a),x)`

[Out]  $x**7*(-a*d**4/(7*b**2) + 4*c*d**3/(7*b)) + x**4*(a**2*d**4/(4*b**3) - a*c*d**3/b**2 + 3*c**2*d**2/(2*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b**3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + \text{RootSum}(27*_t**3*a**2*b**13 - a**12*d**12 + 12*a**11*b*c*d**11 - 66*a**10*b**2*c**2*d**10 + 220*a**9*b**3*c**3*d**9 - 495*a**8*b**4*c**4*d**8 + 792*a**7*b**5*c**5*d**7 - 924*a**6*b**6*c**6*d**6 + 792*a**5*b**7*c**7*d**5 - 495*a**4*b**8*c**8*d**4 + 220*a**3*b**9*c**9*d**3 - 66*a**2*b**10*c**10*d**2 + 12*a*b**11*c**11*d - b**12*c**12, \text{Lambda}(_t, _t*\log(3*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4) + x))) + d**4*x**10/(10*b)$



$$3.15 \quad \int \frac{(c+dx^3)^3}{a+bx^3} dx$$

**Optimal.** Leaf size=208

$$-\frac{(bc-ad)^3 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{10/3}} + \frac{(bc-ad)^3 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{10/3}} - \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{10/3}} + dx (a^2 d^3 x^7 / b + 1/3 (-a*d+b*c)^3 \ln(a^{1/3} + b^{1/3} * x) / a^{2/3} / b^{10/3} - 1/6 (-a*d+b*c)^3 \ln(a^{2/3} - a^{1/3} * b^{1/3} * x + b^{2/3} * x^2) / a^{2/3} / b^{10/3} - 1/3 (-a*d+b*c)^3 \arctan(1/3 * (a^{1/3} - 2 * b^{1/3} * x) / a^{1/3} * 3^{1/2}) / a^{2/3} / b^{10/3} * 3^{1/2})$$

[Out] d\*(a^2\*d^2-3\*a\*b\*c\*d+3\*b^2\*c^2)\*x/b^3+1/4\*d^2\*(-a\*d+3\*b\*c)\*x^4/b^2+1/7\*d^3\*x^7/b+1/3\*(-a\*d+b\*c)^3\*ln(a^(1/3)+b^(1/3)\*x)/a^(2/3)/b^(10/3)-1/6\*(-a\*d+b\*c)^3\*ln(a^(2/3)-a^(1/3)\*b^(1/3)\*x+b^(2/3)\*x^2)/a^(2/3)/b^(10/3)-1/3\*(-a\*d+b\*c)^3\*arctan(1/3\*(a^(1/3)-2\*b^(1/3)\*x)/a^(1/3)\*3^(1/2))/a^(2/3)/b^(10/3)\*3^(1/2)

**Rubi [A]** time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{dx (a^2 d^2 - 3abcd + 3b^2 c^2)}{b^3} - \frac{(bc-ad)^3 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{10/3}} + \frac{(bc-ad)^3 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{10/3}} - \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^3/(a + b\*x^3), x]

[Out] (d\*(3\*b^2\*c^2 - 3\*a\*b\*c\*d + a^2\*d^2)\*x)/b^3 + (d^2\*(3\*b\*c - a\*d)\*x^4)/(4\*b^2) + (d^3\*x^7)/(7\*b) - ((b\*c - a\*d)^3\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(10/3)) + ((b\*c - a\*d)^3\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(10/3)) - ((b\*c - a\*d)^3\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(10/3))

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 390**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

**Rule 617**

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^3}{a + bx^3} dx &= \int \left( \frac{d(3b^2c^2 - 3abcd + a^2d^2)}{b^3} + \frac{d^2(3bc - ad)x^3}{b^2} + \frac{d^3x^6}{b} + \frac{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3}{b^3(a + bx^3)} \right) dx \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc - ad)^3 \int \frac{1}{a + bx^3} dx}{b^3} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc - ad)^3 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}b^3} + \frac{(bc - ad)^3}{3a^{2/3}b^3} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc - ad)^3 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{10/3}} - \frac{(bc - ad)^3}{3a^{2/3}b^{10/3}} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} + \frac{(bc - ad)^3 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{10/3}} - \frac{(bc - ad)^3}{3a^{2/3}b^{10/3}} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^4}{4b^2} + \frac{d^3x^7}{7b} - \frac{(bc - ad)^3 \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{10/3}} + \frac{(bc - ad)^3}{3a^{2/3}b^{10/3}} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 203, normalized size = 0.98

$$\frac{14(ad-bc)^3 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b}x + b^{2/3}x^2)}{a^{2/3}} + \frac{28(bc-ad)^3 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{2/3}} + \frac{28\sqrt{3}(bc-ad)^3 \tan^{-1}\left(\frac{2\sqrt[3]{b}x - \sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{2/3}} + 84\sqrt[3]{b} dx (a^2d^2 - 3abcd + 3b^2c^2) - \frac{84b^{10/3}}{84b^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^3/(a + b*x^3), x]
```

```
[Out] (84*b^(1/3)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x + 21*b^(4/3)*d^2*(3*b*c -
a*d)*x^4 + 12*b^(7/3)*d^3*x^7 + (28*sqrt[3]*(b*c - a*d)^3*ArcTan[(-a^(1/3)
+ 2*b^(1/3)*x)/(sqrt[3]*a^(1/3))])/a^(2/3) + (28*(b*c - a*d)^3*Log[a^(1/3)
+ b^(1/3)*x])/a^(2/3) + (14*(-(b*c) + a*d)^3*Log[a^(2/3) - a^(1/3)*b^(1/3)
*x + b^(2/3)*x^2])/a^(2/3))/(84*b^(10/3))
```

**fricas** [A] time = 0.44, size = 700, normalized size = 3.37

$$\frac{12 a^2 b^3 d^3 x^7 + 21 (3 a^2 b^3 c d^2 - a^3 b^2 d^3) x^4 - 42 \sqrt{\frac{1}{3}} (a b^4 c^3 - 3 a^2 b^3 c^2 d + 3 a^3 b^2 c d^2 - a^4 b d^3) \sqrt{\frac{(-a^2 b)^{\frac{1}{3}}}{b}} \log \left( \frac{2 a b x^2 + (-a^2 b)^{\frac{1}{3}}}{b x^3 + a} \right)}{3 (-a b^2)^{\frac{2}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^3/(b\*x^3+a),x, algorithm="fricas")

[Out] [1/84\*(12\*a^2\*b^3\*d^3\*x^7 + 21\*(3\*a^2\*b^3\*c\*d^2 - a^3\*b^2\*d^3)\*x^4 - 42\*sqrt(1/3)\*(a\*b^4\*c^3 - 3\*a^2\*b^3\*c^2\*d + 3\*a^3\*b^2\*c\*d^2 - a^4\*b\*d^3)\*sqrt((-a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 + 3\*(-a^2\*b)^(1/3)\*a\*x - a^2 - 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt((-a^2\*b)^(1/3)/b))/(b\*x^3 + a) - 14\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 28\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 84\*(3\*a^2\*b^3\*c^2\*d - 3\*a^3\*b^2\*c\*d^2 + a^4\*b\*d^3)\*x)/(a^2\*b^4), 1/84\*(12\*a^2\*b^3\*d^3\*x^7 + 21\*(3\*a^2\*b^3\*c\*d^2 - a^3\*b^2\*d^3)\*x^4 + 84\*sqrt(1/3)\*(a\*b^4\*c^3 - 3\*a^2\*b^3\*c^2\*d + 3\*a^3\*b^2\*c\*d^2 - a^4\*b\*d^3)\*sqrt(-(-a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt(-(-a^2\*b)^(1/3)/b)/a^2) - 14\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 28\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 84\*(3\*a^2\*b^3\*c^2\*d - 3\*a^3\*b^2\*c\*d^2 + a^4\*b\*d^3)\*x)/(a^2\*b^4)]

**giac** [A] time = 0.18, size = 296, normalized size = 1.42

$$\frac{\sqrt{3} (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \arctan \left( \frac{\sqrt{3} \left( 2 x + \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{a}{b} \right)^{\frac{1}{3}}} \right) (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \log \left( x^2 + \frac{2 a b x + \left( -a^2 b \right)^{\frac{1}{3}}}{b x^3 + a} \right)}{3 (-a b^2)^{\frac{2}{3}} b^2 \quad 6 (-a b^2)^{\frac{2}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^3/(b\*x^3+a),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a\*b^2)^(2/3)\*b^2) - 1/6\*(b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a\*b^2)^(2/3)\*b^2) - 1/3\*(b^7\*c^3 - 3\*a\*b^6\*c^2\*d + 3\*a^2\*b^5\*c\*d^2 - a^3\*b^4\*d^3)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a\*b^7) + 1/28\*(4\*b^6\*d^3\*x^7 + 21\*b^6\*c\*d^2\*x^4 - 7\*a\*b^5\*d^3\*x^4 + 84\*b^6\*c^2\*d\*x - 84\*a\*b^5\*c\*d^2\*x + 28\*a^2\*b^4\*d^3\*x)/b^7

**maple [B]** time = 0.05, size = 486, normalized size = 2.34

$$\frac{d^3 x^7}{7b} - \frac{a d^3 x^4}{4b^2} + \frac{3c d^2 x^4}{4b} - \frac{\sqrt{3} a^3 d^3 \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} - \frac{a^3 d^3 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{a^3 d^3 \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}} x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6 \left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{\sqrt{3} a^2 d^3}{3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^3/(b*x^3+a),x)
```

```
[Out] 1/7*d^3*x^7/b-1/4*d^3/b^2*x^4*a+3/4*d^2/b*x^4*c+d^3/b^3*a^2*x-3*d^2/b^2*a*c*x+3*d/b*c^2*x-1/3/b^4/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^3*d^3+1/b^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a^2*c*d^2-1/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*a*c^2*d+1/3/b/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3+1/6/b^4/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^3*d^3-1/2/b^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a^2*c*d^2+1/2/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*a*c^2*d-1/6/b/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3-1/3/b^4/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^3*d^3+1/b^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a^2*c*d^2-1/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*a*c^2*d+1/3/b/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3
```

**maxima [A]** time = 1.32, size = 273, normalized size = 1.31

$$\frac{4b^2 d^3 x^7 + 7(3b^2 c d^2 - a b d^3) x^4 + 28(3b^2 c^2 d - 3 a b c d^2 + a^2 d^3) x}{28 b^3} + \frac{\sqrt{3}(b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \arctan\left(\frac{\sqrt{3} \left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3 b^4 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^3/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] 1/28*(4*b^2*d^3*x^7 + 7*(3*b^2*c*d^2 - a*b*d^3)*x^4 + 28*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3 + 1/3*sqrt(3)*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*arctan(1/3*sqrt(3)*(2*x - (a/b)^(1/3))/(a/b)^(1/3))/(b^4*(a/b)^(2/3)) - 1/6*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x^2 - x*(a/b)^(1/3) + (a/b)^(2/3))/(b^4*(a/b)^(2/3)) + 1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(x + (a/b)^(1/3))/(b^4*(a/b)^(2/3))
```

**mupad [B]** time = 1.40, size = 192, normalized size = 0.92

$$x \left( \frac{3c^2 d}{b} + \frac{a \left( \frac{a d^3}{b^2} - \frac{3c d^2}{b} \right)}{b} \right) - x^4 \left( \frac{a d^3}{4 b^2} - \frac{3c d^2}{4 b} \right) + \frac{d^3 x^7}{7 b} - \frac{\ln(b^{1/3} x + a^{1/3}) (a d - b c)^3}{3 a^{2/3} b^{10/3}} - \frac{\ln(2 b^{1/3} x - a^{1/3} + \sqrt{3} a^{1/3})}{3 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^3/(a + b*x^3),x)
```

```
[Out] x*((3*c^2*d)/b + (a*((a*d^3)/b^2 - (3*c*d^2)/b))/b - x^4*((a*d^3)/(4*b^2) - (3*c*d^2)/(4*b)) + (d^3*x^7)/(7*b) - (log(b^(1/3)*x + a^(1/3))*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) - (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^3)/(3*a^(2/3)*b^(10/3))
```

$a^{1/3}i - 2b^{1/3}x + a^{1/3}) \cdot ((3^{1/2}i)/6 + 1/6) \cdot (ad - bc)^3 / (a^{2/3}b^{10/3})$

**sympy [A]** time = 1.00, size = 257, normalized size = 1.24

$$x^4 \left( -\frac{ad^3}{4b^2} + \frac{3cd^2}{4b} \right) + x \left( \frac{a^2d^3}{b^3} - \frac{3acd^2}{b^2} + \frac{3c^2d}{b} \right) + \text{RootSum} \left( 27t^3a^2b^{10} + a^9d^9 - 9a^8bcd^8 + 36a^7b^2c^2d^7 - 84a^6b^3c^3d^6 + 126a^5b^4c^4d^5 - 126a^4b^5c^5d^4 + 84a^3b^6c^6d^3 - 36a^2b^7c^7d^2 + 9ab^8c^8d - b^9c^9, \text{Lambda}(t, t \cdot \log(-3t^3a^2b^{10}/(a^3d^9 - 3a^2b^3cd^8 + 3ab^2c^2d^7 - b^3c^3d^6) + x)) \right) + d^3x^7/(7b^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*3/(b\*x\*\*3+a),x)

[Out] x\*\*4\*(-a\*d\*\*3/(4\*b\*\*2) + 3\*c\*d\*\*2/(4\*b)) + x\*(a\*\*2\*d\*\*3/b\*\*3 - 3\*a\*c\*d\*\*2/b\*\*2 + 3\*c\*\*2\*d/b) + RootSum(27\*\_t\*\*3\*a\*\*2\*b\*\*10 + a\*\*9\*d\*\*9 - 9\*a\*\*8\*b\*c\*d\*\*8 + 36\*a\*\*7\*b\*\*2\*c\*\*2\*d\*\*7 - 84\*a\*\*6\*b\*\*3\*c\*\*3\*d\*\*6 + 126\*a\*\*5\*b\*\*4\*c\*\*4\*d\*\*5 - 126\*a\*\*4\*b\*\*5\*c\*\*5\*d\*\*4 + 84\*a\*\*3\*b\*\*6\*c\*\*6\*d\*\*3 - 36\*a\*\*2\*b\*\*7\*c\*\*7\*d\*\*2 + 9\*a\*b\*\*8\*c\*\*8\*d - b\*\*9\*c\*\*9, Lambda(\_t, \_t\*log(-3\*\_t\*a\*b\*\*3/(a\*\*3\*d\*\*3 - 3\*a\*\*2\*b\*c\*d\*\*2 + 3\*a\*b\*\*2\*c\*\*2\*d - b\*\*3\*c\*\*3) + x))) + d\*\*3\*x\*\*7/(7\*b\*\*3)

$$3.16 \quad \int \frac{(c+dx^3)^2}{a+bx^3} dx$$

**Optimal.** Leaf size=173

$$\frac{(bc-ad)^2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{7/3}} + \frac{(bc-ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{7/3}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{7/3}} + \frac{dx(2bc-a)}{b^2}$$

[Out] d\*(-a\*d+2\*b\*c)\*x/b^2+1/4\*d^2\*x^4/b+1/3\*(-a\*d+b\*c)^2\*ln(a^(1/3)+b^(1/3)\*x)/a^(2/3)/b^(7/3)-1/6\*(-a\*d+b\*c)^2\*ln(a^(2/3)-a^(1/3)\*b^(1/3)\*x+b^(2/3)\*x^2)/a^(2/3)/b^(7/3)-1/3\*(-a\*d+b\*c)^2\*arctan(1/3\*(a^(1/3)-2\*b^(1/3)\*x)/a^(1/3)\*3^(1/2))/a^(2/3)/b^(7/3)\*3^(1/2)

**Rubi [A]** time = 0.12, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)^2 \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3} b^{7/3}} + \frac{(bc-ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3} b^{7/3}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3} b^{7/3}} + \frac{dx(2bc-a)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3), x]

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^4)/(4\*b) - ((b\*c - a\*d)^2\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))]/(Sqrt[3]\*a^(2/3)\*b^(7/3)) + ((b\*c - a\*d)^2\*Log[a^(1/3) + b^(1/3)\*x]/(3\*a^(2/3)\*b^(7/3)) - ((b\*c - a\*d)^2\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2]/(6\*a^(2/3)\*b^(7/3))

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^2}{a + bx^3} dx &= \int \left( \frac{d(2bc - ad)}{b^2} + \frac{d^2x^3}{b} + \frac{b^2c^2 - 2abcd + a^2d^2}{b^2(a + bx^3)} \right) dx \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^4}{4b} + \frac{(bc - ad)^2 \int \frac{1}{a + bx^3} dx}{b^2} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^4}{4b} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}b^2} + \frac{(bc - ad)^2 \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}b^2} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^4}{4b} + \frac{(bc - ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{7/3}} - \frac{(bc - ad)^2 \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}b^{7/3}} + \dots \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^4}{4b} + \frac{(bc - ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{7/3}} - \frac{(bc - ad)^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{7/3}} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^4}{4b} - \frac{(bc - ad)^2 \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{7/3}} + \frac{(bc - ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{7/3}} - \dots \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 167, normalized size = 0.97

$$\frac{-2(bc - ad)^2 \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 3a^{2/3}b^{4/3}d^2x^4 - 12a^{2/3}\sqrt[3]{b}dx(ad - 2bc) + 4(bc - ad)^2 \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{12a^{2/3}b^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)^2/(a + b*x^3), x]
```

```
[Out] (-12*a^(2/3)*b^(1/3)*d*(-2*b*c + a*d)*x + 3*a^(2/3)*b^(4/3)*d^2*x^4 + 4*Sqr
t[3]*(b*c - a*d)^2*ArcTan[(-a^(1/3) + 2*b^(1/3)*x)/(Sqrt[3]*a^(1/3))] + 4*(
b*c - a*d)^2*Log[a^(1/3) + b^(1/3)*x] - 2*(b*c - a*d)^2*Log[a^(2/3) - a^(1/
3)*b^(1/3)*x + b^(2/3)*x^2])/(12*a^(2/3)*b^(7/3))
```

**fricas** [A] time = 0.45, size = 507, normalized size = 2.93

$$\frac{3 a^2 b^2 d^2 x^4 + 6 \sqrt{\frac{1}{3}} (a b^3 c^2 - 2 a^2 b^2 c d + a^3 b d^2) \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}} \log \left( \frac{2 a b x^3 - 3 (a^2 b)^{\frac{1}{3}} a x - a^2 + 3 \sqrt{\frac{1}{3}} \left( 2 a b x^2 + (a^2 b)^{\frac{2}{3}} x - (a^2 b)^{\frac{1}{3}} a \right) \sqrt{-\frac{(a^2 b)^{\frac{1}{3}}}{b}}}{b x^3 + a} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a),x, algorithm="fricas")

[Out] [1/12\*(3\*a^2\*b^2\*d^2\*x^4 + 6\*sqrt(1/3)\*(a\*b^3\*c^2 - 2\*a^2\*b^2\*c\*d + a^3\*b\*d^2)\*sqrt(-(a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 - 3\*(a^2\*b)^(1/3)\*a\*x - a^2 + 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt(-(a^2\*b)^(1/3)/b))/(b\*x^3 + a)) - 2\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 12\*(2\*a^2\*b^2\*c\*d - a^3\*b\*d^2)\*x)/(a^2\*b^3), 1/12\*(3\*a^2\*b^2\*d^2\*x^4 + 12\*sqrt(1/3)\*(a\*b^3\*c^2 - 2\*a^2\*b^2\*c\*d + a^3\*b\*d^2)\*sqrt((a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt((a^2\*b)^(1/3)/b)/a^2) - 2\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 4\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 12\*(2\*a^2\*b^2\*c\*d - a^3\*b\*d^2)\*x)/(a^2\*b^3)]

**giac** [A] time = 0.32, size = 211, normalized size = 1.22

$$\frac{\sqrt{3} (b^2 c^2 - 2 a b c d + a^2 d^2) \arctan \left( \frac{\sqrt{3} \left( 2 x + \left( -\frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left( -\frac{a}{b} \right)^{\frac{1}{3}}} \right) (b^2 c^2 - 2 a b c d + a^2 d^2) \log \left( x^2 + x \left( -\frac{a}{b} \right)^{\frac{1}{3}} + \left( -\frac{a}{b} \right)^{\frac{2}{3}} \right) (b^4 c^2 - \dots)}{3 (-ab^2)^{\frac{2}{3}} b \quad 6 (-ab^2)^{\frac{2}{3}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a/b)^(1/3))/((-a\*b^2)^(2/3)\*b) - 1/6\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a\*b^2)^(2/3)\*b) - 1/3\*(b^4\*c^2 - 2\*a\*b^3\*c\*d + a^2\*b^2\*d^2)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a\*b^4) + 1/4\*(b^3\*d^2\*x^4 + 8\*b^3\*c\*d\*x - 4\*a\*b^2\*d^2\*x)/b^4

**maple** [B] time = 0.04, size = 334, normalized size = 1.93

$$\frac{d^2 x^4}{4b} + \frac{\sqrt{3} a^2 d^2 \arctan \left( \frac{\sqrt{3} \left( \frac{2x}{\left( \frac{a}{b} \right)^{\frac{1}{3}}} - 1 \right)}{3} \right)}{3 \left( \frac{a}{b} \right)^{\frac{2}{3}} b^3} + \frac{a^2 d^2 \ln \left( x + \left( \frac{a}{b} \right)^{\frac{1}{3}} \right)}{3 \left( \frac{a}{b} \right)^{\frac{2}{3}} b^3} - \frac{a^2 d^2 \ln \left( x^2 - \left( \frac{a}{b} \right)^{\frac{1}{3}} x + \left( \frac{a}{b} \right)^{\frac{2}{3}} \right)}{6 \left( \frac{a}{b} \right)^{\frac{2}{3}} b^3} - \frac{2\sqrt{3} a c d \arctan \left( \frac{\sqrt{3} \left( \frac{2x}{\left( \frac{a}{b} \right)^{\frac{1}{3}}} - 1 \right)}{3} \right)}{3 \left( \frac{a}{b} \right)^{\frac{2}{3}} b^2}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((d\*x^3+c)^2/(b\*x^3+a), x)

[Out]  $\frac{1}{4}d^2x^4/b - d^2/b^2 * a*x + 2*d/b*c*x + 1/3/b^3/(a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * a^2*d^2 - 2/3/b^2/(a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * a*c*d + 1/3/b/(a/b)^{(2/3)} * \ln(x + (a/b)^{(1/3)}) * c^2 - 1/6/b^3/(a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * a^2*d^2 + 1/3/b^2/(a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * a*c*d - 1/6/b/(a/b)^{(2/3)} * \ln(x^2 - (a/b)^{(1/3)} * x + (a/b)^{(2/3)}) * c^2 + 1/3/b^3/(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * a^2*d^2 - 2/3/b^2/(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * a*c*d + 1/3/b/(a/b)^{(2/3)} * 3^{(1/2)} * \arctan(1/3 * 3^{(1/2)} * (2/(a/b)^{(1/3)} * x - 1)) * c^2$

**maxima** [A] time = 1.32, size = 190, normalized size = 1.10

$$\frac{bd^2x^4 + 4(2bcd - ad^2)x}{4b^2} + \frac{\sqrt{3}(b^2c^2 - 2abcd + a^2d^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2 - 2abcd + a^2d^2) \log\left(x^2 - \dots\right)}{6b^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a), x, algorithm="maxima")

[Out]  $\frac{1}{4}*(b*d^2*x^4 + 4*(2*b*c*d - a*d^2)*x)/b^2 + 1/3*\sqrt{3}*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^3*(a/b)^{(2/3)}) - 1/6*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^3*(a/b)^{(2/3)}) + 1/3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*\log(x + (a/b)^{(1/3)})/(b^3*(a/b)^{(2/3)})$

**mupad** [B] time = 1.38, size = 152, normalized size = 0.88

$$\frac{d^2x^4}{4b} - x\left(\frac{ad^2}{b^2} - \frac{2cd}{b}\right) + \frac{\ln(b^{1/3}x + a^{1/3})(ad - bc)^2}{3a^{2/3}b^{7/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)\left(-\frac{1}{6} + \frac{\sqrt{3}i}{6}\right)(ad - bc)^2}{a^{2/3}b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3), x)

[Out]  $(d^2*x^4)/(4*b) - x*((a*d^2)/b^2 - (2*c*d)/b) + (\log(b^{(1/3)}*x + a^{(1/3)}))*(a*d - b*c)^2/(3*a^{(2/3)}*b^{(7/3)}) + (\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)}))*((3^{(1/2)}*1i)/6 - 1/6)*(a*d - b*c)^2/(a^{(2/3)}*b^{(7/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)}))*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^2/(3*a^{(2/3)}*b^{(7/3)})$

**sympy** [A] time = 0.69, size = 156, normalized size = 0.90

$$x\left(-\frac{ad^2}{b^2} + \frac{2cd}{b}\right) + \text{RootSum}\left(27t^3a^2b^7 - a^6d^6 + 6a^5bcd^5 - 15a^4b^2c^2d^4 + 20a^3b^3c^3d^3 - 15a^2b^4c^4d^2 + 6ab^5c^5d - \dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a), x)

[Out]  $x*(-a*d**2/b**2 + 2*c*d/b) + \text{RootSum}(27*_t**3*a**2*b**7 - a**6*d**6 + 6*a**5*b*c*d**5 - 15*a**4*b**2*c**2*d**4 + 20*a**3*b**3*c**3*d**3 - 15*a**2*b**4*c**4*d**2 + 6*a*b**5*c**5*d - b**6*c**6, \text{Lambda}(_t, _t*\log(3*_t*a*b**2/(a**2*d**2 - 2*a*b*c*d + b**2*c**2) + x))) + d**2*x**4/(4*b)$

### 3.17 $\int \frac{c+dx^3}{a+bx^3} dx$

**Optimal.** Leaf size=145

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

[Out] d\*x/b+1/3\*(-a\*d+b\*c)\*ln(a^(1/3)+b^(1/3)\*x)/a^(2/3)/b^(4/3)-1/6\*(-a\*d+b\*c)\*ln(a^(2/3)-a^(1/3)\*b^(1/3)\*x+b^(2/3)\*x^2)/a^(2/3)/b^(4/3)-1/3\*(-a\*d+b\*c)\*arctan(1/3\*(a^(1/3)-2\*b^(1/3)\*x)/a^(1/3)\*3^(1/2))/a^(2/3)/b^(4/3)\*3^(1/2)

**Rubi [A]** time = 0.08, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {388, 200, 31, 634, 617, 204, 628}

$$-\frac{(bc-ad)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{6a^{2/3}b^{4/3}}+\frac{(bc-ad)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{3a^{2/3}b^{4/3}}-\frac{(bc-ad)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}}+\frac{dx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3), x]

[Out] (d\*x)/b - ((b\*c - a\*d)\*ArcTan[(a^(1/3) - 2\*b^(1/3)\*x)/(Sqrt[3]\*a^(1/3))])/(Sqrt[3]\*a^(2/3)\*b^(4/3)) + ((b\*c - a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(3\*a^(2/3)\*b^(4/3)) - ((b\*c - a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(4/3))

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(n\_), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

Q[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := S  
imp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,  
e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := D  
ist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), In  
t[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ  
[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{c + dx^3}{a + bx^3} dx &= \frac{dx}{b} - \frac{(-bc + ad) \int \frac{1}{a+bx^3} dx}{b} \\ &= \frac{dx}{b} + \frac{(bc - ad) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}b} + \frac{(bc - ad) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}b} \\ &= \frac{dx}{b} + \frac{(bc - ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(bc - ad) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}b^{4/3}} + \frac{(bc - ad) \int \frac{1}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2}}{2\sqrt[3]{a}b} \\ &= \frac{dx}{b} + \frac{(bc - ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(bc - ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{4/3}} + \frac{(bc - ad) \text{Subst}}{2\sqrt[3]{a}b} \\ &= \frac{dx}{b} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}b^{4/3}} + \frac{(bc - ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}b^{4/3}} - \frac{(bc - ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}b^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 129, normalized size = 0.89

$$\frac{-(bc - ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) + 6a^{2/3}\sqrt[3]{b}dx + 2(bc - ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{6a^{2/3}b^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3), x]

[Out] (6\*a^(2/3)\*b^(1/3)\*d\*x - 2\*Sqrt[3]\*(b\*c - a\*d)\*ArcTan[(1 - (2\*b^(1/3)\*x)/a^(1/3))/Sqrt[3]] + 2\*(b\*c - a\*d)\*Log[a^(1/3) + b^(1/3)\*x] - (b\*c - a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(6\*a^(2/3)\*b^(4/3))

**fricas** [A] time = 0.43, size = 390, normalized size = 2.69

$$\frac{6a^2bdx - 3\sqrt{\frac{1}{3}}(ab^2c - a^2bd)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}} \log\left(\frac{2abx^3 + 3(-a^2b)^{\frac{1}{3}}ax - a^2 - 3\sqrt{\frac{1}{3}}\left(2abx^2 + (-a^2b)^{\frac{2}{3}}x + (-a^2b)^{\frac{1}{3}}a\right)\sqrt{\frac{(-a^2b)^{\frac{1}{3}}}{b}}}{bx^3 + a}\right) - (-a^2b)^{\frac{2}{3}}}{6a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a),x, algorithm="fricas")

[Out] [1/6\*(6\*a^2\*b\*d\*x - 3\*sqrt(1/3)\*(a\*b^2\*c - a^2\*b\*d)\*sqrt((-a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 + 3\*(-a^2\*b)^(1/3)\*a\*x - a^2 - 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt((-a^2\*b)^(1/3)/b))/(b\*x^3 + a) - (-a^2\*b)^(2/3)\*(b\*c - a\*d)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 2\*(-a^2\*b)^(2/3)\*(b\*c - a\*d)\*log(a\*b\*x + (-a^2\*b)^(2/3)))/(a^2\*b^2), 1/6\*(6\*a^2\*b\*d\*x + 6\*sqrt(1/3)\*(a\*b^2\*c - a^2\*b\*d)\*sqrt(-(-a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt(-(-a^2\*b)^(1/3)/b)/a^2) - (-a^2\*b)^(2/3)\*(b\*c - a\*d)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 2\*(-a^2\*b)^(2/3)\*(b\*c - a\*d)\*log(a\*b\*x + (-a^2\*b)^(2/3)))/(a^2\*b^2)]

**giac** [A] time = 0.19, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) (bc - ad) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right) \frac{dx}{b} + \frac{(bc - ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3ab}}{3(-ab^2)^{\frac{2}{3}} + 6(-ab^2)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b\*c - a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(-a\*b^2)^(2/3) - 1/6\*(b\*c - a\*d)\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/(-a\*b^2)^(2/3) + d\*x/b - 1/3\*(b\*c - a\*d)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a\*b)

**maple** [A] time = 0.05, size = 195, normalized size = 1.34

$$\frac{\sqrt{3} ad \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right) ad \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) ad \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right) \sqrt{3} c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right) c \ln\left(x\right)}{3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2 + 3\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2 + 6\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2 + 3\left(\frac{a}{b}\right)^{\frac{2}{3}}b + 3\left(\frac{a}{b}\right)^{\frac{2}{3}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)/(b\*x^3+a),x)

[Out]  $d*x/b - 1/3/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*a*d + 1/3/b/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c + 1/6/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*a*d - 1/6/b/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c - 1/3/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*a*d + 1/3/b/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c$

**maxima [A]** time = 1.08, size = 128, normalized size = 0.88

$$\frac{dx}{b} + \frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(bc - ad) \log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(bc - ad) \log\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a), x, algorithm="maxima")

[Out]  $d*x/b + 1/3*\sqrt{3}*(b*c - a*d)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(b^2*(a/b)^{(2/3)}) - 1/6*(b*c - a*d)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(b^2*(a/b)^{(2/3)}) + 1/3*(b*c - a*d)*\log(x + (a/b)^{(1/3)})/(b^2*(a/b)^{(2/3)})$

**mupad [B]** time = 1.38, size = 123, normalized size = 0.85

$$\frac{dx}{b} - \frac{\ln(b^{1/3}x + a^{1/3})(ad - bc)}{3a^{2/3}b^{4/3}} + \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(ad - bc)}{3a^{2/3}b^{4/3}} - \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}i)\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)(ad - bc)}{3a^{2/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)/(a + b\*x^3), x)

[Out]  $(d*x)/b - (\log(b^{(1/3)}*x + a^{(1/3)})*(a*d - b*c))/(3*a^{(2/3)}*b^{(4/3)}) + (\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(a*d - b*c))/(3*a^{(2/3)}*b^{(4/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(a*d - b*c))/(3*a^{(2/3)}*b^{(4/3)})$

**sympy [A]** time = 0.44, size = 71, normalized size = 0.49

$$\text{RootSum}\left(27t^3a^2b^4 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t \log\left(-\frac{3tab}{ad - bc} + x\right)\right)\right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a), x)

[Out]  $\text{RootSum}(27*_t**3*a**2*b**4 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3, \text{Lambda}(_t, _t*\log(-3*_t*a*b/(a*d - b*c) + x))) + d*x/b$

$$3.18 \quad \int \frac{1}{(a+bx^3)(c+dx^3)} dx$$

**Optimal.** Leaf size=288

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)} + \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3}(bc - ad)}$$

[Out]  $1/3*b^{(2/3)}*\ln(a^{(1/3)}+b^{(1/3)}*x)/a^{(2/3)/(-a*d+b*c)}-1/3*d^{(2/3)}*\ln(c^{(1/3)}+d^{(1/3)}*x)/c^{(2/3)/(-a*d+b*c)}-1/6*b^{(2/3)}*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*x+b^{(2/3)}*x^2)/a^{(2/3)/(-a*d+b*c)}+1/6*d^{(2/3)}*\ln(c^{(2/3)}-c^{(1/3)}*d^{(1/3)}*x+d^{(2/3)}*x^2)/c^{(2/3)/(-a*d+b*c)}-1/3*b^{(2/3)}*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*x)/a^{(1/3)*3^{(1/2)}}/a^{(2/3)/(-a*d+b*c)}*3^{(1/2)}+1/3*d^{(2/3)}*\arctan(1/3*(c^{(1/3)}-2*d^{(1/3)}*x)/c^{(1/3)*3^{(1/2)}}/c^{(2/3)/(-a*d+b*c)}*3^{(1/2)})$

**Rubi [A]** time = 0.15, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {391, 200, 31, 634, 617, 204, 628}

$$\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)} - \frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)} + \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3} x^2)}{6c^{2/3}(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)\*(c + d\*x^3)), x]

[Out]  $-(b^{(2/3)}*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})]) / (\text{Sqrt}[3]*a^{(2/3)}*(b*c - a*d)) + (d^{(2/3)}*\text{ArcTan}[(c^{(1/3)} - 2*d^{(1/3)}*x)/(\text{Sqrt}[3]*c^{(1/3)})]) / (\text{Sqrt}[3]*c^{(2/3)}*(b*c - a*d)) + (b^{(2/3)}*\text{Log}[a^{(1/3)} + b^{(1/3)}*x]) / (3*a^{(2/3)}*(b*c - a*d)) - (d^{(2/3)}*\text{Log}[c^{(1/3)} + d^{(1/3)}*x]) / (3*c^{(2/3)}*(b*c - a*d)) - (b^{(2/3)}*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2]) / (6*a^{(2/3)}*(b*c - a*d)) + (d^{(2/3)}*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2]) / (6*c^{(2/3)}*(b*c - a*d))$

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 391

Int[1/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^3)(c + dx^3)} dx &= \frac{b \int \frac{1}{a+bx^3} dx}{bc - ad} - \frac{d \int \frac{1}{c+dx^3} dx}{bc - ad} \\ &= \frac{b \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}(bc - ad)} + \frac{b \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}(bc - ad)} - \frac{d \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{2/3}(bc - ad)} - \frac{d \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{2/3}(bc - ad)} \\ &= \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc - ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc - ad)} - \frac{b^{2/3} \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}(bc - ad)} + \frac{b \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{6a^{2/3}(bc - ad)} \\ &= \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc - ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc - ad)} - \frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{6a^{2/3}(bc - ad)} \\ &= -\frac{b^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc - ad)} + \frac{d^{2/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc - ad)} + \frac{b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc - ad)} - \frac{d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc - ad)} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 224, normalized size = 0.78

$$\frac{\frac{b^{2/3} \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{a^{2/3}} - \frac{2b^{2/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{a^{2/3}} + \frac{2\sqrt{3}b^{2/3} \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{a^{2/3}} - \frac{d^{2/3} \log(c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2)}{c^{2/3}} + \frac{2d^{2/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{c^{2/3}}}{6ad - 6bc}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + b*x^3)*(c + d*x^3)), x]
```

```
[Out] ((2*Sqrt[3]*b^(2/3)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]])/a^(2/3) -
(2*Sqrt[3]*d^(2/3)*ArcTan[(1 - (2*d^(1/3)*x)/c^(1/3))/Sqrt[3]])/c^(2/3) - (
2*b^(2/3)*Log[a^(1/3) + b^(1/3)*x])/a^(2/3) + (2*d^(2/3)*Log[c^(1/3) + d^(1
/3)*x])/c^(2/3) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/
a^(2/3) - (d^(2/3)*Log[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/c^(2/3)
/(-6*b*c + 6*a*d)
```

**fricas** [A] time = 0.50, size = 254, normalized size = 0.88

$$2\sqrt{3}\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)+2\sqrt{3}\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}cx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)-\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\log\left(b^2x^2+abx\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c),x, algorithm="fricas")

[Out]  $-1/6*(2*\sqrt{3}*(-b^2/a^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*a*x*(-b^2/a^2)^{(2/3)} - \sqrt{3}*b)/b + 2*\sqrt{3}*(d^2/c^2)^{(1/3)}*\arctan(1/3*(2*\sqrt{3})*c*x*(d^2/c^2)^{(2/3)} - \sqrt{3}*d)/d - (-b^2/a^2)^{(1/3)}*\log(b^2*x^2 + a*b*x*(-b^2/a^2)^{(1/3)} + a^2*(-b^2/a^2)^{(2/3)}) - (d^2/c^2)^{(1/3)}*\log(d^2*x^2 - c*d*x*(d^2/c^2)^{(1/3)} + c^2*(d^2/c^2)^{(2/3)}) + 2*(-b^2/a^2)^{(1/3)}*\log(b*x - a*(-b^2/a^2)^{(1/3)}) + 2*(d^2/c^2)^{(1/3)}*\log(d*x + c*(d^2/c^2)^{(1/3)})/(b*c - a*d)$

**giac** [A] time = 0.27, size = 278, normalized size = 0.97

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(abc-a^2d)}+\frac{d\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(bc^2-acd)}+\frac{(-ab^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}abc-\sqrt{3}a^2d}-\frac{(cd^2)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}bc^2-\sqrt{3}acd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c),x, algorithm="giac")

[Out]  $-1/3*b*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/(a*b*c - a^2*d) + 1/3*d*(-c/d)^{(1/3)}*\log(\text{abs}(x - (-c/d)^{(1/3)}))/(b*c^2 - a*c*d) + (-a*b^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/(\sqrt{3}*a*b*c - \sqrt{3}*a^2*d) - (-c*d^2)^{(1/3)}*\arctan(1/3*\sqrt{3}*(2*x + (-c/d)^{(1/3)})/(-c/d)^{(1/3)})/(\sqrt{3}*b*c^2 - \sqrt{3}*a*c*d) + 1/6*(-a*b^2)^{(1/3)}*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/(a*b*c - a^2*d) - 1/6*(-c*d^2)^{(1/3)}*\log(x^2 + x*(-c/d)^{(1/3)} + (-c/d)^{(2/3)})/(b*c^2 - a*c*d)$

**maple** [A] time = 0.05, size = 222, normalized size = 0.77

$$\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{c}{d}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}-\frac{\ln\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(x+\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3(ad-bc)\left(\frac{c}{d}\right)^{\frac{2}{3}}}+\frac{\ln\left(x^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}x+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ad-bc)\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)/(d\*x^3+c),x)

[Out]  $-1/3/(a*d-b*c)/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})+1/6/(a*d-b*c)/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})-1/3/(a*d-b*c)/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))+1/3/(a*d-b*c)/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})-1/6/(a*d-b*c)/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})+1/3/(a*d-b*c)/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))$



**maxima** [A] time = 1.24, size = 293, normalized size = 1.02

$$\frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{a}{b}\right)^{\frac{1}{3}} - ad\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3\left(bc\left(\frac{c}{d}\right)^{\frac{1}{3}} - ad\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{\log\left(x^2 - x\left(\frac{a}{b}\right)^{\frac{1}{3}} + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{a}{b}\right)^{\frac{2}{3}} - ad\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)} + \frac{\log\left(x^2 - x\left(\frac{c}{d}\right)^{\frac{1}{3}} + \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(bc\left(\frac{c}{d}\right)^{\frac{2}{3}} - ad\left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c),x, algorithm="maxima")

[Out] 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - (a/b)^(1/3))/(a/b)^(1/3))/((b\*c\*(a/b)^(1/3) - a\*d\*(a/b)^(1/3))\*(a/b)^(1/3)) - 1/3\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(2\*x - (c/d)^(1/3))/(c/d)^(1/3))/((b\*c\*(c/d)^(1/3) - a\*d\*(c/d)^(1/3))\*(c/d)^(1/3)) - 1/6\*log(x^2 - x\*(a/b)^(1/3) + (a/b)^(2/3))/(b\*c\*(a/b)^(2/3) - a\*d\*(a/b)^(2/3)) + 1/6\*log(x^2 - x\*(c/d)^(1/3) + (c/d)^(2/3))/(b\*c\*(c/d)^(2/3) - a\*d\*(c/d)^(2/3)) + 1/3\*log(x + (a/b)^(1/3))/(b\*c\*(a/b)^(2/3) - a\*d\*(a/b)^(2/3)) - 1/3\*log(x + (c/d)^(1/3))/(b\*c\*(c/d)^(2/3) - a\*d\*(c/d)^(2/3))

**mupad** [B] time = 7.70, size = 1364, normalized size = 4.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)\*(c + d\*x^3)),x)

[Out] log(((b^2/(a^2\*(a\*d - b\*c)^3))^(1/3)\*(9\*a^2\*b^4\*d^6 + 9\*b^6\*c^2\*d^4 - 18\*a\*b^5\*c\*d^5 - 9\*b^3\*d^3\*(x + a\*c\*(b^2/(a^2\*(a\*d - b\*c)^3))^(1/3))\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(b^2/(a^2\*(a\*d - b\*c)^3))^(2/3)))/3 - 6\*b^5\*d^5\*x\*(b^2/(27\*a^5\*d^3 - 27\*a^2\*b^3\*c^3 + 81\*a^3\*b^2\*c^2\*d - 81\*a^4\*b\*c\*d^2))^(1/3) + log(((d^2/(c^2\*(a\*d - b\*c)^3))^(1/3)\*(9\*a^2\*b^4\*d^6 + 9\*b^6\*c^2\*d^4 - 18\*a\*b^5\*c\*d^5 - 9\*b^3\*d^3\*(x + a\*c\*(d^2/(c^2\*(a\*d - b\*c)^3))^(1/3))\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(d^2/(c^2\*(a\*d - b\*c)^3))^(2/3)))/3 - 6\*b^5\*d^5\*x\*(-d^2/(27\*b^3\*c^5 - 27\*a^3\*c^2\*d^3 + 81\*a^2\*b\*c^3\*d^2 - 81\*a\*b^2\*c^4\*d))^(1/3) + log(6\*b^5\*d^5\*x + ((3^(1/2)\*1i - 1)\*(b^2/(a^2\*(a\*d - b\*c)^3))^(1/3)\*(((3^(1/2)\*1i - 1)^2\*(81\*b^3\*d^3\*x\*(a\*d + b\*c)\*(a\*d - b\*c)^4 + (81\*a\*b^3\*c\*d^3\*(3^(1/2)\*1i - 1)\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(b^2/(a^2\*(a\*d - b\*c)^3))^(1/3))/2) \* (-b^2/(a^2\*(a\*d - b\*c)^3))^(2/3))/36 - 9\*a^2\*b^4\*d^6 - 9\*b^6\*c^2\*d^4 + 18\*a\*b^5\*c\*d^5)/6)\*(b^2/(27\*a^5\*d^3 - 27\*a^2\*b^3\*c^3 + 81\*a^3\*b^2\*c^2\*d - 81\*a^4\*b\*c\*d^2))^(1/3)\*(3^(1/2)\*1i - 1))/2 - (log(6\*b^5\*d^5\*x - ((3^(1/2)\*1i + 1)\*(b^2/(a^2\*(a\*d - b\*c)^3))^(1/3)\*(((3^(1/2)\*1i + 1)^2\*(81\*b^3\*d^3\*x\*(a\*d + b\*c)\*(a\*d - b\*c)^4 - (81\*a\*b^3\*c\*d^3\*(3^(1/2)\*1i + 1)\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(b^2/(a^2\*(a\*d - b\*c)^3))^(1/3))/2) \* (-b^2/(a^2\*(a\*d - b\*c)^3))^(2/3))/36 - 9\*a^2\*b^4\*d^6 - 9\*b^6\*c^2\*d^4 + 18\*a\*b^5\*c\*d^5)/6)\*(b^2/(27\*a^5\*d^3 - 27\*a^2\*b^3\*c^3 + 81\*a^3\*b^2\*c^2\*d - 81\*a^4\*b\*c\*d^2))^(1/3)\*(3^(1/2)\*1i + 1))/2 + (log(6\*b^5\*d^5\*x + ((3^(1/2)\*1i - 1)\*(d^2/(c^2\*(a\*d - b\*c)^3))^(1/3)\*(((3^(1/2)\*1i - 1)^2\*(81\*b^3\*d^3\*x\*(a\*d + b\*c)\*(a\*d - b\*c)^4 + (81\*a\*b^3\*c\*d^3\*(3^(1/2)\*1i - 1)\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(d^2/(c^2\*(a\*d - b\*c)^3))^(1/3))/2) \* (d^2/(c^2\*(a\*d - b\*c)^3))^(2/3))/36 - 9\*a^2\*b^4\*d^6 - 9\*b^6\*c^2\*d^4 + 18\*a\*b^5\*c\*d^5)/6)\*(-d^2/(27\*b^3\*c^5 - 27\*a^3\*c^2\*d^3 + 81\*a^2\*b\*c^3\*d^2 - 81\*a\*b^2\*c^4\*d))^(1/3)\*(3^(1/2)\*1i - 1))/2 - (log(6\*b^5\*d^5\*x - ((3^(1/2)\*1i + 1)\*(d^2/(c^2\*(a\*d - b\*c)^3))^(1/3)\*(((3^(1/2)\*1i + 1)^2\*(81\*b^3\*d^3\*x\*(a\*d + b\*c)\*(a\*d - b\*c)^4 - (81\*a\*b^3\*c\*d^3\*(3^(1/2)\*1i + 1)\*(a\*d + b\*c)\*(a\*d - b\*c)^4\*(d^2/(c^2\*(a\*d - b\*c)^3))^(1/3))/2) \* (d^2/(c^2\*(a\*d - b\*c)^3))^(2/3))/36 - 9\*a^2\*b^4\*d^6 - 9\*b^6\*c^2\*d^4 + 18\*a\*b^5\*c\*d^5)/6)\*(-d^2/(27\*b^3\*c^5 - 27\*a^3\*c^2\*d^3 + 81\*a^2\*b\*c^3\*d^2 - 81\*a\*b^2\*c^4\*d))^(1/3)\*(3^(1/2)\*1i + 1))/2

sympy [A] time = 79.72, size = 447, normalized size = 1.55

$$\text{RootSum}\left(t^3(27a^5d^3 - 81a^4bcd^2 + 81a^3b^2c^2d - 27a^2b^3c^3) + b^2, \left(t \mapsto t \log\left(x + \frac{81t^4a^7c^2d^5 - 243t^4a^6bc^3d^4 + 162t^4a^5b^2c^4d^3 + 162t^4a^4b^3c^5d^2 - 243t^4a^3b^4c^6d + 81t^4a^2b^5c^7 - 3t^4a^4d^4 + 3t^4a^3b^3c^3d - 3t^4b^4c^4}{a^2bd^3 + b^3c^2d}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)/(d\*x\*\*3+c),x)

[Out] RootSum(\_t\*\*3\*(27\*a\*\*5\*d\*\*3 - 81\*a\*\*4\*b\*c\*d\*\*2 + 81\*a\*\*3\*b\*\*2\*c\*\*2\*d - 27\*a\*\*2\*b\*\*3\*c\*\*3) + b\*\*2, Lambda(\_t, \_t\*log(x + (81\*\_t\*\*4\*a\*\*7\*c\*\*2\*d\*\*5 - 243\*\_t\*\*4\*a\*\*6\*b\*c\*\*3\*d\*\*4 + 162\*\_t\*\*4\*a\*\*5\*b\*\*2\*c\*\*4\*d\*\*3 + 162\*\_t\*\*4\*a\*\*4\*b\*\*3\*c\*\*5\*d\*\*2 - 243\*\_t\*\*4\*a\*\*3\*b\*\*4\*c\*\*6\*d + 81\*\_t\*\*4\*a\*\*2\*b\*\*5\*c\*\*7 - 3\*\_t\*\*4\*a\*\*4\*d\*\*4 + 3\*\_t\*\*4\*a\*\*3\*b\*\*3\*c\*\*3\*d - 3\*\_t\*\*4\*b\*\*4\*c\*\*4)/(a\*\*2\*b\*d\*\*3 + b\*\*3\*c\*\*2\*d)))) + RootSum(\_t\*\*3\*(27\*a\*\*3\*c\*\*2\*d\*\*3 - 81\*a\*\*2\*b\*c\*\*3\*d\*\*2 + 81\*a\*b\*\*2\*c\*\*4\*d - 27\*b\*\*3\*c\*\*5) - d\*\*2, Lambda(\_t, \_t\*log(x + (81\*\_t\*\*4\*a\*\*7\*c\*\*2\*d\*\*5 - 243\*\_t\*\*4\*a\*\*6\*b\*c\*\*3\*d\*\*4 + 162\*\_t\*\*4\*a\*\*5\*b\*\*2\*c\*\*4\*d\*\*3 + 162\*\_t\*\*4\*a\*\*4\*b\*\*3\*c\*\*5\*d\*\*2 - 243\*\_t\*\*4\*a\*\*3\*b\*\*4\*c\*\*6\*d + 81\*\_t\*\*4\*a\*\*2\*b\*\*5\*c\*\*7 - 3\*\_t\*\*4\*a\*\*4\*d\*\*4 + 3\*\_t\*\*4\*a\*\*3\*b\*\*3\*c\*\*3\*d - 3\*\_t\*\*4\*b\*\*4\*c\*\*4)/(a\*\*2\*b\*d\*\*3 + b\*\*3\*c\*\*2\*d))))

$$3.19 \quad \int \frac{1}{(a+bx^3)(c+dx^3)^2} dx$$

**Optimal.** Leaf size=346

$$-\frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)^2} + \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)^2} - \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)^2} + \frac{d^{2/3}(5bc - 2ad) \log(c^{2/3} - \dots)}{18c^{5/3}(bc - \dots)}$$

[Out]  $-1/3*d*x/c/(-a*d+b*c)/(d*x^3+c)+1/3*b^(5/3)*\ln(a^(1/3)+b^(1/3)*x)/a^(2/3)/(-a*d+b*c)^2-1/9*d^(2/3)*(-2*a*d+5*b*c)*\ln(c^(1/3)+d^(1/3)*x)/c^(5/3)/(-a*d+b*c)^2-1/6*b^(5/3)*\ln(a^(2/3)-a^(1/3)*b^(1/3)*x+b^(2/3)*x^2)/a^(2/3)/(-a*d+b*c)^2+1/18*d^(2/3)*(-2*a*d+5*b*c)*\ln(c^(2/3)-c^(1/3)*d^(1/3)*x+d^(2/3)*x^2)/c^(5/3)/(-a*d+b*c)^2-1/3*b^(5/3)*\arctan(1/3*(a^(1/3)-2*b^(1/3)*x)/a^(1/3)*3^(1/2))/a^(2/3)/(-a*d+b*c)^2*3^(1/2)+1/9*d^(2/3)*(-2*a*d+5*b*c)*\arctan(1/3*(c^(1/3)-2*d^(1/3)*x)/c^(1/3)*3^(1/2))/c^(5/3)/(-a*d+b*c)^2*3^(1/2)$

**Rubi [A]** time = 0.27, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$-\frac{b^{5/3} \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{6a^{2/3}(bc - ad)^2} + \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{3a^{2/3}(bc - ad)^2} - \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b} x}{\sqrt{3} \sqrt[3]{a}}\right)}{\sqrt{3} a^{2/3}(bc - ad)^2} + \frac{d^{2/3}(5bc - 2ad) \log(c^{2/3} - \dots)}{18c^{5/3}(bc - \dots)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)\*(c + d\*x^3)^2), x]

[Out]  $-(d*x)/(3*c*(b*c - a*d)*(c + d*x^3)) - (b^(5/3)*\text{ArcTan}[(a^(1/3) - 2*b^(1/3)*x)/(\text{Sqrt}[3]*a^(1/3))])/(\text{Sqrt}[3]*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*\text{ArcTan}[(c^(1/3) - 2*d^(1/3)*x)/(\text{Sqrt}[3]*c^(1/3))])/ (3*\text{Sqrt}[3]*c^(5/3)*(b*c - a*d)^2) + (b^(5/3)*\text{Log}[a^(1/3) + b^(1/3)*x])/ (3*a^(2/3)*(b*c - a*d)^2) - (d^(2/3)*(5*b*c - 2*a*d)*\text{Log}[c^(1/3) + d^(1/3)*x])/ (9*c^(5/3)*(b*c - a*d)^2) - (b^(5/3)*\text{Log}[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2])/ (6*a^(2/3)*(b*c - a*d)^2) + (d^(2/3)*(5*b*c - 2*a*d)*\text{Log}[c^(2/3) - c^(1/3)*d^(1/3)*x + d^(2/3)*x^2])/ (18*c^(5/3)*(b*c - a*d)^2)$

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c -

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

### Rule 522

```
Int[((e_) + (f_)*(x_)^(n_))/((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(
n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + bx^3)(c + dx^3)^2} dx &= -\frac{dx}{3c(bc - ad)(c + dx^3)} + \frac{\int \frac{3bc - 2ad - 2bdx^3}{(a + bx^3)(c + dx^3)} dx}{3c(bc - ad)} \\
&= -\frac{dx}{3c(bc - ad)(c + dx^3)} + \frac{b^2 \int \frac{1}{a + bx^3} dx}{(bc - ad)^2} - \frac{(d(5bc - 2ad)) \int \frac{1}{c + dx^3} dx}{3c(bc - ad)^2} \\
&= -\frac{dx}{3c(bc - ad)(c + dx^3)} + \frac{b^2 \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{3a^{2/3}(bc - ad)^2} + \frac{b^2 \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{3a^{2/3}(bc - ad)^2} - \frac{d(5bc - 2ad)}{9c^{5/3}} \\
&= -\frac{dx}{3c(bc - ad)(c + dx^3)} + \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc - ad)^2} - \frac{d^{2/3}(5bc - 2ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}(bc - ad)^2} \\
&= -\frac{dx}{3c(bc - ad)(c + dx^3)} + \frac{b^{5/3} \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{3a^{2/3}(bc - ad)^2} - \frac{d^{2/3}(5bc - 2ad) \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{9c^{5/3}(bc - ad)^2} \\
&= -\frac{dx}{3c(bc - ad)(c + dx^3)} - \frac{b^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{2/3}(bc - ad)^2} + \frac{d^{2/3}(5bc - 2ad) \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{3\sqrt{3}c^{5/3}(bc - ad)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 336, normalized size = 0.97

$$-3b^{5/3}c^{5/3}(c+dx^3)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)+a^{2/3}d^{2/3}(c+dx^3)(5bc-2ad)\log\left(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)\*(c + d\*x^3)^2), x]

[Out] (6\*a^(2/3)\*c^(2/3)\*d\*(-(b\*c) + a\*d)\*x - 6\*Sqrt[3]\*b^(5/3)\*c^(5/3)\*(c + d\*x^3)\*ArcTan[(1 - (2\*b^(1/3)\*x)/a^(1/3))/Sqrt[3]] - 2\*Sqrt[3]\*a^(2/3)\*d^(2/3)\*(-5\*b\*c + 2\*a\*d)\*(c + d\*x^3)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]] + 6\*b^(5/3)\*c^(5/3)\*(c + d\*x^3)\*Log[a^(1/3) + b^(1/3)\*x] + 2\*a^(2/3)\*d^(2/3)\*(-5\*b\*c + 2\*a\*d)\*(c + d\*x^3)\*Log[c^(1/3) + d^(1/3)\*x] - 3\*b^(5/3)\*c^(5/3)\*(c + d\*x^3)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2] + a^(2/3)\*d^(2/3)\*(5\*b\*c - 2\*a\*d)\*(c + d\*x^3)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(18\*a^(2/3)\*c^(5/3)\*(b\*c - a\*d)^2\*(c + d\*x^3))

**fricas [A]** time = 7.44, size = 432, normalized size = 1.25

$$6\sqrt{3}(bcdx^3+bc^2)\left(\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-2\sqrt{3}\left((5bcd-2ad^2)x^3+5bc^2-2acd\right)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}dx\left(\frac{d^2}{c^2}\right)^{\frac{2}{3}}-\sqrt{3}d}{3d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] 1/18\*(6\*sqrt(3)\*(b\*c\*d\*x^3 + b\*c^2)\*(b^2/a^2)^(1/3)\*arctan(1/3\*(2\*sqrt(3)\*a\*x\*(b^2/a^2)^(2/3) - sqrt(3)\*b)/b) - 2\*sqrt(3)\*((5\*b\*c\*d - 2\*a\*d^2)\*x^3 + 5\*b\*c^2 - 2\*a\*c\*d)\*(d^2/c^2)^(1/3)\*arctan(1/3\*(2\*sqrt(3)\*c\*x\*(d^2/c^2)^(2/3) - sqrt(3)\*d)/d) - 3\*(b\*c\*d\*x^3 + b\*c^2)\*(b^2/a^2)^(1/3)\*log(b^2\*x^2 - a\*b\*x\*(b^2/a^2)^(1/3) + a^2\*(b^2/a^2)^(2/3)) + ((5\*b\*c\*d - 2\*a\*d^2)\*x^3 + 5\*b\*c^2 - 2\*a\*c\*d)\*(d^2/c^2)^(1/3)\*log(d^2\*x^2 - c\*d\*x\*(d^2/c^2)^(1/3) + c^2\*(d^2/c^2)^(2/3)) + 6\*(b\*c\*d\*x^3 + b\*c^2)\*(b^2/a^2)^(1/3)\*log(b\*x + a\*(b^2/a^2)^(1/3)) - 2\*((5\*b\*c\*d - 2\*a\*d^2)\*x^3 + 5\*b\*c^2 - 2\*a\*c\*d)\*(d^2/c^2)^(1/3)\*log(d\*x + c\*(d^2/c^2)^(1/3)) - 6\*(b\*c\*d - a\*d^2)\*x/(b^2\*c^4 - 2\*a\*b\*c^3\*d + a^2\*c^2\*d^2 + (b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2 + a^2\*c\*d^3)\*x^3)

**giac [A]** time = 0.20, size = 443, normalized size = 1.28

$$\frac{b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{3(ab^2c^2-2a^2bcd+a^3d^2)}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}ab^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-ab^2\right)^{\frac{1}{3}}b\log\left(x^2+x\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6(ab^2c^2-2a^2bcd+a^3d^2)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}cd^2c^2-2\sqrt{3}a^2bcd+\sqrt{3}a^3d^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6(cd^2c^2-2a^2bcd+a^3d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] -1/3\*b^2\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2) + (-a\*b^2)^(1/3)\*b\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/(sqrt(3)\*a\*b^2\*c^2 - 2\*sqrt(3)\*a^2\*b\*c\*d + sqrt(3)\*a^3\*d^2) + 1/6\*(-a\*b^2)^(1/3)\*b\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2) + 1/9\*(5\*b\*c\*d - 2\*a\*d^2)\*(-c/d)^(1/3)\*log(abs(x - (-c/d)^(1/3)))/((c\*d^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2)^(1/3))

$\sqrt[3]{(1/3))}/(b^2c^4 - 2abc^3d + a^2c^2d^2) - 1/3(5(-cd^2)^{1/3}bc - 2(-cd^2)^{1/3}ad) \arctan(1/3\sqrt{3}(2x + (-c/d)^{1/3})/(-c/d)^{1/3})/(\sqrt{3}b^2c^4 - 2\sqrt{3}abc^3d + \sqrt{3}a^2c^2d^2) - 1/18(5(-cd^2)^{1/3}bc - 2(-cd^2)^{1/3}ad) \log(x^2 + x(-c/d)^{1/3} + (-c/d)^{2/3})/(b^2c^4 - 2abc^3d + a^2c^2d^2) - 1/3dx/((dx^3 + c)(bc^2 - acd))$

**maple [A]** time = 0.05, size = 406, normalized size = 1.17

$$\frac{\frac{ad^2x}{3(ad-bc)^2(dx^3+c)c} - \frac{bdx}{3(ad-bc)^2(dx^3+c)}}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c} + \frac{2\sqrt{3}ad \arctan\left(\frac{\sqrt{3}\left(\frac{2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}-1\right)}{3}\right)}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c} + \frac{2ad \ln\left(x + \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c} - \frac{ad \ln\left(x^2 - \left(\frac{c}{d}\right)^{\frac{2}{3}}\right)}{9(ad-bc)^2\left(\frac{c}{d}\right)^{\frac{2}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)/(d\*x^3+c)^2,x)

[Out]  $1/3b/(a*d-b*c)^2/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})-1/6b/(a*d-b*c)^2/(a/b)^{2/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})+1/3b/(a*d-b*c)^2/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))+1/3*d^2/(a*d-b*c)^2/c*x/(d*x^3+c)*a-1/3*d/(a*d-b*c)^2*x/(d*x^3+c)*b+2/9*d/(a*d-b*c)^2/c/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*a-5/9/(a*d-b*c)^2/(c/d)^{2/3}*\ln(x+(c/d)^{1/3})*b-1/9*d/(a*d-b*c)^2/c/(c/d)^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*a+5/18/(a*d-b*c)^2/(c/d)^{2/3}*\ln(x^2-(c/d)^{1/3}*x+(c/d)^{2/3})*b+2/9*d/(a*d-b*c)^2/c/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*a-5/9/(a*d-b*c)^2/(c/d)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(c/d)^{1/3}*x-1))*b$

**maxima [A]** time = 1.30, size = 489, normalized size = 1.41

$$\frac{\sqrt{3}b \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\left(b^2c^2\left(\frac{a}{b}\right)^{\frac{1}{3}}-2abcd\left(\frac{a}{b}\right)^{\frac{1}{3}}+a^2d^2\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} - \frac{\sqrt{3}(5bc-2ad) \arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^2c^3\left(\frac{c}{d}\right)^{\frac{1}{3}}-2abc^2d\left(\frac{c}{d}\right)^{\frac{1}{3}}+a^2cd^2\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}} - \frac{dx}{3(bc^3-ac^2d+(a/b)^{2/3}*(a/b)^{1/3}-2abc^3d*(a/b)^{1/3}+a^2d^2*(a/b)^{1/3})*(a/b)^{1/3}) - 1/9\sqrt{3}*(5b*c - 2*a*d) \arctan(1/3\sqrt{3}(2x - (c/d)^{1/3})/(c/d)^{1/3})/((b^2c^3*(c/d)^{1/3} - 2abc^2d*(c/d)^{1/3} + a^2cd^2*(c/d)^{1/3})*(c/d)^{1/3}) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6*b*log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(b^2c^2*(a/b)^{2/3} - 2abc^2*d*(a/b)^{2/3} + a^2*d^2*(a/b)^{2/3}) + 1/18*(5b*c - 2*a*d) \log(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3})/(b^2c^3*(c/d)^{2/3} - 2abc^2*d*(c/d)^{2/3} + a^2*c*d^2*(c/d)^{2/3}) + 1/3*b*log(x + (a/b)^{1/3})/(b^2c^2*(a/b)^{2/3} - 2abc^2*d*(a/b)^{2/3} + a^2*d^2*(a/b)^{2/3}) - 1/9*(5b*c - 2*a*d) \log(x + (c/d)^{1/3})/(b^2c^3*(c/d)^{2/3} - 2abc^2*d*(c/d)^{2/3} + a^2*c*d^2*(c/d)^{2/3})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out]  $1/3*\sqrt{3}*b*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3})/(a/b)^{1/3})/((b^2*c^3*(a/b)^{1/3} - 2*a*b*c*d*(a/b)^{1/3} + a^2*d^2*(a/b)^{1/3})*(a/b)^{1/3}) - 1/9*\sqrt{3}*(5*b*c - 2*a*d) \arctan(1/3*\sqrt{3}*(2*x - (c/d)^{1/3})/(c/d)^{1/3})/((b^2*c^3*(c/d)^{1/3} - 2*a*b*c^2*d*(c/d)^{1/3} + a^2*c*d^2*(c/d)^{1/3})*(c/d)^{1/3}) - 1/3*d*x/(b*c^3 - a*c^2*d + (b*c^2*d - a*c*d^2)*x^3) - 1/6*b*log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(b^2*c^2*(a/b)^{2/3} - 2*a*b*c*d*(a/b)^{2/3} + a^2*d^2*(a/b)^{2/3}) + 1/18*(5*b*c - 2*a*d) \log(x^2 - x*(c/d)^{1/3} + (c/d)^{2/3})/(b^2*c^3*(c/d)^{2/3} - 2*a*b*c^2*d*(c/d)^{2/3} + a^2*c*d^2*(c/d)^{2/3}) + 1/3*b*log(x + (a/b)^{1/3})/(b^2*c^2*(a/b)^{2/3} - 2*a*b*c*d*(a/b)^{2/3} + a^2*d^2*(a/b)^{2/3}) - 1/9*(5*b*c - 2*a*d) \log(x + (c/d)^{1/3})/(b^2*c^3*(c/d)^{2/3} - 2*a*b*c^2*d*(c/d)^{2/3} + a^2*c*d^2*(c/d)^{2/3})$

**mupad [B]** time = 16.81, size = 2589, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/((a + b*x^3)*(c + d*x^3)^2), x)$

[Out]  $\log\left(\frac{((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}{(b*c^4 - a*c^3*d)}*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{2/3}}{81} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}{9} + \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^{11} + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^{10}*d))^{1/3}} + \log\left(\frac{((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}{(b*c^4 - a*c^3*d)}*(b^5/(a^2*(a*d - b*c)^6))^{2/3}}{9} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}{3} + \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a^7*b*c*d^5))^{1/3}} + (\log(((3^{1/2})*i - 1)*((3^{1/2})*i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (27*a*b^3*c^4*d^3*(3^{1/2})*i - 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{2/3}}{324} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}{18} + \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*(3^{1/2})*i - 1)*((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^{11} + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^{10}*d))^{1/3}}{2} - (\log(((3^{1/2})*i + 1)*((3^{1/2})*i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (27*a*b^3*c^4*d^3*(3^{1/2})*i + 1)*(a*d + b*c)*(a*d - b*c)^5*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{2/3}}{324} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{1/3}}{18} - \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*(3^{1/2})*i + 1)*((8*a^3*d^5 - 125*b^3*c^3*d^2 + 150*a*b^2*c^2*d^3 - 60*a^2*b*c*d^4)/(729*b^6*c^{11} + 729*a^6*c^5*d^6 - 4374*a^5*b*c^6*d^5 + 10935*a^2*b^4*c^9*d^2 - 14580*a^3*b^3*c^8*d^3 + 10935*a^4*b^2*c^7*d^4 - 4374*a*b^5*c^{10}*d))^{1/3}}{2} + (\log(((3^{1/2})*i - 1)*((3^{1/2})*i - 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c + (81*a*b^3*c^4*d^3*(3^{1/2})*i - 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{2/3}}{36} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}{6} + \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*(3^{1/2})*i - 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a^7*b*c*d^5))^{1/3}}{2} - (\log(((3^{1/2})*i + 1)*((3^{1/2})*i + 1)^2*((27*b^3*d^3*x*(a*d - b*c)^3*(3*b^2*c^2 - 2*a^2*d^2 + 3*a*b*c*d))/c - (81*a*b^3*c^4*d^3*(3^{1/2})*i + 1)*(a*d + b*c)*(a*d - b*c)^5*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}/(2*(b*c^4 - a*c^3*d)))*((d^2*(2*a*d - 5*b*c)^3)/(c^5*(a*d - b*c)^6))^{2/3}}{36} - \frac{(b^4*d^4*(8*a^3*d^3 - 27*b^3*c^3 + 98*a*b^2*c^2*d - 52*a^2*b*c*d^2))/(3*b*c^4 - 3*a*c^3*d)}*(b^5/(a^2*(a*d - b*c)^6))^{1/3}}{6} - \frac{(2*b^6*d^5*x*(4*a^3*d^3 - 85*b^3*c^3 + 84*a*b^2*c^2*d - 30*a^2*b*c*d^2))/(9*c^3*(a*d - b*c)^4)}*(3^{1/2})*i + 1)*(b^5/(27*a^8*d^6 + 27*a^2*b^6*c^6 - 162*a^3*b^5*c^5*d + 405*a^4*b^4*c^4*d^2 - 540*a^5*b^3*c^3*d^3 + 405*a^6*b^2*c^2*d^4 - 162*a$

$(7*b*c*d^5)^{1/3}/2 + (d*x)/(3*c*(c + d*x^3)*(a*d - b*c))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)/(d\*x\*\*3+c)\*\*2,x)

[Out] Timed out



$$3.20 \quad \int \frac{(c+dx^3)^5}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=320

$$\frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}} + \frac{(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{16/3}} - \frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}+\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}$$

[Out]  $d^2*(-4*a^3*d^3+15*a^2*b*c*d^2-20*a*b^2*c^2*d+10*b^3*c^3)*x/b^5+1/4*d^3*(3*a^2*d^2-10*a*b*c*d+10*b^2*c^2)*x^4/b^4+1/7*d^4*(-2*a*d+5*b*c)*x^7/b^3+1/10*d^5*x^10/b^2+1/3*(-a*d+b*c)^5*x/a/b^5/(b*x^3+a)+1/9*(-a*d+b*c)^4*(13*a*d+2*b*c)*\ln(a^{1/3}+b^{1/3}*x)/a^{5/3}/b^{16/3}-1/18*(-a*d+b*c)^4*(13*a*d+2*b*c)*\ln(a^{2/3}-a^{1/3}*b^{1/3}*x+b^{2/3}*x^2)/a^{5/3}/b^{16/3}-1/9*(-a*d+b*c)^4*(13*a*d+2*b*c)*\arctan(1/3*(a^{1/3}-2*b^{1/3}*x)/a^{1/3}*3^{1/2})/a^{5/3}/b^{16/3}*3^{1/2}$

**Rubi [A]** time = 0.30, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^3x^4(3a^2d^2-10abcd+10b^2c^2)}{4b^4} + \frac{d^2x(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3)}{b^5} - \frac{(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{16/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^5/(a + b\*x^3)^2, x]

[Out]  $(d^2*(10*b^3*c^3-20*a*b^2*c^2*d+15*a^2*b*c*d^2-4*a^3*d^3)*x)/b^5+(d^3*(10*b^2*c^2-10*a*b*c*d+3*a^2*d^2)*x^4)/(4*b^4)+(d^4*(5*b*c-2*a*d)*x^7)/(7*b^3)+(d^5*x^10)/(10*b^2)+((b*c-a*d)^5*x)/(3*a*b^5*(a+b*x^3))-((b*c-a*d)^4*(2*b*c+13*a*d)*\text{ArcTan}[(a^{1/3}-2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})])/(3*\text{Sqrt}[3]*a^{5/3}*b^{16/3})+((b*c-a*d)^4*(2*b*c+13*a*d)*\text{Log}[a^{1/3}+b^{1/3}*x])/(9*a^{5/3}*b^{16/3})-((b*c-a*d)^4*(2*b*c+13*a*d)*\text{Log}[a^{2/3}-a^{1/3}*b^{1/3}*x+b^{2/3}*x^2])/(18*a^{5/3}*b^{16/3})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_)^3)^-1, x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^-1, x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 385**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^5}{(a + bx^3)^2} dx &= \int \left( \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^3}{b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^6}{b^3} \right) dx \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{4b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^7}{28b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{4b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^7}{28b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{4b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^7}{28b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{4b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^7}{28b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{4b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^7}{28b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.30, size = 313, normalized size = 0.98

$$-\frac{70(bc-ad)^4(13ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{5/3}} + \frac{140(bc-ad)^4(13ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}} + \frac{140\sqrt{3}(bc-ad)^4(13ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^5/(a + b\*x^3)^2,x]

[Out] (1260\*b^(1/3)\*d^2\*(10\*b^3\*c^3 - 20\*a\*b^2\*c^2\*d + 15\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3)\*x + 315\*b^(4/3)\*d^3\*(10\*b^2\*c^2 - 10\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^4 + 180\*b^(7/3)\*d^4\*(5\*b\*c - 2\*a\*d)\*x^7 + 126\*b^(10/3)\*d^5\*x^10 + (420\*b^(1/3)\*(b\*c - a\*d)^5\*x)/(a\*(a + b\*x^3)) + (140\*sqrt(3)\*(b\*c - a\*d)^4\*(2\*b\*c + 13\*a\*d)\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(sqrt(3)\*a^(1/3))])/a^(5/3) + (140\*(b\*c - a\*d)^4\*(2\*b\*c + 13\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/a^(5/3) - (70\*(b\*c - a\*d)^4\*(2\*b\*c + 13\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(5/3))/(1260\*b^(16/3))

**fricas [B]** time = 0.47, size = 1619, normalized size = 5.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^5/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] [1/1260\*(126\*a^3\*b^5\*d^5\*x^13 + 18\*(50\*a^3\*b^5\*c\*d^4 - 13\*a^4\*b^4\*d^5)\*x^10 + 45\*(70\*a^3\*b^5\*c^2\*d^3 - 50\*a^4\*b^4\*c\*d^4 + 13\*a^5\*b^3\*d^5)\*x^7 + 315\*(40\*a^3\*b^5\*c^3\*d^2 - 70\*a^4\*b^4\*c^2\*d^3 + 50\*a^5\*b^3\*c\*d^4 - 13\*a^6\*b^2\*d^5)\*x^4 + 210\*sqrt(1/3)\*(2\*a^2\*b^6\*c^5 + 5\*a^3\*b^5\*c^4\*d - 40\*a^4\*b^4\*c^3\*d^2 + 70\*a^5\*b^3\*c^2\*d^3 - 50\*a^6\*b^2\*c\*d^4 + 13\*a^7\*b\*d^5 + (2\*a\*b^7\*c^5 + 5\*a

$$\begin{aligned} & ^2b^6c^4d - 40a^3b^5c^3d^2 + 70a^4b^4c^2d^3 - 50a^5b^3cd^4 + \\ & 13a^6b^2d^5)x^3)\sqrt{-(a^2b)^{(1/3)}/b})\log((2abx^3 - 3(a^2b)^{(1/3)} \\ & )ax - a^2 + 3\sqrt{1/3}(2abx^2 + (a^2b)^{(2/3)}x - (a^2b)^{(1/3)}a) \\ & \sqrt{-(a^2b)^{(1/3)}/b}))/ (bx^3 + a) - 70(2a^5b^5c^5 + 5a^2b^4c^4d - \\ & 40a^3b^3c^3d^2 + 70a^4b^2c^2d^3 - 50a^5b^3cd^4 + 13a^6d^5 + (2 \\ & b^6c^5 + 5a^5b^5c^4d - 40a^2b^4c^3d^2 + 70a^3b^3c^2d^3 - 50a^4b \\ & ^2cd^4 + 13a^5bd^5)x^3)(a^2b)^{(2/3)}\log(abx^2 - (a^2b)^{(2/3)}x \\ & + (a^2b)^{(1/3)}a) + 140(2a^5b^5c^5 + 5a^2b^4c^4d - 40a^3b^3c^3d^2 \\ & + 70a^4b^2c^2d^3 - 50a^5b^3cd^4 + 13a^6d^5 + (2b^6c^5 + 5a^5b^5 \\ & c^4d - 40a^2b^4c^3d^2 + 70a^3b^3c^2d^3 - 50a^4b^2cd^4 + 13a^5 \\ & bd^5)x^3)(a^2b)^{(2/3)}\log(abx + (a^2b)^{(2/3)}) + 420(a^2b^6c^5 - \\ & 5a^3b^5c^4d + 40a^4b^4c^3d^2 - 70a^5b^3c^2d^3 + 50a^6b^2cd^4 - \\ & 13a^7bd^5)x)/(a^3b^7x^3 + a^4b^6), 1/1260(126a^3b^5d^5x^13 \\ & + 18(50a^3b^5cd^4 - 13a^4b^4d^5)x^10 + 45(70a^3b^5c^2d^3 - 5 \\ & 0a^4b^4cd^4 + 13a^5bd^5)x^7 + 315(40a^3b^5c^3d^2 - 70a^4b^4 \\ & c^2d^3 + 50a^5b^3cd^4 - 13a^6bd^5)x^4 + 420\sqrt{1/3}(2a^2b^6 \\ & c^5 + 5a^3b^5c^4d - 40a^4b^4c^3d^2 + 70a^5b^3c^2d^3 - 50a^6 \\ & b^2cd^4 + 13a^7bd^5 + (2a^5b^7c^5 + 5a^2b^6c^4d - 40a^3b^5c^3 \\ & d^2 + 70a^4b^4c^2d^3 - 50a^5b^3cd^4 + 13a^6bd^5)x^3)\sqrt{((a \\ & ^2b)^{(1/3)}/b)}\arctan(\sqrt{1/3}(2(a^2b)^{(2/3)}x - (a^2b)^{(1/3)}a)\sqrt{ \\ & (a^2b)^{(1/3)}/b})/a^2) - 70(2a^5b^5c^5 + 5a^2b^4c^4d - 40a^3b^3c^3 \\ & d^2 + 70a^4b^2c^2d^3 - 50a^5b^3cd^4 + 13a^6d^5 + (2b^6c^5 + 5a^5b^5 \\ & c^4d - 40a^2b^4c^3d^2 + 70a^3b^3c^2d^3 - 50a^4b^2cd^4 + 13a^5 \\ & bd^5)x^3)(a^2b)^{(2/3)}\log(abx^2 - (a^2b)^{(2/3)}x + (a^2b)^{(1/3)} \\ & )a) + 140(2a^5b^5c^5 + 5a^2b^4c^4d - 40a^3b^3c^3d^2 + 70a^4b^2c^2 \\ & d^3 - 50a^5b^3cd^4 + 13a^6d^5 + (2b^6c^5 + 5a^5b^5c^4d - 40a^2 \\ & b^4c^3d^2 + 70a^3b^3c^2d^3 - 50a^4b^2cd^4 + 13a^5bd^5)x^3)( \\ & a^2b)^{(2/3)}\log(abx + (a^2b)^{(2/3)}) + 420(a^2b^6c^5 - 5a^3b^5c^4 \\ & d + 40a^4b^4c^3d^2 - 70a^5b^3c^2d^3 + 50a^6b^2cd^4 - 13a^7bd^5) \\ & )x)/(a^3b^7x^3 + a^4b^6)] \end{aligned}$$

**giac [A]** time = 0.19, size = 529, normalized size = 1.65

$$\frac{\sqrt{3}(2b^5c^5 + 5ab^4c^4d - 40a^2b^3c^3d^2 + 70a^3b^2c^2d^3 - 50a^4bcd^4 + 13a^5d^5) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}ab^4} (2b^5c^5 + 5ab^4c^4d - 40a^2b^3c^3d^2 + 70a^3b^2c^2d^3 - 50a^4bcd^4 + 13a^5d^5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^5/(b\*x^3+a)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/9\sqrt{3}(2b^5c^5 + 5a^5b^4c^4d - 40a^2b^3c^3d^2 + 70a^3b^2c^2 \\ & ^2d^3 - 50a^4b^3cd^4 + 13a^5d^5)\arctan(1/3\sqrt{3}(2x + (-a/b)^{(1/3)} \\ & ))/(-a/b)^{(1/3)}/((a^2b)^{(2/3)}ab^4) - 1/18(2b^5c^5 + 5a^5b^4c^4d - \\ & 40a^2b^3c^3d^2 + 70a^3b^2c^2d^3 - 50a^4b^3cd^4 + 13a^5d^5)\log \\ & (x^2 + x(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/((a^2b)^{(2/3)}ab^4) - 1/9(2b^5c^5 \\ & ^5 + 5a^5b^4c^4d - 40a^2b^3c^3d^2 + 70a^3b^2c^2d^3 - 50a^4b^3cd^4 + \\ & 13a^5d^5)(-a/b)^{(1/3)}\log(\text{abs}(x - (-a/b)^{(1/3)}))/ (a^2b^5) + 1/3(b \\ & ^5c^5x - 5a^5b^4c^4dx + 10a^2b^3c^3d^2x - 10a^3b^2c^2d^3x + \\ & 5a^4b^3cd^4x - a^5d^5x)/((bx^3 + a)ab^5) + 1/140(14b^18d^5x^10 \\ & + 100b^18cd^4x^7 - 40a^5b^17d^5x^7 + 350b^18c^2d^3x^4 - 350a^5b^1 \\ & 7cd^4x^4 + 105a^2b^16d^5x^4 + 1400b^18c^3d^2x - 2800a^5b^17c^2d^3 \\ & ^3x + 2100a^2b^16cd^4x - 560a^3b^15d^5x)/b^20 \end{aligned}$$

**maple [B]** time = 0.06, size = 905, normalized size = 2.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^5/(b\*x^3+a)^2,x)

[Out] 
$$-50/9/b^5*a^3/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c*d^4+70/9/b^4*a^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^2*d^3-40/9/b^3*a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^3*d^2-2/7*d^5/b^3*x^7+a+5/7*d^4/b^2*x^7*c+3/4*d^5/b^4*x^4*a^2+5/2*d^3/b^2*x^4*c^2-4*d^5/b^5*a^3*x+10*d^2/b^2*c^3*x+1/3*a*x/(b*x^3+a)*c^5-35/9/b^4*a^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^2*d^3+20/9/b^3*a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^3*d^2-50/9/b^5*a^3/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c*d^4+70/9/b^4*a^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^2*d^3-40/9/b^3*a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^3*d^2+25/9/b^5*a^3/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c*d^4-10/3/b^3*a^2*x/(b*x^3+a)*c^2*d^3+10/3/b^2*a*x/(b*x^3+a)*c^3*d^2+13/9/b^6*a^4/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*d^5+5/9/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^4*d+2/9/b/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^5+5/3/b^4*a^3*x/(b*x^3+a)*c*d^4-13/18/b^6*a^4/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*d^5-5/18/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^4*d-1/9/b/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^5+13/9/b^6*a^4/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*d^5+5/9/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^4*d+2/9/b/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^5-5/2*d^4/b^3*x^4*a*c+15*d^4/b^4*a^2*c*x-1/3/b^5*a^4*x/(b*x^3+a)*d^5-5/3/b*x/(b*x^3+a)*c^4*d-20*d^3/b^3*a*c^2*x+1/10*d^5*x^10/b^2$$

**maxima** [A] time = 1.35, size = 509, normalized size = 1.59

$$\frac{(b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)x^{14} + 14b^3d^5x^{10} + 20(5b^3cd^4 - 2ab^2d^5)x^7 + 35}{3(ab^6x^3 + a^2b^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^5/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] 
$$\frac{1}{3}*(b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*x/(a*b^6*x^3 + a^2*b^5) + \frac{1}{140}*(14*b^3*d^5*x^{10} + 20*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^7 + 35*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^4 + 140*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b*c*d^4 - 4*a^3*d^5)*x)/b^5 + \frac{1}{9}*\sqrt{3}*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^6*(a/b)^{(2/3)}) - \frac{1}{18}*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^6*(a/b)^{(2/3)}) + \frac{1}{9}*(2*b^5*c^5 + 5*a*b^4*c^4*d - 40*a^2*b^3*c^3*d^2 + 70*a^3*b^2*c^2*d^3 - 50*a^4*b*c*d^4 + 13*a^5*d^5)*\log(x + (a/b)^{(1/3)})/(a*b^6*(a/b)^{(2/3)})$$

**mupad** [B] time = 0.39, size = 416, normalized size = 1.30

$$x \left( \frac{10c^3d^2}{b^2} - \frac{2a \left( \frac{2ad^5}{b^3} - \frac{5cd^4}{b^2} \right) - \frac{a^2d^5}{b^4} + \frac{10c^2d^3}{b^2}}{b} + \frac{a^2 \left( \frac{2ad^5}{b^3} - \frac{5cd^4}{b^2} \right)}{b^2} \right) - x^7 \left( \frac{2ad^5}{7b^3} - \frac{5cd^4}{7b^2} \right) + x^4 \left( \frac{a \left( \frac{2ad^5}{b^3} - \frac{5cd^4}{b^2} \right)}{2b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^5/(a + b\*x^3)^2,x)

```
[Out] x*((10*c^3*d^2)/b^2 - (2*a*((2*a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2)))/b - (a^2*d^5)/b^4 + (10*c^2*d^3)/b^2))/b + (a^2*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/b^2) - x^7*((2*a*d^5)/(7*b^3) - (5*c*d^4)/(7*b^2)) + x^4*((a*((2*a*d^5)/b^3 - (5*c*d^4)/b^2))/(2*b) - (a^2*d^5)/(4*b^4) + (5*c^2*d^3)/(2*b^2)) + (d^5*x^10)/(10*b^2) - (x*(a^5*d^5 - b^5*c^5 - 10*a^2*b^3*c^3*d^2 + 10*a^3*b^2*c^2*d^3 + 5*a*b^4*c^4*d - 5*a^4*b*c*d^4))/(3*a*(a*b^5 + b^6*x^3)) + (log(b^(1/3))*x + a^(1/3))*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) - (log(3^(1/2)*a^(1/3)*1i - 2*b^(1/3)*x + a^(1/3))*((3^(1/2)*1i)/2 + 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3)) + (log(3^(1/2)*a^(1/3)*1i + 2*b^(1/3)*x - a^(1/3))*((3^(1/2)*1i)/2 - 1/2)*(a*d - b*c)^4*(13*a*d + 2*b*c))/(9*a^(5/3)*b^(16/3))
```

**sympy [A]** time = 12.43, size = 546, normalized size = 1.71

$$x^7 \left( -\frac{2ad^5}{7b^3} + \frac{5cd^4}{7b^2} \right) + x^4 \left( \frac{3a^2d^5}{4b^4} - \frac{5acd^4}{2b^3} + \frac{5c^2d^3}{2b^2} \right) + x \left( -\frac{4a^3d^5}{b^5} + \frac{15a^2cd^4}{b^4} - \frac{20ac^2d^3}{b^3} + \frac{10c^3d^2}{b^2} \right) + \frac{x(-a^5d^5 + 5a^4b^5 + 10a^3b^4c^4d - 5a^2b^3c^3d^2 + 10a^3b^2c^2d^3 + 5a^4b^4c^4d - 5a^4b^3c^3d^4)}{3a(a^5b + b^6x^3)} + \frac{\log(b^{1/3})x + a^{1/3}}{9a^{5/3}b^{16/3}} (a^4d - b^4c)^4(13ad + 2bc) - \frac{\log(3^{1/2}a^{1/3}i - 2b^{1/3}x + a^{1/3})}{9a^{5/3}b^{16/3}} ((3^{1/2}i)/2 + 1/2)(a^4d - b^4c)^4(13ad + 2bc) + \frac{\log(3^{1/2}a^{1/3}i + 2b^{1/3}x - a^{1/3})}{9a^{5/3}b^{16/3}} ((3^{1/2}i)/2 - 1/2)(a^4d - b^4c)^4(13ad + 2bc)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**5/(b*x**3+a)**2,x)
```

```
[Out] x**7*(-2*a*d**5/(7*b**3) + 5*c*d**4/(7*b**2)) + x**4*(3*a**2*d**5/(4*b**4) - 5*a*c*d**4/(2*b**3) + 5*c**2*d**3/(2*b**2)) + x*(-4*a**3*d**5/b**5 + 15*a**2*c*d**4/b**4 - 20*a*c**2*d**3/b**3 + 10*c**3*d**2/b**2) + x*(-a**5*d**5 + 5*a**4*b*c*d**4 - 10*a**3*b**2*c**2*d**3 + 10*a**2*b**3*c**3*d**2 - 5*a*b**4*c**4*d + b**5*c**5)/(3*a**2*b**5 + 3*a*b**6*x**3) + RootSum(729*_t**3*a**5*b**16 - 2197*a**15*d**15 + 25350*a**14*b*c*d**14 - 132990*a**13*b**2*c**2*d**13 + 418280*a**12*b**3*c**3*d**12 - 874635*a**11*b**4*c**4*d**11 + 1271886*a**10*b**5*c**5*d**10 - 1302400*a**9*b**6*c**6*d**9 + 922680*a**8*b**7*c**7*d**8 - 422235*a**7*b**8*c**8*d**7 + 97570*a**6*b**9*c**9*d**6 + 7194*a**5*b**10*c**10*d**5 - 10200*a**4*b**11*c**11*d**4 + 1435*a**3*b**12*c**12*d**3 + 330*a**2*b**13*c**13*d**2 - 60*a*b**14*c**14*d - 8*b**15*c**15, Lambda(_t, _t*log(9*_t*a**2*b**5/(13*a**5*d**5 - 50*a**4*b*c*d**4 + 70*a**3*b**2*c**2*d**3 - 40*a**2*b**3*c**3*d**2 + 5*a*b**4*c**4*d + 2*b**5*c**5) + x)) + d**5*x**10/(10*b**2))
```

$$3.21 \quad \int \frac{(c+dx^3)^4}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=267

$$\frac{(bc-ad)^3(5ad+bc)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{9a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(5ad+bc)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9a^{5/3}b^{13/3}} - \frac{2(bc-ad)^3(5ad+bc)\log(\sqrt[3]{a}-\sqrt[3]{b}x)}{9a^{5/3}b^{13/3}}$$

[Out]  $d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+1/2*d^3*(-a*d+2*b*c)*x^4/b^3+1/7*d^4*x^7/b^2+1/3*(-a*d+b*c)^4*x/a/b^4/(b*x^3+a)+2/9*(-a*d+b*c)^3*(5*a*d+b*c)*\ln(a^{1/3}+b^{1/3}*x)/a^{5/3}/b^{13/3}-1/9*(-a*d+b*c)^3*(5*a*d+b*c)*\ln(a^{2/3}-a^{1/3}*b^{1/3}*x+b^{2/3}*x^2)/a^{5/3}/b^{13/3}-2/9*(-a*d+b*c)^3*(5*a*d+b*c)*\arctan(1/3*(a^{1/3}-2*b^{1/3}*x)/a^{1/3}*3^{1/2})/a^{5/3}/b^{13/3}*3^{1/2}$

**Rubi [A]** time = 0.23, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} - \frac{(bc-ad)^3(5ad+bc)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)}{9a^{5/3}b^{13/3}} + \frac{2(bc-ad)^3(5ad+bc)\log(\sqrt[3]{a}+\sqrt[3]{b}x)}{9a^{5/3}b^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^4/(a + b\*x^3)^2, x]

[Out]  $(d^2*(6*b^2*c^2-8*a*b*c*d+3*a^2*d^2)*x)/b^4+(d^3*(2*b*c-a*d)*x^4)/(2*b^3)+(d^4*x^7)/(7*b^2)+((b*c-a*d)^4*x)/(3*a*b^4*(a+b*x^3))-2*(b*c-a*d)^3*(b*c+5*a*d)*\text{ArcTan}[(a^{1/3}-2*b^{1/3}*x)/(\text{Sqrt}[3]*a^{1/3})]/(3*\text{Sqrt}[3]*a^{5/3}*b^{13/3})+2*(b*c-a*d)^3*(b*c+5*a*d)*\text{Log}[a^{1/3}+b^{1/3}*x]/(9*a^{5/3}*b^{13/3})-((b*c-a*d)^3*(b*c+5*a*d)*\text{Log}[a^{2/3}-a^{1/3}*b^{1/3}*x+b^{2/3}*x^2])/(9*a^{5/3}*b^{13/3})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_))^(n\_)\*((c\_) + (d\_.)\*(x\_))^(m\_), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + 1, 0])

p, 0])

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^4}{(a + bx^3)^2} dx &= \int \left( \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)}{b^4} + \frac{2d^3(2bc - ad)x^3}{b^3} + \frac{d^4x^6}{b^2} + \frac{(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3}{b^4(a + bx^3)^2} \right) dx \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{\int \frac{(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3}{(a + bx^3)^2} dx}{b^4} \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4x}{3ab^4(a + bx^3)} + \frac{(2(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3)}{3ab^4(a + bx^3)^2} \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4x}{3ab^4(a + bx^3)} + \frac{(2(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3)}{9a^2b^4(a + bx^3)^2} \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4x}{3ab^4(a + bx^3)} + \frac{2(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3}{9a^2b^4(a + bx^3)^2} \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4x}{3ab^4(a + bx^3)} + \frac{2(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3}{9a^2b^4(a + bx^3)^2} \\ &= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{d^3(2bc - ad)x^4}{2b^3} + \frac{d^4x^7}{7b^2} + \frac{(bc - ad)^4x}{3ab^4(a + bx^3)} - \frac{2(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^3}{3\sqrt{3}ab^4(a + bx^3)^2} \end{aligned}$$



**Mathematica [A]** time = 0.24, size = 260, normalized size = 0.97

$$\frac{14(ad-bc)^3(5ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{5/3}} + \frac{28(bc-ad)^3(5ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}} + \frac{28\sqrt{3}(bc-ad)^3(5ad+bc)\tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 126\sqrt[3]{b^3}$$


---


$$126b^{13/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^4/(a + b\*x^3)^2,x]

[Out] (126\*b^(1/3)\*d^2\*(6\*b^2\*c^2 - 8\*a\*b\*c\*d + 3\*a^2\*d^2)\*x + 63\*b^(4/3)\*d^3\*(2\*b\*c - a\*d)\*x^4 + 18\*b^(7/3)\*d^4\*x^7 + (42\*b^(1/3)\*(b\*c - a\*d)^4\*x)/(a\*(a + b\*x^3)) + (28\*sqrt[3]\*(b\*c - a\*d)^3\*(b\*c + 5\*a\*d)\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(sqrt[3]\*a^(1/3))])/a^(5/3) + (28\*(b\*c - a\*d)^3\*(b\*c + 5\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/a^(5/3) + (14\*(-(b\*c) + a\*d)^3\*(b\*c + 5\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(5/3))/(126\*b^(13/3))

**fricas [B]** time = 0.49, size = 1316, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^4/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] [1/126\*(18\*a^3\*b^4\*d^4\*x^10 + 9\*(14\*a^3\*b^4\*c\*d^3 - 5\*a^4\*b^3\*d^4)\*x^7 + 63\*(12\*a^3\*b^4\*c^2\*d^2 - 14\*a^4\*b^3\*c\*d^3 + 5\*a^5\*b^2\*d^4)\*x^4 - 42\*sqrt(1/3)\*(a^2\*b^5\*c^4 + 2\*a^3\*b^4\*c^3\*d - 12\*a^4\*b^3\*c^2\*d^2 + 14\*a^5\*b^2\*c\*d^3 - 5\*a^6\*b\*d^4 + (a\*b^6\*c^4 + 2\*a^2\*b^5\*c^3\*d - 12\*a^3\*b^4\*c^2\*d^2 + 14\*a^4\*b^3\*c\*d^3 - 5\*a^5\*b^2\*d^4)\*x^3)\*sqrt((-a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 + 3\*(-a^2\*b)^(1/3)\*a\*x - a^2 - 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt((-a^2\*b)^(1/3)/b))/(b\*x^3 + a)) - 14\*(a\*b^4\*c^4 + 2\*a^2\*b^3\*c^3\*d - 12\*a^3\*b^2\*c^2\*d^2 + 14\*a^4\*b\*c\*d^3 - 5\*a^5\*d^4 + (b^5\*c^4 + 2\*a\*b^4\*c^3\*d - 12\*a^2\*b^3\*c^2\*d^2 + 14\*a^3\*b^2\*c\*d^3 - 5\*a^4\*b\*d^4)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 28\*(a\*b^4\*c^4 + 2\*a^2\*b^3\*c^3\*d - 12\*a^3\*b^2\*c^2\*d^2 + 14\*a^4\*b\*c\*d^3 - 5\*a^5\*d^4 + (b^5\*c^4 + 2\*a\*b^4\*c^3\*d - 12\*a^2\*b^3\*c^2\*d^2 + 14\*a^3\*b^2\*c\*d^3 - 5\*a^4\*b\*d^4)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 42\*(a^2\*b^5\*c^4 - 4\*a^3\*b^4\*c^3\*d + 24\*a^4\*b^3\*c^2\*d^2 - 28\*a^5\*b^2\*c\*d^3 + 10\*a^6\*b\*d^4)\*x)/(a^3\*b^6\*x^3 + a^4\*b^5), 1/126\*(18\*a^3\*b^4\*d^4\*x^10 + 9\*(14\*a^3\*b^4\*c\*d^3 - 5\*a^4\*b^3\*d^4)\*x^7 + 63\*(12\*a^3\*b^4\*c^2\*d^2 - 14\*a^4\*b^3\*c\*d^3 + 5\*a^5\*b^2\*d^4)\*x^4 + 84\*sqrt(1/3)\*(a^2\*b^5\*c^4 + 2\*a^3\*b^4\*c^3\*d - 12\*a^4\*b^3\*c^2\*d^2 + 14\*a^5\*b^2\*c\*d^3 - 5\*a^6\*b\*d^4 + (a\*b^6\*c^4 + 2\*a^2\*b^5\*c^3\*d - 12\*a^3\*b^4\*c^2\*d^2 + 14\*a^4\*b^3\*c\*d^3 - 5\*a^5\*b^2\*d^4)\*x^3)\*sqrt(-(-a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt(-(-a^2\*b)^(1/3)/b)/a^2) - 14\*(a\*b^4\*c^4 + 2\*a^2\*b^3\*c^3\*d - 12\*a^3\*b^2\*c^2\*d^2 + 14\*a^4\*b\*c\*d^3 - 5\*a^5\*d^4 + (b^5\*c^4 + 2\*a\*b^4\*c^3\*d - 12\*a^2\*b^3\*c^2\*d^2 + 14\*a^3\*b^2\*c\*d^3 - 5\*a^4\*b\*d^4)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 28\*(a\*b^4\*c^4 + 2\*a^2\*b^3\*c^3\*d - 12\*a^3\*b^2\*c^2\*d^2 + 14\*a^4\*b\*c\*d^3 - 5\*a^5\*d^4 + (b^5\*c^4 + 2\*a\*b^4\*c^3\*d - 12\*a^2\*b^3\*c^2\*d^2 + 14\*a^3\*b^2\*c\*d^3 - 5\*a^4\*b\*d^4)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 42\*(a^2\*b^5\*c^4 - 4\*a^3\*b^4\*c^3\*d + 24\*a^4\*b^3\*c^2\*d^2 - 28\*a^5\*b^2\*c\*d^3 + 10\*a^6\*b\*d^4)\*x)/(a^3\*b^6\*x^3 + a^4\*b^5)]

**giac [A]** time = 0.19, size = 412, normalized size = 1.54

$$2\sqrt{3}(b^4c^4 + 2ab^3c^3d - 12a^2b^2c^2d^2 + 14a^3bcd^3 - 5a^4d^4) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right) (b^4c^4 + 2ab^3c^3d - 12a^2b^2c^2d^2 + 14a^3bcd^3 - 5a^4d^4)$$


---


$$9(-ab^2)^{\frac{2}{3}}ab^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="giac")
```

```
[Out] -2/9*sqrt(3)*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*arctan(1/3*sqrt(3)*(2*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a*b^2)^(2/3)*a*b^3) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*log(x^2 + x*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a*b^2)^(2/3)*a*b^3) - 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*(-a/b)^(1/3)*log(abs(x - (-a/b)^(1/3)))/(a^2*b^4) + 1/3*(b^4*c^4*x - 4*a*b^3*c^3*d*x + 6*a^2*b^2*c^2*d^2*x - 4*a^3*b*c*d^3*x + a^4*d^4*x)/((b*x^3 + a)*a*b^4) + 1/14*(2*b^12*d^4*x^7 + 14*b^12*c*d^3*x^4 - 7*a*b^11*d^4*x^4 + 84*b^12*c^2*d^2*x - 112*a*b^11*c*d^3*x + 42*a^2*b^10*d^4*x)/b^14
```

```
maple [B] time = 0.06, size = 708, normalized size = 2.65
```

$$10\sqrt{3} a^3 d^4 a$$

$$\frac{d^4 x^7}{7b^2} - \frac{a d^4 x^4}{2b^3} + \frac{c d^3 x^4}{b^2} + \frac{a^3 d^4 x}{3(bx^3 + a)b^4} - \frac{4a^2 c d^3 x}{3(bx^3 + a)b^3} + \frac{2a c^2 d^2 x}{(bx^3 + a)b^2} + \frac{c^4 x}{3(bx^3 + a)a} - \frac{4c^3 dx}{3(bx^3 + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^4/(b*x^3+a)^2,x)
```

```
[Out] -8/3/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^2*d^2+28/9/b^4*a^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c*d^3+6*d^2/b^2*c^2*x-1/2*d^4/b^3*x^4*a+d^3/b^2*x^4*c+1/3/a*x/(b*x^3+a)*c^4+3*d^4/b^4*a^2*x-10/9/b^5*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*d^4+4/9/b^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^3*d+2/9/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*x-1))*c^4-4/3/b^3*a^2*x/(b*x^3+a)*c*d^3+2/b^2*a*x/(b*x^3+a)*c^2*d^2+28/9/b^4*a^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c*d^3-8/3/b^3*a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^2*d^2-14/9/b^4*a^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c*d^3+4/3/b^3*a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^2*d^2+5/9/b^5*a^3/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*d^4-10/9/b^5*a^3/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*d^4-2/9/b^2/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^3*d-1/9/b/a/(a/b)^(2/3)*ln(x^2-(a/b)^(1/3)*x+(a/b)^(2/3))*c^4+1/3/b^4*a^3*x/(b*x^3+a)*d^4-4/3/b*x/(b*x^3+a)*c^3*d-8*d^3/b^3*a*c*x+4/9/b^2/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^3*d+2/9/b/a/(a/b)^(2/3)*ln(x+(a/b)^(1/3))*c^4+1/7*d^4*x^7/b^2
```

```
maxima [A] time = 1.16, size = 397, normalized size = 1.49
```

$$\frac{(b^4 c^4 - 4 a b^3 c^3 d + 6 a^2 b^2 c^2 d^2 - 4 a^3 b c d^3 + a^4 d^4) x}{3(a b^5 x^3 + a^2 b^4)} + \frac{2 b^2 d^4 x^7 + 7(2 b^2 c d^3 - a b d^4) x^4 + 14(6 b^2 c^2 d^2 - 8 a b c d^3 + 3 a^2 d^4)}{14 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^4/(b*x^3+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*x/(a*b^5*x^3 + a^2*b^4) + 1/14*(2*b^2*d^4*x^7 + 7*(2*b^2*c*d^3 - a*b*d^4)*
```

$$x^4 + 14*(6*b^2*c^2*d^2 - 8*a*b*c*d^3 + 3*a^2*d^4)*x)/b^4 + 2/9*\sqrt{3}*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^5*(a/b)^{(2/3)}) - 1/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^5*(a/b)^{(2/3)}) + 2/9*(b^4*c^4 + 2*a*b^3*c^3*d - 12*a^2*b^2*c^2*d^2 + 14*a^3*b*c*d^3 - 5*a^4*d^4)*\log(x + (a/b)^{(1/3)})/(a*b^5*(a/b)^{(2/3)})$$

**mupad [B]** time = 1.49, size = 302, normalized size = 1.13

$$x \left( \frac{2a \left( \frac{2ad^4}{b^3} - \frac{4cd^3}{b^2} \right)}{b} - \frac{a^2d^4}{b^4} + \frac{6c^2d^2}{b^2} \right) - x^4 \left( \frac{ad^4}{2b^3} - \frac{cd^3}{b^2} \right) + \frac{d^4x^7}{7b^2} + \frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{3a(b^5x^3 + ab^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^4/(a + b\*x^3)^2,x)

[Out]  $x*((2*a*((2*a*d^4)/b^3 - (4*c*d^3)/b^2))/b - (a^2*d^4)/b^4 + (6*c^2*d^2)/b^2) - x^4*((a*d^4)/(2*b^3) - (c*d^3)/b^2) + (d^4*x^7)/(7*b^2) + (x*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3))/(3*a*(a*b^4 + b^5*x^3)) - (2*\log(b^{(1/3)}*x + a^{(1/3)})*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^{(5/3)}*b^{(13/3)}) + (2*\log(3^{(1/2)}*a^{(1/3)}*1i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^{(5/3)}*b^{(13/3)}) - (2*\log(3^{(1/2)}*a^{(1/3)}*1i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(a*d - b*c)^3*(5*a*d + b*c))/(9*a^{(5/3)}*b^{(13/3)})$

**sympy [A]** time = 8.54, size = 405, normalized size = 1.52

$$x^4 \left( -\frac{ad^4}{2b^3} + \frac{cd^3}{b^2} \right) + x \left( \frac{3a^2d^4}{b^4} - \frac{8acd^3}{b^3} + \frac{6c^2d^2}{b^2} \right) + \frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{3a^2b^4 + 3ab^5x^3} + \text{RootSum}(\dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*4/(b\*x\*\*3+a)\*\*2,x)

[Out]  $x**4*(-a*d**4/(2*b**3) + c*d**3/b**2) + x*(3*a**2*d**4/b**4 - 8*a*c*d**3/b**3 + 6*c**2*d**2/b**2) + x*(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4)/(3*a**2*b**4 + 3*a*b**5*x**3) + \text{RootSum}(729*_t**3*a**5*b**13 + 1000*a**12*d**12 - 8400*a**11*b*c*d**11 + 30720*a**10*b**2*c**2*d**10 - 63472*a**9*b**3*c**3*d**9 + 79848*a**8*b**4*c**4*d**8 - 60192*a**7*b**5*c**5*d**7 + 22848*a**6*b**6*c**6*d**6 + 288*a**5*b**7*c**7*d**5 - 3528*a**4*b**8*c**8*d**4 + 752*a**3*b**9*c**9*d**3 + 192*a**2*b**10*c**10*d**2 - 48*a*b**11*c**11*d - 8*b**12*c**12, \text{Lambda}(_t, _t*\log(-9*_t*a**2*b**4/(10*a**4*d**4 - 28*a**3*b*c*d**3 + 24*a**2*b**2*c**2*d**2 - 4*a*b**3*c**3*d - 2*b**4*c**4) + x))) + d**4*x**7/(7*b**2)$

$$3.22 \quad \int \frac{(c+dx^3)^3}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=234

$$\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(7ad+2bc)}{3\sqrt[3]{ab}}$$

[Out]  $d^2*(-2*a*d+3*b*c)*x/b^3+1/4*d^3*x^4/b^2+1/3*(-a*d+b*c)^3*x/a/b^3/(b*x^3+a)$   
 $+1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*\ln(a^{(1/3)}+b^{(1/3)}*x)/a^{(5/3)}/b^{(10/3)}-1/18$   
 $*(-a*d+b*c)^2*(7*a*d+2*b*c)*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*x+b^{(2/3)}*x^2)/a^{(5/3)}/b^{(10/3)}$   
 $-1/9*(-a*d+b*c)^2*(7*a*d+2*b*c)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*x)/a^{(1/3)}*3^{(1/2)})/a^{(5/3)}/b^{(10/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{10/3}} + \frac{(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{10/3}} - \frac{(bc-ad)^2(7ad+2bc)}{3\sqrt[3]{ab}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^3/(a + b\*x^3)^2, x]

[Out]  $(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^4)/(4*b^2) + ((b*c - a*d)^3*x)/(3*a*b^3*(a + b*x^3)) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*\text{ArcTan}[a^{(1/3)} - 2*b^{(1/3)}*x]/(\text{Sqrt}[3]*a^{(1/3)}))/ (3*\text{Sqrt}[3]*a^{(5/3)}*b^{(10/3)}) + ((b*c - a*d)^2*(2*b*c + 7*a*d)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])/ (9*a^{(5/3)}*b^{(10/3)}) - ((b*c - a*d)^2*(2*b*c + 7*a*d)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])/ (18*a^{(5/3)}*b^{(10/3)})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + 1, 0])

p, 0]

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^3}{(a + bx^3)^2} dx &= \int \left( \frac{d^2(3bc - 2ad)}{b^3} + \frac{d^3x^3}{b^2} + \frac{(bc - ad)^2(bc + 2ad) + 3bd(bc - ad)^2x^3}{b^3(a + bx^3)^2} \right) dx \\ &= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{\int \frac{(bc - ad)^2(bc + 2ad) + 3bd(bc - ad)^2x^3}{(a + bx^3)^2} dx}{b^3} \\ &= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc - ad)^3x}{3ab^3(a + bx^3)} + \frac{((bc - ad)^2(2bc + 7ad)) \int \frac{1}{a + bx^3} dx}{3ab^3} \\ &= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc - ad)^3x}{3ab^3(a + bx^3)} + \frac{((bc - ad)^2(2bc + 7ad)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{5/3}b^3} + \frac{(bc - ad)^2(2bc + 7ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{10/3}} \\ &= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^4}{4b^2} + \frac{(bc - ad)^3x}{3ab^3(a + bx^3)} + \frac{(bc - ad)^2(2bc + 7ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{10/3}} - \frac{(bc - ad)^2(2bc + 7ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(bc - ad)^2(2bc + 7ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} + \frac{(bc - ad)^2(2bc + 7ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{10/3}} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 227, normalized size = 0.97

$$\frac{\frac{2(bc-ad)^2(7ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{bx+b^{2/3}x^2}\right)}{a^{5/3}} + \frac{4(bc-ad)^2(7ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{bx}\right)}{a^{5/3}} + \frac{4\sqrt{3}(bc-ad)^2(7ad+2bc)\tan^{-1}\left(\frac{2\sqrt[3]{bx}-\sqrt[3]{a}}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}} + 36\sqrt[3]{b}a}{36b^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^3/(a + b\*x^3)^2,x]

[Out] (36\*b^(1/3)\*d^2\*(3\*b\*c - 2\*a\*d)\*x + 9\*b^(4/3)\*d^3\*x^4 + (12\*b^(1/3)\*(b\*c - a\*d)^3\*x)/(a\*(a + b\*x^3)) + (4\*sqrt[3]\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*ArcTan[(-a^(1/3) + 2\*b^(1/3)\*x)/(sqrt[3]\*a^(1/3))]/a^(5/3) + (4\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/a^(5/3) - (2\*(b\*c - a\*d)^2\*(2\*b\*c + 7\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(5/3))/(36\*b^(10/3))

**fricas [B]** time = 0.46, size = 1027, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^3/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] [1/36\*(9\*a^3\*b^3\*d^3\*x^7 + 9\*(12\*a^3\*b^3\*c\*d^2 - 7\*a^4\*b^2\*d^3)\*x^4 + 6\*sqrt(1/3)\*(2\*a^2\*b^4\*c^3 + 3\*a^3\*b^3\*c^2\*d - 12\*a^4\*b^2\*c\*d^2 + 7\*a^5\*b\*d^3 + (2\*a\*b^5\*c^3 + 3\*a^2\*b^4\*c^2\*d - 12\*a^3\*b^3\*c\*d^2 + 7\*a^4\*b^2\*d^3)\*x^3)\*sqrt(-a^(2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 - 3\*(a^2\*b)^(1/3)\*a\*x - a^2 + 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt(-a^(2\*b)^(1/3)/b))/(b\*x^3 + a) - 2\*(2\*a\*b^3\*c^3 + 3\*a^2\*b^2\*c^2\*d - 12\*a^3\*b\*c\*d^2 + 7\*a^4\*d^3 + (2\*b^4\*c^3 + 3\*a\*b^3\*c^2\*d - 12\*a^2\*b^2\*c^2\*d - 12\*a^3\*b\*d^3)\*x^3)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 4\*(2\*a\*b^3\*c^3 + 3\*a^2\*b^2\*c^2\*d - 12\*a^3\*b\*c\*d^2 + 7\*a^4\*d^3 + (2\*b^4\*c^3 + 3\*a\*b^3\*c^2\*d - 12\*a^2\*b^2\*c^2\*d + 7\*a^3\*b\*d^3)\*x^3)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 12\*(a^2\*b^4\*c^3 - 3\*a^3\*b^3\*c^2\*d + 12\*a^4\*b^2\*c\*d^2 - 7\*a^5\*b\*d^3)\*x)/(a^3\*b^5\*x^3 + a^4\*b^4), 1/36\*(9\*a^3\*b^3\*d^3\*x^7 + 9\*(12\*a^3\*b^3\*c\*d^2 - 7\*a^4\*b^2\*d^3)\*x^4 + 12\*sqrt(1/3)\*(2\*a^2\*b^4\*c^3 + 3\*a^3\*b^3\*c^2\*d - 12\*a^4\*b^2\*c\*d^2 + 7\*a^5\*b\*d^3 + (2\*a\*b^5\*c^3 + 3\*a^2\*b^4\*c^2\*d - 12\*a^3\*b^3\*c\*d^2 + 7\*a^4\*b^2\*d^3)\*x^3)\*sqrt((a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt((a^2\*b)^(1/3)/b)/a^2) - 2\*(2\*a\*b^3\*c^3 + 3\*a^2\*b^2\*c^2\*d - 12\*a^3\*b\*c\*d^2 + 7\*a^4\*d^3 + (2\*b^4\*c^3 + 3\*a\*b^3\*c^2\*d - 12\*a^2\*b^2\*c^2\*d + 7\*a^3\*b\*d^3)\*x^3)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 4\*(2\*a\*b^3\*c^3 + 3\*a^2\*b^2\*c^2\*d - 12\*a^3\*b\*c\*d^2 + 7\*a^4\*d^3 + (2\*b^4\*c^3 + 3\*a\*b^3\*c^2\*d - 12\*a^2\*b^2\*c^2\*d + 7\*a^3\*b\*d^3)\*x^3)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 12\*(a^2\*b^4\*c^3 - 3\*a^3\*b^3\*c^2\*d + 12\*a^4\*b^2\*c\*d^2 - 7\*a^5\*b\*d^3)\*x)/(a^3\*b^5\*x^3 + a^4\*b^4)]

**giac [A]** time = 0.20, size = 319, normalized size = 1.36

$$\frac{\sqrt{3}\left(2b^3c^3 + 3ab^2c^2d - 12a^2bcd^2 + 7a^3d^3\right)\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(-ab^2\right)^{\frac{2}{3}}ab^2} - \frac{\left(2b^3c^3 + 3ab^2c^2d - 12a^2bcd^2 + 7a^3d^3\right)\log\left(\frac{\sqrt{3}\left(2x+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{18\left(-ab^2\right)^{\frac{2}{3}}ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^3/(b\*x^3+a)^2,x, algorithm="giac")

[Out]  $-1/9*\sqrt{3}*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*\arctan(1/3*\sqrt{3}*(2*x + (-a/b)^{(1/3)})/(-a/b)^{(1/3)})/((-a*b^2)^{(2/3)}*a*b^2) - 1/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*\log(x^2 + x*(-a/b)^{(1/3)} + (-a/b)^{(2/3)})/((-a*b^2)^{(2/3)}*a*b^2) - 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*(-a/b)^{(1/3)}*\log(\text{abs}(x - (-a/b)^{(1/3)}))/((b*x^3 + a)*a*b^3) + 1/3*(b^3*c^3*x - 3*a*b^2*c^2*d*x + 3*a^2*b*c*d^2*x - a^3*d^3*x)/((b*x^3 + a)*a*b^3) + 1/4*(b^6*d^3*x^4 + 12*b^6*c*d^2*x - 8*a*b^5*d^3*x)/b^8$

**maple** [B] time = 0.05, size = 529, normalized size = 2.26

$$\frac{d^3 x^4}{4b^2} - \frac{a^2 d^3 x}{3(bx^3 + a)b^3} + \frac{ac d^2 x}{(bx^3 + a)b^2} + \frac{c^3 x}{3(bx^3 + a)a} - \frac{c^2 dx}{(bx^3 + a)b} + \frac{7\sqrt{3} a^2 d^3 \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^3} - 1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^4} + \frac{7a^2 d^3 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((d*x^3+c)^3/(b*x^3+a)^2, x)$

[Out]  $1/4*d^3*x^4/b^2 - 2*d^3/b^3*a*x + 3*d^2/b^2*c*x - 1/3/b^3*a^2*x/(b*x^3+a)*d^3 + 1/b^2*a*x/(b*x^3+a)*c*d^2 - 1/b*x/(b*x^3+a)*c^2*d + 1/3/a*x/(b*x^3+a)*c^3 + 7/9/b^4*a^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*d^3 - 4/3/b^3*a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c*d^2 + 1/3/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^2*d + 2/9/b/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c^3 - 7/18/b^4*a^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*d^3 + 2/3/b^3*a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c*d^2 - 1/6/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^2*d - 1/9/b/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c^3 + 7/9/b^4*a^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*d^3 - 4/3/b^3*a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c*d^2 + 1/3/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^2*d + 2/9/b/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c^3$

**maxima** [A] time = 1.22, size = 306, normalized size = 1.31

$$\frac{(b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) x}{3 (a b^4 x^3 + a^2 b^3)} + \frac{b d^3 x^4 + 4 (3 b c d^2 - 2 a d^3) x}{4 b^3} + \frac{\sqrt{3} (2 b^3 c^3 + 3 a b^2 c^2 d - 12 a^2 b c d^2 + 7 a^3 d^3)}{9 a b^4 \left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d*x^3+c)^3/(b*x^3+a)^2, x, \text{algorithm}="maxima")$

[Out]  $1/3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x/(a*b^4*x^3 + a^2*b^3) + 1/4*(b*d^3*x^4 + 4*(3*b*c*d^2 - 2*a*d^3)*x)/b^3 + 1/9*\sqrt{3}*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^4*(a/b)^{(2/3)}) - 1/18*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^4*(a/b)^{(2/3)}) + 1/9*(2*b^3*c^3 + 3*a*b^2*c^2*d - 12*a^2*b*c*d^2 + 7*a^3*d^3)*\log(x + (a/b)^{(1/3)})/(a*b^4*(a/b)^{(2/3)})$

**mupad** [B] time = 0.30, size = 240, normalized size = 1.03

$$\frac{d^3 x^4}{4 b^2} - x \left( \frac{2 a d^3}{b^3} - \frac{3 c d^2}{b^2} \right) - \frac{x (a^3 d^3 - 3 a^2 b c d^2 + 3 a b^2 c^2 d - b^3 c^3)}{3 a (b^4 x^3 + a b^3)} + \frac{\ln(b^{1/3} x + a^{1/3}) (a d - b c)^2 (7 a d + 2 b c)}{9 a^{5/3} b^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^3/(a + b*x^3)^2,x)`

[Out]  $(d^3x^4)/(4b^2) - x((2ad^3)/b^3 - (3cd^2)/b^2) - (x(a^3d^3 - b^3c^3 + 3ab^2c^2d - 3a^2b^2cd^2))/(3a(a^3b^3 + b^4x^3)) + (\log(b^{1/3}x + a^{1/3}))(ad - bc)^2(7ad + 2bc)/(9a^{5/3}b^{10/3}) - (\log(3^{1/2}a^{1/3}i - 2b^{1/3}x + a^{1/3}))((3^{1/2}i)/2 + 1/2)(ad - bc)^2(7ad + 2bc)/(9a^{5/3}b^{10/3}) + (\log(3^{1/2}a^{1/3}i + 2b^{1/3}x - a^{1/3}))((3^{1/2}i)/2 - 1/2)(ad - bc)^2(7ad + 2bc)/(9a^{5/3}b^{10/3})$

**sympy** [A] time = 4.33, size = 291, normalized size = 1.24

$$x\left(-\frac{2ad^3}{b^3} + \frac{3cd^2}{b^2}\right) + \frac{x(-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3)}{3a^2b^3 + 3ab^4x^3} + \text{RootSum}\left(729t^3a^5b^{10} - 343a^9d^9 + 1764a^8bcd^8 - 343a^7b^2c^2d^7 + 2946a^6b^3c^3d^6 - 477a^5b^4c^4d^5 - 792a^4b^5c^5d^4 + 321a^3b^6c^6d^3 + 90a^2b^7c^7d^2 - 36ab^8c^8d - 8b^9c^9, \text{Lambda}(t, t \log(9t^2a^3d^3 - 12a^2b^2cd^2 + 3ab^2c^2d + 2b^3c^3)) + x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**3/(b*x**3+a)**2,x)`

[Out]  $x(-2ad^3/b^3 + 3cd^2/b^2) + x(-a^3d^3 + 3a^2bcd^2 - 3a^2b^2c^2d + b^3c^3)/(3a^2b^3 + 3ab^4x^3) + \text{RootSum}(729t^3a^5b^{10} - 343a^9d^9 + 1764a^8bcd^8 - 3465a^7b^2c^2d^7 + 2946a^6b^3c^3d^6 - 477a^5b^4c^4d^5 - 792a^4b^5c^5d^4 + 321a^3b^6c^6d^3 + 90a^2b^7c^7d^2 - 36ab^8c^8d - 8b^9c^9, \text{Lambda}(t, t \log(9t^2a^3d^3 - 12a^2b^2cd^2 + 3ab^2c^2d + 2b^3c^3)) + x)) + d^3x^4/(4b^2)$



$$3.23 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=203

$$\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{2(bc-ad)(2ad+bc)}{3\sqrt{3}}$$

[Out]  $d^2x/b^2+1/3*(-a*d+b*c)^2*x/a/b^2/(b*x^3+a)+2/9*(-a*d+b*c)*(2*a*d+b*c)*\ln(a^{1/3}+b^{1/3}*x)/a^{5/3}/b^{7/3}-1/9*(-a*d+b*c)*(2*a*d+b*c)*\ln(a^{2/3}-a^{1/3}*b^{1/3}*x+b^{2/3}*x^2)/a^{5/3}/b^{7/3}-2/9*(-a*d+b*c)*(2*a*d+b*c)*\arctan(1/3*(a^{1/3}-2*b^{1/3}*x)/a^{1/3}*3^{1/2})/a^{5/3}/b^{7/3}*3^{1/2}$

**Rubi [A]** time = 0.23, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 200, 31, 634, 617, 204, 628}

$$\frac{(bc-ad)(2ad+bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{9a^{5/3}b^{7/3}} + \frac{2(bc-ad)(2ad+bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{7/3}} - \frac{2(bc-ad)(2ad+bc)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^2,x]

[Out]  $(d^2x)/b^2 + ((b*c - a*d)^2*x)/(3*a*b^2*(a + b*x^3)) - (2*(b*c - a*d)*(b*c + 2*a*d)*\text{ArcTan}[a^{1/3} - 2*b^{1/3}*x]/(\text{Sqrt}[3]*a^{1/3}))/ (3*\text{Sqrt}[3]*a^{5/3}*b^{7/3}) + (2*(b*c - a*d)*(b*c + 2*a*d)*\text{Log}[a^{1/3} + b^{1/3}*x])/ (9*a^{5/3}*b^{7/3}) - ((b*c - a*d)*(b*c + 2*a*d)*\text{Log}[a^{2/3} - a^{1/3}*b^{1/3}*x + b^{2/3}*x^2])/ (9*a^{5/3}*b^{7/3})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^-1, x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 200**

Int[((a\_) + (b\_.)\*(x\_)^3)^-1, x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^-1, x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^n)^p\*((c\_) + (d\_.)\*(x\_)^n), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^2}{(a + bx^3)^2} dx &= \int \left( \frac{d^2}{b^2} + \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^3}{b^2(a + bx^3)^2} \right) dx \\
&= \frac{d^2x}{b^2} + \frac{\int \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^3}{(a + bx^3)^2} dx}{b^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{3ab^2(a + bx^3)} + \frac{(2(bc - ad)(bc + 2ad)) \int \frac{1}{a + bx^3} dx}{3ab^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{3ab^2(a + bx^3)} + \frac{(2(bc - ad)(bc + 2ad)) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{5/3}b^2} + \frac{(2(bc - ad)(bc + 2ad)) \int \frac{1}{a^{2/3} - \sqrt[3]{b}x} dx}{9a^{5/3}b^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{3ab^2(a + bx^3)} + \frac{2(bc - ad)(bc + 2ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{7/3}} - \frac{((bc - ad)(bc + 2ad)) \int \frac{1}{a^{2/3} - \sqrt[3]{b}x} dx}{9a^{5/3}b^{7/3}} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{3ab^2(a + bx^3)} + \frac{2(bc - ad)(bc + 2ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{7/3}} - \frac{(bc - ad)(bc + 2ad) \log(a^{2/3} - \sqrt[3]{b}x)}{9a^{5/3}b^{7/3}} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{3ab^2(a + bx^3)} - \frac{2(bc - ad)(bc + 2ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{7/3}} + \frac{2(bc - ad)(bc + 2ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{7/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.20, size = 205, normalized size = 1.01

$$\frac{2(-2a^2d^2+abcd+b^2c^2)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{a^{5/3}} - \frac{2\sqrt{3}(-2a^2d^2+abcd+b^2c^2)\tan^{-1}\left(\frac{1-2\sqrt[3]{b}x}{\sqrt[3]{a}}\right)}{a^{5/3}} - \frac{(-2a^2d^2+abcd+b^2c^2)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{a^{5/3}} + \frac{3\sqrt{3}(-2a^2d^2+abcd+b^2c^2)}{9b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^2,x]

[Out] (9\*b^(1/3)\*d^2\*x + (3\*b^(1/3)\*(b\*c - a\*d)^2\*x)/(a\*(a + b\*x^3)) - (2\*sqrt[3]\*(b^2\*c^2 + a\*b\*c\*d - 2\*a^2\*d^2)\*ArcTan[(1 - (2\*b^(1/3)\*x)/a^(1/3))/sqrt[3]])/a^(5/3) + (2\*(b^2\*c^2 + a\*b\*c\*d - 2\*a^2\*d^2)\*Log[a^(1/3) + b^(1/3)\*x])/a^(5/3) - ((b^2\*c^2 + a\*b\*c\*d - 2\*a^2\*d^2)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/a^(5/3))/(9\*b^(7/3))

**fricas [B]** time = 0.47, size = 768, normalized size = 3.78

$$\left[ \frac{9a^3b^2d^2x^4 - 3\sqrt{\frac{1}{3}}(a^2b^3c^2 + a^3b^2cd - 2a^4bd^2 + (ab^4c^2 + a^2b^3cd - 2a^3b^2d^2)x^3)\sqrt{\frac{(-a^2b)^{1/3}}{b}}\log\left(\frac{2abx^3 + 3(-a^2b)^{1/3}a}{\dots}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] [1/9\*(9\*a^3\*b^2\*d^2\*x^4 - 3\*sqrt(1/3)\*(a^2\*b^3\*c^2 + a^3\*b^2\*c\*d - 2\*a^4\*b\*d^2 + (a\*b^4\*c^2 + a^2\*b^3\*c\*d - 2\*a^3\*b^2\*d^2)\*x^3)\*sqrt((-a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 + 3\*(-a^2\*b)^(1/3)\*a\*x - a^2 - 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt((-a^2\*b)^(1/3)/b))/(b\*x^3 + a)) - (a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 2\*(a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 3\*(a^2\*b^3\*c^2 - 2\*a^3\*b^2\*c\*d + 4\*a^4\*b\*d^2)\*x)/(a^3\*b^4\*x^3 + a^4\*b^3), 1/9\*(9\*a^3\*b^2\*d^2\*x^4 + 6\*sqrt(1/3)\*(a^2\*b^3\*c^2 + a^3\*b^2\*c\*d - 2\*a^4\*b\*d^2 + (a\*b^4\*c^2 + a^2\*b^3\*c\*d - 2\*a^3\*b^2\*d^2)\*x^3)\*sqrt(-(-a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(-a^2\*b)^(2/3)\*x + (-a^2\*b)^(1/3)\*a)\*sqrt(-(-a^2\*b)^(1/3)/b)/a^2) - (a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x^2 - (-a^2\*b)^(2/3)\*x - (-a^2\*b)^(1/3)\*a) + 2\*(a\*b^2\*c^2 + a^2\*b\*c\*d - 2\*a^3\*d^2 + (b^3\*c^2 + a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*(-a^2\*b)^(2/3)\*log(a\*b\*x + (-a^2\*b)^(2/3)) + 3\*(a^2\*b^3\*c^2 - 2\*a^3\*b^2\*c\*d + 4\*a^4\*b\*d^2)\*x)/(a^3\*b^4\*x^3 + a^4\*b^3)]

**giac [A]** time = 0.21, size = 227, normalized size = 1.12

$$\frac{d^2x}{b^2} - \frac{2\sqrt{3}(b^2c^2 + abcd - 2a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}ab} - \frac{(b^2c^2 + abcd - 2a^2d^2)\log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9(-ab^2)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^2,x, algorithm="giac")

[Out]  $d^2x/b^2 - 2/9\sqrt{3}*(b^2c^2 + a*b*c*d - 2a^2d^2)*\arctan(1/3\sqrt{3}*(2*x + (-a/b)^{1/3})/(-a/b)^{1/3})/((-a*b^2)^{2/3}*a*b) - 1/9*(b^2c^2 + a*b*c*d - 2a^2d^2)*\log(x^2 + x*(-a/b)^{1/3} + (-a/b)^{2/3})/((-a*b^2)^{2/3}*a*b) - 2/9*(b^2c^2 + a*b*c*d - 2a^2d^2)*(-a/b)^{1/3}*\log(\text{abs}(x - (-a/b)^{1/3}))/((a^2*b^2) + 1/3*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^3 + a)*a*b^2)$

**maple [B]** time = 0.05, size = 367, normalized size = 1.81

$$\frac{\frac{a d^2 x}{3(b x^3 + a) b^2} + \frac{c^2 x}{3(b x^3 + a) a} - \frac{2 c d x}{3(b x^3 + a) b} - \frac{4 \sqrt{3} a d^2 \arctan\left(\frac{\sqrt{3}\left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} - \frac{4 a d^2 \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3} + \frac{2 a d^2 \ln\left(x^2 - \dots\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}} b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^2,x)

[Out]  $d^2x/b^2 + 1/3/b^2*a*x/(b*x^3+a)*d^2-2/3/b*x/(b*x^3+a)*c*d + 1/3*a*x/(b*x^3+a)*c^2 - 4/9/b^3*a/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*d^2 + 2/9/b^2/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*c*d + 2/9/b/a/(a/b)^{2/3}*\ln(x+(a/b)^{1/3})*c^2 + 2/9/b^3*a/(a/b)^{2/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})*d^2 - 1/9/b^2/(a/b)^{2/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})*c*d - 1/9/b/a/(a/b)^{2/3}*\ln(x^2-(a/b)^{1/3}*x+(a/b)^{2/3})*c^2 - 4/9/b^3*a/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*d^2 + 2/9/b^2/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*c*d + 2/9/b/a/(a/b)^{2/3}*3^{1/2}*\arctan(1/3*3^{1/2}*(2/(a/b)^{1/3}*x-1))*c^2$

**maxima [A]** time = 1.39, size = 220, normalized size = 1.08

$$\frac{\frac{(b^2c^2 - 2abcd + a^2d^2)x}{3(ab^3x^3 + a^2b^2)} + \frac{d^2x}{b^2} + \frac{2\sqrt{3}(b^2c^2 + abcd - 2a^2d^2)\arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(b^2c^2 + abcd - 2a^2d^2)\log\left(x^2 - \dots\right)}{9ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^2,x, algorithm="maxima")

[Out]  $1/3*(b^2c^2 - 2a*b*c*d + a^2d^2)*x/(a*b^3*x^3 + a^2*b^2) + d^2*x/b^2 + 2/9*\sqrt{3}*(b^2c^2 + a*b*c*d - 2a^2d^2)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{1/3})/(a/b)^{1/3})/(a*b^3*(a/b)^{2/3}) - 1/9*(b^2c^2 + a*b*c*d - 2a^2d^2)*\log(x^2 - x*(a/b)^{1/3} + (a/b)^{2/3})/(a*b^3*(a/b)^{2/3}) + 2/9*(b^2c^2 + a*b*c*d - 2a^2d^2)*\log(x + (a/b)^{1/3})/(a*b^3*(a/b)^{2/3})$

**mupad [B]** time = 1.47, size = 191, normalized size = 0.94

$$\frac{d^2x}{b^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{3a(b^3x^3 + a^2b^2)} - \frac{2\ln(b^{1/3}x + a^{1/3})(ad - bc)(2ad + bc)}{9a^{5/3}b^{7/3}} - \frac{2\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}a^{1/3}i)}{9a^{5/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^2,x)

[Out] (d^2\*x)/b^2 + (x\*(a^2\*d^2 + b^2\*c^2 - 2\*a\*b\*c\*d))/(3\*a\*(a\*b^2 + b^3\*x^3)) - (2\*log(b^(1/3)\*x + a^(1/3))\*(a\*d - b\*c)\*(2\*a\*d + b\*c))/(9\*a^(5/3)\*b^(7/3)) - (2\*log(3^(1/2)\*a^(1/3)\*1i + 2\*b^(1/3)\*x - a^(1/3))\*((3^(1/2)\*1i)/2 - 1/2)\*(a\*d - b\*c)\*(2\*a\*d + b\*c))/(9\*a^(5/3)\*b^(7/3)) + (2\*log(3^(1/2)\*a^(1/3)\*1i - 2\*b^(1/3)\*x + a^(1/3))\*((3^(1/2)\*1i)/2 + 1/2)\*(a\*d - b\*c)\*(2\*a\*d + b\*c))/(9\*a^(5/3)\*b^(7/3))

**sympy [A]** time = 2.56, size = 189, normalized size = 0.93

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{3a^2b^2 + 3ab^3x^3} + \text{RootSum}\left(729t^3a^5b^7 + 64a^6d^6 - 96a^5bcd^5 - 48a^4b^2c^2d^4 + 88a^3b^3c^3d^3 + 24a^2b^4c^4d^2 - 8a^2b^5c^5d - 8b^6c^6, \text{Lambda}(t, t \log(-9t^2a^2b^2/(4a^2d^2 - 2ab^3cd - 2b^2c^2) + x))\right) + d^2x/b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*2,x)

[Out] x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(3\*a\*\*2\*b\*\*2 + 3\*a\*b\*\*3\*x\*\*3) + RootSum(729\*\_t\*\*3\*a\*\*5\*b\*\*7 + 64\*a\*\*6\*d\*\*6 - 96\*a\*\*5\*b\*c\*d\*\*5 - 48\*a\*\*4\*b\*\*2\*c\*\*2\*d\*\*4 + 88\*a\*\*3\*b\*\*3\*c\*\*3\*d\*\*3 + 24\*a\*\*2\*b\*\*4\*c\*\*4\*d\*\*2 - 24\*a\*b\*\*5\*c\*\*5\*d - 8\*b\*\*6\*c\*\*6, Lambda(\_t, \_t\*log(-9\*\_t\*a\*\*2\*b\*\*2/(4\*a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d - 2\*b\*\*2\*c\*\*2) + x))) + d\*\*2\*x/b\*\*2

$$3.24 \quad \int \frac{c+dx^3}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=169

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-a)}{3ab(a+bx^3)}$$

[Out]  $\frac{1}{3}*(-a*d+b*c)*x/a/b/(b*x^3+a)+\frac{1}{9}*(a*d+2*b*c)*\ln(a^{(1/3)}+b^{(1/3)}*x)/a^{(5/3)}/b^{(4/3)}-\frac{1}{18}*(a*d+2*b*c)*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*x+b^{(2/3)}*x^2)/a^{(5/3)}/b^{(4/3)}-\frac{1}{9}*(a*d+2*b*c)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*x)/a^{(1/3)}*3^{(1/2)})/a^{(5/3)}/b^{(4/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {385, 200, 31, 634, 617, 204, 628}

$$-\frac{(ad+2bc)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2\right)}{18a^{5/3}b^{4/3}}+\frac{(ad+2bc)\log\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}{9a^{5/3}b^{4/3}}-\frac{(ad+2bc)\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}}+\frac{x(bc-a)}{3ab(a+bx^3)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^2, x]

[Out]  $\frac{(b*c - a*d)*x}{(3*a*b*(a + b*x^3))} - \frac{((2*b*c + a*d)*\text{ArcTan}[(a^{(1/3)} - 2*b^{(1/3)}*x)/(\text{Sqrt}[3]*a^{(1/3)})])}{(3*\text{Sqrt}[3]*a^{(5/3)}*b^{(4/3)})} + \frac{((2*b*c + a*d)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])}{(9*a^{(5/3)}*b^{(4/3)})} - \frac{((2*b*c + a*d)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])}{(18*a^{(5/3)}*b^{(4/3)})}$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{c + dx^3}{(a + bx^3)^2} dx &= \frac{(bc - ad)x}{3ab(a + bx^3)} + \frac{(2bc + ad) \int \frac{1}{a + bx^3} dx}{3ab} \\ &= \frac{(bc - ad)x}{3ab(a + bx^3)} + \frac{(2bc + ad) \int \frac{1}{\sqrt[3]{a} + \sqrt[3]{b}x} dx}{9a^{5/3}b} + \frac{(2bc + ad) \int \frac{2\sqrt[3]{a} - \sqrt[3]{b}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{9a^{5/3}b} \\ &= \frac{(bc - ad)x}{3ab(a + bx^3)} + \frac{(2bc + ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2bc + ad) \int \frac{-\sqrt[3]{a}\sqrt[3]{b} + 2b^{2/3}x}{a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2} dx}{18a^{5/3}b^{4/3}} + \frac{(2bc + ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{18a^{5/3}b^{4/3}} \\ &= \frac{(bc - ad)x}{3ab(a + bx^3)} + \frac{(2bc + ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2bc + ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{18a^{5/3}b^{4/3}} + \frac{(2bc + ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}b^{4/3}} + \frac{(2bc + ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}b^{4/3}} - \frac{(2bc + ad) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2)}{18a^{5/3}b^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 145, normalized size = 0.86

$$\frac{-(ad + 2bc) \log(a^{2/3} - \sqrt[3]{a}\sqrt[3]{b}x + b^{2/3}x^2) - \frac{6a^{2/3}\sqrt[3]{b}x(ad - bc)}{a + bx^3} + 2(ad + 2bc) \log(\sqrt[3]{a} + \sqrt[3]{b}x) - 2\sqrt{3}(ad + 2bc) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{18a^{5/3}b^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^3)/(a + b*x^3)^2, x]
```

```
[Out] ((-6*a^(2/3)*b^(1/3)*(-(b*c) + a*d)*x)/(a + b*x^3) - 2*Sqrt[3]*(2*b*c + a*d)*ArcTan[(1 - (2*b^(1/3)*x)/a^(1/3))/Sqrt[3]] + 2*(2*b*c + a*d)*Log[a^(1/3) + b^(1/3)*x] - (2*b*c + a*d)*Log[a^(2/3) - a^(1/3)*b^(1/3)*x + b^(2/3)*x^2]/(18*a^(5/3)*b^(4/3))
```

**fricas** [A] time = 0.47, size = 537, normalized size = 3.18

$$3 \sqrt{\frac{1}{3}} (2a^2b^2c + a^3bd + (2ab^3c + a^2b^2d)x^3) \sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}} \log \left( \frac{2abx^3 - 3(a^2b)^{\frac{1}{3}}ax - a^2 + 3\sqrt{\frac{1}{3}} \left( 2abx^2 + (a^2b)^{\frac{2}{3}}x - (a^2b)^{\frac{1}{3}}a \right) \sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}}{bx^3 + a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] [1/18\*(3\*sqrt(1/3)\*(2\*a^2\*b^2\*c + a^3\*b\*d + (2\*a\*b^3\*c + a^2\*b^2\*d)\*x^3)\*sqrt(-(a^2\*b)^(1/3)/b)\*log((2\*a\*b\*x^3 - 3\*(a^2\*b)^(1/3)\*a\*x - a^2 + 3\*sqrt(1/3)\*(2\*a\*b\*x^2 + (a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt(-(a^2\*b)^(1/3)/b))/(b\*x^3 + a)) - ((2\*b^2\*c + a\*b\*d)\*x^3 + 2\*a\*b\*c + a^2\*d)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 2\*((2\*b^2\*c + a\*b\*d)\*x^3 + 2\*a\*b\*c + a^2\*d)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 6\*(a^2\*b^2\*c - a^3\*b\*d)\*x)/(a^3\*b^3\*x^3 + a^4\*b^2), 1/18\*(6\*sqrt(1/3)\*(2\*a^2\*b^2\*c + a^3\*b\*d + (2\*a\*b^3\*c + a^2\*b^2\*d)\*x^3)\*sqrt((a^2\*b)^(1/3)/b)\*arctan(sqrt(1/3)\*(2\*(a^2\*b)^(2/3)\*x - (a^2\*b)^(1/3)\*a)\*sqrt((a^2\*b)^(1/3)/b)/a^2 - ((2\*b^2\*c + a\*b\*d)\*x^3 + 2\*a\*b\*c + a^2\*d)\*(a^2\*b)^(2/3)\*log(a\*b\*x^2 - (a^2\*b)^(2/3)\*x + (a^2\*b)^(1/3)\*a) + 2\*((2\*b^2\*c + a\*b\*d)\*x^3 + 2\*a\*b\*c + a^2\*d)\*(a^2\*b)^(2/3)\*log(a\*b\*x + (a^2\*b)^(2/3)) + 6\*(a^2\*b^2\*c - a^3\*b\*d)\*x)/(a^3\*b^3\*x^3 + a^4\*b^2)]

**giac** [A] time = 0.17, size = 160, normalized size = 0.95

$$\frac{\sqrt{3}(2bc + ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9(-ab^2)^{\frac{2}{3}}a} + \frac{(2bc + ad) \log\left(x^2 + x\left(-\frac{a}{b}\right)^{\frac{1}{3}} + \left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18(-ab^2)^{\frac{2}{3}}a} + \frac{(2bc + ad)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^2,x, algorithm="giac")

[Out] -1/9\*sqrt(3)\*(2\*b\*c + a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-a/b)^(1/3))/(-a/b)^(1/3))/((-a\*b^2)^(2/3)\*a) - 1/18\*(2\*b\*c + a\*d)\*log(x^2 + x\*(-a/b)^(1/3) + (-a/b)^(2/3))/((-a\*b^2)^(2/3)\*a) - 1/9\*(2\*b\*c + a\*d)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)^(1/3)))/(a^2\*b) + 1/3\*(b\*c\*x - a\*d\*x)/((b\*x^3 + a)\*a\*b)

**maple** [A] time = 0.05, size = 221, normalized size = 1.31

$$\frac{2\sqrt{3}c \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} + \frac{2c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} - \frac{c \ln\left(x^2 - \left(\frac{a}{b}\right)^{\frac{1}{3}}x + \left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab} + \frac{\sqrt{3}d \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}b^2} + \frac{d \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9\left(\frac{a}{b}\right)^{\frac{2}{3}}ab}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((d\*x^3+c)/(b\*x^3+a)^2,x)

[Out] 
$$-1/3*(a*d-b*c)/a/b*x/(b*x^3+a)+1/9/b^2/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*d+2/9/b/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c-1/18/b^2/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*d-1/9/b/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c+1/9/b^2/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*d+2/9/b/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c$$

**maxima** [A] time = 1.44, size = 158, normalized size = 0.93

$$\frac{(bc-ad)x}{3(ab^2x^3+a^2b)} + \frac{\sqrt{3}(2bc+ad)\arctan\left(\frac{\sqrt{3}\left(2x-\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} - \frac{(2bc+ad)\log\left(x^2-x\left(\frac{a}{b}\right)^{\frac{1}{3}}+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{18ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}} + \frac{(2bc+ad)\log\left(x+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] 
$$1/3*(b*c - a*d)*x/(a*b^2*x^3 + a^2*b) + 1/9*\sqrt{3}*(2*b*c + a*d)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)}) - 1/18*(2*b*c + a*d)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^2*(a/b)^{(2/3)}) + 1/9*(2*b*c + a*d)*\log(x + (a/b)^{(1/3)})/(a*b^2*(a/b)^{(2/3)})$$

**mupad** [B] time = 1.43, size = 143, normalized size = 0.85

$$\frac{\ln(b^{1/3}x + a^{1/3})(ad + 2bc)}{9a^{5/3}b^{4/3}} - \frac{\ln(a^{1/3} - 2b^{1/3}x + \sqrt{3}a^{1/3}i)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)(ad + 2bc)}{9a^{5/3}b^{4/3}} + \frac{\ln(2b^{1/3}x - a^{1/3} + \sqrt{3}i)\left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)(ad + 2bc)}{9a^{5/3}b^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)/(a + b\*x^3)^2,x)

[Out] 
$$\frac{(\log(b^{(1/3)}*x + a^{(1/3)})*(a*d + 2*b*c))/(9*a^{(5/3)}*b^{(4/3)}) - (\log(3^{(1/2)}*a^{(1/3)}*i - 2*b^{(1/3)}*x + a^{(1/3)})*((3^{(1/2)}*i)/2 + 1/2)*(a*d + 2*b*c))/(9*a^{(5/3)}*b^{(4/3)}) + (\log(3^{(1/2)}*a^{(1/3)}*i + 2*b^{(1/3)}*x - a^{(1/3)})*((3^{(1/2)}*i)/2 - 1/2)*(a*d + 2*b*c))/(9*a^{(5/3)}*b^{(4/3)}) - (x*(a*d - b*c))/(3*a*b*(a + b*x^3))$$

**sympy** [A] time = 1.42, size = 97, normalized size = 0.57

$$\frac{x(-ad + bc)}{3a^2b + 3ab^2x^3} + \text{RootSum}\left(729t^3a^5b^4 - a^3d^3 - 6a^2bcd^2 - 12ab^2c^2d - 8b^3c^3, \left(t \mapsto t \log\left(\frac{9ta^2b}{ad + 2bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a)\*\*2,x)

[Out] 
$$x*(-a*d + b*c)/(3*a**2*b + 3*a*b**2*x**3) + \text{RootSum}(729*_t**3*a**5*b**4 - a**3*d**3 - 6*a**2*b*c*d**2 - 12*a*b**2*c**2*d - 8*b**3*c**3, \text{Lambda}(_t, _t*\log(9*_t*a**2*b/(a*d + 2*b*c) + x)))$$

$$3.25 \quad \int \frac{1}{(a+bx^3)^2(c+dx^3)} dx$$

**Optimal.** Leaf size=346

$$\frac{b^{2/3}(2bc-5ad) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}$$

[Out]  $\frac{1}{3} \frac{b^2 x}{a(-ad+bc)(bx^3+a)} + \frac{1}{9} b^{2/3} (-5ad+2bc) \ln(a^{1/3} + b^{1/3} x) / a^{5/3} (-ad+bc)^2 + \frac{1}{3} d^{5/3} \ln(c^{1/3} + d^{1/3} x) / c^{2/3} (-ad+bc)^2 - \frac{1}{18} b^{2/3} (-5ad+2bc) \ln(a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2) / a^{5/3} (-ad+bc)^2 - \frac{1}{6} d^{5/3} \ln(c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2) / c^{2/3} (-ad+bc)^2 - \frac{1}{9} b^{2/3} (-5ad+2bc) \arctan(1/3(a^{1/3} - 2b^{1/3} x) / a^{1/3})^3 / a^{5/3} (-ad+bc)^2 - \frac{1}{3} d^{5/3} \arctan(1/3(c^{1/3} - 2d^{1/3} x) / c^{1/3})^3 / c^{2/3} (-ad+bc)^2$

**Rubi [A]** time = 0.25, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {414, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{2/3}(2bc-5ad) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{18a^{5/3}(bc-ad)^2} + \frac{b^{2/3}(2bc-5ad) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3}(bc-ad)^2} - \frac{b^{2/3}(2bc-5ad) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^2\*(c + d\*x^3)), x]

[Out]  $\frac{b^2 x}{3a(b^2 c - a^2 d)(a + b^2 x^3)} - \frac{b^{2/3}(2bc - 5ad) \text{ArcTan}\left[\frac{a^{1/3} - 2b^{1/3} x}{\sqrt{3} a^{1/3}}\right]}{3 \sqrt{3} a^{5/3} (bc - ad)^2} - \frac{d^{5/3} \text{ArcTan}\left[\frac{c^{1/3} - 2d^{1/3} x}{\sqrt{3} c^{1/3}}\right]}{\sqrt{3} c^{2/3} (bc - ad)^2} + \frac{b^{2/3}(2bc - 5ad) \text{Log}[a^{1/3} + b^{1/3} x]}{9a^{5/3} (bc - ad)^2} + \frac{d^{5/3} \text{Log}[c^{1/3} + d^{1/3} x]}{3c^{2/3} (bc - ad)^2} - \frac{b^{2/3}(2bc - 5ad) \text{Log}[a^{2/3} - a^{1/3} b^{1/3} x + b^{2/3} x^2]}{18a^{5/3} (bc - ad)^2} - \frac{d^{5/3} \text{Log}[c^{2/3} - c^{1/3} d^{1/3} x + d^{2/3} x^2]}{6c^{2/3} (bc - ad)^2}$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)<sup>(-1)</sup>, x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(bc -

$a*d)), x] + \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*\text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& !( \text{IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

### Rule 522

$\text{Int}[(e + f*x^n)/((a + b*x^n)*(c + d*x^n)), x\_Symbol] := \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x]$

### Rule 617

$\text{Int}[(a + b*x + c*x^2)^{-1}, x\_Symbol] := \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] || !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[(d + e*x)/(a + b*x + c*x^2), x\_Symbol] := \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

### Rule 634

$\text{Int}[(d + e*x)/(a + b*x + c*x^2), x\_Symbol] := \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& !\text{NiceSqrtQ}[b^2 - 4*a*c]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^3)^2 (c + dx^3)} dx &= \frac{bx}{3a(bc - ad)(a + bx^3)} - \frac{\int \frac{-2bc + 3ad - 2bdx^3}{(a + bx^3)(c + dx^3)} dx}{3a(bc - ad)} \\ &= \frac{bx}{3a(bc - ad)(a + bx^3)} + \frac{d^2 \int \frac{1}{c + dx^3} dx}{(bc - ad)^2} + \frac{(b(2bc - 5ad)) \int \frac{1}{a + bx^3} dx}{3a(bc - ad)^2} \\ &= \frac{bx}{3a(bc - ad)(a + bx^3)} + \frac{d^2 \int \frac{1}{\sqrt[3]{c} + \sqrt[3]{d}x} dx}{3c^{2/3}(bc - ad)^2} + \frac{d^2 \int \frac{2\sqrt[3]{c} - \sqrt[3]{d}x}{c^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}x^2} dx}{3c^{2/3}(bc - ad)^2} + \frac{(b(2bc - 5ad)) \int \frac{1}{a + bx^3} dx}{9a^{5/3}(bc - ad)^2} \\ &= \frac{bx}{3a(bc - ad)(a + bx^3)} + \frac{b^{2/3}(2bc - 5ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}(bc - ad)^2} + \frac{d^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc - ad)^2} \\ &= \frac{bx}{3a(bc - ad)(a + bx^3)} + \frac{b^{2/3}(2bc - 5ad) \log(\sqrt[3]{a} + \sqrt[3]{b}x)}{9a^{5/3}(bc - ad)^2} + \frac{d^{5/3} \log(\sqrt[3]{c} + \sqrt[3]{d}x)}{3c^{2/3}(bc - ad)^2} \\ &= \frac{bx}{3a(bc - ad)(a + bx^3)} - \frac{b^{2/3}(2bc - 5ad) \tan^{-1}\left(\frac{\sqrt[3]{a} - 2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc - ad)^2} - \frac{d^{5/3} \tan^{-1}\left(\frac{\sqrt[3]{c} - 2\sqrt[3]{d}x}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc - ad)^2} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 337, normalized size = 0.97

$$-b^{2/3}c^{2/3}(a+bx^3)(2bc-5ad)\log(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x^2)-3a^{5/3}d^{5/3}(a+bx^3)\log(c^{2/3}-\sqrt[3]{c}\sqrt[3]{d}x+d^{2/3}x^2)+$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^2\*(c + d\*x^3)),x]

[Out] (6\*a^(2/3)\*b\*c^(2/3)\*(b\*c - a\*d)\*x - 2\*Sqrt[3]\*b^(2/3)\*c^(2/3)\*(2\*b\*c - 5\*a\*d)\*(a + b\*x^3)\*ArcTan[(1 - (2\*b^(1/3)\*x)/a^(1/3))/Sqrt[3]] - 6\*Sqrt[3]\*a^(5/3)\*d^(5/3)\*(a + b\*x^3)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]] + 2\*b^(2/3)\*c^(2/3)\*(2\*b\*c - 5\*a\*d)\*(a + b\*x^3)\*Log[a^(1/3) + b^(1/3)\*x] + 6\*a^(5/3)\*d^(5/3)\*(a + b\*x^3)\*Log[c^(1/3) + d^(1/3)\*x] - b^(2/3)\*c^(2/3)\*(2\*b\*c - 5\*a\*d)\*(a + b\*x^3)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2] - 3\*a^(5/3)\*d^(5/3)\*(a + b\*x^3)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2]/(18\*a^(5/3)\*c^(2/3)\*(b\*c - a\*d)^2\*(a + b\*x^3))

**fricas [A]** time = 7.38, size = 440, normalized size = 1.27

$$2\sqrt{3}\left((2b^2c-5abd)x^3+2abc-5a^2d\right)\left(-\frac{b^2}{a^2}\right)^{\frac{1}{3}}\arctan\left(\frac{2\sqrt{3}ax\left(-\frac{b^2}{a^2}\right)^{\frac{2}{3}}-\sqrt{3}b}{3b}\right)-6\sqrt{3}(abdx^3+a^2d)\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{d^2}{c^2}\right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^2/(d\*x^3+c),x, algorithm="fricas")

[Out] -1/18\*(2\*sqrt(3)\*((2\*b^2\*c - 5\*a\*b\*d)\*x^3 + 2\*a\*b\*c - 5\*a^2\*d)\*(-b^2/a^2)^(1/3)\*arctan(1/3\*(2\*sqrt(3)\*a\*x\*(-b^2/a^2)^(2/3) - sqrt(3)\*b)/b) - 6\*sqrt(3)\*(a\*b\*d\*x^3 + a^2\*d)\*(d^2/c^2)^(1/3)\*arctan(1/3\*(2\*sqrt(3)\*c\*x\*(d^2/c^2)^(2/3) - sqrt(3)\*d)/d) - ((2\*b^2\*c - 5\*a\*b\*d)\*x^3 + 2\*a\*b\*c - 5\*a^2\*d)\*(-b^2/a^2)^(1/3)\*log(b^2\*x^2 + a\*b\*x\*(-b^2/a^2)^(1/3) + a^2\*(-b^2/a^2)^(2/3)) + 3\*(a\*b\*d\*x^3 + a^2\*d)\*(d^2/c^2)^(1/3)\*log(d^2\*x^2 - c\*d\*x\*(d^2/c^2)^(1/3) + c^2\*(d^2/c^2)^(2/3)) + 2\*((2\*b^2\*c - 5\*a\*b\*d)\*x^3 + 2\*a\*b\*c - 5\*a^2\*d)\*(-b^2/a^2)^(1/3)\*log(b\*x - a\*(-b^2/a^2)^(1/3)) - 6\*(a\*b\*d\*x^3 + a^2\*d)\*(d^2/c^2)^(1/3)\*log(d\*x + c\*(d^2/c^2)^(1/3)) - 6\*(b^2\*c - a\*b\*d)\*x/(a^2\*b^2\*c^2 - 2\*a^3\*b\*c\*d + a^4\*d^2 + (a\*b^3\*c^2 - 2\*a^2\*b^2\*c\*d + a^3\*b\*d^2)\*x^3)

**giac [A]** time = 0.20, size = 443, normalized size = 1.28

$$\frac{d^2\left(-\frac{c}{d}\right)^{\frac{1}{3}}\log\left(\left|x-\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{3\left(b^2c^3-2abc^2d+a^2cd^2\right)}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\arctan\left(\frac{\sqrt{3}\left(2x+\left(-\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{\sqrt{3}b^2c^3-2\sqrt{3}abc^2d+\sqrt{3}a^2cd^2}+\frac{\left(-cd^2\right)^{\frac{1}{3}}d\log\left(x^2+x\left(-\frac{c}{d}\right)^{\frac{1}{3}}+\left(-\frac{c}{d}\right)^{\frac{2}{3}}\right)}{6\left(b^2c^3-2abc^2d+a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^2/(d\*x^3+c),x, algorithm="giac")

[Out] -1/3\*d^2\*(-c/d)^(1/3)\*log(abs(x - (-c/d)^(1/3)))/(b^2\*c^3 - 2\*a\*b\*c^2\*d + a^2\*c\*d^2) + (-c\*d^2)^(1/3)\*d\*arctan(1/3\*sqrt(3)\*(2\*x + (-c/d)^(1/3))/(-c/d)^(1/3))/(sqrt(3)\*b^2\*c^3 - 2\*sqrt(3)\*a\*b\*c^2\*d + sqrt(3)\*a^2\*c\*d^2) + 1/6\*(-c\*d^2)^(1/3)\*d\*log(x^2 + x\*(-c/d)^(1/3) + (-c/d)^(2/3))/(b^2\*c^3 - 2\*a\*b\*c^2\*d + a^2\*c\*d^2) - 1/9\*(2\*b^2\*c - 5\*a\*b\*d)\*(-a/b)^(1/3)\*log(abs(x - (-a/b)

$$\begin{aligned} & \left. \frac{1}{(a^2 b^2 c^2 - 2 a^3 b c d + a^4 d^2)} + \frac{1}{3} (2 (-a b^2)^{1/3} b c - 5 (-a b^2)^{1/3} a d) \arctan\left(\frac{1}{3} \sqrt{3} (2 x + (-a/b)^{1/3}) / (-a/b)^{1/3}\right) \right. \\ & \left. \frac{1}{(\sqrt{3} a^2 b^2 c^2 - 2 \sqrt{3} a^3 b c d + \sqrt{3} a^4 d^2)} + \frac{1}{18} (2 (-a b^2)^{1/3} b c - 5 (-a b^2)^{1/3} a d) \log(x^2 + x (-a/b)^{1/3} + (-a/b)^{2/3}) \right. \\ & \left. \frac{1}{(a^2 b^2 c^2 - 2 a^3 b c d + a^4 d^2)} + \frac{1}{3} b x / ((b x^3 + a) (a b c - a^2 d)) \right) \end{aligned}$$

**maple [A]** time = 0.06, size = 406, normalized size = 1.17

$$\frac{\frac{b^2 c x}{3 (a d - b c)^2 (b x^3 + a) a} - \frac{b d x}{3 (a d - b c)^2 (b x^3 + a)}}{9 (a d - b c)^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} a} + \frac{2 \sqrt{3} b c \arctan\left(\frac{\sqrt{3} \left(\frac{2 x}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{9 (a d - b c)^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} a} + \frac{2 b c \ln\left(x + \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9 (a d - b c)^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} a} - \frac{b c \ln\left(x^2\right)}{9 (a d - b c)^2 \left(\frac{a}{b}\right)^{\frac{2}{3}} a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^2/(d\*x^3+c), x)

[Out] 
$$\begin{aligned} & -1/3 b / (a d - b c)^2 x / (b x^3 + a) d + 1/3 b^2 / (a d - b c)^2 a x / (b x^3 + a) c - 5/9 / (a d - b c)^2 / (a/b)^{2/3} * \ln(x + (a/b)^{1/3}) * d + 2/9 b / (a d - b c)^2 a / (a/b)^{2/3} * \ln(x + (a/b)^{1/3}) * c \\ & + 5/18 / (a d - b c)^2 / (a/b)^{2/3} * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * d - 1/9 b / (a d - b c)^2 a / (a/b)^{2/3} * \ln(x^2 - (a/b)^{1/3} * x + (a/b)^{2/3}) * c \\ & - 5/9 / (a d - b c)^2 / (a/b)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2 / (a/b)^{1/3} * x - 1)) * d + 2/9 b / (a d - b c)^2 a / (a/b)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2 / (a/b)^{1/3} * x - 1)) * c \\ & + 1/3 d / (a d - b c)^2 / (c/d)^{2/3} * \ln(x + (c/d)^{1/3}) - 1/6 d / (a d - b c)^2 / (c/d)^{2/3} * \ln(x^2 - (c/d)^{1/3} * x + (c/d)^{2/3}) \\ & + 1/3 d / (a d - b c)^2 / (c/d)^{2/3} * 3^{1/2} * \arctan(1/3 * 3^{1/2} * (2 / (c/d)^{1/3} * x - 1)) \end{aligned}$$

**maxima [A]** time = 1.29, size = 489, normalized size = 1.41

$$\frac{\frac{\sqrt{3} (2 b c - 5 a d) \arctan\left(\frac{\sqrt{3} \left(2 x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9 \left(a b^2 c^2 \left(\frac{a}{b}\right)^{\frac{1}{3}} - 2 a^2 b c d \left(\frac{a}{b}\right)^{\frac{1}{3}} + a^3 d^2 \left(\frac{a}{b}\right)^{\frac{1}{3}}\right) \left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{\sqrt{3} d \arctan\left(\frac{\sqrt{3} \left(2 x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3 \left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{3 \left(b^2 c^2 \left(\frac{c}{d}\right)^{\frac{1}{3}} - 2 a b c d \left(\frac{c}{d}\right)^{\frac{1}{3}} + a^2 d^2 \left(\frac{c}{d}\right)^{\frac{1}{3}}\right) \left(\frac{c}{d}\right)^{\frac{1}{3}}} + \frac{1}{3 \left(a^2 b c - a^3 d\right) \left(\frac{c}{d}\right)^{\frac{1}{3}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^2/(d\*x^3+c), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & \frac{1}{9} \sqrt{3} (2 b c - 5 a d) \arctan\left(\frac{1}{3} \sqrt{3} (2 x - (a/b)^{1/3}) / (a/b)^{1/3}\right) / ((a b^2 c^2 (a/b)^{1/3} - 2 a^2 b c d (a/b)^{1/3} + a^3 d^2 (a/b)^{1/3}) (a/b)^{1/3}) \\ & + \frac{1}{3} \sqrt{3} d \arctan\left(\frac{1}{3} \sqrt{3} (2 x - (c/d)^{1/3}) / (c/d)^{1/3}\right) / ((b^2 c^2 (c/d)^{1/3} - 2 a b c d (c/d)^{1/3} + a^2 d^2 (c/d)^{1/3}) (c/d)^{1/3}) \\ & + \frac{1}{3} b x / (a^2 b c - a^3 d + (a b^2 c - a^2 b d) x^3) - \frac{1}{18} (2 b c - 5 a d) \log(x^2 - x (a/b)^{1/3} + (a/b)^{2/3}) / (a b^2 c^2 (a/b)^{2/3} - 2 a^2 b c d (a/b)^{2/3} + a^3 d^2 (a/b)^{2/3}) \\ & - \frac{1}{6} d \log(x^2 - x (c/d)^{1/3} + (c/d)^{2/3}) / (b^2 c^2 (c/d)^{2/3} - 2 a b c d (c/d)^{2/3} + a^2 d^2 (c/d)^{2/3}) \\ & + \frac{1}{9} (2 b c - 5 a d) \log(x + (a/b)^{1/3}) / (a b^2 c^2 (a/b)^{2/3} - 2 a^2 b c d (a/b)^{2/3} + a^3 d^2 (a/b)^{2/3}) \\ & + \frac{1}{3} d \log(x + (c/d)^{1/3}) / (b^2 c^2 (c/d)^{2/3} - 2 a b c d (c/d)^{2/3} + a^2 d^2 (c/d)^{2/3}) \end{aligned}$$

**mupad [B]** time = 15.93, size = 2492, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/((a + b*x^3)^2*(c + d*x^3)),x)$

[Out]  $\log\left(\frac{\left(\frac{27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d)}{a + 27a^2b^3c^3d^3(a d + b c)(a d - b c)^4(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6)}\right)^{1/3}(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6)}{81} - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6)}\right)^{1/3} + \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(8b^5c^3 - 125a^3b^2d^3 + 150a^2b^3c^2d^2 - 60a^2b^4c^2d)/(729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5)}{729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5)}\right)^{1/3} + \log\left(\frac{\left(\frac{27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d)}{a + 81a^2b^3c^3d^3(a d + b c)(a d - b c)^4(d^5/(c^2(a d - b c)^6))^{1/3}}\right)^{1/3}(d^5/(c^2(a d - b c)^6))^{2/3}}{9} - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(d^5/(c^2(a d - b c)^6))^{1/3}}\right)/3 + \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}}{(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}} + (\log((3^{1/2}) * 1i - 1) * ((3^{1/2}) * 1i - 1)^2 * ((27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d))/a + (27a^2b^3c^3d^3(3^{1/2}) * 1i - 1)(a d + b c)(a d - b c)^4(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6))^{1/3})/2 * (-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6))^{2/3})/324 - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6)}\right)^{1/3}/18 + \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(3^{1/2} * 1i - 1)((8b^5c^3 - 125a^3b^2d^3 + 150a^2b^3c^2d^2 - 60a^2b^4c^2d)/(729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5))^{1/3}}{729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5)}\right)^{1/3}/2 - (\log((3^{1/2}) * 1i + 1) * ((3^{1/2}) * 1i + 1)^2 * ((27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d))/a - (27a^2b^3c^3d^3(3^{1/2}) * 1i + 1)(a d + b c)(a d - b c)^4(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6))^{1/3})/2 * (-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6))^{2/3})/324 - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(-b^2(5a d - 2b^2c^2)^3)/(a^5(a d - b c)^6)}\right)^{1/3}/18 - \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(3^{1/2} * 1i + 1)((8b^5c^3 - 125a^3b^2d^3 + 150a^2b^3c^2d^2 - 60a^2b^4c^2d)/(729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5))^{1/3}}{729a^{11}d^6 + 729a^5b^6c^6 - 4374a^6b^5c^5d + 10935a^7b^4c^4d^2 - 14580a^8b^3c^3d^3 + 10935a^9b^2c^2d^4 - 4374a^{10}b^2c^2d^5)}\right)^{1/3}/2 + (\log((3^{1/2}) * 1i - 1) * ((3^{1/2}) * 1i - 1)^2 * ((27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d))/a + (81a^2b^3c^3d^3(3^{1/2}) * 1i - 1)(a d + b c)(a d - b c)^4(d^5/(c^2(a d - b c)^6))^{1/3})/2 * (d^5/(c^2(a d - b c)^6))^{2/3})/36 - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(d^5/(c^2(a d - b c)^6))^{1/3}}\right)/6 + \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(3^{1/2} * 1i - 1)(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}}{(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}}\right)^{1/3}/2 - (\log((3^{1/2}) * 1i + 1) * ((3^{1/2}) * 1i + 1)^2 * ((27b^3d^3x(a d - b c)^3(3a^2d^2 - 2b^2c^2 + 3a b c d))/a - (81a^2b^3c^3d^3(3^{1/2}) * 1i + 1)(a d + b c)(a d - b c)^4(d^5/(c^2(a d - b c)^6))^{1/3})/2 * (d^5/(c^2(a d - b c)^6))^{2/3})/36 - \frac{b^4d^4(27a^3d^3 - 8b^3c^3 + 52a^2b^2c^2d - 98a^2b^2c^2d^2)/(3a^4d - 3a^3b^2c)}{(d^5/(c^2(a d - b c)^6))^{1/3}}\right)/6 - \frac{(2b^5d^6x(85a^3d^3 - 4b^3c^3 + 30a^2b^2c^2d - 84a^2b^2c^2d^2))/(9a^3(a d - b c)^4)(3^{1/2} * 1i + 1)(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}}{(d^5/(27b^6c^8 + 27a^6c^2d^6 - 162a^5b^2c^3d^5 + 405a^4b^4c^6d^2 - 540a^3b^3c^5d^3 + 405a^4b^2c^4d^4 - 162a^4b^5c^7d))^{1/3}}\right)^{1/3}/2 - (b*x)/(3*a*(a + b*x^3)*(a*d - b*c))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*2/(d\*x\*\*3+c), x)

[Out] Timed out

$$3.26 \quad \int \frac{1}{(a+bx^3)^2(c+dx^3)^2} dx$$

**Optimal.** Leaf size=419

$$\frac{b^{5/3}(bc-4ad) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{9a^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3}(bc-ad)^3} - \frac{2b^{5/3}(bc-4ad) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}$$

[Out]  $\frac{1}{3}d*(a*d+b*c)*x/a/c/(-a*d+b*c)^2/(d*x^3+c)+\frac{1}{3}b*x/a/(-a*d+b*c)/(b*x^3+a)/(d*x^3+c)+\frac{2}{9}b^{(5/3)}*(-4*a*d+b*c)*\ln(a^{(1/3)}+b^{(1/3)}*x)/a^{(5/3)}/(-a*d+b*c)^3+\frac{2}{9}d^{(5/3)}*(-a*d+4*b*c)*\ln(c^{(1/3)}+d^{(1/3)}*x)/c^{(5/3)}/(-a*d+b*c)^3-\frac{1}{9}b^{(5/3)}*(-4*a*d+b*c)*\ln(a^{(2/3)}-a^{(1/3)}*b^{(1/3)}*x+b^{(2/3)}*x^2)/a^{(5/3)}/(-a*d+b*c)^3-\frac{1}{9}d^{(5/3)}*(-a*d+4*b*c)*\ln(c^{(2/3)}-c^{(1/3)}*d^{(1/3)}*x+d^{(2/3)}*x^2)/c^{(5/3)}/(-a*d+b*c)^3-\frac{2}{9}b^{(5/3)}*(-4*a*d+b*c)*\arctan(1/3*(a^{(1/3)}-2*b^{(1/3)}*x)/a^{(1/3)}*3^{(1/2)})/a^{(5/3)}/(-a*d+b*c)^3*3^{(1/2)}-\frac{2}{9}d^{(5/3)}*(-a*d+4*b*c)*\arctan(1/3*(c^{(1/3)}-2*d^{(1/3)}*x)/c^{(1/3)}*3^{(1/2)})/c^{(5/3)}/(-a*d+b*c)^3*3^{(1/2)}$

**Rubi [A]** time = 0.49, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 19, number of rules / integrand size = 0.474, Rules used = {414, 527, 522, 200, 31, 634, 617, 204, 628}

$$\frac{b^{5/3}(bc-4ad) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{9a^{5/3}(bc-ad)^3} + \frac{2b^{5/3}(bc-4ad) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{9a^{5/3}(bc-ad)^3} - \frac{2b^{5/3}(bc-4ad) \tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}x}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^2\*(c + d\*x^3)^2), x]

[Out]  $\frac{d*(b*c + a*d)*x}{3*a*c*(b*c - a*d)^2*(c + d*x^3)} + \frac{b*x}{3*a*(b*c - a*d)*(a + b*x^3)*(c + d*x^3)} - \frac{(2*b^{(5/3)}*(b*c - 4*a*d)*\text{ArcTan}[a^{(1/3)} - 2*b^{(1/3)}*x]/(\text{Sqrt}[3]*a^{(1/3)})]}{3*\text{Sqrt}[3]*a^{(5/3)}*(b*c - a*d)^3} - \frac{(2*d^{(5/3)}*(4*b*c - a*d)*\text{ArcTan}[c^{(1/3)} - 2*d^{(1/3)}*x]/(\text{Sqrt}[3]*c^{(1/3)})]}{3*\text{Sqrt}[3]*c^{(5/3)}*(b*c - a*d)^3} + \frac{(2*b^{(5/3)}*(b*c - 4*a*d)*\text{Log}[a^{(1/3)} + b^{(1/3)}*x])}{(9*a^{(5/3)}*(b*c - a*d)^3)} + \frac{(2*d^{(5/3)}*(4*b*c - a*d)*\text{Log}[c^{(1/3)} + d^{(1/3)}*x])}{(9*c^{(5/3)}*(b*c - a*d)^3)} - \frac{(b^{(5/3)}*(b*c - 4*a*d)*\text{Log}[a^{(2/3)} - a^{(1/3)}*b^{(1/3)}*x + b^{(2/3)}*x^2])}{(9*a^{(5/3)}*(b*c - a*d)^3)} - \frac{(d^{(5/3)}*(4*b*c - a*d)*\text{Log}[c^{(2/3)} - c^{(1/3)}*d^{(1/3)}*x + d^{(2/3)}*x^2])}{(9*c^{(5/3)}*(b*c - a*d)^3)}$

### Rule 31

Int[((a\_) + (b\_)\*(x\_))<sup>(-1)</sup>, x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_)\*(x\_)^3)<sup>(-1)</sup>, x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])



Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^3)^2(c+dx^3)^2} dx &= \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{\int \frac{-2bc+3ad-5bdx^3}{(a+bx^3)(c+dx^3)^2} dx}{3a(bc-ad)} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{\int \frac{-6(b^2c^2-3abcd+a^2d^2)-6b}{(a+bx^3)(c+dx^3)} dx}{9ac(bc-ad)} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{(2b^2(bc-4ad)) \int \frac{1}{a+bx^3} dx}{3a(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{(2b^2(bc-4ad)) \int \frac{1}{\sqrt[3]{a}} dx}{9a^{5/3}(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{2b^{5/3}(bc-4ad) \log(\sqrt[3]{a+bx^3})}{9a^{5/3}(bc-ad)} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} + \frac{2b^{5/3}(bc-4ad) \log(\sqrt[3]{a+bx^3})}{9a^{5/3}(bc-ad)} \\
&= \frac{d(bc+ad)x}{3ac(bc-ad)^2(c+dx^3)} + \frac{bx}{3a(bc-ad)(a+bx^3)(c+dx^3)} - \frac{2b^{5/3}(bc-4ad) \tan^{-1}\left(\frac{\sqrt[3]{a+bx^3}-\sqrt[3]{a}}{\sqrt[3]{a+bx^3}+\sqrt[3]{a}}\right)}{3\sqrt{3}a^{5/3}(bc-ad)}
\end{aligned}$$

**Mathematica [A]** time = 0.63, size = 381, normalized size = 0.91

$$\frac{1}{9} \left( \frac{b^{5/3}(bc-4ad) \log(a^{2/3} - \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2)}{a^{5/3}(ad-bc)^3} + \frac{2b^{5/3}(4ad-bc) \log(\sqrt[3]{a} + \sqrt[3]{b} x)}{a^{5/3}(ad-bc)^3} + \frac{2\sqrt{3} b^{5/3}(bc-4ad) \tan^{-1}\left(\frac{\sqrt[3]{a+bx^3}-\sqrt[3]{a}}{\sqrt[3]{a+bx^3}+\sqrt[3]{a}}\right)}{a^{5/3}(ad-bc)^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^2\*(c + d\*x^3)^2), x]

[Out] ((3\*b^2\*x)/(a\*(b\*c - a\*d)^2\*(a + b\*x^3)) + (3\*d^2\*x)/(c\*(b\*c - a\*d)^2\*(c + d\*x^3)) + (2\*Sqrt[3]\*b^(5/3)\*(b\*c - 4\*a\*d)\*ArcTan[(1 - (2\*b^(1/3)\*x)/a^(1/3))/Sqrt[3]])/(a^(5/3)\*(-b\*c) + a\*d)^3 + (2\*Sqrt[3]\*d^(5/3)\*(-4\*b\*c + a\*d)\*ArcTan[(1 - (2\*d^(1/3)\*x)/c^(1/3))/Sqrt[3]])/(c^(5/3)\*(b\*c - a\*d)^3 + (2\*b^(5/3)\*(-b\*c) + 4\*a\*d)\*Log[a^(1/3) + b^(1/3)\*x])/(a^(5/3)\*(-b\*c) + a\*d)^3 + (2\*d^(5/3)\*(4\*b\*c - a\*d)\*Log[c^(1/3) + d^(1/3)\*x])/(c^(5/3)\*(b\*c - a\*d)^3 + (b^(5/3)\*(b\*c - 4\*a\*d)\*Log[a^(2/3) - a^(1/3)\*b^(1/3)\*x + b^(2/3)\*x^2])/(a^(5/3)\*(-b\*c) + a\*d)^3 + (d^(5/3)\*(-4\*b\*c + a\*d)\*Log[c^(2/3) - c^(1/3)\*d^(1/3)\*x + d^(2/3)\*x^2])/(c^(5/3)\*(b\*c - a\*d)^3))/9

**fricas [B]** time = 85.30, size = 897, normalized size = 2.14

$$3(b^3c^2d - a^2bd^3)x^4 + 2\sqrt{3}((b^3c^2d - 4ab^2cd^2)x^6 + ab^2c^3 - 4a^2bc^2d + (b^3c^3 - 3ab^2c^2d - 4a^2bcd^2)x^3) \left(\frac{b^2}{a^2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt[3]{a+bx^3}-\sqrt[3]{a}}{\sqrt[3]{a+bx^3}+\sqrt[3]{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^2/(d\*x^3+c)^2,x, algorithm="fricas")

[Out]  $\frac{1}{9} \cdot (3 \cdot (b^3 \cdot c^2 \cdot d - a^2 \cdot b \cdot d^3) \cdot x^4 + 2 \cdot \sqrt{3} \cdot ((b^3 \cdot c^2 \cdot d - 4 \cdot a \cdot b^2 \cdot c \cdot d^2) \cdot x^6 + a \cdot b^2 \cdot c^3 - 4 \cdot a^2 \cdot b \cdot c^2 \cdot d + (b^3 \cdot c^3 - 3 \cdot a \cdot b^2 \cdot c^2 \cdot d - 4 \cdot a^2 \cdot b \cdot c \cdot d^2) \cdot x^3) \cdot (b^2/a^2)^{1/3} \cdot \arctan(1/3 \cdot (2 \cdot \sqrt{3} \cdot a \cdot x \cdot (b^2/a^2)^{2/3} - \sqrt{3} \cdot b)/b) + 2 \cdot \sqrt{3} \cdot ((4 \cdot a \cdot b^2 \cdot c \cdot d^2 - a^2 \cdot b \cdot d^3) \cdot x^6 + 4 \cdot a^2 \cdot b \cdot c^2 \cdot d - a^3 \cdot c \cdot d^2 + (4 \cdot a \cdot b^2 \cdot c^2 \cdot d + 3 \cdot a^2 \cdot b \cdot c \cdot d^2 - a^3 \cdot d^3) \cdot x^3) \cdot (d^2/c^2)^{1/3} \cdot \arctan(1/3 \cdot (2 \cdot \sqrt{3} \cdot c \cdot x \cdot (d^2/c^2)^{2/3} - \sqrt{3} \cdot d)/d) - ((b^3 \cdot c^2 \cdot d - 4 \cdot a \cdot b^2 \cdot c \cdot d^2) \cdot x^6 + a \cdot b^2 \cdot c^3 - 4 \cdot a^2 \cdot b \cdot c^2 \cdot d + (b^3 \cdot c^3 - 3 \cdot a \cdot b^2 \cdot c^2 \cdot d - 4 \cdot a^2 \cdot b \cdot c \cdot d^2) \cdot x^3) \cdot (b^2/a^2)^{1/3} \cdot \log(b^2 \cdot x^2 - a \cdot b \cdot x \cdot (b^2/a^2)^{1/3} + a^2 \cdot (b^2/a^2)^{2/3}) - ((4 \cdot a \cdot b^2 \cdot c \cdot d^2 - a^2 \cdot b \cdot d^3) \cdot x^6 + 4 \cdot a^2 \cdot b \cdot c^2 \cdot d - a^3 \cdot c \cdot d^2 + (4 \cdot a \cdot b^2 \cdot c^2 \cdot d + 3 \cdot a^2 \cdot b \cdot c \cdot d^2 - a^3 \cdot d^3) \cdot x^3) \cdot (d^2/c^2)^{1/3} \cdot \log(d^2 \cdot x^2 - c \cdot d \cdot x \cdot (d^2/c^2)^{1/3} + c^2 \cdot (d^2/c^2)^{2/3}) + 2 \cdot ((b^3 \cdot c^2 \cdot d - 4 \cdot a \cdot b^2 \cdot c \cdot d^2) \cdot x^6 + a \cdot b^2 \cdot c^3 - 4 \cdot a^2 \cdot b \cdot c^2 \cdot d + (b^3 \cdot c^3 - 3 \cdot a \cdot b^2 \cdot c^2 \cdot d - 4 \cdot a^2 \cdot b \cdot c \cdot d^2) \cdot x^3) \cdot (b^2/a^2)^{1/3} \cdot \log(b \cdot x + a \cdot (b^2/a^2)^{1/3}) + 2 \cdot ((4 \cdot a \cdot b^2 \cdot c \cdot d^2 - a^2 \cdot b \cdot d^3) \cdot x^6 + 4 \cdot a^2 \cdot b \cdot c^2 \cdot d - a^3 \cdot c \cdot d^2 + (4 \cdot a \cdot b^2 \cdot c^2 \cdot d + 3 \cdot a^2 \cdot b \cdot c \cdot d^2 - a^3 \cdot d^3) \cdot x^3) \cdot (d^2/c^2)^{1/3} \cdot \log(d \cdot x + c \cdot (d^2/c^2)^{1/3}) + 3 \cdot (b^3 \cdot c^3 - a \cdot b^2 \cdot c^2 \cdot d + a^2 \cdot b \cdot c \cdot d^2 - a^3 \cdot d^3) \cdot x) / (a^2 \cdot b^3 \cdot c^5 - 3 \cdot a^3 \cdot b^2 \cdot c^4 \cdot d + 3 \cdot a^4 \cdot b \cdot c^3 \cdot d^2 - a^5 \cdot c^2 \cdot d^3 + (a \cdot b^4 \cdot c^4 \cdot d - 3 \cdot a^2 \cdot b^3 \cdot c^3 \cdot d^2 + 3 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d^3 - a^4 \cdot b \cdot c \cdot d^4) \cdot x^6 + (a \cdot b^4 \cdot c^5 - 2 \cdot a^2 \cdot b^3 \cdot c^4 \cdot d + 2 \cdot a^4 \cdot b \cdot c^2 \cdot d^3 - a^5 \cdot c \cdot d^4) \cdot x^3)$

**giac** [A] time = 0.22, size = 664, normalized size = 1.58

$$\frac{2(b^3c - 4ab^2d)\left(-\frac{a}{b}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{a}{b}\right)^{\frac{1}{3}}\right|\right)}{9(a^2b^3c^3 - 3a^3b^2c^2d + 3a^4bcd^2 - a^5d^3)} + \frac{2(4bcd^2 - ad^3)\left(-\frac{c}{d}\right)^{\frac{1}{3}} \log\left(\left|x - \left(-\frac{c}{d}\right)^{\frac{1}{3}}\right|\right)}{9(b^3c^5 - 3ab^2c^4d + 3a^2bc^3d^2 - a^3c^2d^3)} + \frac{2\left(\left(-ab^2\right)^{\frac{1}{3}}b^2c - 4(-a\right)}{3(\sqrt{3}a^2b^3c^3 - 3\sqrt{3}a^3b^2c^2d + 3a^4b^2c^2d^3 - a^5c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^2/(d\*x^3+c)^2,x, algorithm="giac")

[Out]  $-\frac{2}{9} \cdot (b^3 \cdot c - 4 \cdot a \cdot b^2 \cdot d) \cdot (-a/b)^{1/3} \cdot \log(\text{abs}(x - (-a/b)^{1/3})) / (a^2 \cdot b^3 \cdot c^3 - 3 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \cdot a^4 \cdot b \cdot c \cdot d^2 - a^5 \cdot d^3) - \frac{2}{9} \cdot (4 \cdot b \cdot c \cdot d^2 - a \cdot d^3) \cdot (-c/d)^{1/3} \cdot \log(\text{abs}(x - (-c/d)^{1/3})) / (b^3 \cdot c^5 - 3 \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - a^3 \cdot c^2 \cdot d^3) + \frac{2}{3} \cdot ((-a \cdot b^2)^{1/3} \cdot b^2 \cdot c - 4 \cdot (-a \cdot b^2)^{1/3} \cdot a \cdot b \cdot d) \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot x + (-a/b)^{1/3}) / (-a/b)^{1/3}) / (\sqrt{3} \cdot a^2 \cdot b^3 \cdot c^3 - 3 \cdot \sqrt{3} \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \cdot \sqrt{3} \cdot a^4 \cdot b \cdot c \cdot d^2 - \sqrt{3} \cdot a^5 \cdot d^3) + \frac{2}{3} \cdot (4 \cdot (-c \cdot d^2)^{1/3} \cdot b \cdot c \cdot d - (-c \cdot d^2)^{1/3} \cdot a \cdot d^2) \cdot \arctan(1/3 \cdot \sqrt{3} \cdot (2 \cdot x + (-c/d)^{1/3}) / (-c/d)^{1/3}) / (\sqrt{3} \cdot b^3 \cdot c^5 - 3 \cdot \sqrt{3} \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \cdot \sqrt{3} \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - \sqrt{3} \cdot a^3 \cdot c^2 \cdot d^3) + \frac{1}{9} \cdot ((-a \cdot b^2)^{1/3} \cdot b^2 \cdot c - 4 \cdot (-a \cdot b^2)^{1/3} \cdot a \cdot b \cdot d) \cdot \log(x^2 + x \cdot (-a/b)^{1/3} + (-a/b)^{2/3}) / (a^2 \cdot b^3 \cdot c^3 - 3 \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \cdot a^4 \cdot b \cdot c \cdot d^2 - a^5 \cdot d^3) + \frac{1}{9} \cdot (4 \cdot (-c \cdot d^2)^{1/3} \cdot b \cdot c \cdot d - (-c \cdot d^2)^{1/3} \cdot a \cdot d^2) \cdot \log(x^2 + x \cdot (-c/d)^{1/3} + (-c/d)^{2/3}) / (b^3 \cdot c^5 - 3 \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - a^3 \cdot c^2 \cdot d^3) + \frac{1}{3} \cdot (b^2 \cdot c \cdot d \cdot x^4 + a \cdot b \cdot d^2 \cdot x^4 + b^2 \cdot c^2 \cdot x + a^2 \cdot d^2 \cdot x) / ((b \cdot d \cdot x^6 + b \cdot c \cdot x^3 + a \cdot d \cdot x^3 + a \cdot c) \cdot (a \cdot b^2 \cdot c^3 - 2 \cdot a^2 \cdot b \cdot c^2 \cdot d + a^3 \cdot c \cdot d^2))$

**maple** [A] time = 0.06, size = 606, normalized size = 1.45

$$\frac{2\sqrt{3} a d^2 a x}{9(ad - bc)^3 (dx^3 + c)c} - \frac{a d^3 x}{3(ad - bc)^3 (bx^3 + a)a} + \frac{b^2 dx}{3(ad - bc)^3 (bx^3 + a)} - \frac{b d^2 x}{3(ad - bc)^3 (dx^3 + c)} + \frac{b^2 dx}{9(ad - bc)^3 (dx^3 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^3+a)^2/(d*x^3+c)^2,x)`

[Out]  $\frac{1}{3}b^2/(a*d-b*c)^3*x/(b*x^3+a)*d-1/3*b^3/(a*d-b*c)^3/a*x/(b*x^3+a)*c+8/9*b/(a*d-b*c)^3/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^{(2/3)}*\ln(x+(a/b)^{(1/3)})*c-4/9*b/(a*d-b*c)^3/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*d+1/9*b^2/(a*d-b*c)^3/a/(a/b)^{(2/3)}*\ln(x^2-(a/b)^{(1/3)}*x+(a/b)^{(2/3)})*c+8/9*b/(a*d-b*c)^3/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*d-2/9*b^2/(a*d-b*c)^3/a/(a/b)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(a/b)^{(1/3)}*x-1))*c+1/3*d^3/(a*d-b*c)^3/c*x/(d*x^3+c)*a-1/3*d^2/(a*d-b*c)^3*x/(d*x^3+c)*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*a-8/9*d/(a*d-b*c)^3/(c/d)^{(2/3)}*\ln(x+(c/d)^{(1/3)})*b-1/9*d^2/(a*d-b*c)^3/c/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*a+4/9*d/(a*d-b*c)^3/(c/d)^{(2/3)}*\ln(x^2-(c/d)^{(1/3)}*x+(c/d)^{(2/3)})*b+2/9*d^2/(a*d-b*c)^3/c/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*a-8/9*d/(a*d-b*c)^3/(c/d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(c/d)^{(1/3)}*x-1))*b$

**maxima** [B] time = 1.26, size = 784, normalized size = 1.87

$$\frac{2\sqrt{3}(b^2c - 4abd) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9\left(ab^3c^3\left(\frac{a}{b}\right)^{\frac{1}{3}} - 3a^2b^2c^2d\left(\frac{a}{b}\right)^{\frac{1}{3}} + 3a^3bcd^2\left(\frac{a}{b}\right)^{\frac{1}{3}} - a^4d^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}} + \frac{2\sqrt{3}(4bcd - ad^2) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{c}{d}\right)^{\frac{1}{3}}\right)}{3\left(\frac{c}{d}\right)^{\frac{1}{3}}}\right)}{9\left(b^3c^4\left(\frac{c}{d}\right)^{\frac{1}{3}} - 3ab^2c^3d\left(\frac{c}{d}\right)^{\frac{1}{3}} + 3a^2bc^2d^2\left(\frac{c}{d}\right)^{\frac{1}{3}} - a^3c^2d^3\left(\frac{c}{d}\right)^{\frac{1}{3}}\right)\left(\frac{c}{d}\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x^3+a)^2/(d*x^3+c)^2,x, algorithm="maxima")`

[Out]  $\frac{2/9*\sqrt{3}*(b^2*c - 4*a*b*d)*\arctan(1/3*\sqrt{3}*(2*x - (a/b)^{(1/3)})/(a/b)^{(1/3)})/((a*b^3*c^3*(a/b)^{(1/3)} - 3*a^2*b^2*c^2*d*(a/b)^{(1/3)} + 3*a^3*b*c*d^2*(a/b)^{(1/3)} - a^4*d^3*(a/b)^{(1/3)})*(a/b)^{(1/3)}) + 2/9*\sqrt{3}*(4*b*c*d - a*d^2)*\arctan(1/3*\sqrt{3}*(2*x - (c/d)^{(1/3)})/(c/d)^{(1/3)})/((b^3*c^4*(c/d)^{(1/3)} - 3*a*b^2*c^3*d*(c/d)^{(1/3)} + 3*a^2*b*c^2*d^2*(c/d)^{(1/3)} - a^3*c*d^3*(c/d)^{(1/3)})*(c/d)^{(1/3)}) - 1/9*(b^2*c - 4*a*b*d)*\log(x^2 - x*(a/b)^{(1/3)} + (a/b)^{(2/3)})/(a*b^3*c^3*(a/b)^{(2/3)} - 3*a^2*b^2*c^2*d*(a/b)^{(2/3)} + 3*a^3*b*c*d^2*(a/b)^{(2/3)} - a^4*d^3*(a/b)^{(2/3)}) - 1/9*(4*b*c*d - a*d^2)*\log(x^2 - x*(c/d)^{(1/3)} + (c/d)^{(2/3)})/(b^3*c^4*(c/d)^{(2/3)} - 3*a*b^2*c^3*d*(c/d)^{(2/3)} + 3*a^2*b*c^2*d^2*(c/d)^{(2/3)} - a^3*c*d^3*(c/d)^{(2/3)}) + 2/9*(b^2*c - 4*a*b*d)*\log(x + (a/b)^{(1/3)})/(a*b^3*c^3*(a/b)^{(2/3)} - 3*a^2*b^2*c^2*d*(a/b)^{(2/3)} + 3*a^3*b*c*d^2*(a/b)^{(2/3)} - a^4*d^3*(a/b)^{(2/3)}) + 2/9*(4*b*c*d - a*d^2)*\log(x + (c/d)^{(1/3)})/(b^3*c^4*(c/d)^{(2/3)} - 3*a*b^2*c^3*d*(c/d)^{(2/3)} + 3*a^2*b*c^2*d^2*(c/d)^{(2/3)} - a^3*c*d^3*(c/d)^{(2/3)}) + 1/3*((b^2*c*d + a*b*d^2)*x^4 + (b^2*c^2 + a^2*d^2)*x)/(a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2 + (a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + a^3*b*c*d^3)*x^6 + (a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + a^4*c*d^3)*x^3)$

**mupad** [B] time = 24.31, size = 3637, normalized size = 8.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^2*(c + d*x^3)^2),x)`

[Out]  $\frac{(x*(a^2*d^2 + b^2*c^2))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (b*d*x^4*(a*d + b*c))/(3*a*c*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(a*c + x^3*(a*d + b*c) + b*d*x^6) + \log((2*((4*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(a*c) + 54*a*b^3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9)))^{(1/3)}*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9)))^{(1/3)}$

$$\begin{aligned}
& *c)^3)/(a^5*(a*d - b*c)^9))^{(2/3)}/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37* \\
& a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d \\
& - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*( \\
& a*d - b*c)^9))^{(1/3)}/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^ \\
& 4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49 \\
& *a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + \\
& 384*a^2*b^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561 \\
& *a^6*b^8*c^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9* \\
& b^5*c^5*d^4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12* \\
& b^2*c^2*d^7 - 6561*a^13*b*c*d^8))^{(1/3)} + \log((2*((4*((54*b^3*d^3*x*(a*d - \\
& b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 54*a*b^ \\
& 3*c*d^3*(a*d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9 \\
& ))^{(1/3)}*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^{(2/3)}/81 - (8*b^4*d^ \\
& 4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2 \\
& *c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d \\
& ^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^{(1/3)}/9 - (16*b^6*d^6*x*(4*a^6*d^ \\
& 6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2 \\
& *d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(-(8*a \\
& ^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^1 \\
& 4 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a \\
& ^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^ \\
& 6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^{(1/3)} + (\log((( \\
& 3^{(1/2)}*1i - 1)*((3^{(1/2)}*1i - 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 \\
& + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^{(1/2)} \\
& *1i - 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^ \\
& 9))^{(1/3)}*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^{(2/3)}/81 - (8*b^4*d \\
& ^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^ \\
& 2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*(( \\
& b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^{(1/3)}/9 - (16*b^6*d^6*x*(4*a^6*d \\
& ^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^ \\
& 2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*((3^{(1 \\
& /2)}*1i - 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b^6*c*d^2 - 96*a*b^7*c \\
& ^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c^8*d - 26244*a^7*b^7* \\
& c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^4 + 91854*a^10*b^4*c^ \\
& 4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^7 - 6561*a^13*b*c*d^8 \\
& ))^{(1/3)})/2 - (\log(((3^{(1/2)}*1i + 1)*((3^{(1/2)}*1i + 1)^2*((54*b^3*d^3*x*(a \\
& *d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(a*c) - 27 \\
& *a*b^3*c*d^3*(3^{(1/2)}*1i + 1)*(a*d + b*c)*(a*d - b*c)^4*((b^5*(4*a*d - b*c) \\
& ^3)/(a^5*(a*d - b*c)^9))^{(1/3)}*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9)) \\
& ^{(2/3)}/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^ \\
& 3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c \\
& ^3*(a*d - b*c)^4))*((b^5*(4*a*d - b*c)^3)/(a^5*(a*d - b*c)^9))^{(1/3)}/9 + ( \\
& 16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3 \\
& *d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3* \\
& (a*d - b*c)^8))*((3^{(1/2)}*1i + 1)*(-(8*b^8*c^3 - 512*a^3*b^5*d^3 + 384*a^2*b \\
& ^6*c*d^2 - 96*a*b^7*c^2*d)/(729*a^14*d^9 - 729*a^5*b^9*c^9 + 6561*a^6*b^8*c \\
& ^8*d - 26244*a^7*b^7*c^7*d^2 + 61236*a^8*b^6*c^6*d^3 - 91854*a^9*b^5*c^5*d^ \\
& 4 + 91854*a^10*b^4*c^4*d^5 - 61236*a^11*b^3*c^3*d^6 + 26244*a^12*b^2*c^2*d^ \\
& 7 - 6561*a^13*b*c*d^8))^{(1/3)})/2 + (\log(((3^{(1/2)}*1i - 1)*((3^{(1/2)}*1i - 1 \\
& )^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 - 3*a*b^2*c^2*d - 3*a^2 \\
& *b*c*d^2))/(a*c) + 27*a*b^3*c*d^3*(3^{(1/2)}*1i - 1)*(a*d + b*c)*(a*d - b*c)^ \\
& 4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^{(1/3)}*((d^5*(a*d - 4*b*c)^3) \\
& /c^5*(a*d - b*c)^9))^{(2/3)}/81 - (8*b^4*d^4*(a^6*d^6 + b^6*c^6 + 37*a^2*b^ \\
& 4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 - 11*a*b^5*c^5*d - 11*a \\
& ^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - \\
& b*c)^9))^{(1/3)}/9 - (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c^6 + 268*a^2*b^4*c^4* \\
& d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a*b^5*c^5*d - 49*a^5*b \\
& *c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*((3^{(1/2)}*1i - 1)*(-(8*a^3*d^8 - 512*b^ \\
& 3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b^9*c^14 - 729*a^9*c^5
\end{aligned}$$

```

*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 61236*a^3*b^6*c^11*d^3
+ 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61236*a^6*b^3*c^8*d^6 -
26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2 - (log(((3^(1/2)*1i +
1)*((3^(1/2)*1i + 1)^2*((54*b^3*d^3*x*(a*d - b*c)^2*(a^3*d^3 + b^3*c^3 -
3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/(a*c) - 27*a*b^3*c*d^3*(3^(1/2)*1i + 1)*(a*
d + b*c)*(a*d - b*c)^4*((d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))*
(d^5*(a*d - 4*b*c)^3)/(c^5*(a*d - b*c)^9))^(2/3))/81 - (8*b^4*d^4*(a^6*d^6
+ b^6*c^6 + 37*a^2*b^4*c^4*d^2 - 27*a^3*b^3*c^3*d^3 + 37*a^4*b^2*c^2*d^4 -
11*a*b^5*c^5*d - 11*a^5*b*c*d^5))/(3*a^3*c^3*(a*d - b*c)^4))*((d^5*(a*d - 4
*b*c)^3)/(c^5*(a*d - b*c)^9))^(1/3))/9 + (16*b^6*d^6*x*(4*a^6*d^6 + 4*b^6*c
^6 + 268*a^2*b^4*c^4*d^2 - 608*a^3*b^3*c^3*d^3 + 268*a^4*b^2*c^2*d^4 - 49*a
*b^5*c^5*d - 49*a^5*b*c*d^5))/(27*a^3*c^3*(a*d - b*c)^8))*(3^(1/2)*1i + 1)*
(-(8*a^3*d^8 - 512*b^3*c^3*d^5 + 384*a*b^2*c^2*d^6 - 96*a^2*b*c*d^7)/(729*b
^9*c^14 - 729*a^9*c^5*d^9 + 6561*a^8*b*c^6*d^8 + 26244*a^2*b^7*c^12*d^2 - 6
1236*a^3*b^6*c^11*d^3 + 91854*a^4*b^5*c^10*d^4 - 91854*a^5*b^4*c^9*d^5 + 61
236*a^6*b^3*c^8*d^6 - 26244*a^7*b^2*c^7*d^7 - 6561*a*b^8*c^13*d))^(1/3))/2
sympy [F(-1)]    time = 0.00, size = 0, normalized size = 0.00

```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**2/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

### 3.27 $\int (a - bx^3)(a + bx^3)^{2/3} dx$

**Optimal.** Leaf size=112

$$-\frac{7a^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18\sqrt[3]{b}} + \frac{7a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} + \frac{7}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a+bx^3)^{5/3}$$

[Out]  $7/18*a*x*(b*x^3+a)^{(2/3)}-1/6*x*(b*x^3+a)^{(5/3)}-7/18*a^2*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}+7/27*a^2*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {388, 195, 239}

$$-\frac{7a^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18\sqrt[3]{b}} + \frac{7a^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} + \frac{7}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a+bx^3)^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)\*(a + b\*x^3)^(2/3), x]

[Out]  $(7*a*x*(a + b*x^3)^{(2/3)})/18 - (x*(a + b*x^3)^{(5/3)})/6 + (7*a^2*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(9*\text{Sqrt}[3]*b^{(1/3)}) - (7*a^2*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(18*b^{(1/3)})$

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int (a - bx^3)(a + bx^3)^{2/3} dx &= -\frac{1}{6}x(a + bx^3)^{5/3} + \frac{1}{6}(7a) \int (a + bx^3)^{2/3} dx \\
&= \frac{7}{18}ax(a + bx^3)^{2/3} - \frac{1}{6}x(a + bx^3)^{5/3} + \frac{1}{9}(7a^2) \int \frac{1}{\sqrt[3]{a + bx^3}} dx \\
&= \frac{7}{18}ax(a + bx^3)^{2/3} - \frac{1}{6}x(a + bx^3)^{5/3} + \frac{7a^2 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{7a^2 \log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{18\sqrt[3]{b}}
\end{aligned}$$

**Mathematica [C]** time = 0.11, size = 62, normalized size = 0.55

$$\frac{1}{6}x(a + bx^3)^{2/3} \left( \frac{7a {}_2F_1\left(-\frac{2}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a} + 1\right)^{2/3}} - a - bx^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)\*(a + b\*x^3)^(2/3), x]

[Out] (x\*(a + b\*x^3)^(2/3)\*(-a - b\*x^3 + (7\*a\*Hypergeometric2F1[-2/3, 1/3, 4/3, -(b\*x^3)/a]))/(1 + (b\*x^3)/a)^(2/3))/6

**fricas [B]** time = 0.44, size = 399, normalized size = 3.56

$$\frac{21 \sqrt{\frac{1}{3}} a^2 b \sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 + 2(bx^3 + a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)\*(b\*x^3+a)^(2/3), x, algorithm="fricas")

[Out] [1/54\*(21\*sqrt(1/3)\*a^2\*b\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 14\*a^2\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 7\*a^2\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(3\*b^2\*x^4 - 4\*a\*b\*x)\*(b\*x^3 + a)^(2/3))/b, -1/54\*(42\*sqrt(1/3)\*a^2\*b\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt((-b)^(1/3)/b)/x) + 14\*a^2\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - 7\*a^2\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*(3\*b^2\*x^4 - 4\*a\*b\*x)\*(b\*x^3 + a)^(2/3))/b]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -(bx^3 + a)^{\frac{2}{3}}(bx^3 - a) dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-b\*x^3+a)\*(b\*x^3+a)^(2/3),x, algorithm="giac")

[Out] integrate(-(b\*x^3 + a)^(2/3)\*(b\*x^3 - a), x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int (-bx^3 + a)(bx^3 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)\*(b\*x^3+a)^(2/3),x)

[Out] int((-b\*x^3+a)\*(b\*x^3+a)^(2/3),x)

**maxima** [B] time = 1.22, size = 322, normalized size = 2.88

$$\frac{1}{9} \left( \frac{2\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} + \frac{3(bx^3+a)^{\frac{2}{3}}}{b-x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)\*(b\*x^3+a)^(2/3),x, algorithm="maxima")

[Out] -1/9\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3) + 3\*(b\*x^3 + a)^(2/3)\*a/((b - (b\*x^3 + a)/x^3)\*x^2)\*a - 1/54\*(2\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + 3\*((b\*x^3 + a)^(2/3)\*a^2\*b/x^2 + 2\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^3 - 2\*(b\*x^3 + a)\*b^2/x^3 + (b\*x^3 + a)^2\*b/x^6))\*b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{2/3} (a - bx^3) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)\*(a - b\*x^3),x)

[Out] int((a + b\*x^3)^(2/3)\*(a - b\*x^3), x)

**sympy** [C] time = 4.98, size = 80, normalized size = 0.71

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)*(b*x**3+a)**(2/3),x)
```

```
[Out] a**(5/3)*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/  
(3*gamma(4/3)) - a**(2/3)*b*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*x*  
*3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

$$3.28 \quad \int \frac{a-bx^3}{\sqrt[3]{a+bx^3}} dx$$

**Optimal.** Leaf size=91

$$-\frac{1}{3}x(a+bx^3)^{2/3} - \frac{2a \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{3\sqrt[3]{b}} + \frac{4a \tan^{-1}\left(\frac{2\sqrt[3]{b}x + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

[Out]  $-1/3*x*(b*x^3+a)^{(2/3)}-2/3*a*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}+4/9*a*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.02, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {388, 239}

$$-\frac{1}{3}x(a+bx^3)^{2/3} - \frac{2a \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{3\sqrt[3]{b}} + \frac{4a \tan^{-1}\left(\frac{2\sqrt[3]{b}x + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(1/3), x]

[Out]  $-(x*(a + b*x^3)^{(2/3)})/3 + (4*a*ArcTan[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/Sqrt[3])/(3*Sqrt[3]*b^{(1/3)}) - (2*a*Log[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(3*b^{(1/3)})$

**Rule 239**

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

**Rule 388**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{a-bx^3}{\sqrt[3]{a+bx^3}} dx &= -\frac{1}{3}x(a+bx^3)^{2/3} + \frac{1}{3}(4a) \int \frac{1}{\sqrt[3]{a+bx^3}} dx \\ &= -\frac{1}{3}x(a+bx^3)^{2/3} + \frac{4a \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} - \frac{2a \log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{3\sqrt[3]{b}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 134, normalized size = 1.47

$$\frac{2a \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right) - 3\sqrt[3]{b}x(a+bx^3)^{2/3} - 4a \log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right) + 4\sqrt{3}a \tan^{-1}\left(\frac{2\sqrt[3]{b}x + 1}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(1/3), x]

[Out] (-3\*b^(1/3)\*x\*(a + b\*x^3)^(2/3) + 4\*Sqrt[3]\*a\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]] - 4\*a\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + 2\*a\*Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(9\*b^(1/3))

**fricas** [B] time = 0.44, size = 363, normalized size = 3.99

$$\frac{6\sqrt{\frac{1}{3}}ab\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 + 2(bx^3 + a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\right)}{9b^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(1/3), x, algorithm="fricas")

[Out] [1/9\*(6\*sqrt(1/3)\*a\*b\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a - 3\*(b\*x^3 + a)^(2/3)\*b\*x - 4\*a\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 2\*a\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)/b, -1/9\*(12\*sqrt(1/3)\*a\*b\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt((-b)^(1/3)/b)/x) + 3\*(b\*x^3 + a)^(2/3)\*b\*x + 4\*a\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - 2\*a\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)/b]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(1/3), x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(1/3), x)

**maple** [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-bx^3 + a}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)/(b\*x^3+a)^(1/3), x)

[Out] int((-b\*x^3+a)/(b\*x^3+a)^(1/3), x)

**maxima [B]** time = 1.22, size = 244, normalized size = 2.68

$$\frac{1}{6} \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} \right) a - \frac{1}{18} \left( \frac{2\sqrt{3}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] -1/6\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3)))/b^(1/3) - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3)\*a - 1/18\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) - 6\*(b\*x^3 + a)^(2/3)\*a/((b^2 - (b\*x^3 + a)\*b/x^3)\*x^2))\*b

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a - bx^3}{(bx^3 + a)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)/(a + b\*x^3)^(1/3), x)

[Out] int((a - b\*x^3)/(a + b\*x^3)^(1/3), x)

**sympy [C]** time = 5.53, size = 76, normalized size = 0.84

$$\frac{a^{\frac{2}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)/(b\*x\*\*3+a)\*\*(1/3),x)

[Out] a\*\*(2/3)\*x\*gamma(1/3)\*hyper((1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) - b\*x\*\*4\*gamma(4/3)\*hyper((1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*a\*\*(1/3)\*gamma(7/3))

$$3.29 \quad \int \frac{a-bx^3}{(a+bx^3)^{4/3}} dx$$

**Optimal.** Leaf size=85

$$\frac{2x}{\sqrt[3]{a+bx^3}} + \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2\sqrt[3]{b}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3} + 1}\right)}{\sqrt{3}\sqrt[3]{b}}$$

[Out]  $2*x/(b*x^3+a)^{(1/3)}+1/2*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}-1/3*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.01, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {385, 239}

$$\frac{2x}{\sqrt[3]{a+bx^3}} + \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2\sqrt[3]{b}} - \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3} + 1}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(4/3), x]

[Out]  $(2*x)/(a + b*x^3)^{(1/3)} - \text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]]/(\text{Sqrt}[3]*b^{(1/3)}) + \text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3})]/(2*b^{(1/3)})$

**Rule 239**

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{a-bx^3}{(a+bx^3)^{4/3}} dx &= \frac{2x}{\sqrt[3]{a+bx^3}} - \int \frac{1}{\sqrt[3]{a+bx^3}} dx \\ &= \frac{2x}{\sqrt[3]{a+bx^3}} - \frac{\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} + \frac{\log\left(-\sqrt[3]{b}x + \sqrt[3]{a+bx^3}\right)}{2\sqrt[3]{b}} \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 62, normalized size = 0.73

$$\frac{4ax - bx^4 \sqrt[3]{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(4/3), x]

[Out] (4\*a\*x - b\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*Hypergeometric2F1[4/3, 4/3, 7/3, -((b\*x^3)/a)])/(4\*a\*(a + b\*x^3)^(1/3))

**fricas** [B] time = 0.44, size = 372, normalized size = 4.38

$$\frac{3 \sqrt{\frac{1}{3}} (b^2 x^3 + ab) \sqrt{-\frac{1}{2}} \log \left( 3 b x^3 - 3 (b x^3 + a)^{\frac{1}{3}} b^{\frac{2}{3}} x^2 - 3 \sqrt{\frac{1}{3}} \left( b^{\frac{4}{3}} x^3 + (b x^3 + a)^{\frac{1}{3}} b x^2 - 2 (b x^3 + a)^{\frac{2}{3}} b^{\frac{2}{3}} x \right) \sqrt{-\frac{1}{2}} \right)}{6 (b^2 x^3 + ab)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(4/3), x, algorithm="fricas")

[Out] [1/6\*(3\*sqrt(1/3)\*(b^2\*x^3 + a\*b)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x)\*sqrt(-1/b^(2/3)) + 2\*a) + 12\*(b\*x^3 + a)^(2/3)\*b\*x + 2\*(b\*x^3 + a)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (b\*x^3 + a)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)/(b^2\*x^3 + a\*b), 1/6\*(12\*(b\*x^3 + a)^(2/3)\*b\*x + 2\*(b\*x^3 + a)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (b\*x^3 + a)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 6\*sqrt(1/3)\*(b^2\*x^3 + a\*b)\*arctan(sqrt(1/3)\*(b^(1/3)\*x + 2\*(b\*x^3 + a)^(1/3))/(b^(1/3)\*x))/b^(1/3))/(b^2\*x^3 + a\*b)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(4/3), x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(4/3), x)

**maple** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{-bx^3 + a}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)/(b\*x^3+a)^(4/3), x)

[Out] int((-b\*x^3+a)/(b\*x^3+a)^(4/3), x)

**maxima** [A] time = 1.25, size = 130, normalized size = 1.53

$$\frac{1}{6} b \frac{\left( 2 \sqrt{3} \arctan \left( \frac{\sqrt{3} \left( \frac{1}{b^{\frac{1}{3}}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x} \right)}{3 b^{\frac{1}{3}}} \right) \right)}{b^{\frac{4}{3}}} + \frac{6x}{(bx^3+a)^{\frac{1}{3}} b} - \frac{\log \left( b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}} b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2} \right)}{b^{\frac{4}{3}}} + \frac{2 \log \left( -b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x} \right)}{b^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(4/3),x, algorithm="maxima")

[Out] 1/6\*b\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6\*x/((b\*x^3 + a)^(1/3)\*b) - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + x/(b\*x^3 + a)^(1/3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a - b x^3}{(b x^3 + a)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)/(a + b\*x^3)^(4/3),x)

[Out] int((a - b\*x^3)/(a + b\*x^3)^(4/3), x)

**sympy** [C] time = 16.14, size = 70, normalized size = 0.82

$$\frac{x \Gamma\left(\frac{1}{3}\right)}{3 \sqrt[3]{a} \sqrt[3]{1 + \frac{bx^3}{a}} \Gamma\left(\frac{4}{3}\right)} - \frac{bx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3 a^{\frac{4}{3}} \Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)/(b\*x\*\*3+a)\*\*(4/3),x)

[Out] x\*gamma(1/3)/(3\*a\*\*(1/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(4/3)) - b\*x\*\*4\*gamma(4/3)\*hyper((4/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*a\*\*(4/3)\*gamma(7/3))



$$3.30 \quad \int \frac{a-bx^3}{(a+bx^3)^{7/3}} dx$$

**Optimal.** Leaf size=47

$$\frac{3x}{4a\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}$$

[Out] 1/4\*x\*(-b\*x^3+a)/a/(b\*x^3+a)^(4/3)+3/4\*x/a/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$ , Rules used = {378, 191}

$$\frac{3x}{4a\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(7/3), x]

[Out] (x\*(a - b\*x^3))/(4\*a\*(a + b\*x^3)^(4/3)) + (3\*x)/(4\*a\*(a + b\*x^3)^(1/3))

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{a-bx^3}{(a+bx^3)^{7/3}} dx &= \frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}} + \frac{3}{4} \int \frac{1}{(a+bx^3)^{4/3}} dx \\ &= \frac{x(a-bx^3)}{4a(a+bx^3)^{4/3}} + \frac{3x}{4a\sqrt[3]{a+bx^3}} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 0.60

$$\frac{x(2a+bx^3)}{2a(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(7/3), x]

[Out] (x\*(2\*a + b\*x^3))/(2\*a\*(a + b\*x^3)^(4/3))

**fricas** [A] time = 0.42, size = 44, normalized size = 0.94

$$\frac{(bx^4 + 2ax)(bx^3 + a)^{\frac{2}{3}}}{2(ab^2x^6 + 2a^2bx^3 + a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(7/3),x, algorithm="fricas")

[Out] 1/2\*(b\*x^4 + 2\*a\*x)\*(b\*x^3 + a)^(2/3)/(a\*b^2\*x^6 + 2\*a^2\*b\*x^3 + a^3)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(7/3),x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(7/3), x)

**maple** [A] time = 0.05, size = 25, normalized size = 0.53

$$\frac{(bx^3 + 2a)x}{2(bx^3 + a)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)/(b\*x^3+a)^(7/3),x)

[Out] 1/2\*x\*(b\*x^3+2\*a)/(b\*x^3+a)^(4/3)/a

**maxima** [A] time = 0.52, size = 50, normalized size = 1.06

$$-\frac{\left(b - \frac{4(bx^3+a)}{x^3}\right)x^4}{4(bx^3 + a)^{\frac{4}{3}}a} - \frac{bx^4}{4(bx^3 + a)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(7/3),x, algorithm="maxima")

[Out] -1/4\*(b - 4\*(b\*x^3 + a)/x^3)\*x^4/((b\*x^3 + a)^(4/3)\*a) - 1/4\*b\*x^4/((b\*x^3 + a)^(4/3)\*a)

**mupad** [B] time = 1.35, size = 27, normalized size = 0.57

$$\frac{x(bx^3 + a) + ax}{2a(bx^3 + a)^{4/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)/(a + b\*x^3)^(7/3),x)

[Out] (x\*(a + b\*x^3) + a\*x)/(2\*a\*(a + b\*x^3)^(4/3))

**sympy [B]** time = 91.68, size = 190, normalized size = 4.04

$$a \left( \frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right) + 9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right) + 9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} \right) - \frac{1}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)/(b\*x\*\*3+a)\*\*(7/3),x)

[Out] a\*(4\*a\*x\*gamma(1/3)/(9\*a\*\*(10/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 9\*a\*\*(7/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3)) + 3\*b\*x\*\*4\*gamma(1/3)/(9\*a\*\*(10/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 9\*a\*\*(7/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3)) - b\*x\*\*4\*gamma(4/3)/(3\*a\*\*(7/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 3\*a\*\*(4/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3))

$$3.31 \quad \int \frac{a-bx^3}{(a+bx^3)^{10/3}} dx$$

**Optimal.** Leaf size=55

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{2x}{7(a+bx^3)^{7/3}}$$

[Out]  $2/7*x/(b*x^3+a)^{(7/3)}+5/28*x/a/(b*x^3+a)^{(4/3)}+15/28*x/a^2/(b*x^3+a)^{(1/3)}$

**Rubi [A]** time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {385, 192, 191}

$$\frac{15x}{28a^2\sqrt[3]{a+bx^3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{2x}{7(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(10/3), x]

[Out]  $(2*x)/(7*(a + b*x^3)^{(7/3)}) + (5*x)/(28*a*(a + b*x^3)^{(4/3)}) + (15*x)/(28*a^2*(a + b*x^3)^{(1/3)})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 192**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{a-bx^3}{(a+bx^3)^{10/3}} dx &= \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5}{7} \int \frac{1}{(a+bx^3)^{7/3}} dx \\ &= \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{15}{28a} \int \frac{1}{(a+bx^3)^{4/3}} dx \\ &= \frac{2x}{7(a+bx^3)^{7/3}} + \frac{5x}{28a(a+bx^3)^{4/3}} + \frac{15x}{28a^2\sqrt[3]{a+bx^3}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 40, normalized size = 0.73

$$\frac{x(28a^2 + 35abx^3 + 15b^2x^6)}{28a^2(a + bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(10/3), x]

[Out] (x\*(28\*a^2 + 35\*a\*b\*x^3 + 15\*b^2\*x^6))/(28\*a^2\*(a + b\*x^3)^(7/3))

**fricas [A]** time = 0.42, size = 69, normalized size = 1.25

$$\frac{(15b^2x^7 + 35abx^4 + 28a^2x)(bx^3 + a)^{\frac{2}{3}}}{28(a^2b^3x^9 + 3a^3b^2x^6 + 3a^4bx^3 + a^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(10/3), x, algorithm="fricas")

[Out] 1/28\*(15\*b^2\*x^7 + 35\*a\*b\*x^4 + 28\*a^2\*x)\*(b\*x^3 + a)^(2/3)/(a^2\*b^3\*x^9 + 3\*a^3\*b^2\*x^6 + 3\*a^4\*b\*x^3 + a^5)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(10/3), x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(10/3), x)

**maple [A]** time = 0.04, size = 37, normalized size = 0.67

$$\frac{(15b^2x^6 + 35abx^3 + 28a^2)x}{28(bx^3 + a)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)/(b\*x^3+a)^(10/3), x)

[Out] 1/28\*x\*(15\*b^2\*x^6+35\*a\*b\*x^3+28\*a^2)/(b\*x^3+a)^(7/3)/a^2

**maxima [A]** time = 0.62, size = 85, normalized size = 1.55

$$\frac{\left(4b - \frac{7(bx^3+a)}{x^3}\right)bx^7}{28(bx^3 + a)^{\frac{7}{3}}a^2} + \frac{\left(2b^2 - \frac{7(bx^3+a)b}{x^3} + \frac{14(bx^3+a)^2}{x^6}\right)x^7}{14(bx^3 + a)^{\frac{7}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(10/3), x, algorithm="maxima")

[Out] 1/28\*(4\*b - 7\*(b\*x^3 + a)/x^3)\*b\*x^7/((b\*x^3 + a)^(7/3)\*a^2) + 1/14\*(2\*b^2 - 7\*(b\*x^3 + a)\*b/x^3 + 14\*(b\*x^3 + a)^2/x^6)\*x^7/((b\*x^3 + a)^(7/3)\*a^2)

**mupad [B]** time = 1.42, size = 44, normalized size = 0.80

$$\frac{15x(bx^3 + a)^2 + 8a^2x + 5ax(bx^3 + a)}{28a^2(bx^3 + a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)/(a + b*x^3)^(10/3), x)`

[Out] `(15*x*(a + b*x^3)^2 + 8*a^2*x + 5*a*x*(a + b*x^3))/(28*a^2*(a + b*x^3)^(7/3))`

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)/(b*x**3+a)**(10/3), x)`

[Out] Timed out

$$3.32 \quad \int \frac{a-bx^3}{(a+bx^3)^{13/3}} dx$$

**Optimal.** Leaf size=74

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}} + \frac{6x}{35a^2(a+bx^3)^{4/3}} + \frac{4x}{35a(a+bx^3)^{7/3}} + \frac{x}{5(a+bx^3)^{10/3}}$$

[Out] 1/5\*x/(b\*x^3+a)^(10/3)+4/35\*x/a/(b\*x^3+a)^(7/3)+6/35\*x/a^2/(b\*x^3+a)^(4/3)+18/35\*x/a^3/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.02, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {385, 192, 191}

$$\frac{18x}{35a^3\sqrt[3]{a+bx^3}} + \frac{6x}{35a^2(a+bx^3)^{4/3}} + \frac{4x}{35a(a+bx^3)^{7/3}} + \frac{x}{5(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(13/3), x]

[Out] x/(5\*(a + b\*x^3)^(10/3)) + (4\*x)/(35\*a\*(a + b\*x^3)^(7/3)) + (6\*x)/(35\*a^2\*(a + b\*x^3)^(4/3)) + (18\*x)/(35\*a^3\*(a + b\*x^3)^(1/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{a - bx^3}{(a + bx^3)^{13/3}} dx &= \frac{x}{5(a + bx^3)^{10/3}} + \frac{4}{5} \int \frac{1}{(a + bx^3)^{10/3}} dx \\
&= \frac{x}{5(a + bx^3)^{10/3}} + \frac{4x}{35a(a + bx^3)^{7/3}} + \frac{24}{35a} \int \frac{1}{(a + bx^3)^{7/3}} dx \\
&= \frac{x}{5(a + bx^3)^{10/3}} + \frac{4x}{35a(a + bx^3)^{7/3}} + \frac{6x}{35a^2(a + bx^3)^{4/3}} + \frac{18}{35a^2} \int \frac{1}{(a + bx^3)^{4/3}} dx \\
&= \frac{x}{5(a + bx^3)^{10/3}} + \frac{4x}{35a(a + bx^3)^{7/3}} + \frac{6x}{35a^2(a + bx^3)^{4/3}} + \frac{18x}{35a^3 \sqrt[3]{a + bx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 51, normalized size = 0.69

$$\frac{x(35a^3 + 70a^2bx^3 + 60ab^2x^6 + 18b^3x^9)}{35a^3(a + bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(13/3), x]

[Out] (x\*(35\*a^3 + 70\*a^2\*b\*x^3 + 60\*a\*b^2\*x^6 + 18\*b^3\*x^9))/(35\*a^3\*(a + b\*x^3)^(10/3))

**fricas [A]** time = 0.43, size = 91, normalized size = 1.23

$$\frac{(18b^3x^{10} + 60ab^2x^7 + 70a^2bx^4 + 35a^3x)(bx^3 + a)^{\frac{2}{3}}}{35(a^3b^4x^{12} + 4a^4b^3x^9 + 6a^5b^2x^6 + 4a^6bx^3 + a^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(13/3), x, algorithm="fricas")

[Out] 1/35\*(18\*b^3\*x^10 + 60\*a\*b^2\*x^7 + 70\*a^2\*b\*x^4 + 35\*a^3\*x)\*(b\*x^3 + a)^(2/3)/(a^3\*b^4\*x^12 + 4\*a^4\*b^3\*x^9 + 6\*a^5\*b^2\*x^6 + 4\*a^6\*b\*x^3 + a^7)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{13}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(13/3), x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(13/3), x)

**maple [A]** time = 0.05, size = 48, normalized size = 0.65

$$\frac{(18b^3x^9 + 60ab^2x^6 + 70a^2bx^3 + 35a^3)x}{35(bx^3 + a)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] int((-b\*x^3+a)/(b\*x^3+a)^(13/3),x)

[Out] 1/35\*x\*(18\*b^3\*x^9+60\*a\*b^2\*x^6+70\*a^2\*b\*x^3+35\*a^3)/(b\*x^3+a)^(10/3)/a^3

**maxima [B]** time = 0.58, size = 119, normalized size = 1.61

$$\frac{\left(14b^2 - \frac{40(bx^3+a)b}{x^3} + \frac{35(bx^3+a)^2}{x^6}\right)bx^{10}}{140(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14b^3 - \frac{60(bx^3+a)b^2}{x^3} + \frac{105(bx^3+a)^2b}{x^6} - \frac{140(bx^3+a)^3}{x^9}\right)x^{10}}{140(bx^3+a)^{\frac{10}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(13/3),x, algorithm="maxima")

[Out] -1/140\*(14\*b^2 - 40\*(b\*x^3 + a)\*b/x^3 + 35\*(b\*x^3 + a)^2/x^6)\*b\*x^10/((b\*x^3 + a)^(10/3)\*a^3) - 1/140\*(14\*b^3 - 60\*(b\*x^3 + a)\*b^2/x^3 + 105\*(b\*x^3 + a)^2\*b/x^6 - 140\*(b\*x^3 + a)^3/x^9)\*x^10/((b\*x^3 + a)^(10/3)\*a^3)

**mupad [B]** time = 1.39, size = 58, normalized size = 0.78

$$\frac{x}{5(bx^3+a)^{10/3}} + \frac{18x}{35a^3(bx^3+a)^{1/3}} + \frac{6x}{35a^2(bx^3+a)^{4/3}} + \frac{4x}{35a(bx^3+a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)/(a + b\*x^3)^(13/3),x)

[Out] x/(5\*(a + b\*x^3)^(10/3)) + (18\*x)/(35\*a^3\*(a + b\*x^3)^(1/3)) + (6\*x)/(35\*a^2\*(a + b\*x^3)^(4/3)) + (4\*x)/(35\*a\*(a + b\*x^3)^(7/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)/(b\*x\*\*3+a)\*\*(13/3),x)

[Out] Timed out

$$3.33 \quad \int \frac{a-bx^3}{(a+bx^3)^{16/3}} dx$$

**Optimal.** Leaf size=93

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}} + \frac{297x}{1820a^3(a+bx^3)^{4/3}} + \frac{99x}{910a^2(a+bx^3)^{7/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{2x}{13(a+bx^3)^{13/3}}$$

[Out]  $2/13*x/(b*x^3+a)^{(13/3)}+11/130*x/a/(b*x^3+a)^{(10/3)}+99/910*x/a^2/(b*x^3+a)^{(7/3)}+297/1820*x/a^3/(b*x^3+a)^{(4/3)}+891/1820*x/a^4/(b*x^3+a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {385, 192, 191}

$$\frac{891x}{1820a^4\sqrt[3]{a+bx^3}} + \frac{297x}{1820a^3(a+bx^3)^{4/3}} + \frac{99x}{910a^2(a+bx^3)^{7/3}} + \frac{11x}{130a(a+bx^3)^{10/3}} + \frac{2x}{13(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)/(a + b\*x^3)^(16/3), x]

[Out]  $(2*x)/(13*(a + b*x^3)^{(13/3)}) + (11*x)/(130*a*(a + b*x^3)^{(10/3)}) + (99*x)/(910*a^2*(a + b*x^3)^{(7/3)}) + (297*x)/(1820*a^3*(a + b*x^3)^{(4/3)}) + (891*x)/(1820*a^4*(a + b*x^3)^{(1/3)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{a - bx^3}{(a + bx^3)^{16/3}} dx &= \frac{2x}{13(a + bx^3)^{13/3}} + \frac{11}{13} \int \frac{1}{(a + bx^3)^{13/3}} dx \\
&= \frac{2x}{13(a + bx^3)^{13/3}} + \frac{11x}{130a(a + bx^3)^{10/3}} + \frac{99 \int \frac{1}{(a+bx^3)^{10/3}} dx}{130a} \\
&= \frac{2x}{13(a + bx^3)^{13/3}} + \frac{11x}{130a(a + bx^3)^{10/3}} + \frac{99x}{910a^2(a + bx^3)^{7/3}} + \frac{297 \int \frac{1}{(a+bx^3)^{7/3}} dx}{455a^2} \\
&= \frac{2x}{13(a + bx^3)^{13/3}} + \frac{11x}{130a(a + bx^3)^{10/3}} + \frac{99x}{910a^2(a + bx^3)^{7/3}} + \frac{297x}{1820a^3(a + bx^3)^{4/3}} + \frac{891}{1820a^4} \\
&= \frac{2x}{13(a + bx^3)^{13/3}} + \frac{11x}{130a(a + bx^3)^{10/3}} + \frac{99x}{910a^2(a + bx^3)^{7/3}} + \frac{297x}{1820a^3(a + bx^3)^{4/3}} + \frac{891}{1820a^4}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 62, normalized size = 0.67

$$\frac{x(1820a^4 + 5005a^3bx^3 + 6435a^2b^2x^6 + 3861ab^3x^9 + 891b^4x^{12})}{1820a^4(a + bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)/(a + b\*x^3)^(16/3), x]

[Out] (x\*(1820\*a^4 + 5005\*a^3\*b\*x^3 + 6435\*a^2\*b^2\*x^6 + 3861\*a\*b^3\*x^9 + 891\*b^4\*x^12))/(1820\*a^4\*(a + b\*x^3)^(13/3))

**fricas [A]** time = 0.43, size = 113, normalized size = 1.22

$$\frac{(891b^4x^{13} + 3861ab^3x^{10} + 6435a^2b^2x^7 + 5005a^3bx^4 + 1820a^4x)(bx^3 + a)^{\frac{2}{3}}}{1820(a^4b^5x^{15} + 5a^5b^4x^{12} + 10a^6b^3x^9 + 10a^7b^2x^6 + 5a^8bx^3 + a^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(16/3), x, algorithm="fricas")

[Out] 1/1820\*(891\*b^4\*x^13 + 3861\*a\*b^3\*x^10 + 6435\*a^2\*b^2\*x^7 + 5005\*a^3\*b\*x^4 + 1820\*a^4\*x)\*(b\*x^3 + a)^(2/3)/(a^4\*b^5\*x^15 + 5\*a^5\*b^4\*x^12 + 10\*a^6\*b^3\*x^9 + 10\*a^7\*b^2\*x^6 + 5\*a^8\*b\*x^3 + a^9)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{bx^3 - a}{(bx^3 + a)^{\frac{16}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)/(b\*x^3+a)^(16/3), x, algorithm="giac")

[Out] integrate(-(b\*x^3 - a)/(b\*x^3 + a)^(16/3), x)

**maple** [A] time = 0.05, size = 59, normalized size = 0.63

$$\frac{(891b^4x^{12} + 3861b^3x^9a + 6435b^2x^6a^2 + 5005bx^3a^3 + 1820a^4)x}{1820(bx^3 + a)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)/(b*x^3+a)^(16/3),x)`

[Out] `1/1820*x*(891*b^4*x^12+3861*a*b^3*x^9+6435*a^2*b^2*x^6+5005*a^3*b*x^3+1820*a^4)/(b*x^3+a)^(13/3)/a^4`

**maxima** [B] time = 0.54, size = 153, normalized size = 1.65

$$\frac{\left(140b^3 - \frac{546(bx^3+a)b^2}{x^3} + \frac{780(bx^3+a)^2b}{x^6} - \frac{455(bx^3+a)^3}{x^9}\right)bx^{13}}{1820(bx^3 + a)^{\frac{13}{3}}a^4} + \frac{\left(35b^4 - \frac{182(bx^3+a)b^3}{x^3} + \frac{390(bx^3+a)^2b^2}{x^6} - \frac{455(bx^3+a)^3b}{x^9} + \frac{455(bx^3+a)^4}{x^{12}}\right)x^{13}}{455(bx^3 + a)^{\frac{13}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)/(b*x^3+a)^(16/3),x, algorithm="maxima")`

[Out] `1/1820*(140*b^3 - 546*(b*x^3 + a)*b^2/x^3 + 780*(b*x^3 + a)^2*b/x^6 - 455*(b*x^3 + a)^3/x^9)*b*x^13/((b*x^3 + a)^(13/3)*a^4) + 1/455*(35*b^4 - 182*(b*x^3 + a)*b^3/x^3 + 390*(b*x^3 + a)^2*b^2/x^6 - 455*(b*x^3 + a)^3*b/x^9 + 455*(b*x^3 + a)^4/x^12)*x^13/((b*x^3 + a)^(13/3)*a^4)`

**mupad** [B] time = 1.37, size = 73, normalized size = 0.78

$$\frac{2x}{13(bx^3 + a)^{13/3}} + \frac{891x}{1820a^4(bx^3 + a)^{1/3}} + \frac{297x}{1820a^3(bx^3 + a)^{4/3}} + \frac{99x}{910a^2(bx^3 + a)^{7/3}} + \frac{11x}{130a(bx^3 + a)^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)/(a + b*x^3)^(16/3),x)`

[Out] `(2*x)/(13*(a + b*x^3)^(13/3)) + (891*x)/(1820*a^4*(a + b*x^3)^(1/3)) + (297*x)/(1820*a^3*(a + b*x^3)^(4/3)) + (99*x)/(910*a^2*(a + b*x^3)^(7/3)) + (11*x)/(130*a*(a + b*x^3)^(10/3))`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)/(b*x**3+a)**(16/3),x)`

[Out] Timed out

$$3.34 \quad \int \frac{(a+bx^3)^{7/3}}{a-bx^3} dx$$

**Optimal.** Leaf size=483

$$\frac{2\sqrt[3]{2} a^{5/3} \log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{2\sqrt[3]{2} a^{5/3} \log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt[3]{b}} - \frac{4\sqrt[3]{2} a^{5/3} \log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}}$$

[Out]  $-7/5*a*x*(b*x^3+a)^{(1/3)}-1/5*x*(b*x^3+a)^{(4/3)}-7/5*a^2*x*(1+b*x^3/a)^{(2/3)}*$   
 $\text{hypergeom}([1/3, 2/3], [4/3], -b*x^3/a)/(b*x^3+a)^{(2/3)}-2/3*2^{(1/3)}*a^{(5/3)}*\ln$   
 $(2^{(2/3)}+(-a^{(1/3)}-b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})/b^{(1/3)}+2/3*2^{(1/3)}*a^{(5/3)}*$   
 $\ln(1+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)}*x)^2/(b*x^3+a)^{(2/3)}-2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}$   
 $*x)/(b*x^3+a)^{(1/3)})/b^{(1/3)}-4/3*2^{(1/3)}*a^{(5/3)}*\ln(1+2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}$   
 $*x)/(b*x^3+a)^{(1/3)})/b^{(1/3)}+1/3*2^{(1/3)}*a^{(5/3)}*\ln(2*2^{(1/3)}+(a^{(1/3)}+b^{(1/3)}$   
 $*x)^2/(b*x^3+a)^{(2/3)}+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})/b^{(1/3)}$   
 $-4/3*2^{(1/3)}*a^{(5/3)}*\arctan(1/3*(1-2*2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})$   
 $*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}-2/3*2^{(1/3)}*a^{(5/3)}*\arctan(1/3*(1+2^{(1/3)}$   
 $*x)/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 56, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; 1, -\frac{7}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(7/3)/(a - b\*x^3), x]

[Out] (a\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, 1, -7/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)]/(1 + (b\*x^3)/a)^(1/3)

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{(a + bx^3)^{7/3}}{a - bx^3} dx = \frac{\left(a^2 \sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{7/3}}{a - bx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{ax \sqrt[3]{a + bx^3} F_1\left(\frac{1}{3}; 1, -\frac{7}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

**Mathematica [C]** time = 0.38, size = 232, normalized size = 0.48

$$4x \left( \frac{52a^4 F_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a - bx^3) \left( 3F_1\left(\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 2F_1\left(\frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) + 4a F_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right)} - 8a^2 - 9abx^3 - b^2x^6 \right) + 27abx^4 \left( \frac{bx^3}{a} + \dots \right)$$


---


$$20(a + bx^3)^{2/3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(7/3)/(a - b\*x^3), x]

[Out] (27\*a\*b\*x^4\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a] + 4\*x\*(-8\*a^2 - 9\*a\*b\*x^3 - b^2\*x^6 + (52\*a^4\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a])/((a - b\*x^3)\*(4\*a\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), (b\*x^3)/a] - 2\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a]))))/((20\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)/(-b\*x^3+a), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{7/3}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)/(-b\*x^3+a), x, algorithm="giac")

[Out] integrate(-(b\*x^3 + a)^(7/3)/(b\*x^3 - a), x)

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{7/3}}{-bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(7/3)/(-b\*x^3+a), x)

[Out] `int((b*x^3+a)^(7/3)/(-b*x^3+a),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{7}{3}}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(7/3)/(-b*x^3+a),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(7/3)/(b*x^3 - a), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{7/3}}{a - bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(7/3)/(a - b*x^3),x)`

[Out] `int((a + b*x^3)^(7/3)/(a - b*x^3), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2 \sqrt[3]{a + bx^3}}{-a + bx^3} dx - \int \frac{b^2 x^6 \sqrt[3]{a + bx^3}}{-a + bx^3} dx - \int \frac{2abx^3 \sqrt[3]{a + bx^3}}{-a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(7/3)/(-b*x**3+a),x)`

[Out] `-Integral(a**2*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b**2*x**6*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(2*a*b*x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)`

$$3.35 \quad \int \frac{(a+bx^3)^{4/3}}{a-bx^3} dx$$

**Optimal.** Leaf size=464

$$\frac{\sqrt[3]{2} a^{2/3} \log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt[3]{b}} + \frac{\sqrt[3]{2} a^{2/3} \log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt[3]{b}} - \frac{2\sqrt[3]{2} a^{2/3} \log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3\sqrt[3]{b}}$$

[Out]  $-1/2*x*(b*x^3+a)^{(1/3)}-1/2*a*x*(1+b*x^3/a)^{(2/3)}*\text{hypergeom}([1/3, 2/3], [4/3], -b*x^3/a)/(b*x^3+a)^{(2/3)}-1/3*2^{(1/3)}*a^{(2/3)}*\ln(2^{(2/3)}+(-a^{(1/3)}-b^{(1/3)})*x)/(b*x^3+a)^{(1/3))/b^{(1/3)}+1/3*2^{(1/3)}*a^{(2/3)}*\ln(1+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)}*x)^2/(b*x^3+a)^{(2/3)}-2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3))/b^{(1/3)}-2/3*2^{(1/3)}*a^{(2/3)}*\ln(1+2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3))/b^{(1/3)}+1/6*a^{(2/3)}*\ln(2*2^{(1/3)}+(a^{(1/3)}+b^{(1/3)}*x)^2/(b*x^3+a)^{(2/3)}+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})*2^{(1/3)}/b^{(1/3)}-2/3*2^{(1/3)}*a^{(2/3)}*\arctan(1/3*(1-2*2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}-1/3*2^{(1/3)}*a^{(2/3)}*\arctan(1/3*(1+2^{(1/3)}*(a^{(1/3)}+b^{(1/3)}*x)/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 55, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; 1, -\frac{4}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(4/3)/(a - b\*x^3), x]

[Out] (x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, 1, -4/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)]/(1 + (b\*x^3)/a)^(1/3)

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**



$$\int \frac{(a + bx^3)^{4/3}}{a - bx^3} dx = \frac{\left(a \sqrt[3]{a + bx^3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{4/3}}{a - bx^3} dx}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

$$= \frac{x \sqrt[3]{a + bx^3} F_1\left(\frac{1}{3}; 1, -\frac{4}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{\sqrt[3]{1 + \frac{bx^3}{a}}}$$

**Mathematica [C]** time = 0.12, size = 217, normalized size = 0.47

$$x \frac{\left( \frac{48a^3 F_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a - bx^3) \left( bx^3 \left( 3F_1\left(\frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 2F_1\left(\frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right) + 4a F_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right)}{8(a + bx^3)^{2/3}} + 5bx^3 \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}\right) \right)}{8(a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(4/3)/(a - b\*x^3), x]

[Out] (x\*(-4\*(a + b\*x^3) + 5\*b\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -(b\*x^3)/a, (b\*x^3)/a] + (48\*a^3\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, (b\*x^3)/a])/((a - b\*x^3)\*(4\*a\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, 2/3, 2, 7/3, -(b\*x^3)/a, (b\*x^3)/a] - 2\*AppellF1[4/3, 5/3, 1, 7/3, -(b\*x^3)/a, (b\*x^3)/a]))))/(8\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 34.20, size = 0, normalized size = 0.00

$$\text{integral} \left( -\frac{(bx^3 + a)^{4/3}}{bx^3 - a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(-b\*x^3+a), x, algorithm="fricas")

[Out] integral(-(b\*x^3 + a)^(4/3)/(b\*x^3 - a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{4/3}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(-b\*x^3+a), x, algorithm="giac")

[Out] integrate(-(b\*x^3 + a)^(4/3)/(b\*x^3 - a), x)

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{4/3}}{-bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(4/3)/(-b*x^3+a),x)`

[Out] `int((b*x^3+a)^(4/3)/(-b*x^3+a),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(bx^3 + a)^{\frac{4}{3}}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(4/3)/(-b*x^3+a),x, algorithm="maxima")`

[Out] `-integrate((b*x^3 + a)^(4/3)/(b*x^3 - a), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{a - bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(4/3)/(a - b*x^3),x)`

[Out] `int((a + b*x^3)^(4/3)/(a - b*x^3), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt[3]{a + bx^3}}{-a + bx^3} dx - \int \frac{bx^3\sqrt[3]{a + bx^3}}{-a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(4/3)/(-b*x**3+a),x)`

[Out] `-Integral(a*(a + b*x**3)**(1/3)/(-a + b*x**3), x) - Integral(b*x**3*(a + b*x**3)**(1/3)/(-a + b*x**3), x)`

$$3.36 \quad \int \frac{\sqrt[3]{a+bx^3}}{a-bx^3} dx$$

Optimal. Leaf size=398

$$\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{3 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{b}} - \frac{\sqrt[3]{2} \log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3 \sqrt[3]{a} \sqrt[3]{b}} + \frac{\log\left(\frac{(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}}\right)}{6 \cdot 2^{2/3} \sqrt[3]{a} \sqrt[3]{b}}$$

[Out]  $-1/6 \cdot \ln(2^{2/3} + (-a^{1/3} - b^{1/3}x)/(bx^3+a)^{1/3}) \cdot 2^{1/3}/a^{1/3}/b^{1/3} + 1/6 \cdot \ln(1+2^{2/3} \cdot (a^{1/3} + b^{1/3}x)^2/(bx^3+a)^{2/3}) - 2^{1/3} \cdot (a^{1/3} + b^{1/3}x)/(bx^3+a)^{1/3}) \cdot 2^{1/3}/a^{1/3}/b^{1/3} - 1/3 \cdot 2^{1/3} \cdot \ln(1+2^{1/3} \cdot (a^{1/3} + b^{1/3}x)/(bx^3+a)^{1/3})/a^{1/3}/b^{1/3} + 1/12 \cdot \ln(2 \cdot 2^{1/3} + (a^{1/3} + b^{1/3}x)^2/(bx^3+a)^{2/3}) + 2^{2/3} \cdot (a^{1/3} + b^{1/3}x)/(bx^3+a)^{1/3}) \cdot 2^{1/3}/a^{1/3}/b^{1/3} - 1/3 \cdot 2^{1/3} \cdot \arctan(1/3 \cdot (1-2 \cdot 2^{1/3}) \cdot (a^{1/3} + b^{1/3}x)/(bx^3+a)^{1/3}) \cdot 3^{1/2}/a^{1/3}/b^{1/3} \cdot 3^{1/2} - 1/6 \cdot \arctan(1/3 \cdot (1+2^{1/3}) \cdot (a^{1/3} + b^{1/3}x)/(bx^3+a)^{1/3}) \cdot 3^{1/2}) \cdot 2^{1/3}/a^{1/3}/b^{1/3} \cdot 3^{1/2}$

Rubi [C] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.15, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{x \sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; 1, -\frac{1}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(1/3)/(a - b\*x^3), x]

[Out] (x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, 1, -1/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)]/(a\*(1 + (b\*x^3)/a)^(1/3))

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{\sqrt[3]{a+bx^3}}{a-bx^3} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{a-bx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} = \frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; 1, -\frac{1}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a\sqrt[3]{1+\frac{bx^3}{a}}}$$

**Mathematica** [C] time = 0.13, size = 151, normalized size = 0.38

$$\frac{4ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a-bx^3)\left(bx^3\left(3F_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) + F_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)\right) + 4aF_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/3)/(a - b\*x^3), x]

[Out] (4\*a\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, -1/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a])/((a - b\*x^3)\*(4\*a\*AppellF1[1/3, -1/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, -1/3, 2, 7/3, -((b\*x^3)/a), (b\*x^3)/a] + AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a])))

**fricas** [B] time = 29.48, size = 644, normalized size = 1.62

$$-\frac{1}{18} \sqrt{3} 2^{\frac{1}{3}} \left(-\frac{1}{ab}\right)^{\frac{1}{3}} \arctan \left( \frac{6 \sqrt{3} 2^{\frac{2}{3}} (ab^6 x^{16} + 33 a^2 b^5 x^{13} + 110 a^3 b^4 x^{10} + 110 a^4 b^3 x^7 + 33 a^5 b^2 x^4 + a^6 b x)}{(bx^3 + a)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(-b\*x^3+a), x, algorithm="fricas")

[Out] -1/18\*sqrt(3)\*2^(1/3)\*(-1/(a\*b))^(1/3)\*arctan(1/3\*(6\*sqrt(3)\*2^(2/3)\*(a\*b^6\*x^16 + 33\*a^2\*b^5\*x^13 + 110\*a^3\*b^4\*x^10 + 110\*a^4\*b^3\*x^7 + 33\*a^5\*b^2\*x^4 + a^6\*b\*x)\*(b\*x^3 + a)^(1/3)\*(-1/(a\*b))^(2/3) + 24\*sqrt(3)\*2^(1/3)\*(a\*b^5\*x^14 + 2\*a^2\*b^4\*x^11 - 6\*a^3\*b^3\*x^8 + 2\*a^4\*b^2\*x^5 + a^5\*b\*x^2)\*(b\*x^3 + a)^(2/3)\*(-1/(a\*b))^(1/3) - sqrt(3)\*(b^6\*x^18 - 42\*a\*b^5\*x^15 - 417\*a^2\*b^4\*x^12 - 812\*a^3\*b^3\*x^9 - 417\*a^4\*b^2\*x^6 - 42\*a^5\*b\*x^3 + a^6))/(b^6\*x^18 + 102\*a\*b^5\*x^15 + 447\*a^2\*b^4\*x^12 + 628\*a^3\*b^3\*x^9 + 447\*a^4\*b^2\*x^6 + 102\*a^5\*b\*x^3 + a^6)) - 1/36\*2^(1/3)\*(-1/(a\*b))^(1/3)\*log((12\*2^(2/3)\*(a\*b^3\*x^8 + 4\*a^2\*b^2\*x^5 + a^3\*b\*x^2)\*(b\*x^3 + a)^(2/3)\*(-1/(a\*b))^(2/3) - 2^(1/3)\*(b^4\*x^12 + 32\*a\*b^3\*x^9 + 78\*a^2\*b^2\*x^6 + 32\*a^3\*b\*x^3 + a^4)\*(-1/(a\*b))^(1/3) + 6\*(b^3\*x^10 + 11\*a\*b^2\*x^7 + 11\*a^2\*b\*x^4 + a^3\*x)\*(b\*x^3 + a)^(1/3))/(b^4\*x^12 - 4\*a\*b^3\*x^9 + 6\*a^2\*b^2\*x^6 - 4\*a^3\*b\*x^3 + a^4)) + 1/18\*2^(1/3)\*(-1/(a\*b))^(1/3)\*log(-(12\*(b\*x^3 + a)^(2/3)\*x^2 + 2^(2/3)\*(b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(-1/(a\*b))^(2/3) + 6\*2^(1/3)\*(b\*x^4 + a\*x)\*(b\*x^3 + a)^(1/3)\*(-1/(a\*b))^(1/3))/(b^2\*x^6 - 2\*a\*b\*x^3 + a^2))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(bx^3 + a)^{\frac{1}{3}}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(-b\*x^3+a),x, algorithm="giac")

[Out] integrate(-(b\*x^3 + a)^(1/3)/(b\*x^3 - a), x)

**maple** [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{-bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/3)/(-b\*x^3+a),x)

[Out] int((b\*x^3+a)^(1/3)/(-b\*x^3+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{(bx^3 + a)^{\frac{1}{3}}}{bx^3 - a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(-b\*x^3+a),x, algorithm="maxima")

[Out] -integrate((b\*x^3 + a)^(1/3)/(b\*x^3 - a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{1/3}}{a - bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)/(a - b\*x^3),x)

[Out] int((a + b\*x^3)^(1/3)/(a - b\*x^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt[3]{a + bx^3}}{-a + bx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/3)/(-b\*x\*\*3+a),x)

[Out] -Integral((a + b\*x\*\*3)\*\*(1/3)/(-a + b\*x\*\*3), x)

$$3.37 \quad \int \frac{1}{(a-bx^3)(a+bx^3)^{2/3}} dx$$

**Optimal.** Leaf size=452

$$\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{6 \cdot 2^{2/3} a^{4/3} \sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{6 \cdot 2^{2/3} a^{4/3} \sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{3 \cdot 2^{2/3} a^{4/3} \sqrt[3]{b}} + \frac{\log\left(\frac{(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} + \frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}}\right)}{12 \cdot 2^{2/3} a^{4/3}}$$

[Out]  $\frac{1}{2} x (1 + b x^3/a)^{2/3} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{2}{3}\right], \left[\frac{4}{3}\right], -b x^3/a\right) / a / (b x^3 + a)^{2/3} - \frac{1}{12} \ln\left(2^{2/3} + (-a^{1/3} - b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 2^{1/3} / a^{4/3} / b^{1/3} + \frac{1}{12} \ln\left(1 + 2^{2/3} * (a^{1/3} + b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 2^{1/3} / a^{4/3} / b^{1/3} - \frac{1}{6} \ln\left(1 + 2^{1/3} * (a^{1/3} + b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 2^{1/3} / a^{4/3} / b^{1/3} + \frac{1}{24} \ln\left(2 * 2^{1/3} + (a^{1/3} + b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 2^{1/3} / a^{4/3} / b^{1/3} - \frac{1}{6} \arctan\left(\frac{1}{3} * (1 - 2 * 2^{1/3}) * (a^{1/3} + b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 3^{1/2} * 2^{1/3} / a^{4/3} / b^{1/3} * 3^{1/2} - \frac{1}{12} \arctan\left(\frac{1}{3} * (1 + 2^{1/3}) * (a^{1/3} + b^{1/3}) x / (b x^3 + a)^{1/3}\right) * 3^{1/2} * 2^{1/3} / a^{4/3} / b^{1/3} * 3^{1/2}$

**Rubi [C]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.13, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{2}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a (a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a - b\*x^3)\*(a + b\*x^3)^(2/3)),x]

[Out] (x\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[1/3, 1, 2/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)])/ (a\*(a + b\*x^3)^(2/3))

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{(a - bx^3)(a + bx^3)^{2/3}} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{(a - bx^3)\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{(a + bx^3)^{2/3}}$$

$$= \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{2}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a (a + bx^3)^{2/3}}$$

**Mathematica [C]** time = 0.13, size = 153, normalized size = 0.34

$$\frac{4ax F_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a - bx^3)(a + bx^3)^{2/3} \left(bx^3 \left(3F_1\left(\frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 2F_1\left(\frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)\right) + 4a F_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^3)\*(a + b\*x^3)^(2/3)),x]

[Out] (4\*a\*x\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a])/((a - b\*x^3)\*(a + b\*x^3)^(2/3)\*(4\*a\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), (b\*x^3)/a] - 2\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a])))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(2/3),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(bx^3 + a)^{\frac{2}{3}}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(2/3),x, algorithm="giac")

[Out] integrate(-1/((b\*x^3 + a)^(2/3)\*(b\*x^3 - a)), x)

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^3 + a)(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^3+a)/(b\*x^3+a)^(2/3),x)

[Out] int(1/(-b\*x^3+a)/(b\*x^3+a)^(2/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(2/3),x, algorithm="maxima")

[Out] -integrate(1/((b\*x^3 + a)^(2/3)\*(b\*x^3 - a)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(a - bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(2/3)\*(a - b\*x^3)),x)

[Out] int(1/((a + b\*x^3)^(2/3)\*(a - b\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a(a + bx^3)^{\frac{2}{3}} + bx^3(a + bx^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*3+a)/(b\*x\*\*3+a)\*\*(2/3),x)

[Out] -Integral(1/(-a\*(a + b\*x\*\*3)\*\*(2/3) + b\*x\*\*3\*(a + b\*x\*\*3)\*\*(2/3)), x)



$$3.38 \quad \int \frac{1}{(a-bx^3)(a+bx^3)^{5/3}} dx$$

**Optimal.** Leaf size=473

$$\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{12 \cdot 2^{2/3} a^{7/3} \sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{12 \cdot 2^{2/3} a^{7/3} \sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{6 \cdot 2^{2/3} a^{7/3} \sqrt[3]{b}} + \frac{\log\left(\frac{(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} + \frac{2^{2/3}}{\sqrt[3]{a+bx^3}}\right)}{24 \cdot 2^{2/3} a^{7/3} \sqrt[3]{b}}$$

[Out]  $\frac{1}{4} x/a^2/(b x^3+a)^{(2/3)}+1/2 x*(1+b x^3/a)^{(2/3)} \operatorname{hypergeom}([1/3, 2/3], [4/3], -b x^3/a)/a^2/(b x^3+a)^{(2/3)}-1/24 \ln(2^{(2/3)}+(-a^{(1/3)}-b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)}+1/24 \ln(1+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)} x)^2/(b x^3+a)^{(2/3)}-2^{(1/3)}*(a^{(1/3)}+b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)}-1/12 \ln(1+2^{(1/3)}*(a^{(1/3)}+b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)}+1/48 \ln(2 * 2^{(1/3)}+(a^{(1/3)}+b^{(1/3)} x)^2/(b x^3+a)^{(2/3)}+2^{(2/3)}*(a^{(1/3)}+b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)}-1/12 \operatorname{arctan}(1/3*(1-2 * 2^{(1/3)}*(a^{(1/3)}+b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 3^{(1/2)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)} * 3^{(1/2)}-1/24 \operatorname{arctan}(1/3*(1+2^{(1/3)}*(a^{(1/3)}+b^{(1/3)} x)/(b x^3+a)^{(1/3)}) * 3^{(1/2)}) * 2^{(1/3)}/a^{(7/3)}/b^{(1/3)} * 3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{5}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a^2 (a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a - b\*x^3)\*(a + b\*x^3)^(5/3)), x]

[Out] (x\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[1/3, 1, 5/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)])/ (a^2\*(a + b\*x^3)^(2/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a - bx^3)(a + bx^3)^{5/3}} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{(a - bx^3)\left(1 + \frac{bx^3}{a}\right)^{5/3}} dx}{a(a + bx^3)^{2/3}}$$

$$= \frac{x\left(1 + \frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{5}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a^2(a + bx^3)^{2/3}}$$

**Mathematica** [C] time = 0.14, size = 213, normalized size = 0.45

$$x \left( -\frac{bx^3 \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{a^3} + \frac{4}{a^2} + \frac{48F_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a - bx^3) \left( bx^3 \left( 3F_1\left(\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 2F_1\left(\frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) \right) + 4aF_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)} \right) \right) / (16(a + bx^3)^{2/3})$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^3)\*(a + b\*x^3)^(5/3)),x]

[Out] (x\*(4/a^2 - (b\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a])/a^3 + (48\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a])/((a - b\*x^3)\*(4\*a\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), (b\*x^3)/a] - 2\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a]))))/(16\*(a + b\*x^3)^(2/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(5/3),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(bx^3 + a)^{5/3}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(5/3),x, algorithm="giac")

[Out] integrate(-1/((b\*x^3 + a)^(5/3)\*(b\*x^3 - a)), x)

**maple** [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^3 + a)(bx^3 + a)^{5/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^3+a)/(b\*x^3+a)^(5/3),x)

[Out] `int(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x^3+a)/(b*x^3+a)^(5/3),x, algorithm="maxima")`

[Out] `-integrate(1/((b*x^3 + a)^(5/3)*(b*x^3 - a)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(a - bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(5/3)*(a - b*x^3)),x)`

[Out] `int(1/((a + b*x^3)^(5/3)*(a - b*x^3)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-a^2(a + bx^3)^{\frac{2}{3}} + b^2x^6(a + bx^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**3+a)/(b*x**3+a)**(5/3),x)`

[Out] `-Integral(1/(-a**2*(a + b*x**3)**(2/3) + b**2*x**6*(a + b*x**3)**(2/3)), x)`

$$3.39 \quad \int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}} dx$$

**Optimal.** Leaf size=492

$$\frac{\log\left(2^{2/3} - \frac{\sqrt[3]{a} + \sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right)}{24 \cdot 2^{2/3} a^{10/3} \sqrt[3]{b}} + \frac{\log\left(\frac{2^{2/3}(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} - \frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{24 \cdot 2^{2/3} a^{10/3} \sqrt[3]{b}} - \frac{\log\left(\frac{\sqrt[3]{2}(\sqrt[3]{a} + \sqrt[3]{bx^3})}{\sqrt[3]{a+bx^3}} + 1\right)}{12 \cdot 2^{2/3} a^{10/3} \sqrt[3]{b}} + \frac{\log\left(\frac{(\sqrt[3]{a} + \sqrt[3]{bx^3})^2}{(a+bx^3)^{2/3}} + \frac{2^{2/3}}{\sqrt[3]{a+bx^3}}\right)}{48 \cdot 2^{2/3} a^{10/3}}$$

[Out]  $\frac{1}{10} x/a^2/(b*x^3+a)^{(5/3)} + \frac{13}{40} x/a^3/(b*x^3+a)^{(2/3)} + \frac{9}{20} x*(1+b*x^3/a)^{(2/3)} * \text{hypergeom}([1/3, 2/3], [4/3], -b*x^3/a)/a^3/(b*x^3+a)^{(2/3)} - \frac{1}{48} \ln(2^{(2/3)} + (-a^{(1/3)} - b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} + \frac{1}{48} \ln(1 + 2^{(2/3)}*(a^{(1/3)} + b^{(1/3)}*x)^2/(b*x^3+a)^{(2/3)} - 2^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} - \frac{1}{24} \ln(1 + 2^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} + \frac{1}{96} \ln(2*2^{(1/3)} + (a^{(1/3)} + b^{(1/3)}*x)^2/(b*x^3+a)^{(2/3)} + 2^{(2/3)}*(a^{(1/3)} + b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} - \frac{1}{24} \arctan(1/3*(1 - 2*2^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 3^{(1/2)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} * 3^{(1/2)} - \frac{1}{48} \arctan(1/3*(1 + 2^{(1/3)}*(a^{(1/3)} + b^{(1/3)}*x)/(b*x^3+a)^{(1/3)}) * 3^{(1/2)}) * 2^{(1/3)}/a^{(10/3)}/b^{(1/3)} * 3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 58, normalized size of antiderivative = 0.12, number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {430, 429}

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{8}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a^3 (a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a - b\*x^3)\*(a + b\*x^3)^(8/3)),x]

[Out] (x\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[1/3, 1, 8/3, 4/3, (b\*x^3)/a, -((b\*x^3)/a)])/((a^3\*(a + b\*x^3)^(2/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/((1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a-bx^3)(a+bx^3)^{8/3}} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{(a-bx^3)\left(1 + \frac{bx^3}{a}\right)^{8/3}} dx}{a^2 (a+bx^3)^{2/3}}$$

$$= \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1\left(\frac{1}{3}; 1, \frac{8}{3}; \frac{4}{3}; \frac{bx^3}{a}, -\frac{bx^3}{a}\right)}{a^3 (a+bx^3)^{2/3}}$$

**Mathematica [C]** time = 0.18, size = 240, normalized size = 0.49

$$x \frac{\left( \frac{368a^3(a+bx^3)F_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)}{(a-bx^3)\left(3F_1\left(\frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right) - 2F_1\left(\frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)\right) + 4aF_1\left(\frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, \frac{bx^3}{a}\right)} \right)}{160a^4 (a+bx^3)^{5/3}} + 16a^2 - 13bx^3 (a+bx^3) \left(\frac{bx^3}{a} + 1\right)^{2/3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^3)\*(a + b\*x^3)^(8/3)),x]

[Out] (x\*(16\*a^2 + 52\*a\*(a + b\*x^3) - 13\*b\*x^3\*(a + b\*x^3)\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a] + (368\*a^3\*(a + b\*x^3)\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a])/((a - b\*x^3)\*(4\*a\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), (b\*x^3)/a] + b\*x^3\*(3\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), (b\*x^3)/a] - 2\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), (b\*x^3)/a]))))/((160\*a^4\*(a + b\*x^3)^(5/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(8/3),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(bx^3 + a)^{\frac{8}{3}}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^3+a)/(b\*x^3+a)^(8/3),x, algorithm="giac")

[Out] integrate(-1/((b\*x^3 + a)^(8/3)\*(b\*x^3 - a)), x)

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^3 + a)(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^3+a)/(b\*x^3+a)^(8/3),x)

[Out] `int(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(bx^3 - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x^3+a)/(b*x^3+a)^(8/3),x, algorithm="maxima")`

[Out] `-integrate(1/((b*x^3 + a)^(8/3)*(b*x^3 - a)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(a - bx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(8/3)*(a - b*x^3)),x)`

[Out] `int(1/((a + b*x^3)^(8/3)*(a - b*x^3)), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**3+a)/(b*x**3+a)**(8/3),x)`

[Out] Timed out

### 3.40 $\int (a - bx^3)^2 (a + bx^3)^{2/3} dx$

**Optimal.** Leaf size=139

$$\frac{38a^3 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{81\sqrt[3]{b}} + \frac{76a^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} + \frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{8}{27}ax(a+bx^3)^{5/3} - \frac{1}{9}x(a-bx^3)$$

[Out]  $38/81*a^2*x*(b*x^3+a)^{(2/3)}-8/27*a*x*(b*x^3+a)^{(5/3)}-1/9*x*(-b*x^3+a)*(b*x^3+a)^{(5/3)}-38/81*a^3*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}+76/243*a^3*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {416, 388, 195, 239}

$$\frac{38}{81}a^2x(a+bx^3)^{2/3} - \frac{38a^3 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{81\sqrt[3]{b}} + \frac{76a^3 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{81\sqrt{3}\sqrt[3]{b}} - \frac{8}{27}ax(a+bx^3)^{5/3} - \frac{1}{9}x(a-bx^3)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2\*(a + b\*x^3)^(2/3), x]

[Out]  $(38*a^2*x*(a + b*x^3)^{(2/3)})/81 - (8*a*x*(a + b*x^3)^{(5/3)})/27 - (x*(a - b*x^3)*(a + b*x^3)^{(5/3)})/9 + (76*a^3*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(81*\text{Sqrt}[3]*b^{(1/3)}) - (38*a^3*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(81*b^{(1/3)})$

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d,

0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a - bx^3)^2 (a + bx^3)^{2/3} dx &= -\frac{1}{9}x(a - bx^3)(a + bx^3)^{5/3} + \frac{\int (a + bx^3)^{2/3} (10a^2b - 16ab^2x^3) dx}{9b} \\ &= -\frac{8}{27}ax(a + bx^3)^{5/3} - \frac{1}{9}x(a - bx^3)(a + bx^3)^{5/3} + \frac{1}{27}(38a^2) \int (a + bx^3)^{2/3} dx \\ &= \frac{38}{81}a^2x(a + bx^3)^{2/3} - \frac{8}{27}ax(a + bx^3)^{5/3} - \frac{1}{9}x(a - bx^3)(a + bx^3)^{5/3} + \frac{1}{81}(76a^3) \int (a + bx^3)^{2/3} dx \\ &= \frac{38}{81}a^2x(a + bx^3)^{2/3} - \frac{8}{27}ax(a + bx^3)^{5/3} - \frac{1}{9}x(a - bx^3)(a + bx^3)^{5/3} + \frac{76a^3 \tan^{-1}\left(\frac{\sqrt[3]{bx^3 + a}}{\sqrt[3]{a + bx^3}}\right)}{81\sqrt[3]{b}} \end{aligned}$$

**Mathematica** [A] time = 0.14, size = 151, normalized size = 1.09

$$\frac{1}{243} \left( \frac{38a^3 \left( \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}} + 1\right)\right)}{\sqrt[3]{b}} + 3(a + bx^3)^{2/3} (5a^2x - \dots) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2\*(a + b\*x^3)^(2/3),x]

[Out] (3\*(a + b\*x^3)^(2/3)\*(5\*a^2\*x - 24\*a\*b\*x^4 + 9\*b^2\*x^7) + (38\*a^3\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3)]/sqrt[3]) - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/b^(1/3))/243

**fricas** [A] time = 0.88, size = 421, normalized size = 3.03

$$\left[ \frac{114 \sqrt{\frac{1}{3}} a^3 b \sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 + 2(bx^3 + a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}\right)\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(2/3),x, algorithm="fricas")

[Out] [1/243\*(114\*sqrt(1/3)\*a^3\*b\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 76\*a^3\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 38\*a^3\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2



) + 3\*(9\*b^3\*x^7 - 24\*a\*b^2\*x^4 + 5\*a^2\*b\*x)\*(b\*x^3 + a)^(2/3))/b, -1/243\*(228\*sqrt(1/3)\*a^3\*b\*sqrt(-(-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt(-(-b)^(1/3)/b)/x) + 76\*a^3\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - 38\*a^3\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(9\*b^3\*x^7 - 24\*a\*b^2\*x^4 + 5\*a^2\*b\*x)\*(b\*x^3 + a)^(2/3))/b]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{2}{3}}(bx^3 - a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(2/3),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)\*(b\*x^3 - a)^2, x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (-bx^3 + a)^2 (bx^3 + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2\*(b\*x^3+a)^(2/3),x)

[Out] int((-b\*x^3+a)^2\*(b\*x^3+a)^(2/3),x)

**maxima** [B] time = 1.53, size = 552, normalized size = 3.97

$$\frac{1}{9} \left( \frac{2\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} + \frac{3(bx^3+a)^{\frac{2}{3}}}{(b-x)^{\frac{1}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(2/3),x, algorithm="maxima")

[Out] -1/9\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3) + 3\*(b\*x^3 + a)^(2/3)\*a/((b - (b\*x^3 + a)/x^3)\*x^2)\*a^2 - 1/27\*(2\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + 3\*((b\*x^3 + a)^(2/3)\*a^2\*b/x^2 + 2\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^3 - 2\*(b\*x^3 + a)\*b^2/x^3 + (b\*x^3 + a)^2\*b/x^6)\*a\*b - 1/243\*(4\*sqrt(3)\*a^3\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2\*a^3\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*a^3\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3) + 3\*(2\*(b\*x^3 + a)^(2/3)\*a^3\*b^2/x^2 + 11\*(b\*x^3 + a)^(5/3)\*a^3\*b/x^5 - 4\*(b\*x^3 + a)^(8/3)\*a^3/x^8)/(b^5 - 3\*(b\*x^3 + a)\*b^4/x^3 + 3\*(b\*x^3 + a)^2\*b^3/x^6 - (b\*x^3 + a)^3\*b^2/x^9))\*b^2

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{2/3} (a - bx^3)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)\*(a - b\*x^3)^2,x)

[Out] int((a + b\*x^3)^(2/3)\*(a - b\*x^3)^2, x)

**sympy [C]** time = 9.17, size = 126, normalized size = 0.91

$$\frac{a^{\frac{8}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{5}{3}} bx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}} b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2\*(b\*x\*\*3+a)\*\*(2/3),x)

[Out] a\*\*(8/3)\*x\*gamma(1/3)\*hyper((-2/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) - 2\*a\*\*(5/3)\*b\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(2/3)\*b\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-2/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3))

$$3.41 \quad \int \frac{(a-bx^3)^2}{\sqrt[3]{a+bx^3}} dx$$

Optimal. Leaf size=120

$$-\frac{17a^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{18\sqrt[3]{b}} + \frac{17a^2 \tan^{-1}\left(\frac{2\sqrt[3]{bx^3} + 1}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)(a+bx^3)^{2/3}$$

[Out]  $-13/18*a*x*(b*x^3+a)^{(2/3)} - 1/6*x*(-b*x^3+a)*(b*x^3+a)^{(2/3)} - 17/18*a^2*\ln(-b^{(1/3)*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}+17/27*a^2*\arctan(1/3*(1+2*b^{(1/3)*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)*3^{(1/2)}}$

Rubi [A] time = 0.04, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {416, 388, 239}

$$-\frac{17a^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{18\sqrt[3]{b}} + \frac{17a^2 \tan^{-1}\left(\frac{2\sqrt[3]{bx^3} + 1}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{13}{18}ax(a+bx^3)^{2/3} - \frac{1}{6}x(a-bx^3)(a+bx^3)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(1/3), x]

[Out]  $(-13*a*x*(a + b*x^3)^{(2/3)})/18 - (x*(a - b*x^3)*(a + b*x^3)^{(2/3)})/6 + (17*a^2*\text{ArcTan}[(1 + (2*b^{(1/3)*x})/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(9*\text{Sqrt}[3]*b^{(1/3)}) - (17*a^2*\text{Log}[-(b^{(1/3)*x}) + (a + b*x^3)^{(1/3)})]/(18*b^{(1/3)})$

Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned} \int \frac{(a - bx^3)^2}{\sqrt[3]{a + bx^3}} dx &= -\frac{1}{6}x(a - bx^3)(a + bx^3)^{2/3} + \frac{\int \frac{7a^2b - 13ab^2x^3}{\sqrt[3]{a + bx^3}} dx}{6b} \\ &= -\frac{13}{18}ax(a + bx^3)^{2/3} - \frac{1}{6}x(a - bx^3)(a + bx^3)^{2/3} + \frac{1}{9}(17a^2) \int \frac{1}{\sqrt[3]{a + bx^3}} dx \\ &= -\frac{13}{18}ax(a + bx^3)^{2/3} - \frac{1}{6}x(a - bx^3)(a + bx^3)^{2/3} + \frac{17a^2 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}\sqrt[3]{b}} - \frac{17a^2 \log\left(-\sqrt[3]{b}x + \dots\right)}{18\sqrt[3]{b}} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 141, normalized size = 1.18

$$\frac{17a^2 \left( \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right) \right)}{54\sqrt[3]{b}} + (a + bx^3)^{2/3} \left( \frac{bx^4}{6} - \frac{8ax}{9} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(1/3), x]

[Out] (a + b\*x^3)^(2/3)\*((-8\*a\*x)/9 + (b\*x^4)/6) + (17\*a^2\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/54\*b^(1/3))

**fricas [A]** time = 0.81, size = 399, normalized size = 3.32

$$\left[ \frac{51 \sqrt{\frac{1}{3}} a^2 b \sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 + 2(bx^3 + a)^{\frac{2}{3}}(-b)^{\frac{2}{3}}x\right)\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(1/3), x, algorithm="fricas")

[Out] [1/54\*(51\*sqrt(1/3)\*a^2\*b\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 34\*a^2\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 17\*a^2\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*(3\*b^2\*x^4 - 16\*a\*b\*x)\*(b\*x^3 + a)^(2/3))/b, -1/54\*(102\*sqrt(1/3)\*a^2\*b\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt((-b)^(1/3)/b)/x) + 34\*a^2\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - 17\*a^2\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(3\*b^2\*x^4 - 16\*a\*b\*x)\*(b\*x^3 + a)^(2/3))/b]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(1/3),x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(1/3), x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(1/3),x)

[Out] int((-b\*x^3+a)^2/(b\*x^3+a)^(1/3),x)

**maxima** [B] time = 1.40, size = 436, normalized size = 3.63

$$\frac{1}{6} \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} \right) a^2 - \frac{1}{9} \left( 2\sqrt{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] -1/6\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3)))/b^(1/3) - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3))\*a^2 - 1/9\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) - 6\*(b\*x^3 + a)^(2/3)\*a/((b^2 - (b\*x^3 + a)\*b/x^3)\*x^2))\*a\*b - 1/54\*(4\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2\*a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3) - 3\*(7\*(b\*x^3 + a)^(2/3)\*a^2\*b/x^2 - 4\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^4 - 2\*(b\*x^3 + a)\*b^3/x^3 + (b\*x^3 + a)^2\*b^2/x^6))\*b^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(1/3),x)

[Out] int((a - b\*x^3)^2/(a + b\*x^3)^(1/3), x)

sympy [C] time = 7.60, size = 121, normalized size = 1.01

$$\frac{a^{\frac{5}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{2}{3}}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{b^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{7}{3} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(1/3),x)

[Out] a\*\*(5/3)\*x\*gamma(1/3)\*hyper((1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) - 2\*a\*\*(2/3)\*b\*x\*\*4\*gamma(4/3)\*hyper((1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + b\*\*2\*x\*\*7\*gamma(7/3)\*hyper((1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*a\*\*(1/3)\*gamma(10/3))

$$3.42 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{4/3}} dx$$

Optimal. Leaf size=113

$$\frac{7}{3}x(a+bx^3)^{2/3} + \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{5a \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{3\sqrt[3]{b}} - \frac{10a \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

[Out]  $2*x*(-b*x^3+a)/(b*x^3+a)^{(1/3)}+7/3*x*(b*x^3+a)^{(2/3)}+5/3*a*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(1/3)}-10/9*a*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(1/3)}*3^{(1/2)}$

Rubi [A] time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {413, 388, 239}

$$\frac{7}{3}x(a+bx^3)^{2/3} + \frac{2x(a-bx^3)}{\sqrt[3]{a+bx^3}} + \frac{5a \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{3\sqrt[3]{b}} - \frac{10a \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(4/3), x]

[Out]  $(2*x*(a - b*x^3))/(a + b*x^3)^{(1/3)} + (7*x*(a + b*x^3)^{(2/3)})/3 - (10*a*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]*b^{(1/3)}) + (5*a*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(3*b^{(1/3)})$

Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1)+1)) + d\*(a\*d\*(n\*(q-1)+1) - b\*c\*(n\*(p+q)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{4/3}} dx &= \frac{2x(a - bx^3)}{\sqrt[3]{a + bx^3}} + \frac{\int \frac{-a^2b + 7ab^2x^3}{\sqrt[3]{a + bx^3}} dx}{ab} \\
&= \frac{2x(a - bx^3)}{\sqrt[3]{a + bx^3}} + \frac{7}{3}x(a + bx^3)^{2/3} - \frac{1}{3}(10a) \int \frac{1}{\sqrt[3]{a + bx^3}} dx \\
&= \frac{2x(a - bx^3)}{\sqrt[3]{a + bx^3}} + \frac{7}{3}x(a + bx^3)^{2/3} - \frac{10a \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}\sqrt[3]{b}} + \frac{5a \log\left(-\sqrt[3]{b}x + \sqrt[3]{a + bx^3}\right)}{3\sqrt[3]{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.08, size = 137, normalized size = 1.21

$$\frac{x(13a + bx^3)}{3\sqrt[3]{a + bx^3}} - \frac{5a \left( \log\left(\frac{b^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}} + 1}{\sqrt{3}}\right) \right)}{9\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(4/3), x]

[Out] (x\*(13\*a + b\*x^3))/(3\*(a + b\*x^3)^(1/3)) - (5\*a\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3)]/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/(9\*b^(1/3))

**fricas [B]** time = 0.84, size = 412, normalized size = 3.65

$$\frac{15 \sqrt{\frac{1}{3}} (ab^2x^3 + a^2b) \sqrt{-\frac{1}{b^3}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3 + a)^{\frac{1}{3}}bx^2 - 2(bx^3 + a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\sqrt{-\frac{1}{b^3}}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(4/3), x, algorithm="fricas")

[Out] [1/9\*(15\*sqrt(1/3)\*(a\*b^2\*x^3 + a^2\*b)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x)\*sqrt(-1/b^(2/3)) + 2\*a) + 10\*(a\*b\*x^3 + a^2)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - 5\*(a\*b\*x^3 + a^2)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*(b^2\*x^4 + 13\*a\*b\*x)\*(b\*x^3 + a)^(2/3)/(b^2\*x^3 + a\*b), 1/9\*(10\*(a\*b\*x^3 + a^2)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - 5\*(a\*b\*x^3 + a^2)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 30\*sqrt(1/3)\*(a\*b^2\*x^3 + a^2\*b)\*arctan(sqrt(1/3)\*(b^(1/3)\*x + 2\*(b\*x^3 + a)^(1/3))/(b^(1/3)\*x))/b^(1/3) + 3\*(b^2\*x^4 + 13\*a\*b\*x)\*(b\*x^3 + a)^(2/3)/(b^2\*x^3 + a\*b)]



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(4/3),x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(4/3), x)

**maple** [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(4/3),x)

[Out] int((-b\*x^3+a)^2/(b\*x^3+a)^(4/3),x)

**maxima** [B] time = 1.37, size = 296, normalized size = 2.62

$$\frac{1}{9} b^2 \left( \frac{4 \sqrt{3} a \arctan \left( \frac{\sqrt{3} \left( b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x} \right)}{3 b^{\frac{1}{3}}} \right)}{b^{\frac{7}{3}}} + \frac{3 \left( 3 ab - \frac{4(bx^3+a)a}{x^3} \right)}{\frac{(bx^3+a)^{\frac{1}{3}} b^3}{x} - \frac{(bx^3+a)^{\frac{4}{3}} b^2}{x^4}} - \frac{2 a \log \left( b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}} b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2} \right)}{b^{\frac{7}{3}}} + \frac{4 a \log \left( b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}} b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2} \right)}{b^{\frac{7}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(4/3),x, algorithm="maxima")

[Out] 1/9\*b^2\*(4\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) + 3\*(3\*a\*b - 4\*(b\*x^3 + a)\*a/x^3)/((b\*x^3 + a)^(1/3)\*b^3/x - (b\*x^3 + a)^(4/3)\*b^2/x^4) - 2\*a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3) + 1/3\*a\*b\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6\*x/((b\*x^3 + a)^(1/3)\*b - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + a\*x/(b\*x^3 + a)^(1/3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(4/3),x)

[Out] int((a - b\*x^3)^2/(a + b\*x^3)^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^3)^2}{(a + bx^3)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(4/3),x)

[Out] Integral((-a + b\*x\*\*3)\*\*2/(a + b\*x\*\*3)\*\*(4/3), x)

$$3.43 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{7/3}} dx$$

**Optimal.** Leaf size=110

$$-\frac{x}{2\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2\sqrt[3]{b}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+1}\right)}{\sqrt{3}\sqrt[3]{b}}$$

[Out] 1/2\*x\*(-b\*x^3+a)/(b\*x^3+a)^(4/3)-1/2\*x/(b\*x^3+a)^(1/3)-1/2\*ln(-b^(1/3)\*x+(b\*x^3+a)^(1/3))/b^(1/3)+1/3\*arctan(1/3\*(1+2\*b^(1/3)\*x/(b\*x^3+a)^(1/3))\*3^(1/2))/b^(1/3)\*3^(1/2)

**Rubi [A]** time = 0.04, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {413, 385, 239}

$$-\frac{x}{2\sqrt[3]{a+bx^3}} + \frac{x(a-bx^3)}{2(a+bx^3)^{4/3}} - \frac{\log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2\sqrt[3]{b}} + \frac{\tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+1}\right)}{\sqrt{3}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(7/3), x]

[Out] (x\*(a - b\*x^3))/(2\*(a + b\*x^3)^(4/3)) - x/(2\*(a + b\*x^3)^(1/3)) + ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*b^(1/3)) - Log[-(b^(1/3)\*x) + (a + b\*x^3)^(1/3)]/(2\*b^(1/3))

**Rule 239**

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 413**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1)+1)) + d\*(a\*d\*(n\*(q-1)+1) - b\*c\*(n\*(p+q)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

**Rubi steps**

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{7/3}} dx &= \frac{x(a - bx^3)}{2(a + bx^3)^{4/3}} + \frac{\int \frac{2a^2b + 4ab^2x^3}{(a + bx^3)^{4/3}} dx}{4ab} \\
&= \frac{x(a - bx^3)}{2(a + bx^3)^{4/3}} - \frac{x}{2\sqrt[3]{a + bx^3}} + \int \frac{1}{\sqrt[3]{a + bx^3}} dx \\
&= \frac{x(a - bx^3)}{2(a + bx^3)^{4/3}} - \frac{x}{2\sqrt[3]{a + bx^3}} + \frac{\tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{b}x}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}\sqrt[3]{b}} - \frac{\log\left(-\sqrt[3]{b}x + \sqrt[3]{a + bx^3}\right)}{2\sqrt[3]{b}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 131, normalized size = 1.19

$$\frac{-\frac{6b^{4/3}x^4}{(a+bx^3)^{4/3}} + \log\left(\frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1\right) - 2\log\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}\right) + 2\sqrt{3}\tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{6\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(7/3), x]

[Out] ((-6\*b^(4/3)\*x^4)/(a + b\*x^3)^(4/3) + 2\*Sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(6\*b^(1/3))

**fricas [B]** time = 0.91, size = 521, normalized size = 4.74

$$\frac{6(bx^3 + a)^{\frac{2}{3}}b^2x^4 - 3\sqrt{\frac{1}{3}}(b^3x^6 + 2ab^2x^3 + a^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - \dots\right)\right)}{6(bx^3 + a)^{\frac{2}{3}}b^2x^4 - 3\sqrt{\frac{1}{3}}(b^3x^6 + 2ab^2x^3 + a^2b)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - \dots\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(7/3), x, algorithm="fricas")

[Out] [-1/6\*(6\*(b\*x^3 + a)^(2/3)\*b^2\*x^4 - 3\*sqrt(1/3)\*(b^3\*x^6 + 2\*a\*b^2\*x^3 + a^2\*b)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) + 2\*(b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - (b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)]/(b^3\*x^6 + 2\*a\*b^2\*x^3 + a^2\*b), -1/6\*(6\*(b\*x^3 + a)^(2/3)\*b^2\*x^4 + 6\*sqrt(1/3)\*(b^3\*x^6 + 2\*a\*b^2\*x^3 + a^2\*b)\*sqrt(-(-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt(-(-b)^(1/3)/b)/x) + 2\*(b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - (b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)]/(b^3\*x^6 + 2\*a\*b^2\*x^3 + a^2\*b)

$3) * x^2 - (b * x^3 + a)^{1/3} * (-b)^{1/3} * x + (b * x^3 + a)^{2/3} / x^2) / (b^3 * x^6 + 2 * a * b^2 * x^3 + a^2 * b)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(7/3),x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(7/3), x)

**maple** [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(7/3),x)

[Out] int((-b\*x^3+a)^2/(b\*x^3+a)^(7/3),x)

**maxima** [B] time = 1.35, size = 180, normalized size = 1.64

$$\frac{\left( b - \frac{4(bx^3+a)}{x^3} \right) x^4}{4(bx^3+a)^{4/3}} - \frac{bx^4}{2(bx^3+a)^{4/3}} - \frac{1}{12} \left( \frac{3 \left( b + \frac{4(bx^3+a)}{x^3} \right) x^4}{(bx^3+a)^{4/3} b^2} + \frac{4\sqrt{3} \arctan \left( \frac{\sqrt{3} \left( \frac{1}{b^{1/3}} + \frac{2(bx^3+a)^{1/3}}{x} \right)}{3b^{1/3}} \right)}{b^{7/3}} - 2 \log \left( b^{2/3} + \frac{bx^3}{b^{1/3}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(7/3),x, algorithm="maxima")

[Out]  $-1/4 * (b - 4 * (b * x^3 + a) / x^3) * x^4 / (b * x^3 + a)^{4/3} - 1/2 * b * x^4 / (b * x^3 + a)^{4/3} - 1/12 * (3 * (b + 4 * (b * x^3 + a) / x^3) * x^4 / ((b * x^3 + a)^{4/3} * b^2) + 4 * \text{sqrt}(3) * \arctan(1/3 * \text{sqrt}(3) * (b^{1/3} + 2 * (b * x^3 + a)^{1/3} / x) / b^{1/3})) / b^{7/3} - 2 * \log(b^{2/3} + (b * x^3 + a)^{1/3} * b^{1/3} / x + (b * x^3 + a)^{2/3} / x^2) / b^{7/3} + 4 * \log(-b^{1/3} + (b * x^3 + a)^{1/3} / x) / b^{7/3}) * b^2$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(7/3),x)

[Out] int((a - b\*x^3)^2/(a + b\*x^3)^(7/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^3)^2}{(a + bx^3)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(7/3),x)

[Out] Integral((-a + b\*x\*\*3)\*\*2/(a + b\*x\*\*3)\*\*(7/3), x)

$$3.44 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} dx$$

Optimal. Leaf size=76

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}} + \frac{9x}{14a\sqrt[3]{a+bx^3}}$$

[Out] 1/7\*x\*(-b\*x^3+a)^2/a/(b\*x^3+a)^(7/3)+3/14\*x\*(-b\*x^3+a)/a/(b\*x^3+a)^(4/3)+9/14\*x/a/(b\*x^3+a)^(1/3)

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$ , Rules used = {378, 191}

$$\frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}} + \frac{9x}{14a\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(10/3), x]

[Out] (x\*(a - b\*x^3)^2)/(7\*a\*(a + b\*x^3)^(7/3)) + (3\*x\*(a - b\*x^3))/(14\*a\*(a + b\*x^3)^(4/3)) + (9\*x)/(14\*a\*(a + b\*x^3)^(1/3))

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(a-bx^3)^2}{(a+bx^3)^{10/3}} dx &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{6}{7} \int \frac{a-bx^3}{(a+bx^3)^{7/3}} dx \\ &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}} + \frac{9}{14} \int \frac{1}{(a+bx^3)^{4/3}} dx \\ &= \frac{x(a-bx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{3x(a-bx^3)}{14a(a+bx^3)^{4/3}} + \frac{9x}{14a\sqrt[3]{a+bx^3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.53

$$\frac{x(7a^2 + 7abx^3 + 4b^2x^6)}{7a(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(10/3), x]

[Out] (x\*(7\*a^2 + 7\*a\*b\*x^3 + 4\*b^2\*x^6))/(7\*a\*(a + b\*x^3)^(7/3))

**fricas** [A] time = 0.86, size = 67, normalized size = 0.88

$$\frac{(4b^2x^7 + 7abx^4 + 7a^2x)(bx^3 + a)^{\frac{2}{3}}}{7(ab^3x^9 + 3a^2b^2x^6 + 3a^3bx^3 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(10/3), x, algorithm="fricas")

[Out] 1/7\*(4\*b^2\*x^7 + 7\*a\*b\*x^4 + 7\*a^2\*x)\*(b\*x^3 + a)^(2/3)/(a\*b^3\*x^9 + 3\*a^2\*b^2\*x^6 + 3\*a^3\*b\*x^3 + a^4)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(10/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(10/3), x)

**maple** [A] time = 0.04, size = 37, normalized size = 0.49

$$\frac{(4b^2x^6 + 7abx^3 + 7a^2)x}{7(bx^3 + a)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(10/3), x)

[Out] 1/7\*x\*(4\*b^2\*x^6+7\*a\*b\*x^3+7\*a^2)/(b\*x^3+a)^(7/3)/a

**maxima** [A] time = 0.50, size = 105, normalized size = 1.38

$$\frac{\left(4b - \frac{7(bx^3+a)}{x^3}\right)bx^7}{14(bx^3 + a)^{\frac{7}{3}}a} + \frac{b^2x^7}{7(bx^3 + a)^{\frac{7}{3}}a} + \frac{\left(2b^2 - \frac{7(bx^3+a)b}{x^3} + \frac{14(bx^3+a)^2}{x^6}\right)x^7}{14(bx^3 + a)^{\frac{7}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(10/3), x, algorithm="maxima")

[Out] 1/14\*(4\*b - 7\*(b\*x^3 + a)/x^3)\*b\*x^7/((b\*x^3 + a)^(7/3)\*a) + 1/7\*b^2\*x^7/((b\*x^3 + a)^(7/3)\*a) + 1/14\*(2\*b^2 - 7\*(b\*x^3 + a)\*b/x^3 + 14\*(b\*x^3 + a)^2/x^6)\*x^7/((b\*x^3 + a)^(7/3)\*a)

**mupad** [B] time = 1.43, size = 44, normalized size = 0.58

$$\frac{4x(bx^3 + a)^2 + 4a^2x - ax(bx^3 + a)}{7a(bx^3 + a)^{\frac{7}{3}}}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^3)^2/(a + b*x^3)^(10/3),x)
```

```
[Out] (4*x*(a + b*x^3)^2 + 4*a^2*x - a*x*(a + b*x^3))/(7*a*(a + b*x^3)^(7/3))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**3+a)**2/(b*x**3+a)**(10/3),x)
```

```
[Out] Timed out
```

$$3.45 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{13/3}} dx$$

**Optimal.** Leaf size=105

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

[Out]  $1/20*x*(-b*x^3+a)^3/a^2/(b*x^3+a)^{(10/3)}+19/140*x*(-b*x^3+a)^2/a^2/(b*x^3+a)^{(7/3)}+57/280*x*(-b*x^3+a)/a^2/(b*x^3+a)^{(4/3)}+171/280*x/a^2/(b*x^3+a)^{(1/3)}$

**Rubi [A]** time = 0.04, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$ , Rules used = {382, 378, 191}

$$\frac{x(a-bx^3)^3}{20a^2(a+bx^3)^{10/3}} + \frac{19x(a-bx^3)^2}{140a^2(a+bx^3)^{7/3}} + \frac{57x(a-bx^3)}{280a^2(a+bx^3)^{4/3}} + \frac{171x}{280a^2\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(13/3), x]

[Out]  $(x*(a - b*x^3)^3)/(20*a^2*(a + b*x^3)^{(10/3)}) + (19*x*(a - b*x^3)^2)/(140*a^2*(a + b*x^3)^{(7/3)}) + (57*x*(a - b*x^3))/(280*a^2*(a + b*x^3)^{(4/3)}) + (171*x)/(280*a^2*(a + b*x^3)^{(1/3)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{13/3}} dx &= \frac{x(a - bx^3)^3}{20a^2(a + bx^3)^{10/3}} + \frac{19 \int \frac{(a - bx^3)^2}{(a + bx^3)^{10/3}} dx}{20a} \\
&= \frac{x(a - bx^3)^3}{20a^2(a + bx^3)^{10/3}} + \frac{19x(a - bx^3)^2}{140a^2(a + bx^3)^{7/3}} + \frac{57 \int \frac{a - bx^3}{(a + bx^3)^{7/3}} dx}{70a} \\
&= \frac{x(a - bx^3)^3}{20a^2(a + bx^3)^{10/3}} + \frac{19x(a - bx^3)^2}{140a^2(a + bx^3)^{7/3}} + \frac{57x(a - bx^3)}{280a^2(a + bx^3)^{4/3}} + \frac{171 \int \frac{1}{(a + bx^3)^{4/3}} dx}{280a} \\
&= \frac{x(a - bx^3)^3}{20a^2(a + bx^3)^{10/3}} + \frac{19x(a - bx^3)^2}{140a^2(a + bx^3)^{7/3}} + \frac{57x(a - bx^3)}{280a^2(a + bx^3)^{4/3}} + \frac{171x}{280a^2 \sqrt[3]{a + bx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 51, normalized size = 0.49

$$\frac{x(140a^3 + 245a^2bx^3 + 230ab^2x^6 + 69b^3x^9)}{140a^2(a + bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(13/3), x]

[Out] (x\*(140\*a^3 + 245\*a^2\*b\*x^3 + 230\*a\*b^2\*x^6 + 69\*b^3\*x^9))/(140\*a^2\*(a + b\*x^3)^(10/3))

**fricas [A]** time = 0.91, size = 91, normalized size = 0.87

$$\frac{(69b^3x^{10} + 230ab^2x^7 + 245a^2bx^4 + 140a^3x)(bx^3 + a)^{\frac{2}{3}}}{140(a^2b^4x^{12} + 4a^3b^3x^9 + 6a^4b^2x^6 + 4a^5bx^3 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(13/3), x, algorithm="fricas")

[Out] 1/140\*(69\*b^3\*x^10 + 230\*a\*b^2\*x^7 + 245\*a^2\*b\*x^4 + 140\*a^3\*x)\*(b\*x^3 + a)^(2/3)/(a^2\*b^4\*x^12 + 4\*a^3\*b^3\*x^9 + 6\*a^4\*b^2\*x^6 + 4\*a^5\*b\*x^3 + a^6)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{13}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(13/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(13/3), x)

**maple [A]** time = 0.05, size = 48, normalized size = 0.46

$$\frac{(69b^3x^9 + 230ab^2x^6 + 245a^2bx^3 + 140a^3)x}{140(bx^3 + a)^{\frac{10}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)^2/(b*x^3+a)^(13/3),x)`

[Out]  $1/140*x*(69*b^3*x^9+230*a*b^2*x^6+245*a^2*b*x^3+140*a^3)/(b*x^3+a)^(10/3)/a^2$

**maxima** [A] time = 0.59, size = 155, normalized size = 1.48

$$\frac{\left(7b - \frac{10(bx^3+a)}{x^3}\right)b^2x^{10}}{70(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14b^2 - \frac{40(bx^3+a)b}{x^3} + \frac{35(bx^3+a)^2}{x^6}\right)b^2x^{10}}{70(bx^3+a)^{\frac{10}{3}}a^2} - \frac{\left(14b^3 - \frac{60(bx^3+a)b^2}{x^3} + \frac{105(bx^3+a)^2b}{x^6} - \frac{140(bx^3+a)^3}{x^9}\right)x^{10}}{140(bx^3+a)^{\frac{10}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2/(b*x^3+a)^(13/3),x, algorithm="maxima")`

[Out]  $-1/70*(7*b - 10*(b*x^3 + a)/x^3)*b^2*x^{10}/((b*x^3 + a)^(10/3)*a^2) - 1/70*(14*b^2 - 40*(b*x^3 + a)*b/x^3 + 35*(b*x^3 + a)^2/x^6)*b*x^{10}/((b*x^3 + a)^(10/3)*a^2) - 1/140*(14*b^3 - 60*(b*x^3 + a)*b^2/x^3 + 105*(b*x^3 + a)^2*b/x^6 - 140*(b*x^3 + a)^3/x^9)*x^{10}/((b*x^3 + a)^(10/3)*a^2)$

**mupad** [B] time = 1.39, size = 56, normalized size = 0.53

$$\frac{69x}{140a^2(bx^3+a)^{1/3}} - \frac{2x}{35(bx^3+a)^{7/3}} + \frac{23x}{140a(bx^3+a)^{4/3}} + \frac{2ax}{5(bx^3+a)^{10/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)^2/(a + b*x^3)^(13/3),x)`

[Out]  $(69*x)/(140*a^2*(a + b*x^3)^(1/3)) - (2*x)/(35*(a + b*x^3)^(7/3)) + (23*x)/(140*a*(a + b*x^3)^(4/3)) + (2*a*x)/(5*(a + b*x^3)^(10/3))$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2/(b*x**3+a)**(13/3),x)`

[Out] Timed out

$$3.46 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{16/3}} dx$$

**Optimal.** Leaf size=98

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

[Out] 2/13\*x\*(-b\*x^3+a)/(b\*x^3+a)^(13/3)+8/65\*x/(b\*x^3+a)^(10/3)+47/455\*x/a/(b\*x^3+a)^(7/3)+141/910\*x/a^2/(b\*x^3+a)^(4/3)+423/910\*x/a^3/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.04, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 385, 192, 191}

$$\frac{423x}{910a^3\sqrt[3]{a+bx^3}} + \frac{141x}{910a^2(a+bx^3)^{4/3}} + \frac{47x}{455a(a+bx^3)^{7/3}} + \frac{8x}{65(a+bx^3)^{10/3}} + \frac{2x(a-bx^3)}{13(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(16/3), x]

[Out] (2\*x\*(a - b\*x^3))/(13\*(a + b\*x^3)^(13/3)) + (8\*x)/(65\*(a + b\*x^3)^(10/3)) + (47\*x)/(455\*a\*(a + b\*x^3)^(7/3)) + (141\*x)/(910\*a^2\*(a + b\*x^3)^(4/3)) + (423\*x)/(910\*a^3\*(a + b\*x^3)^(1/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{16/3}} dx &= \frac{2x(a - bx^3)}{13(a + bx^3)^{13/3}} + \frac{\int \frac{11a^2b - 5ab^2x^3}{(a + bx^3)^{13/3}} dx}{13ab} \\
&= \frac{2x(a - bx^3)}{13(a + bx^3)^{13/3}} + \frac{8x}{65(a + bx^3)^{10/3}} + \frac{47}{65} \int \frac{1}{(a + bx^3)^{10/3}} dx \\
&= \frac{2x(a - bx^3)}{13(a + bx^3)^{13/3}} + \frac{8x}{65(a + bx^3)^{10/3}} + \frac{47x}{455a(a + bx^3)^{7/3}} + \frac{282}{455a} \int \frac{1}{(a + bx^3)^{7/3}} dx \\
&= \frac{2x(a - bx^3)}{13(a + bx^3)^{13/3}} + \frac{8x}{65(a + bx^3)^{10/3}} + \frac{47x}{455a(a + bx^3)^{7/3}} + \frac{141x}{910a^2(a + bx^3)^{4/3}} + \frac{423}{910a^3} \int \frac{1}{(a + bx^3)} dx \\
&= \frac{2x(a - bx^3)}{13(a + bx^3)^{13/3}} + \frac{8x}{65(a + bx^3)^{10/3}} + \frac{47x}{455a(a + bx^3)^{7/3}} + \frac{141x}{910a^2(a + bx^3)^{4/3}} + \frac{423x}{910a^3\sqrt[3]{a + bx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 62, normalized size = 0.63

$$\frac{x(910a^4 + 2275a^3bx^3 + 3055a^2b^2x^6 + 1833ab^3x^9 + 423b^4x^{12})}{910a^3(a + bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(16/3), x]

[Out] (x\*(910\*a^4 + 2275\*a^3\*b\*x^3 + 3055\*a^2\*b^2\*x^6 + 1833\*a\*b^3\*x^9 + 423\*b^4\*x^12))/(910\*a^3\*(a + b\*x^3)^(13/3))

**fricas [A]** time = 0.91, size = 113, normalized size = 1.15

$$\frac{(423 b^4 x^{13} + 1833 a b^3 x^{10} + 3055 a^2 b^2 x^7 + 2275 a^3 b x^4 + 910 a^4 x)(b x^3 + a)^{\frac{2}{3}}}{910 (a^3 b^5 x^{15} + 5 a^4 b^4 x^{12} + 10 a^5 b^3 x^9 + 10 a^6 b^2 x^6 + 5 a^7 b x^3 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(16/3), x, algorithm="fricas")

[Out] 1/910\*(423\*b^4\*x^13 + 1833\*a\*b^3\*x^10 + 3055\*a^2\*b^2\*x^7 + 2275\*a^3\*b\*x^4 + 910\*a^4\*x)\*(b\*x^3 + a)^(2/3)/(a^3\*b^5\*x^15 + 5\*a^4\*b^4\*x^12 + 10\*a^5\*b^3\*x^9 + 10\*a^6\*b^2\*x^6 + 5\*a^7\*b\*x^3 + a^8)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{16}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(16/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(16/3), x)

**maple [A]** time = 0.05, size = 59, normalized size = 0.60

$$\frac{(423b^4x^{12} + 1833b^3x^9a + 3055b^2x^6a^2 + 2275bx^3a^3 + 910a^4)x}{910(bx^3 + a)^{\frac{13}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(16/3), x)

[Out] 1/910\*x\*(423\*b^4\*x^12+1833\*a\*b^3\*x^9+3055\*a^2\*b^2\*x^6+2275\*a^3\*b\*x^3+910\*a^4)/(b\*x^3+a)^(13/3)/a^3

**maxima [B]** time = 0.57, size = 206, normalized size = 2.10

$$\frac{\left(35b^2 - \frac{91(bx^3+a)b}{x^3} + \frac{65(bx^3+a)^2}{x^6}\right)b^2x^{13}}{455(bx^3 + a)^{\frac{13}{3}}a^3} + \frac{\left(140b^3 - \frac{546(bx^3+a)b^2}{x^3} + \frac{780(bx^3+a)^2b}{x^6} - \frac{455(bx^3+a)^3}{x^9}\right)bx^{13}}{910(bx^3 + a)^{\frac{13}{3}}a^3} + \frac{\left(35b^4 - \frac{182(bx^3+a)b^3}{x^3} + \frac{390(bx^3+a)^2b^2}{x^6} - \frac{455(bx^3+a)^3b}{x^9} + \frac{455(bx^3+a)^4}{x^{12}}\right)x^{13}}{910(bx^3 + a)^{\frac{13}{3}}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(16/3), x, algorithm="maxima")

[Out] 1/455\*(35\*b^2 - 91\*(b\*x^3 + a)\*b/x^3 + 65\*(b\*x^3 + a)^2/x^6)\*b^2\*x^13/((b\*x^3 + a)^(13/3)\*a^3) + 1/910\*(140\*b^3 - 546\*(b\*x^3 + a)\*b^2/x^3 + 780\*(b\*x^3 + a)^2\*b/x^6 - 455\*(b\*x^3 + a)^3/x^9)\*b\*x^13/((b\*x^3 + a)^(13/3)\*a^3) + 1/455\*(35\*b^4 - 182\*(b\*x^3 + a)\*b^3/x^3 + 390\*(b\*x^3 + a)^2\*b^2/x^6 - 455\*(b\*x^3 + a)^3\*b/x^9 + 455\*(b\*x^3 + a)^4/x^12)\*x^13/((b\*x^3 + a)^(13/3)\*a^3)

**mupad [B]** time = 1.44, size = 71, normalized size = 0.72

$$\frac{423x}{910a^3(bx^3 + a)^{1/3}} - \frac{2x}{65(bx^3 + a)^{10/3}} + \frac{141x}{910a^2(bx^3 + a)^{4/3}} + \frac{47x}{455a(bx^3 + a)^{7/3}} + \frac{4ax}{13(bx^3 + a)^{13/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(16/3), x)

[Out] (423\*x)/(910\*a^3\*(a + b\*x^3)^(1/3)) - (2\*x)/(65\*(a + b\*x^3)^(10/3)) + (141\*x)/(910\*a^2\*(a + b\*x^3)^(4/3)) + (47\*x)/(455\*a\*(a + b\*x^3)^(7/3)) + (4\*a\*x)/(13\*(a + b\*x^3)^(13/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(16/3), x)

[Out] Timed out

$$3.47 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{19/3}} dx$$

**Optimal.** Leaf size=117

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

[Out] 1/8\*x\*(-b\*x^3+a)/(b\*x^3+a)^(16/3)+11/104\*x/(b\*x^3+a)^(13/3)+1/13\*x/a/(b\*x^3+a)^(10/3)+9/91\*x/a^2/(b\*x^3+a)^(7/3)+27/182\*x/a^3/(b\*x^3+a)^(4/3)+81/182\*x/a^4/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.04, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 385, 192, 191}

$$\frac{81x}{182a^4\sqrt[3]{a+bx^3}} + \frac{27x}{182a^3(a+bx^3)^{4/3}} + \frac{9x}{91a^2(a+bx^3)^{7/3}} + \frac{x}{13a(a+bx^3)^{10/3}} + \frac{11x}{104(a+bx^3)^{13/3}} + \frac{x(a-bx^3)}{8(a+bx^3)^{16/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(19/3), x]

[Out] (x\*(a - b\*x^3))/(8\*(a + b\*x^3)^(16/3)) + (11\*x)/(104\*(a + b\*x^3)^(13/3)) + x/(13\*a\*(a + b\*x^3)^(10/3)) + (9\*x)/(91\*a^2\*(a + b\*x^3)^(7/3)) + (27\*x)/(182\*a^3\*(a + b\*x^3)^(4/3)) + (81\*x)/(182\*a^4\*(a + b\*x^3)^(1/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps



$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{19/3}} dx &= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{\int \frac{14a^2b - 8ab^2x^3}{(a + bx^3)^{16/3}} dx}{16ab} \\
&= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{11x}{104(a + bx^3)^{13/3}} + \frac{10}{13} \int \frac{1}{(a + bx^3)^{13/3}} dx \\
&= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{11x}{104(a + bx^3)^{13/3}} + \frac{x}{13a(a + bx^3)^{10/3}} + \frac{9 \int \frac{1}{(a + bx^3)^{10/3}} dx}{13a} \\
&= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{11x}{104(a + bx^3)^{13/3}} + \frac{x}{13a(a + bx^3)^{10/3}} + \frac{9x}{91a^2(a + bx^3)^{7/3}} + \frac{54 \int \frac{1}{(a + bx^3)^{7/3}} dx}{91a^2} \\
&= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{11x}{104(a + bx^3)^{13/3}} + \frac{x}{13a(a + bx^3)^{10/3}} + \frac{9x}{91a^2(a + bx^3)^{7/3}} + \frac{27}{182a^3(a + bx^3)^{4/3}} \\
&= \frac{x(a - bx^3)}{8(a + bx^3)^{16/3}} + \frac{11x}{104(a + bx^3)^{13/3}} + \frac{x}{13a(a + bx^3)^{10/3}} + \frac{9x}{91a^2(a + bx^3)^{7/3}} + \frac{27}{182a^3(a + bx^3)^{4/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 73, normalized size = 0.62

$$\frac{x(364a^5 + 1183a^4bx^3 + 2080a^3b^2x^6 + 1872a^2b^3x^9 + 864ab^4x^{12} + 162b^5x^{15})}{364a^4(a + bx^3)^{16/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(19/3), x]

[Out] (x\*(364\*a^5 + 1183\*a^4\*b\*x^3 + 2080\*a^3\*b^2\*x^6 + 1872\*a^2\*b^3\*x^9 + 864\*a\*b^4\*x^12 + 162\*b^5\*x^15))/(364\*a^4\*(a + b\*x^3)^(16/3))

**fricas [A]** time = 0.99, size = 135, normalized size = 1.15

$$\frac{(162b^5x^{16} + 864ab^4x^{13} + 1872a^2b^3x^{10} + 2080a^3b^2x^7 + 1183a^4bx^4 + 364a^5x)(bx^3 + a)^{2/3}}{364(a^4b^6x^{18} + 6a^5b^5x^{15} + 15a^6b^4x^{12} + 20a^7b^3x^9 + 15a^8b^2x^6 + 6a^9bx^3 + a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(19/3), x, algorithm="fricas")

[Out] 1/364\*(162\*b^5\*x^16 + 864\*a\*b^4\*x^13 + 1872\*a^2\*b^3\*x^10 + 2080\*a^3\*b^2\*x^7 + 1183\*a^4\*b\*x^4 + 364\*a^5\*x)\*(b\*x^3 + a)^(2/3)/(a^4\*b^6\*x^18 + 6\*a^5\*b^5\*x^15 + 15\*a^6\*b^4\*x^12 + 20\*a^7\*b^3\*x^9 + 15\*a^8\*b^2\*x^6 + 6\*a^9\*b\*x^3 + a^10)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{19/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(19/3),x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(19/3), x)

**maple [A]** time = 0.05, size = 70, normalized size = 0.60

$$\frac{(162b^5x^{15} + 864ab^4x^{12} + 1872a^2b^3x^9 + 2080a^3b^2x^6 + 1183a^4bx^3 + 364a^5)x}{364(bx^3 + a)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(19/3),x)

[Out] 1/364\*x\*(162\*b^5\*x^15+864\*a\*b^4\*x^12+1872\*a^2\*b^3\*x^9+2080\*a^3\*b^2\*x^6+1183\*a^4\*b\*x^3+364\*a^5)/(b\*x^3+a)^(16/3)/a^4

**maxima [B]** time = 0.51, size = 257, normalized size = 2.20

$$\frac{\left(455b^3 - \frac{1680(bx^3+a)b^2}{x^3} + \frac{2184(bx^3+a)^2b}{x^6} - \frac{1040(bx^3+a)^3}{x^9}\right)b^2x^{16}}{7280(bx^3 + a)^{\frac{16}{3}}a^4} - \frac{\left(455b^4 - \frac{2240(bx^3+a)b^3}{x^3} + \frac{4368(bx^3+a)^2b^2}{x^6} - \frac{4160(bx^3+a)^3}{x^9}\right)}{3640(bx^3 + a)^{\frac{16}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(19/3),x, algorithm="maxima")

[Out] -1/7280\*(455\*b^3 - 1680\*(b\*x^3 + a)\*b^2/x^3 + 2184\*(b\*x^3 + a)^2\*b/x^6 - 1040\*(b\*x^3 + a)^3/x^9)\*b^2\*x^16/((b\*x^3 + a)^(16/3)\*a^4) - 1/3640\*(455\*b^4 - 2240\*(b\*x^3 + a)\*b^3/x^3 + 4368\*(b\*x^3 + a)^2\*b^2/x^6 - 4160\*(b\*x^3 + a)^3\*b/x^9 + 1820\*(b\*x^3 + a)^4/x^12)\*b\*x^16/((b\*x^3 + a)^(16/3)\*a^4) - 1/1456\*(91\*b^5 - 560\*(b\*x^3 + a)\*b^4/x^3 + 1456\*(b\*x^3 + a)^2\*b^3/x^6 - 2080\*(b\*x^3 + a)^3\*b^2/x^9 + 1820\*(b\*x^3 + a)^4\*b/x^12 - 1456\*(b\*x^3 + a)^5/x^15)\*x^16/((b\*x^3 + a)^(16/3)\*a^4)

**mupad [B]** time = 1.43, size = 86, normalized size = 0.74

$$\frac{81x}{182a^4(bx^3 + a)^{1/3}} - \frac{x}{52(bx^3 + a)^{13/3}} + \frac{27x}{182a^3(bx^3 + a)^{4/3}} + \frac{9x}{91a^2(bx^3 + a)^{7/3}} + \frac{x}{13a(bx^3 + a)^{10/3}} + \frac{ax}{4(bx^3 + a)^{16/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(19/3),x)

[Out] (81\*x)/(182\*a^4\*(a + b\*x^3)^(1/3)) - x/(52\*(a + b\*x^3)^(13/3)) + (27\*x)/(182\*a^3\*(a + b\*x^3)^(4/3)) + (9\*x)/(91\*a^2\*(a + b\*x^3)^(7/3)) + x/(13\*a\*(a + b\*x^3)^(10/3)) + (a\*x)/(4\*(a + b\*x^3)^(16/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(19/3),x)

[Out] Timed out

$$3.48 \quad \int (a - bx^3)^2 (a + bx^3)^{4/3} dx$$

**Optimal.** Leaf size=94

$$\frac{57a^3x\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{9}{44}ax(a+bx^3)^{7/3} - \frac{1}{11}x(a-bx^3)(a+bx^3)^{7/3}$$

[Out]  $-9/44*a*x*(b*x^3+a)^{(7/3)}-1/11*x*(-b*x^3+a)*(b*x^3+a)^{(7/3)}+57/44*a^3*x*(b*x^3+a)^{(1/3)}*\text{hypergeom}([-4/3, 1/3], [4/3], -b*x^3/a)/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {416, 388, 246, 245}

$$\frac{57a^3x\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{44\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{9}{44}ax(a+bx^3)^{7/3} - \frac{1}{11}x(a-bx^3)(a+bx^3)^{7/3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2\*(a + b\*x^3)^(4/3), x]

[Out]  $(-9*a*x*(a + b*x^3)^{(7/3)})/44 - (x*(a - b*x^3)*(a + b*x^3)^{(7/3)})/11 + (57*a^3*x*(a + b*x^3)^{(1/3)}*\text{Hypergeometric2F1}[-4/3, 1/3, 4/3, -(b*x^3)/a])/ (4* (1 + (b*x^3)/a)^{(1/3)})$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int (a - bx^3)^2 (a + bx^3)^{4/3} dx &= -\frac{1}{11}x(a - bx^3)(a + bx^3)^{7/3} + \frac{\int (a + bx^3)^{4/3} (12a^2b - 18ab^2x^3) dx}{11b} \\
&= -\frac{9}{44}ax(a + bx^3)^{7/3} - \frac{1}{11}x(a - bx^3)(a + bx^3)^{7/3} + \frac{1}{44}(57a^2) \int (a + bx^3)^{4/3} dx \\
&= -\frac{9}{44}ax(a + bx^3)^{7/3} - \frac{1}{11}x(a - bx^3)(a + bx^3)^{7/3} + \frac{\left(57a^3 \sqrt[3]{a + bx^3}\right) \int \left(1 + \frac{bx^3}{a}\right)^{4/3}}{44 \sqrt[3]{1 + \frac{bx^3}{a}}} \\
&= -\frac{9}{44}ax(a + bx^3)^{7/3} - \frac{1}{11}x(a - bx^3)(a + bx^3)^{7/3} + \frac{57a^3x \sqrt[3]{a + bx^3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{44 \sqrt[3]{1 + \frac{bx^3}{a}}}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 97, normalized size = 1.03

$$\frac{x \left( 114a^4 \left( \frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + 106a^4 + 53a^3bx^3 - 78a^2b^2x^6 - 5ab^3x^9 + 20b^4x^{12} \right)}{220(a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2\*(a + b\*x^3)^(4/3), x]

[Out] (x\*(106\*a^4 + 53\*a^3\*b\*x^3 - 78\*a^2\*b^2\*x^6 - 5\*a\*b^3\*x^9 + 20\*b^4\*x^12 + 14\*a^4\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(220\*(a + b\*x^3)^(2/3))

**fricas** [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3x^9 - ab^2x^6 - a^2bx^3 + a^3\right)(bx^3 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(4/3), x, algorithm="fricas")

[Out] integral((b^3\*x^9 - a\*b^2\*x^6 - a^2\*b\*x^3 + a^3)\*(b\*x^3 + a)^(1/3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}}(bx^3 - a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(4/3), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)\*(b\*x^3 - a)^2, x)

**maple** [F] time = 0.41, size = 0, normalized size = 0.00

$$\int (-bx^3 + a)^2 (bx^3 + a)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2\*(b\*x^3+a)^(4/3), x)

[Out] `int((-b*x^3+a)^2*(b*x^3+a)^(4/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}} (bx^3 - a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2*(b*x^3+a)^(4/3),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(4/3)*(b*x^3 - a)^2, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{\frac{4}{3}} (a - bx^3)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(4/3)*(a - b*x^3)^2,x)`

[Out] `int((a + b*x^3)^(4/3)*(a - b*x^3)^2, x)`

**sympy** [C] time = 8.41, size = 168, normalized size = 1.79

$$\frac{a^{\frac{10}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{a^{\frac{7}{3}} bx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} - \frac{a^{\frac{4}{3}} b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a} b^3 x^{10} \Gamma\left(\frac{10}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2*(b*x**3+a)**(4/3),x)`

[Out] `a**(10/3)*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - a**(7/3)*b*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) - a**(4/3)*b**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b**3*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3))`

### 3.49 $\int (a - bx^3)^2 \sqrt[3]{a + bx^3} dx$

**Optimal.** Leaf size=94

$$\frac{3a^2x\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{3}{8}ax(a+bx^3)^{4/3} - \frac{1}{8}x(a-bx^3)(a+bx^3)^{4/3}$$

[Out]  $-3/8*a*x*(b*x^3+a)^{(4/3)}-1/8*x*(-b*x^3+a)*(b*x^3+a)^{(4/3)}+3/2*a^2*x*(b*x^3+a)^{(1/3)}*\text{hypergeom}([-1/3, 1/3], [4/3], -b*x^3/a)/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {416, 388, 246, 245}

$$\frac{3a^2x\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2\sqrt[3]{\frac{bx^3}{a}+1}} - \frac{3}{8}ax(a+bx^3)^{4/3} - \frac{1}{8}x(a-bx^3)(a+bx^3)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2\*(a + b\*x^3)^(1/3), x]

[Out]  $(-3*a*x*(a+b*x^3)^{(4/3)})/8 - (x*(a-b*x^3)*(a+b*x^3)^{(4/3)})/8 + (3*a^2*x*(a+b*x^3)^{(1/3)}*\text{Hypergeometric2F1}[-1/3, 1/3, 4/3, -(b*x^3/a)])/(2*(1+(b*x^3/a)^{(1/3)})$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int (a - bx^3)^2 \sqrt[3]{a + bx^3} dx &= -\frac{1}{8}x(a - bx^3)(a + bx^3)^{4/3} + \frac{\int \sqrt[3]{a + bx^3} (9a^2b - 15ab^2x^3) dx}{8b} \\
&= -\frac{3}{8}ax(a + bx^3)^{4/3} - \frac{1}{8}x(a - bx^3)(a + bx^3)^{4/3} + \frac{1}{2}(3a^2) \int \sqrt[3]{a + bx^3} dx \\
&= -\frac{3}{8}ax(a + bx^3)^{4/3} - \frac{1}{8}x(a - bx^3)(a + bx^3)^{4/3} + \frac{(3a^2\sqrt[3]{a + bx^3}) \int \sqrt[3]{1 + \frac{bx^3}{a}} dx}{2\sqrt[3]{1 + \frac{bx^3}{a}}} \\
&= -\frac{3}{8}ax(a + bx^3)^{4/3} - \frac{1}{8}x(a - bx^3)(a + bx^3)^{4/3} + \frac{3a^2x\sqrt[3]{a + bx^3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2\sqrt[3]{1 + \frac{bx^3}{a}}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 85, normalized size = 0.90

$$\frac{x \left( 6a^3 \left( \frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) + 2a^3 - a^2bx^3 - 2ab^2x^6 + b^3x^9 \right)}{8(a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2\*(a + b\*x^3)^(1/3), x]

[Out] (x\*(2\*a^3 - a^2\*b\*x^3 - 2\*a\*b^2\*x^6 + b^3\*x^9 + 6\*a^3\*(1 + (b\*x^3)/a)^(2/3) \*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(8\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral} \left( (b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{\frac{1}{3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(1/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}}(bx^3 - a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(1/3), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)\*(b\*x^3 - a)^2, x)

**maple [F]** time = 0.38, size = 0, normalized size = 0.00

$$\int (-bx^3 + a)^2 (bx^3 + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2\*(b\*x^3+a)^(1/3), x)

[Out] int((-b\*x^3+a)^2\*(b\*x^3+a)^(1/3), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}}(bx^3 - a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2\*(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/3)\*(b\*x^3 - a)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{1/3} (a - bx^3)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)\*(a - b\*x^3)^2,x)

[Out] int((a + b\*x^3)^(1/3)\*(a - b\*x^3)^2, x)

**sympy** [C] time = 5.59, size = 126, normalized size = 1.34

$$\frac{a^{\frac{7}{3}}x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} - \frac{2a^{\frac{4}{3}}bx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}b^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2\*(b\*x\*\*3+a)\*\*(1/3),x)

[Out] a\*\*(7/3)\*x\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) - 2\*a\*\*(4/3)\*b\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(1/3)\*b\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3))



$$3.50 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{2/3}} dx$$

**Optimal.** Leaf size=94

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} - \frac{6}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

[Out]  $-6/5*a*x*(b*x^3+a)^{(1/3)}-1/5*x*(-b*x^3+a)*(b*x^3+a)^{(1/3)}+12/5*a^2*x*(1+b*x^3/a)^{(2/3)}*\text{hypergeom}([1/3, 2/3], [4/3], -b*x^3/a)/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {416, 388, 246, 245}

$$\frac{12a^2x\left(\frac{bx^3}{a}+1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} - \frac{6}{5}ax\sqrt[3]{a+bx^3} - \frac{1}{5}x(a-bx^3)\sqrt[3]{a+bx^3}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(2/3), x]

[Out]  $(-6*a*x*(a + b*x^3)^{(1/3)})/5 - (x*(a - b*x^3)*(a + b*x^3)^{(1/3)})/5 + (12*a^2*x*(1 + (b*x^3)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/3, 2/3, 4/3, -((b*x^3)/a)])/(5*(a + b*x^3)^{(2/3)})$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{2/3}} dx &= -\frac{1}{5}x(a - bx^3)\sqrt[3]{a + bx^3} + \frac{\int \frac{6a^2b - 12ab^2x^3}{(a + bx^3)^{2/3}} dx}{5b} \\
&= -\frac{6}{5}ax\sqrt[3]{a + bx^3} - \frac{1}{5}x(a - bx^3)\sqrt[3]{a + bx^3} + \frac{1}{5}(12a^2) \int \frac{1}{(a + bx^3)^{2/3}} dx \\
&= -\frac{6}{5}ax\sqrt[3]{a + bx^3} - \frac{1}{5}x(a - bx^3)\sqrt[3]{a + bx^3} + \frac{\left(12a^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{5(a + bx^3)^{2/3}} \\
&= -\frac{6}{5}ax\sqrt[3]{a + bx^3} - \frac{1}{5}x(a - bx^3)\sqrt[3]{a + bx^3} + \frac{12a^2x \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 75, normalized size = 0.80

$$\frac{12a^2x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) - 7a^2x - 6abx^4 + b^2x^7}{5(a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(2/3), x]

[Out] (-7\*a^2\*x - 6\*a\*b\*x^4 + b^2\*x^7 + 12\*a^2\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(5\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^6 - 2abx^3 + a^2}{(bx^3 + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(2/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)/(b\*x^3 + a)^(2/3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(2/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(2/3), x)

**maple [F]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)`

[Out] `int((-b*x^3+a)^2/(b*x^3+a)^(2/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2/(b*x^3+a)^(2/3),x, algorithm="maxima")`

[Out] `integrate((b*x^3 - a)^2/(b*x^3 + a)^(2/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)^2/(a + b*x^3)^(2/3),x)`

[Out] `int((a - b*x^3)^2/(a + b*x^3)^(2/3), x)`

**sympy** [C] time = 5.76, size = 121, normalized size = 1.29

$$\frac{a^{\frac{4}{3}} x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{4}{3}\right)} - \frac{2 \sqrt[3]{a} b x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3 \Gamma\left(\frac{7}{3}\right)} + \frac{b^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{7}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3 a^{\frac{2}{3}} \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2/(b*x**3+a)**(2/3),x)`

[Out] `a**(4/3)*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) - 2*a**(1/3)*b*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + b**2*x**7*gamma(7/3)*hyper((2/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(10/3))`

$$3.51 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{5/3}} dx$$

**Optimal.** Leaf size=74

$$\frac{3bx^4 \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4(a+bx^3)^{2/3}} + \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

[Out] x\*(-b\*x^3+a)/(b\*x^3+a)^(2/3)+3/4\*b\*x^4\*(1+b\*x^3/a)^(2/3)\*hypergeom([2/3, 4/3], [7/3], -b\*x^3/a)/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 12, 365, 364}

$$\frac{3bx^4 \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4(a+bx^3)^{2/3}} + \frac{x(a-bx^3)}{(a+bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(5/3), x]

[Out] (x\*(a - b\*x^3))/(a + b\*x^3)^(2/3) + (3\*b\*x^4\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[2/3, 4/3, 7/3, -((b\*x^3)/a)])/(4\*(a + b\*x^3)^(2/3))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 364

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(a^p\*(c\*x)^(m+1)\*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n+1, -((b\*x^n)/a)])/((c\*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

#### Rule 365

Int[((c\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^p\*IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(c\*x)^m\*(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && !(ILtQ[p, 0] || GtQ[a, 0])

#### Rule 413

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1)+1)) + d\*(a\*d\*(n\*(q-1)+1) - b\*c\*(n\*(p+q)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{5/3}} dx &= \frac{x(a - bx^3)}{(a + bx^3)^{2/3}} + \frac{\int \frac{6ab^2x^3}{(a+bx^3)^{2/3}} dx}{2ab} \\
&= \frac{x(a - bx^3)}{(a + bx^3)^{2/3}} + (3b) \int \frac{x^3}{(a + bx^3)^{2/3}} dx \\
&= \frac{x(a - bx^3)}{(a + bx^3)^{2/3}} + \frac{\left(3b \left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{x^3}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{(a + bx^3)^{2/3}} \\
&= \frac{x(a - bx^3)}{(a + bx^3)^{2/3}} + \frac{3bx^4 \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right)}{4(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 62, normalized size = 0.84

$$\frac{-3ax \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 5ax + bx^4}{2(a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(5/3), x]

[Out] (5\*a\*x + b\*x^4 - 3\*a\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(2\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{\frac{1}{3}}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(5/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3)/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(5/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(5/3), x)

**maple [F]** time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)`

[Out] `int((-b*x^3+a)^2/(b*x^3+a)^(5/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2/(b*x^3+a)^(5/3),x, algorithm="maxima")`

[Out] `integrate((b*x^3 - a)^2/(b*x^3 + a)^(5/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)^2/(a + b*x^3)^(5/3),x)`

[Out] `int((a - b*x^3)^2/(a + b*x^3)^(5/3), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-a + bx^3)^2}{(a + bx^3)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2/(b*x**3+a)**(5/3),x)`

[Out] `Integral((-a + b*x**3)**2/(a + b*x**3)**(5/3), x)`

$$3.52 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{8/3}} dx$$

**Optimal.** Leaf size=74

$$\frac{3x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} + \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

[Out] 2/5\*x\*(-b\*x^3+a)/(b\*x^3+a)^(5/3)+3/5\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 2/3], [4/3], -b\*x^3/a)/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 21, 246, 245}

$$\frac{3x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5(a+bx^3)^{2/3}} + \frac{2x(a-bx^3)}{5(a+bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(8/3), x]

[Out] (2\*x\*(a - b\*x^3))/(5\*(a + b\*x^3)^(5/3)) + (3\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(5\*(a + b\*x^3)^(2/3))

#### Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{8/3}} dx &= \frac{2x(a - bx^3)}{5(a + bx^3)^{5/3}} + \frac{\int \frac{3a^2b + 3ab^2x^3}{(a + bx^3)^{5/3}} dx}{5ab} \\
&= \frac{2x(a - bx^3)}{5(a + bx^3)^{5/3}} + \frac{3}{5} \int \frac{1}{(a + bx^3)^{2/3}} dx \\
&= \frac{2x(a - bx^3)}{5(a + bx^3)^{5/3}} + \frac{\left(3\left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{5(a + bx^3)^{2/3}} \\
&= \frac{2x(a - bx^3)}{5(a + bx^3)^{5/3}} + \frac{3x\left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{bx^3}{a}\right)}{5(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica** [A] time = 0.07, size = 70, normalized size = 0.95

$$\frac{3x(a + bx^3)\left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, -\frac{bx^3}{a}\right) + 2x(a - bx^3)}{5(a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(8/3), x]

[Out] (2\*x\*(a - b\*x^3) + 3\*x\*(a + b\*x^3)\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(5\*(a + b\*x^3)^(5/3))

**fricas** [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{1/3}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(8/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3)/(b^3\*x^9 + 3\*a\*b^2\*x^6 + 3\*a^2\*b\*x^3 + a^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{8/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(8/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(8/3), x)

**maple** [F] time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{8/3}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)`

[Out] `int((-b*x^3+a)^2/(b*x^3+a)^(8/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2/(b*x^3+a)^(8/3),x, algorithm="maxima")`

[Out] `integrate((b*x^3 - a)^2/(b*x^3 + a)^(8/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)^2/(a + b*x^3)^(8/3),x)`

[Out] `int((a - b*x^3)^2/(a + b*x^3)^(8/3), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2/(b*x**3+a)**(8/3),x)`

[Out] Timed out

$$3.53 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{11/3}} dx$$

**Optimal.** Leaf size=77

$$\frac{3x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}} + \frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

[Out] 1/4\*x\*(-b\*x^3+a)/(b\*x^3+a)^(8/3)+3/4\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 8/3], [4/3], -b\*x^3/a)/a/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 12, 246, 245}

$$\frac{3x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{4a(a+bx^3)^{2/3}} + \frac{x(a-bx^3)}{4(a+bx^3)^{8/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(11/3), x]

[Out] (x\*(a - b\*x^3))/(4\*(a + b\*x^3)^(8/3)) + (3\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 8/3, 4/3, -((b\*x^3)/a)]/(4\*a\*(a + b\*x^3)^(2/3))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{11/3}} dx &= \frac{x(a - bx^3)}{4(a + bx^3)^{8/3}} + \frac{\int \frac{6a^2b}{(a+bx^3)^{8/3}} dx}{8ab} \\
&= \frac{x(a - bx^3)}{4(a + bx^3)^{8/3}} + \frac{1}{4}(3a) \int \frac{1}{(a + bx^3)^{8/3}} dx \\
&= \frac{x(a - bx^3)}{4(a + bx^3)^{8/3}} + \frac{\left(3\left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{8/3}} dx}{4a(a + bx^3)^{2/3}} \\
&= \frac{x(a - bx^3)}{4(a + bx^3)^{8/3}} + \frac{3x\left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{4a(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.12, size = 85, normalized size = 1.10

$$\frac{7a^2x + 3x(a + bx^3)^2 \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 5abx^4 + 3b^2x^7}{10a(a + bx^3)^{8/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(11/3), x]

[Out] (7\*a^2\*x + 5\*a\*b\*x^4 + 3\*b^2\*x^7 + 3\*x\*(a + b\*x^3)^2\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(10\*a\*(a + b\*x^3)^(8/3))

**fricas [F]** time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{\frac{1}{3}}}{b^4x^{12} + 4ab^3x^9 + 6a^2b^2x^6 + 4a^3bx^3 + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(11/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3)/(b^4\*x^12 + 4\*a\*b^3\*x^9 + 6\*a^2\*b^2\*x^6 + 4\*a^3\*b\*x^3 + a^4), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(11/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(11/3), x)

**maple [F]** time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)`

[Out] `int((-b*x^3+a)^2/(b*x^3+a)^(11/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x^3+a)^2/(b*x^3+a)^(11/3),x, algorithm="maxima")`

[Out] `integrate((b*x^3 - a)^2/(b*x^3 + a)^(11/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{11}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^3)^2/(a + b*x^3)^(11/3),x)`

[Out] `int((a - b*x^3)^2/(a + b*x^3)^(11/3), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**3+a)**2/(b*x**3+a)**(11/3),x)`

[Out] Timed out

$$3.54 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{14/3}} dx$$

**Optimal.** Leaf size=93

$$\frac{15x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{22a^2 (a+bx^3)^{2/3}} + \frac{3x}{22(a+bx^3)^{8/3}} + \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

[Out] 2/11\*x\*(-b\*x^3+a)/(b\*x^3+a)^(11/3)+3/22\*x/(b\*x^3+a)^(8/3)+15/22\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 8/3], [4/3], -b\*x^3/a)/a^2/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 385, 246, 245}

$$\frac{15x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{22a^2 (a+bx^3)^{2/3}} + \frac{3x}{22(a+bx^3)^{8/3}} + \frac{2x(a-bx^3)}{11(a+bx^3)^{11/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(14/3), x]

[Out] (2\*x\*(a - b\*x^3))/(11\*(a + b\*x^3)^(11/3)) + (3\*x)/(22\*(a + b\*x^3)^(8/3)) + (15\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 8/3, 4/3, -((b\*x^3)/a)])/(22\*a^2\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{14/3}} dx &= \frac{2x(a - bx^3)}{11(a + bx^3)^{11/3}} + \frac{\int \frac{9a^2b - 3ab^2x^3}{(a + bx^3)^{11/3}} dx}{11ab} \\
&= \frac{2x(a - bx^3)}{11(a + bx^3)^{11/3}} + \frac{3x}{22(a + bx^3)^{8/3}} + \frac{15}{22} \int \frac{1}{(a + bx^3)^{8/3}} dx \\
&= \frac{2x(a - bx^3)}{11(a + bx^3)^{11/3}} + \frac{3x}{22(a + bx^3)^{8/3}} + \frac{\left(15 \left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{8/3}} dx}{22a^2(a + bx^3)^{2/3}} \\
&= \frac{2x(a - bx^3)}{11(a + bx^3)^{11/3}} + \frac{3x}{22(a + bx^3)^{8/3}} + \frac{15x \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{22a^2(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.13, size = 95, normalized size = 1.02

$$\frac{x \left( 16a^3 + 23a^2bx^3 + 21ab^2x^6 + 6(a + bx^3)^3 \left( \frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 6b^3x^9 \right)}{22a^2(a + bx^3)^{11/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(14/3), x]

[Out] (x\*(16\*a^3 + 23\*a^2\*b\*x^3 + 21\*a\*b^2\*x^6 + 6\*b^3\*x^9 + 6\*(a + b\*x^3)^3\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(22\*a^2\*(a + b\*x^3)^(11/3))

**fricas [F]** time = 0.65, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{\frac{1}{3}}}{b^5x^{15} + 5ab^4x^{12} + 10a^2b^3x^9 + 10a^3b^2x^6 + 5a^4bx^3 + a^5}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(14/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3)/(b^5\*x^15 + 5\*a\*b^4\*x^12 + 10\*a^2\*b^3\*x^9 + 10\*a^3\*b^2\*x^6 + 5\*a^4\*b\*x^3 + a^5), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(14/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(14/3), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(14/3), x)

[Out] int((-b\*x^3+a)^2/(b\*x^3+a)^(14/3), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(14/3), x, algorithm="maxima")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(14/3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{14}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(14/3), x)

[Out] int((a - b\*x^3)^2/(a + b\*x^3)^(14/3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(14/3), x)

[Out] Timed out

$$3.55 \quad \int \frac{(a-bx^3)^2}{(a+bx^3)^{17/3}} dx$$

**Optimal.** Leaf size=93

$$\frac{57x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{11}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{77a^3 (a+bx^3)^{2/3}} + \frac{9x}{77(a+bx^3)^{11/3}} + \frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

[Out] 1/7\*x\*(-b\*x^3+a)/(b\*x^3+a)^(14/3)+9/77\*x/(b\*x^3+a)^(11/3)+57/77\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 11/3], [4/3], -b\*x^3/a)/a^3/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$ , Rules used = {413, 385, 246, 245}

$$\frac{57x \left(\frac{bx^3}{a} + 1\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{11}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{77a^3 (a+bx^3)^{2/3}} + \frac{9x}{77(a+bx^3)^{11/3}} + \frac{x(a-bx^3)}{7(a+bx^3)^{14/3}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^3)^2/(a + b\*x^3)^(17/3), x]

[Out] (x\*(a - b\*x^3))/(7\*(a + b\*x^3)^(14/3)) + (9\*x)/(77\*(a + b\*x^3)^(11/3)) + (57\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 11/3, 4/3, -(b\*x^3)/a])/(77\*a^3\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]



Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^3)^2}{(a + bx^3)^{17/3}} dx &= \frac{x(a - bx^3)}{7(a + bx^3)^{14/3}} + \frac{\int \frac{12a^2b - 6ab^2x^3}{(a + bx^3)^{14/3}} dx}{14ab} \\
&= \frac{x(a - bx^3)}{7(a + bx^3)^{14/3}} + \frac{9x}{77(a + bx^3)^{11/3}} + \frac{57}{77} \int \frac{1}{(a + bx^3)^{11/3}} dx \\
&= \frac{x(a - bx^3)}{7(a + bx^3)^{14/3}} + \frac{9x}{77(a + bx^3)^{11/3}} + \frac{\left(57 \left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{11/3}} dx}{77a^3(a + bx^3)^{2/3}} \\
&= \frac{x(a - bx^3)}{7(a + bx^3)^{14/3}} + \frac{9x}{77(a + bx^3)^{11/3}} + \frac{57x \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{11}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{77a^3(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 106, normalized size = 1.14

$$\frac{x \left( 2282a^4 + 4879a^3bx^3 + 6270a^2b^2x^6 + 3591ab^3x^9 + 798(a + bx^3)^4 \left( \frac{bx^3}{a} + 1 \right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + 798b^4x^{12} \right)}{3080a^3(a + bx^3)^{14/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^3)^2/(a + b\*x^3)^(17/3), x]

[Out] (x\*(2282\*a^4 + 4879\*a^3\*b\*x^3 + 6270\*a^2\*b^2\*x^6 + 3591\*a\*b^3\*x^9 + 798\*b^4\*x^12 + 798\*(a + b\*x^3)^4\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(3080\*a^3\*(a + b\*x^3)^(14/3))

**fricas [F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(b^2x^6 - 2abx^3 + a^2)(bx^3 + a)^{1/3}}{b^6x^{18} + 6ab^5x^{15} + 15a^2b^4x^{12} + 20a^3b^3x^9 + 15a^4b^2x^6 + 6a^5bx^3 + a^6}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(17/3), x, algorithm="fricas")

[Out] integral((b^2\*x^6 - 2\*a\*b\*x^3 + a^2)\*(b\*x^3 + a)^(1/3)/(b^6\*x^18 + 6\*a\*b^5\*x^15 + 15\*a^2\*b^4\*x^12 + 20\*a^3\*b^3\*x^9 + 15\*a^4\*b^2\*x^6 + 6\*a^5\*b\*x^3 + a^6), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{17/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(17/3), x, algorithm="giac")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(17/3), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(-bx^3 + a)^2}{(bx^3 + a)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^3+a)^2/(b\*x^3+a)^(17/3),x)

[Out] int((-b\*x^3+a)^2/(b\*x^3+a)^(17/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 - a)^2}{(bx^3 + a)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^3+a)^2/(b\*x^3+a)^(17/3),x, algorithm="maxima")

[Out] integrate((b\*x^3 - a)^2/(b\*x^3 + a)^(17/3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a - bx^3)^2}{(bx^3 + a)^{\frac{17}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^3)^2/(a + b\*x^3)^(17/3),x)

[Out] int((a - b\*x^3)^2/(a + b\*x^3)^(17/3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*3+a)\*\*2/(b\*x\*\*3+a)\*\*(17/3),x)

[Out] Timed out

### 3.56 $\int (a + bx^3)^{5/3} (c + dx^3) dx$

**Optimal.** Leaf size=174

$$\frac{5a^2(9bc - ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x\right)}{162b^{4/3}} + \frac{5a^2(9bc - ad) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+1}\right)}{81\sqrt{3}b^{4/3}} + \frac{x(a + bx^3)^{5/3}(9bc - ad)}{54b} + \frac{5ax(a + bx^3)^{5/3}}{54b}$$

[Out] 5/162\*a\*(-a\*d+9\*b\*c)\*x\*(b\*x^3+a)^(2/3)/b+1/54\*(-a\*d+9\*b\*c)\*x\*(b\*x^3+a)^(5/3)/b+1/9\*d\*x\*(b\*x^3+a)^(8/3)/b-5/162\*a^2\*(-a\*d+9\*b\*c)\*ln(-b^(1/3)\*x+(b\*x^3+a)^(1/3))/b^(4/3)+5/243\*a^2\*(-a\*d+9\*b\*c)\*arctan(1/3\*(1+2\*b^(1/3)\*x/(b\*x^3+a)^(1/3))\*3^(1/2))/b^(4/3)\*3^(1/2)

**Rubi [A]** time = 0.06, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 195, 239}

$$\frac{5a^2(9bc - ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x\right)}{162b^{4/3}} + \frac{5a^2(9bc - ad) \tan^{-1}\left(\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}+1}\right)}{81\sqrt{3}b^{4/3}} + \frac{x(a + bx^3)^{5/3}(9bc - ad)}{54b} + \frac{5ax(a + bx^3)^{5/3}}{54b}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(5/3)\*(c + d\*x^3), x]

[Out] (5\*a\*(9\*b\*c - a\*d)\*x\*(a + b\*x^3)^(2/3))/(162\*b) + ((9\*b\*c - a\*d)\*x\*(a + b\*x^3)^(5/3))/(54\*b) + (d\*x\*(a + b\*x^3)^(8/3))/(9\*b) + (5\*a^2\*(9\*b\*c - a\*d)\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]])/(81\*Sqrt[3]\*b^(4/3)) - (5\*a^2\*(9\*b\*c - a\*d)\*Log[-(b^(1/3)\*x) + (a + b\*x^3)^(1/3)])/(162\*b^(4/3))

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] :> Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int (a + bx^3)^{5/3} (c + dx^3) dx &= \frac{dx (a + bx^3)^{8/3}}{9b} - \frac{(-9bc + ad) \int (a + bx^3)^{5/3} dx}{9b} \\
&= \frac{(9bc - ad)x (a + bx^3)^{5/3}}{54b} + \frac{dx (a + bx^3)^{8/3}}{9b} + \frac{(5a(9bc - ad)) \int (a + bx^3)^{2/3} dx}{54b} \\
&= \frac{5a(9bc - ad)x (a + bx^3)^{2/3}}{162b} + \frac{(9bc - ad)x (a + bx^3)^{5/3}}{54b} + \frac{dx (a + bx^3)^{8/3}}{9b} + \frac{(5a^2(9bc - ad)) \int (a + bx^3)^{-1/3} dx}{54b} \\
&= \frac{5a(9bc - ad)x (a + bx^3)^{2/3}}{162b} + \frac{(9bc - ad)x (a + bx^3)^{5/3}}{54b} + \frac{dx (a + bx^3)^{8/3}}{9b} + \frac{5a^2(9bc - ad)}{54b} \int (a + bx^3)^{-1/3} dx
\end{aligned}$$

**Mathematica [C]** time = 0.07, size = 75, normalized size = 0.43

$$\frac{x (a + bx^3)^{2/3} \left( d (a + bx^3)^2 - \frac{a(ad-9bc) {}_2F_1\left(-\frac{5}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a} + 1\right)^{2/3}} \right)}{9b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(5/3)\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^(2/3)\*(d\*(a + b\*x^3)^2 - (a\*(-9\*b\*c + a\*d)\*Hypergeometric2F1[-5/3, 1/3, 4/3, -(b\*x^3)/a]))/(1 + (b\*x^3)/a)^(2/3))/(9\*b)

**fricas [A]** time = 0.71, size = 482, normalized size = 2.77

$$\frac{15 \sqrt{\frac{1}{3}} (9 a^2 b^2 c - a^3 b d) \sqrt{-\frac{1}{2}} \log\left(3 b x^3 - 3 (b x^3 + a)^{\frac{1}{3}} b^{\frac{2}{3}} x^2 - 3 \sqrt{\frac{1}{3}} \left(b^{\frac{4}{3}} x^3 + (b x^3 + a)^{\frac{1}{3}} b x^2 - 2 (b x^3 + a)^{\frac{2}{3}} b^{\frac{2}{3}} x\right)\right)}{9 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c), x, algorithm="fricas")

[Out] [-1/486\*(15\*sqrt(1/3)\*(9\*a^2\*b^2\*c - a^3\*b\*d)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x)\*sqrt(-1/b^(2/3)) + 2\*a) + 10\*(9\*a^2\*b\*c - a^3\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - 5\*(9\*a^2\*b\*c - a^3\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(18\*b^3\*d\*x^7 + 3\*(9\*b^3\*c + 11\*a\*b^2\*d)\*x^4 + 2\*(36\*a\*b^2\*c + 5\*a^2\*b\*d)\*x)\*(b\*x^3 + a)^(2/3))/b^2, -1/486\*(10\*(9\*a^2\*b\*c - a^3\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - 5\*(9\*a^2\*b\*c - a^3\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 30\*sqrt(1/3)\*(9\*a^2\*b^2\*c - a^3\*b\*d)\*arctan(sqrt(1/3)\*(b^(1/3)\*x + 2\*(b\*x^3 + a)^(1/3))/(b^(1/3)\*x))/b^(1/3) - 3\*(18\*b^3\*d\*x^7 + 3\*(9\*b^3\*c + 11\*a\*b^2\*d)\*x^4 + 2\*(36\*a\*b^2\*c + 5\*a^2\*b\*d)\*x)\*(b\*x^3 + a)^(2/3))/b^2]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{5}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/3)\*(d\*x^3 + c), x)

**maple** [F] time = 0.37, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{5}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(5/3)\*(d\*x^3+c),x)

[Out] int((b\*x^3+a)^(5/3)\*(d\*x^3+c),x)

**maxima** [B] time = 1.68, size = 406, normalized size = 2.33

$$\frac{1}{54} \left( \frac{10\sqrt{3}a^2 \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{5a^2 \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{10a^2 \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c),x, algorithm="maxima")

[Out] -1/54\*(10\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - 5\*a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 10\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3) + 3\*(5\*(b\*x^3 + a)^(2/3)\*a^2\*b/x^2 - 8\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^2 - 2\*(b\*x^3 + a)\*b/x^3 + (b\*x^3 + a)^2/x^6)\*c + 1/486\*(10\*sqrt(3)\*a^3\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - 5\*a^3\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 10\*a^3\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + 3\*(5\*(b\*x^3 + a)^(2/3)\*a^3\*b^2/x^2 - 13\*(b\*x^3 + a)^(5/3)\*a^3\*b/x^5 - 10\*(b\*x^3 + a)^(8/3)\*a^3/x^8)/(b^4 - 3\*(b\*x^3 + a)\*b^3/x^3 + 3\*(b\*x^3 + a)^2\*b^2/x^6 - (b\*x^3 + a)^3\*b/x^9))\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{\frac{5}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(5/3)\*(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(5/3)\*(c + d\*x^3), x)

sympy [C] time = 10.42, size = 170, normalized size = 0.98

$$\frac{a^{\frac{5}{3}}cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{5}{3}}dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{2}{3}}bdx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(5/3)\*(d\*x\*\*3+c),x)

[Out] a\*\*(5/3)\*c\*x\*gamma(1/3)\*hyper((-2/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + a\*\*(5/3)\*d\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(2/3)\*b\*c\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(2/3)\*b\*d\*x\*\*7\*gamma(7/3)\*hyper((-2/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3))

### 3.57 $\int (a + bx^3)^{2/3} (c + dx^3) dx$

**Optimal.** Leaf size=141

$$\frac{a(6bc - ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{bx}\right)}{18b^{4/3}} + \frac{a(6bc - ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} + \frac{x(a + bx^3)^{2/3} (6bc - ad)}{18b} + \frac{dx(a + bx^3)^{5/3}}{6b}$$

[Out]  $1/18*(-a*d+6*b*c)*x*(b*x^3+a)^{(2/3)}/b+1/6*d*x*(b*x^3+a)^{(5/3)}/b-1/18*a*(-a*d+6*b*c)*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(4/3)}+1/27*a*(-a*d+6*b*c)*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(4/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 195, 239}

$$\frac{a(6bc - ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{bx}\right)}{18b^{4/3}} + \frac{a(6bc - ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}b^{4/3}} + \frac{x(a + bx^3)^{2/3} (6bc - ad)}{18b} + \frac{dx(a + bx^3)^{5/3}}{6b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^{(2/3)}*(c + d*x^3), x]$

[Out]  $((6*b*c - a*d)*x*(a + b*x^3)^{(2/3)})/(18*b) + (d*x*(a + b*x^3)^{(5/3)})/(6*b) + (a*(6*b*c - a*d)*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(9*\text{Sqrt}[3]*b^{(4/3)}) - (a*(6*b*c - a*d)*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(18*b^{(4/3)})$

#### Rule 195

$\text{Int}[(a + b*x^n)^p, x] \rightarrow \text{Simp}[(x*(a + b*x^n)^p)/(n*p + 1), x] + \text{Dist}[(a*n*p)/(n*p + 1), \text{Int}[(a + b*x^n)^{p-1}, x], x] /;$  FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

$\text{Int}[(a + b*x^3)^{-1/3}, x] \rightarrow \text{Simp}[\text{ArcTan}[(1 + (2*\text{Rt}[b, 3]*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]]/(\text{Sqrt}[3]*\text{Rt}[b, 3]), x] - \text{Simp}[\text{Log}[(a + b*x^3)^{(1/3)} - \text{Rt}[b, 3]*x]/(2*\text{Rt}[b, 3]), x] /;$  FreeQ[{a, b}, x]

#### Rule 388

$\text{Int}[(a + b*x^n)^p*(c + d*x^n), x] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{p+1})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /;$  FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int (a + bx^3)^{2/3} (c + dx^3) dx &= \frac{dx (a + bx^3)^{5/3}}{6b} - \frac{(-6bc + ad) \int (a + bx^3)^{2/3} dx}{6b} \\
&= \frac{(6bc - ad)x (a + bx^3)^{2/3}}{18b} + \frac{dx (a + bx^3)^{5/3}}{6b} + \frac{(a(6bc - ad)) \int \frac{1}{\sqrt[3]{a+bx^3}} dx}{9b} \\
&= \frac{(6bc - ad)x (a + bx^3)^{2/3}}{18b} + \frac{dx (a + bx^3)^{5/3}}{6b} + \frac{a(6bc - ad) \tan^{-1} \left( \frac{1 + \frac{2\sqrt[3]{bx^3}}{\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{9\sqrt{3}b^{4/3}} - \frac{a(6bc - ad)}{9\sqrt{3}b^{4/3}}
\end{aligned}$$

**Mathematica** [C] time = 0.10, size = 72, normalized size = 0.51

$$\frac{x (a + bx^3)^{2/3} \left( \frac{(6bc - ad) {}_2F_1 \left( -\frac{2}{3}, \frac{4}{3}; -\frac{bx^3}{a} \right)}{\left( \frac{bx^3}{a} + 1 \right)^{2/3}} + d (a + bx^3) \right)}{6b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(2/3)\*(c + d\*x^3),x]

[Out] (x\*(a + b\*x^3)^(2/3)\*(d\*(a + b\*x^3) + ((6\*b\*c - a\*d)\*Hypergeometric2F1[-2/3, 1/3, 4/3, -(b\*x^3)/a]))/(1 + (b\*x^3)/a)^(2/3))/(6\*b)

**fricas** [A] time = 0.87, size = 424, normalized size = 3.01

$$\frac{3\sqrt{\frac{1}{3}}(6ab^2c - a^2bd)\sqrt{-\frac{1}{2}}\log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3 + a)^{\frac{1}{3}}bx^2 - 2(bx^3 + a)^{\frac{2}{3}}b^{\frac{2}{3}}x\right)\right)}{6b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c),x, algorithm="fricas")

[Out] [-1/54\*(3\*sqrt(1/3)\*(6\*a\*b^2\*c - a^2\*b\*d)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x) + 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x)\*sqrt(-1/b^(2/3)) + 2\*(6\*a\*b\*c - a^2\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (6\*a\*b\*c - a^2\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(3\*b^2\*d\*x^4 + 2\*(3\*b^2\*c + a\*b\*d)\*x)\*(b\*x^3 + a)^(2/3))/b^2, -1/54\*(2\*(6\*a\*b\*c - a^2\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (6\*a\*b\*c - a^2\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 6\*sqrt(1/3)\*(6\*a\*b^2\*c - a^2\*b\*d)\*arctan(sqrt(1/3)\*(b^(1/3)\*x + 2\*(b\*x^3 + a)^(1/3))/(b^(1/3)\*x))/b^(1/3) - 3\*(3\*b^2\*d\*x^4 + 2\*(3\*b^2\*c + a\*b\*d)\*x)\*(b\*x^3 + a)^(2/3))/b^2]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{2}{3}}(dx^3 + c) dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)\*(d\*x^3 + c), x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{2}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(2/3)\*(d\*x^3+c),x)

[Out] int((b\*x^3+a)^(2/3)\*(d\*x^3+c),x)

**maxima** [B] time = 1.44, size = 322, normalized size = 2.28

$$\frac{1}{9} \left( \frac{2\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(\frac{1}{b^{\frac{1}{3}}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} + \frac{3(bx^3+a)^{\frac{2}{3}}}{b^{\frac{1}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c),x, algorithm="maxima")

[Out] -1/9\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3) + 3\*(b\*x^3 + a)^(2/3)\*a/((b - (b\*x^3 + a)/x^3)\*x^2)\*c + 1/54\*(2\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) + 3\*((b\*x^3 + a)^(2/3)\*a^2\*b/x^2 + 2\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^3 - 2\*(b\*x^3 + a)\*b^2/x^3 + (b\*x^3 + a)^2\*b/x^6))\*d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{2/3} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)\*(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(2/3)\*(c + d\*x^3), x)

**sympy** [C] time = 5.35, size = 82, normalized size = 0.58

$$\frac{a^{\frac{2}{3}}cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{2}{3}}dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(2/3)*(d*x**3+c),x)
```

```
[Out] a**(2/3)*c*x*gamma(1/3)*hyper((-2/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a
)/(3*gamma(4/3)) + a**(2/3)*d*x**4*gamma(4/3)*hyper((-2/3, 4/3), (7/3,), b*
x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))
```

$$3.58 \quad \int \frac{c+dx^3}{\sqrt[3]{a+bx^3}} dx$$

Optimal. Leaf size=111

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{bx}\right)}{6b^{4/3}} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{dx(a+bx^3)^{2/3}}{3b}$$

[Out] 1/3\*d\*x\*(b\*x^3+a)^(2/3)/b-1/6\*(-a\*d+3\*b\*c)\*ln(-b^(1/3)\*x+(b\*x^3+a)^(1/3))/b^(4/3)+1/9\*(-a\*d+3\*b\*c)\*arctan(1/3\*(1+2\*b^(1/3)\*x/(b\*x^3+a)^(1/3))\*3^(1/2))/b^(4/3)\*3^(1/2)

Rubi [A] time = 0.03, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {388, 239}

$$-\frac{(3bc-ad)\log\left(\sqrt[3]{a+bx^3}-\sqrt[3]{bx}\right)}{6b^{4/3}} + \frac{(3bc-ad)\tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} + \frac{dx(a+bx^3)^{2/3}}{3b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(1/3), x]

[Out] (d\*x\*(a + b\*x^3)^(2/3))/(3\*b) + ((3\*b\*c - a\*d)\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]])/(3\*Sqrt[3]\*b^(4/3)) - ((3\*b\*c - a\*d)\*Log[-(b^(1/3)\*x) + (a + b\*x^3)^(1/3)])/(6\*b^(4/3))

Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] :> Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rubi steps

$$\begin{aligned} \int \frac{c+dx^3}{\sqrt[3]{a+bx^3}} dx &= \frac{dx(a+bx^3)^{2/3}}{3b} - \frac{(-3bc+ad)\int \frac{1}{\sqrt[3]{a+bx^3}} dx}{3b} \\ &= \frac{dx(a+bx^3)^{2/3}}{3b} + \frac{(3bc-ad)\tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{4/3}} - \frac{(3bc-ad)\log\left(-\sqrt[3]{bx}+\sqrt[3]{a+bx^3}\right)}{6b^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 141, normalized size = 1.27

$$\frac{(3bc-ad) \left( \log \left( \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}} + 1 \right) - 2 \log \left( 1 - \frac{\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}} \right) \right)}{6\sqrt[3]{b}} + dx(a+bx^3)^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(1/3), x]

[Out] (d\*x\*(a + b\*x^3)^(2/3) + ((3\*b\*c - a\*d)\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3)]/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/(6\*b^(1/3)))/(3\*b)

**fricas [A]** time = 0.65, size = 362, normalized size = 3.26

$$\frac{6(bx^3 + a)^{\frac{2}{3}} b dx - 3\sqrt{\frac{1}{3}}(3b^2c - abd)\sqrt{-\frac{1}{\frac{2}{b^3}}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3 + a)^{\frac{1}{3}}bx^2 - 2\left(bx^3 + a\right)^{\frac{1}{3}}\right)\right)}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(1/3), x, algorithm="fricas")

[Out] [1/18\*(6\*(b\*x^3 + a)^(2/3)\*b\*d\*x - 3\*sqrt(1/3)\*(3\*b^2\*c - a\*b\*d)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(1/3)\*b\*x - 2\*a) - 2\*(3\*b\*c - a\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) + (3\*b\*c - a\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2), 1/18\*(6\*(b\*x^3 + a)^(2/3)\*b\*d\*x - 2\*(3\*b\*c - a\*d)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) + (3\*b\*c - a\*d)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 6\*sqrt(1/3)\*(3\*b^2\*c - a\*b\*d)\*arctan(sqrt(1/3)\*(b^(1/3)\*x + 2\*(b\*x^3 + a)^(1/3))/(b^(1/3)\*x))/b^(1/3))/b^2]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(1/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(1/3), x)

**maple [F]** time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(1/3),x)`

[Out] `int((d*x^3+c)/(b*x^3+a)^(1/3),x)`

**maxima** [B] time = 1.20, size = 244, normalized size = 2.20

$$\frac{1}{6} \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3} \left( \frac{1}{b^{\frac{1}{3}}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x} \right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2 \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} \right) c + \frac{1}{18} \left( \frac{2\sqrt{3}}{b^{\frac{1}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(1/3),x, algorithm="maxima")`

[Out] `-1/6*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3)))/b^(1/3) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(1/3)*c + 1/18*(2*sqrt(3)*a*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a*log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*a*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) - 6*(b*x^3 + a)^(2/3)*a/((b^2 - (b*x^3 + a)*b/x^3)*x^2))*d`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{dx^3 + c}{(bx^3 + a)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(1/3),x)`

[Out] `int((c + d*x^3)/(a + b*x^3)^(1/3),x)`

**sympy** [C] time = 4.44, size = 78, normalized size = 0.70

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(1/3),x)`

[Out] `c*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(1/3)*gamma(7/3))`

$$3.59 \quad \int \frac{c+dx^3}{(a+bx^3)^{4/3}} dx$$

**Optimal.** Leaf size=99

$$-\frac{d \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{2b^{4/3}} + \frac{d \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} + \frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

[Out]  $(-a*d+b*c)*x/a/b/(b*x^3+a)^{(1/3)}-1/2*d*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(4/3)}+1/3*d*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(4/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.02, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {385, 239}

$$-\frac{d \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{2b^{4/3}} + \frac{d \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} + \frac{x(bc-ad)}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(4/3), x]

[Out]  $((b*c - a*d)*x)/(a*b*(a + b*x^3)^{(1/3)}) + (d*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(\text{Sqrt}[3]*b^{(4/3)}) - (d*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(2*b^{(4/3)})$

**Rule 239**

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{c+dx^3}{(a+bx^3)^{4/3}} dx &= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d \int \frac{1}{\sqrt[3]{a+bx^3}} dx}{b} \\ &= \frac{(bc-ad)x}{ab\sqrt[3]{a+bx^3}} + \frac{d \tan^{-1}\left(\frac{1+\frac{2\sqrt[3]{bx}}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{4/3}} - \frac{d \log\left(-\sqrt[3]{bx} + \sqrt[3]{a+bx^3}\right)}{2b^{4/3}} \end{aligned}$$

**Mathematica [C]** time = 0.05, size = 61, normalized size = 0.62

$$\frac{dx^4 \sqrt[3]{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{4}{3}, \frac{4}{3}; \frac{7}{3}; -\frac{bx^3}{a}\right) + 4cx}{4a \sqrt[3]{a + bx^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(4/3), x]

[Out] (4\*c\*x + d\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*Hypergeometric2F1[4/3, 4/3, 7/3, -(b\*x^3)/a])/(4\*a\*(a + b\*x^3)^(1/3))

**fricas [B]** time = 0.69, size = 488, normalized size = 4.93

$$\frac{3 \sqrt{\frac{1}{3}} (ab^2 dx^3 + a^2 bd) \sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3 bx^3 - 3 (bx^3 + a)^{\frac{1}{3}} (-b)^{\frac{2}{3}} x^2 - 3 \sqrt{\frac{1}{3}} \left((-b)^{\frac{1}{3}} bx^3 - (bx^3 + a)^{\frac{1}{3}} bx^2 + 2 (bx^3 + a)^{\frac{1}{3}}\right)\right)}{4 a \sqrt[3]{a + b x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(4/3), x, algorithm="fricas")

[Out] [1/6\*(3\*sqrt(1/3)\*(a\*b^2\*d\*x^3 + a^2\*b\*d)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) + 6\*(b\*x^3 + a)^(2/3)\*(b^2\*c - a\*b\*d)\*x - 2\*(a\*b\*d\*x^3 + a^2\*d)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + (a\*b\*d\*x^3 + a^2\*d)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)]/(a\*b^3\*x^3 + a^2\*b^2), -1/6\*(6\*sqrt(1/3)\*(a\*b^2\*d\*x^3 + a^2\*b\*d)\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt((-b)^(1/3)/b)/x - 6\*(b\*x^3 + a)^(2/3)\*(b^2\*c - a\*b\*d)\*x + 2\*(a\*b\*d\*x^3 + a^2\*d)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - (a\*b\*d\*x^3 + a^2\*d)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2)]/(a\*b^3\*x^3 + a^2\*b^2)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(4/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(4/3), x)

**maple [F]** time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(4/3),x)`

[Out] `int((d*x^3+c)/(b*x^3+a)^(4/3),x)`

**maxima** [A] time = 1.22, size = 134, normalized size = 1.35

$$-\frac{1}{6}d \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{4}{3}}} + \frac{6x}{(bx^3+a)^{\frac{1}{3}}b} - \frac{\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{4}{3}}} + \frac{2 \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{4}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(4/3),x, algorithm="maxima")`

[Out] `-1/6*d*(2*sqrt(3)*arctan(1/3*sqrt(3)*(b^(1/3) + 2*(b*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6*x/((b*x^3 + a)^(1/3)*b) - log(b^(2/3) + (b*x^3 + a)^(1/3)*b^(1/3)/x + (b*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2*log(-b^(1/3) + (b*x^3 + a)^(1/3)/x)/b^(4/3) + c*x/((b*x^3 + a)^(1/3)*a)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{dx^3 + c}{(bx^3 + a)^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(4/3),x)`

[Out] `int((c + d*x^3)/(a + b*x^3)^(4/3), x)`

**sympy** [C] time = 12.83, size = 71, normalized size = 0.72

$$\frac{cx\Gamma\left(\frac{1}{3}\right)}{3a^{\frac{4}{3}}\sqrt[3]{1 + \frac{bx^3}{a}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{4}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(4/3),x)`

[Out] `c*x*gamma(1/3)/(3*a**(4/3)*(1 + b*x**3/a)**(1/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 4/3), (7/3, ), b*x**3*exp_polar(I*pi)/a)/(3*a**(4/3)*gamma(7/3))`



$$3.60 \quad \int \frac{c+dx^3}{(a+bx^3)^{7/3}} dx$$

**Optimal.** Leaf size=47

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

[Out]  $3/4*c*x/a^2/(b*x^3+a)^{(1/3)}+1/4*x*(d*x^3+c)/a/(b*x^3+a)^{(4/3)}$

**Rubi [A]** time = 0.01, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {378, 191}

$$\frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(7/3), x]

[Out]  $(3*c*x)/(4*a^2*(a + b*x^3)^{(1/3)}) + (x*(c + d*x^3))/(4*a*(a + b*x^3)^{(4/3)})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{c+dx^3}{(a+bx^3)^{7/3}} dx &= \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}} + \frac{(3c) \int \frac{1}{(a+bx^3)^{4/3}} dx}{4a} \\ &= \frac{3cx}{4a^2\sqrt[3]{a+bx^3}} + \frac{x(c+dx^3)}{4a(a+bx^3)^{4/3}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 37, normalized size = 0.79

$$\frac{x(4ac + adx^3 + 3bcx^3)}{4a^2(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(7/3), x]

[Out]  $(x*(4*a*c + 3*b*c*x^3 + a*d*x^3))/(4*a^2*(a + b*x^3)^{(4/3)})$

**fricas** [A] time = 0.83, size = 54, normalized size = 1.15

$$\frac{((3bc + ad)x^4 + 4acx)(bx^3 + a)^{\frac{2}{3}}}{4(a^2b^2x^6 + 2a^3bx^3 + a^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(7/3),x, algorithm="fricas")

[Out] 1/4\*((3\*b\*c + a\*d)\*x^4 + 4\*a\*c\*x)\*(b\*x^3 + a)^(2/3)/(a^2\*b^2\*x^6 + 2\*a^3\*b\*x^3 + a^4)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(7/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(7/3), x)

**maple** [A] time = 0.05, size = 34, normalized size = 0.72

$$\frac{(adx^3 + 3bcx^3 + 4ac)x}{4(bx^3 + a)^{\frac{4}{3}}a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)/(b\*x^3+a)^(7/3),x)

[Out] 1/4\*x\*(a\*d\*x^3+3\*b\*c\*x^3+4\*a\*c)/(b\*x^3+a)^(4/3)/a^2

**maxima** [A] time = 0.47, size = 51, normalized size = 1.09

$$-\frac{\left(b - \frac{4(bx^3+a)}{x^3}\right)cx^4}{4(bx^3 + a)^{\frac{4}{3}}a^2} + \frac{dx^4}{4(bx^3 + a)^{\frac{4}{3}}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(7/3),x, algorithm="maxima")

[Out] -1/4\*(b - 4\*(b\*x^3 + a)/x^3)\*c\*x^4/((b\*x^3 + a)^(4/3)\*a^2) + 1/4\*d\*x^4/((b\*x^3 + a)^(4/3)\*a)

**mupad** [B] time = 1.37, size = 33, normalized size = 0.70

$$\frac{4acx + adx^4 + 3bcx^4}{4a^2(bx^3 + a)^{\frac{4}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)/(a + b\*x^3)^(7/3),x)

[Out] (4\*a\*c\*x + a\*d\*x^4 + 3\*b\*c\*x^4)/(4\*a^2\*(a + b\*x^3)^(4/3))

sympy [B] time = 82.05, size = 190, normalized size = 4.04

$$c \left( \frac{4ax\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right) + 9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} + \frac{3bx^4\Gamma\left(\frac{1}{3}\right)}{9a^{\frac{10}{3}}\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right) + 9a^{\frac{7}{3}}bx^3\sqrt[3]{1+\frac{bx^3}{a}}\Gamma\left(\frac{7}{3}\right)} \right) + \frac{1}{3a^{\frac{7}{3}}\sqrt[3]{1+\frac{bx^3}{a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a)\*\*(7/3),x)

[Out] c\*(4\*a\*x\*gamma(1/3)/(9\*a\*\*(10/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 9\*a\*\*(7/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3)) + 3\*b\*x\*\*4\*gamma(1/3)/(9\*a\*\*(10/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 9\*a\*\*(7/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3)) + d\*x\*\*4\*gamma(4/3)/(3\*a\*\*(7/3)\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3) + 3\*a\*\*(4/3)\*b\*x\*\*3\*(1 + b\*x\*\*3/a)\*\*(1/3)\*gamma(7/3))

$$3.61 \quad \int \frac{c+dx^3}{(a+bx^3)^{10/3}} dx$$

**Optimal.** Leaf size=91

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}} + \frac{x(ad+6bc)}{28a^2b(a+bx^3)^{4/3}} + \frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

[Out]  $1/7*(-a*d+b*c)*x/a/b/(b*x^3+a)^{(7/3)}+1/28*(a*d+6*b*c)*x/a^2/b/(b*x^3+a)^{(4/3)}+3/28*(a*d+6*b*c)*x/a^3/b/(b*x^3+a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {385, 192, 191}

$$\frac{3x(ad+6bc)}{28a^3b\sqrt[3]{a+bx^3}} + \frac{x(ad+6bc)}{28a^2b(a+bx^3)^{4/3}} + \frac{x(bc-ad)}{7ab(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(10/3), x]

[Out]  $((b*c - a*d)*x)/(7*a*b*(a + b*x^3)^{(7/3)}) + ((6*b*c + a*d)*x)/(28*a^2*b*(a + b*x^3)^{(4/3)}) + (3*(6*b*c + a*d)*x)/(28*a^3*b*(a + b*x^3)^{(1/3)})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 192**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{c+dx^3}{(a+bx^3)^{10/3}} dx &= \frac{(bc-ad)x}{7ab(a+bx^3)^{7/3}} + \frac{(6bc+ad) \int \frac{1}{(a+bx^3)^{7/3}} dx}{7ab} \\ &= \frac{(bc-ad)x}{7ab(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b(a+bx^3)^{4/3}} + \frac{(3(6bc+ad)) \int \frac{1}{(a+bx^3)^{4/3}} dx}{28a^2b} \\ &= \frac{(bc-ad)x}{7ab(a+bx^3)^{7/3}} + \frac{(6bc+ad)x}{28a^2b(a+bx^3)^{4/3}} + \frac{3(6bc+ad)x}{28a^3b\sqrt[3]{a+bx^3}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 59, normalized size = 0.65

$$\frac{7a^2(4cx + dx^4) + 3abx^4(14c + dx^3) + 18b^2cx^7}{28a^3(a + bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(10/3), x]

[Out] (18\*b^2\*c\*x^7 + 3\*a\*b\*x^4\*(14\*c + d\*x^3) + 7\*a^2\*(4\*c\*x + d\*x^4))/(28\*a^3\*(a + b\*x^3)^(7/3))

**fricas [A]** time = 0.72, size = 87, normalized size = 0.96

$$\frac{(3(6b^2c + abd)x^7 + 7(6abc + a^2d)x^4 + 28a^2cx)(bx^3 + a)^{2/3}}{28(a^3b^3x^9 + 3a^4b^2x^6 + 3a^5bx^3 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(10/3), x, algorithm="fricas")

[Out] 1/28\*(3\*(6\*b^2\*c + a\*b\*d)\*x^7 + 7\*(6\*a\*b\*c + a^2\*d)\*x^4 + 28\*a^2\*c\*x)\*(b\*x^3 + a)^(2/3)/(a^3\*b^3\*x^9 + 3\*a^4\*b^2\*x^6 + 3\*a^5\*b\*x^3 + a^6)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{10/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(10/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(10/3), x)

**maple [A]** time = 0.04, size = 57, normalized size = 0.63

$$\frac{(3abd x^6 + 18b^2c x^6 + 7a^2d x^3 + 42abc x^3 + 28a^2c)x}{28(b x^3 + a)^{7/3} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)/(b\*x^3+a)^(10/3), x)

[Out] 1/28\*x\*(3\*a\*b\*d\*x^6+18\*b^2\*c\*x^6+7\*a^2\*d\*x^3+42\*a\*b\*c\*x^3+28\*a^2\*c)/(b\*x^3+a)^(7/3)/a^3

**maxima [A]** time = 0.61, size = 86, normalized size = 0.95

$$\frac{\left(4b - \frac{7(bx^3+a)}{x^3}\right)dx^7}{28(bx^3 + a)^{7/3}a^2} + \frac{\left(2b^2 - \frac{7(bx^3+a)b}{x^3} + \frac{14(bx^3+a)^2}{x^6}\right)cx^7}{14(bx^3 + a)^{7/3}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(10/3), x, algorithm="maxima")

[Out]  $-1/28*(4*b - 7*(b*x^3 + a)/x^3)*d*x^7/((b*x^3 + a)^{(7/3)}*a^2) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c*x^7/((b*x^3 + a)^{(7/3)}*a^3)$

**mupad [B]** time = 1.42, size = 87, normalized size = 0.96

$$\frac{3adx(bx^3 + a)^2 - 4a^3dx + 18bcx(bx^3 + a)^2 + a^2dx(bx^3 + a) + 4a^2bcx + 6abcx(bx^3 + a)}{28a^3b(bx^3 + a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(10/3), x)`

[Out]  $(3*a*d*x*(a + b*x^3)^2 - 4*a^3*d*x + 18*b*c*x*(a + b*x^3)^2 + a^2*d*x*(a + b*x^3) + 4*a^2*b*c*x + 6*a*b*c*x*(a + b*x^3))/(28*a^3*b*(a + b*x^3)^{(7/3)})$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(10/3), x)`

[Out] Timed out

$$3.62 \quad \int \frac{c+dx^3}{(a+bx^3)^{13/3}} dx$$

**Optimal.** Leaf size=121

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}} + \frac{3x(ad+9bc)}{140a^3b(a+bx^3)^{4/3}} + \frac{x(ad+9bc)}{70a^2b(a+bx^3)^{7/3}} + \frac{x(bc-ad)}{10ab(a+bx^3)^{10/3}}$$

[Out] 1/10\*(-a\*d+b\*c)\*x/a/b/(b\*x^3+a)^(10/3)+1/70\*(a\*d+9\*b\*c)\*x/a^2/b/(b\*x^3+a)^(7/3)+3/140\*(a\*d+9\*b\*c)\*x/a^3/b/(b\*x^3+a)^(4/3)+9/140\*(a\*d+9\*b\*c)\*x/a^4/b/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {385, 192, 191}

$$\frac{9x(ad+9bc)}{140a^4b\sqrt[3]{a+bx^3}} + \frac{3x(ad+9bc)}{140a^3b(a+bx^3)^{4/3}} + \frac{x(ad+9bc)}{70a^2b(a+bx^3)^{7/3}} + \frac{x(bc-ad)}{10ab(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(13/3), x]

[Out] ((b\*c - a\*d)\*x)/(10\*a\*b\*(a + b\*x^3)^(10/3)) + ((9\*b\*c + a\*d)\*x)/(70\*a^2\*b\*(a + b\*x^3)^(7/3)) + (3\*(9\*b\*c + a\*d)\*x)/(140\*a^3\*b\*(a + b\*x^3)^(4/3)) + (9\*(9\*b\*c + a\*d)\*x)/(140\*a^4\*b\*(a + b\*x^3)^(1/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3}{(a + bx^3)^{13/3}} dx &= \frac{(bc - ad)x}{10ab(a + bx^3)^{10/3}} + \frac{(9bc + ad) \int \frac{1}{(a+bx^3)^{10/3}} dx}{10ab} \\
&= \frac{(bc - ad)x}{10ab(a + bx^3)^{10/3}} + \frac{(9bc + ad)x}{70a^2b(a + bx^3)^{7/3}} + \frac{(3(9bc + ad)) \int \frac{1}{(a+bx^3)^{7/3}} dx}{35a^2b} \\
&= \frac{(bc - ad)x}{10ab(a + bx^3)^{10/3}} + \frac{(9bc + ad)x}{70a^2b(a + bx^3)^{7/3}} + \frac{3(9bc + ad)x}{140a^3b(a + bx^3)^{4/3}} + \frac{(9(9bc + ad)) \int \frac{1}{(a+bx^3)^{4/3}} dx}{140a^3b} \\
&= \frac{(bc - ad)x}{10ab(a + bx^3)^{10/3}} + \frac{(9bc + ad)x}{70a^2b(a + bx^3)^{7/3}} + \frac{3(9bc + ad)x}{140a^3b(a + bx^3)^{4/3}} + \frac{9(9bc + ad)x}{140a^4b\sqrt[3]{a + bx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 80, normalized size = 0.66

$$\frac{x(35a^3(4c + dx^3) + 15a^2bx^3(21c + 2dx^3) + 9ab^2x^6(30c + dx^3) + 81b^3cx^9)}{140a^4(a + bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(13/3), x]

[Out] (x\*(81\*b^3\*c\*x^9 + 35\*a^3\*(4\*c + d\*x^3) + 9\*a\*b^2\*x^6\*(30\*c + d\*x^3) + 15\*a^2\*b\*x^3\*(21\*c + 2\*d\*x^3)))/(140\*a^4\*(a + b\*x^3)^(10/3))

**fricas [A]** time = 0.58, size = 121, normalized size = 1.00

$$\frac{(9(9b^3c + ab^2d)x^{10} + 30(9ab^2c + a^2bd)x^7 + 140a^3cx + 35(9a^2bc + a^3d)x^4)(bx^3 + a)^{\frac{2}{3}}}{140(a^4b^4x^{12} + 4a^5b^3x^9 + 6a^6b^2x^6 + 4a^7bx^3 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(13/3), x, algorithm="fricas")

[Out] 1/140\*(9\*(9\*b^3\*c + a\*b^2\*d)\*x^10 + 30\*(9\*a\*b^2\*c + a^2\*b\*d)\*x^7 + 140\*a^3\*c\*x + 35\*(9\*a^2\*b\*c + a^3\*d)\*x^4)\*(b\*x^3 + a)^(2/3)/(a^4\*b^4\*x^12 + 4\*a^5\*b^3\*x^9 + 6\*a^6\*b^2\*x^6 + 4\*a^7\*b\*x^3 + a^8)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{13}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(13/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(13/3), x)

**maple [A]** time = 0.04, size = 81, normalized size = 0.67

$$\frac{(9ab^2dx^9 + 81b^3cx^9 + 30a^2bdx^6 + 270ab^2cx^6 + 35a^3dx^3 + 315a^2bcx^3 + 140c a^3)x}{140(bx^3 + a)^{\frac{10}{3}} a^4}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(13/3),x)`

[Out]  $\frac{1}{140}x(9ab^2dx^9+81b^3cx^9+30a^2b^2dx^6+270ab^2cx^6+35a^3d^2x^3+315a^2b^2cx^3+140a^3c)/(b^3x^3+a)^{10/3}/a^4$

**maxima** [A] time = 0.50, size = 120, normalized size = 0.99

$$\frac{\left(14b^2 - \frac{40(bx^3+a)b}{x^3} + \frac{35(bx^3+a)^2}{x^6}\right)dx^{10}}{140(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14b^3 - \frac{60(bx^3+a)b^2}{x^3} + \frac{105(bx^3+a)^2b}{x^6} - \frac{140(bx^3+a)^3}{x^9}\right)cx^{10}}{140(bx^3+a)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(13/3),x, algorithm="maxima")`

[Out]  $\frac{1}{140}(14b^2 - 40(bx^3 + a)b/x^3 + 35(bx^3 + a)^2/x^6)dx^{10}/((bx^3 + a)^{10/3}a^3) - \frac{1}{140}(14b^3 - 60(bx^3 + a)b^2/x^3 + 105(bx^3 + a)^2b/x^6 - 140(bx^3 + a)^3/x^9)cx^{10}/((bx^3 + a)^{10/3}a^4)$

**mupad** [B] time = 1.46, size = 105, normalized size = 0.87

$$\frac{x\left(\frac{c}{10a} - \frac{d}{10b}\right)}{(bx^3 + a)^{10/3}} + \frac{x(ad + 9bc)}{70a^2b(bx^3 + a)^{7/3}} + \frac{x(3ad + 27bc)}{140a^3b(bx^3 + a)^{4/3}} + \frac{x(9ad + 81bc)}{140a^4b(bx^3 + a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(13/3),x)`

[Out]  $(x(c/(10a) - d/(10b)))/(a + bx^3)^{10/3} + (x(ad + 9bc))/(70a^2b(a + bx^3)^{7/3}) + (x(3ad + 27bc))/(140a^3b(a + bx^3)^{4/3}) + (x(9ad + 81bc))/(140a^4b(a + bx^3)^{1/3})$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(13/3),x)`

[Out] Timed out

$$3.63 \quad \int \frac{c+dx^3}{(a+bx^3)^{16/3}} dx$$

**Optimal.** Leaf size=151

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}} + \frac{27x(ad+12bc)}{1820a^4b(a+bx^3)^{4/3}} + \frac{9x(ad+12bc)}{910a^3b(a+bx^3)^{7/3}} + \frac{x(ad+12bc)}{130a^2b(a+bx^3)^{10/3}} + \frac{x(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

[Out] 1/13\*(-a\*d+b\*c)\*x/a/b/(b\*x^3+a)^(13/3)+1/130\*(a\*d+12\*b\*c)\*x/a^2/b/(b\*x^3+a)^(10/3)+9/910\*(a\*d+12\*b\*c)\*x/a^3/b/(b\*x^3+a)^(7/3)+27/1820\*(a\*d+12\*b\*c)\*x/a^4/b/(b\*x^3+a)^(4/3)+81/1820\*(a\*d+12\*b\*c)\*x/a^5/b/(b\*x^3+a)^(1/3)

**Rubi [A]** time = 0.05, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {385, 192, 191}

$$\frac{81x(ad+12bc)}{1820a^5b\sqrt[3]{a+bx^3}} + \frac{27x(ad+12bc)}{1820a^4b(a+bx^3)^{4/3}} + \frac{9x(ad+12bc)}{910a^3b(a+bx^3)^{7/3}} + \frac{x(ad+12bc)}{130a^2b(a+bx^3)^{10/3}} + \frac{x(bc-ad)}{13ab(a+bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(16/3), x]

[Out] ((b\*c - a\*d)\*x)/(13\*a\*b\*(a + b\*x^3)^(13/3)) + ((12\*b\*c + a\*d)\*x)/(130\*a^2\*b\*(a + b\*x^3)^(10/3)) + (9\*(12\*b\*c + a\*d)\*x)/(910\*a^3\*b\*(a + b\*x^3)^(7/3)) + (27\*(12\*b\*c + a\*d)\*x)/(1820\*a^4\*b\*(a + b\*x^3)^(4/3)) + (81\*(12\*b\*c + a\*d)\*x)/(1820\*a^5\*b\*(a + b\*x^3)^(1/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3}{(a + bx^3)^{16/3}} dx &= \frac{(bc - ad)x}{13ab(a + bx^3)^{13/3}} + \frac{(12bc + ad) \int \frac{1}{(a+bx^3)^{13/3}} dx}{13ab} \\
&= \frac{(bc - ad)x}{13ab(a + bx^3)^{13/3}} + \frac{(12bc + ad)x}{130a^2b(a + bx^3)^{10/3}} + \frac{(9(12bc + ad)) \int \frac{1}{(a+bx^3)^{10/3}} dx}{130a^2b} \\
&= \frac{(bc - ad)x}{13ab(a + bx^3)^{13/3}} + \frac{(12bc + ad)x}{130a^2b(a + bx^3)^{10/3}} + \frac{9(12bc + ad)x}{910a^3b(a + bx^3)^{7/3}} + \frac{(27(12bc + ad)) \int \frac{1}{(a+bx^3)^{7/3}} dx}{455a^3b} \\
&= \frac{(bc - ad)x}{13ab(a + bx^3)^{13/3}} + \frac{(12bc + ad)x}{130a^2b(a + bx^3)^{10/3}} + \frac{9(12bc + ad)x}{910a^3b(a + bx^3)^{7/3}} + \frac{27(12bc + ad)x}{1820a^4b(a + bx^3)^{4/3}} \\
&= \frac{(bc - ad)x}{13ab(a + bx^3)^{13/3}} + \frac{(12bc + ad)x}{130a^2b(a + bx^3)^{10/3}} + \frac{9(12bc + ad)x}{910a^3b(a + bx^3)^{7/3}} + \frac{27(12bc + ad)x}{1820a^4b(a + bx^3)^{4/3}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 100, normalized size = 0.66

$$\frac{x(455a^4(4c + dx^3) + 195a^3bx^3(28c + 3dx^3) + 351a^2b^2x^6(20c + dx^3) + 81ab^3x^9(52c + dx^3) + 972b^4cx^{12})}{1820a^5(a + bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(16/3), x]

[Out] (x\*(972\*b^4\*c\*x^12 + 455\*a^4\*(4\*c + d\*x^3) + 351\*a^2\*b^2\*x^6\*(20\*c + d\*x^3) + 81\*a\*b^3\*x^9\*(52\*c + d\*x^3) + 195\*a^3\*b\*x^3\*(28\*c + 3\*d\*x^3)))/(1820\*a^5\*(a + b\*x^3)^(13/3))

**fricas [A]** time = 0.67, size = 155, normalized size = 1.03

$$\frac{(81(12b^4c + ab^3d)x^{13} + 351(12ab^3c + a^2b^2d)x^{10} + 585(12a^2b^2c + a^3bd)x^7 + 1820a^4cx + 455(12a^3bc + a^4c^2))}{1820(a^5b^5x^{15} + 5a^6b^4x^{12} + 10a^7b^3x^9 + 10a^8b^2x^6 + 5a^9bx^3 + a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(16/3), x, algorithm="fricas")

[Out] 1/1820\*(81\*(12\*b^4\*c + a\*b^3\*d)\*x^13 + 351\*(12\*a\*b^3\*c + a^2\*b^2\*d)\*x^10 + 585\*(12\*a^2\*b^2\*c + a^3\*b\*d)\*x^7 + 1820\*a^4\*c\*x + 455\*(12\*a^3\*b\*c + a^4\*d)\*x^4\*(b\*x^3 + a)^(2/3)/(a^5\*b^5\*x^15 + 5\*a^6\*b^4\*x^12 + 10\*a^7\*b^3\*x^9 + 10\*a^8\*b^2\*x^6 + 5\*a^9\*b\*x^3 + a^10)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{16/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(16/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(16/3), x)

**maple [A]** time = 0.05, size = 105, normalized size = 0.70

$$\frac{(81ab^3dx^{12} + 972b^4cx^{12} + 351a^2b^2dx^9 + 4212ab^3cx^9 + 585a^3bdx^6 + 7020a^2b^2cx^6 + 455a^4dx^3 + 5460a^3bcx^3)}{1820(bx^3 + a)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)/(b\*x^3+a)^(16/3),x)

[Out] 1/1820\*x\*(81\*a\*b^3\*d\*x^12+972\*b^4\*c\*x^12+351\*a^2\*b^2\*d\*x^9+4212\*a\*b^3\*c\*x^9+585\*a^3\*b\*d\*x^6+7020\*a^2\*b^2\*c\*x^6+455\*a^4\*d\*x^3+5460\*a^3\*b\*c\*x^3+1820\*a^4\*c)/(b\*x^3+a)^(13/3)/a^5

**maxima [A]** time = 0.66, size = 154, normalized size = 1.02

$$\frac{\left(140b^3 - \frac{546(bx^3+a)b^2}{x^3} + \frac{780(bx^3+a)^2b}{x^6} - \frac{455(bx^3+a)^3}{x^9}\right)dx^{13}}{1820(bx^3+a)^{\frac{13}{3}}a^4} + \frac{\left(35b^4 - \frac{182(bx^3+a)b^3}{x^3} + \frac{390(bx^3+a)^2b^2}{x^6} - \frac{455(bx^3+a)^3b}{x^9} + \frac{455}{x^9}\right)dx^{13}}{455(bx^3+a)^{\frac{13}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(16/3),x, algorithm="maxima")

[Out] -1/1820\*(140\*b^3 - 546\*(b\*x^3 + a)\*b^2/x^3 + 780\*(b\*x^3 + a)^2\*b/x^6 - 455\*(b\*x^3 + a)^3/x^9)\*d\*x^13/((b\*x^3 + a)^(13/3)\*a^4) + 1/455\*(35\*b^4 - 182\*(b\*x^3 + a)\*b^3/x^3 + 390\*(b\*x^3 + a)^2\*b^2/x^6 - 455\*(b\*x^3 + a)^3\*b/x^9 + 455\*(b\*x^3 + a)^4/x^12)\*c\*x^13/((b\*x^3 + a)^(13/3)\*a^5)

**mupad [B]** time = 1.45, size = 132, normalized size = 0.87

$$\frac{x\left(\frac{c}{13a} - \frac{d}{13b}\right)}{(bx^3 + a)^{13/3}} + \frac{x(ad + 12bc)}{130a^2b(bx^3 + a)^{10/3}} + \frac{x(9ad + 108bc)}{910a^3b(bx^3 + a)^{7/3}} + \frac{x(27ad + 324bc)}{1820a^4b(bx^3 + a)^{4/3}} + \frac{x(81ad + 972bc)}{1820a^5b(bx^3 + a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)/(a + b\*x^3)^(16/3),x)

[Out] (x\*(c/(13\*a) - d/(13\*b)))/(a + b\*x^3)^(13/3) + (x\*(a\*d + 12\*b\*c))/(130\*a^2\*b\*(a + b\*x^3)^(10/3)) + (x\*(9\*a\*d + 108\*b\*c))/(910\*a^3\*b\*(a + b\*x^3)^(7/3)) + (x\*(27\*a\*d + 324\*b\*c))/(1820\*a^4\*b\*(a + b\*x^3)^(4/3)) + (x\*(81\*a\*d + 972\*b\*c))/(1820\*a^5\*b\*(a + b\*x^3)^(1/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)/(b\*x\*\*3+a)\*\*(16/3),x)

[Out] Timed out

### 3.64 $\int (a + bx^3)^{7/3} (c + dx^3) dx$

**Optimal.** Leaf size=85

$$\frac{a^2 x \sqrt[3]{a + bx^3} (11bc - ad) {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{11b \sqrt[3]{\frac{bx^3}{a}} + 1} + \frac{dx (a + bx^3)^{10/3}}{11b}$$

[Out] 1/11\*d\*x\*(b\*x^3+a)^(10/3)/b+1/11\*a^2\*(-a\*d+11\*b\*c)\*x\*(b\*x^3+a)^(1/3)\*hypergeometric2F1[-7/3, 1/3, 4/3, -(b\*x^3/a)]/b/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.02, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 246, 245}

$$\frac{a^2 x \sqrt[3]{a + bx^3} (11bc - ad) {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{11b \sqrt[3]{\frac{bx^3}{a}} + 1} + \frac{dx (a + bx^3)^{10/3}}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(7/3)\*(c + d\*x^3), x]

[Out] (d\*x\*(a + b\*x^3)^(10/3))/(11\*b) + (a^2\*(11\*b\*c - a\*d)\*x\*(a + b\*x^3)^(1/3)\*Hypergeometric2F1[-7/3, 1/3, 4/3, -(b\*x^3/a)]/(11\*b\*(1 + (b\*x^3/a)^(1/3)))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n/a)^FracPart[p]), Int[(1 + (b\*x^n/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (a + bx^3)^{7/3} (c + dx^3) dx &= \frac{dx (a + bx^3)^{10/3}}{11b} - \frac{(-11bc + ad) \int (a + bx^3)^{7/3} dx}{11b} \\ &= \frac{dx (a + bx^3)^{10/3}}{11b} - \frac{\left(a^2(-11bc + ad) \sqrt[3]{a + bx^3}\right) \int \left(1 + \frac{bx^3}{a}\right)^{7/3} dx}{11b \sqrt[3]{1 + \frac{bx^3}{a}}} \\ &= \frac{dx (a + bx^3)^{10/3}}{11b} + \frac{a^2(11bc - ad)x \sqrt[3]{a + bx^3} {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{11b \sqrt[3]{1 + \frac{bx^3}{a}}} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 77, normalized size = 0.91

$$\frac{x \sqrt[3]{a + bx^3} \left( d (a + bx^3)^3 - \frac{a^2(ad - 11bc) {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a} + 1}} \right)}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(7/3)\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^(1/3)\*(d\*(a + b\*x^3)^3 - (a^2\*(-11\*b\*c + a\*d)\*Hypergeometric2F1[-7/3, 1/3, 4/3, -(b\*x^3)/a]))/(1 + (b\*x^3)/a)^(1/3))/(11\*b)

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 dx^9 + (b^2c + 2abd)x^6 + (2abc + a^2d)x^3 + a^2c\right)(bx^3 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c), x, algorithm="fricas")

[Out] integral((b^2\*d\*x^9 + (b^2\*c + 2\*a\*b\*d)\*x^6 + (2\*a\*b\*c + a^2\*d)\*x^3 + a^2\*c)\*(b\*x^3 + a)^(1/3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(7/3)\*(d\*x^3 + c), x)

**maple** [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(7/3)\*(d\*x^3+c), x)

[Out] int((b\*x^3+a)^(7/3)\*(d\*x^3+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(7/3)\*(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(7/3)\*(c + d\*x^3), x)

[Out] int((a + b\*x^3)^(7/3)\*(c + d\*x^3), x)

**sympy** [C] time = 10.99, size = 265, normalized size = 3.12

$$\frac{a^{\frac{7}{3}} cx \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{7}{3}} dx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}} bcx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{2a^{\frac{4}{3}} bdx^7 \Gamma\left(\frac{7}{3}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(7/3)\*(d\*x\*\*3+c), x)

[Out] a\*\*(7/3)\*c\*x\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + a\*\*(7/3)\*d\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + 2\*a\*\*(4/3)\*b\*c\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + 2\*a\*\*(4/3)\*b\*d\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + a\*\*(1/3)\*b\*\*2\*c\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + a\*\*(1/3)\*b\*\*2\*d\*x\*\*10\*gamma(10/3)\*hyper((-1/3, 10/3), (13/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(13/3))

### 3.65 $\int (a + bx^3)^{4/3} (c + dx^3) dx$

**Optimal.** Leaf size=83

$$\frac{ax\sqrt[3]{a+bx^3}(8bc-ad) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}}+1} + \frac{dx(a+bx^3)^{7/3}}{8b}$$

[Out]  $1/8*d*x*(b*x^3+a)^{(7/3)}/b+1/8*a*(-a*d+8*b*c)*x*(b*x^3+a)^{(1/3)}*\text{hypergeom}([-4/3, 1/3], [4/3], -b*x^3/a)/b/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3}(8bc-ad) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{8b\sqrt[3]{\frac{bx^3}{a}}+1} + \frac{dx(a+bx^3)^{7/3}}{8b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^{(4/3)}*(c + d*x^3), x]$

[Out]  $(d*x*(a + b*x^3)^{(7/3)})/(8*b) + (a*(8*b*c - a*d)*x*(a + b*x^3)^{(1/3)}*\text{Hypergeometric2F1}[-4/3, 1/3, 4/3, -((b*x^3)/a)])/(8*b*(1 + (b*x^3)/a)^{(1/3)})$

#### Rule 245

$\text{Int}[(a_ + (b_.)*(x_ )^{(n_ )})^{(p_ )}, x\_Symbol] := \text{Simp}[a^{p_}*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& !\text{IGtQ}[p, 0] \&\& !\text{IntegerQ}[1/n] \&\& !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \&\& (\text{IntegerQ}[p] || \text{GtQ}[a, 0])$

#### Rule 246

$\text{Int}[(a_ + (b_.)*(x_ )^{(n_ )})^{(p_ )}, x\_Symbol] := \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}[\{a, b, n, p\}, x] \&\& !\text{IGtQ}[p, 0] \&\& !\text{IntegerQ}[1/n] \&\& !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \&\& !(\text{IntegerQ}[p] || \text{GtQ}[a, 0])$

#### Rule 388

$\text{Int}[(a_ + (b_.)*(x_ )^{(n_ )})^{(p_ )}*((c_ ) + (d_.)*(x_ )^{(n_ )}), x\_Symbol] := \text{Simp}[(d*x*(a + b*x^n)^{(p+1)})/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n*(p+1) + 1, 0]$

#### Rubi steps



$$\begin{aligned}
\int (a + bx^3)^{4/3} (c + dx^3) dx &= \frac{dx (a + bx^3)^{7/3}}{8b} - \frac{(-8bc + ad) \int (a + bx^3)^{4/3} dx}{8b} \\
&= \frac{dx (a + bx^3)^{7/3}}{8b} - \frac{\left(a(-8bc + ad)\sqrt[3]{a + bx^3}\right) \int \left(1 + \frac{bx^3}{a}\right)^{4/3} dx}{8b\sqrt[3]{1 + \frac{bx^3}{a}}} \\
&= \frac{dx (a + bx^3)^{7/3}}{8b} + \frac{a(8bc - ad)x\sqrt[3]{a + bx^3} {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{8b\sqrt[3]{1 + \frac{bx^3}{a}}}
\end{aligned}$$

**Mathematica** [A] time = 0.14, size = 75, normalized size = 0.90

$$\frac{x\sqrt[3]{a + bx^3} \left( d(a + bx^3)^2 - \frac{a(ad - 8bc) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a} + 1}} \right)}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(4/3)\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^(1/3)\*(d\*(a + b\*x^3)^2 - (a\*(-8\*b\*c + a\*d)\*Hypergeometric2F1[-4/3, 1/3, 4/3, -((b\*x^3)/a)])/(1 + (b\*x^3)/a)^(1/3))/(8\*b)

**fricas** [F] time = 0.71, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bdx^6 + (bc + ad)x^3 + ac\right)(bx^3 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)\*(d\*x^3+c), x, algorithm="fricas")

[Out] integral((b\*d\*x^6 + (b\*c + a\*d)\*x^3 + a\*c)\*(b\*x^3 + a)^(1/3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)\*(d\*x^3+c), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)\*(d\*x^3 + c), x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(4/3)\*(d\*x^3+c), x)

[Out] int((b\*x^3+a)^(4/3)\*(d\*x^3+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}}(dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)\*(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(4/3)\*(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{\frac{4}{3}}(dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(4/3)\*(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(4/3)\*(c + d\*x^3), x)

**sympy** [C] time = 7.40, size = 170, normalized size = 2.05

$$\frac{a^{\frac{4}{3}}cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^{\frac{4}{3}}dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bcx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a}bdx^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(4/3)\*(d\*x\*\*3+c),x)

[Out] a\*\*(4/3)\*c\*x\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + a\*\*(4/3)\*d\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(1/3)\*b\*c\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(1/3)\*b\*d\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3))

### 3.66 $\int \sqrt[3]{a + bx^3} (c + dx^3) dx$

Optimal. Leaf size=82

$$\frac{x\sqrt[3]{a + bx^3} (5bc - ad) {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}} + 1} + \frac{dx(a + bx^3)^{4/3}}{5b}$$

[Out] 1/5\*d\*x\*(b\*x^3+a)^(4/3)/b+1/5\*(-a\*d+5\*b\*c)\*x\*(b\*x^3+a)^(1/3)\*hypergeom([-1/3, 1/3], [4/3], -b\*x^3/a)/b/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 246, 245}

$$\frac{x\sqrt[3]{a + bx^3} (5bc - ad) {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5b\sqrt[3]{\frac{bx^3}{a}} + 1} + \frac{dx(a + bx^3)^{4/3}}{5b}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(1/3)\*(c + d\*x^3), x]

[Out] (d\*x\*(a + b\*x^3)^(4/3))/(5\*b) + ((5\*b\*c - a\*d)\*x\*(a + b\*x^3)^(1/3)\*Hypergeometric2F1[-1/3, 1/3, 4/3, -((b\*x^3)/a)])/(5\*b\*(1 + (b\*x^3)/a)^(1/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \sqrt[3]{a+bx^3} (c+dx^3) dx &= \frac{dx(a+bx^3)^{4/3}}{5b} - \frac{(-5bc+ad) \int \sqrt[3]{a+bx^3} dx}{5b} \\
&= \frac{dx(a+bx^3)^{4/3}}{5b} - \frac{\left((-5bc+ad)\sqrt[3]{a+bx^3}\right) \int \sqrt[3]{1+\frac{bx^3}{a}} dx}{5b\sqrt[3]{1+\frac{bx^3}{a}}} \\
&= \frac{dx(a+bx^3)^{4/3}}{5b} + \frac{(5bc-ad)x\sqrt[3]{a+bx^3} {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5b\sqrt[3]{1+\frac{bx^3}{a}}}
\end{aligned}$$

**Mathematica [A]** time = 0.14, size = 72, normalized size = 0.88

$$\frac{x\sqrt[3]{a+bx^3} \left( \frac{(5bc-ad) {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\sqrt[3]{\frac{bx^3}{a}+1}} + d(a+bx^3) \right)}{5b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(1/3)\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^(1/3)\*(d\*(a + b\*x^3) + ((5\*b\*c - a\*d)\*Hypergeometric2F1[-1/3, 1/3, 4/3, -(b\*x^3)/a]))/(1 + (b\*x^3)/a)^(1/3))/(5\*b)

**fricas [F]** time = 0.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3+a\right)^{\frac{1}{3}}(dx^3+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)\*(d\*x^3+c), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/3)\*(d\*x^3 + c), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)\*(d\*x^3+c), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)\*(d\*x^3 + c), x)

**maple [F]** time = 0.40, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/3)\*(d\*x^3+c), x)

[Out] int((b\*x^3+a)^(1/3)\*(d\*x^3+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)\*(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/3)\*(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{1/3} (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)\*(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(1/3)\*(c + d\*x^3), x)

**sympy** [C] time = 4.97, size = 82, normalized size = 1.00

$$\frac{\sqrt[3]{a} cx \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{\sqrt[3]{a} dx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/3)\*(d\*x\*\*3+c),x)

[Out] a\*\*(1/3)\*c\*x\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + a\*\*(1/3)\*d\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3))

$$3.67 \quad \int \frac{c+dx^3}{(a+bx^3)^{2/3}} dx$$

**Optimal.** Leaf size=82

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (2bc - ad) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{2b (a + bx^3)^{2/3}} + \frac{dx \sqrt[3]{a + bx^3}}{2b}$$

[Out] 1/2\*d\*x\*(b\*x^3+a)^(1/3)/b+1/2\*(-a\*d+2\*b\*c)\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 2/3], [4/3], -b\*x^3/a)/b/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.02, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {388, 246, 245}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (2bc - ad) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{2b (a + bx^3)^{2/3}} + \frac{dx \sqrt[3]{a + bx^3}}{2b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(2/3), x]

[Out] (d\*x\*(a + b\*x^3)^(1/3))/(2\*b) + ((2\*b\*c - a\*d)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(2\*b\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{c + dx^3}{(a + bx^3)^{2/3}} dx &= \frac{dx \sqrt[3]{a + bx^3}}{2b} - \frac{(-2bc + ad) \int \frac{1}{(a+bx^3)^{2/3}} dx}{2b} \\ &= \frac{dx \sqrt[3]{a + bx^3}}{2b} - \frac{\left( (-2bc + ad) \left( 1 + \frac{bx^3}{a} \right)^{2/3} \right) \int \frac{1}{\left( 1 + \frac{bx^3}{a} \right)^{2/3}} dx}{2b (a + bx^3)^{2/3}} \\ &= \frac{dx \sqrt[3]{a + bx^3}}{2b} + \frac{(2bc - ad)x \left( 1 + \frac{bx^3}{a} \right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2b (a + bx^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 73, normalized size = 0.89

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (2bc - ad) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + dx (a + bx^3)}{2b (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(2/3), x]

[Out] (d\*x\*(a + b\*x^3) + (2\*b\*c - a\*d)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a])/(2\*b\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(2/3), x, algorithm="fricas")

[Out] integral((d\*x^3 + c)/(b\*x^3 + a)^(2/3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(2/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(2/3), x)

**maple [F]** time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(2/3),x)`

[Out] `int((d*x^3+c)/(b*x^3+a)^(2/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(2/3),x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)/(b*x^3 + a)^(2/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{dx^3 + c}{(bx^3 + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(2/3),x)`

[Out] `int((c + d*x^3)/(a + b*x^3)^(2/3), x)`

**sympy** [C] time = 5.07, size = 78, normalized size = 0.95

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(2/3),x)`

[Out] `c*x*gamma(1/3)*hyper((1/3, 2/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((2/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(2/3)*gamma(7/3))`



$$3.68 \quad \int \frac{c+dx^3}{(a+bx^3)^{5/3}} dx$$

**Optimal.** Leaf size=93

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (ad + bc) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{2ab (a + bx^3)^{2/3}} + \frac{x(bc - ad)}{2ab (a + bx^3)^{2/3}}$$

[Out] 1/2\*(-a\*d+b\*c)\*x/a/b/(b\*x^3+a)^(2/3)+1/2\*(a\*d+b\*c)\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 2/3], [4/3], -b\*x^3/a)/a/b/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {385, 246, 245}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (ad + bc) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{2ab (a + bx^3)^{2/3}} + \frac{x(bc - ad)}{2ab (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(5/3), x]

[Out] ((b\*c - a\*d)\*x)/(2\*a\*b\*(a + b\*x^3)^(2/3)) + ((b\*c + a\*d)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -((b\*x^3)/a)]/(2\*a\*b\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{c + dx^3}{(a + bx^3)^{5/3}} dx &= \frac{(bc - ad)x}{2ab(a + bx^3)^{2/3}} + \frac{(bc + ad) \int \frac{1}{(a+bx^3)^{2/3}} dx}{2ab} \\
&= \frac{(bc - ad)x}{2ab(a + bx^3)^{2/3}} + \frac{\left( (bc + ad) \left(1 + \frac{bx^3}{a}\right)^{2/3} \right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{2ab(a + bx^3)^{2/3}} \\
&= \frac{(bc - ad)x}{2ab(a + bx^3)^{2/3}} + \frac{(bc + ad)x \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2ab(a + bx^3)^{2/3}}
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 66, normalized size = 0.71

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} (ad + bc) {}_2F_1\left(\frac{1}{3}, \frac{5}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) - adx}{ab(a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(5/3), x]

[Out]  $(-(a*d*x) + (b*c + a*d)*x*(1 + (b*x^3)/a)^{(2/3)}*Hypergeometric2F1[1/3, 5/3, 4/3, -(b*x^3)/a])/(a*b*(a + b*x^3)^{(2/3)})$

**fricas** [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(5/3), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(5/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(5/3), x)

**maple** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(5/3),x)`

[Out] `int((d*x^3+c)/(b*x^3+a)^(5/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(5/3),x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)/(b*x^3 + a)^(5/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(5/3),x)`

[Out] `int((c + d*x^3)/(a + b*x^3)^(5/3), x)`

**sympy** [C] time = 16.36, size = 78, normalized size = 0.84

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{5}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{5}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(5/3),x)`

[Out] `c*x*gamma(1/3)*hyper((1/3, 5/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 5/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(5/3)*gamma(7/3))`

$$3.69 \quad \int \frac{c+dx^3}{(a+bx^3)^{8/3}} dx$$

**Optimal.** Leaf size=94

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (ad + 4bc) {}_2F_1 \left( \frac{1}{3}, \frac{5}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{5a^2b (a + bx^3)^{2/3}} + \frac{x(bc - ad)}{5ab (a + bx^3)^{5/3}}$$

[Out] 1/5\*(-a\*d+b\*c)\*x/a/b/(b\*x^3+a)^(5/3)+1/5\*(a\*d+4\*b\*c)\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 5/3], [4/3], -b\*x^3/a)/a^2/b/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {385, 246, 245}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} (ad + 4bc) {}_2F_1 \left( \frac{1}{3}, \frac{5}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right)}{5a^2b (a + bx^3)^{2/3}} + \frac{x(bc - ad)}{5ab (a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)/(a + b\*x^3)^(8/3), x]

[Out] ((b\*c - a\*d)\*x)/(5\*a\*b\*(a + b\*x^3)^(5/3)) + ((4\*b\*c + a\*d)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 5/3, 4/3, -(b\*x^3)/a])/(5\*a^2\*b\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{c + dx^3}{(a + bx^3)^{8/3}} dx &= \frac{(bc - ad)x}{5ab(a + bx^3)^{5/3}} + \frac{(4bc + ad) \int \frac{1}{(a+bx^3)^{5/3}} dx}{5ab} \\ &= \frac{(bc - ad)x}{5ab(a + bx^3)^{5/3}} + \frac{\left( (4bc + ad) \left(1 + \frac{bx^3}{a}\right)^{2/3} \right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{5/3}} dx}{5a^2b(a + bx^3)^{2/3}} \\ &= \frac{(bc - ad)x}{5ab(a + bx^3)^{5/3}} + \frac{(4bc + ad)x \left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{5}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5a^2b(a + bx^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 75, normalized size = 0.80

$$\frac{x \left( \frac{(a+bx^3) \left(\frac{bx^3}{a} + 1\right)^{2/3} (ad+4bc) {}_2F_1\left(\frac{1}{3}, \frac{8}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{a^2} - d \right)}{4b(a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)/(a + b\*x^3)^(8/3), x]

[Out] (x\*(-d + ((4\*b\*c + a\*d)\*(a + b\*x^3)\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 8/3, 4/3, -(b\*x^3)/a])/a^2))/(4\*b\*(a + b\*x^3)^(5/3))

**fricas [F]** time = 0.66, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{1/3} (dx^3 + c)}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(8/3), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)/(b^3\*x^9 + 3\*a\*b^2\*x^6 + 3\*a^2\*b\*x^3 + a^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{8/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)/(b\*x^3+a)^(8/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)/(b\*x^3 + a)^(8/3), x)

**maple [F]** time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{8/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)/(b*x^3+a)^(8/3),x)`

[Out] `int((d*x^3+c)/(b*x^3+a)^(8/3),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)/(b*x^3+a)^(8/3),x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)/(b*x^3 + a)^(8/3), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{dx^3 + c}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)/(a + b*x^3)^(8/3),x)`

[Out] `int((c + d*x^3)/(a + b*x^3)^(8/3), x)`

**sympy** [C] time = 117.70, size = 78, normalized size = 0.83

$$\frac{cx\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{8}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{4}{3}\right)} + \frac{dx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, \frac{8}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{8}{3}}\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)/(b*x**3+a)**(8/3),x)`

[Out] `c*x*gamma(1/3)*hyper((1/3, 8/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8/3)*gamma(4/3)) + d*x**4*gamma(4/3)*hyper((4/3, 8/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**(8/3)*gamma(7/3))`

$$3.70 \quad \int (a + bx^3)^{5/3} (c + dx^3)^2 dx$$

**Optimal.** Leaf size=262

$$\frac{x(a + bx^3)^{5/3} (a^2d^2 - 6abcd + 27b^2c^2)}{162b^2} + \frac{5ax(a + bx^3)^{2/3} (a^2d^2 - 6abcd + 27b^2c^2)}{486b^2} - \frac{5a^2(a^2d^2 - 6abcd + 27b^2c^2)}{486b^2}$$

[Out]  $5/486*a*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+1/162*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(5/3)/b^2+1/108*d*(-4*a*d+15*b*c)*x*(b*x^3+a)^(8/3)/b^2+1/12*d*x*(b*x^3+a)^(8/3)*(d*x^3+c)/b-5/486*a^2*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*\ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+5/729*a^2*(a^2*d^2-6*a*b*c*d+27*b^2*c^2)*\arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)$

**Rubi [A]** time = 0.16, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 195, 239}

$$\frac{x(a + bx^3)^{5/3} (a^2d^2 - 6abcd + 27b^2c^2)}{162b^2} + \frac{5ax(a + bx^3)^{2/3} (a^2d^2 - 6abcd + 27b^2c^2)}{486b^2} - \frac{5a^2(a^2d^2 - 6abcd + 27b^2c^2)}{486b^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(5/3)\*(c + d\*x^3)^2,x]

[Out]  $(5*a*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(2/3))/(486*b^2) + ((27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(5/3))/(162*b^2) + (d*(15*b*c - 4*a*d)*x*(a + b*x^3)^(8/3))/(108*b^2) + (d*x*(a + b*x^3)^(8/3)*(c + d*x^3))/(12*b) + (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*\text{ArcTan}[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/\text{Sqrt}[3]])/(243*\text{Sqrt}[3]*b^(7/3)) - (5*a^2*(27*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*\text{Log}[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(486*b^(7/3))$

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^{5/3} (c + dx^3)^2 dx &= \frac{dx (a + bx^3)^{8/3} (c + dx^3)}{12b} + \frac{\int (a + bx^3)^{5/3} (c(12bc - ad) + d(15bc - 4ad)x^3) dx}{12b} \\ &= \frac{d(15bc - 4ad)x (a + bx^3)^{8/3}}{108b^2} + \frac{dx (a + bx^3)^{8/3} (c + dx^3)}{12b} + \frac{(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{5/3}}{27b^2} \\ &= \frac{(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{5/3}}{162b^2} + \frac{d(15bc - 4ad)x (a + bx^3)^{8/3}}{108b^2} + \frac{dx (a + bx^3)^{5/3}}{12b} \\ &= \frac{5a(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{5/3}}{162b^2} \\ &= \frac{5a(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{2/3}}{486b^2} + \frac{(27b^2c^2 - 6abcd + a^2d^2)x (a + bx^3)^{5/3}}{162b^2} \end{aligned}$$

**Mathematica** [A] time = 5.19, size = 238, normalized size = 0.91

$$10a^2 (a^2d^2 - 6abcd + 27b^2c^2) \left( \log \left( \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1 \right) - 2 \log \left( 1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}} \right) \right) + 3\sqrt[3]{b}x^3$$

2916

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(5/3)\*(c + d\*x^3)^2,x]

[Out] (3\*b^(1/3)\*x\*(a + b\*x^3)^(2/3)\*(-20\*a^3\*d^2 + 15\*a^2\*b\*d\*(8\*c + d\*x^3) + 27\*b^3\*x^3\*(6\*c^2 + 8\*c\*d\*x^3 + 3\*d^2\*x^6) + 18\*a\*b^2\*(24\*c^2 + 22\*c\*d\*x^3 + 7\*d^2\*x^6)) + 10\*a^2\*(27\*b^2\*c^2 - 6\*a\*b\*c\*d + a^2\*d^2)\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(2916\*b^(7/3))

**fricas** [A] time = 0.85, size = 717, normalized size = 2.74

$$\left[ \frac{30 \sqrt{\frac{1}{3}} (27 a^2 b^3 c^2 - 6 a^3 b^2 c d + a^4 b d^2) \sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log \left( 3 b x^3 - 3 (b x^3 + a)^{\frac{1}{3}} (-b)^{\frac{2}{3}} x^2 - 3 \sqrt{\frac{1}{3}} \left( (-b)^{\frac{1}{3}} b x^3 - (b x^3 + a)^{\frac{1}{3}} \right) \right)}{2916 b^{\frac{7}{3}}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] [1/2916\*(30\*sqrt(1/3)\*(27\*a^2\*b^3\*c^2 - 6\*a^3\*b^2\*c\*d + a^4\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 20\*(27\*a^2\*b^2\*c^2 - 6\*a^3\*b\*c\*d + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 10\*(27\*a^2\*b^2\*c^2 - 6\*a^3\*b\*c\*d + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*(81\*b^4\*d^2\*x^10 + 18\*(12\*b^4\*c\*d + 7\*a\*b^3\*d^2)\*x^7 + 3\*(54\*b^4\*c^2 + 132\*a\*b^3\*c\*d + 5\*a^2\*b^2\*d^2)\*x^4 + 4\*(108\*a\*b^3\*c^2 + 30\*a^2\*b^2\*c\*d - 5\*a^3\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3))/b^3, -1/2916\*(60\*sqrt(1/3)\*(27\*a^2\*b^3\*c^2 - 6\*a^3\*b^2\*c\*d + a^4\*b\*d^2)\*sqrt(-(-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt(-(-b)^(1/3)/b)/x) + 20\*(27\*a^2\*b^2\*c^2 - 6\*a^3\*b\*c\*d + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - 10\*(27\*a^2\*b^2\*c^2 - 6\*a^3\*b\*c\*d + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(81\*b^4\*d^2\*x^10 + 18\*(12\*b^4\*c\*d + 7\*a\*b^3\*d^2)\*x^7 + 3\*(54\*b^4\*c^2 + 132\*a\*b^3\*c\*d + 5\*a^2\*b^2\*d^2)\*x^4 + 4\*(108\*a\*b^3\*c^2 + 30\*a^2\*b^2\*c\*d - 5\*a^3\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3))/b^3]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{5}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)^2, x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{5}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(5/3)\*(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(5/3)\*(d\*x^3+c)^2,x)

**maxima** [B] time = 1.29, size = 672, normalized size = 2.56

$$\frac{1}{54} \left( \frac{10 \sqrt{3} a^2 \arctan \left( \frac{\sqrt{3} \left( \frac{1}{b^{\frac{1}{3}}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x} \right)}{3 b^{\frac{1}{3}}} \right)}{b^{\frac{1}{3}}} - \frac{5 a^2 \log \left( b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}} b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2} \right)}{b^{\frac{1}{3}}} + \frac{10 a^2 \log \left( -b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x} \right)}{b^{\frac{1}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)\*(d\*x^3+c)^2,x, algorithm="maxima")

[Out] -1/54\*(10\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(1/3) - 5\*a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 10\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3)

$3) + 3*(5*(b*x^3 + a)^{(2/3)}*a^2*b/x^2 - 8*(b*x^3 + a)^{(5/3)}*a^2/x^5)/(b^2 - 2*(b*x^3 + a)*b/x^3 + (b*x^3 + a)^2/x^6)*c^2 + 1/243*(10*\sqrt{3}*a^3*\arctan(1/3*\sqrt{3}*(b^{(1/3)} + 2*(b*x^3 + a)^{(1/3)}/x)/b^{(1/3)})/b^{(4/3)} - 5*a^3*\log(b^{(2/3)} + (b*x^3 + a)^{(1/3)}*b^{(1/3)}/x + (b*x^3 + a)^{(2/3)}/x^2)/b^{(4/3)} + 10*a^3*\log(-b^{(1/3)} + (b*x^3 + a)^{(1/3)}/x)/b^{(4/3)} + 3*(5*(b*x^3 + a)^{(2/3)}*a^3*b^2/x^2 - 13*(b*x^3 + a)^{(5/3)}*a^3*b/x^5 - 10*(b*x^3 + a)^{(8/3)}*a^3/x^8)/(b^4 - 3*(b*x^3 + a)*b^3/x^3 + 3*(b*x^3 + a)^2*b^2/x^6 - (b*x^3 + a)^3*b/x^9))*c*d - 1/2916*(20*\sqrt{3}*a^4*\arctan(1/3*\sqrt{3}*(b^{(1/3)} + 2*(b*x^3 + a)^{(1/3)}/x)/b^{(1/3)})/b^{(7/3)} - 10*a^4*\log(b^{(2/3)} + (b*x^3 + a)^{(1/3)}*b^{(1/3)}/x + (b*x^3 + a)^{(2/3)}/x^2)/b^{(7/3)} + 20*a^4*\log(-b^{(1/3)} + (b*x^3 + a)^{(1/3)}/x)/b^{(7/3)} + 3*(10*(b*x^3 + a)^{(2/3)}*a^4*b^3/x^2 - 36*(b*x^3 + a)^{(5/3)}*a^4*b^2/x^5 - 75*(b*x^3 + a)^{(8/3)}*a^4*b/x^8 + 20*(b*x^3 + a)^{(11/3)}*a^4/x^11)/(b^6 - 4*(b*x^3 + a)*b^5/x^3 + 6*(b*x^3 + a)^2*b^4/x^6 - 4*(b*x^3 + a)^3*b^3/x^9 + (b*x^3 + a)^4*b^2/x^12))*d^2$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (bx^3 + a)^{5/3} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(5/3)\*(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(5/3)\*(c + d\*x^3)^2, x)

**sympy [C]** time = 13.13, size = 270, normalized size = 1.03

$$\frac{a^{\frac{5}{3}}c^2x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{5}{3}}cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{5}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{a^{\frac{2}{3}}bc^2x^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{10}{3} \\ \frac{13}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{13}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(5/3)\*(d\*x\*\*3+c)\*\*2,x)

[Out] a\*\*(5/3)\*c\*\*2\*x\*gamma(1/3)\*hyper((-2/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + 2\*a\*\*(5/3)\*c\*d\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(5/3)\*d\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-2/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + a\*\*(2/3)\*b\*c\*\*2\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + 2\*a\*\*(2/3)\*b\*c\*d\*x\*\*7\*gamma(7/3)\*hyper((-2/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + a\*\*(2/3)\*b\*d\*\*2\*x\*\*10\*gamma(10/3)\*hyper((-2/3, 10/3), (13/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(13/3))

### 3.71 $\int (a + bx^3)^{2/3} (c + dx^3)^2 dx$

**Optimal.** Leaf size=219

$$\frac{x(a + bx^3)^{2/3} (2a^2d^2 - 9abcd + 27b^2c^2)}{81b^2} - \frac{a(2a^2d^2 - 9abcd + 27b^2c^2) \log(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x)}{81b^{7/3}} + \frac{2a(2a^2d^2 - 9abcd + 27b^2c^2)}{81b^2}$$

```
[Out] 1/81*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*x*(b*x^3+a)^(2/3)/b^2+2/27*d*(-a*d+3*b*c)*x*(b*x^3+a)^(5/3)/b^2+1/9*d*x*(b*x^3+a)^(5/3)*(d*x^3+c)/b-1/81*a*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*ln(-b^(1/3)*x+(b*x^3+a)^(1/3))/b^(7/3)+2/243*a*(2*a^2*d^2-9*a*b*c*d+27*b^2*c^2)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/b^(7/3)*3^(1/2)
```

**Rubi [A]** time = 0.17, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 195, 239}

$$\frac{x(a + bx^3)^{2/3} (2a^2d^2 - 9abcd + 27b^2c^2)}{81b^2} - \frac{a(2a^2d^2 - 9abcd + 27b^2c^2) \log(\sqrt[3]{a + bx^3} - \sqrt[3]{b}x)}{81b^{7/3}} + \frac{2a(2a^2d^2 - 9abcd + 27b^2c^2)}{81b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^3)^(2/3)*(c + d*x^3)^2,x]
```

```
[Out] ((27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(2/3))/(81*b^2) + (2*d*(3*b*c - a*d)*x*(a + b*x^3)^(5/3))/(27*b^2) + (d*x*(a + b*x^3)^(5/3)*(c + d*x^3))/(9*b) + (2*a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*ArcTan[(1 + (2*b^(1/3)*x)/(a + b*x^3)^(1/3))/Sqrt[3]])/(81*Sqrt[3]*b^(7/3)) - (a*(27*b^2*c^2 - 9*a*b*c*d + 2*a^2*d^2)*Log[-(b^(1/3)*x) + (a + b*x^3)^(1/3)])/(81*b^(7/3))
```

#### Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

#### Rule 239

```
Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]
```

#### Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

#### Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
```

x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^{2/3} (c + dx^3)^2 dx &= \frac{dx (a + bx^3)^{5/3} (c + dx^3)}{9b} + \frac{\int (a + bx^3)^{2/3} (c(9bc - ad) + 4d(3bc - ad)x^3) dx}{9b} \\ &= \frac{2d(3bc - ad)x (a + bx^3)^{5/3}}{27b^2} + \frac{dx (a + bx^3)^{5/3} (c + dx^3)}{9b} - \frac{(4ad(3bc - ad) - 6bc(9b^2c^2 - 9abcd + 2a^2d^2)) (a + bx^3)^{2/3}}{81b^2} \\ &= \frac{(27b^2c^2 - 9abcd + 2a^2d^2) x (a + bx^3)^{2/3}}{81b^2} + \frac{2d(3bc - ad)x (a + bx^3)^{5/3}}{27b^2} + \frac{dx (a + bx^3)^{5/3} (c + dx^3)}{9b} \\ &= \frac{(27b^2c^2 - 9abcd + 2a^2d^2) x (a + bx^3)^{2/3}}{81b^2} + \frac{2d(3bc - ad)x (a + bx^3)^{5/3}}{27b^2} + \frac{dx (a + bx^3)^{5/3} (c + dx^3)}{9b} \end{aligned}$$

**Mathematica** [A] time = 5.17, size = 203, normalized size = 0.93

$$\frac{3\sqrt[3]{b} x (a + bx^3)^{2/3} (-4a^2d^2 + 3abd(6c + dx^3) + 9b^2(3c^2 + 3cdx^3 + d^2x^6)) + a(2a^2d^2 - 9abcd + 27b^2c^2) \log\left(\frac{3bx^3 - 3(bx^3 + a)^{1/3}(-b)^{2/3}x^2 - 3\sqrt[3]{1/3}\left((-b)^{1/3}bx^3 - (bx^3 + a)^{1/3}\right)}{(a + bx^3)^{2/3}}\right)}{243b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(2/3)\*(c + d\*x^3)^2,x]

[Out] (3\*b^(1/3)\*x\*(a + b\*x^3)^(2/3)\*(-4\*a^2\*d^2 + 3\*a\*b\*d\*(6\*c + d\*x^3) + 9\*b^2\*(3\*c^2 + 3\*c\*d\*x^3 + d^2\*x^6)) + a\*(27\*b^2\*c^2 - 9\*a\*b\*c\*d + 2\*a^2\*d^2)\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(243\*b^(7/3))

**fricas** [A] time = 0.63, size = 634, normalized size = 2.89

$$\left[ \frac{3\sqrt{\frac{1}{3}}(27ab^3c^2 - 9a^2b^2cd + 2a^3bd^2)\sqrt{\frac{(-b)^{1/3}}{b}} \log\left(3bx^3 - 3(bx^3 + a)^{1/3}(-b)^{2/3}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{1/3}bx^3 - (bx^3 + a)^{1/3}\right)\right)}{(a + bx^3)^{2/3}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] [1/243\*(3\*sqrt(1/3)\*(27\*a\*b^3\*c^2 - 9\*a^2\*b^2\*c\*d + 2\*a^3\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-

$$b^{1/3}bx^3 - (bx^3 + a)^{1/3}bx^2 + 2(bx^3 + a)^{2/3}(-b)^{2/3}x) \sqrt{(-b)^{1/3}/b} + 2a - 2(27ab^2c^2 - 9a^2b^2cd + 2a^3d^2)(-b)^{2/3} \log\left(\frac{(-b)^{1/3}x + (bx^3 + a)^{1/3}}{x}\right) + (27ab^2c^2 - 9a^2b^2cd + 2a^3d^2)(-b)^{2/3} \log\left(\frac{(-b)^{2/3}x^2 - (bx^3 + a)^{1/3}(-b)^{1/3}x + (bx^3 + a)^{2/3}}{x^2}\right) + 3(9b^3d^2x^7 + 3(9b^3cd + ab^2d^2)x^4 + (27b^3c^2 + 18ab^2cd - 4a^2bd^2)x)(bx^3 + a)^{2/3} / b^3, -1/243(6\sqrt{1/3})(27ab^3c^2 - 9a^2b^2cd + 2a^3bd^2)\sqrt{(-b)^{1/3}/b} \arctan\left(\frac{-\sqrt{1/3}((-b)^{1/3}x - 2(bx^3 + a)^{1/3})\sqrt{(-b)^{1/3}/b}}{x}\right) + 2(27ab^2c^2 - 9a^2b^2cd + 2a^3d^2)(-b)^{2/3} \log\left(\frac{(-b)^{1/3}x + (bx^3 + a)^{1/3}}{x}\right) - (27ab^2c^2 - 9a^2b^2cd + 2a^3d^2)(-b)^{2/3} \log\left(\frac{(-b)^{2/3}x^2 - (bx^3 + a)^{1/3}(-b)^{1/3}x + (bx^3 + a)^{2/3}}{x^2}\right) - 3(9b^3d^2x^7 + 3(9b^3cd + ab^2d^2)x^4 + (27b^3c^2 + 18ab^2cd - 4a^2bd^2)x)(bx^3 + a)^{2/3} / b^3]$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{2}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)^2, x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{2}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(2/3)\*(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(2/3)\*(d\*x^3+c)^2,x)

**maxima** [B] time = 1.48, size = 552, normalized size = 2.52

$$\frac{1}{9} \left( \frac{2\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2a \log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} + \frac{3(bx^3+a)^{\frac{2}{3}}}{(b - (bx^3+a)/x^3)x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)\*(d\*x^3+c)^2,x, algorithm="maxima")

[Out]  $-1/9(2\sqrt{3}a \arctan(1/3\sqrt{3}(b^{1/3} + 2(bx^3 + a)^{1/3}/x)/b^{1/3})/b^{1/3} - a \log(b^{2/3} + (bx^3 + a)^{1/3}b^{1/3}/x + (bx^3 + a)^{2/3}/x^2)/b^{1/3} + 2a \log(-b^{1/3} + (bx^3 + a)^{1/3}/x)/b^{1/3} + 3(bx^3 + a)^{2/3}a/((b - (bx^3 + a)/x^3)x^2) + 1/27(2\sqrt{3}a^2 \arctan(1/3\sqrt{3}(b^{1/3} + 2(bx^3 + a)^{1/3}/x)/b^{1/3})/b^{4/3} - a^2 \log(b^{2/3} + (bx^3 + a)^{1/3}b^{1/3}/x + (bx^3 + a)^{2/3}/x^2)/b^{4/3} + 2a^2 \log(-b^{1/3} + (bx^3 + a)^{1/3}/x)/b^{4/3} + 3((bx^3 + a)^{2/3}a^2 * b/x^2 + 2(bx^3 + a)^{5/3}a^2/x^5)/(b^3 - 2(bx^3 + a)b^2/x^3 + (bx^3 + a)^2/x^3)$

+ a)<sup>2</sup>\*b/x<sup>6</sup>))\*c\*d - 1/243\*(4\*sqrt(3)\*a<sup>3</sup>\*arctan(1/3\*sqrt(3)\*(b<sup>1/3</sup> + 2\*(b\*x<sup>3</sup> + a)<sup>1/3</sup>)/x)/b<sup>1/3</sup>)/b<sup>7/3</sup> - 2\*a<sup>3</sup>\*log(b<sup>2/3</sup> + (b\*x<sup>3</sup> + a)<sup>1/3</sup>)/b<sup>7/3</sup> + 4\*a<sup>3</sup>\*log(-b<sup>1/3</sup> + (b\*x<sup>3</sup> + a)<sup>1/3</sup>)/x)/b<sup>7/3</sup> + 3\*(2\*(b\*x<sup>3</sup> + a)<sup>2/3</sup>\*a<sup>3</sup>\*b<sup>2</sup>/x<sup>2</sup> + 11\*(b\*x<sup>3</sup> + a)<sup>5/3</sup>\*a<sup>3</sup>\*b/x<sup>5</sup> - 4\*(b\*x<sup>3</sup> + a)<sup>8/3</sup>\*a<sup>3</sup>/x<sup>8</sup>)/(b<sup>5</sup> - 3\*(b\*x<sup>3</sup> + a)\*b<sup>4</sup>/x<sup>3</sup> + 3\*(b\*x<sup>3</sup> + a)<sup>2</sup>\*b<sup>3</sup>/x<sup>6</sup> - (b\*x<sup>3</sup> + a)<sup>3</sup>\*b<sup>2</sup>/x<sup>9</sup>))\*d<sup>2</sup>

**mupad [F]** time = 0.00, size = -1, normalized size = -0.00

$$\int (bx^3 + a)^{2/3} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)\*(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(2/3)\*(c + d\*x^3)^2, x)

**sympy [C]** time = 7.27, size = 131, normalized size = 0.60

$$\frac{a^{2/3}c^2x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{2/3}cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{2/3}d^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(2/3)\*(d\*x\*\*3+c)\*\*2,x)

[Out] a\*\*(2/3)\*c\*\*2\*x\*gamma(1/3)\*hyper((-2/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + 2\*a\*\*(2/3)\*c\*d\*x\*\*4\*gamma(4/3)\*hyper((-2/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(2/3)\*d\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-2/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3))

$$3.72 \quad \int \frac{(c+dx^3)^2}{\sqrt[3]{a+bx^3}} dx$$

Optimal. Leaf size=175

$$\frac{(2a^2d^2 - 6abcd + 9b^2c^2) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18b^{7/3}} + \frac{(2a^2d^2 - 6abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} + \frac{dx(a+bx^3)^{2/3}}{18b^{7/3}}$$

[Out] 1/18\*d\*(-4\*a\*d+9\*b\*c)\*x\*(b\*x^3+a)^(2/3)/b^2+1/6\*d\*x\*(b\*x^3+a)^(2/3)\*(d\*x^3+c)/b-1/18\*(2\*a^2\*d^2-6\*a\*b\*c\*d+9\*b^2\*c^2)\*ln(-b^(1/3)\*x+(b\*x^3+a)^(1/3))/b^(7/3)+1/27\*(2\*a^2\*d^2-6\*a\*b\*c\*d+9\*b^2\*c^2)\*arctan(1/3\*(1+2\*b^(1/3)\*x/(b\*x^3+a)^(1/3))\*3^(1/2))/b^(7/3)\*3^(1/2)

Rubi [A] time = 0.10, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {416, 388, 239}

$$\frac{(2a^2d^2 - 6abcd + 9b^2c^2) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18b^{7/3}} + \frac{(2a^2d^2 - 6abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3}b^{7/3}} + \frac{dx(a+bx^3)^{2/3}}{18b^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(1/3), x]

[Out] (d\*(9\*b\*c - 4\*a\*d)\*x\*(a + b\*x^3)^(2/3))/(18\*b^2) + (d\*x\*(a + b\*x^3)^(2/3)\*(c + d\*x^3))/(6\*b) + ((9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]])/(9\*Sqrt[3]\*b^(7/3)) - ((9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*Log[-(b^(1/3)\*x) + (a + b\*x^3)^(1/3)])/(18\*b^(7/3))

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q)+1)), x] + Dist[1/(b\*(n\*(p+q)+1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q)+1) - a\*d) + d\*(b\*c\*(n\*(p+2\*q-1)+1) - a\*d\*(n\*(q-1)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^2}{\sqrt[3]{a + bx^3}} dx &= \frac{dx (a + bx^3)^{2/3} (c + dx^3)}{6b} + \frac{\int \frac{c(6bc - ad) + d(9bc - 4ad)x^3}{\sqrt[3]{a + bx^3}} dx}{6b} \\ &= \frac{d(9bc - 4ad)x (a + bx^3)^{2/3}}{18b^2} + \frac{dx (a + bx^3)^{2/3} (c + dx^3)}{6b} + \frac{(9b^2c^2 - 6abcd + 2a^2d^2) \int \frac{1}{\sqrt[3]{a + bx^3}} dx}{9b^2} \\ &= \frac{d(9bc - 4ad)x (a + bx^3)^{2/3}}{18b^2} + \frac{dx (a + bx^3)^{2/3} (c + dx^3)}{6b} + \frac{(9b^2c^2 - 6abcd + 2a^2d^2) \tan^{-1} \left( \frac{1 + \frac{2}{\sqrt[3]{a + bx^3}}}{\sqrt[3]{a + bx^3}} \right)}{9\sqrt{3} b^{7/3}} \end{aligned}$$

**Mathematica [A]** time = 5.15, size = 172, normalized size = 0.98

$$\frac{(2a^2d^2 - 6abcd + 9b^2c^2) \left( \log \left( \frac{b^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1 \right) - 2 \log \left( 1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}} \right) \right) + 3\sqrt[3]{b} dx}{54b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(1/3), x]

[Out] (3\*b^(1/3)\*d\*x\*(a + b\*x^3)^(2/3)\*(-4\*a\*d + 3\*b\*(4\*c + d\*x^3)) + (9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(54\*b^(7/3))

**fricas [A]** time = 0.73, size = 554, normalized size = 3.17

$$\left[ \frac{3\sqrt{\frac{1}{3}}(9b^3c^2 - 6ab^2cd + 2a^2bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log \left( 3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}} \left( (-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)^{\frac{1}{3}}bx^2 \right) \right)}{54b^{7/3}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(1/3), x, algorithm="fricas")

[Out] [1/54\*(3\*sqrt(1/3)\*(9\*b^3\*c^2 - 6\*a\*b^2\*c\*d + 2\*a^2\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 2\*(9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + (9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*(3\*b^2\*d^2\*x^4 + 4\*(3\*b^2\*c\*d - a\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3)/b^3, -1/54\*(6\*sqrt(1/3)\*(9\*b^3\*c^2 - 6\*a\*b^2\*c\*d + 2\*a^2\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*((-b)^(1/3)\*x - 2\*(b\*x^3 + a)^(1/3))\*sqrt((-b)^(1/3)/b)/x) + 2\*(9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) - (9\*b^2\*c^2 - 6\*a\*b\*c\*d + 2\*a^2\*d^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(3\*b^2\*d^2\*x^4 + 4\*(3\*b^2\*c\*d - a\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3)/b^3]



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(1/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(1/3), x)

**maple** [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(1/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(1/3),x)

**maxima** [B] time = 1.28, size = 436, normalized size = 2.49

$$\frac{1}{6} \left( \frac{2\sqrt{3} \arctan\left(\frac{\sqrt{3}\left(\frac{1}{b^{\frac{1}{3}}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}} - \frac{\log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{1}{3}}} + \frac{2\log\left(-b^{\frac{1}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}}{x}\right)}{b^{\frac{1}{3}}} \right) c^2 + \frac{1}{9} \left( \frac{2\sqrt{3}}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(1/3),x, algorithm="maxima")

[Out] -1/6\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3)))/b^(1/3) - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(1/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(1/3))\*c^2 + 1/9\*(2\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) - a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3) - 6\*(b\*x^3 + a)^(2/3)\*a/((b^2 - (b\*x^3 + a)\*b/x^3)\*x^2))\*c\*d - 1/54\*(4\*sqrt(3)\*a^2\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2\*a^2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*a^2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3) - 3\*(7\*(b\*x^3 + a)^(2/3)\*a^2\*b/x^2 - 4\*(b\*x^3 + a)^(5/3)\*a^2/x^5)/(b^4 - 2\*(b\*x^3 + a)\*b^3/x^3 + (b\*x^3 + a)^2\*b^2/x^6))\*d^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^2/(a + b*x^3)^(1/3),x)`

[Out] `int((c + d*x^3)^2/(a + b*x^3)^(1/3), x)`

**sympy** [C] time = 6.44, size = 126, normalized size = 0.72

$$\frac{c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{1}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a} \Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a} \Gamma\left(\frac{7}{3}\right)} + \frac{d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{7}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\sqrt[3]{a} \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**2/(b*x**3+a)**(1/3),x)`

[Out] `c**2*x*gamma(1/3)*hyper((1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*a**  
*(1/3)*gamma(4/3)) + 2*c*d*x**4*gamma(4/3)*hyper((1/3, 4/3), (7/3,), b*x**3  
*exp_polar(I*pi)/a)/(3*a** (1/3)*gamma(7/3)) + d**2*x**7*gamma(7/3)*hyper((1  
/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*a** (1/3)*gamma(10/3))`

$$3.73 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{4/3}} dx$$

**Optimal.** Leaf size=159

$$\frac{d(3bc - 2ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{bx}\right)}{3b^{7/3}} + \frac{2d(3bc - 2ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{dx(a + bx^3)^{2/3}(3bc - 4ad)}{3ab^2} + \frac{x(c + d}{ab\sqrt[3]{a+bx^3}}$$

[Out]  $-1/3*d*(-4*a*d+3*b*c)*x*(b*x^3+a)^{(2/3)}/a/b^2+(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^{(1/3)}-1/3*d*(-2*a*d+3*b*c)*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/b^{(7/3)}+2/9*d*(-2*a*d+3*b*c)*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/b^{(7/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {413, 388, 239}

$$\frac{dx(a + bx^3)^{2/3}(3bc - 4ad)}{3ab^2} - \frac{d(3bc - 2ad) \log\left(\sqrt[3]{a + bx^3} - \sqrt[3]{bx}\right)}{3b^{7/3}} + \frac{2d(3bc - 2ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} + \frac{x(c + d}{ab\sqrt[3]{a+bx^3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(4/3), x]

[Out]  $-(d*(3*b*c - 4*a*d)*x*(a + b*x^3)^{(2/3)})/(3*a*b^2) + ((b*c - a*d)*x*(c + d*x^3))/(a*b*(a + b*x^3)^{(1/3)}) + (2*d*(3*b*c - 2*a*d)*\text{ArcTan}[(1 + (2*b^{(1/3)}*x)/(a + b*x^3)^{(1/3)})/\text{Sqrt}[3]])/(3*\text{Sqrt}[3]*b^{(7/3)}) - (d*(3*b*c - 2*a*d)*\text{Log}[-(b^{(1/3)}*x) + (a + b*x^3)^{(1/3)})]/(3*b^{(7/3)})$

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3], x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{4/3}} dx = \frac{(bc - ad)x(c + dx^3)}{ab\sqrt[3]{a + bx^3}} + \frac{\int \frac{acd - d(3bc - 4ad)x^3}{\sqrt[3]{a + bx^3}} dx}{ab}$$

$$= -\frac{d(3bc - 4ad)x(a + bx^3)^{2/3}}{3ab^2} + \frac{(bc - ad)x(c + dx^3)}{ab\sqrt[3]{a + bx^3}} + \frac{(2d(3bc - 2ad)) \int \frac{1}{\sqrt[3]{a + bx^3}} dx}{3b^2}$$

$$= -\frac{d(3bc - 4ad)x(a + bx^3)^{2/3}}{3ab^2} + \frac{(bc - ad)x(c + dx^3)}{ab\sqrt[3]{a + bx^3}} + \frac{2d(3bc - 2ad) \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}b^{7/3}} - \frac{d(3bc - 2ad)}{3b^2}$$

**Mathematica [A]** time = 5.16, size = 168, normalized size = 1.06

$$\frac{d(3bc - 2ad) \left( \log\left(\frac{b^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}} + 1}{\sqrt{3}}\right) \right)}{9b^{7/3}} + \frac{x(a + bx^3)^{2/3} \left(\frac{3(bc - ad)}{a(a + bx^3)}\right)}{3b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(4/3), x]

[Out] (x\*(a + b\*x^3)^(2/3)\*(d^2 + (3\*(b\*c - a\*d)^2)/(a\*(a + b\*x^3))))/(3\*b^2) + (d\*(3\*b\*c - 2\*a\*d)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3)]/Sqrt[3]) - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/(9\*b^(7/3))

**fricas [B]** time = 0.58, size = 652, normalized size = 4.10

$$\frac{3\sqrt{\frac{1}{3}}(3a^2b^2cd - 2a^3bd^2 + (3ab^3cd - 2a^2b^2d^2)x^3)\sqrt{-\frac{1}{b^3}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left(b^{\frac{4}{3}}x^3 + (bx^3 + a)^{\frac{1}{3}}b^{\frac{2}{3}}x^2\right)\right)}{9b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(4/3), x, algorithm="fricas")

[Out] [-1/9\*(3\*sqrt(1/3)\*(3\*a^2\*b^2\*c\*d - 2\*a^3\*b\*d^2 + (3\*a\*b^3\*c\*d - 2\*a^2\*b^2\*d^2)\*x^3)\*sqrt(-1/b^(2/3))\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*b^(2/3)\*x^2 - 3\*sqrt(1/3)\*(b^(4/3)\*x^3 + (b\*x^3 + a)^(1/3)\*b\*x^2 - 2\*(b\*x^3 + a)^(2/3)\*b^(2/3)\*x)\*sqrt(-1/b^(2/3)) + 2\*a) + 2\*(3\*a^2\*b\*c\*d - 2\*a^3\*d^2 + (3\*a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (3\*a^2\*b\*c\*d - 2\*a^3\*d^2 + (3\*a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) - 3\*(a\*b^2\*d^2\*x^4 + (3\*b^3\*c^2 - 6\*a\*b^2\*c\*d + 4\*a^2\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3))/(a\*b^4\*x^3 + a^2\*b^3), -1/9\*(2\*(3\*a^2\*b\*c\*d - 2\*a^3\*d^2 + (3\*a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*b^(2/3)\*log(-(b^(1/3)\*x - (b\*x^3 + a)^(1/3))/x) - (3\*a^2\*b\*c\*d - 2\*a^3\*d^2 + (3\*a\*b^2\*c\*d - 2\*a^2\*b\*d^2)\*x^3)\*b^(2/3)\*log((b^(2/3)\*x^2 + (b\*x^3 + a)^(1/3)\*b^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 6\*sqrt(1/3)\*(3\*a^2\*b

$$\begin{aligned} &^2*c*d - 2*a^3*b*d^2 + (3*a*b^3*c*d - 2*a^2*b^2*d^2)*x^3)*\arctan(\sqrt{1/3}* \\ &(b^{(1/3)}*x + 2*(b*x^3 + a)^{(1/3)})/(b^{(1/3)}*x))/b^{(1/3)} - 3*(a*b^2*d^2*x^4 + \\ &(3*b^3*c^2 - 6*a*b^2*c*d + 4*a^2*b*d^2)*x)*(b*x^3 + a)^{(2/3)})/(a*b^4*x^3 + \\ &a^2*b^3) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(4/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(4/3), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(4/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(4/3),x)

**maxima** [B] time = 1.14, size = 301, normalized size = 1.89

$$\frac{1}{9}d^2 \left( \frac{4\sqrt{3}a \arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}} + \frac{2(bx^3+a)^{\frac{1}{3}}}{x}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}} + \frac{3\left(3ab - \frac{4(bx^3+a)a}{x^3}\right)}{\frac{(bx^3+a)^{\frac{1}{3}}b^3}{x} - \frac{(bx^3+a)^{\frac{4}{3}}b^2}{x^4}} - \frac{2a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} + \frac{4a \log\left(b^{\frac{2}{3}} + \frac{(bx^3+a)^{\frac{1}{3}}b^{\frac{1}{3}}}{x} + \frac{(bx^3+a)^{\frac{2}{3}}}{x^2}\right)}{b^{\frac{7}{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(4/3),x, algorithm="maxima")

[Out] 1/9\*d^2\*(4\*sqrt(3)\*a\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) + 3\*(3\*a\*b - 4\*(b\*x^3 + a)\*a/x^3)/((b\*x^3 + a)^(1/3)\*b^3/x - (b\*x^3 + a)^(4/3)\*b^2/x^4) - 2\*a\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*a\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3) - 1/3\*c\*d\*(2\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(4/3) + 6\*x/((b\*x^3 + a)^(1/3)\*b - log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(4/3) + 2\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(4/3)) + c^2\*x/((b\*x^3 + a)^(1/3)\*a)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)`

[Out] `int((c + d*x^3)^2/(a + b*x^3)^(4/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**2/(b*x**3+a)**(4/3), x)`

[Out] `Integral((c + d*x**3)**2/(a + b*x**3)**(4/3), x)`

$$3.74 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{7/3}} dx$$

**Optimal.** Leaf size=152

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}} - \frac{d^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2b^{7/3}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} + \frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

[Out] 1/4\*(-a\*d+b\*c)\*(4\*a\*d+3\*b\*c)\*x/a^2/b^2/(b\*x^3+a)^(1/3)+1/4\*(-a\*d+b\*c)\*x\*(d\*x^3+c)/a/b/(b\*x^3+a)^(4/3)-1/2\*d^2\*ln(-b^(1/3)\*x+(b\*x^3+a)^(1/3))/b^(7/3)+1/3\*d^2\*arctan(1/3\*(1+2\*b^(1/3)\*x/(b\*x^3+a)^(1/3))\*3^(1/2))/b^(7/3)\*3^(1/2)

**Rubi [A]** time = 0.07, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, number of rules / integrand size = 0.143, Rules used = {413, 385, 239}

$$\frac{x(bc-ad)(4ad+3bc)}{4a^2b^2\sqrt[3]{a+bx^3}} - \frac{d^2 \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{2b^{7/3}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}}+1}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} + \frac{x(c+dx^3)(bc-ad)}{4ab(a+bx^3)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(7/3), x]

[Out] ((b\*c - a\*d)\*(3\*b\*c + 4\*a\*d)\*x)/(4\*a^2\*b^2\*(a + b\*x^3)^(1/3)) + ((b\*c - a\*d)\*x\*(c + d\*x^3))/(4\*a\*b\*(a + b\*x^3)^(4/3)) + (d^2\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*b^(7/3)) - (d^2\*Log[-(b^(1/3)\*x) + (a + b\*x^3)^(1/3)])/(2\*b^(7/3)))

#### Rule 239

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1/3), x\_Symbol] := Simp[ArcTan[(1 + (2\*Rt[b, 3]\*x)/(a + b\*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]\*Rt[b, 3]), x] - Simp[Log[(a + b\*x^3)^(1/3) - Rt[b, 3]\*x]/(2\*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1) + 1)) + d\*(a\*d\*(n\*(q-1) + 1) - b\*c\*(n\*(p+q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^2}{(a + bx^3)^{7/3}} dx &= \frac{(bc - ad)x(c + dx^3)}{4ab(a + bx^3)^{4/3}} + \frac{\int \frac{c(3bc + ad) + 4ad^2x^3}{(a + bx^3)^{4/3}} dx}{4ab} \\
&= \frac{(bc - ad)(3bc + 4ad)x}{4a^2b^2\sqrt[3]{a + bx^3}} + \frac{(bc - ad)x(c + dx^3)}{4ab(a + bx^3)^{4/3}} + \frac{d^2 \int \frac{1}{\sqrt[3]{a + bx^3}} dx}{b^2} \\
&= \frac{(bc - ad)(3bc + 4ad)x}{4a^2b^2\sqrt[3]{a + bx^3}} + \frac{(bc - ad)x(c + dx^3)}{4ab(a + bx^3)^{4/3}} + \frac{d^2 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{\sqrt{3}b^{7/3}} - \frac{d^2 \log\left(-\sqrt[3]{b}x + \sqrt[3]{a}\right)}{2b^{7/3}}
\end{aligned}$$

**Mathematica [A]** time = 5.25, size = 180, normalized size = 1.18

$$\frac{x((a + bx^3)(-5a^2d^2 + 2abcd + 3b^2c^2) + a(bc - ad)^2)}{4a^2b^2(a + bx^3)^{4/3}} + \frac{d^2 \left( \log\left(\frac{b^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}} + 1\right) - 2 \log\left(1 - \frac{\sqrt[3]{bx}}{\sqrt[3]{a + bx^3}}\right) + 2 \right)}{6b^{7/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(7/3), x]

[Out] (x\*(a\*(b\*c - a\*d)^2 + (3\*b^2\*c^2 + 2\*a\*b\*c\*d - 5\*a^2\*d^2)\*(a + b\*x^3)))/(4\*a^2\*b^2\*(a + b\*x^3)^(4/3)) + (d^2\*(2\*sqrt[3]\*ArcTan[(1 + (2\*b^(1/3)\*x)/(a + b\*x^3)^(1/3))/sqrt[3]] - 2\*Log[1 - (b^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[1 + (b^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (b^(1/3)\*x)/(a + b\*x^3)^(1/3)]))/(6\*b^(7/3))

**fricas [B]** time = 0.74, size = 719, normalized size = 4.73

$$\frac{6\sqrt{\frac{1}{3}}(a^2b^3d^2x^6 + 2a^3b^2d^2x^3 + a^4bd^2)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}} \log\left(3bx^3 - 3(bx^3 + a)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x^2 - 3\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^3 - (bx^3 + a)\right)\right)}{6b^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(7/3), x, algorithm="fricas")

[Out] [1/12\*(6\*sqrt(1/3)\*(a^2\*b^3\*d^2\*x^6 + 2\*a^3\*b^2\*d^2\*x^3 + a^4\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*log(3\*b\*x^3 - 3\*(b\*x^3 + a)^(1/3)\*(-b)^(2/3)\*x^2 - 3\*sqrt(1/3)\*((-b)^(1/3)\*b\*x^3 - (b\*x^3 + a)^(1/3)\*b\*x^2 + 2\*(b\*x^3 + a)^(2/3)\*(-b)^(2/3)\*x)\*sqrt((-b)^(1/3)/b) + 2\*a) - 4\*(a^2\*b^2\*d^2\*x^6 + 2\*a^3\*b\*d^2\*x^3 + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(1/3)\*x + (b\*x^3 + a)^(1/3))/x) + 2\*(a^2\*b^2\*d^2\*x^6 + 2\*a^3\*b\*d^2\*x^3 + a^4\*d^2)\*(-b)^(2/3)\*log(((b)^(2/3)\*x^2 - (b\*x^3 + a)^(1/3)\*(-b)^(1/3)\*x + (b\*x^3 + a)^(2/3))/x^2) + 3\*((3\*b^4\*c^2 + 2\*a\*b^3\*c\*d - 5\*a^2\*b^2\*d^2)\*x^4 + 4\*(a\*b^3\*c^2 - a^3\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3))/(a^2\*b^5\*x^6 + 2\*a^3\*b^4\*x^3 + a^4\*b^3), -1/12\*(12\*sqrt(1/3)\*(a^2\*b^3\*d^2\*x^6 + 2\*a^3\*b^2\*d^2\*x^3 + a^4\*b\*d^2)\*sqrt((-b)^(1/3)/b)\*arctan(-sqrt(1/3)\*



$$\begin{aligned} & ((-b)^{1/3}x - 2*(b*x^3 + a)^{1/3})*\text{sqrt}(-(-b)^{1/3}/b)/x + 4*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^{2/3}*\log(((b*x^3 + a)^{1/3})/x) \\ & - 2*(a^2*b^2*d^2*x^6 + 2*a^3*b*d^2*x^3 + a^4*d^2)*(-b)^{2/3}*\log(((b*x^3 + a)^{2/3})/x^2) \\ & - 3*((3*b^4*c^2 + 2*a*b^3*c*d - 5*a^2*b^2*d^2)*x^4 + 4*(a*b^3*c^2 - a^3*b*d^2)*x)*(b*x^3 + a)^{2/3}/(a^2*b^5*x^6 + 2*a^3*b^4*x^3 + a^4*b^3) \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(7/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(7/3), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(7/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(7/3),x)

**maxima** [A] time = 1.19, size = 190, normalized size = 1.25

$$\frac{\left( b - \frac{4(bx^3+a)}{x^3} \right) c^2 x^4}{4(bx^3+a)^{4/3} a^2} + \frac{cdx^4}{2(bx^3+a)^{4/3} a} - \frac{1}{12} \left( \frac{3 \left( b + \frac{4(bx^3+a)}{x^3} \right) x^4}{(bx^3+a)^{4/3} b^2} + \frac{4\sqrt{3} \arctan \left( \frac{\sqrt{3} \left( \frac{1}{b^3} + \frac{2(bx^3+a)^{1/3}}{x} \right)}{3b^{1/3}} \right)}{b^{7/3}} - 2 \log \left( b^{2/3} + \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(7/3),x, algorithm="maxima")

[Out] -1/4\*(b - 4\*(b\*x^3 + a)/x^3)\*c^2\*x^4/((b\*x^3 + a)^(4/3)\*a^2) + 1/2\*c\*d\*x^4/((b\*x^3 + a)^(4/3)\*a) - 1/12\*(3\*(b + 4\*(b\*x^3 + a)/x^3)\*x^4/((b\*x^3 + a)^(4/3)\*b^2) + 4\*sqrt(3)\*arctan(1/3\*sqrt(3)\*(b^(1/3) + 2\*(b\*x^3 + a)^(1/3)/x)/b^(1/3))/b^(7/3) - 2\*log(b^(2/3) + (b\*x^3 + a)^(1/3)\*b^(1/3)/x + (b\*x^3 + a)^(2/3)/x^2)/b^(7/3) + 4\*log(-b^(1/3) + (b\*x^3 + a)^(1/3)/x)/b^(7/3))\*d^2

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{7/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)`

[Out] `int((c + d*x^3)^2/(a + b*x^3)^(7/3), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{7}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**2/(b*x**3+a)**(7/3), x)`

[Out] `Integral((c + d*x**3)**2/(a + b*x**3)**(7/3), x)`

$$3.75 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}} dx$$

Optimal. Leaf size=78

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

[Out]  $9/14*c^2*x/a^3/(b*x^3+a)^{(1/3)}+3/14*c*x*(d*x^3+c)/a^2/(b*x^3+a)^{(4/3)}+1/7*x*(d*x^3+c)^2/a/(b*x^3+a)^{(7/3)}$

Rubi [A] time = 0.02, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {378, 191}

$$\frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(10/3), x]

[Out]  $(9*c^2*x)/(14*a^3*(a + b*x^3)^{(1/3)}) + (3*c*x*(c + d*x^3))/(14*a^2*(a + b*x^3)^{(4/3)}) + (x*(c + d*x^3)^2)/(7*a*(a + b*x^3)^{(7/3)})$

Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}} dx &= \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{(6c) \int \frac{c+dx^3}{(a+bx^3)^{7/3}} dx}{7a} \\ &= \frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}} + \frac{(9c^2) \int \frac{1}{(a+bx^3)^{4/3}} dx}{14a^2} \\ &= \frac{9c^2x}{14a^3\sqrt[3]{a+bx^3}} + \frac{3cx(c+dx^3)}{14a^2(a+bx^3)^{4/3}} + \frac{x(c+dx^3)^2}{7a(a+bx^3)^{7/3}} \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 126, normalized size = 1.62

$$\frac{x^3 \sqrt{\frac{bx^3}{a}} + 1 \left( a^2 (14c^2 + 7cdx^3 + 2d^2x^6) + 3abcx^3 (7c + dx^3) + 9b^2c^2x^6 \right)}{14a^3 (a + bx^3)^{7/3} \sqrt[3]{\frac{dx^3}{c}} + 1 \sqrt[3]{\frac{c(a+bx^3)}{a(c+dx^3)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(10/3), x]

[Out] (x\*(1 + (b\*x^3)/a)^(1/3)\*(9\*b^2\*c^2\*x^6 + 3\*a\*b\*c\*x^3\*(7\*c + d\*x^3) + a^2\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6)))/(14\*a^3\*(a + b\*x^3)^(7/3)\*((c\*(a + b\*x^3))/(a\*(c + d\*x^3)))^(1/3)\*(1 + (d\*x^3)/c)^(1/3))

**fricas** [A] time = 0.62, size = 103, normalized size = 1.32

$$\frac{\left( (9b^2c^2 + 3abcd + 2a^2d^2)x^7 + 14a^2c^2x + 7(3abc^2 + a^2cd)x^4 \right) (bx^3 + a)^{\frac{2}{3}}}{14(a^3b^3x^9 + 3a^4b^2x^6 + 3a^5bx^3 + a^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(10/3), x, algorithm="fricas")

[Out] 1/14\*((9\*b^2\*c^2 + 3\*a\*b\*c\*d + 2\*a^2\*d^2)\*x^7 + 14\*a^2\*c^2\*x + 7\*(3\*a\*b\*c^2 + a^2\*c\*d)\*x^4)\*(b\*x^3 + a)^(2/3)/(a^3\*b^3\*x^9 + 3\*a^4\*b^2\*x^6 + 3\*a^5\*b\*x^3 + a^6)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{10}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(10/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(10/3), x)

**maple** [A] time = 0.05, size = 76, normalized size = 0.97

$$\frac{\left( 2a^2d^2x^6 + 3abcdx^6 + 9b^2c^2x^6 + 7a^2cdx^3 + 21abc^2x^3 + 14a^2c^2 \right) x}{14(bx^3 + a)^{\frac{7}{3}} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(10/3), x)

[Out] 1/14\*x\*(2\*a^2\*d^2\*x^6+3\*a\*b\*c\*d\*x^6+9\*b^2\*c^2\*x^6+7\*a^2\*c\*d\*x^3+21\*a\*b\*c^2\*x^3+14\*a^2\*c^2)/(b\*x^3+a)^(7/3)/a^3

**maxima** [A] time = 0.51, size = 109, normalized size = 1.40

$$-\frac{\left( 4b - \frac{7(bx^3+a)}{x^3} \right) cdx^7}{14(bx^3 + a)^{\frac{7}{3}} a^2} + \frac{d^2x^7}{7(bx^3 + a)^{\frac{7}{3}} a} + \frac{\left( 2b^2 - \frac{7(bx^3+a)b}{x^3} + \frac{14(bx^3+a)^2}{x^6} \right) c^2x^7}{14(bx^3 + a)^{\frac{7}{3}} a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(10/3),x, algorithm="maxima")

[Out] 
$$-1/14*(4*b - 7*(b*x^3 + a)/x^3)*c*d*x^7/((b*x^3 + a)^(7/3)*a^2) + 1/7*d^2*x^7/((b*x^3 + a)^(7/3)*a) + 1/14*(2*b^2 - 7*(b*x^3 + a)*b/x^3 + 14*(b*x^3 + a)^2/x^6)*c^2*x^7/((b*x^3 + a)^(7/3)*a^3)$$

**mupad [B]** time = 1.43, size = 148, normalized size = 1.90

$$\frac{2a^4d^2x + 2a^2d^2x(bx^3 + a)^2 + 9b^2c^2x(bx^3 + a)^2 + 2a^2b^2c^2x - 4a^3d^2x(bx^3 + a) + 3ab^2c^2x(bx^3 + a)}{14a^3b^2(bx^3 + a)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(10/3),x)

[Out] 
$$(2*a^4*d^2*x + 2*a^2*d^2*x*(a + b*x^3)^2 + 9*b^2*c^2*x*(a + b*x^3)^2 + 2*a^2*b^2*c^2*x - 4*a^3*d^2*x*(a + b*x^3) + 3*a*b^2*c^2*x*(a + b*x^3) - 4*a^3*b*c*d*x + 3*a*b*c*d*x*(a + b*x^3)^2 + a^2*b*c*d*x*(a + b*x^3))/(14*a^3*b^2*(a + b*x^3)^(7/3))$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*(10/3),x)

[Out] Timed out

$$3.76 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}} dx$$

**Optimal.** Leaf size=174

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}(bc-ad)} + \frac{3cx(c+dx^3)(9bc-10ad)}{140a^3(a+bx^3)^{4/3}(bc-ad)} + \frac{x(c+dx^3)^2(9bc-10ad)}{70a^2(a+bx^3)^{7/3}(bc-ad)} + \frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

[Out]  $9/140*c^2*(-10*a*d+9*b*c)*x/a^4/(-a*d+b*c)/(b*x^3+a)^{(1/3)}+3/140*c*(-10*a*d+9*b*c)*x*(d*x^3+c)/a^3/(-a*d+b*c)/(b*x^3+a)^{(4/3)}+1/70*(-10*a*d+9*b*c)*x*(d*x^3+c)^2/a^2/(-a*d+b*c)/(b*x^3+a)^{(7/3)}+1/10*b*x*(d*x^3+c)^3/a/(-a*d+b*c)/(b*x^3+a)^{(10/3)}$

**Rubi [A]** time = 0.07, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {382, 378, 191}

$$\frac{9c^2x(9bc-10ad)}{140a^4\sqrt[3]{a+bx^3}(bc-ad)} + \frac{x(c+dx^3)^2(9bc-10ad)}{70a^2(a+bx^3)^{7/3}(bc-ad)} + \frac{3cx(c+dx^3)(9bc-10ad)}{140a^3(a+bx^3)^{4/3}(bc-ad)} + \frac{bx(c+dx^3)^3}{10a(a+bx^3)^{10/3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(13/3), x]

[Out]  $(9*c^2*(9*b*c - 10*a*d)*x)/(140*a^4*(b*c - a*d)*(a + b*x^3)^{(1/3)}) + (3*c*(9*b*c - 10*a*d)*x*(c + d*x^3))/(140*a^3*(b*c - a*d)*(a + b*x^3)^{(4/3)}) + ((9*b*c - 10*a*d)*x*(c + d*x^3)^2)/(70*a^2*(b*c - a*d)*(a + b*x^3)^{(7/3)}) + (b*x*(c + d*x^3)^3)/(10*a*(b*c - a*d)*(a + b*x^3)^{(10/3)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{(c+dx^3)^2}{(a+bx^3)^{13/3}} dx &= \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \frac{(9bc-10ad) \int \frac{(c+dx^3)^2}{(a+bx^3)^{10/3}} dx}{10a(bc-ad)} \\
&= \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \frac{(3c(9bc-10ad)) \int \frac{c+dx^3}{(a+bx^3)^{7/3}} dx}{35a^2(bc-ad)} \\
&= \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \frac{bx(c+dx^3)^3}{10a(bc-ad)(a+bx^3)^{10/3}} + \\
&= \frac{9c^2(9bc-10ad)x}{140a^4(bc-ad)\sqrt[3]{a+bx^3}} + \frac{3c(9bc-10ad)x(c+dx^3)}{140a^3(bc-ad)(a+bx^3)^{4/3}} + \frac{(9bc-10ad)x(c+dx^3)^2}{70a^2(bc-ad)(a+bx^3)^{7/3}} + \dots
\end{aligned}$$

**Mathematica [A]** time = 5.11, size = 106, normalized size = 0.61

$$\frac{x(10a^3(14c^2 + 7cdx^3 + 2d^2x^6) + 3a^2bx^3(105c^2 + 20cdx^3 + 2d^2x^6) + 18ab^2cx^6(15c + dx^3) + 81b^3c^2x^9)}{140a^4(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(13/3), x]

[Out] (x\*(81\*b^3\*c^2\*x^9 + 18\*a\*b^2\*c\*x^6\*(15\*c + d\*x^3) + 10\*a^3\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6) + 3\*a^2\*b\*x^3\*(105\*c^2 + 20\*c\*d\*x^3 + 2\*d^2\*x^6)))/(140\*a^4\*(a + b\*x^3)^(10/3))

**fricas [A]** time = 0.73, size = 152, normalized size = 0.87

$$\frac{(3(27b^3c^2 + 6ab^2cd + 2a^2bd^2)x^{10} + 10(27ab^2c^2 + 6a^2bcd + 2a^3d^2)x^7 + 140a^3c^2x + 35(9a^2bc^2 + 2a^3cd)x^4)}{140(a^4b^4x^{12} + 4a^5b^3x^9 + 6a^6b^2x^6 + 4a^7bx^3 + a^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(13/3), x, algorithm="fricas")

[Out] 1/140\*(3\*(27\*b^3\*c^2 + 6\*a\*b^2\*c\*d + 2\*a^2\*b\*d^2)\*x^10 + 10\*(27\*a\*b^2\*c^2 + 6\*a^2\*b\*c\*d + 2\*a^3\*d^2)\*x^7 + 140\*a^3\*c^2\*x + 35\*(9\*a^2\*b\*c^2 + 2\*a^3\*c\*d)\*x^4)\*(b\*x^3 + a)^(2/3)/(a^4\*b^4\*x^12 + 4\*a^5\*b^3\*x^9 + 6\*a^6\*b^2\*x^6 + 4\*a^7\*b\*x^3 + a^8)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{13/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(13/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(13/3), x)

**maple [A]** time = 0.05, size = 115, normalized size = 0.66

$$\frac{(6a^2bd^2x^9 + 18ab^2cdx^9 + 81b^3c^2x^9 + 20a^3d^2x^6 + 60a^2bcdx^6 + 270ab^2c^2x^6 + 70a^3cdx^3 + 315a^2b^2c^2x^3 + 140c^2a^2d^2x^3 + 140c^2a^2d^2x^3)}{140(bx^3 + a)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(13/3),x)

[Out] 1/140\*x\*(6\*a^2\*b\*d^2\*x^9+18\*a\*b^2\*c\*d\*x^9+81\*b^3\*c^2\*x^9+20\*a^3\*d^2\*x^6+60\*a^2\*b\*c\*d\*x^6+270\*a\*b^2\*c^2\*x^6+70\*a^3\*c\*d\*x^3+315\*a^2\*b\*c^2\*x^3+140\*a^3\*c^2\*d^2\*x^3)/(b\*x^3+a)^(10/3)/a^4

**maxima [A]** time = 0.71, size = 159, normalized size = 0.91

$$\frac{\left(7b - \frac{10(bx^3+a)}{x^3}\right)d^2x^{10}}{70(bx^3+a)^{\frac{10}{3}}a^2} + \frac{\left(14b^2 - \frac{40(bx^3+a)b}{x^3} + \frac{35(bx^3+a)^2}{x^6}\right)cdx^{10}}{70(bx^3+a)^{\frac{10}{3}}a^3} - \frac{\left(14b^3 - \frac{60(bx^3+a)b^2}{x^3} + \frac{105(bx^3+a)^2b}{x^6} - \frac{140(bx^3+a)^3}{x^9}\right)c^2d^2x^{10}}{140(bx^3+a)^{\frac{10}{3}}a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(13/3),x, algorithm="maxima")

[Out] -1/70\*(7\*b - 10\*(b\*x^3 + a)/x^3)\*d^2\*x^10/((b\*x^3 + a)^(10/3)\*a^2) + 1/70\*(14\*b^2 - 40\*(b\*x^3 + a)\*b/x^3 + 35\*(b\*x^3 + a)^2/x^6)\*c\*d\*x^10/((b\*x^3 + a)^(10/3)\*a^3) - 1/140\*(14\*b^3 - 60\*(b\*x^3 + a)\*b^2/x^3 + 105\*(b\*x^3 + a)^2\*b/x^6 - 140\*(b\*x^3 + a)^3/x^9)\*c^2\*x^10/((b\*x^3 + a)^(10/3)\*a^4)

**mupad [B]** time = 1.45, size = 176, normalized size = 1.01

$$\frac{x \left( \frac{c^2}{10a} + \frac{a \left( \frac{d^2}{10b} - \frac{cd}{5a} \right)}{b} \right)}{(bx^3 + a)^{10/3}} - \frac{x \left( \frac{d^2}{7b^2} - \frac{-a^2d^2 + 2abcd + 9b^2c^2}{70a^2b^2} \right)}{(bx^3 + a)^{7/3}} + \frac{x (2a^2d^2 + 6abcd + 27b^2c^2)}{140a^3b^2(bx^3 + a)^{4/3}} + \frac{x (6a^2d^2 + 18abcd + 81b^3c^2)}{140a^4b^2(bx^3 + a)^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(13/3),x)

[Out] (x\*(c^2/(10\*a) + (a\*(d^2/(10\*b) - (c\*d)/(5\*a)))/b))/(a + b\*x^3)^(10/3) - (x\*(d^2/(7\*b^2) - (9\*b^2\*c^2 - a^2\*d^2 + 2\*a\*b\*c\*d)/(70\*a^2\*b^2)))/(a + b\*x^3)^(7/3) + (x\*(2\*a^2\*d^2 + 27\*b^2\*c^2 + 6\*a\*b\*c\*d))/(140\*a^3\*b^2\*(a + b\*x^3)^(4/3)) + (x\*(6\*a^2\*d^2 + 81\*b^2\*c^2 + 18\*a\*b\*c\*d))/(140\*a^4\*b^2\*(a + b\*x^3)^(1/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*(13/3),x)

[Out] Timed out



$$3.77 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{16/3}} dx$$

**Optimal.** Leaf size=211

$$\frac{2x(bc-ad)(ad+3bc)}{65a^2b^2(a+bx^3)^{10/3}} + \frac{9x(2a^2d^2+9abcd+54b^2c^2)}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{3x(2a^2d^2+9abcd+54b^2c^2)}{910a^4b^2(a+bx^3)^{4/3}} + \frac{x(2a^2d^2+9abcd+54b^2c^2)}{455a^3b^2(a+bx^3)^{7/3}}$$

[Out]  $2/65*(-a*d+b*c)*(a*d+3*b*c)*x/a^2/b^2/(b*x^3+a)^{(10/3)}+1/455*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^3/b^2/(b*x^3+a)^{(7/3)}+3/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^4/b^2/(b*x^3+a)^{(4/3)}+9/910*(2*a^2*d^2+9*a*b*c*d+54*b^2*c^2)*x/a^5/b^2/(b*x^3+a)^{(1/3)}+1/13*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^{(13/3)}$

**Rubi [A]** time = 0.13, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {413, 385, 192, 191}

$$\frac{9x(2a^2d^2+9abcd+54b^2c^2)}{910a^5b^2\sqrt[3]{a+bx^3}} + \frac{3x(2a^2d^2+9abcd+54b^2c^2)}{910a^4b^2(a+bx^3)^{4/3}} + \frac{x(2a^2d^2+9abcd+54b^2c^2)}{455a^3b^2(a+bx^3)^{7/3}} + \frac{2x(bc-ad)(ad+3bc)}{65a^2b^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(16/3), x]

[Out]  $(2*(b*c - a*d)*(3*b*c + a*d)*x)/(65*a^2*b^2*(a + b*x^3)^{(10/3)}) + ((54*b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(455*a^3*b^2*(a + b*x^3)^{(7/3)}) + (3*(54*b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^4*b^2*(a + b*x^3)^{(4/3)}) + (9*(54*b^2*c^2 + 9*a*b*c*d + 2*a^2*d^2)*x)/(910*a^5*b^2*(a + b*x^3)^{(1/3)}) + ((b*c - a*d)*x*(c + d*x^3))/(13*a*b*(a + b*x^3)^{(13/3)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d,

0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx^3)^2}{(a + bx^3)^{16/3}} dx &= \frac{(bc - ad)x(c + dx^3)}{13ab(a + bx^3)^{13/3}} + \frac{\int \frac{c(12bc+ad)+d(9bc+4ad)x^3}{(a+bx^3)^{13/3}} dx}{13ab} \\
 &= \frac{2(bc - ad)(3bc + ad)x}{65a^2b^2(a + bx^3)^{10/3}} + \frac{(bc - ad)x(c + dx^3)}{13ab(a + bx^3)^{13/3}} + \frac{(54b^2c^2 + 9abcd + 2a^2d^2) \int \frac{1}{(a+bx^3)^{10/3}} dx}{65a^2b^2} \\
 &= \frac{2(bc - ad)(3bc + ad)x}{65a^2b^2(a + bx^3)^{10/3}} + \frac{(54b^2c^2 + 9abcd + 2a^2d^2)x}{455a^3b^2(a + bx^3)^{7/3}} + \frac{(bc - ad)x(c + dx^3)}{13ab(a + bx^3)^{13/3}} + \frac{(6(54b^2c^2 - 9abcd + 2a^2d^2)) \int \frac{1}{(a+bx^3)^{7/3}} dx}{65a^2b^2} \\
 &= \frac{2(bc - ad)(3bc + ad)x}{65a^2b^2(a + bx^3)^{10/3}} + \frac{(54b^2c^2 + 9abcd + 2a^2d^2)x}{455a^3b^2(a + bx^3)^{7/3}} + \frac{3(54b^2c^2 + 9abcd + 2a^2d^2)x}{910a^4b^2(a + bx^3)^{4/3}} + \frac{(bc - ad)x(c + dx^3)}{13ab(a + bx^3)^{13/3}} \\
 &= \frac{2(bc - ad)(3bc + ad)x}{65a^2b^2(a + bx^3)^{10/3}} + \frac{(54b^2c^2 + 9abcd + 2a^2d^2)x}{455a^3b^2(a + bx^3)^{7/3}} + \frac{3(54b^2c^2 + 9abcd + 2a^2d^2)x}{910a^4b^2(a + bx^3)^{4/3}} + \frac{(bc - ad)x(c + dx^3)}{13ab(a + bx^3)^{13/3}}
 \end{aligned}$$

**Mathematica [A]** time = 5.18, size = 138, normalized size = 0.65

$$\frac{x(65a^4(14c^2 + 7cdx^3 + 2d^2x^6) + 39a^3bx^3(70c^2 + 15cdx^3 + 2d^2x^6) + 9a^2b^2x^6(390c^2 + 39cdx^3 + 2d^2x^6) + 81ab^3c^2x^9)}{910a^5(a + bx^3)^{13/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(16/3), x]

[Out] (x\*(486\*b^4\*c^2\*x^12 + 81\*a\*b^3\*c\*x^9\*(26\*c + d\*x^3) + 65\*a^4\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6) + 39\*a^3\*b\*x^3\*(70\*c^2 + 15\*c\*d\*x^3 + 2\*d^2\*x^6) + 9\*a^2\*b^2\*x^6\*(390\*c^2 + 39\*c\*d\*x^3 + 2\*d^2\*x^6)))/(910\*a^5\*(a + b\*x^3)^(13/3))

**fricas [A]** time = 0.64, size = 200, normalized size = 0.95

$$\frac{(9(54b^4c^2 + 9ab^3cd + 2a^2b^2d^2)x^{13} + 39(54ab^3c^2 + 9a^2b^2cd + 2a^3bd^2)x^{10} + 65(54a^2b^2c^2 + 9a^3bcd + 2a^4d^2)x^7 + 81ab^3c^2x^9)}{910(a^5b^5x^{15} + 5a^6b^4x^{12} + 10a^7b^3x^9 + 10a^8b^2x^6 + 5a^9bx^3 + a^{10})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(16/3), x, algorithm="fricas")

[Out] 1/910\*(9\*(54\*b^4\*c^2 + 9\*a\*b^3\*c\*d + 2\*a^2\*b^2\*d^2)\*x^13 + 39\*(54\*a\*b^3\*c^2 + 9\*a^2\*b^2\*c\*d + 2\*a^3\*b\*d^2)\*x^10 + 65\*(54\*a^2\*b^2\*c^2 + 9\*a^3\*b\*c\*d + 2\*a^4\*d^2)\*x^7 + 910\*a^4\*c^2\*x + 455\*(6\*a^3\*b\*c^2 + a^4\*c\*d)\*x^4)\*(b\*x^3 + a)^(2/3)/(a^5\*b^5\*x^15 + 5\*a^6\*b^4\*x^12 + 10\*a^7\*b^3\*x^9 + 10\*a^8\*b^2\*x^6 + 5\*a^9\*b\*x^3 + a^10)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{16/3}} dx$$



```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(16/3),x)
```

```
[Out] Timed out
```

$$3.78 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{19/3}} dx$$

**Optimal.** Leaf size=253

$$\frac{x(bc-ad)(4ad+15bc)}{208a^2b^2(a+bx^3)^{13/3}} + \frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}} + \frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}}$$

[Out] 1/208\*(-a\*d+b\*c)\*(4\*a\*d+15\*b\*c)\*x/a^2/b^2/(b\*x^3+a)^(13/3)+1/520\*(a^2\*d^2+6\*a\*b\*c\*d+45\*b^2\*c^2)\*x/a^3/b^2/(b\*x^3+a)^(10/3)+9/3640\*(a^2\*d^2+6\*a\*b\*c\*d+45\*b^2\*c^2)\*x/a^4/b^2/(b\*x^3+a)^(7/3)+27/7280\*(a^2\*d^2+6\*a\*b\*c\*d+45\*b^2\*c^2)\*x/a^5/b^2/(b\*x^3+a)^(4/3)+81/7280\*(a^2\*d^2+6\*a\*b\*c\*d+45\*b^2\*c^2)\*x/a^6/b^2/(b\*x^3+a)^(1/3)+1/16\*(-a\*d+b\*c)\*x\*(d\*x^3+c)/a/b/(b\*x^3+a)^(16/3)

**Rubi [A]** time = 0.21, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {413, 385, 192, 191}

$$\frac{81x(a^2d^2+6abcd+45b^2c^2)}{7280a^6b^2\sqrt[3]{a+bx^3}} + \frac{27x(a^2d^2+6abcd+45b^2c^2)}{7280a^5b^2(a+bx^3)^{4/3}} + \frac{9x(a^2d^2+6abcd+45b^2c^2)}{3640a^4b^2(a+bx^3)^{7/3}} + \frac{x(a^2d^2+6abcd+45b^2c^2)}{520a^3b^2(a+bx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(19/3), x]

[Out] ((b\*c - a\*d)\*(15\*b\*c + 4\*a\*d)\*x)/(208\*a^2\*b^2\*(a + b\*x^3)^(13/3)) + ((45\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x)/(520\*a^3\*b^2\*(a + b\*x^3)^(10/3)) + (9\*(45\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x)/(3640\*a^4\*b^2\*(a + b\*x^3)^(7/3)) + (27\*(45\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x)/(7280\*a^5\*b^2\*(a + b\*x^3)^(4/3)) + (81\*(45\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x)/(7280\*a^6\*b^2\*(a + b\*x^3)^(1/3)) + ((b\*c - a\*d)\*x\*(c + d\*x^3))/(16\*a\*b\*(a + b\*x^3)^(16/3))

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + 1) + 1)), x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + 1) + 1)), 0]

+ q) + 1)) \* x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned}
 \int \frac{(c + dx^3)^2}{(a + bx^3)^{19/3}} dx &= \frac{(bc - ad)x(c + dx^3)}{16ab(a + bx^3)^{16/3}} + \frac{\int \frac{c(15bc + ad) + 4d(3bc + ad)x^3}{(a + bx^3)^{16/3}} dx}{16ab} \\
 &= \frac{(bc - ad)(15bc + 4ad)x}{208a^2b^2(a + bx^3)^{13/3}} + \frac{(bc - ad)x(c + dx^3)}{16ab(a + bx^3)^{16/3}} + \frac{(45b^2c^2 + 6abcd + a^2d^2) \int \frac{1}{(a + bx^3)^{13/3}} dx}{52a^2b^2} \\
 &= \frac{(bc - ad)(15bc + 4ad)x}{208a^2b^2(a + bx^3)^{13/3}} + \frac{(45b^2c^2 + 6abcd + a^2d^2)x}{520a^3b^2(a + bx^3)^{10/3}} + \frac{(bc - ad)x(c + dx^3)}{16ab(a + bx^3)^{16/3}} + \frac{9(45b^2c^2 + 6abcd + a^2d^2) \int \frac{1}{(a + bx^3)^{10/3}} dx}{520a^3b^2} \\
 &= \frac{(bc - ad)(15bc + 4ad)x}{208a^2b^2(a + bx^3)^{13/3}} + \frac{(45b^2c^2 + 6abcd + a^2d^2)x}{520a^3b^2(a + bx^3)^{10/3}} + \frac{9(45b^2c^2 + 6abcd + a^2d^2)x}{3640a^4b^2(a + bx^3)^{7/3}} + \frac{(bc - ad)x(c + dx^3)}{16ab(a + bx^3)^{16/3}} \\
 &= \frac{(bc - ad)(15bc + 4ad)x}{208a^2b^2(a + bx^3)^{13/3}} + \frac{(45b^2c^2 + 6abcd + a^2d^2)x}{520a^3b^2(a + bx^3)^{10/3}} + \frac{9(45b^2c^2 + 6abcd + a^2d^2)x}{3640a^4b^2(a + bx^3)^{7/3}} + \frac{27(45b^2c^2 + 6abcd + a^2d^2) \int \frac{1}{(a + bx^3)^{7/3}} dx}{3640a^4b^2} \\
 &= \frac{(bc - ad)(15bc + 4ad)x}{208a^2b^2(a + bx^3)^{13/3}} + \frac{(45b^2c^2 + 6abcd + a^2d^2)x}{520a^3b^2(a + bx^3)^{10/3}} + \frac{9(45b^2c^2 + 6abcd + a^2d^2)x}{3640a^4b^2(a + bx^3)^{7/3}} + \frac{27(45b^2c^2 + 6abcd + a^2d^2) \int \frac{1}{(a + bx^3)^{4/3}} dx}{3640a^4b^2}
 \end{aligned}$$

**Mathematica [A]** time = 5.15, size = 169, normalized size = 0.67

$$\frac{x(520a^5(14c^2 + 7cdx^3 + 2d^2x^6) + 156a^4bx^3(175c^2 + 40cdx^3 + 6d^2x^6) + 144a^3b^2x^6(325c^2 + 39cdx^3 + 3d^2x^6) + 156a^4b^2x^9(175c^2 + 40cdx^3 + 6d^2x^6))}{7280a^6(a + bx^3)^{16/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(19/3), x]

[Out] (x\*(3645\*b^5\*c^2\*x^15 + 486\*a\*b^4\*c\*x^12\*(40\*c + d\*x^3) + 81\*a^2\*b^3\*x^9\*(520\*c^2 + 32\*c\*d\*x^3 + d^2\*x^6) + 520\*a^5\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6) + 144\*a^3\*b^2\*x^6\*(325\*c^2 + 39\*c\*d\*x^3 + 3\*d^2\*x^6) + 156\*a^4\*b\*x^3\*(175\*c^2 + 40\*c\*d\*x^3 + 6\*d^2\*x^6)))/(7280\*a^6\*(a + b\*x^3)^(16/3))

**fricas [A]** time = 0.83, size = 246, normalized size = 0.97

$$\frac{(81(45b^5c^2 + 6ab^4cd + a^2b^3d^2)x^{16} + 432(45ab^4c^2 + 6a^2b^3cd + a^3b^2d^2)x^{13} + 936(45a^2b^3c^2 + 6a^3b^2cd + a^4bd^2)x^{10} + 520a^5(14c^2 + 7cdx^3 + 2d^2x^6) + 144a^3b^2x^6(325c^2 + 39cdx^3 + 3d^2x^6) + 156a^4b^2x^9(175c^2 + 40cdx^3 + 6d^2x^6))}{7280(a^6b^6x^{18} + 6a^7b^5x^{15} + 15a^8b^4x^{12} + 20a^9b^3x^9 + 15a^{10}b^2x^6 + 6a^{11}bx^3 + a^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(19/3),x, algorithm="fricas")

[Out] 1/7280\*(81\*(45\*b^5\*c^2 + 6\*a\*b^4\*c\*d + a^2\*b^3\*d^2)\*x^16 + 432\*(45\*a\*b^4\*c^2 + 6\*a^2\*b^3\*c\*d + a^3\*b^2\*d^2)\*x^13 + 936\*(45\*a^2\*b^3\*c^2 + 6\*a^3\*b^2\*c\*d + a^4\*b\*d^2)\*x^10 + 7280\*a^5\*c^2\*x + 1040\*(45\*a^3\*b^2\*c^2 + 6\*a^4\*b\*c\*d + a^5\*d^2)\*x^7 + 1820\*(15\*a^4\*b\*c^2 + 2\*a^5\*c\*d)\*x^4)\*(b\*x^3 + a)^(2/3)/(a^6)

$b^6x^{18} + 6a^7b^5x^{15} + 15a^8b^4x^{12} + 20a^9b^3x^9 + 15a^{10}b^2x^6 + 6a^{11}bx^3 + a^{12}$ )

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{19}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(19/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(19/3), x)

**maple** [A] time = 0.05, size = 197, normalized size = 0.78

$(81a^2b^3d^2x^{15} + 486ab^4cdx^{15} + 3645b^5c^2x^{15} + 432a^3b^2d^2x^{12} + 2592a^2b^3cdx^{12} + 19440ab^4c^2x^{12} + 936a^4bd^2x^9 + 616a^3b^2cdx^9 + 42120a^2b^3c^2x^9 + 1040a^5d^2x^6 + 6240a^4b^2cdx^6 + 46800a^3b^2c^2x^6 + 3640a^5c^2dx^3 + 27300a^4b^2c^2x^3 + 7280a^5c^2) / (bx^3 + a)^{\frac{16}{3}} / a^6$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(19/3),x)

[Out]  $1/7280*x*(81*a^2*b^3*d^2*x^{15}+486*a*b^4*c*d*x^{15}+3645*b^5*c^2*x^{15}+432*a^3*b^2*d^2*x^{12}+2592*a^2*b^3*c*d*x^{12}+19440*a*b^4*c^2*x^{12}+936*a^4*b*d^2*x^9+5616*a^3*b^2*c*d*x^9+42120*a^2*b^3*c^2*x^9+1040*a^5*d^2*x^6+6240*a^4*b*c*d*x^6+46800*a^3*b^2*c^2*x^6+3640*a^5*c*d*x^3+27300*a^4*b*c^2*x^3+7280*a^5*c^2) / (bx^3+a)^{\frac{16}{3}} / a^6$

**maxima** [A] time = 0.64, size = 261, normalized size = 1.03

$$\frac{\left(455b^3 - \frac{1680(bx^3+a)b^2}{x^3} + \frac{2184(bx^3+a)^2b}{x^6} - \frac{1040(bx^3+a)^3}{x^9}\right)d^2x^{16} + \left(455b^4 - \frac{2240(bx^3+a)b^3}{x^3} + \frac{4368(bx^3+a)^2b^2}{x^6} - \frac{4160(bx^3+a)b}{x^9}\right)d^2x^{16}}{7280(bx^3+a)^{\frac{16}{3}}a^4 + 3640(bx^3+a)^{\frac{16}{3}}a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(19/3),x, algorithm="maxima")

[Out]  $-1/7280*(455*b^3 - 1680*(bx^3 + a)*b^2/x^3 + 2184*(bx^3 + a)^2*b/x^6 - 1040*(bx^3 + a)^3/x^9)*d^2*x^{16}/((bx^3 + a)^{\frac{16}{3}}*a^4) + 1/3640*(455*b^4 - 2240*(bx^3 + a)*b^3/x^3 + 4368*(bx^3 + a)^2*b^2/x^6 - 4160*(bx^3 + a)^3*b/x^9 + 1820*(bx^3 + a)^4/x^{12})*c*d*x^{16}/((bx^3 + a)^{\frac{16}{3}}*a^5) - 1/1456*(91*b^5 - 560*(bx^3 + a)*b^4/x^3 + 1456*(bx^3 + a)^2*b^3/x^6 - 2080*(bx^3 + a)^3*b^2/x^9 + 1820*(bx^3 + a)^4*b/x^{12} - 1456*(bx^3 + a)^5/x^{15})*c^2*x^{16}/((bx^3 + a)^{\frac{16}{3}}*a^6)$

**mupad** [B] time = 1.48, size = 257, normalized size = 1.02

$$\frac{x \left( \frac{c^2}{16a} + \frac{a \left( \frac{d^2}{16b} - \frac{cd}{8a} \right)}{b} \right)}{(bx^3 + a)^{\frac{16}{3}}} - \frac{x \left( \frac{d^2}{13b^2} - \frac{-a^2d^2 + 2abcd + 15b^2c^2}{208a^2b^2} \right)}{(bx^3 + a)^{\frac{13}{3}}} + \frac{x (a^2d^2 + 6abcd + 45b^2c^2)}{520a^3b^2(bx^3 + a)^{\frac{10}{3}}} + \frac{x (9a^2d^2 + 54abcd + 36a^4b^2)}{3640a^4b^2(bx^3 + a)^{\frac{7}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(19/3),x)

```
[Out] (x*(c^2/(16*a) + (a*(d^2/(16*b) - (c*d)/(8*a)))/b))/(a + b*x^3)^(16/3) - (x
*(d^2/(13*b^2) - (15*b^2*c^2 - a^2*d^2 + 2*a*b*c*d)/(208*a^2*b^2)))/(a + b*
x^3)^(13/3) + (x*(a^2*d^2 + 45*b^2*c^2 + 6*a*b*c*d))/(520*a^3*b^2*(a + b*x^
3)^(10/3)) + (x*(9*a^2*d^2 + 405*b^2*c^2 + 54*a*b*c*d))/(3640*a^4*b^2*(a +
b*x^3)^(7/3)) + (x*(27*a^2*d^2 + 1215*b^2*c^2 + 162*a*b*c*d))/(7280*a^5*b^2
*(a + b*x^3)^(4/3)) + (x*(81*a^2*d^2 + 3645*b^2*c^2 + 486*a*b*c*d))/(7280*a
^6*b^2*(a + b*x^3)^(1/3))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**2/(b*x**3+a)**(19/3),x)
```

```
[Out] Timed out
```



### 3.79 $\int (a + bx^3)^{7/3} (c + dx^3)^2 dx$

**Optimal.** Leaf size=135

$$\frac{a^2 x^3 \sqrt[3]{a + bx^3} (2a^2 d^2 - 14abcd + 77b^2 c^2) {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) dx (a + bx^3)^{10/3} (17bc - 4ad) dx (a + bx^3)^{10/3}}{77b^2 \sqrt[3]{\frac{bx^3}{a} + 1} + \frac{dx (a + bx^3)^{10/3} (17bc - 4ad)}{154b^2} + \frac{dx (a + bx^3)^{10/3}}{14b}}$$

[Out]  $1/154*d*(-4*a*d+17*b*c)*x*(b*x^3+a)^(10/3)/b^2+1/14*d*x*(b*x^3+a)^(10/3)*(d*x^3+c)/b+1/77*a^2*(2*a^2*d^2-14*a*b*c*d+77*b^2*c^2)*x*(b*x^3+a)^(1/3)*\text{hypergeom}([-7/3, 1/3], [4/3], -b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)$

**Rubi [A]** time = 0.07, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 246, 245}

$$\frac{a^2 x^3 \sqrt[3]{a + bx^3} (2a^2 d^2 - 14abcd + 77b^2 c^2) {}_2F_1\left(-\frac{7}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) dx (a + bx^3)^{10/3} (17bc - 4ad) dx (a + bx^3)^{10/3}}{77b^2 \sqrt[3]{\frac{bx^3}{a} + 1} + \frac{dx (a + bx^3)^{10/3} (17bc - 4ad)}{154b^2} + \frac{dx (a + bx^3)^{10/3}}{14b}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^(7/3)*(c + d*x^3)^2, x]$

[Out]  $(d*(17*b*c - 4*a*d)*x*(a + b*x^3)^(10/3))/(154*b^2) + (d*x*(a + b*x^3)^(10/3)*(c + d*x^3))/(14*b) + (a^2*(77*b^2*c^2 - 14*a*b*c*d + 2*a^2*d^2)*x*(a + b*x^3)^(1/3)*\text{Hypergeometric2F1}[-7/3, 1/3, 4/3, -(b*x^3)/a])/((77*b^2*(1 + (b*x^3)/a)^(1/3))$

#### Rule 245

$\text{Int}[(a + b*x^n)^p, x\_Symbol] \rightarrow \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

#### Rule 246

$\text{Int}[(a + b*x^n)^p, x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ !(\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

#### Rule 388

$\text{Int}[(a + b*x^n)^p*((c + d*x^n)^q), x\_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{p+1}/(b*(n*(p+1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p+1) + 1))/(b*(n*(p+1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n*(p+1) + 1, 0]$

#### Rule 416

$\text{Int}[(a + b*x^n)^p*((c + d*x^n)^q), x\_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{p+1}*(c + d*x^n)^{q-1}/(b*(n*(p+q) + 1)), x] + \text{Dist}[1/(b*(n*(p+q) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^{q-2}*\text{Simp}[c*(b*c*(n*(p+q) + 1) - a*d) + d*(b*c*(n*(p+2*q-1) + 1) - a*d*(n*(q-1) + 1))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, p\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{NeQ}[n*(p+q) + 1, 0] \ \&\& \ !\text{IGtQ}[p, 1] \ \&\& \ \text{IntBinomialQ}[a$

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^{7/3} (c + dx^3)^2 dx &= \frac{dx (a + bx^3)^{10/3} (c + dx^3)}{14b} + \frac{\int (a + bx^3)^{7/3} (c(14bc - ad) + d(17bc - 4ad)x^3) dx}{14b} \\ &= \frac{d(17bc - 4ad)x (a + bx^3)^{10/3}}{154b^2} + \frac{dx (a + bx^3)^{10/3} (c + dx^3)}{14b} - \frac{(ad(17bc - 4ad) - 11ad^2)}{14b} \\ &= \frac{d(17bc - 4ad)x (a + bx^3)^{10/3}}{154b^2} + \frac{dx (a + bx^3)^{10/3} (c + dx^3)}{14b} - \frac{(a^2(ad(17bc - 4ad) - 11ad^2))}{14b} \\ &= \frac{d(17bc - 4ad)x (a + bx^3)^{10/3}}{154b^2} + \frac{dx (a + bx^3)^{10/3} (c + dx^3)}{14b} + \frac{a^2 (77b^2c^2 - 14abcd)}{14b} \end{aligned}$$

**Mathematica** [A] time = 3.59, size = 177, normalized size = 1.31

$$\frac{ax^3\sqrt[3]{a+bx^3} \left(-9bx^3\Gamma\left(-\frac{4}{3}\right)(c+dx^3)^2 {}_3F_2\left(-\frac{4}{3}, \frac{4}{3}, 2; 1, \frac{13}{3}; -\frac{bx^3}{a}\right) - 3bx^3\Gamma\left(-\frac{4}{3}\right)(11c^2 + 16cdx^3 + 5d^2x^6) {}_2F_1\left(-\frac{4}{3}, \frac{4}{3}; -\frac{bx^3}{a}\right) - 9b^2x^6\Gamma\left(-\frac{4}{3}\right)(c+dx^3)^2\right)}{280\Gamma\left(-\frac{7}{3}\right)\sqrt[3]{\frac{bx^3}{a}+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(7/3)\*(c + d\*x^3)^2,x]

[Out] (a\*x\*(a + b\*x^3)^(1/3)\*(20\*a\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6)\*Gamma[-7/3]\*Hypergeometric2F1[-7/3, 1/3, 10/3, -((b\*x^3)/a)] - 3\*b\*x^3\*(11\*c^2 + 16\*c\*d\*x^3 + 5\*d^2\*x^6)\*Gamma[-4/3]\*Hypergeometric2F1[-4/3, 4/3, 13/3, -((b\*x^3)/a)]) - 9\*b\*x^6\*(c + d\*x^3)^2\*Gamma[-4/3]\*HypergeometricPFQ[{-4/3, 4/3, 2}, {1, 13/3}, -((b\*x^3)/a)])/(280\*(1 + (b\*x^3)/a)^(1/3)\*Gamma[-7/3])

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2d^2x^{12} + 2(b^2cd + abd^2)x^9 + (b^2c^2 + 4abcd + a^2d^2)x^6 + a^2c^2 + 2(abc^2 + a^2cd)x^3\right)(bx^3 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] integral((b^2\*d^2\*x^12 + 2\*(b^2\*c\*d + a\*b\*d^2)\*x^9 + (b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*x^6 + a^2\*c^2 + 2\*(a\*b\*c^2 + a^2\*c\*d)\*x^3)\*(b\*x^3 + a)^(1/3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^2, x)

maple [F] time = 0.42, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(7/3)\*(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(7/3)\*(d\*x^3+c)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/3)\*(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{\frac{7}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(7/3)\*(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(7/3)\*(c + d\*x^3)^2, x)

sympy [C] time = 12.60, size = 418, normalized size = 3.10

$$\frac{a^{\frac{7}{3}} c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{7}{3}} c dx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{7}{3}} d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{2a^{\frac{4}{3}} bc^2 x^4 \Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(7/3)\*(d\*x\*\*3+c)\*\*2,x)

[Out] a\*\*(7/3)\*c\*\*2\*x\*gamma(1/3)\*hyper((-1/3, 1/3), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3)) + 2\*a\*\*(7/3)\*c\*d\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + a\*\*(7/3)\*d\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + 2\*a\*\*(4/3)\*b\*c\*\*2\*x\*\*4\*gamma(4/3)\*hyper((-1/3, 4/3), (7/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(7/3)) + 4\*a\*\*(4/3)\*b\*c\*d\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + 2\*a\*\*(4/3)\*b\*d\*\*2\*x\*\*10\*gamma(10/3)\*hyper((-1/3, 10/3), (13/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(13/3)) + a\*\*(1/3)\*b\*\*2\*c\*\*2\*x\*\*7\*gamma(7/3)\*hyper((-1/3, 7/3), (10/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(10/3)) + 2\*a\*\*(1/3)\*b\*\*2\*c\*d\*x\*\*10\*gamma(10/3)\*hyper((-1/3, 10/3), (13/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(13/3)) + a\*\*(1/3)\*b\*\*2\*d\*\*2\*x\*\*13\*gamma(13/3)\*hyper((-1/3, 13/3), (16/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(16/3))

### 3.80 $\int (a + bx^3)^{4/3} (c + dx^3)^2 dx$

**Optimal.** Leaf size=133

$$\frac{ax\sqrt[3]{a+bx^3} (2a^2d^2 - 11abcd + 44b^2c^2) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{7/3} (7bc - 2ad)}{44b^2} + \frac{dx(a+bx^3)^{7/3} (c+dx^3)}{11b}$$

[Out] 1/44\*d\*(-2\*a\*d+7\*b\*c)\*x\*(b\*x^3+a)^(7/3)/b^2+1/11\*d\*x\*(b\*x^3+a)^(7/3)\*(d\*x^3+c)/b+1/44\*a\*(2\*a^2\*d^2-11\*a\*b\*c\*d+44\*b^2\*c^2)\*x\*(b\*x^3+a)^(1/3)\*hypergeom([-4/3, 1/3], [4/3], -b\*x^3/a)/b^2/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 246, 245}

$$\frac{ax\sqrt[3]{a+bx^3} (2a^2d^2 - 11abcd + 44b^2c^2) {}_2F_1\left(-\frac{4}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{44b^2\sqrt[3]{\frac{bx^3}{a}+1}} + \frac{dx(a+bx^3)^{7/3} (7bc - 2ad)}{44b^2} + \frac{dx(a+bx^3)^{7/3} (c+dx^3)}{11b}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(4/3)\*(c + d\*x^3)^2,x]

[Out] (d\*(7\*b\*c - 2\*a\*d)\*x\*(a + b\*x^3)^(7/3))/(44\*b^2) + (d\*x\*(a + b\*x^3)^(7/3)\*(c + d\*x^3))/(11\*b) + (a\*(44\*b^2\*c^2 - 11\*a\*b\*c\*d + 2\*a^2\*d^2)\*x\*(a + b\*x^3)^(1/3)\*Hypergeometric2F1[-4/3, 1/3, 4/3, -(b\*x^3)/a])/(44\*b^2\*(1 + (b\*x^3)/a)^(1/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^{4/3} (c + dx^3)^2 dx &= \frac{dx (a + bx^3)^{7/3} (c + dx^3)}{11b} + \frac{\int (a + bx^3)^{4/3} (c(11bc - ad) + 2d(7bc - 2ad)x^3) dx}{11b} \\ &= \frac{d(7bc - 2ad)x (a + bx^3)^{7/3}}{44b^2} + \frac{dx (a + bx^3)^{7/3} (c + dx^3)}{11b} - \frac{(2ad(7bc - 2ad) - 8bd^2)}{11b} \\ &= \frac{d(7bc - 2ad)x (a + bx^3)^{7/3}}{44b^2} + \frac{dx (a + bx^3)^{7/3} (c + dx^3)}{11b} - \frac{(a(2ad(7bc - 2ad) - 8bd^2))}{11b} \\ &= \frac{d(7bc - 2ad)x (a + bx^3)^{7/3}}{44b^2} + \frac{dx (a + bx^3)^{7/3} (c + dx^3)}{11b} + \frac{a(44b^2c^2 - 11abcd + 8bd^2)}{11b} \end{aligned}$$

**Mathematica [A]** time = 5.19, size = 171, normalized size = 1.29

$$\frac{x \left( 2a^2 \left( \frac{bx^3}{a} + 1 \right)^{2/3} (2a^2d^2 - 11abcd + 44b^2c^2) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) - (a + bx^3) (4a^3d^2 - 2a^2bd(11c + dx^3) - 3abd^2) \right)}{220b^2 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(4/3)\*(c + d\*x^3)^2,x]

[Out] (x\*(-((a + b\*x^3)\*(4\*a^3\*d^2 - 2\*a^2\*b\*d\*(11\*c + d\*x^3) - 3\*a\*b^2\*(44\*c^2 + 33\*c\*d\*x^3 + 10\*d^2\*x^6) - b^3\*x^3\*(44\*c^2 + 55\*c\*d\*x^3 + 20\*d^2\*x^6)))) + 2\*a^2\*(44\*b^2\*c^2 - 11\*a\*b\*c\*d + 2\*a^2\*d^2)\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -((b\*x^3)/a)]))/(220\*b^2\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.67, size = 0, normalized size = 0.00

$$\text{integral} \left( (bd^2x^9 + (2bcd + ad^2)x^6 + (bc^2 + 2acd)x^3 + ac^2)(bx^3 + a)^{\frac{1}{3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] integral((b\*d^2\*x^9 + (2\*b\*c\*d + a\*d^2)\*x^6 + (b\*c^2 + 2\*a\*c\*d)\*x^3 + a\*c^2)\*(b\*x^3 + a)^(1/3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)^2, x)

**maple [F]** time = 0.37, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{4}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
[Out] int((b*x^3+a)^(4/3)*(d*x^3+c)^2,x)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int (bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(4/3)*(d*x^3+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^(4/3)*(d*x^3 + c)^2, x)
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.01
```

$$\int (bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)*(c + d*x^3)^2,x)
```

```
[Out] int((a + b*x^3)^(4/3)*(c + d*x^3)^2, x)
```

```
sympy [C] time = 7.06, size = 270, normalized size = 2.03
```

$$\frac{a^{\frac{4}{3}}c^2x\Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2a^{\frac{4}{3}}cdx^4\Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{a^{\frac{4}{3}}d^2x^7\Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3e^{i\pi}}{a} \right)}{3\Gamma\left(\frac{10}{3}\right)} + \frac{\sqrt[3]{a}bc^2x^4\Gamma\left(\frac{4}{3}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)*(d*x**3+c)**2,x)
```

```
[Out] a**(4/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a**(4/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(4/3)*d**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*c**2*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + 2*a**(1/3)*b*c*d*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3)) + a**(1/3)*b*d**2*x**10*gamma(10/3)*hyper((-1/3, 10/3), (13/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(13/3))
```

### 3.81 $\int \sqrt[3]{a + bx^3} (c + dx^3)^2 dx$

**Optimal.** Leaf size=131

$$\frac{x\sqrt[3]{a + bx^3} (a^2d^2 - 4abcd + 10b^2c^2) {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a} + 1}} + \frac{dx(a + bx^3)^{4/3} (11bc - 4ad)}{40b^2} + \frac{dx(a + bx^3)^{4/3} (c + dx^3)}{8b}$$

[Out]  $1/40*d*(-4*a*d+11*b*c)*x*(b*x^3+a)^(4/3)/b^2+1/8*d*x*(b*x^3+a)^(4/3)*(d*x^3+c)/b+1/10*(a^2*d^2-4*a*b*c*d+10*b^2*c^2)*x*(b*x^3+a)^(1/3)*\text{hypergeom}([-1/3, 1/3], [4/3], -b*x^3/a)/b^2/(1+b*x^3/a)^(1/3)$

**Rubi [A]** time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 246, 245}

$$\frac{x\sqrt[3]{a + bx^3} (a^2d^2 - 4abcd + 10b^2c^2) {}_2F_1\left(-\frac{1}{3}, \frac{1}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{10b^2\sqrt[3]{\frac{bx^3}{a} + 1}} + \frac{dx(a + bx^3)^{4/3} (11bc - 4ad)}{40b^2} + \frac{dx(a + bx^3)^{4/3} (c + dx^3)}{8b}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^(1/3)*(c + d*x^3)^2, x]$

[Out]  $(d*(11*b*c - 4*a*d)*x*(a + b*x^3)^(4/3))/(40*b^2) + (d*x*(a + b*x^3)^(4/3)*(c + d*x^3))/(8*b) + ((10*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x*(a + b*x^3)^(1/3))*\text{Hypergeometric2F1}[-1/3, 1/3, 4/3, -((b*x^3)/a)]/(10*b^2*(1 + (b*x^3)/a)^(1/3))$

#### Rule 245

$\text{Int}(((a_) + (b_.)*(x_)^(n_))^(p_), x\_Symbol] :> \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /;$   $\text{FreeQ}\{a, b, n, p\}, x$  &&  $!\text{IGtQ}[p, 0]$  &&  $!\text{IntegerQ}[1/n]$  &&  $!\text{ILtQ}[\text{Simplify}[1/n + p], 0]$  &&  $(\text{IntegerQ}[p] || \text{GtQ}[a, 0])$

#### Rule 246

$\text{Int}(((a_) + (b_.)*(x_)^(n_))^(p_), x\_Symbol] :> \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p, x], x] /;$   $\text{FreeQ}\{a, b, n, p\}, x$  &&  $!\text{IGtQ}[p, 0]$  &&  $!\text{IntegerQ}[1/n]$  &&  $!\text{ILtQ}[\text{Simplify}[1/n + p], 0]$  &&  $!(\text{IntegerQ}[p] || \text{GtQ}[a, 0])$

#### Rule 388

$\text{Int}(((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x\_Symbol] :> \text{Simp}[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - \text{Dist}[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), \text{Int}[(a + b*x^n)^p, x], x] /;$   $\text{FreeQ}\{a, b, c, d, n\}, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{NeQ}[n*(p + 1) + 1, 0]$

#### Rule 416

$\text{Int}(((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x\_Symbol] :> \text{Simp}[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)), x] + \text{Dist}[1/(b*(n*(p + q) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*\text{Simp}[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /;$   $\text{FreeQ}\{a, b, c, d, n, p\}, x$  &&  $\text{NeQ}[b*c - a*d, 0]$  &&  $\text{GtQ}[q, 1]$  &&  $\text{NeQ}[n*(p + q) + 1, 0]$  &&  $!\text{IGtQ}[p, 1]$  &&  $\text{IntBinomialQ}[a$

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int \sqrt[3]{a+bx^3} (c+dx^3)^2 dx &= \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{\int \sqrt[3]{a+bx^3} (c(8bc-ad) + d(11bc-4ad)x^3) dx}{8b} \\ &= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{(10b^2c^2-4abcd+a^2d^2)}{10b^2} \\ &= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{((10b^2c^2-4abcd+a^2d^2))}{10b^2} \\ &= \frac{d(11bc-4ad)x(a+bx^3)^{4/3}}{40b^2} + \frac{dx(a+bx^3)^{4/3}(c+dx^3)}{8b} + \frac{(10b^2c^2-4abcd+a^2d^2)}{10b^2} \end{aligned}$$

**Mathematica** [A] time = 4.00, size = 179, normalized size = 1.37

$$\frac{x^3 \sqrt[3]{a+bx^3} \left( -9bx^3 \Gamma\left(\frac{2}{3}\right) (c+dx^3)^2 {}_3F_2\left(\frac{2}{3}, \frac{4}{3}, 2; 1, \frac{13}{3}; -\frac{bx^3}{a}\right) - 3bx^3 \Gamma\left(\frac{2}{3}\right) (11c^2 + 16cdx^3 + 5d^2x^6) {}_2F_1\left(\frac{2}{3}, \frac{4}{3}; \frac{13}{3}; -\frac{bx^3}{a}\right) \right)}{280a \Gamma\left(-\frac{1}{3}\right) \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/3)\*(c + d\*x^3)^2,x]

[Out] (x\*(a + b\*x^3)^(1/3)\*(20\*a\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6)\*Gamma[-1/3]\*Hypergeometric2F1[-1/3, 1/3, 10/3, -((b\*x^3)/a)] - 3\*b\*x^3\*(11\*c^2 + 16\*c\*d\*x^3 + 5\*d^2\*x^6)\*Gamma[2/3]\*Hypergeometric2F1[2/3, 4/3, 13/3, -((b\*x^3)/a)] - 9\*b\*x^3\*(c + d\*x^3)^2\*Gamma[2/3]\*HypergeometricPFQ[{2/3, 4/3, 2}, {1, 13/3}, -((b\*x^3)/a)))/(280\*a\*(1 + (b\*x^3)/a)^(1/3)\*Gamma[-1/3])

**fricas** [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^6 + 2cdx^3 + c^2\right)(bx^3 + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)\*(b\*x^3 + a)^(1/3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)^2, x)

**maple** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2 dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)`

[Out] `int((b*x^3+a)^(1/3)*(d*x^3+c)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{\frac{1}{3}}(dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(1/3)*(d*x^3+c)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(1/3)*(d*x^3 + c)^2, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^{1/3} (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(1/3)*(c + d*x^3)^2,x)`

[Out] `int((a + b*x^3)^(1/3)*(c + d*x^3)^2, x)`

**sympy** [C] time = 3.94, size = 131, normalized size = 1.00

$$\frac{\sqrt[3]{a} c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{1}{3} \\ \frac{4}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{2\sqrt[3]{a} c d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{4}{3} \\ \frac{7}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)} + \frac{\sqrt[3]{a} d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, \frac{7}{3} \\ \frac{10}{3} \end{matrix} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(1/3)*(d*x**3+c)**2,x)`

[Out] `a**(1/3)*c**2*x*gamma(1/3)*hyper((-1/3, 1/3), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + 2*a**(1/3)*c*d*x**4*gamma(4/3)*hyper((-1/3, 4/3), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3)) + a**(1/3)*d**2*x**7*gamma(7/3)*hyper((-1/3, 7/3), (10/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(10/3))`

$$3.82 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{2/3}} dx$$

**Optimal.** Leaf size=132

$$\frac{x\left(\frac{bx^3}{a} + 1\right)^{2/3} (2a^2d^2 - 5abcd + 5b^2c^2) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5b^2 (a + bx^3)^{2/3}} + \frac{2dx\sqrt[3]{a + bx^3} (2bc - ad)}{5b^2} + \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b}$$

[Out] 2/5\*d\*(-a\*d+2\*b\*c)\*x\*(b\*x^3+a)^(1/3)/b^2+1/5\*d\*x\*(b\*x^3+a)^(1/3)\*(d\*x^3+c)/b+1/5\*(2\*a^2\*d^2-5\*a\*b\*c\*d+5\*b^2\*c^2)\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 2/3], [4/3], -b\*x^3/a)/b^2/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.08, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {416, 388, 246, 245}

$$\frac{x\left(\frac{bx^3}{a} + 1\right)^{2/3} (2a^2d^2 - 5abcd + 5b^2c^2) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5b^2 (a + bx^3)^{2/3}} + \frac{2dx\sqrt[3]{a + bx^3} (2bc - ad)}{5b^2} + \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(2/3), x]

[Out] (2\*d\*(2\*b\*c - a\*d)\*x\*(a + b\*x^3)^(1/3))/(5\*b^2) + (d\*x\*(a + b\*x^3)^(1/3)\*(c + d\*x^3))/(5\*b) + ((5\*b^2\*c^2 - 5\*a\*b\*c\*d + 2\*a^2\*d^2)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -((b\*x^3)/a)]/(5\*b^2\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d] + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d,

0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^2}{(a + bx^3)^{2/3}} dx &= \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b} + \frac{\int \frac{c(5bc - ad) + 4d(2bc - ad)x^3}{(a + bx^3)^{2/3}} dx}{5b} \\ &= \frac{2d(2bc - ad)x\sqrt[3]{a + bx^3}}{5b^2} + \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b} - \frac{(4ad(2bc - ad) - 2bc(5bc - ad)) \int \frac{1}{(a + bx^3)^{2/3}} dx}{10b^2} \\ &= \frac{2d(2bc - ad)x\sqrt[3]{a + bx^3}}{5b^2} + \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b} - \frac{\left( (4ad(2bc - ad) - 2bc(5bc - ad)) \left( 1 - \frac{3\sqrt[3]{a + bx^3}}{a} \right) \right)}{10b^2 (a + bx^3)^{2/3}} \\ &= \frac{2d(2bc - ad)x\sqrt[3]{a + bx^3}}{5b^2} + \frac{dx\sqrt[3]{a + bx^3} (c + dx^3)}{5b} + \frac{(5b^2c^2 - 5abcd + 2a^2d^2)x \left( 1 + \frac{bx^3}{a} \right)^2}{5b^2 (a + bx^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 5.14, size = 104, normalized size = 0.79

$$\frac{x \left( \left( \frac{bx^3}{a} + 1 \right)^{2/3} \left( 2a^2d^2 - 5abcd + 5b^2c^2 \right) {}_2F_1 \left( \frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a} \right) - d(a + bx^3) (2ad - b(5c + dx^3)) \right)}{5b^2 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(2/3), x]

[Out] (x\*(-(d\*(a + b\*x^3)\*(2\*a\*d - b\*(5\*c + d\*x^3))) + (5\*b^2\*c^2 - 5\*a\*b\*c\*d + 2\*a^2\*d^2)\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(5\*b^2\*(a + b\*x^3)^(2/3))

**fricas [F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{d^2x^6 + 2cdx^3 + c^2}{(bx^3 + a)^{2/3}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(2/3), x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)/(b\*x^3 + a)^(2/3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(2/3), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(2/3), x)

**maple** [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(2/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(2/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(2/3),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(2/3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(2/3),x)

[Out] int((c + d\*x^3)^2/(a + b\*x^3)^(2/3), x)

**sympy** [C] time = 4.27, size = 126, normalized size = 0.95

$$\frac{c^2 x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} \Gamma\left(\frac{4}{3}\right)} + \frac{2cdx^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{4}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} \Gamma\left(\frac{7}{3}\right)} + \frac{d^2 x^7 \Gamma\left(\frac{7}{3}\right) {}_2F_1\left(\frac{2}{3}, \frac{7}{3} \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3a^{\frac{2}{3}} \Gamma\left(\frac{10}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*(2/3),x)

[Out] c\*\*2\*x\*gamma(1/3)\*hyper((1/3, 2/3), (4/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*a\*\*  
 (2/3)\*gamma(4/3)) + 2\*c\*d\*x\*\*4\*gamma(4/3)\*hyper((2/3, 4/3), (7/3, ), b\*x\*\*3  
 \*exp\_polar(I\*pi)/a)/(3\*a\*\*  
 (2/3)\*gamma(7/3)) + d\*\*2\*x\*\*7\*gamma(7/3)\*hyper((2  
 /3, 7/3), (10/3, ), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*a\*\*  
 (2/3)\*gamma(10/3))

$$3.83 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{5/3}} dx$$

**Optimal.** Leaf size=146

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} \left(-2a^2d^2 + 2abcd + b^2c^2\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2ab^2 (a + bx^3)^{2/3}} - \frac{dx \sqrt[3]{a + bx^3} (bc - 2ad)}{2ab^2} + \frac{x (c + dx^3) (bc - ad)}{2ab (a + bx^3)^{2/3}}$$

[Out]  $-1/2*d*(-2*a*d+b*c)*x*(b*x^3+a)^{(1/3)}/a/b^2+1/2*(-a*d+b*c)*x*(d*x^3+c)/a/b/(b*x^3+a)^{(2/3)}+1/2*(-2*a^2*d^2+2*a*b*c*d+b^2*c^2)*x*(1+b*x^3/a)^{(2/3)}*\text{hypergeom}([1/3, 2/3], [4/3], -b*x^3/a)/a/b^2/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {413, 388, 246, 245}

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{2/3} \left(-2a^2d^2 + 2abcd + b^2c^2\right) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{2ab^2 (a + bx^3)^{2/3}} - \frac{dx \sqrt[3]{a + bx^3} (bc - 2ad)}{2ab^2} + \frac{x (c + dx^3) (bc - ad)}{2ab (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(5/3), x]

[Out]  $-(d*(b*c - 2*a*d)*x*(a + b*x^3)^{(1/3)})/(2*a*b^2) + ((b*c - a*d)*x*(c + d*x^3))/(2*a*b*(a + b*x^3)^{(2/3)}) + ((b^2*c^2 + 2*a*b*c*d - 2*a^2*d^2)*x*(1 + (b*x^3)/a)^{(2/3)}*\text{Hypergeometric2F1}[1/3, 2/3, 4/3, -((b*x^3)/a)])/(2*a*b^2*(a + b*x^3)^{(2/3)})$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d,

0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^2}{(a + bx^3)^{5/3}} dx &= \frac{(bc - ad)x(c + dx^3)}{2ab(a + bx^3)^{2/3}} + \frac{\int \frac{c(bc+ad) - 2d(bc-2ad)x^3}{(a+bx^3)^{2/3}} dx}{2ab} \\ &= -\frac{d(bc - 2ad)x\sqrt[3]{a + bx^3}}{2ab^2} + \frac{(bc - ad)x(c + dx^3)}{2ab(a + bx^3)^{2/3}} + \frac{(-2ad(bc - 2ad) - 2bc(bc + ad)) \int \frac{1}{(a+bx^3)^{2/3}} dx}{4ab^2} \\ &= -\frac{d(bc - 2ad)x\sqrt[3]{a + bx^3}}{2ab^2} + \frac{(bc - ad)x(c + dx^3)}{2ab(a + bx^3)^{2/3}} + \frac{\left((-2ad(bc - 2ad) - 2bc(bc + ad))\left(1 + \frac{bx^3}{a}\right)\right)^{2/3}}{4ab^2(a + bx^3)^{2/3}} \\ &= -\frac{d(bc - 2ad)x\sqrt[3]{a + bx^3}}{2ab^2} + \frac{(bc - ad)x(c + dx^3)}{2ab(a + bx^3)^{2/3}} + \frac{(b^2c^2 + 2abcd - 2a^2d^2)x\left(1 + \frac{bx^3}{a}\right)^{2/3}}{2ab^2(a + bx^3)^{2/3}} {}_2F_1 \end{aligned}$$

**Mathematica** [A] time = 3.76, size = 171, normalized size = 1.17

$$\frac{x\Gamma\left(\frac{2}{3}\right)\left(\frac{bx^3}{a} + 1\right)^{2/3}\left(-3bx^3(c + dx^3)^2 {}_3F_2\left(\frac{4}{3}, 2, \frac{8}{3}; 1, \frac{13}{3}; -\frac{bx^3}{a}\right) - bx^3(11c^2 + 16cdx^3 + 5d^2x^6) {}_2F_1\left(\frac{4}{3}, \frac{8}{3}, \frac{13}{3}; -\frac{bx^3}{a}\right)\right)}{84a^2\Gamma\left(\frac{5}{3}\right)(a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(5/3), x]

[Out] (x\*(1 + (b\*x^3)/a)^(2/3)\*Gamma[2/3]\*(4\*a\*(14\*c^2 + 7\*c\*d\*x^3 + 2\*d^2\*x^6)\*Hypergeometric2F1[1/3, 5/3, 10/3, -((b\*x^3)/a)] - b\*x^3\*(11\*c^2 + 16\*c\*d\*x^3 + 5\*d^2\*x^6)\*Hypergeometric2F1[4/3, 8/3, 13/3, -((b\*x^3)/a)] - 3\*b\*x^3\*(c + d\*x^3)^2\*HypergeometricPFQ[{4/3, 2, 8/3}, {1, 13/3}, -((b\*x^3)/a)]))/(84\*a^2\*(a + b\*x^3)^(2/3)\*Gamma[5/3])

**fricas** [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(d^2x^6 + 2cdx^3 + c^2)(bx^3 + a)^{\frac{1}{3}}}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(5/3), x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)\*(b\*x^3 + a)^(1/3)/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{5/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(5/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(5/3), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(5/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(5/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(5/3),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(5/3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(5/3),x)

[Out] int((c + d\*x^3)^2/(a + b\*x^3)^(5/3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{5}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*(5/3),x)

[Out] Integral((c + d\*x\*\*3)\*\*2/(a + b\*x\*\*3)\*\*(5/3), x)

$$3.84 \quad \int \frac{(c+dx^3)^2}{(a+bx^3)^{8/3}} dx$$

**Optimal.** Leaf size=147

$$\frac{x\left(\frac{bx^3}{a} + 1\right)^{2/3} (2a^2d^2 + abcd + 2b^2c^2) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5a^2b^2 (a + bx^3)^{2/3}} + \frac{2x\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)}{5(a + bx^3)^{2/3}} + \frac{x(c + dx^3)(bc - ad)}{5ab(a + bx^3)^{5/3}}$$

[Out] 2/5\*(c^2/a^2-d^2/b^2)\*x/(b\*x^3+a)^(2/3)+1/5\*(-a\*d+b\*c)\*x\*(d\*x^3+c)/a/b/(b\*x^3+a)^(5/3)+1/5\*(2\*a^2\*d^2+a\*b\*c\*d+2\*b^2\*c^2)\*x\*(1+b\*x^3/a)^(2/3)\*hypergeom([1/3, 2/3], [4/3], -b\*x^3/a)/a^2/b^2/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {413, 385, 246, 245}

$$\frac{x\left(\frac{bx^3}{a} + 1\right)^{2/3} (2a^2d^2 + abcd + 2b^2c^2) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5a^2b^2 (a + bx^3)^{2/3}} + \frac{2x\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)}{5(a + bx^3)^{2/3}} + \frac{x(c + dx^3)(bc - ad)}{5ab(a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^2/(a + b\*x^3)^(8/3), x]

[Out] (2\*(c^2/a^2 - d^2/b^2)\*x)/(5\*(a + b\*x^3)^(2/3)) + ((b\*c - a\*d)\*x\*(c + d\*x^3))/(5\*a\*b\*(a + b\*x^3)^(5/3)) + ((2\*b^2\*c^2 + a\*b\*c\*d + 2\*a^2\*d^2)\*x\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -((b\*x^3)/a)])/(5\*a^2\*b^2\*(a + b\*x^3)^(2/3))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + 1) + 1)), x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && (LtQ[p, -1] || ILtQ[1/n + p, 0])



+ q) + 1)) \* x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^2}{(a + bx^3)^{8/3}} dx &= \frac{(bc - ad)x(c + dx^3)}{5ab(a + bx^3)^{5/3}} + \frac{\int \frac{c(4bc + ad) + d(bc + 4ad)x^3}{(a + bx^3)^{5/3}} dx}{5ab} \\ &= \frac{2\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)x}{5(a + bx^3)^{2/3}} + \frac{(bc - ad)x(c + dx^3)}{5ab(a + bx^3)^{5/3}} + \frac{(2b^2c^2 + abcd + 2a^2d^2) \int \frac{1}{(a + bx^3)^{2/3}} dx}{5a^2b^2} \\ &= \frac{2\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)x}{5(a + bx^3)^{2/3}} + \frac{(bc - ad)x(c + dx^3)}{5ab(a + bx^3)^{5/3}} + \frac{\left((2b^2c^2 + abcd + 2a^2d^2)\left(1 + \frac{bx^3}{a}\right)^{2/3}\right) \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} dx}{5a^2b^2(a + bx^3)^{2/3}} \\ &= \frac{2\left(\frac{c^2}{a^2} - \frac{d^2}{b^2}\right)x}{5(a + bx^3)^{2/3}} + \frac{(bc - ad)x(c + dx^3)}{5ab(a + bx^3)^{5/3}} + \frac{(2b^2c^2 + abcd + 2a^2d^2)x\left(1 + \frac{bx^3}{a}\right)^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right)}{5a^2b^2(a + bx^3)^{2/3}} \end{aligned}$$

**Mathematica [A]** time = 5.16, size = 128, normalized size = 0.87

$$\frac{x \left( (a + bx^3) \left( \frac{bx^3}{a} + 1 \right)^{2/3} (2a^2d^2 + abcd + 2b^2c^2) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -\frac{bx^3}{a}\right) + (a + bx^3)(-3a^2d^2 + abcd + 2b^2c^2) + a(b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3) \right)}{5a^2b^2(a + bx^3)^{5/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^3)^2/(a + b\*x^3)^(8/3), x]

[Out] (x\*(a\*(b\*c - a\*d)^2 + (2\*b^2\*c^2 + a\*b\*c\*d - 3\*a^2\*d^2)\*(a + b\*x^3) + (2\*b^2\*c^2 + a\*b\*c\*d + 2\*a^2\*d^2)\*(a + b\*x^3)\*(1 + (b\*x^3)/a)^(2/3)\*Hypergeometric2F1[1/3, 2/3, 4/3, -(b\*x^3)/a]))/(5\*a^2\*b^2\*(a + b\*x^3)^(5/3))

**fricas [F]** time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(d^2x^6 + 2cdx^3 + c^2)(bx^3 + a)^{\frac{1}{3}}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(8/3), x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)\*(b\*x^3 + a)^(1/3)/(b^3\*x^9 + 3\*a\*b^2\*x^6 + 3\*a^2\*b\*x^3 + a^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(8/3),x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(8/3), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^2/(b\*x^3+a)^(8/3),x)

[Out] int((d\*x^3+c)^2/(b\*x^3+a)^(8/3),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^2/(b\*x^3+a)^(8/3),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^2/(b\*x^3 + a)^(8/3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^2}{(bx^3 + a)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^2/(a + b\*x^3)^(8/3),x)

[Out] int((c + d\*x^3)^2/(a + b\*x^3)^(8/3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^2}{(a + bx^3)^{\frac{8}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*2/(b\*x\*\*3+a)\*\*(8/3),x)

[Out] Integral((c + d\*x\*\*3)\*\*2/(a + b\*x\*\*3)\*\*(8/3), x)

$$3.85 \quad \int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} dx$$

**Optimal.** Leaf size=109

$$\frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

[Out]  $1/10*x*(b*x^3+a)^3/c/(d*x^3+c)^{(10/3)}+9/70*a*x*(b*x^3+a)^2/c^2/(d*x^3+c)^{(7/3)}+27/140*a^2*x*(b*x^3+a)/c^3/(d*x^3+c)^{(4/3)}+81/140*a^3*x/c^4/(d*x^3+c)^{(1/3)}$

**Rubi [A]** time = 0.04, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {378, 191}

$$\frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^3/(c + d\*x^3)^(13/3), x]

[Out]  $(x*(a + b*x^3)^3)/(10*c*(c + d*x^3)^{(10/3)}) + (9*a*x*(a + b*x^3)^2)/(70*c^2*(c + d*x^3)^{(7/3)}) + (27*a^2*x*(a + b*x^3))/(140*c^3*(c + d*x^3)^{(4/3)}) + (81*a^3*x)/(140*c^4*(c + d*x^3)^{(1/3)})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned}
\int \frac{(a+bx^3)^3}{(c+dx^3)^{13/3}} dx &= \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}} + \frac{(9a) \int \frac{(a+bx^3)^2}{(c+dx^3)^{10/3}} dx}{10c} \\
&= \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{(27a^2) \int \frac{a+bx^3}{(c+dx^3)^{7/3}} dx}{35c^2} \\
&= \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{(81a^3) \int \frac{1}{(c+dx^3)^{4/3}} dx}{140c^3} \\
&= \frac{x(a+bx^3)^3}{10c(c+dx^3)^{10/3}} + \frac{9ax(a+bx^3)^2}{70c^2(c+dx^3)^{7/3}} + \frac{27a^2x(a+bx^3)}{140c^3(c+dx^3)^{4/3}} + \frac{81a^3x}{140c^4\sqrt[3]{c+dx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 120, normalized size = 1.10

$$\frac{x(a^3(140c^3 + 315c^2dx^3 + 270cd^2x^6 + 81d^3x^9) + 3a^2bcx^3(35c^2 + 30cdx^3 + 9d^2x^6) + 6ab^2c^2x^6(10c + 3dx^3) + 14a^3c^3x^9)}{140c^4(c+dx^3)^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^3/(c + d\*x^3)^(13/3), x]

[Out] (x\*(14\*b^3\*c^3\*x^9 + 6\*a\*b^2\*c^2\*x^6\*(10\*c + 3\*d\*x^3) + 3\*a^2\*b\*c\*x^3\*(35\*c^2 + 30\*c\*d\*x^3 + 9\*d^2\*x^6) + a^3\*(140\*c^3 + 315\*c^2\*d\*x^3 + 270\*c\*d^2\*x^6 + 81\*d^3\*x^9)))/(140\*c^4\*(c + d\*x^3)^(10/3))

**fricas [A]** time = 0.67, size = 166, normalized size = 1.52

$$\frac{((14b^3c^3 + 18ab^2c^2d + 27a^2bcd^2 + 81a^3d^3)x^{10} + 30(2ab^2c^3 + 3a^2bc^2d + 9a^3cd^2)x^7 + 140a^3c^3x + 105(a^2bc^3 + 3a^3c^2d)x^4)(d^3x^3 + c)^{2/3}}{140(c^4d^4x^{12} + 4c^5d^3x^9 + 6c^6d^2x^6 + 4c^7dx^3 + c^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^3/(d\*x^3+c)^(13/3), x, algorithm="fricas")

[Out] 1/140\*((14\*b^3\*c^3 + 18\*a\*b^2\*c^2\*d + 27\*a^2\*b\*c\*d^2 + 81\*a^3\*d^3)\*x^10 + 30\*(2\*a\*b^2\*c^3 + 3\*a^2\*b\*c^2\*d + 9\*a^3\*c\*d^2)\*x^7 + 140\*a^3\*c^3\*x + 105\*(a^2\*b\*c^3 + 3\*a^3\*c^2\*d)\*x^4)\*(d\*x^3 + c)^(2/3)/(c^4\*d^4\*x^12 + 4\*c^5\*d^3\*x^9 + 6\*c^6\*d^2\*x^6 + 4\*c^7\*d\*x^3 + c^8)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^3}{(dx^3 + c)^{13/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^3/(d\*x^3+c)^(13/3), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^3/(d\*x^3 + c)^(13/3), x)

**maple [A]** time = 0.05, size = 134, normalized size = 1.23

$$\frac{(81a^3d^3x^9 + 27a^2bcd^2x^9 + 18ab^2c^2dx^9 + 14b^3c^3x^9 + 270a^3cd^2x^6 + 90a^2b^2c^2dx^6 + 60ab^2c^3x^6 + 315a^3c^2dx^3 + 105a^2b^3c^3x^3 + 140a^3c^3x^3)}{140(dx^3 + c)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^3/(d\*x^3+c)^(13/3), x)

[Out] 1/140\*x\*(81\*a^3\*d^3\*x^9+27\*a^2\*b\*c\*d^2\*x^9+18\*a\*b^2\*c^2\*d\*x^9+14\*b^3\*c^3\*x^9+270\*a^3\*c\*d^2\*x^6+90\*a^2\*b\*c^2\*d\*x^6+60\*a\*b^2\*c^3\*x^6+315\*a^3\*c^2\*d\*x^3+105\*a^2\*b\*c^3\*x^3+140\*a^3\*c^3)/(d\*x^3+c)^(10/3)/c^4

**maxima [A]** time = 0.63, size = 182, normalized size = 1.67

$$\frac{b^3x^{10}}{10(dx^3 + c)^{\frac{10}{3}}c} - \frac{3ab^2\left(7d - \frac{10(dx^3+c)}{x^3}\right)x^{10}}{70(dx^3 + c)^{\frac{10}{3}}c^2} + \frac{3\left(14d^2 - \frac{40(dx^3+c)d}{x^3} + \frac{35(dx^3+c)^2}{x^6}\right)a^2bx^{10}}{140(dx^3 + c)^{\frac{10}{3}}c^3} - \frac{\left(14d^3 - \frac{60(dx^3+c)d^2}{x^3} + \frac{35(dx^3+c)^2d}{x^6} - \frac{10(dx^3+c)^3}{x^9}\right)a^3x^{10}}{140(dx^3 + c)^{\frac{10}{3}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^3/(d\*x^3+c)^(13/3), x, algorithm="maxima")

[Out] 1/10\*b^3\*x^10/((d\*x^3 + c)^(10/3)\*c) - 3/70\*a\*b^2\*(7\*d - 10\*(d\*x^3 + c)/x^3)\*x^10/((d\*x^3 + c)^(10/3)\*c^2) + 3/140\*(14\*d^2 - 40\*(d\*x^3 + c)\*d/x^3 + 35\*(d\*x^3 + c)^2/x^6)\*a^2\*b\*x^10/((d\*x^3 + c)^(10/3)\*c^3) - 1/140\*(14\*d^3 - 60\*(d\*x^3 + c)\*d^2/x^3 + 105\*(d\*x^3 + c)^2\*d/x^6 - 140\*(d\*x^3 + c)^3/x^9)\*a^3\*x^10/((d\*x^3 + c)^(10/3)\*c^4)

**mupad [B]** time = 1.56, size = 271, normalized size = 2.49

$$x \frac{\left(\frac{a^3}{10c} - \frac{c\left(\frac{b^3}{10d} - \frac{3ab^2}{10c}\right) + \frac{3a^2b}{10c}}{d}\right)}{(dx^3 + c)^{10/3}} - x \frac{\left(\frac{b^3}{4d^3} - \frac{27a^3d^3 + 9a^2bcd^2 + 6ab^2c^2d - 7b^3c^3}{140c^3d^3}\right)}{(dx^3 + c)^{4/3}} + x \frac{\left(\frac{c\left(\frac{b^3}{7d^2} - \frac{b^2(3ad-bc)}{7cd^2}\right) + \frac{9a^3d^3 + 3a^2bcd^2 - 3a^3d^3}{70c^2d^3}}{d}\right)}{(dx^3 + c)^{7/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^3/(c + d\*x^3)^(13/3), x)

[Out] (x\*(a^3/(10\*c) - (c\*((c\*(b^3/(10\*d) - (3\*a\*b^2)/(10\*c)))/d + (3\*a^2\*b)/(10\*c))))/d)/(c + d\*x^3)^(10/3) - (x\*(b^3/(4\*d^3) - (27\*a^3\*d^3 - 7\*b^3\*c^3 + 6\*a\*b^2\*c^2\*d + 9\*a^2\*b\*c\*d^2)/(140\*c^3\*d^3)))/d)/(c + d\*x^3)^(4/3) + (x\*((c\*(b^3/(7\*d^2) - (b^2\*(3\*a\*d - b\*c))/(7\*c\*d^2)))/d + (9\*a^3\*d^3 + b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2)/(70\*c^2\*d^3)))/d)/(c + d\*x^3)^(7/3) + (x\*(81\*a^3\*d^3 + 14\*b^3\*c^3 + 18\*a\*b^2\*c^2\*d + 27\*a^2\*b\*c\*d^2))/(140\*c^4\*d^3\*(c + d\*x^3)^(1/3))

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*3/(d\*x\*\*3+c)\*\*(13/3), x)

[Out] Timed out

$$3.86 \quad \int \frac{(a+bx^3)^{8/3}}{c+dx^3} dx$$

**Optimal.** Leaf size=331

$$\frac{b^{2/3} (20a^2d^2 - 24abcd + 9b^2c^2) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{b}x\right)}{18d^3} + \frac{b^{2/3} (20a^2d^2 - 24abcd + 9b^2c^2) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{b}x}{\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}}\right)}{9\sqrt{3}d^3} (bc -$$

[Out]  $-1/18*b*(-11*a*d+6*b*c)*x*(b*x^3+a)^{(2/3)}/d^2+1/6*b*x*(b*x^3+a)^{(5/3)}/d-1/6*(-a*d+b*c)^{(8/3)*\ln(d*x^3+c)/c^{(2/3)}/d^3+1/2*(-a*d+b*c)^{(8/3)*\ln((-a*d+b*c)^{(1/3)*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(2/3)}/d^3-1/18*b^{(2/3)*(20*a^2*d^2-24*a*b*c*d+9*b^2*c^2)*\ln(-b^{(1/3)*x+(b*x^3+a)^{(1/3)})/d^3+1/27*b^{(2/3)*(20*a^2*d^2-24*a*b*c*d+9*b^2*c^2)*\arctan(1/3*(1+2*b^{(1/3)*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/d^3*3^{(1/2)}-1/3*(-a*d+b*c)^{(8/3)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)*x/c^{(1/3)/(b*x^3+a)^{(1/3)})*3^{(1/2)})/c^{(2/3)}/d^3*3^{(1/2)}}$

**Rubi [C]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.19, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{a^2x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(8/3)/(c + d\*x^3), x]

[Out] (a^2\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -8/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c\*(1 + (b\*x^3)/a)^(2/3))

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 >: Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 >: Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{(a+bx^3)^{8/3}}{c+dx^3} dx = \frac{\left(a^2(a+bx^3)^{2/3}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^{8/3}}{c+dx^3} dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} = \frac{a^2x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 1.23, size = 655, normalized size = 1.98

$$3bx^4\sqrt[3]{\frac{bx^3}{a}} + 1\sqrt[3]{bc-ad} (20a^2d^2 - 24abcd + 9b^2c^2) F_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 2\sqrt[3]{c} \left( 9a^3d^2\sqrt[3]{a+bx^3} \log\left(\frac{\sqrt[3]{cx^3}}{\sqrt[3]{a+bx^3}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(8/3)/(c + d\*x^3), x]

[Out] (3\*b\*(b\*c - a\*d)^(1/3)\*(9\*b^2\*c^2 - 24\*a\*b\*c\*d + 20\*a^2\*d^2)\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*c^(1/3)\*(-18\*a\*b^2\*c^(5/3)\*(b\*c - a\*d)^(1/3)\*x + 42\*a^2\*b\*c^(2/3)\*d\*(b\*c - a\*d)^(1/3)\*x - 18\*b^3\*c^(5/3)\*(b\*c - a\*d)^(1/3)\*x^4 + 51\*a\*b^2\*c^(2/3)\*d\*(b\*c - a\*d)^(1/3)\*x^4 + 9\*b^3\*c^(2/3)\*d\*(b\*c - a\*d)^(1/3)\*x^7 + 2\*Sqrt[3]\*a\*(3\*b^2\*c^2 - 7\*a\*b\*c\*d + 9\*a^2\*d^2)\*(a + b\*x^3)^(1/3)\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*a\*(3\*b^2\*c^2 - 7\*a\*b\*c\*d + 9\*a^2\*d^2)\*(a + b\*x^3)^(1/3)\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + 3\*a\*b^2\*c^2\*(a + b\*x^3)^(1/3)\*Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] - 7\*a^2\*b\*c\*d\*(a + b\*x^3)^(1/3)\*Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + 9\*a^3\*d^2\*(a + b\*x^3)^(1/3)\*Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)))/(108\*c\*d^2\*(b\*c - a\*d)^(1/3)\*(a + b\*x^3)^(1/3))

**fricas [B]** time = 15.64, size = 643, normalized size = 1.94

$$18\sqrt{3}(b^2c^2 - 2abcd + a^2d^2) \left( \frac{b^2c^2 - 2abcd + a^2d^2}{c^2} \right)^{\frac{1}{3}} \arctan \left( \frac{\sqrt{3}(bc-ad)x + 2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c \left( \frac{b^2c^2 - 2abcd + a^2d^2}{c^2} \right)^{\frac{1}{3}}}{3(bc-ad)x} \right) + 2\sqrt{3}(9b^2c^2 - 24abcd + 20a^2d^2) \sqrt[3]{a+bx^3} \log\left(\frac{\sqrt[3]{cx^3}}{\sqrt[3]{a+bx^3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c), x, algorithm="fricas")

[Out] -1/54\*(18\*sqrt(3)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*(b\*c - a\*d)\*x + 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*c\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3))/((b\*c - a\*d)\*x)) + 2\*sqrt(3)\*(9\*b^2\*c^2 - 24\*a\*b\*c\*d + 20\*a^2\*d^2)\*(-b^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*b\*x - 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*(-b^2)^(1/3))/(b\*x)) - 18\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log((c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) - (b\*x^3 + a)^(1/3)\*(b\*c - a\*d))/x) - 2\*(9\*b^2\*c^2 - 24\*a\*b\*c\*d + 20\*a^2\*d^2)\*(-b^2)^(1/3)\*log(-((-b^2)^(2/3)\*x - (b\*x^3 + a)^(1/3)\*b)/x) + (9\*b^2\*c^2 - 24\*a\*b\*c\*d + 20\*a^2\*d^2)\*(-b^2)^(1/3)\*log(-((-b^2)^(1/3)\*b\*x^2 - (b\*x^3 + a)^(1/3)\*(-b^2)^(2/3)\*x - (b\*x^3 + a)^(2/3)\*b)/x^2) + 9\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log(-((b\*c - a\*d)\*x^2\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3) + (b\*x^3 + a)^(1/3)\*c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) + (b\*x^3 + a)^(2/3)\*(b\*c - a\*d))/x^2) - 3\*(3\*b^2\*d^2\*x^4 - 2\*(3\*b^2\*c\*d - 7\*a\*b\*d^2)\*x)\*(b\*x^3 + a)^(2/3))/d^3

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(8/3)/(d\*x^3 + c), x)

maple [F] time = 0.70, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(8/3)/(d\*x^3+c),x)

[Out] int((b\*x^3+a)^(8/3)/(d\*x^3+c),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(8/3)/(d\*x^3 + c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(8/3)/(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(8/3)/(c + d\*x^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(8/3)/(d\*x\*\*3+c),x)

[Out] Timed out



$$3.87 \quad \int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx$$

**Optimal.** Leaf size=273

$$\frac{b^{2/3}(3bc - 5ad) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{6d^2} - \frac{b^{2/3}(3bc - 5ad) \tan^{-1}\left(\frac{2\sqrt[3]{bx} + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}d^2} + \frac{(bc - ad)^{5/3} \log(c + dx^3)}{6c^{2/3}d^2} - \frac{(bc - ad)^{5/3}}{6c^{2/3}d^2}$$

[Out]  $\frac{1}{3}bx^3(a+bx^3)^{2/3}/d + \frac{1}{6}(-ad+bc)^{5/3} \ln(d^2x^3+c)/c^{2/3}/d^2 - \frac{1}{2}(-ad+bc)^{5/3} \ln((a+bx^3)^{1/3}x/c^{1/3} - (a+bx^3)^{1/3})/c^{2/3}/d^2 + \frac{1}{6}b^{2/3}(-5ad+3bc) \ln(-b^{1/3}x + (a+bx^3)^{1/3})/d^2 - \frac{1}{9}b^{2/3}(-5ad+3bc) \arctan(1/3(1+2b^{1/3}x/(a+bx^3)^{1/3})^3)^{1/2}/d^2 + \frac{1}{3}(-ad+bc)^{5/3} \arctan(1/3(1+2(-ad+bc)^{1/3}x/c^{1/3})/(a+bx^3)^{1/3})^3)^{1/2}/c^{2/3}/d^2$

**Rubi [C]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.22, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{ax(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{5}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(5/3)/(c + d\*x^3), x]

[Out] (a\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -5/3, 1, 4/3, -(b\*x^3)/a, -(d\*x^3)/c])/(c\*(1 + (b\*x^3)/a)^(2/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -(b\*x^n)/a, -(d\*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^{5/3}}{c+dx^3} dx &= \frac{\left(a(a+bx^3)^{2/3}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^{5/3}}{c+dx^3} dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} \\ &= \frac{ax(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{5}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}} \end{aligned}$$

**Mathematica** [C] time = 0.71, size = 443, normalized size = 1.62

$$2\sqrt[3]{c} \left( 3a^2d\sqrt[3]{a+bx^3} \log\left(\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3}\right) + 6b^2c^{2/3}x^4\sqrt[3]{bc-ad} - abc\sqrt[3]{a+bx^3} \log\left(\frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \dots \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(5/3)/(c + d\*x^3), x]

[Out] (3\*b\*(b\*c - a\*d)^(1/3)\*(-3\*b\*c + 5\*a\*d)\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*c^(1/3)\*(6\*a\*b\*c^(2/3)\*(b\*c - a\*d)^(1/3)\*x + 6\*b^2\*c^(2/3)\*(b\*c - a\*d)^(1/3)\*x^4 + 2\*Sqrt[3]\*a\*(-(b\*c) + 3\*a\*d)\*(a + b\*x^3)^(1/3)\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] + 2\*a\*(b\*c - 3\*a\*d)\*(a + b\*x^3)^(1/3)\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] - a\*b\*c\*(a + b\*x^3)^(1/3)\*Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + 3\*a^2\*d\*(a + b\*x^3)^(1/3)\*Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)])/(36\*c\*d\*(b\*c - a\*d)^(1/3)\*(a + b\*x^3)^(1/3))

**fricas** [B] time = 1.75, size = 535, normalized size = 1.96

$$6(bx^3 + a)^{\frac{2}{3}}bdx + 6\sqrt{3}(bc - ad)\left(\frac{b^2c^2 - 2abcd + a^2d^2}{c^2}\right)^{\frac{1}{3}} \arctan\left(\frac{\sqrt{3}(bc-ad)x + 2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2 - 2abcd + a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right) + 2\sqrt{3}(-b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c), x, algorithm="fricas")

[Out] 1/18\*(6\*(b\*x^3 + a)^(2/3)\*b\*d\*x + 6\*sqrt(3)\*(b\*c - a\*d)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*(b\*c - a\*d)\*x + 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*c\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3))/((b\*c - a\*d)\*x)) + 2\*sqrt(3)\*(-b^2)^(1/3)\*(3\*b\*c - 5\*a\*d)\*arctan(-1/3\*(sqrt(3)\*b\*x - 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*(-b^2)^(1/3))/(b\*x)) - 6\*(b\*c - a\*d)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log((c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) - (b\*x^3 + a)^(1/3)\*(b\*c - a\*d))/x) - 2\*(-b^2)^(1/3)\*(3\*b\*c - 5\*a\*d)\*log(-((-b^2)^(2/3)\*x - (b\*x^3 + a)^(1/3)\*b)/x) + (-b^2)^(1/3)\*(3\*b\*c - 5\*a\*d)\*log(-((-b^2)^(1/3)\*b\*x^2 - (b\*x^3 + a)^(1/3)\*(-b^2)^(2/3)\*x - (b\*x^3 + a)^(2/3)\*b)/x^2) + 3\*(b\*c - a\*d)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log(-((b\*c - a\*d)\*x^2\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3) + (b\*x^3 + a)^(1/3)\*c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) + (b\*x^3 + a)^(2/3)\*(b\*c - a\*d))/x^2))/d^2

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(5/3)/(d\*x^3+c), x)

[Out] int((b\*x^3+a)^(5/3)/(d\*x^3+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c), x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(5/3)/(c + d\*x^3), x)

[Out] int((a + b\*x^3)^(5/3)/(c + d\*x^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{5}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(5/3)/(d\*x\*\*3+c), x)

[Out] Integral((a + b\*x\*\*3)\*\*(5/3)/(c + d\*x\*\*3), x)

$$3.88 \quad \int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx$$

**Optimal.** Leaf size=233

$$\frac{b^{2/3} \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx^3}\right)}{2d} + \frac{b^{2/3} \tan^{-1}\left(\frac{2\sqrt[3]{bx^3} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}d} - \frac{(bc-ad)^{2/3} \log(c+dx^3)}{6c^{2/3}d} + \frac{(bc-ad)^{2/3} \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a}\right)}{2c^{2/3}d}$$

[Out]  $-1/6*(-a*d+b*c)^{(2/3)*\ln(d*x^3+c)/c^{(2/3)/d}+1/2*(-a*d+b*c)^{(2/3)*\ln((-a*d+b*c)^{(1/3)*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(2/3)/d}-1/2*b^{(2/3)*\ln(-b^{(1/3)*x+(b*x^3+a)^{(1/3)})/d+1/3*b^{(2/3)*\arctan(1/3*(1+2*b^{(1/3)*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/d*3^{(1/2)}-1/3*(-a*d+b*c)^{(2/3)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)*x/c^{(1/3)/(b*x^3+a)^{(1/3)})*3^{(1/2)})/c^{(2/3)/d*3^{(1/2)}}$

**Rubi [C]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 0.25, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(2/3)/(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)])/((c\*(1 + (b\*x^3)/a)^(2/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/((1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{(a+bx^3)^{2/3}}{c+dx^3} dx = \frac{(a+bx^3)^{2/3} \int \frac{\left(1+\frac{bx^3}{a}\right)^{2/3}}{c+dx^3} dx}{\left(1+\frac{bx^3}{a}\right)^{2/3}} = \frac{x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\left(1+\frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 0.22, size = 161, normalized size = 0.69

$$\frac{4acx(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(2bcF_1\left(\frac{4}{3}; \frac{1}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 3adF_1\left(\frac{4}{3}; -\frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) + 4acF_1\left(\frac{1}{3}; -\frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(2/3)/(c + d\*x^3), x]

[Out] (4\*a\*c\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c + d\*x^3)\*(4\*a\*c\*AppellF1[1/3, -2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(-3\*a\*d\*AppellF1[4/3, -2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas [B]** time = 1.02, size = 469, normalized size = 2.01

$$2\sqrt{3}\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}(bc-ad)x+2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c\left(\frac{b^2c^2-2abcd+a^2d^2}{c^2}\right)^{\frac{1}{3}}}{3(bc-ad)x}\right)+2\sqrt{3}(-b^2)^{\frac{1}{3}}\arctan\left(-\frac{\sqrt{3}bx-2\sqrt{3}}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c), x, algorithm="fricas")

[Out] -1/6\*(2\*sqrt(3)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*(b\*c - a\*d)\*x + 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*c\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3))/((b\*c - a\*d)\*x)) + 2\*sqrt(3)\*(-b^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*b\*x - 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*(-b^2)^(1/3))/(b\*x)) - 2\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log((c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) - (b\*x^3 + a)^(1/3)\*(b\*c - a\*d))/x) - 2\*(-b^2)^(1/3)\*log(-((-b^2)^(2/3)\*x - (b\*x^3 + a)^(1/3)\*b)/x) + (-b^2)^(1/3)\*log(-((-b^2)^(1/3)\*b\*x^2 - (b\*x^3 + a)^(1/3)\*(-b^2)^(2/3)\*x - (b\*x^3 + a)^(2/3)\*b)/x^2) + ((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*log(-((b\*c - a\*d)\*x^2\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3) + (b\*x^3 + a)^(1/3)\*c\*x\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(2/3) + (b\*x^3 + a)^(2/3)\*(b\*c - a\*d))/x^2))/d

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c), x)

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(2/3)/(d\*x^3+c), x)

[Out] int((b\*x^3+a)^(2/3)/(d\*x^3+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)/(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(2/3)/(c + d\*x^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(2/3)/(d\*x\*\*3+c),x)

[Out] Integral((a + b\*x\*\*3)\*\*(2/3)/(c + d\*x\*\*3), x)

$$3.89 \quad \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx$$

Optimal. Leaf size=148

$$\frac{\log(c+dx^3)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}\sqrt[3]{bc-ad}} + \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

[Out]  $1/6*\ln(d*x^3+c)/c^{(2/3)/(-a*d+b*c)^{(1/3)}-1/2*\ln((-a*d+b*c)^{(1/3)*x/c^{(1/3)}-(b*x^3+a)^{(1/3)}/c^{(2/3)/(-a*d+b*c)^{(1/3)}+1/3*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)*x/c^{(1/3)}/(b*x^3+a)^{(1/3)})*3^{(1/2)})/c^{(2/3)/(-a*d+b*c)^{(1/3)}*3^{(1/2)}$

Rubi [A] time = 0.19, antiderivative size = 207, normalized size of antiderivative = 1.40, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {377, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{2/3}\sqrt[3]{bc-ad}} + \frac{\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)), x]

[Out] ArcTan[(c^(1/3) + (2\*(b\*c - a\*d)^(1/3)\*x)/(a + b\*x^3)^(1/3))/(Sqrt[3]\*c^(1/3))]/(Sqrt[3]\*c^(2/3)\*(b\*c - a\*d)^(1/3)) - Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(a + b\*x^3)^(1/3)]/(3\*c^(2/3)\*(b\*c - a\*d)^(1/3)) + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(a + b\*x^3)^(1/3)]/(6\*c^(2/3)\*(b\*c - a\*d)^(1/3))

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx &= \text{Subst} \left( \int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right) \\ &= \frac{\text{Subst} \left( \int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}} + \frac{\text{Subst} \left( \int \frac{2\sqrt[3]{c}+\sqrt[3]{bc-ad}x}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}} \\ &= -\frac{\log \left( \sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\text{Subst} \left( \int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x+(bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{2\sqrt[3]{c}} + \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^{2/3}\sqrt[3]{bc-ad}} \\ &= -\frac{\log \left( \sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log \left( c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6c^{2/3}\sqrt[3]{bc-ad}} - \frac{\text{Subst} \left( \int \frac{1}{-3-x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}} \right)}{c^{2/3}\sqrt[3]{bc-ad}} \\ &= \frac{\tan^{-1} \left( \frac{1+\frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}} \right)}{\sqrt{3}c^{2/3}\sqrt[3]{bc-ad}} - \frac{\log \left( \sqrt[3]{c}-\frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{3c^{2/3}\sqrt[3]{bc-ad}} + \frac{\log \left( c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}} \right)}{6c^{2/3}\sqrt[3]{bc-ad}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 168, normalized size = 1.14

$$\frac{\log \left( \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + c^{2/3} \right) - 2 \log \left( \sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}\sqrt[3]{a+bx^3}} + 1}{\sqrt{3}} \right)}{6c^{2/3}\sqrt[3]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)), x]

[Out] (2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(a + b\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(a + b\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(a + b\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(a + b\*x^3)^(1/3)])/(6\*c^(2/3)\*(b\*c - a\*d)^(1/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)), x)

**maple** [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{1/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)),x)

[Out] int(1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(1/3)/(d\*x\*\*3+c),x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(1/3)\*(c + d\*x\*\*3)), x)

$$3.90 \quad \int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)} dx$$

**Optimal.** Leaf size=179

$$-\frac{d \log(c+dx^3)}{6c^{2/3}(bc-ad)^{4/3}} + \frac{d \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{2c^{2/3}(bc-ad)^{4/3}} - \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

[Out]  $b*x/a/(-a*d+b*c)/(b*x^3+a)^{(1/3)}-1/6*d*\ln(d*x^3+c)/c^{(2/3)/(-a*d+b*c)^{(4/3)}$   
 $+1/2*d*\ln((-a*d+b*c)^{(1/3)*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(2/3)/(-a*d+b*c)^{(4/3)}$   
 $-1/3*d*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)*x/c^{(1/3)}(b*x^3+a)^{(1/3)})*3^{(1/2)})/c^{(2/3)/(-a*d+b*c)^{(4/3)*3^{(1/2)}}$

**Rubi [A]** time = 0.19, antiderivative size = 238, normalized size of antiderivative = 1.33, number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$\frac{d \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc-ad)^{4/3}} - \frac{d \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{6c^{2/3}(bc-ad)^{4/3}} - \frac{d \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3}\sqrt[3]{c}}\right)}{\sqrt{3}c^{2/3}(bc-ad)^{4/3}} + \frac{bx}{a\sqrt[3]{a+bx^3}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)), x]

[Out]  $(b*x)/(a*(b*c - a*d)*(a + b*x^3)^{(1/3)} - (d*\text{ArcTan}[(c^{(1/3)} + (2*(b*c - a*d)^{(1/3)*x})/(a + b*x^3)^{(1/3)})]/(\text{Sqrt}[3]*c^{(1/3)})))/(\text{Sqrt}[3]*c^{(2/3)}*(b*c - a*d)^{(4/3)} + (d*\text{Log}[c^{(1/3)} - ((b*c - a*d)^{(1/3)*x})/(a + b*x^3)^{(1/3)}])/ (3*c^{(2/3)}*(b*c - a*d)^{(4/3)} - (d*\text{Log}[c^{(2/3)} + ((b*c - a*d)^{(2/3)*x^2})/(a + b*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)*x})/(a + b*x^3)^{(1/3)}])/ (6*c^{(2/3)}*(b*c - a*d)^{(4/3)})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)], I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + bx^3)^{4/3} (c + dx^3)} dx &= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} - \frac{d \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx}{bc - ad} \\
&= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{c - (bc - ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{bc - ad} \\
&= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{c} - \sqrt[3]{bc - ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{2/3}(bc - ad)} - \frac{d \operatorname{Subst}\left(\int \frac{1}{c^{2/3} + \sqrt[3]{c} \sqrt[3]{bc - ad}x + (bc - ad)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}(bc - ad)} \\
&= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} + \frac{d \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc - ad}x}{\sqrt[3]{a + bx^3}}\right)}{3c^{2/3}(bc - ad)^{4/3}} - \frac{d \operatorname{Subst}\left(\int \frac{\sqrt[3]{c} \sqrt[3]{bc - ad} + 2(bc - ad)}{c^{2/3} + \sqrt[3]{c} \sqrt[3]{bc - ad}x + (bc - ad)} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{6c^{2/3}(bc - ad)} \\
&= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} + \frac{d \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc - ad}x}{\sqrt[3]{a + bx^3}}\right)}{3c^{2/3}(bc - ad)^{4/3}} - \frac{d \log\left(c^{2/3} + \frac{(bc - ad)^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{\sqrt[3]{c} \sqrt[3]{bc - ad}}{\sqrt[3]{a + bx^3}}\right)}{6c^{2/3}(bc - ad)^{4/3}} \\
&= \frac{bx}{a(bc - ad)\sqrt[3]{a + bx^3}} - \frac{d \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bc - ad}x}{\sqrt[3]{c} \sqrt[3]{a + bx^3}}}{\sqrt{3}}\right)}{\sqrt{3} c^{2/3}(bc - ad)^{4/3}} + \frac{d \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc - ad}x}{\sqrt[3]{a + bx^3}}\right)}{3c^{2/3}(bc - ad)^{4/3}} - \frac{d \log\left(c^{2/3} + \frac{(bc - ad)^{2/3}x^2}{(a + bx^3)^{2/3}} + \frac{\sqrt[3]{c} \sqrt[3]{bc - ad}}{\sqrt[3]{a + bx^3}}\right)}{6c^{2/3}(bc - ad)^{4/3}}
\end{aligned}$$

**Mathematica** [C] time = 0.80, size = 256, normalized size = 1.43

$$\frac{28c^3(a+bx^3)^2 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 28c^3(a+bx^3)^2 + 21c^2dx^3(a+bx^3)^2 {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 21c^2dx^3(a+bx^3)^2}{7c^3x^2(a+bx^3)^{7/3}(ad-bc)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)),x]

[Out]  $-1/7*(-28*c^3*(a + b*x^3)^2 - 21*c^2*d*x^3*(a + b*x^3)^2 + 28*c^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21*c^2*d*x^3*(a + b*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*c*(b*c - a*d)^2*x^6*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3*d*(b*c - a*d)^2*x^9*Hypergeometric2F1[2, 7/3, 10/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(c^3*(-(b*c) + a*d)*x^2*(a + b*x^3)^(7/3))$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{4/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)), x)

[Out] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{4/3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(4/3)/(d\*x\*\*3+c), x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(4/3)\*(c + d\*x\*\*3)), x)

**3.91**  $\int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)} dx$

**Optimal.** Leaf size=226

$$\frac{bx(3bc - 7ad)}{4a^2 \sqrt[3]{a + bx^3} (bc - ad)^2} + \frac{d^2 \log(c + dx^3)}{6c^{2/3}(bc - ad)^{7/3}} - \frac{d^2 \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a + bx^3}\right)}{2c^{2/3}(bc - ad)^{7/3}} + \frac{d^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{\sqrt{3} c^{2/3}(bc - ad)^{7/3}} + \frac{bx}{4a(a + bx^3)^{4/3}}$$

[Out]  $\frac{1}{4}bx/a/(-a*d+b*c)/(b*x^3+a)^{(4/3)} + \frac{1}{4}b*(-7*a*d+3*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^3+a)^{(1/3)} + \frac{1}{6}d^2*\ln(d*x^3+c)/c^{(2/3)}/(-a*d+b*c)^{(7/3)} - \frac{1}{2}d^2*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)} - (b*x^3+a)^{(1/3)})/c^{(2/3)}/(-a*d+b*c)^{(7/3)} + \frac{1}{3}d^2*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})/c^{(2/3)}/(-a*d+b*c)^{(7/3)}$

**Rubi [C]** time = 2.58, antiderivative size = 621, normalized size of antiderivative = 2.75, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$-9c^2x^9(bc - ad)^3 {}_3F_2\left(2, 2, \frac{10}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 9d^2x^{15}(bc - ad)^3 {}_3F_2\left(2, 2, \frac{10}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) - 18cdx^{12}(bc - ad)$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)), x]

[Out]  $-(70*c^4*(b*c - a*d)*x^3*(a + b*x^3)^2 + 105*c^3*d*(b*c - a*d)*x^6*(a + b*x^3)^2 + 45*c^2*d^2*(b*c - a*d)*x^9*(a + b*x^3)^2 + 280*c^5*(a + b*x^3)^3 + 420*c^4*d*x^3*(a + b*x^3)^3 + 180*c^3*d^2*x^6*(a + b*x^3)^3 - 280*c^5*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 420*c^4*d*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 180*c^3*d^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 33*c^2*(b*c - a*d)^3*x^9*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 60*c*d*(b*c - a*d)^3*x^12*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 27*d^2*(b*c - a*d)^3*x^15*Hypergeometric2F1[2, 10/3, 13/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*c^2*(b*c - a*d)^3*x^9*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 18*c*d*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 9*d^2*(b*c - a*d)^3*x^15*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(40*c^4*(b*c - a*d)^2*x^5*(a + b*x^3)^(10/3))$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{1}{(a + bx^3)^{7/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{7/3} (c + dx^3)} dx}{a^2 \sqrt[3]{a + bx^3}}$$

$$= -\frac{70c^4(bc - ad)x^3 (a + bx^3)^2 + 105c^3d(bc - ad)x^6 (a + bx^3)^2 + 45c^2d^2(bc - ad)x^9 (a + bx^3)^2 + 15cd^3(bc - ad)x^{12} (a + bx^3)^2 + 5d^4(bc - ad)x^{15}}{a^2 \sqrt[3]{a + bx^3}}$$

**Mathematica [C]** time = 2.67, size = 621, normalized size = 2.75

$$9c^2x^9(bc - ad)^3 {}_3F_2\left(2, 2, \frac{10}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 9d^2x^{15}(bc - ad)^3 {}_3F_2\left(2, 2, \frac{10}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right) + 18cdx^{12}(bc - ad)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)),x]

[Out] (-70\*c^4\*(b\*c - a\*d)\*x^3\*(a + b\*x^3)^2 - 105\*c^3\*d\*(b\*c - a\*d)\*x^6\*(a + b\*x^3)^2 - 45\*c^2\*d^2\*(b\*c - a\*d)\*x^9\*(a + b\*x^3)^2 - 280\*c^5\*(a + b\*x^3)^3 - 420\*c^4\*d\*x^3\*(a + b\*x^3)^3 - 180\*c^3\*d^2\*x^6\*(a + b\*x^3)^3 + 280\*c^5\*(a + b\*x^3)^3\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 420\*c^4\*d\*x^3\*(a + b\*x^3)^3\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 180\*c^3\*d^2\*x^6\*(a + b\*x^3)^3\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 33\*c^2\*(b\*c - a\*d)^3\*x^9\*Hypergeometric2F1[2, 10/3, 13/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 60\*c\*d\*(b\*c - a\*d)^3\*x^12\*Hypergeometric2F1[2, 10/3, 13/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 27\*d^2\*(b\*c - a\*d)^3\*x^15\*Hypergeometric2F1[2, 10/3, 13/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 9\*c^2\*(b\*c - a\*d)^3\*x^9\*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 18\*c\*d\*(b\*c - a\*d)^3\*x^12\*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 9\*d^2\*(b\*c - a\*d)^3\*x^15\*HypergeometricPFQ[{2, 2, 10/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]/(40\*c^4\*(b\*c - a\*d)^2\*x^5\*(a + b\*x^3)^(10/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{7/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)),x)

[Out] int(1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{7}{3}}(c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(7/3)/(d\*x\*\*3+c),x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(7/3)\*(c + d\*x\*\*3)), x)



$$3.92 \quad \int \frac{1}{(a+bx^3)^{10/3}(c+dx^3)} dx$$

**Optimal.** Leaf size=280

$$\frac{bx(6bc - 13ad)}{28a^2(a + bx^3)^{4/3}(bc - ad)^2} + \frac{bx(67a^2d^2 - 57abcd + 18b^2c^2)}{28a^3\sqrt[3]{a + bx^3}(bc - ad)^3} - \frac{d^3 \log(c + dx^3)}{6c^{2/3}(bc - ad)^{10/3}} + \frac{d^3 \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a + bx^3}\right)}{2c^{2/3}(bc - ad)^{10/3}}$$

[Out]  $1/7*b*x/a/(-a*d+b*c)/(b*x^3+a)^{(7/3)}+1/28*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^3+a)^{(4/3)}+1/28*b*(67*a^2*d^2-57*a*b*c*d+18*b^2*c^2)*x/a^3/(-a*d+b*c)^3/(b*x^3+a)^{(1/3)}-1/6*d^3*\ln(d*x^3+c)/c^{(2/3)}/(-a*d+b*c)^{(10/3)}+1/2*d^3*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(2/3)}/(-a*d+b*c)^{(10/3)}-1/3*d^3*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})*3^{(1/2)}/c^{(2/3)}/(-a*d+b*c)^{(10/3)}*3^{(1/2)}$

**Rubi [C]** time = 6.64, antiderivative size = 1172, normalized size of antiderivative = 4.19, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$-4158d^3(bc - ad)^4 {}_2F_1\left(2, \frac{13}{3}; \frac{16}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{21} - 2268d^3(bc - ad)^4 {}_3F_2\left(2, 2, \frac{13}{3}; 1, \frac{16}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{21} - 378d^3(bc - ad)^4 {}_4F_3\left(2, 2, 2, \frac{13}{3}; 1, 1, \frac{16}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{21}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(10/3)\*(c + d\*x^3)),x]

[Out]  $-(7280*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 16380*c^4*d*(b*c - a*d)^2*x^9*(a + b*x^3)^2 + 14040*c^3*d^2*(b*c - a*d)^2*x^{12}*(a + b*x^3)^2 + 4212*c^2*d^3*(b*c - a*d)^2*x^{15}*(a + b*x^3)^2 + 12740*c^6*(b*c - a*d)*x^3*(a + b*x^3)^3 + 28665*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 24570*c^4*d^2*(b*c - a*d)*x^9*(a + b*x^3)^3 + 7371*c^3*d^3*(b*c - a*d)*x^{12}*(a + b*x^3)^3 + 50960*c^7*(a + b*x^3)^4 + 114660*c^6*d*x^3*(a + b*x^3)^4 + 98280*c^5*d^2*x^6*(a + b*x^3)^4 + 29484*c^4*d^3*x^9*(a + b*x^3)^4 - 50960*c^7*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 114660*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 98280*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 29484*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5796*c^3*(b*c - a*d)^4*x^{12}*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 15246*c^2*d*(b*c - a*d)^4*x^{15}*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 13608*c*d^2*(b*c - a*d)^4*x^{18}*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 4158*d^3*(b*c - a*d)^4*x^{21}*Hypergeometric2F1[2, 13/3, 16/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2646*c^3*(b*c - a*d)^4*x^{12}*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 7560*c^2*d*(b*c - a*d)^4*x^{15}*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 7182*c*d^2*(b*c - a*d)^4*x^{18}*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 2268*d^3*(b*c - a*d)^4*x^{21}*HypergeometricPFQ[{2, 2, 13/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*c^3*(b*c - a*d)^4*x^{12}*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1134*c^2*d*(b*c - a*d)^4*x^{15}*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1134*c*d^2*(b*c - a*d)^4*x^{18}*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 378*d^3*(b*c - a*d)^4*x^{21}*HypergeometricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]$

tricPFQ[{2, 2, 2, 13/3}, {1, 1, 16/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]/  
(5096\*c^5\*(b\*c - a\*d)^3\*x^8\*(a + b\*x^3)^(13/3))

#### Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rubi steps

$$\int \frac{1}{(a + bx^3)^{10/3} (c + dx^3)} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{10/3} (c + dx^3)} dx}{a^3 \sqrt[3]{a + bx^3}}$$

$$= -\frac{7280c^5(bc - ad)^2x^6(a + bx^3)^2 + 16380c^4d(bc - ad)^2x^9(a + bx^3)^2 + 14040c^3d^2(a + bx^3)^2}{28a^3(a + bx^3)^{7/3}(bc - ad)^3}$$

**Mathematica [A]** time = 5.75, size = 277, normalized size = 0.99

$$\frac{bx \left( (a + bx^3)^2 (67a^2d^2 - 57abcd + 18b^2c^2) + 4a^2(bc - ad)^2 + a(a + bx^3)(ad - bc)(13ad - 6bc) \right)}{28a^3(a + bx^3)^{7/3}(bc - ad)^3} d^3 \left( \log \left( \frac{\sqrt[3]{cx} \sqrt[3]{b}}{\sqrt[3]{ax^3}} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(10/3)\*(c + d\*x^3)),x]

```
[Out] (b*x*(4*a^2*(b*c - a*d)^2 + a*(-(b*c) + a*d)*(-6*b*c + 13*a*d)*(a + b*x^3)
+ (18*b^2*c^2 - 57*a*b*c*d + 67*a^2*d^2)*(a + b*x^3)^2)/(28*a^3*(b*c - a*d)
)^3*(a + b*x^3)^(7/3) - (d^3*(2*sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)
)/(c^(1/3)*(b + a*x^3)^(1/3))]/sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)
)*x]/(b + a*x^3)^(1/3] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)
^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)])/(6*c^(2/3)*(b*c
- a*d)^(10/3))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(10/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{10}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(10/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(10/3)\*(d\*x^3 + c)), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{10}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(10/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(10/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{10}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(10/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(10/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{10}{3}} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(10/3)\*(c + d\*x^3)),x)

[Out] int(1/((a + b\*x^3)^(10/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{10}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(10/3)/(d\*x\*\*3+c),x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(10/3)\*(c + d\*x\*\*3)), x)

$$3.93 \quad \int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx$$

**Optimal.** Leaf size=60

$$\frac{ax^3\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

[Out] a\*x\*(b\*x^3+a)^(1/3)\*AppellF1(1/3,-4/3,1,4/3,-b\*x^3/a,-d\*x^3/c)/c/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{ax^3\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(4/3)/(c + d\*x^3),x]

[Out] (a\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, -4/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c\*(1 + (b\*x^3)/a)^(1/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^{4/3}}{c+dx^3} dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{c+dx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{ax^3\sqrt[3]{a+bx^3}F_1\left(\frac{1}{3};-\frac{4}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$

**Mathematica [B]** time = 0.61, size = 346, normalized size = 5.77

$$x \left( \frac{4\left(bx^3(a+bx^3)(c+dx^3)\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4ac(2a^2d+abdx^3+b^2x^3(c+dx^3))F_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(c+dx^3)\left(x^3\left(3adF_1\left(\frac{4}{3};\frac{2}{3},2;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)+2bcF_1\left(\frac{4}{3};\frac{5}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)\right)-4acF_1\left(\frac{1}{3};\frac{2}{3},1;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}\right) + \dots}{8d(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(4/3)/(c + d\*x^3),x]

[Out] (x\*((b\*(-2\*b\*c + 3\*a\*d)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])/c + (4\*(-4\*a\*c\*(2\*a^2\*d + a\*b\*d\*x^3 + b^2\*x^3\*(c + d\*x^3))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + b\*x^3\*(a + b\*x^3)\*(c + d\*x^3)\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((c + d\*x^3)\*(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(8\*d\*(a + b\*x^3)^(2/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(4/3)/(d\*x^3+c),x)

[Out] int((b\*x^3+a)^(4/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

```
[Out] int((a + b*x^3)^(4/3)/(c + d*x^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{4}{3}}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(4/3)/(d*x**3+c), x)
```

```
[Out] Integral((a + b*x**3)**(4/3)/(c + d*x**3), x)
```

$$3.94 \quad \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} dx$$

Optimal. Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}} + 1}$$

[Out]  $x*(b*x^3+a)^{(1/3)}*AppellF1(1/3,-1/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\sqrt[3]{\frac{bx^3}{a}} + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(1/3)/(c + d\*x^3), x]

[Out]  $(x*(a + b*x^3)^{(1/3)}*AppellF1[1/3, -1/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(1 + (b*x^3)/a)^{(1/3)})$

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{a+bx^3}}{c+dx^3} dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{c+dx^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c\sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$

**Mathematica [B]** time = 0.17, size = 160, normalized size = 2.71

$$\frac{4acx\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c+dx^3) \left( x^3 \left( bcF_1\left(\frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 3adF_1\left(\frac{4}{3}; -\frac{1}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) + 4acF_1\left(\frac{1}{3}; -\frac{1}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/3)/(c + d\*x^3),x]

[Out] (4\*a\*c\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, -1/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/((c + d\*x^3)\*(4\*a\*c\*AppellF1[1/3, -1/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(-3\*a\*d\*AppellF1[4/3, -1/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + b\*c\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/3)/(d\*x^3+c),x)

[Out] int((b\*x^3+a)^(1/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)/(c + d\*x^3),x)

[Out] int((a + b\*x^3)^(1/3)/(c + d\*x^3), x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{c + dx^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/3)/(d\*x\*\*3+c), x)

[Out] Integral((a + b\*x\*\*3)\*\*(1/3)/(c + d\*x\*\*3), x)

$$3.95 \quad \int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx$$

**Optimal.** Leaf size=59

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)}*AppellF1(1/3,2/3,1,4/3,-b*x^3/a,-d*x^3/c)/c/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)),x]

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)}*AppellF1[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(a + b*x^3)^{(2/3)})$

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3}(c+dx^3)} dx}{(a+bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c (a + bx^3)^{2/3}}$$

**Mathematica [B]** time = 0.04, size = 161, normalized size = 2.73

$$\frac{4acx F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{(a + bx^3)^{2/3} (c + dx^3) \left( x^3 \left( 3ad F_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bc F_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4ac F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)),x]

[Out] (-4\*a\*c\*x\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/((a + b\*x^3)^(2/3)\*(c + d\*x^3)\*(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)),x)

[Out] `int(1/((a + b*x^3)^(2/3)*(c + d*x^3)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{2}{3}} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c),x)`

[Out] `Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)), x)`

$$3.96 \quad \int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)}*AppellF1(1/3,5/3,1,4/3,-b*x^3/a,-d*x^3/c)/a/c/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)),x]

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)}*AppellF1[1/3, 5/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*c*(a + b*x^3)^{(2/3)})$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{5/3}(c+dx^3)} dx}{a(a+bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{5}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{ac(a+bx^3)^{2/3}}$$

**Mathematica [B]** time = 0.28, size = 332, normalized size = 5.35

$$x \frac{\left( 4 \left( b x^3 (c + d x^3) \left( 3 {}_3F_1 \left( \frac{4}{3}, \frac{2}{3}, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) + 2 b c {}_1F_1 \left( \frac{4}{3}; \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right) + 4 a c (2 a d - b (2 c + d x^3)) {}_1F_1 \left( \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right)}{(c + d x^3) \left( 4 a c {}_1F_1 \left( \frac{1}{3}; \frac{2}{3}, \frac{4}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) - x^3 \left( 3 {}_3F_1 \left( \frac{4}{3}, \frac{2}{3}, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) + 2 b c {}_1F_1 \left( \frac{4}{3}; \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right) \right)} - \frac{b d x^3 \left( \frac{b x^3}{a} + 1 \right)^{2/3} {}_1F_1 \left( \frac{4}{3}; \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right)}{c} \right) \frac{1}{8 a (a + b x^3)^{2/3} (a d - b c)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)),x]

[Out] (x\*(-((b\*d\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])/c) + (4\*(4\*a\*c\*(2\*a\*d - b\*(2\*c + d\*x^3))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + b\*x^3\*(c + d\*x^3)\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(c + d\*x^3)\*(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(8\*a\*(-(b\*c) + a\*d)\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{5}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)), x)

**maple [F]** time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{5}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{5}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{5/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)), x)

[Out] int(1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{5/3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(5/3)/(d\*x\*\*3+c), x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(5/3)\*(c + d\*x\*\*3)), x)

$$3.97 \quad \int \frac{1}{(a+bx^3)^{8/3}(c+dx^3)} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)*AppellF1(1/3,8/3,1,4/3,-b*x^3/a,-d*x^3/c)/a^2/c/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)),x]

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)*AppellF1[1/3, 8/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)]})/(a^2*c*(a + b*x^3)^{(2/3)})$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a + bx^3)^{8/3}(c + dx^3)} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{8/3}(c + dx^3)} dx}{a^2 (a + bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{8}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2 c (a + bx^3)^{2/3}}$$



**Mathematica [B]** time = 0.87, size = 429, normalized size = 6.92

$$x \frac{\left( 4 \left( b x^3 (c + d x^3) (11 a^2 d + a b (9 d x^3 - 6 c) - 4 b^2 c x^3) \left( 3 \operatorname{ad} F_1 \left( \frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) + 2 b c F_1 \left( \frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right) + 4 a c (10 a^3 d^2 - a^2 b d (20 c + d x^3) + a b^2 (10 a^2 d + a b (9 d x^3 - 6 c) - 4 b^2 c x^3)) \right)}{(a + b x^3)(c + d x^3) \left( x^3 \left( 3 \operatorname{ad} F_1 \left( \frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) + 2 b c F_1 \left( \frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right) - 4 a c F_1 \left( \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{b x^3}{a}, -\frac{d x^3}{c} \right) \right)}{40 a^2 (a + b x^3)^{2/3} (b c - a d)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)),x]

[Out] 
$$-1/40 * (x * ((b*d*(-4*b*c + 9*a*d)) * x^3 * (1 + (b*x^3)/a)^{2/3} * \operatorname{AppellF1}[4/3, 2/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]) / c + (4 * (4*a*c*(10*a^3*d^2 + 4*b^3*c*x^3*(2*c + d*x^3) - a^2*b*d*(20*c + d*x^3) + a*b^2*(10*c^2 - 12*c*d*x^3 - 9*d^2*x^6)) * \operatorname{AppellF1}[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + b*x^3*(c + d*x^3)*(11*a^2*d - 4*b^2*c*x^3 + a*b*(-6*c + 9*d*x^3)) * (3*a*d*\operatorname{AppellF1}[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*\operatorname{AppellF1}[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)])) / ((a + b*x^3)*(c + d*x^3)*(-4*a*c*\operatorname{AppellF1}[1/3, 2/3, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)] + x^3*(3*a*d*\operatorname{AppellF1}[4/3, 2/3, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)] + 2*b*c*\operatorname{AppellF1}[4/3, 5/3, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))))) / (a^2*(b*c - a*d)^2*(a + b*x^3)^{2/3})$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{8}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)), x)

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{8}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c),x)

[Out] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b x^3 + a)^{\frac{8}{3}} (d x^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{8/3} (dx^3 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)),x)

[Out] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{8/3} (c + dx^3)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(8/3)/(d\*x\*\*3+c),x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(8/3)\*(c + d\*x\*\*3)), x)

$$3.98 \quad \int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=351

$$\frac{b^{5/3}(3bc - 4ad) \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{3d^3} - \frac{2b^{5/3}(3bc - 4ad) \tan^{-1}\left(\frac{\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}d^3} + \frac{(bc - ad)^{5/3}(ad + 3bc) \log(c + dx^3)}{9c^{5/3}d^3}$$

```
[Out] 1/3*b*(-a*d+2*b*c)*x*(b*x^3+a)^(2/3)/c/d^2-1/3*(-a*d+b*c)*x*(b*x^3+a)^(5/3)
/c/d/(d*x^3+c)+1/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln(d*x^3+c)/c^(5/3)/d^3-1/3
*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3)
)/c^(5/3)/d^3+1/3*b^(5/3)*(-4*a*d+3*b*c)*ln(-b^(1/3)*x/(b*x^3+a)^(1/3))/d^3
-2/9*b^(5/3)*(-4*a*d+3*b*c)*arctan(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1
/2))/d^3*3^(1/2)+2/9*(-a*d+b*c)^(5/3)*(a*d+3*b*c)*arctan(1/3*(1+2*(-a*d+b*c)
)^(1/3)*x/c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(5/3)/d^3*3^(1/2)
```

**Rubi [C]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.18, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{a^2 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(8/3)/(c + d*x^3)^2, x]
```

```
[Out] (a^2*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3
)/c)]/(c^2*(1 + (b*x^3)/a)^(2/3))
```

**Rule 429**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

**Rule 430**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

**Rubi steps**

$$\int \frac{(a + bx^3)^{8/3}}{(c + dx^3)^2} dx = \frac{\left(a^2 (a + bx^3)^{2/3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{8/3}}{\left(c + dx^3\right)^2} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^2 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 1.08, size = 698, normalized size = 1.99

$$\frac{1}{18} \left( \frac{2a^3 \left( \log \left( \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3} \right) - 2 \log \left( \sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{ax^3+b}} + 1}{\sqrt{3}} \right) \right)}{c^{5/3} \sqrt[3]{bc-ad}} + \frac{2a^2 b \left( \log \left( \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3} \right) - 2 \log \left( \sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}} \right) + 2\sqrt{3} \tan^{-1} \left( \frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c} \sqrt[3]{ax^3+b}} + 1}{\sqrt{3}} \right) \right)}{c^{5/3} \sqrt[3]{bc-ad}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(8/3)/(c + d\*x^3)^2,x]

[Out] ((6\*x\*(a + b\*x^3)^(2/3)\*(b^2 + (b\*c - a\*d)^2/(c\*(c + d\*x^3))))/d^2 - (9\*b^3\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(d^2\*(a + b\*x^3)^(1/3)) + (12\*a\*b^2\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c\*d\*(a + b\*x^3)^(1/3)) + (2\*a^3\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(5/3)\*(b\*c - a\*d)^(1/3)) - (2\*a\*b^2\*c^(1/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(d^2\*(b\*c - a\*d)^(1/3)) + (2\*a^2\*b\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(2/3)\*d\*(b\*c - a\*d)^(1/3))/18

**fricas [B]** time = 12.13, size = 819, normalized size = 2.33

$$2\sqrt{3} \left( 3b^2c^3 - 2abc^2d - a^2cd^2 + (3b^2c^2d - 2abcd^2 - a^2d^3)x^3 \right) \left( \frac{b^2c^2 - 2abcd + a^2d^2}{c^2} \right)^{\frac{1}{3}} \arctan \left( \frac{\sqrt{3}(bc-ad)x + 2\sqrt{3}(bx^3+a)^{\frac{1}{3}}}{3(bc-ad)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] 1/9\*(2\*sqrt(3)\*(3\*b^2\*c^3 - 2\*a\*b\*c^2\*d - a^2\*c\*d^2 + (3\*b^2\*c^2\*d - 2\*a\*b\*c\*d^2 - a^2\*d^3)\*x^3)\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3)\*arctan(-1/3\*(sqrt(3)\*(b\*c - a\*d)\*x + 2\*sqrt(3)\*(b\*x^3 + a)^(1/3)\*c\*((b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)/c^2)^(1/3))/((b\*c - a\*d)\*x)) + 2\*sqrt(3)\*(3\*b^2\*c^3 - 4\*a\*b

$c^2d + (3b^2c^2d - 4abc^2d^2)x^3)(-b^2)^{1/3} \arctan(-1/3(\sqrt{3}bx - 2\sqrt{3}(bx^3 + a)^{1/3})(-b^2)^{1/3})/(bx) - 2(3b^2c^3 - 2abc^2d - a^2c^2d^2 + (3b^2c^2d - 2abc^2d^2 - a^2d^3)x^3)((b^2c^2 - 2abc^2d + a^2d^2)/c^2)^{1/3} \log((c^2x((b^2c^2 - 2abc^2d + a^2d^2)/c^2)^{2/3} - (bx^3 + a)^{1/3}(bc - ad))/x) - 2(3b^2c^3 - 4abc^2d + (3b^2c^2d - 4abc^2d^2)x^3)(-b^2)^{1/3} \log(-((-b^2)^{2/3})x - (bx^3 + a)^{1/3}b)/x + (3b^2c^3 - 4abc^2d + (3b^2c^2d - 4abc^2d^2)x^3)(-b^2)^{1/3} \log(-((-b^2)^{1/3})bx^2 - (bx^3 + a)^{1/3}(-b^2)^{2/3}x - (bx^3 + a)^{2/3}b)/x^2 + (3b^2c^3 - 2abc^2d - a^2c^2d^2 + (3b^2c^2d - 2abc^2d^2 - a^2d^3)x^3)((b^2c^2 - 2abc^2d + a^2d^2)/c^2)^{1/3} \log(-((bc - ad)x^2((b^2c^2 - 2abc^2d + a^2d^2)/c^2)^{1/3} + (bx^3 + a)^{1/3})c^2x((b^2c^2 - 2abc^2d + a^2d^2)/c^2)^{2/3} + (bx^3 + a)^{2/3}(bc - ad))/x^2) + 3(b^2c^2d^2x^4 + (2b^2c^2d - 2abc^2d^2 + a^2d^3)x)(bx^3 + a)^{2/3})/(c^2d^4x^3 + c^2d^3)$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(8/3)/(d\*x^3 + c)^2, x)

**maple** [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{8}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(8/3)/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{8/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(8/3)/(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(8/3)/(c + d\*x^3)^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(8/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Timed out

$$3.99 \quad \int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=301

$$\frac{b^{5/3} \log\left(\sqrt[3]{a+bx^3} - \sqrt[3]{bx}\right)}{2d^2} + \frac{b^{5/3} \tan^{-1}\left(\frac{2\sqrt[3]{bx}+1}{\sqrt[3]{a+bx^3}}\right)}{\sqrt{3}d^2} - \frac{(bc-ad)^{2/3}(2ad+3bc) \log(c+dx^3)}{18c^{5/3}d^2} + \dots$$

[Out]  $-1/3*(-a*d+b*c)*x*(b*x^3+a)^{(2/3)}/c/d/(d*x^3+c)-1/18*(-a*d+b*c)^{(2/3)}*(2*a*d+3*b*c)*\ln(d*x^3+c)/c^{(5/3)}/d^2+1/6*(-a*d+b*c)^{(2/3)}*(2*a*d+3*b*c)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(5/3)}/d^2-1/2*b^{(5/3)}*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/d^2+1/3*b^{(5/3)}*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)}))*3^{(1/2)}/d^2*3^{(1/2)}-1/9*(-a*d+b*c)^{(2/3)}*(2*a*d+3*b*c)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})*3^{(1/2)}/c^{(5/3)}/d^2*3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 0.20, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{ax(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{5}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(5/3)/(c + d\*x^3)^2, x]

[Out] (a\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -5/3, 2, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c^2\*(1 + (b\*x^3)/a)^(2/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{(a + bx^3)^{5/3}}{(c + dx^3)^2} dx = \frac{\left(a(a + bx^3)^{2/3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{5/3}}{(c + dx^3)^2} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{ax(a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{5}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 0.64, size = 450, normalized size = 1.50

$$\frac{4a^2 \left( \log\left(\frac{\sqrt[3]{c} x \sqrt[3]{bc-ad} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3}}{\sqrt[3]{ax^3+b}}\right) - 2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2x \sqrt[3]{bc-ad} + 1}{\sqrt[3]{c} \sqrt[3]{ax^3+b}}\right) \right)}{\sqrt[3]{bc-ad}} + \frac{9b^2 c^{2/3} x^4 \sqrt{\frac{bx^3}{a} + 1} F_1\left(\frac{4}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{d \sqrt[3]{a+bx^3}} - \frac{12c^{2/3}}{36c^{5/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(5/3)/(c + d*x^3)^2,x]
```

```
[Out] ((-12*c^(2/3)*(b*c - a*d)*x*(a + b*x^3)^(2/3))/(d*(c + d*x^3)) + (9*b^2*c^(2/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, -(d*x^3)/c])/(d*(a + b*x^3)^(1/3)) + (4*a^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))]/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (2*a*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3))]/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3)))/(36*c^(5/3))
```

**fricas [B]** time = 1.34, size = 631, normalized size = 2.10

$$2\sqrt{3} \left( (3bcd + 2ad^2)x^3 + 3bc^2 + 2acd \right) \left( \frac{b^2c^2 - 2abcd + a^2d^2}{c^2} \right)^{\frac{1}{3}} \arctan \left( -\frac{\sqrt{3}(bc-ad)x + 2\sqrt{3}(bx^3+a)^{\frac{1}{3}}c \left( \frac{b^2c^2 - 2abcd + a^2d^2}{c^2} \right)^{\frac{1}{3}}}{3(bc-ad)x} \right) + 6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(5/3)/(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -1/18*(2*sqrt(3)*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3))/(b*x)) + 6*(b*x^3 + a)^(2/3)*(b*c*d - a*d^2)*x - 2*((3*b*c*d + 2*a*d^2)*x^3 + 3*b*c^2 + 2*a*c*d)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)/x) + 3*(b*c*d*x^3 + b*c^2)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(1/3))
```



$$2)^{(2/3)} * x - (b * x^3 + a)^{(2/3)} * b / x^2) + ((3 * b * c * d + 2 * a * d^2) * x^3 + 3 * b * c^2 + 2 * a * c * d) * ((b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) / c^2)^{(1/3)} * \log(-((b * c - a * d) * x^2 * ((b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) / c^2)^{(1/3)} + (b * x^3 + a)^{(1/3)} * c * x * ((b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) / c^2)^{(2/3)} + (b * x^3 + a)^{(2/3)} * (b * c - a * d)) / x^2)) / (c * d^3 * x^3 + c^2 * d^2)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c)^2, x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{5/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(5/3)/(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(5/3)/(c + d\*x^3)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(5/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Timed out

$$3.100 \quad \int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=182

$$\frac{a \log(c+dx^3)}{9c^{5/3} \sqrt[3]{bc-ad}} - \frac{a \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3} \sqrt[3]{bc-ad}} + \frac{2a \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{3\sqrt{3} c^{5/3} \sqrt[3]{bc-ad}} + \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

[Out]  $1/3*x*(b*x^3+a)^{(2/3)}/c/(d*x^3+c)+1/9*a*\ln(d*x^3+c)/c^{(5/3)}/(-a*d+b*c)^{(1/3)}$   
 $-1/3*a*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(5/3)}/(-a*d+b*c)^{(1/3)}$   
 $+2/9*a*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})*3^{(1/2)}/c^{(5/3)}/(-a*d+b*c)^{(1/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 241, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{2a \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3} \sqrt[3]{bc-ad}} + \frac{a \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c} x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{9c^{5/3} \sqrt[3]{bc-ad}} + \frac{2a \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt{3} \sqrt[3]{c}}\right)}{3\sqrt{3} c^{5/3} \sqrt[3]{bc-ad}} + \frac{x(a+bx^3)^{2/3}}{3c(c+dx^3)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(2/3)/(c + d\*x^3)^2, x]

[Out]  $(x*(a + b*x^3)^{(2/3)})/(3*c*(c + d*x^3)) + (2*a*ArcTan[(c^{(1/3)} + (2*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(3*sqrt[3]*c^{(5/3)}*(b*c - a*d)^{(1/3)}) - (2*a*Log[c^{(1/3)} - ((b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(9*c^{(5/3)}*(b*c - a*d)^{(1/3)}) + (a*Log[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(a + b*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(9*c^{(5/3)}*(b*c - a*d)^{(1/3)})$

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1)), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b}

, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c))] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simplify[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^3)^{2/3}}{(c + dx^3)^2} dx &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} + \frac{(2a) \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx}{3c} \\
 &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c} \\
 &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} + \frac{(2a) \operatorname{Subst}\left(\int \frac{2\sqrt[3]{c} + \sqrt[3]{bc-ad}}{c^{2/3} + \sqrt[3]{c}\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}} \\
 &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} - \frac{2a \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a \operatorname{Subst}\left(\int \frac{1}{c^{2/3} + \sqrt[3]{c}\sqrt[3]{bc-ad}x + (bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}} \\
 &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} - \frac{2a \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a \log\left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} - \frac{a \operatorname{Subst}\left(\int \frac{1}{c^{2/3} + \sqrt[3]{c}\sqrt[3]{bc-ad}x + (bc-ad)^{2/3}x^2} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c^{4/3}} \\
 &= \frac{x(a + bx^3)^{2/3}}{3c(c + dx^3)} + \frac{2a \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}\sqrt[3]{bc-ad}} - \frac{2a \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}} + \frac{a \log\left(c^{2/3} + \frac{(bc-ad)^{2/3}}{(a+bx^3)^{2/3}}\right)}{9c^{5/3}\sqrt[3]{bc-ad}}
 \end{aligned}$$

**Mathematica** [C] time = 0.04, size = 78, normalized size = 0.43

$$\frac{x (a + bx^3)^{2/3} {}_2F_1\left(-\frac{2}{3}, \frac{1}{3}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2 \left(\frac{bx^3}{a} + 1\right)^{2/3} \sqrt[3]{\frac{dx^3}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(2/3)/(c + d\*x^3)^2,x]

[Out] (x\*(a + b\*x^3)^(2/3)\*Hypergeometric2F1[-2/3, 1/3, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))]/(c^2\*(1 + (b\*x^3)/a)^(2/3)\*(1 + (d\*x^3)/c)^(1/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c)^2, x)

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)`

[Out] `int((a + b*x^3)^(2/3)/(c + d*x^3)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{\frac{2}{3}}}{(c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(2/3)/(d*x**3+c)**2, x)`

[Out] `Integral((a + b*x**3)**(2/3)/(c + d*x**3)**2, x)`

$$3.101 \quad \int \frac{1}{\sqrt[3]{a+bx^3} (c+dx^3)^2} dx$$

**Optimal.** Leaf size=217

$$\frac{(3bc - 2ad) \log(c + dx^3)}{18c^{5/3}(bc - ad)^{4/3}} - \frac{(3bc - 2ad) \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc - ad)^{4/3}} + \frac{(3bc - 2ad) \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}c^{5/3}(bc - ad)^{4/3}} - \frac{dx(a + b}{3c(c + dx^3)}$$

[Out]  $-1/3*d*x*(b*x^3+a)^{(2/3)}/c/(-a*d+b*c)/(d*x^3+c)+1/18*(-2*a*d+3*b*c)*\ln(d*x^3+c)/c^{(5/3)}/(-a*d+b*c)^{(4/3)}-1/6*(-2*a*d+3*b*c)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(5/3)}/(-a*d+b*c)^{(4/3)}+1/9*(-2*a*d+3*b*c)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})/c^{(5/3)}/(-a*d+b*c)^{(4/3)}$

**Rubi [A]** time = 0.20, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {382, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{(3bc - 2ad) \log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc - ad)^{4/3}} + \frac{(3bc - 2ad) \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{18c^{5/3}(bc - ad)^{4/3}} + \frac{(3bc - 2ad) \tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + 1}{\sqrt[3]{a+bx^3}}\right)}{3\sqrt{3}c^{5/3}(bc - ad)^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^2), x]

[Out]  $-(d*x*(a + b*x^3)^{(2/3)})/(3*c*(b*c - a*d)*(c + d*x^3)) + ((3*b*c - 2*a*d)*\text{ArcTan}[(c^{(1/3)} + (2*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)})]/(\text{Sqrt}[3]*c^{(1/3)}))/((3*\text{Sqrt}[3]*c^{(5/3)}*(b*c - a*d)^{(4/3)}) - ((3*b*c - 2*a*d)*\text{Log}[c^{(1/3)} - ((b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)}]))/(9*c^{(5/3)}*(b*c - a*d)^{(4/3)}) + ((3*b*c - 2*a*d)*\text{Log}[c^{(2/3)} + ((b*c - a*d)^{(2/3)}*x^2)/(a + b*x^3)^{(2/3)} + (c^{(1/3)}*(b*c - a*d)^{(1/3)}*x)/(a + b*x^3)^{(1/3)}])/(18*c^{(5/3)}*(b*c - a*d)^{(4/3)})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b,

, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c))] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt[3]{a+bx^3} (c+dx^3)^2} dx &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad) \int \frac{1}{\sqrt[3]{a+bx^3} (c+dx^3)} dx}{3c(bc-ad)} \\
 &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad) \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{3c(bc-ad)} \\
 &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad) \text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)} + \dots \\
 &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} - \frac{(3bc-2ad) \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad) \text{Subst}\left(\dots\right)}{18c^5} \\
 &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} - \frac{(3bc-2ad) \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{9c^{5/3}(bc-ad)^{4/3}} + \frac{(3bc-2ad) \log\left(c^2/\dots\right)}{18c^5} \\
 &= -\frac{dx (a+bx^3)^{2/3}}{3c(bc-ad)(c+dx^3)} + \frac{(3bc-2ad) \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{4/3}} - \frac{(3bc-2ad) \log\left(\sqrt[3]{c}/\dots\right)}{9c^{5/3}(bc-ad)^{4/3}}
 \end{aligned}$$

**Mathematica** [C] time = 0.14, size = 99, normalized size = 0.46

$$\frac{x \left( (c + dx^3) (3bc - 2ad) {}_2F_1 \left( \frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)} \right) - cd (a + bx^3) \right)}{3c^2 \sqrt[3]{a + bx^3} (c + dx^3) (bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^2),x]

[Out] (x\*(-(c\*d\*(a + b\*x^3)) + (3\*b\*c - 2\*a\*d)\*(c + d\*x^3)\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]))/(3\*c^2\*(b\*c - a\*d)\*(a + b\*x^3)^(1/3)\*(c + d\*x^3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)^2), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}} (dx^3 + c)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)`

[Out] `int(1/((a + b*x^3)^(1/3)*(c + d*x^3)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3} (c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(1/3)/(d*x**3+c)**2, x)`

[Out] `Integral(1/((a + b*x**3)**(1/3)*(c + d*x**3)**2), x)`

$$3.102 \quad \int \frac{1}{(a+bx^3)^{4/3} (c+dx^3)^2} dx$$

**Optimal.** Leaf size=261

$$\frac{d(3bc-ad)\log(c+dx^3)}{9c^{5/3}(bc-ad)^{7/3}} + \frac{d(3bc-ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{3c^{5/3}(bc-ad)^{7/3}} - \frac{2d(3bc-ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt{3}}\right)}{3\sqrt{3}c^{5/3}(bc-ad)^{7/3}} + \frac{bx(a+bx^3)^{1/3}}{3ac\sqrt[3]{a+bx^3}}$$

[Out]  $\frac{1}{3} b^* (a*d+3*b*c) * x/a/c/(-a*d+b*c)^2/(b*x^3+a)^{(1/3)} - \frac{1}{3} d*x/c/(-a*d+b*c)/(b*x^3+a)^{(1/3)}/(d*x^3+c) - \frac{1}{9} d*(-a*d+3*b*c)*\ln(d*x^3+c)/c^{(5/3)}/(-a*d+b*c)^{(7/3)} + \frac{1}{3} d*(-a*d+3*b*c)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)} - (b*x^3+a)^{(1/3)})/c^{(5/3)}/(-a*d+b*c)^{(7/3)} - \frac{2}{9} d*(-a*d+3*b*c)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})/c^{(5/3)}/(-a*d+b*c)^{(7/3)} * 3^{(1/2)}$

**Rubi [C]** time = 1.93, antiderivative size = 625, normalized size of antiderivative = 2.39, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$c(a+bx^3)^{2/3} \left( -\frac{54d^2x^{15}(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^5(a+bx^3)^3} - \frac{108dx^{12}(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^4(a+bx^3)^3} - \frac{54x^9(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^3(a+bx^3)^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^2), x]

[Out]  $-(c*(a + b*x^3)^{(2/3)}*(6860 + (13720*d*x^3)/c + (6300*d^2*x^6)/c^2 - (525*(b*c - a*d)*x^3)/(c*(a + b*x^3)) - (1890*d*(b*c - a*d)*x^6)/(c^2*(a + b*x^3)) - (945*d^2*(b*c - a*d)*x^9)/(c^3*(a + b*x^3)) - 6860*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - (13720*d*x^3*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c - (6300*d^2*x^6*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2 + (2240*(b*c - a*d)*x^3*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c*(a + b*x^3) + (5320*d*(b*c - a*d)*x^6*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^2*(a + b*x^3) + (2520*d^2*(b*c - a*d)*x^9*\text{Hypergeometric2F1}[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^3*(a + b*x^3) - (54*(b*c - a*d)^3*x^9*\text{HypergeometricPFQ}[\{2, 2, 7/3\}, \{1, 13/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^3*(a + b*x^3)^3 - (108*d*(b*c - a*d)^3*x^12*\text{HypergeometricPFQ}[\{2, 2, 7/3\}, \{1, 13/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^4*(a + b*x^3)^3 - (54*d^2*(b*c - a*d)^3*x^15*\text{HypergeometricPFQ}[\{2, 2, 7/3\}, \{1, 13/3\}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/c^5*(a + b*x^3)^3)/(420*(b*c - a*d)^2*x^5*(c + d*x^3))$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}

, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

### Rubi steps

$$\int \frac{1}{(a + bx^3)^{4/3} (c + dx^3)^2} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)^2} dx}{a \sqrt[3]{a + bx^3}}$$

$$= \frac{c (a + bx^3)^{2/3} \left( 6860 + \frac{13720dx^3}{c} + \frac{6300d^2x^6}{c^2} - \frac{525(bc-ad)x^3}{c(a+bx^3)} - \frac{1890d(bc-ad)x^6}{c^2(a+bx^3)} - \frac{945d^2}{c^3} \right)}{c^5(a+bx^3)^3}$$

**Mathematica [C]** time = 2.03, size = 625, normalized size = 2.39

$$c (a + bx^3)^{2/3} \left( \frac{54d^2x^{15}(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^5(a+bx^3)^3} + \frac{108dx^{12}(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^4(a+bx^3)^3} + \frac{54x^9(bc-ad)^3 {}_3F_2\left(2, 2, \frac{7}{3}; 1, \frac{13}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)}{c^3(a+bx^3)^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^2), x]

[Out] (c\*(a + b\*x^3)^(2/3)\*(-6860 - (13720\*d\*x^3)/c - (6300\*d^2\*x^6)/c^2 + (525\*(b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3)) + (1890\*d\*(b\*c - a\*d)\*x^6)/(c^2\*(a + b\*x^3)) + (945\*d^2\*(b\*c - a\*d)\*x^9)/(c^3\*(a + b\*x^3)) + 6860\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + (13720\*d\*x^3\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c + (6300\*d^2\*x^6\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^2 - (2240\*(b\*c - a\*d)\*x^3\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^3 + (5320\*d\*(-(b\*c) + a\*d)\*x^6\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^2\*(a + b\*x^3) + (2520\*d^2\*(-(b\*c) + a\*d)\*x^9\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^3\*(a + b\*x^3) + (54\*(b\*c - a\*d)^3\*x^9\*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^3\*(a + b\*x^3)^3 + (108\*d\*(b\*c - a\*d)^3\*x^12\*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^4\*(a + b\*x^3)^3 + (54\*d^2\*(b\*c - a\*d)^3\*x^15\*HypergeometricPFQ[{2, 2, 7/3}, {1, 13/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/c^5\*(a + b\*x^3)^3)/(420\*(b\*c - a\*d)^2\*x^5\*(c + d\*x^3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)^2), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^2),x)

[Out] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{4}{3}}(c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(4/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(4/3)\*(c + d\*x\*\*3)\*\*2), x)

$$3.103 \quad \int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^2} dx$$

**Optimal.** Leaf size=324

$$\frac{bx(-4a^2d^2 - 33abcd + 9b^2c^2)}{12a^2c\sqrt[3]{a+bx^3}(bc-ad)^3} + \frac{d^2(9bc-2ad)\log(c+dx^3)}{18c^{5/3}(bc-ad)^{10/3}} - \frac{d^2(9bc-2ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{6c^{5/3}(bc-ad)^{10/3}} + \frac{d^2(9b}{$$

[Out]  $1/12*b*(4*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^{4/3}+1/12*b*(-4*a^2*d^2-33*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^{1/3}-1/3*d*x/c/(-a*d+b*c)/(b*x^3+a)^{4/3}/(d*x^3+c)+1/18*d^2*(-2*a*d+9*b*c)*\ln(d*x^3+c)/c^{5/3}/(-a*d+b*c)^{10/3}-1/6*d^2*(-2*a*d+9*b*c)*\ln((-a*d+b*c)^{1/3}*x/c^{1/3}-(b*x^3+a)^{1/3})/c^{5/3}/(-a*d+b*c)^{10/3}+1/9*d^2*(-2*a*d+9*b*c)*\arctan(1/3*(1+2*(-a*d+b*c)^{1/3}*x/c^{1/3})/(b*x^3+a)^{1/3})*3^{1/2}/c^{5/3}/(-a*d+b*c)^{10/3}*3^{1/2}$

**Rubi [C]** time = 5.69, antiderivative size = 1214, normalized size of antiderivative = 3.75, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$6804d^3(bc-ad)^4 {}_3F_2\left(2, 2, \frac{10}{3}; 1, \frac{16}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{21} + 1134d^3(bc-ad)^4 {}_4F_3\left(2, 2, 2, \frac{10}{3}; 1, 1, \frac{16}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)}\right)x^{21} + 215$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)^2), x]

[Out]  $(26130*c^5*(b*c - a*d)^2*x^6*(a + b*x^3)^2 + 89505*c^4*d*(b*c - a*d)^2*x^9*(a + b*x^3)^2 + 84240*c^3*d^2*(b*c - a*d)^2*x^{12}*(a + b*x^3)^2 + 26325*c^2*d^3*(b*c - a*d)^2*x^{15}*(a + b*x^3)^2 + 748020*c^6*(b*c - a*d)*x^3*(a + b*x^3)^3 + 2113020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3 + 1916460*c^4*d^2*(b*c - a*d)*x^9*(a + b*x^3)^3 + 589680*c^3*d^3*(b*c - a*d)*x^{12}*(a + b*x^3)^3 - 2002000*c^7*(a + b*x^3)^4 - 5460000*c^6*d*x^3*(a + b*x^3)^4 - 4914000*c^5*d^2*x^6*(a + b*x^3)^4 - 1506960*c^4*d^3*x^9*(a + b*x^3)^4 - 1248520*c^6*(b*c - a*d)*x^3*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3478020*c^5*d*(b*c - a*d)*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3144960*c^4*d^2*(b*c - a*d)*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 966420*c^3*d^3*(b*c - a*d)*x^{12}*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 2002000*c^7*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 5460000*c^6*d*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 4914000*c^5*d^2*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1506960*c^4*d^3*x^9*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 7938*c^3*(b*c - a*d)^4*x^{12}*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 22680*c^2*d*(b*c - a*d)^4*x^{15}*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 21546*c*d^2*(b*c - a*d)^4*x^{18}*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 6804*d^3*(b*c - a*d)^4*x^{21}*HypergeometricPFQ[{2, 2, 10/3}, {1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1134*c^3*(b*c - a*d)^4*x^{12}*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3402*c^2*d*(b*c - a*d)^4*x^{15}*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 3$

402\*c\*d^2\*(b\*c - a\*d)^4\*x^18\*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 1134\*d^3\*(b\*c - a\*d)^4\*x^21\*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 16/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]/(21840\*c^5\*(b\*c - a\*d)^3\*x^8\*(a + b\*x^3)^(10/3)\*(c + d\*x^3))

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{1}{(a + bx^3)^{7/3} (c + dx^3)^2} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{7/3} (c+dx^3)^2} dx}{a^2 \sqrt[3]{a + bx^3}}$$

$$= \frac{26130c^5(bc - ad)^2x^6 (a + bx^3)^2 + 89505c^4d(bc - ad)^2x^9 (a + bx^3)^2 + 84240c^3d^2}{...}$$

**Mathematica** [A] time = 5.94, size = 288, normalized size = 0.89

$$\frac{1}{36} \left( 3x (a + bx^3)^{2/3} \left( \frac{3b^2(11ad - 3bc)}{a^2 (a + bx^3) (ad - bc)^3} + \frac{3b^2}{a (a + bx^3)^2 (bc - ad)^2} - \frac{4d^3}{c (c + dx^3) (bc - ad)^3} \right) + \frac{2d^2(9bc - 2ad)}{...} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^3)^(7/3)*(c + d*x^3)^2),x]
[Out] (3*x*(a + b*x^3)^(2/3)*((3*b^2)/(a*(b*c - a*d)^2*(a + b*x^3)^2) + (3*b^2*(-3*b*c + 11*a*d))/(a^2*(-(b*c) + a*d)^3*(a + b*x^3)) - (4*d^3)/(c*(b*c - a*d)^3*(c + d*x^3))) + (2*d^2*(9*b*c - 2*a*d)*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))]/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(c^(5/3)*(b*c - a*d)^(10/3))/36
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^2), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)^2),x)

[Out] int(1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{7}{3}}(c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(7/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(7/3)\*(c + d\*x\*\*3)\*\*2), x)

$$3.104 \quad \int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

[Out] a\*x\*(b\*x^3+a)^(1/3)\*AppellF1(1/3,-4/3,2,4/3,-b\*x^3/a,-d\*x^3/c)/c^2/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(4/3)/(c + d\*x^3)^2,x]

[Out] (a\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, -4/3, 2, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c^2\*(1 + (b\*x^3)/a)^(1/3))

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^2} dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^2} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$



**Mathematica [B]** time = 0.32, size = 341, normalized size = 5.68

$$x \frac{\left( 4c \left( x^3 (a+bx^3) (ad-bc) \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4ac(3a^2d+abdx^3-b^2cx^3)F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)}{(c+dx^3) \left( x^3 \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4acF_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)} + bx^3 \frac{\left( 4c \left( x^3 (a+bx^3) (ad-bc) \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4ac(3a^2d+abdx^3-b^2cx^3)F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)}{12c^2d(a+bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(4/3)/(c + d\*x^3)^2,x]

[Out] (x\*(b\*(2\*b\*c + a\*d)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -(b\*x^3)/a, -((d\*x^3)/c)] + (4\*c\*(-4\*a\*c\*(3\*a^2\*d - b^2\*c\*x^3 + a\*b\*d\*x^3)\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + (-b\*c) + a\*d)\*x^3\*(a + b\*x^3)\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))/((c + d\*x^3)\*(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((12\*c^2\*d\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c)^2, x)

**maple [F]** time = 0.51, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{4/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(4/3)/(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(4/3)/(c + d\*x^3)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^3)^{4/3}}{(c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(4/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Integral((a + b\*x\*\*3)\*\*(4/3)/(c + d\*x\*\*3)\*\*2, x)

$$3.105 \quad \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx$$

**Optimal.** Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

[Out]  $x*(b*x^3+a)^{(1/3)}*AppellF1(1/3,-1/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{\frac{bx^3}{a}+1}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^{(1/3)}/(c + d*x^3)^2, x]$

[Out]  $(x*(a + b*x^3)^{(1/3)}*AppellF1[1/3, -1/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/c^2*(1 + (b*x^3)/a)^{(1/3)}$

**Rule 429**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:= \text{Simp}[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1]$   
 $\&\& (\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] \parallel \text{GtQ}[c, 0])$

**Rule 430**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:= \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]},$   
 $\text{Int}[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1] \&\& !(\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0])$

**Rubi steps**

$$\int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^2} dx = \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^2} dx}{\sqrt[3]{1+\frac{bx^3}{a}}}$$

$$= \frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2\sqrt[3]{1+\frac{bx^3}{a}}}$$

**Mathematica [B]** time = 0.21, size = 232, normalized size = 3.93

$$x \left( \frac{4 \left( \frac{a+bx^3}{c} \frac{8a^2 F_1 \left( \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{x^3 \left( 3ad F_1 \left( \frac{4}{3}, \frac{2}{3}, 2, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bc F_1 \left( \frac{4}{3}, \frac{5}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4ac F_1 \left( \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c+dx^3} \right) + \frac{bx^3 \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{4}{3}, \frac{2}{3}, 1, \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c^2} \right) \\ 12 (a + bx^3)^{2/3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/3)/(c + d\*x^3)^2,x]

[Out] (x\*((b\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])/c^2 + (4\*((a + b\*x^3)/c - (8\*a^2\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, -((d\*x^3)/c)])/(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -(b\*x^3)/a, -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -(b\*x^3)/a, -((d\*x^3)/c)])))/(c + d\*x^3))/(12\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c)^2, x)

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{1/3}}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)/(c + d\*x^3)^2,x)

[Out] int((a + b\*x^3)^(1/3)/(c + d\*x^3)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a + bx^3}}{(c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Integral((a + b\*x\*\*3)\*\*(1/3)/(c + d\*x\*\*3)\*\*2, x)

$$3.106 \quad \int \frac{1}{(a+bx^3)^{2/3} (c+dx^3)^2} dx$$

**Optimal.** Leaf size=59

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c^2 (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)}*AppellF1(1/3,2/3,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c^2 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)^2),x]

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)}*AppellF1[1/3, 2/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^2*(a + b*x^3)^{(2/3)})$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a+bx^3)^{2/3} (c+dx^3)^2} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3} (c+dx^3)^2} dx}{(a+bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{2}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2 (a+bx^3)^{2/3}}$$

**Mathematica [B]** time = 0.27, size = 393, normalized size = 6.66

$$\frac{4acx F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \left( bdx^3 \left( \frac{bx^3}{a} + 1 \right)^{2/3} (c + dx^3) F_1 \left( \frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 4c (3ad - 3bc + bdx^3) \right) - dx^3}{12c^2 (a + bx^3)^{2/3} (c + dx^3) (bc - ad) \left( x^3 \left( 3ad F_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 4c (3ad - 3bc + bdx^3) \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)^2),x]

[Out] (4\*a\*c\*x\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]\*(4\*c\*(-3\*b\*c + 3\*a\*d + b\*d\*x^3) + b\*d\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*(c + d\*x^3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]) - d\*x^4\*(4\*c\*(a + b\*x^3) + b\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*(c + d\*x^3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(12\*c^2\*(b\*c - a\*d)\*(a + b\*x^3)^(2/3)\*(c + d\*x^3)\*(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)^2), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)`

[Out] `int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{2}{3}} (c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**2, x)`

[Out] `Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**2), x)`



$$3.107 \quad \int \frac{1}{(a+bx^3)^{5/3} (c+dx^3)^2} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac^2 (a + bx^3)^{2/3}}$$

[Out] x\*(1+b\*x^3/a)^(2/3)\*AppellF1(1/3,5/3,2,4/3,-b\*x^3/a,-d\*x^3/c)/a/c^2/(b\*x^3+a)^(2/3)

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac^2 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^2),x]

[Out] (x\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[1/3, 5/3, 2, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(a\*c^2\*(a + b\*x^3)^(2/3))

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^3)^{5/3} (c+dx^3)^2} dx &= \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{5/3} (c+dx^3)^2} dx}{a (a + bx^3)^{2/3}} \\ &= \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{5}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{ac^2 (a + bx^3)^{2/3}} \end{aligned}$$

**Mathematica [B]** time = 0.57, size = 386, normalized size = 6.23

$$x \left( \frac{c \left( 16ac(6a^2d^2 + 2abd(dx^3 - 6c)) + 3b^2c(2c + dx^3) \right) F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) - 4x^3(2a^2d^2 + 2abd^2x^3 + 3b^2c(c + dx^3)) \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)}{(c+dx^3) \left( 4acF_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) - x^3 \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)} \right) \Bigg/ 24ac^2 (a + bx^3)^{2/3} (bc - ad)^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^2), x]

[Out] (x\*(b\*d\*(3\*b\*c + 2\*a\*d)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + (c\*(16\*a\*c\*(6\*a^2\*d^2 + 2\*a\*b\*d\*(-6\*c + d\*x^3) + 3\*b^2\*c\*(2\*c + d\*x^3))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 4\*x^3\*(2\*a^2\*d^2 + 2\*a\*b\*d^2\*x^3 + 3\*b^2\*c\*(c + d\*x^3))\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((c + d\*x^3)\*(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((24\*a\*c^2\*(b\*c - a\*d)^2\*(a + b\*x^3)^(2/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)^2), x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)`

[Out] `int(1/((a + b*x^3)^(5/3)*(c + d*x^3)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{5}{3}} (c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(5/3)/(d*x**3+c)**2, x)`

[Out] `Integral(1/((a + b*x**3)**(5/3)*(c + d*x**3)**2), x)`

$$3.108 \quad \int \frac{1}{(a+bx^3)^{8/3} (c+dx^3)^2} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c^2 (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)*AppellF1(1/3,8/3,2,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^2/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c^2 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + b*x^3)^(8/3)*(c + d*x^3)^2),x]`

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)*AppellF1[1/3, 8/3, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]})/(a^2*c^2*(a + b*x^3)^{(2/3)})$

**Rule 429**

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])`

**Rule 430**

`Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])`

**Rubi steps**

$$\int \frac{1}{(a + bx^3)^{8/3} (c + dx^3)^2} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{8/3} (c + dx^3)^2} dx}{a^2 (a + bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{8}{3}, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2 c^2 (a + bx^3)^{2/3}}$$

**Mathematica [B]** time = 0.99, size = 550, normalized size = 8.87

$$\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2+21abcd-6b^2c^2\right)F_1\left(\frac{4}{3};\frac{2}{3},1;\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3} + \frac{4c\left(x^4\left(5a^4d^3+10a^3bd^3x^3+a^2b^2d\left(24c^2+24cdx^3+5d^2x^6\right)+3ab^3c\left(-3c^2+4cdx^3+7d^2x^6\right)\right)\right)}{(ad-bc)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^2), x]

[Out] ((b\*d\*(-6\*b^2\*c^2 + 21\*a\*b\*c\*d + 5\*a^2\*d^2)\*x^4\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(-(b\*c) + a\*d)^3 + (4\*c\*(-4\*a\*c\*x\*(15\*a^4\*d^3 - 6\*b^4\*c^2\*x^3\*(2\*c + d\*x^3) + 5\*a^3\*b\*d^2\*(-9\*c + 4\*d\*x^3) + a^2\*b^2\*d\*(45\*c^2 - 21\*c\*d\*x^3 + 5\*d^2\*x^6) + 3\*a\*b^3\*c\*(-5\*c^2 + 11\*c\*d\*x^3 + 7\*d^2\*x^6))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^4\*(5\*a^4\*d^3 + 10\*a^3\*b\*d^3\*x^3 - 6\*b^4\*c^2\*x^3\*(c + d\*x^3) + a^2\*b^2\*d\*(24\*c^2 + 24\*c\*d\*x^3 + 5\*d^2\*x^6) + 3\*a\*b^3\*c\*(-3\*c^2 + 4\*c\*d\*x^3 + 7\*d^2\*x^6))\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(b\*c - a\*d)^3\*(a + b\*x^3)\*(c + d\*x^3)\*(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(60\*a^2\*c^2\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)^2), x)

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x)

[Out] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{8/3} (dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^2),x)

[Out] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{8/3} (c + dx^3)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(8/3)/(d\*x\*\*3+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(8/3)\*(c + d\*x\*\*3)\*\*2), x)

$$3.109 \quad \int \frac{(a+bx^3)^{14/3}}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=541

$$\frac{bx(a+bx^3)^{2/3}(2bc-ad)(-5a^2d^2-18abcd+18b^2c^2)}{18c^2d^4} + \frac{bx(a+bx^3)^{5/3}(-5a^2d^2-10abcd+18b^2c^2)}{18c^2d^3} - \frac{(bc-a^2)}{18c^2d^3}$$

```
[Out] -1/18*b*(-a*d+2*b*c)*(-5*a^2*d^2-18*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(2/3)/c
^2/d^4+1/18*b*(-5*a^2*d^2-10*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^(5/3)/c^2/d^3-
1/6*(-a*d+b*c)*x*(b*x^3+a)^(11/3)/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+12
*b*c)*x*(b*x^3+a)^(8/3)/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^(8/3)*(5*a^2*d^2+
18*a*b*c*d+54*b^2*c^2)*ln(d*x^3+c)/c^(8/3)/d^5+1/18*(-a*d+b*c)^(8/3)*(5*a^2
*d^2+18*a*b*c*d+54*b^2*c^2)*ln((-a*d+b*c)^(1/3)*x/c^(1/3)-(b*x^3+a)^(1/3))/
c^(8/3)/d^5-1/18*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*ln(-b^(1/3)*x+
(b*x^3+a)^(1/3))/d^5+1/27*b^(8/3)*(77*a^2*d^2-126*a*b*c*d+54*b^2*c^2)*arcta
n(1/3*(1+2*b^(1/3)*x/(b*x^3+a)^(1/3))*3^(1/2))/d^5*3^(1/2)-1/27*(-a*d+b*c)^(
8/3)*(5*a^2*d^2+18*a*b*c*d+54*b^2*c^2)*arctan(1/3*(1+2*(-a*d+b*c)^(1/3)*x/
c^(1/3)/(b*x^3+a)^(1/3))*3^(1/2))/c^(8/3)/d^5*3^(1/2)
```

**Rubi [C]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.11, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{a^4 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{14}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + b*x^3)^(14/3)/(c + d*x^3)^3, x]
```

```
[Out] (a^4*x*(a + b*x^3)^(2/3)*AppellF1[1/3, -14/3, 3, 4/3, -((b*x^3)/a), -((d*x^
3)/c)]/(c^3*(1 + (b*x^3)/a)^(2/3))
```

**Rule 429**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
&& (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

**Rule 430**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

**Rubi steps**

$$\int \frac{(a + bx^3)^{14/3}}{(c + dx^3)^3} dx = \frac{\left(a^4 (a + bx^3)^{2/3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{14/3}}{(c + dx^3)^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^4 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{14}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 2.34, size = 1171, normalized size = 2.16

$$\frac{1}{108} \left( \frac{10 \left( 2\sqrt{3} \tan^{-1} \left( \frac{2\sqrt[3]{bc-ad}x}{\sqrt{3} \sqrt[3]{ax^3+b}} + 1 \right) - 2 \log \left( \sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} \right) + \log \left( \frac{(bc-ad)^{2/3}x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{c} \sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} + c^{2/3} \right) \right) a^5 + 6b \left( 2\sqrt{3} \right)}{c^{8/3} \sqrt[3]{bc-ad}} \right) + \dots$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(14/3)/(c + d\*x^3)^3,x]

[Out] ((6\*x\*(a + b\*x^3)^(2/3)\*(-2\*b^3\*(9\*b\*c - 13\*a\*d) + 3\*b^4\*d\*x^3 + (3\*(b\*c - a\*d)^4)/(c\*(c + d\*x^3)^2) - ((b\*c - a\*d)^3\*(21\*b\*c + 5\*a\*d))/(c^2\*(c + d\*x^3))))/d^4 + (162\*b^5\*c\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(d^4\*(a + b\*x^3)^(1/3)) - (378\*a\*b^4\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(d^3\*(a + b\*x^3)^(1/3)) + (231\*a^2\*b^3\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c\*d^2\*(a + b\*x^3)^(1/3)) + (10\*a^5\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(8/3)\*(b\*c - a\*d)^(1/3)) + (36\*a\*b^4\*c^(4/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(d^4\*(b\*c - a\*d)^(1/3)) - (72\*a^2\*b^3\*c^(1/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(d^3\*(b\*c - a\*d)^(1/3)) + (30\*a^3\*b^2\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(2/3)\*d^2\*(b\*c - a\*d)^(1/3)) + (6\*a^4\*b\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3)))/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(5/3)\*d\*(b\*c - a\*d)^(1/3))/108

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((b\*x^3+a)^(14/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{14}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(14/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(14/3)/(d\*x^3 + c)^3, x)

**maple** [F] time = 0.40, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{14}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(14/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(14/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{14}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(14/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(14/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{14}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(14/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(14/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(14/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.110 \quad \int \frac{(a+bx^3)^{11/3}}{(c+dx^3)^3} dx$$

Optimal. Leaf size=458

$$\frac{bx(a+bx^3)^{2/3}(-5a^2d^2-7abcd+18b^2c^2)}{18c^2d^3} + \frac{(bc-ad)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}d^4} - \frac{(bc-ad)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\log(c+dx^3)}{54c^{8/3}d^4}$$

[Out]  $\frac{1}{18}b(-5a^2d^2-7abcd+18b^2c^2)x(bx^3+a)^{2/3}/c^2/d^3-1/6(-ad+bc)x(bx^3+a)^{8/3}/c/d/(dx^3+c)^2-1/18(-ad+bc)(5ad+9bc)x(bx^3+a)^{5/3}/c^2/d^2/(dx^3+c)+1/54(-ad+bc)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\ln(dx^3+c)/c^{8/3}/d^4-1/18(-ad+bc)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\ln(-ad+bc)^{1/3}x/c^{1/3}-(bx^3+a)^{1/3}/c^{8/3}/d^4+1/6b^{8/3}(-11ad+9bc)\ln(-b^{1/3}x+(bx^3+a)^{1/3})/d^4-1/9b^{8/3}(-11ad+9bc)\arctan(1/3(1+2b^{1/3}x)/(bx^3+a)^{1/3})3^{1/2}/d^43^{1/2}+1/27(-ad+bc)^{5/3}(5a^2d^2+12abcd+27b^2c^2)\arctan(1/3(1+2(-ad+bc)^{1/3}x/c^{1/3})/(bx^3+a)^{1/3})3^{1/2}/c^{8/3}/d^43^{1/2}$

**Rubi [C]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.14, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{a^3x(a+bx^3)^{2/3}F_1\left(\frac{1}{3};-\frac{11}{3},3;\frac{4}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a}+1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(11/3)/(c + d\*x^3)^3,x]

[Out] (a^3\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -11/3, 3, 4/3, -(b\*x^3)/a, -(d\*x^3)/c])/(c^3\*(1 + (b\*x^3)/a)^(2/3))

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
:= Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -(b\*x^n)/a, -(d\*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
:= Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{11/3}}{(c + dx^3)^3} dx = \frac{\left(a^3 (a + bx^3)^{2/3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{11/3}}{(c + dx^3)^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

$$= \frac{a^3 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{11}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 1.84, size = 908, normalized size = 1.98

$$\frac{1}{108} \left( \frac{10 \left( 2\sqrt{3} \tan^{-1} \left( \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{ax^3+b}} + 1 \right) - 2 \log \left( \sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} \right) + \log \left( \frac{(bc-ad)^{2/3}x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} + c^{2/3} \right) \right) a^4}{c^{8/3}\sqrt[3]{bc-ad}} + \frac{4b \left( 2\sqrt{3} \tan^{-1} \left( \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{ax^3+b}} + 1 \right) - 2 \log \left( \sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} \right) + \log \left( \frac{(bc-ad)^{2/3}x^2}{(ax^3+b)^{2/3}} + \frac{\sqrt[3]{c}\sqrt[3]{bc-ad}x}{\sqrt[3]{ax^3+b}} + c^{2/3} \right) \right) b^4}{c^{8/3}\sqrt[3]{bc-ad}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(11/3)/(c + d\*x^3)^3,x]

[Out] ((6\*x\*(a + b\*x^3)^(2/3)\*(6\*b^3 - (3\*(b\*c - a\*d)^3)/(c\*(c + d\*x^3)^2) + (5\*(b\*c - a\*d)^2\*(3\*b\*c + a\*d))/(c^2\*(c + d\*x^3)))/d^3 - (81\*b^4\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(d^3\*(a + b\*x^3)^(1/3)) + (99\*a\*b^3\*x^4\*(1 + (b\*x^3)/a)^(1/3)\*AppellF1[4/3, 1/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c\*d^2\*(a + b\*x^3)^(1/3)) + (10\*a^4\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(8/3)\*(b\*c - a\*d)^(1/3)) - (18\*a\*b^3\*c^(1/3)\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(d^3\*(b\*c - a\*d)^(1/3)) + (16\*a^2\*b^2\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(2/3)\*d^2\*(b\*c - a\*d)^(1/3)) + (4\*a^3\*b\*(2\*Sqrt[3]\*ArcTan[(1 + (2\*(b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/Sqrt[3]] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(c^(5/3)\*d\*(b\*c - a\*d)^(1/3))/108

**fricas [B]** time = 79.23, size = 1246, normalized size = 2.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(11/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] 1/54\*(2\*sqrt(3)\*(27\*b^3\*c^5 - 15\*a\*b^2\*c^4\*d - 7\*a^2\*b\*c^3\*d^2 - 5\*a^3\*c^2\*d^3 + (27\*b^3\*c^3\*d^2 - 15\*a\*b^2\*c^2\*d^3 - 7\*a^2\*b\*c\*d^4 - 5\*a^3\*d^5)\*x^6 + 2\*(27\*b^3\*c^4\*d - 15\*a\*b^2\*c^3\*d^2 - 7\*a^2\*b\*c^2\*d^3 - 5\*a^3\*c\*d^4)\*x^3)\*(

```
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)
*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1
/3))/((b*c - a*d)*x)) + 6*sqrt(3)*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*
d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2
)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*(-b^2)^(1/3)
)/(b*x)) - 2*(27*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3
+ (27*b^3*c^3*d^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*
(27*b^3*c^4*d - 15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*log((c*x*((b^2*c^2 - 2*a*b*c*d + a^
2*d^2)/c^2)^(2/3) - (b*x^3 + a)^(1/3)*(b*c - a*d))/x) - 6*(9*b^3*c^5 - 11*a
*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^6 + 2*(9*b^3*c^4*d - 11*a
*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(2/3)*x - (b*x^3 + a)^(1/3)*b)
/x) + 3*(9*b^3*c^5 - 11*a*b^2*c^4*d + (9*b^3*c^3*d^2 - 11*a*b^2*c^2*d^3)*x^
6 + 2*(9*b^3*c^4*d - 11*a*b^2*c^3*d^2)*x^3)*(-b^2)^(1/3)*log(-((-b^2)^(1/3)
*b*x^2 - (b*x^3 + a)^(1/3)*(-b^2)^(2/3)*x - (b*x^3 + a)^(2/3)*b)/x^2) + (27
*b^3*c^5 - 15*a*b^2*c^4*d - 7*a^2*b*c^3*d^2 - 5*a^3*c^2*d^3 + (27*b^3*c^3*d
^2 - 15*a*b^2*c^2*d^3 - 7*a^2*b*c*d^4 - 5*a^3*d^5)*x^6 + 2*(27*b^3*c^4*d -
15*a*b^2*c^3*d^2 - 7*a^2*b*c^2*d^3 - 5*a^3*c*d^4)*x^3)*((b^2*c^2 - 2*a*b*c*
d + a^2*d^2)/c^2)^(1/3)*log(-((b*c - a*d)*x^2*((b^2*c^2 - 2*a*b*c*d + a^2*d
^2)/c^2)^(1/3) + (b*x^3 + a)^(1/3)*c*x*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2
)^(2/3) + (b*x^3 + a)^(2/3)*(b*c - a*d))/x^2) + 3*(6*b^3*c^2*d^3*x^7 + (27*
b^3*c^3*d^2 - 25*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 + 5*a^3*d^5)*x^4 + 2*(9*b^3*
c^4*d - 8*a*b^2*c^3*d^2 - 2*a^2*b*c^2*d^3 + 4*a^3*c*d^4)*x)*(b*x^3 + a)^(2/
3))/(c^2*d^6*x^6 + 2*c^3*d^5*x^3 + c^4*d^4)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{11}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(11/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(11/3)/(d\*x^3 + c)^3, x)

**maple** [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{11}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(11/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(11/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{11}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(11/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(11/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{11/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(11/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(11/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(11/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.111 \quad \int \frac{(a+bx^3)^{8/3}}{(c+dx^3)^3} dx$$

Optimal. Leaf size=391

$$\frac{(bc-ad)^{2/3} (5a^2d^2 + 6abcd + 9b^2c^2) \log(c+dx^3)}{54c^{8/3}d^3} + \frac{(bc-ad)^{2/3} (5a^2d^2 + 6abcd + 9b^2c^2) \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}d^3}$$

[Out]  $-1/6*(-a*d+b*c)*x*(b*x^3+a)^{(5/3)}/c/d/(d*x^3+c)^2-1/18*(-a*d+b*c)*(5*a*d+6*b*c)*x*(b*x^3+a)^{(2/3)}/c^2/d^2/(d*x^3+c)-1/54*(-a*d+b*c)^{(2/3)}*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*\ln(d*x^3+c)/c^{(8/3)}/d^3+1/18*(-a*d+b*c)^{(2/3)}*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(8/3)}/d^3-1/2*b^{(8/3)}*\ln(-b^{(1/3)}*x+(b*x^3+a)^{(1/3)})/d^3+1/3*b^{(8/3)}*\arctan(1/3*(1+2*b^{(1/3)}*x/(b*x^3+a)^{(1/3)})*3^{(1/2)})/d^3*3^{(1/2)}-1/27*(-a*d+b*c)^{(2/3)}*(5*a^2*d^2+6*a*b*c*d+9*b^2*c^2)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})*3^{(1/2)})/c^{(8/3)}/d^3*3^{(1/2)}$

**Rubi [C]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 0.16, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{a^2x(a+bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3\left(\frac{bx^3}{a} + 1\right)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^3)^(8/3)/(c + d\*x^3)^3, x]

[Out] (a^2\*x\*(a + b\*x^3)^(2/3)\*AppellF1[1/3, -8/3, 3, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c^3\*(1 + (b\*x^3)/a)^(2/3))

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{(a + bx^3)^{8/3}}{(c + dx^3)^3} dx = \frac{\left(a^2 (a + bx^3)^{2/3}\right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^{8/3}}{(c + dx^3)^3} dx}{\left(1 + \frac{bx^3}{a}\right)^{2/3}} = \frac{a^2 x (a + bx^3)^{2/3} F_1\left(\frac{1}{3}; -\frac{8}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \left(1 + \frac{bx^3}{a}\right)^{2/3}}$$

**Mathematica [C]** time = 1.16, size = 651, normalized size = 1.66

$$\frac{10a^3 \left( \log\left(\frac{\sqrt[3]{c} x \sqrt[3]{bc-ad} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3}}{\sqrt[3]{ax^3+b}}\right) - 2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2x \sqrt[3]{bc-ad} + 1}{\frac{\sqrt[3]{c} \sqrt[3]{ax^3+b}}{\sqrt{3}}}\right) \right)}{\sqrt[3]{bc-ad}} + \frac{2a^2bc \left( \log\left(\frac{\sqrt[3]{c} x \sqrt[3]{bc-ad} + \frac{x^2(bc-ad)^{2/3}}{(ax^3+b)^{2/3}} + c^{2/3}}{\sqrt[3]{ax^3+b}}\right) - 2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{ax^3+b}}\right) + 2\sqrt{3} \tan^{-1}\left(\frac{2x \sqrt[3]{bc-ad} + 1}{\frac{\sqrt[3]{c} \sqrt[3]{ax^3+b}}{\sqrt{3}}}\right) \right)}{d \sqrt[3]{b}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^3)^(8/3)/(c + d*x^3)^3,x]
```

```
[Out] ((6*c^(2/3)*(-(b*c) + a*d)*x*(a + b*x^3)^(2/3)*(3*b*c*(2*c + 3*d*x^3) + a*d*(8*c + 5*d*x^3)))/(d^2*(c + d*x^3)^2) + (27*b^3*c^(5/3)*x^4*(1 + (b*x^3)/a)^(1/3)*AppellF1[4/3, 1/3, 1, 7/3, -(b*x^3)/a, -(d*x^3)/c])/(d^2*(a + b*x^3)^(1/3)) + (10*a^3*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(b*c - a*d)^(1/3) + (6*a*b^2*c^2*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d^2*(b*c - a*d)^(1/3)) + (2*a^2*b*c*(2*Sqrt[3]*ArcTan[(1 + (2*(b*c - a*d)^(1/3)*x)/(c^(1/3)*(b + a*x^3)^(1/3)))/Sqrt[3]] - 2*Log[c^(1/3) - ((b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)] + Log[c^(2/3) + ((b*c - a*d)^(2/3)*x^2)/(b + a*x^3)^(2/3) + (c^(1/3)*(b*c - a*d)^(1/3)*x)/(b + a*x^3)^(1/3)]))/(d*(b*c - a*d)^(1/3))/(108*c^(8/3))
```

**fricas [B]** time = 8.89, size = 954, normalized size = 2.44

$$2\sqrt{3} \left( (9b^2c^2d^2 + 6abcd^3 + 5a^2d^4)x^6 + 9b^2c^4 + 6abc^3d + 5a^2c^2d^2 + 2(9b^2c^3d + 6abc^2d^2 + 5a^2cd^3)x^3 \right) \left( \frac{b^2}{d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^(8/3)/(d*x^3+c)^3,x, algorithm="fricas")
```

```
[Out] -1/54*(2*sqrt(3)*((9*b^2*c^2*d^2 + 6*a*b*c*d^3 + 5*a^2*d^4)*x^6 + 9*b^2*c^4 + 6*a*b*c^3*d + 5*a^2*c^2*d^2 + 2*(9*b^2*c^3*d + 6*a*b*c^2*d^2 + 5*a^2*c*d^3)*x^3)*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3)*arctan(-1/3*(sqrt(3)*(b*c - a*d)*x + 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x)) + 18*sqrt(3)*(b^2*c^2*d^2*x^6 + 2*b^2*c^3*d*x^3 + b^2*c^4)*(-b^2)^(1/3)*arctan(-1/3*(sqrt(3)*b*x - 2*sqrt(3)*(b*x^3 + a)^(1/3)*c*((b^2*c^2 - 2*a*b*c*d + a^2*d^2)/c^2)^(1/3))/((b*c - a*d)*x))
```

$$a^{1/3}(-b^2)^{1/3}/(bx) - 2((9b^2c^2d^2 + 6abc^3d + 5a^2d^4)x^6 + 9b^2c^4 + 6abc^3d + 5a^2c^2d^2 + 2(9b^2c^3d + 6abc^2d^2 + 5a^2cd^3)x^3)((b^2c^2 - 2abc^2 + a^2d^2)/c^2)^{1/3} \log((c^2x((b^2c^2 - 2abc^2 + a^2d^2)/c^2)^{2/3} - (bx^3 + a)^{1/3}(bc - a^2d))/x) - 18(b^2c^2d^2x^6 + 2b^2c^3d^2x^3 + b^2c^4)(-b^2)^{1/3} \log(-((-b^2)^{2/3}x - (bx^3 + a)^{1/3}b)/x) + 9(b^2c^2d^2x^6 + 2b^2c^3d^2x^3 + b^2c^4)(-b^2)^{1/3} \log(-((-b^2)^{1/3}bx^2 - (bx^3 + a)^{1/3})(-b^2)^{2/3}x - (bx^3 + a)^{2/3}b)/x^2) + ((9b^2c^2d^2 + 6abc^3d + 5a^2d^4)x^6 + 9b^2c^4 + 6abc^3d + 5a^2c^2d^2 + 2(9b^2c^3d + 6abc^2d^2 + 5a^2cd^3)x^3)((b^2c^2 - 2abc^2 + a^2d^2)/c^2)^{1/3} \log(-((bc - a^2d)x^2((b^2c^2 - 2abc^2 + a^2d^2)/c^2)^{1/3} + (bx^3 + a)^{1/3}c^2x((b^2c^2 - 2abc^2 + a^2d^2)/c^2)^{2/3} + (bx^3 + a)^{2/3}(bc - a^2d))/x^2) + 3((9b^2c^2d^2 - 4abc^3d - 5a^2d^4)x^4 + 2(3b^2c^3d + abc^2d^2 - 4a^2cd^3)x)(bx^3 + a)^{2/3})/(c^2d^5x^6 + 2c^3d^4x^3 + c^4d^3)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{8/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((bx^3+a)^(8/3)/(dx^3+c)^3,x, algorithm="giac")

[Out] integrate((bx^3 + a)^(8/3)/(dx^3 + c)^3, x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{8/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((bx^3+a)^(8/3)/(dx^3+c)^3,x)

[Out] int((bx^3+a)^(8/3)/(dx^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{8/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((bx^3+a)^(8/3)/(dx^3+c)^3,x, algorithm="maxima")

[Out] integrate((bx^3 + a)^(8/3)/(dx^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{8/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + bx^3)^(8/3)/(c + dx^3)^3,x)



```
[Out] int((a + b*x^3)^(8/3)/(c + d*x^3)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(8/3)/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

$$3.112 \quad \int \frac{(a+bx^3)^{5/3}}{(c+dx^3)^3} dx$$

Optimal. Leaf size=217

$$\frac{5a^2 \log(c+dx^3)}{54c^{8/3} \sqrt[3]{bc-ad}} - \frac{5a^2 \log\left(\frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3} \sqrt[3]{bc-ad}} + \frac{5a^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{c}} + 1}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}c^{8/3} \sqrt[3]{bc-ad}} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} + \frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2}$$

[Out]  $\frac{1}{6}x(bx^3+a)^{5/3}/c/(dx^3+c)^2 + \frac{5}{18}a^2x(bx^3+a)^{2/3}/c^2/(dx^3+c) + \frac{5}{54}a^2 \ln(dx^3+c)/c^{8/3}/(-a+d+bc)^{1/3} - \frac{5}{18}a^2 \ln((-a+d+bc)^{1/3})x/c^{1/3} - (bx^3+a)^{1/3}/c^{8/3}/(-a+d+bc)^{1/3} + \frac{5}{27}a^2 \arctan(1/3*(1+2*(-a+d+bc)^{1/3})x/c^{1/3}/(bx^3+a)^{1/3})*3^{1/2}/c^{8/3}/(-a+d+bc)^{1/3}*3^{1/2}$

**Rubi [A]** time = 0.24, antiderivative size = 276, normalized size of antiderivative = 1.27, number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {378, 377, 200, 31, 634, 617, 204, 628}

$$-\frac{5a^2 \log\left(\sqrt[3]{c} - \frac{x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3} \sqrt[3]{bc-ad}} + \frac{5a^2 \log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{54c^{8/3} \sqrt[3]{bc-ad}} + \frac{5a^2 \tan^{-1}\left(\frac{\frac{2x \sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + \sqrt[3]{c}}{\sqrt[3]{a+bx^3}}\right)}{9\sqrt{3}c^{8/3} \sqrt[3]{bc-ad}} + \frac{5ax(a+bx^3)^{2/3}}{18c^2(c+dx^3)} +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(5/3)/(c + d\*x^3)^3,x]

[Out]  $\frac{x(a+bx^3)^{5/3}}{6c(c+dx^3)^2} + \frac{5a^2x(a+bx^3)^{2/3}}{(18c^2(c+dx^3) + 5a^2 \text{ArcTan}[\frac{c^{1/3} + (2(bc-a)d)^{1/3}x}{(a+bx^3)^{1/3}}])/(9\sqrt{3}c^{8/3}(bc-a)d)^{1/3}} - \frac{5a^2 \text{Log}[c^{1/3} - ((bc-a)d)^{1/3}x/(a+bx^3)^{1/3}]}{(27c^{8/3}(bc-a)d)^{1/3}} + \frac{5a^2 \text{Log}[c^{2/3} + ((bc-a)d)^{2/3}x^2/(a+bx^3)^{2/3} + c^{1/3}(bc-a)d)^{1/3}x/(a+bx^3)^{1/3}]}{(54c^{8/3}(bc-a)d)^{1/3}}$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_)^(-1), x\_Symbol] :> Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(-1), x\_Symbol] :> Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

### Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

### Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 634

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^3)^{5/3}}{(c + dx^3)^3} dx &= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{(5a) \int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx}{6c} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} + \frac{(5a^2) \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx}{9c^2} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} + \frac{(5a^2) \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^3} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{9c^2} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} + \frac{(5a^2) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}} + \frac{(5a^2) \operatorname{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{18c^2} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} - \frac{5a^2 \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{(5a^2) \operatorname{Subst}\left(\int \frac{1}{c^{2/3}+\sqrt[3]{c}\sqrt[3]{bc-ad}x} dx, x, \frac{x}{\sqrt[3]{a+bx^3}}\right)}{18c^2} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} - \frac{5a^2 \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2 \log\left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{bc-ad}{a}\right)}{54c^{8/3}\sqrt[3]{bc-ad}} \\
&= \frac{x(a + bx^3)^{5/3}}{6c(c + dx^3)^2} + \frac{5ax(a + bx^3)^{2/3}}{18c^2(c + dx^3)} + \frac{5a^2 \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}\sqrt[3]{bc-ad}} - \frac{5a^2 \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}\sqrt[3]{bc-ad}} + \frac{5a^2 \log\left(c^{2/3} + \frac{(bc-ad)^{2/3}x^2}{(a+bx^3)^{2/3}} + \frac{bc-ad}{a}\right)}{54c^{8/3}\sqrt[3]{bc-ad}}
\end{aligned}$$

**Mathematica [C]** time = 0.03, size = 79, normalized size = 0.36

$$\frac{ax(a + bx^3)^{2/3} {}_2F_1\left(-\frac{5}{3}, \frac{1}{3}, \frac{4}{3}, \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^3 \left(\frac{bx^3}{a} + 1\right)^{2/3} \sqrt[3]{\frac{dx^3}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(5/3)/(c + d\*x^3)^3,x]

[Out] (a\*x\*(a + b\*x^3)^(2/3)\*Hypergeometric2F1[-5/3, 1/3, 4/3, ((-b\*c) + a\*d)\*x^3/(a\*(c + d\*x^3))]/(c^3\*(1 + (b\*x^3)/a)^(2/3)\*(1 + (d\*x^3)/c)^(1/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c)^3, x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(5/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(5/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(5/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(5/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.113 \quad \int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=267

$$\frac{a(6bc-5ad)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{4/3}} - \frac{a(6bc-5ad)\log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a+bx^3}\right)}{18c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc-5ad)\tan^{-1}\left(\frac{\frac{2x\sqrt[3]{bc-ad}}{\sqrt[3]{c}}+1}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} + \frac{x(a+bx^3)^{2/3}}{18c^2(bc-ad)}$$

[Out]  $-1/6*d*x*(b*x^3+a)^{(5/3)}/c/(-a*d+b*c)/(d*x^3+c)^2+1/18*(-5*a*d+6*b*c)*x*(b*x^3+a)^{(2/3)}/c^2/(-a*d+b*c)/(d*x^3+c)+1/54*a*(-5*a*d+6*b*c)*\ln(d*x^3+c)/c^{(8/3)}/(-a*d+b*c)^{(4/3)}-1/18*a*(-5*a*d+6*b*c)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(8/3)}/(-a*d+b*c)^{(4/3)}+1/27*a*(-5*a*d+6*b*c)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3}))*3^{(1/2)})/c^{(8/3)}/(-a*d+b*c)^{(4/3)}*3^{(1/2)}$

**Rubi [A]** time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.22, number of steps used = 9, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {382, 378, 377, 200, 31, 634, 617, 204, 628}

$$\frac{x(a+bx^3)^{2/3}(6bc-5ad)}{18c^2(c+dx^3)(bc-ad)} - \frac{a(6bc-5ad)\log\left(\sqrt[3]{c} - \frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + \frac{a(6bc-5ad)\log\left(\frac{x^2(bc-ad)^{2/3}}{(a+bx^3)^{2/3}} + \frac{\sqrt[3]{c}x\sqrt[3]{bc-ad}}{\sqrt[3]{a+bx^3}} + c^{2/3}\right)}{54c^{8/3}(bc-ad)^{4/3}} +$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(2/3)/(c + d\*x^3)^3,x]

[Out]  $-(d*x*(a+b*x^3)^{(5/3)})/(6*c*(b*c-a*d)*(c+d*x^3)^2)+((6*b*c-5*a*d)*x*(a+b*x^3)^{(2/3)})/(18*c^2*(b*c-a*d)*(c+d*x^3))+ (a*(6*b*c-5*a*d)*\text{ArcTan}[(c^{(1/3)}+(2*(b*c-a*d)^{(1/3)}*x)/(a+b*x^3)^{(1/3)})]/(\text{Sqrt}[3]*c^{(1/3)}))/ (9*\text{Sqrt}[3]*c^{(8/3)}*(b*c-a*d)^{(4/3)}) - (a*(6*b*c-5*a*d)*\text{Log}[c^{(1/3)} - ((b*c-a*d)^{(1/3)}*x)/(a+b*x^3)^{(1/3)}])/ (27*c^{(8/3)}*(b*c-a*d)^{(4/3)}) + (a*(6*b*c-5*a*d)*\text{Log}[c^{(2/3)} + ((b*c-a*d)^{(2/3)}*x^2)/(a+b*x^3)^{(2/3)} + (c^{(1/3)}*(b*c-a*d)^{(1/3)}*x)/(a+b*x^3)^{(1/3)}])/ (54*c^{(8/3)}*(b*c-a*d)^{(4/3)})$

### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 634

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^3} dx &= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad) \int \frac{(a+bx^3)^{2/3}}{(c+dx^3)^2} dx}{6c(bc-ad)} \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad)) \int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)} dx}{9c^2(bc-ad)} \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad)) \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^3} dx\right)}{9c^2(bc-ad)} \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{(a(6bc-5ad)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{c}-\sqrt[3]{bc-ad}x} dx\right)}{27c^{8/3}(bc-ad)} \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} - \frac{a(6bc-5ad) \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + \dots \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} - \frac{a(6bc-5ad) \log\left(\sqrt[3]{c} - \frac{\sqrt[3]{bc-ad}x}{\sqrt[3]{a+bx^3}}\right)}{27c^{8/3}(bc-ad)^{4/3}} + \dots \\
&= -\frac{dx(a+bx^3)^{5/3}}{6c(bc-ad)(c+dx^3)^2} + \frac{(6bc-5ad)x(a+bx^3)^{2/3}}{18c^2(bc-ad)(c+dx^3)} + \frac{a(6bc-5ad) \tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{bc-ad}x}{\sqrt[3]{c}\sqrt[3]{a+bx^3}}}{\sqrt{3}}\right)}{9\sqrt{3}c^{8/3}(bc-ad)^{4/3}} - \dots
\end{aligned}$$

**Mathematica [C]** time = 0.22, size = 153, normalized size = 0.57

$$\frac{x \left( c(-a^2d(8c+5dx^3) + ab(6c^2-5cdx^3-5d^2x^6) + 3b^2cx^3(2c+dx^3)) - 2a(c+dx^3)^2(5ad-6bc) {}_2F_1\left(\frac{1}{3}, 1; \frac{4}{3}; \frac{(b*c-a*d)*x^3}{c*(a+b*x^3)}\right) \right)}{18c^3\sqrt[3]{a+bx^3}(c+dx^3)^2(bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^(2/3)/(c + d\*x^3)^3,x]

[Out] (x\*(c\*(3\*b^2\*c\*x^3\*(2\*c + d\*x^3) - a^2\*d\*(8\*c + 5\*d\*x^3) + a\*b\*(6\*c^2 - 5\*c\*d\*x^3 - 5\*d^2\*x^6)) - 2\*a\*(-6\*b\*c + 5\*a\*d)\*(c + d\*x^3)^2\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))])/(18\*c^3\*(b\*c - a\*d)\*(a + b\*x^3)^(1/3)\*(c + d\*x^3)^2)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c)^3, x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(2/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^3 + a)^{\frac{2}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(2/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(2/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(2/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

**3.114**  $\int \frac{1}{\sqrt[3]{a+bx^3}(c+dx^3)^3} dx$

**Optimal.** Leaf size=307

$$\frac{(5a^2d^2 - 12abcd + 9b^2c^2) \log(c + dx^3)}{54c^{8/3}(bc - ad)^{7/3}} - \frac{(5a^2d^2 - 12abcd + 9b^2c^2) \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} - \sqrt[3]{a + bx^3}\right)}{18c^{8/3}(bc - ad)^{7/3}} + \frac{(5a^2d^2 - 12abcd + 9b^2c^2) \log\left(\frac{x\sqrt[3]{bc-ad}}{\sqrt[3]{c}} + \sqrt[3]{a + bx^3}\right)}{9\sqrt[3]{c^2(bc - ad)^{7/3}}}$$

[Out]  $-1/6*d*x*(b*x^3+a)^{(2/3)}/c/(-a*d+b*c)/(d*x^3+c)^2-1/18*d*(-5*a*d+9*b*c)*x*(b*x^3+a)^{(2/3)}/c^2/(-a*d+b*c)^2/(d*x^3+c)+1/54*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*\ln(d*x^3+c)/c^{(8/3)}/(-a*d+b*c)^{(7/3)}-1/18*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(8/3)}/(-a*d+b*c)^{(7/3)}+1/27*(5*a^2*d^2-12*a*b*c*d+9*b^2*c^2)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})/3^{(1/2)}/c^{(8/3)}/(-a*d+b*c)^{(7/3)}*3^{(1/2)}$

**Rubi [C]** time = 0.31, antiderivative size = 167, normalized size of antiderivative = 0.54, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( cd(-a^2d(8c + 5dx^3) + ab(12c^2 + cdx^3 - 5d^2x^6) + 3b^2cx^3(4c + 3dx^3)) - 2(c + dx^3)^2(5a^2d^2 - 12abcd + 9b^2c^2) \right)}{18c^3\sqrt[3]{a + bx^3}(c + dx^3)^2(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^3), x]

[Out]  $-(x*(c*d*(3*b^2*c*x^3*(4*c + 3*d*x^3) - a^2*d*(8*c + 5*d*x^3) + a*b*(12*c^2 + c*d*x^3 - 5*d^2*x^6)) - 2*(9*b^2*c^2 - 12*a*b*c*d + 5*a^2*d^2)*(c + d*x^3)^2*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))])/(18*c^3*(b*c - a*d)^2*(a + b*x^3)^(1/3)*(c + d*x^3)^2)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\int \frac{1}{\sqrt[3]{a+bx^3} (c+dx^3)^3} dx = \frac{\sqrt[3]{1+\frac{bx^3}{a}} \int \frac{1}{\sqrt[3]{1+\frac{bx^3}{a}} (c+dx^3)^3} dx}{\sqrt[3]{a+bx^3}}$$

$$= \frac{x \left( cd(3b^2cx^3(4c+3dx^3)) - a^2d(8c+5dx^3) + ab(12c^2+cdx^3-5d^2x^6) \right) - 2(9d^2c^2+5cd^2x^3)}{18c^3(bc-ad)^2\sqrt[3]{a+bx^3} (c+dx^3)^2}$$

**Mathematica [C]** time = 0.38, size = 168, normalized size = 0.55

$$\frac{x \left( 2(c+dx^3)^2 (5a^2d^2 - 12abcd + 9b^2c^2) {}_2F_1 \left( \frac{1}{3}, 1; \frac{4}{3}; \frac{(bc-ad)x^3}{c(bx^3+a)} \right) - cd(-a^2d(8c+5dx^3) + ab(12c^2+cdx^3-5d^2x^6)) \right)}{18c^3\sqrt[3]{a+bx^3} (c+dx^3)^2 (bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^3), x]

[Out] (x\*(-(c\*d\*(3\*b^2\*c\*x^3\*(4\*c + 3\*d\*x^3) - a^2\*d\*(8\*c + 5\*d\*x^3) + a\*b\*(12\*c^2 + c\*d\*x^3 - 5\*d^2\*x^6))) + 2\*(9\*b^2\*c^2 - 12\*a\*b\*c\*d + 5\*a^2\*d^2)\*(c + d\*x^3)^2\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]))/(18\*c^3\*(b\*c - a\*d)^2\*(a + b\*x^3)^(1/3)\*(c + d\*x^3)^2)

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)^3), x)

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3+a)^{\frac{1}{3}}(dx^3+c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(1/3)\*(d\*x^3 + c)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^3), x)

[Out] int(1/((a + b\*x^3)^(1/3)\*(c + d\*x^3)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{a + bx^3} (c + dx^3)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(1/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Integral(1/((a + b\*x\*\*3)\*\*(1/3)\*(c + d\*x\*\*3)\*\*3), x)

$$3.115 \quad \int \frac{1}{(a+bx^3)^{4/3}(c+dx^3)^3} dx$$

**Optimal.** Leaf size=377

$$\frac{dx (a + bx^3)^{2/3} (-5a^2d^2 + 15abcd + 18b^2c^2)}{18ac^2 (c + dx^3) (bc - ad)^3} - \frac{d (5a^2d^2 - 18abcd + 27b^2c^2) \log(c + dx^3)}{54c^{8/3} (bc - ad)^{10/3}} + \frac{d (5a^2d^2 - 18abcd + 27b^2c^2)}{54c^{8/3} (bc - ad)^{10/3}}$$

[Out]  $-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^{(1/3)}/(d*x^3+c)^2+1/6*b*(a*d+6*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^{(1/3)}/(d*x^3+c)+1/18*d*(-5*a^2*d^2+15*a*b*c*d+18*b^2*c^2)*x*(b*x^3+a)^{(2/3)}/a/c^2/(-a*d+b*c)^3/(d*x^3+c)-1/54*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*\ln(d*x^3+c)/c^{(8/3)}/(-a*d+b*c)^{(10/3)}+1/18*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*\ln((-a*d+b*c)^{(1/3)}*x/c^{(1/3)}-(b*x^3+a)^{(1/3)})/c^{(8/3)}/(-a*d+b*c)^{(10/3)}-1/27*d*(5*a^2*d^2-18*a*b*c*d+27*b^2*c^2)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)}*x/c^{(1/3)})/(b*x^3+a)^{(1/3)})/c^{(8/3)}/(-a*d+b*c)^{(10/3)}*3^{(1/2)}$

**Rubi [C]** time = 2.74, antiderivative size = 428, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$65c^2 (a + bx^3)^2 \left( -28 (c + dx^3)^2 (a^2 (843c^2 dx^3 + 500c^3 + 375cd^2 x^6 + 27d^3 x^9) + 9abcx^3 (73c^2 + 104cdx^3 + 37d^2 x^6)) \right)$$

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^3), x]

[Out]  $-(65*c^2*(a + b*x^3)^2*(14000*a^2*c^5 + 21896*a*b*c^5*x^3 + 48104*a^2*c^4*d*x^3 + 8391*b^2*c^5*x^6 + 70802*a*b*c^4*d*x^6 + 60807*a^2*c^3*d^2*x^6 + 24417*b^2*c^4*d*x^9 + 81534*a*b*c^3*d^2*x^9 + 33657*a^2*c^2*d^3*x^9 + 23409*b^2*c^3*d^2*x^12 + 38652*a*b*c^2*d^3*x^12 + 7155*a^2*c*d^4*x^12 + 7425*b^2*c^2*d^3*x^15 + 5940*a*b*c*d^4*x^15 + 243*a^2*d^5*x^15 - 28*(c + d*x^3)^2*(27*b^2*c^2*x^6*(7*c + 6*d*x^3) + 9*a*b*c*x^3*(73*c^2 + 104*c*d*x^3 + 33*d^2*x^6) + a^2*(500*c^3 + 843*c^2*d*x^3 + 375*c*d^2*x^6 + 27*d^3*x^9))*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]) - 486*(b*c - a*d)^4*x^12*(c + d*x^3)^3*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(16380*c^5*(b*c - a*d)^3*x^8*(a + b*x^3)^(7/3))*(c + d*x^3)^2)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -(b\*x^n)/a, -(d\*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a + bx^3)^{4/3} (c + dx^3)^3} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{4/3} (c + dx^3)^3} dx}{a \sqrt[3]{a + bx^3}}$$

$$= -\frac{65c^2 (a + bx^3)^2 \left(14000a^2c^5 + 21896abc^5x^3 + 48104a^2c^4dx^3 + 8391b^2c^5x^6 + 70802a^2c^4d^2x^9 + 24417b^2c^4d^2x^9 + 81534a^2b^2c^3d^2x^9 + 33657a^2c^2d^3x^9 + 23409b^2c^3d^2x^{12} + 38652a^2b^2c^2d^3x^{12} + 7155a^2c^2d^4x^{12} + 7425b^2c^2d^3x^{15} + 5940a^2b^2c^2d^4x^{15} + 243a^2d^5x^{15} + 28(c + dx^3)^2 (a^2(500c^3 + 843c^2dx^3 + 375c^2d^2x^6 + 27d^3x^9)) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(bc - ad)x^3}{c(a + bx^3)}\right] + 486(bc - ad)^4 x^{12} (c + dx^3)^3 \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{16}{3}\}, \frac{(bc - ad)x^3}{c(a + bx^3)}\right] / (c^5(-bc) + a^2d)^3 x^8 (a + bx^3)^{(7/3)} (c + dx^3)^2\right)}{a^3 \sqrt[3]{a + bx^3}}$$

**Mathematica** [C] time = 3.08, size = 428, normalized size = 1.14

$$486x^{12} (c + dx^3)^3 (bc - ad)^4 {}_4F_3\left(2, 2, 2, \frac{7}{3}; 1, 1, \frac{16}{3}; \frac{(bc - ad)x^3}{c(bx^3 + a)}\right) + 65c^2 (a + bx^3)^2 \left(28 (c + dx^3)^2 (a^2 (500c^3 + 843c^2dx^3 + 375c^2d^2x^6 + 27d^3x^9)) \operatorname{Hypergeometric2F1}\left[\frac{1}{3}, 1, \frac{4}{3}, \frac{(bc - ad)x^3}{c(a + bx^3)}\right] + 486(bc - ad)^4 x^{12} (c + dx^3)^3 \operatorname{HypergeometricPFQ}\left[\{2, 2, 2, \frac{7}{3}\}, \{1, 1, \frac{16}{3}\}, \frac{(bc - ad)x^3}{c(a + bx^3)}\right] / (c^5(-bc) + a^2d)^3 x^8 (a + bx^3)^{(7/3)} (c + dx^3)^2\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^3), x]

[Out] -1/16380\*(65\*c^2\*(a + b\*x^3)^2\*(-14000\*a^2\*c^5 - 21896\*a\*b\*c^5\*x^3 - 48104\*a^2\*c^4\*d\*x^3 - 8391\*b^2\*c^5\*x^6 - 70802\*a\*b\*c^4\*d\*x^6 - 60807\*a^2\*c^3\*d^2\*x^6 - 24417\*b^2\*c^4\*d\*x^9 - 81534\*a\*b\*c^3\*d^2\*x^9 - 33657\*a^2\*c^2\*d^3\*x^9 - 23409\*b^2\*c^3\*d^2\*x^12 - 38652\*a\*b\*c^2\*d^3\*x^12 - 7155\*a^2\*c\*d^4\*x^12 - 7425\*b^2\*c^2\*d^3\*x^15 - 5940\*a\*b\*c\*d^4\*x^15 - 243\*a^2\*d^5\*x^15 + 28\*(c + d\*x^3)^2\*(27\*b^2\*c^2\*x^6\*(7\*c + 6\*d\*x^3) + 9\*a\*b\*c\*x^3\*(73\*c^2 + 104\*c\*d\*x^3 + 33\*d^2\*x^6) + a^2\*(500\*c^3 + 843\*c^2\*d\*x^3 + 375\*c\*d^2\*x^6 + 27\*d^3\*x^9))\*Hypergeometric2F1[1/3, 1, 4/3, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))] + 486\*(b\*c - a\*d)^4\*x^12\*(c + d\*x^3)^3\*HypergeometricPFQ[{2, 2, 2, 7/3}, {1, 1, 16/3}, ((b\*c - a\*d)\*x^3)/(c\*(a + b\*x^3))]/(c^5\*(-(b\*c) + a\*d)^3\*x^8\*(a + b\*x^3)^(7/3)\*(c + d\*x^3)^2)

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)^3), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}} (dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(4/3)\*(d\*x^3 + c)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{4}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^3),x)

[Out] int(1/((a + b\*x^3)^(4/3)\*(c + d\*x^3)^3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(4/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.116 \quad \int \frac{1}{(a+bx^3)^{7/3}(c+dx^3)^3} dx$$

Optimal. Leaf size=463

$$\frac{bx(-2a^2d^2 - 42abcd + 9b^2c^2)}{12a^2c\sqrt[3]{a+bx^3}(c+dx^3)(bc-ad)^3} + \frac{d^2(5a^2d^2 - 24abcd + 54b^2c^2)\log(c+dx^3)}{54c^{8/3}(bc-ad)^{13/3}} - \frac{d^2(5a^2d^2 - 24abcd + 54b^2c^2)}{18c^{8/3}(bc-ad)^{13/3}}$$

[Out]  $-1/6*d*x/c/(-a*d+b*c)/(b*x^3+a)^{(4/3)/(d*x^3+c)^2+1/12*b*(2*a*d+3*b*c)*x/a/c/(-a*d+b*c)^2/(b*x^3+a)^{(4/3)/(d*x^3+c)+1/12*b*(-2*a^2*d^2-42*a*b*c*d+9*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(b*x^3+a)^{(1/3)/(d*x^3+c)+1/36*d*(10*a^3*d^3-42*a^2*b*c*d^2-135*a*b^2*c^2*d+27*b^3*c^3)*x*(b*x^3+a)^{(2/3)/a^2/c^2/(-a*d+b*c)^4/(d*x^3+c)+1/54*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*\ln(d*x^3+c)/c^{(8/3)/(-a*d+b*c)^{(13/3)-1/18*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*\ln((-a*d+b*c)^{(1/3)*x/c^{(1/3)-(b*x^3+a)^{(1/3)})/c^{(8/3)/(-a*d+b*c)^{(13/3)+1/27*d^2*(5*a^2*d^2-24*a*b*c*d+54*b^2*c^2)*\arctan(1/3*(1+2*(-a*d+b*c)^{(1/3)*x/c^{(1/3)/(b*x^3+a)^{(1/3)})^3^{(1/2)})/c^{(8/3)/(-a*d+b*c)^{(13/3)*3^{(1/2)}}}$

**Rubi [C]** time = 8.66, antiderivative size = 1990, normalized size of antiderivative = 4.30, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)^3), x]

[Out]  $-(522756*c^6*(b*c - a*d)^3*x^9*(a + b*x^3)^2 + 1516320*c^5*d*(b*c - a*d)^3*x^{12}*(a + b*x^3)^2 + 2198664*c^4*d^2*(b*c - a*d)^3*x^{15}*(a + b*x^3)^2 + 1415232*c^3*d^3*(b*c - a*d)^3*x^{18}*(a + b*x^3)^2 + 341172*c^2*d^4*(b*c - a*d)^3*x^{21}*(a + b*x^3)^2 + 28042560*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3 + 107602560*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3 + 157697280*c^5*d^2*(b*c - a*d)^2*x^{12}*(a + b*x^3)^3 + 101088000*c^4*d^3*(b*c - a*d)^2*x^{15}*(a + b*x^3)^3 + 24261120*c^3*d^4*(b*c - a*d)^2*x^{18}*(a + b*x^3)^3 - 265470660*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4 - 1019636800*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4 - 1466086440*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4 - 930252960*c^5*d^3*(b*c - a*d)*x^{12}*(a + b*x^3)^4 - 221899860*c^4*d^4*(b*c - a*d)*x^{15}*(a + b*x^3)^4 + 335877360*c^9*(a + b*x^3)^5 + 1279532800*c^8*d*x^3*(a + b*x^3)^5 + 1823334240*c^7*d^2*x^6*(a + b*x^3)^5 + 1151579520*c^6*d^3*x^9*(a + b*x^3)^5 + 273939120*c^5*d^4*x^{12}*(a + b*x^3)^5 - 67420080*c^7*(b*c - a*d)^2*x^6*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 259692160*c^6*d*(b*c - a*d)^2*x^9*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 377700960*c^5*d^2*(b*c - a*d)^2*x^{12}*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 241113600*c^4*d^3*(b*c - a*d)^2*x^{15}*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57723120*c^3*d^4*(b*c - a*d)^2*x^{18}*(a + b*x^3)^3*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 349440000*c^8*(b*c - a*d)*x^3*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1339520000*c^7*d*(b*c - a*d)*x^6*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1921920000*c^6*d^2*(b*c - a*d)*x^9*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 1218147840*c^5*d^3*(b*c - a*d)*x^{12}*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] + 290384640*c^4*d^4*(b*c - a*d)*x^{15}*(a + b*x^3)^4*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 3$



$35877360*c^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3)) - 1279532800*c^8*d*x^3*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1823334240*c^7*d^2*x^6*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 1151579520*c^6*d^3*x^9*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 273939120*c^5*d^4*x^12*(a + b*x^3)^5*Hypergeometric2F1[1/3, 1, 4/3, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 57834*c^4*(b*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 224532*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 326592*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 210924*c*d^3*(b*c - a*d)^5*x^24*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 51030*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 10/3}, {1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5103*c^4*(b*c - a*d)^5*x^15*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 20412*c^3*d*(b*c - a*d)^5*x^18*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 30618*c^2*d^2*(b*c - a*d)^5*x^21*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 20412*c*d^3*(b*c - a*d)^5*x^24*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))] - 5103*d^4*(b*c - a*d)^5*x^27*HypergeometricPFQ[{2, 2, 2, 2, 10/3}, {1, 1, 1, 19/3}, ((b*c - a*d)*x^3)/(c*(a + b*x^3))]/(524160*c^6*(b*c - a*d)^4*x^11*(a + b*x^3)^(10/3)*(c + d*x^3)^2)$

#### Rule 429

$\text{Int}[(a_+ + (b_+)*(x_+)^{(n_+)})^{(p_+)}*((c_+ + (d_+)*(x_+)^{(n_+)})^{(q_+)}, x\_Symbol]$   
 $\rightarrow \text{Simp}[a^p*c^q*x*\text{AppellF1}[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /;$ 
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1]$   
 $\&\& (\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0]) \&\& (\text{IntegerQ}[q] \parallel \text{GtQ}[c, 0])$

#### Rule 430

$\text{Int}[(a_+ + (b_+)*(x_+)^{(n_+)})^{(p_+)}*((c_+ + (d_+)*(x_+)^{(n_+)})^{(q_+)}, x\_Symbol]$   
 $\rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]}]/(1 + (b*x^n)/a)^{\text{FracPart}[p]},$   
 $\text{Int}[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$ 
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{NeQ}[n, -1] \&\& !(\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0])$

#### Rubi steps

$$\int \frac{1}{(a + bx^3)^{7/3} (c + dx^3)^3} dx = \frac{\sqrt[3]{1 + \frac{bx^3}{a}} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{7/3} (c + dx^3)^3} dx}{a^2 \sqrt[3]{a + bx^3}}$$

$$= - \frac{522756c^6(bc - ad)^3x^9(a + bx^3)^2 + 1516320c^5d(bc - ad)^3x^{12}(a + bx^3)^2 + 21}{\dots}$$

**Mathematica [A]** time = 5.88, size = 337, normalized size = 0.73

$$\frac{1}{36}x(a + bx^3)^{2/3} \left( \frac{27b^3(bc - 5ad)}{a^2(a + bx^3)(bc - ad)^4} - \frac{9b^3}{a(a + bx^3)^2(ad - bc)^3} + \frac{2d^3(5ad - 21bc)}{c^2(c + dx^3)(bc - ad)^4} - \frac{6d^3}{c(c + dx^3)^2(bc - ad)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(7/3)\*(c + d\*x^3)^3), x]

[Out] (x\*(a + b\*x^3)^(2/3)\*((-9\*b^3)/(a\*(-(b\*c) + a\*d)^3\*(a + b\*x^3)^2) + (27\*b^3\*(b\*c - 5\*a\*d))/(a^2\*(b\*c - a\*d)^4\*(a + b\*x^3)) - (6\*d^3)/(c\*(b\*c - a\*d)^3\*(c + d\*x^3)^2) + (2\*d^3\*(-21\*b\*c + 5\*a\*d))/(c^2\*(b\*c - a\*d)^4\*(c + d\*x^3)))/36 + (d^2\*(54\*b^2\*c^2 - 24\*a\*b\*c\*d + 5\*a^2\*d^2)\*(2\*sqrt(3)\*ArcTan[(1 + (b\*c - a\*d)^(1/3)\*x)/(c^(1/3)\*(b + a\*x^3)^(1/3))]/sqrt(3)] - 2\*Log[c^(1/3) - ((b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)] + Log[c^(2/3) + ((b\*c - a\*d)^(2/3)\*x^2)/(b + a\*x^3)^(2/3) + (c^(1/3)\*(b\*c - a\*d)^(1/3)\*x)/(b + a\*x^3)^(1/3)]))/(54\*c^(8/3)\*(b\*c - a\*d)^(13/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^3), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(7/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(7/3)\*(d\*x^3 + c)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{7}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

```
[Out] int(1/((a + b*x^3)^(7/3)*(c + d*x^3)^3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(7/3)/(d*x**3+c)**3, x)
```

```
[Out] Timed out
```

$$3.117 \quad \int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=60

$$\frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

[Out] a\*x\*(b\*x^3+a)^(1/3)\*AppellF1(1/3,-4/3,3,4/3,-b\*x^3/a,-d\*x^3/c)/c^3/(1+b\*x^3/a)^(1/3)

**Rubi [A]** time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(4/3)/(c + d\*x^3)^3,x]

[Out] (a\*x\*(a + b\*x^3)^(1/3)\*AppellF1[1/3, -4/3, 3, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(c^3\*(1 + (b\*x^3)/a)^(1/3))

**Rule 429**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

**Rule 430**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^{4/3}}{(c+dx^3)^3} dx &= \frac{\left(a\sqrt[3]{a+bx^3}\right) \int \frac{\left(1+\frac{bx^3}{a}\right)^{4/3}}{(c+dx^3)^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{ax\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{4}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$

**Mathematica [B]** time = 0.52, size = 316, normalized size = 5.27

$$x \left[ \frac{4c \left( \frac{4a^2c(c+dx^3)(10ad+bc)F_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{4acF_1\left(\frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)} - x^3 \left( 3adF_1\left(\frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 2bcF_1\left(\frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) + 8a^2cd + 5a^2d^2x^3 - abc^2 + 10abcdx^3 + 5abd^2x^6 - b^2c^2x^3 + 2b^2c^2x^3 \right]}{(c+dx^3)^2} \right]$$


---


$$72c^3d(a+bx^3)^{2/3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(4/3)/(c + d\*x^3)^3,x]

[Out] (x\*(b\*(2\*b\*c + 5\*a\*d)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + (4\*c\*(-(a\*b\*c^2) + 8\*a^2\*c\*d - b^2\*c^2\*x^3 + 10\*a\*b\*c\*d\*x^3 + 5\*a^2\*d^2\*x^3 + 2\*b^2\*c\*d\*x^6 + 5\*a\*b\*d^2\*x^6 + (4\*a^2\*c\*(b\*c + 10\*a\*d)\*(c + d\*x^3)\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)])/(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/(c + d\*x^3)^2)/(72\*c^3\*d\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c)^3, x)

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{4}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(4/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(4/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{4/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(4/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(4/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(4/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.118 \quad \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=59

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

[Out]  $x*(b*x^3+a)^{(1/3)}*AppellF1(1/3,-1/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(1+b*x^3/a)^{(1/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{\frac{bx^3}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(1/3)/(c + d\*x^3)^3,x]

[Out]  $(x*(a + b*x^3)^{(1/3)}*AppellF1[1/3, -1/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/c^3*(1 + (b*x^3)/a)^{(1/3)}$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{\sqrt[3]{a+bx^3}}{(c+dx^3)^3} dx &= \frac{\sqrt[3]{a+bx^3} \int \frac{\sqrt[3]{1+\frac{bx^3}{a}}}{(c+dx^3)^3} dx}{\sqrt[3]{1+\frac{bx^3}{a}}} \\ &= \frac{x\sqrt[3]{a+bx^3} F_1\left(\frac{1}{3}; -\frac{1}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 \sqrt[3]{1+\frac{bx^3}{a}}} \end{aligned}$$

**Mathematica [B]** time = 0.62, size = 431, normalized size = 7.31

$$\frac{c \left( 16acx(3a^2d(6c+5dx^3)+ab(-18c^2-7cdx^3+5d^2x^6))-b^2cx^3(7c+4dx^3) \right) F_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) - 4x^4 \left( a^2d(8c+5dx^3)+ab(-7c^2+4cdx^3+5d^2x^6) \right) - b^2cx^3(7c+4dx^3)}{(c+dx^3)^2 \left( x^3 \left( 3adF_1 \left( \frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + 2bcF_1 \left( \frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right) - 4acF_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) \right)}{72c^3 (a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/3)/(c + d\*x^3)^3,x]

[Out]  $(-b(-4bc + 5ad)x^4(1 + (bx^3)/a)^{2/3} \text{AppellF1}[4/3, 2/3, 1, 7/3, -(bx^3)/a, -(dx^3)/c]) + (c(16acx^3(-b^2cx^3(7c + 4dx^3)) + 3a^2d(6c + 5dx^3) + ab(-18c^2 - 7cdx^3 + 5d^2x^6)) \text{AppellF1}[1/3, 2/3, 1, 4/3, -(bx^3)/a, -(dx^3)/c] - 4x^4(-b^2cx^3(7c + 4dx^3)) + a^2d(8c + 5dx^3) + ab(-7c^2 + 4cdx^3 + 5d^2x^6)) \text{AppellF1}[4/3, 2/3, 2, 7/3, -(bx^3)/a, -(dx^3)/c] + 2bc \text{AppellF1}[4/3, 5/3, 1, 7/3, -(bx^3)/a, -(dx^3)/c]) / ((c + dx^3)^2 (-4ac \text{AppellF1}[1/3, 2/3, 1, 4/3, -(bx^3)/a, -(dx^3)/c] + x^3(3ad \text{AppellF1}[4/3, 2/3, 2, 7/3, -(bx^3)/a, -(dx^3)/c] + 2bc \text{AppellF1}[4/3, 5/3, 1, 7/3, -(bx^3)/a, -(dx^3)/c])) / (72c^3(b^2c - ad)(a + bx^3)^{2/3})$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c)^3, x)

**maple [F]** time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{3}}}{(dx^3 + c)^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/3)/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^{1/3}}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/3)/(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^(1/3)/(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.119 \quad \int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3} dx$$

**Optimal.** Leaf size=59

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c^3 (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)*AppellF1(1/3,2/3,3,4/3,-b*x^3/a,-d*x^3/c)/c^3/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{2}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{c^3 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x]$

[Out]  $(x*(1 + (b*x^3)/a)^(2/3)*AppellF1[1/3, 2/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^3*(a + b*x^3)^(2/3))$

#### Rule 429

$\text{Int}[(a_ + (b_ )*(x_ )^(n_ ))^(p_ )*((c_ ) + (d_ )*(x_ )^(n_ ))^(q_ ), x\_Symbol]$   
 $\rightarrow \text{Simp}[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0]) \ \&\& \ (\text{IntegerQ}[q] \ || \ \text{GtQ}[c, 0])$

#### Rule 430

$\text{Int}[(a_ + (b_ )*(x_ )^(n_ ))^(p_ )*((c_ ) + (d_ )*(x_ )^(n_ ))^(q_ ), x\_Symbol]$   
 $\rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ !(\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

#### Rubi steps

$$\int \frac{1}{(a+bx^3)^{2/3}(c+dx^3)^3} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{2/3} (c+dx^3)^3} dx}{(a+bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{2}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3 (a+bx^3)^{2/3}}$$

**Mathematica [B]** time = 0.77, size = 442, normalized size = 7.49

$$x \left( \frac{4c(dx^3(a^2d(8c+5dx^3)+ab(-13c^2-2cdx^3+5d^2x^6))-b^2cx^3(13c+10dx^3)) \left( 3adF_1\left(\frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 2bcF_1\left(\frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) - 4ac(3a^2d^2(6c+5d^2x^3))}{(c+dx^3)^2 \left( x^3 \left( 3adF_1\left(\frac{4}{3}; \frac{2}{3}, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + 2bcF_1\left(\frac{4}{3}; \frac{5}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) - 4acF_1\left(\frac{1}{3}; \frac{2}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)} \right)}{72c^3 (a + bx^3)^{2/3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(2/3)\*(c + d\*x^3)^3),x]

[Out] (x\*(5\*b\*d\*(-2\*b\*c + a\*d)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + (4\*c\*(-4\*a\*c\*(3\*a^2\*d^2\*(6\*c + 5\*d\*x^3) + b^2\*c\*(18\*c^2 + 5\*c\*d\*x^3 - 10\*d^2\*x^6) + a\*b\*d\*(-36\*c^2 - 25\*c\*d\*x^3 + 5\*d^2\*x^6))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + d\*x^3\*(a^2\*d\*(8\*c + 5\*d\*x^3) - b^2\*c\*x^3\*(13\*c + 10\*d\*x^3) + a\*b\*(-13\*c^2 - 2\*c\*d\*x^3 + 5\*d^2\*x^6))\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]) + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((c + d\*x^3)^2\*(-4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((72\*c^3\*(b\*c - a\*d)^2\*(a + b\*x^3)^(2/3))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)^3), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(2/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(2/3)\*(d\*x^3 + c)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{\frac{2}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)`

[Out] `int(1/((a + b*x^3)^(2/3)*(c + d*x^3)^3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{2}{3}} (c + dx^3)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**3+a)**(2/3)/(d*x**3+c)**3, x)`

[Out] `Integral(1/((a + b*x**3)**(2/3)*(c + d*x**3)**3), x)`

$$3.120 \quad \int \frac{1}{(a+bx^3)^{5/3}(c+dx^3)^3} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac^3 (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)}*AppellF1(1/3,5/3,3,4/3,-b*x^3/a,-d*x^3/c)/a/c^3/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{5}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{ac^3 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^3),x]

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)}*AppellF1[1/3, 5/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a*c^3*(a + b*x^3)^{(2/3)})$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\int \frac{1}{(a + bx^3)^{5/3}(c + dx^3)^3} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{5/3}(c + dx^3)^3} dx}{a(a + bx^3)^{2/3}} = \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{5}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{ac^3 (a + bx^3)^{2/3}}$$

**Mathematica [B]** time = 1.10, size = 531, normalized size = 8.56

$$\frac{bdx^4\left(\frac{bx^3}{a}+1\right)^{2/3}\left(5a^2d^2-16abcd-9b^2c^2\right)F_1\left(\frac{4}{3},\frac{2}{3},1,\frac{7}{3};-\frac{bx^3}{a},-\frac{dx^3}{c}\right)}{(ad-bc)^3} - \frac{4c\left(4acx(3a^3d^3(6c+5dx^3)+a^2bd^2(-54c^2-43cdx^3+5d^2x^6))+ab^2cd(54c^2+35cdx^3-16d^2x^6)\right)}{(ad-bc)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^3),x]

[Out] ((b\*d\*(-9\*b^2\*c^2 - 16\*a\*b\*c\*d + 5\*a^2\*d^2)\*x^4\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(-(b\*c) + a\*d)^3 - (4\*c\*(4\*a\*c\*x\*(3\*a^3\*d^3\*(6\*c + 5\*d\*x^3) + a\*b^2\*c\*d\*(54\*c^2 + 35\*c\*d\*x^3 - 16\*d^2\*x^6) - 9\*b^3\*c^2\*(2\*c^2 + 3\*c\*d\*x^3 + d^2\*x^6) + a^2\*b\*d^2\*(-54\*c^2 - 43\*c\*d\*x^3 + 5\*d^2\*x^6))\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + x^4\*(9\*b^3\*c^2\*(c + d\*x^3)^2 - a^3\*d^3\*(8\*c + 5\*d\*x^3) + a\*b^2\*c\*d^2\*x^3\*(19\*c + 16\*d\*x^3) + a^2\*b\*d^2\*(19\*c^2 + 8\*c\*d\*x^3 - 5\*d^2\*x^6))\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((b\*c - a\*d)^3\*(c + d\*x^3)^2\*(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)])))/((72\*a\*c^3\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)^3), x)

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(5/3)\*(d\*x^3 + c)^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{5/3} (dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^3),x)

[Out] int(1/((a + b\*x^3)^(5/3)\*(c + d\*x^3)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(5/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.121 \quad \int \frac{1}{(a+bx^3)^{8/3} (c+dx^3)^3} dx$$

**Optimal.** Leaf size=62

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c^3 (a + bx^3)^{2/3}}$$

[Out]  $x*(1+b*x^3/a)^{(2/3)}*AppellF1(1/3,8/3,3,4/3,-b*x^3/a,-d*x^3/c)/a^2/c^3/(b*x^3+a)^{(2/3)}$

**Rubi [A]** time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {430, 429}

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{2/3} F_1 \left( \frac{1}{3}; \frac{8}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)}{a^2 c^3 (a + bx^3)^{2/3}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((a + b*x^3)^{(8/3)}*(c + d*x^3)^3), x]$

[Out]  $(x*(1 + (b*x^3)/a)^{(2/3)}*AppellF1[1/3, 8/3, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^2*c^3*(a + b*x^3)^{(2/3)})$

**Rule 429**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:= \text{Simp}[a^{p_}c^{q_}x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0]) \ \&\& \ (\text{IntegerQ}[q] \ || \ \text{GtQ}[c, 0])$

**Rule 430**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:= \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ !(\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

**Rubi steps**

$$\int \frac{1}{(a + bx^3)^{8/3} (c + dx^3)^3} dx = \frac{\left(1 + \frac{bx^3}{a}\right)^{2/3} \int \frac{1}{\left(1 + \frac{bx^3}{a}\right)^{8/3} (c + dx^3)^3} dx}{a^2 (a + bx^3)^{2/3}}$$

$$= \frac{x \left(1 + \frac{bx^3}{a}\right)^{2/3} F_1 \left(\frac{1}{3}; \frac{8}{3}, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2 c^3 (a + bx^3)^{2/3}}$$



**Mathematica [B]** time = 1.62, size = 515, normalized size = 8.31

$$x \left( bdx^3 \left( \frac{bx^3}{a} + 1 \right)^{2/3} (25a^3d^3 - 110a^2bcd^2 - 171ab^2c^2d + 36b^3c^3) F_1 \left( \frac{4}{3}; \frac{2}{3}, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right) + \frac{4c}{4acF_1 \left( \frac{1}{3}; \frac{2}{3}, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c} \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^3), x]

[Out] (x\*(b\*d\*(36\*b^3\*c^3 - 171\*a\*b^2\*c^2\*d - 110\*a^2\*b\*c\*d^2 + 25\*a^3\*d^3)\*x^3\*(1 + (b\*x^3)/a)^(2/3)\*AppellF1[4/3, 2/3, 1, 7/3, -(b\*x^3)/a, -(d\*x^3)/c]) + (4\*c\*((36\*b^5\*c^3\*x^3\*(c + d\*x^3)^2 + 9\*a\*b^4\*c^2\*(6\*c - 19\*d\*x^3)\*(c + d\*x^3)^2 + 5\*a^5\*d^4\*(8\*c + 5\*d\*x^3) + 5\*a^3\*b^2\*d^3\*x^3\*(-50\*c^2 - 36\*c\*d\*x^3 + 5\*d^2\*x^6) + 5\*a^4\*b\*d^3\*(-25\*c^2 - 6\*c\*d\*x^3 + 10\*d^2\*x^6) - a^2\*b^3\*c\*d\*(189\*c^3 + 378\*c^2\*d\*x^3 + 314\*c\*d^2\*x^6 + 110\*d^3\*x^9))/(a + b\*x^3) + (4\*a\*c\*(36\*b^4\*c^4 - 171\*a\*b^3\*c^3\*d + 540\*a^2\*b^2\*c^2\*d^2 - 235\*a^3\*b\*c\*d^3 + 50\*a^4\*d^4)\*(c + d\*x^3)\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, -(d\*x^3)/c])/(4\*a\*c\*AppellF1[1/3, 2/3, 1, 4/3, -(b\*x^3)/a, -(d\*x^3)/c]) - x^3\*(3\*a\*d\*AppellF1[4/3, 2/3, 2, 7/3, -(b\*x^3)/a, -(d\*x^3)/c]) + 2\*b\*c\*AppellF1[4/3, 5/3, 1, 7/3, -(b\*x^3)/a, -(d\*x^3)/c])/(c + d\*x^3)^2)/(360\*a^2\*c^3\*(b\*c - a\*d)^4\*(a + b\*x^3)^(2/3))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)^3), x)

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^3,x)

[Out] int(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^3,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{8}{3}}(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(8/3)/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(8/3)\*(d\*x^3 + c)^3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^3 + a)^{8/3} (dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^3),x)

[Out] int(1/((a + b\*x^3)^(8/3)\*(c + d\*x^3)^3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*3+a)\*\*(8/3)/(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

$$3.122 \quad \int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} dx$$

**Optimal.** Leaf size=155

$$\frac{189a^2x^4\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}{}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}}$$

[Out]  $4/25*x*(b*x^3+a)^(7/4)/c/(d*x^3+c)^(25/12)+84/325*a*x*(b*x^3+a)^(3/4)/c^2/(d*x^3+c)^(13/12)+189/325*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^(1/4)*hypergeom([1/4, 1/3], [4/3], -(a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^(1/4)/(d*x^3+c)^(1/12)$

**Rubi [A]** time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{189a^2x^4\sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}}{}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{325c^3\sqrt[4]{a+bx^3}\sqrt[12]{c+dx^3}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(7/4)/(c + d\*x^3)^(37/12), x]

[Out]  $(4*x*(a + b*x^3)^(7/4))/(25*c*(c + d*x^3)^(25/12)) + (84*a*x*(a + b*x^3)^(3/4))/(325*c^2*(c + d*x^3)^(13/12)) + (189*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(1/4)*Hypergeometric2F1[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))]/(325*c^3*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12))$

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{7/4}}{(c+dx^3)^{37/12}} dx &= \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{(21a) \int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx}{25c} \\
&= \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{(189a^2) \int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}} dx}{325c^2} \\
&= \frac{4x(a+bx^3)^{7/4}}{25c(c+dx^3)^{25/12}} + \frac{84ax(a+bx^3)^{3/4}}{325c^2(c+dx^3)^{13/12}} + \frac{189a^2 x \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{325c^3 \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 90, normalized size = 0.58

$$\frac{ax(a+bx^3)^{3/4} {}_2F_1\left(-\frac{7}{4}, \frac{1}{3}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^3 \left(\frac{bx^3}{a} + 1\right)^{3/4} \sqrt[12]{c+dx^3} \sqrt[4]{\frac{dx^3}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(7/4)/(c + d\*x^3)^(37/12), x]

[Out] (a\*x\*(a + b\*x^3)^(3/4)\*Hypergeometric2F1[-7/4, 1/3, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))])/(c^3\*(1 + (b\*x^3)/a)^(3/4)\*(c + d\*x^3)^(1/12)\*(1 + (d\*x^3)/c)^(1/4))

**fricas [F]** time = 77.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3+a)^{\frac{7}{4}}(dx^3+c)^{\frac{11}{12}}}{d^4x^{12}+4cd^3x^9+6c^2d^2x^6+4c^3dx^3+c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/4)/(d\*x^3+c)^(37/12), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(7/4)\*(d\*x^3 + c)^(11/12)/(d^4\*x^12 + 4\*c\*d^3\*x^9 + 6\*c^2\*d^2\*x^6 + 4\*c^3\*d\*x^3 + c^4), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(7/4)/(d\*x^3+c)^(37/12), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(7/4)/(d\*x^3 + c)^(37/12), x)

**maple [F]** time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(bx^3+a)^{\frac{7}{4}}}{(dx^3+c)^{\frac{37}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)`

[Out] `int((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{7}{4}}}{(dx^3 + c)^{\frac{37}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(7/4)/(d*x^3+c)^(37/12),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(7/4)/(d*x^3 + c)^(37/12), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{7/4}}{(dx^3 + c)^{37/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12),x)`

[Out] `int((a + b*x^3)^(7/4)/(c + d*x^3)^(37/12), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(7/4)/(d*x**3+c)**(37/12),x)`

[Out] Timed out

$$3.123 \quad \int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} dx$$

**Optimal.** Leaf size=155

$$\frac{45a^2x(c+dx^3)^{5/12} \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right) + \frac{60ax^4 \sqrt{a+bx^3}}{133c^2 (c+dx^3)^{7/12}} + \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

[Out]  $4/19*x*(b*x^3+a)^{(5/4)}/c/(d*x^3+c)^{(19/12)}+60/133*a*x*(b*x^3+a)^{(1/4)}/c^2/(d*x^3+c)^{(7/12)}+45/133*a^2*x*(c*(b*x^3+a)/a/(d*x^3+c))^{(3/4)}*(d*x^3+c)^{(5/12)}*\text{hypergeom}([1/3, 3/4], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/c^3/(b*x^3+a)^{(3/4)}$

**Rubi [A]** time = 0.06, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{45a^2x(c+dx^3)^{5/12} \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right) + \frac{60ax^4 \sqrt{a+bx^3}}{133c^2 (c+dx^3)^{7/12}} + \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(5/4)/(c + d\*x^3)^(31/12), x]

[Out]  $(4*x*(a + b*x^3)^{(5/4)})/(19*c*(c + d*x^3)^{(19/12)}) + (60*a*x*(a + b*x^3)^{(1/4)})/(133*c^2*(c + d*x^3)^{(7/12)}) + (45*a^2*x*((c*(a + b*x^3))/(a*(c + d*x^3)))^{(3/4)}*(c + d*x^3)^{(5/12)}*\text{Hypergeometric2F1}[1/3, 3/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))]/(133*c^3*(a + b*x^3)^{(3/4)})$

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(a\*n\*(p+1)), x] - Dist[(c\*q)/(a\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^3)^{5/4}}{(c+dx^3)^{31/12}} dx &= \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}} + \frac{(15a) \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx}{19c} \\
&= \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2(c+dx^3)^{7/12}} + \frac{(45a^2) \int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}} dx}{133c^2} \\
&= \frac{4x(a+bx^3)^{5/4}}{19c(c+dx^3)^{19/12}} + \frac{60ax\sqrt[4]{a+bx^3}}{133c^2(c+dx^3)^{7/12}} + \frac{45a^2x \left(\frac{c(a+bx^3)}{a(c+dx^3)}\right)^{3/4} (c+dx^3)^{5/12} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}\right)}{133c^3(a+bx^3)^{3/4}}
\end{aligned}$$

**Mathematica** [A] time = 0.03, size = 90, normalized size = 0.58

$$\frac{ax\sqrt[4]{a+bx^3} \sqrt[4]{\frac{dx^3}{c}+1} {}_2F_1\left(-\frac{5}{4}, \frac{1}{3}, \frac{4}{3}, \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2 \sqrt[4]{\frac{bx^3}{a}+1} (c+dx^3)^{7/12}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(5/4)/(c + d\*x^3)^(31/12), x]

[Out] (a\*x\*(a + b\*x^3)^(1/4)\*(1 + (d\*x^3)/c)^(1/4)\*Hypergeometric2F1[-5/4, 1/3, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))]/(c^2\*(1 + (b\*x^3)/a)^(1/4)\*(c + d\*x^3)^(7/12))

**fricas** [F] time = 3.01, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{5/4} (dx^3 + c)^{5/12}}{d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/4)/(d\*x^3+c)^(31/12), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(5/4)\*(d\*x^3 + c)^(5/12)/(d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/4}}{(dx^3 + c)^{31/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(5/4)/(d\*x^3+c)^(31/12), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(5/4)/(d\*x^3 + c)^(31/12), x)

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{5/4}}{(dx^3 + c)^{31/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)`

[Out] `int((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{5}{4}}}{(dx^3 + c)^{\frac{31}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^(5/4)/(d*x^3+c)^(31/12),x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^(5/4)/(d*x^3 + c)^(31/12), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{5/4}}{(dx^3 + c)^{31/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12),x)`

[Out] `int((a + b*x^3)^(5/4)/(c + d*x^3)^(31/12), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**(5/4)/(d*x**3+c)**(31/12),x)`

[Out] Timed out



$$3.124 \quad \int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx$$

**Optimal.** Leaf size=122

$$\frac{9ax \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2 \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}} + \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

[Out]  $4/13*x*(b*x^3+a)^{(3/4)}/c/(d*x^3+c)^{(13/12)}+9/13*a*x*(c*(b*x^3+a)/a/(d*x^3+c))^{(1/4)}*\text{hypergeom}([1/4, 1/3], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x^3+a)^{(1/4)}/(d*x^3+c)^{(1/12)}$

**Rubi [A]** time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{9ax \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{13c^2 \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}} + \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(3/4)/(c + d\*x^3)^(25/12), x]

[Out]  $(4*x*(a + b*x^3)^{(3/4)})/(13*c*(c + d*x^3)^{(13/12)}) + (9*a*x*((c*(a + b*x^3))/(a*(c + d*x^3)))^{(1/4)}*\text{Hypergeometric2F1}[1/4, 1/3, 4/3, -((b*c - a*d)*x^3)/(a*(c + d*x^3))])/(13*c^2*(a + b*x^3)^{(1/4)}*(c + d*x^3)^{(1/12)})$

### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

### Rubi steps

$$\int \frac{(a+bx^3)^{3/4}}{(c+dx^3)^{25/12}} dx = \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}} + \frac{(9a) \int \frac{1}{\sqrt[4]{a+bx^3}(c+dx^3)^{13/12}} dx}{13c}$$

$$= \frac{4x(a+bx^3)^{3/4}}{13c(c+dx^3)^{13/12}} + \frac{9ax \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{13c^2 \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}}$$

**Mathematica** [A] time = 0.02, size = 89, normalized size = 0.73

$$\frac{x(a + bx^3)^{3/4} {}_2F_1\left(-\frac{3}{4}, \frac{1}{3}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c^2\left(\frac{bx^3}{a} + 1\right)^{3/4} \sqrt[12]{c + dx^3} \sqrt[4]{\frac{dx^3}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(3/4)/(c + d\*x^3)^(25/12), x]

[Out] (x\*(a + b\*x^3)^(3/4)\*Hypergeometric2F1[-3/4, 1/3, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))]/(c^2\*(1 + (b\*x^3)/a)^(3/4)\*(c + d\*x^3)^(1/12)\*(1 + (d\*x^3)/c)^(1/4))

**fricas** [F] time = 47.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{11}{12}}}{d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(3/4)/(d\*x^3+c)^(25/12), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(11/12)/(d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{3}{4}}}{(dx^3 + c)^{\frac{25}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(3/4)/(d\*x^3+c)^(25/12), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(3/4)/(d\*x^3 + c)^(25/12), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{3}{4}}}{(dx^3 + c)^{\frac{25}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(3/4)/(d\*x^3+c)^(25/12), x)

[Out] int((b\*x^3+a)^(3/4)/(d\*x^3+c)^(25/12), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{3}{4}}}{(dx^3 + c)^{\frac{25}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(3/4)/(d\*x^3+c)^(25/12),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(3/4)/(d\*x^3 + c)^(25/12), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{3/4}}{(dx^3 + c)^{25/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(3/4)/(c + d\*x^3)^(25/12),x)

[Out] int((a + b\*x^3)^(3/4)/(c + d\*x^3)^(25/12), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(3/4)/(d\*x\*\*3+c)\*\*(25/12),x)

[Out] Timed out

$$3.125 \quad \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx$$

**Optimal.** Leaf size=122

$$\frac{3ax(c+dx^3)^{5/12} \left( \frac{c+bx^3}{a+dx^3} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right)}{7c^2(a+bx^3)^{3/4}} + \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}}$$

[Out]  $4/7*x*(b*x^3+a)^{(1/4)}/c/(d*x^3+c)^{(7/12)}+3/7*a*x*(c*(b*x^3+a)/a/(d*x^3+c))^{(3/4)}*(d*x^3+c)^{(5/12)}*hypergeom([1/3, 3/4], [4/3], -(a*d+b*c)*x^3/a/(d*x^3+c))/c^2/(b*x^3+a)^{(3/4)}$

**Rubi [A]** time = 0.04, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{3ax(c+dx^3)^{5/12} \left( \frac{c+bx^3}{a+dx^3} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right)}{7c^2(a+bx^3)^{3/4}} + \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^(1/4)/(c + d\*x^3)^(19/12), x]

[Out]  $(4*x*(a + b*x^3)^{(1/4)})/(7*c*(c + d*x^3)^{(7/12)}) + (3*a*x*((c*(a + b*x^3))/(a*(c + d*x^3)))^{(3/4)}*(c + d*x^3)^{(5/12)}*Hypergeometric2F1[1/3, 3/4, 4/3, -((b*c - a*d)*x^3)/(a*(c + d*x^3))]/(7*c^2*(a + b*x^3)^{(3/4)})$

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :- Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :- Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt[4]{a+bx^3}}{(c+dx^3)^{19/12}} dx &= \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}} + \frac{(3a) \int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}} dx}{7c} \\ &= \frac{4x\sqrt[4]{a+bx^3}}{7c(c+dx^3)^{7/12}} + \frac{3ax \left( \frac{c+bx^3}{a+dx^3} \right)^{3/4} (c+dx^3)^{5/12} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)} \right)}{7c^2(a+bx^3)^{3/4}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 89, normalized size = 0.73

$$\frac{x\sqrt[4]{a+bx^3}\sqrt[4]{\frac{dx^3}{c}+1}{}_2F_1\left(-\frac{1}{4}, \frac{1}{3}, \frac{4}{3}, \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{c\sqrt[4]{\frac{bx^3}{a}+1}(c+dx^3)^{7/12}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^(1/4)/(c + d\*x^3)^(19/12), x]

[Out] (x\*(a + b\*x^3)^(1/4)\*(1 + (d\*x^3)/c)^(1/4)\*Hypergeometric2F1[-1/4, 1/3, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))]/(c\*(1 + (b\*x^3)/a)^(1/4)\*(c + d\*x^3)^(7/12))

**fricas [F]** time = 2.25, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^{\frac{1}{4}}(dx^3 + c)^{\frac{5}{12}}}{d^2x^6 + 2cdx^3 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/4)/(d\*x^3+c)^(19/12), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(5/12)/(d^2\*x^6 + 2\*c\*d\*x^3 + c^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{4}}}{(dx^3 + c)^{\frac{19}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/4)/(d\*x^3+c)^(19/12), x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(1/4)/(d\*x^3 + c)^(19/12), x)

**maple [F]** time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{4}}}{(dx^3 + c)^{\frac{19}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(1/4)/(d\*x^3+c)^(19/12), x)

[Out] int((b\*x^3+a)^(1/4)/(d\*x^3+c)^(19/12), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^{\frac{1}{4}}}{(dx^3 + c)^{\frac{19}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(1/4)/(d\*x^3+c)^(19/12), x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(1/4)/(d\*x^3 + c)^(19/12), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^3 + a)^{1/4}}{(dx^3 + c)^{19/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^(1/4)/(c + d\*x^3)^(19/12), x)

[Out] int((a + b\*x^3)^(1/4)/(c + d\*x^3)^(19/12), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*(1/4)/(d\*x\*\*3+c)\*\*(19/12), x)

[Out] Timed out

$$3.126 \quad \int \frac{1}{\sqrt[4]{a+bx^3} (c+dx^3)^{13/12}} dx$$

**Optimal.** Leaf size=87

$$\frac{x \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}}$$

[Out]  $x*(c*(b*x^3+a)/a/(d*x^3+c))^{(1/4)}*\text{hypergeom}([1/4, 1/3], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^{(1/4)}/(d*x^3+c)^{(1/12)}$

**Rubi [A]** time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {380}

$$\frac{x \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(1/4)\*(c + d\*x^3)^(13/12)), x]

[Out]  $(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^{(1/4)}*\text{Hypergeometric2F1}[1/4, 1/3, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))]/(c*(a + b*x^3)^{(1/4)}*(c + d*x^3)^{(1/12)})$

**Rule 380**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

**Rubi steps**

$$\int \frac{1}{\sqrt[4]{a+bx^3} (c+dx^3)^{13/12}} dx = \frac{x \sqrt[4]{\frac{c(a+bx^3)}{a(c+dx^3)}} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3}}$$

**Mathematica [A]** time = 0.06, size = 86, normalized size = 0.99

$$\frac{x \sqrt[4]{\frac{bx^3}{a} + 1} \left(\frac{dx^3}{c} + 1\right)^{3/4} {}_2F_1\left(\frac{1}{4}, \frac{1}{3}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{\sqrt[4]{a+bx^3} (c+dx^3)^{13/12}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(1/4)\*(c + d\*x^3)^(13/12)), x]

[Out]  $(x*(1 + (b*x^3)/a)^{(1/4)}*(1 + (d*x^3)/c)^{(3/4)}*\text{Hypergeometric2F1}[1/4, 1/3, 4/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))]/((a + b*x^3)^{(1/4)}*(c + d*x^3)^{(13/12)})$

**fricas** [F] time = 51.16, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{\frac{3}{4}} (dx^3 + c)^{\frac{11}{12}}}{bd^2x^9 + (2bcd + ad^2)x^6 + (bc^2 + 2acd)x^3 + ac^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/4)/(d\*x^3+c)^(13/12),x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(11/12)/(b\*d^2\*x^9 + (2\*b\*c\*d + a\*d^2)\*x^6 + (b\*c^2 + 2\*a\*c\*d)\*x^3 + a\*c^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{4}} (dx^3 + c)^{\frac{13}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/4)/(d\*x^3+c)^(13/12),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(13/12)), x)

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{4}} (dx^3 + c)^{\frac{13}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(1/4)/(d\*x^3+c)^(13/12),x)

[Out] int(1/(b\*x^3+a)^(1/4)/(d\*x^3+c)^(13/12),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{1}{4}} (dx^3 + c)^{\frac{13}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(1/4)/(d\*x^3+c)^(13/12),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(13/12)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{1/4} (dx^3 + c)^{13/12}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(1/4)\*(c + d\*x^3)^(13/12)),x)

[Out] int(1/((a + b\*x^3)^(1/4)\*(c + d\*x^3)^(13/12)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^3} (c + dx^3)^{\frac{13}{12}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(1/4)/(d*x**3+c)**(13/12), x)
```

```
[Out] Integral(1/((a + b*x**3)**(1/4)*(c + d*x**3)**(13/12)), x)
```

$$3.127 \quad \int \frac{1}{(a+bx^3)^{3/4} (c+dx^3)^{7/12}} dx$$

**Optimal.** Leaf size=87

$$\frac{x(c+dx^3)^{5/12} \left(\frac{c+bx^3}{a(c+dx^3)}\right)^{3/4} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{3/4}}$$

[Out] x\*(c\*(b\*x^3+a)/a/(d\*x^3+c))^(3/4)\*(d\*x^3+c)^(5/12)\*hypergeom([1/3, 3/4], [4/3], -(a\*d+b\*c)\*x^3/a/(d\*x^3+c))/c/(b\*x^3+a)^(3/4)

**Rubi [A]** time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {380}

$$\frac{x(c+dx^3)^{5/12} \left(\frac{c+bx^3}{a(c+dx^3)}\right)^{3/4} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(3/4)\*(c + d\*x^3)^(7/12)), x]

[Out] (x\*((c\*(a + b\*x^3))/(a\*(c + d\*x^3)))^(3/4)\*(c + d\*x^3)^(5/12)\*Hypergeometric2F1[1/3, 3/4, 4/3, -(((b\*c - a\*d)\*x^3)/(a\*(c + d\*x^3)))]/(c\*(a + b\*x^3)^(3/4))

### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

### Rubi steps

$$\int \frac{1}{(a+bx^3)^{3/4} (c+dx^3)^{7/12}} dx = \frac{x \left(\frac{c+bx^3}{a(c+dx^3)}\right)^{3/4} (c+dx^3)^{5/12} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c(a+bx^3)^{3/4}}$$

**Mathematica [A]** time = 0.02, size = 86, normalized size = 0.99

$$\frac{x \left(\frac{bx^3}{a} + 1\right)^{3/4} \sqrt[4]{\frac{dx^3}{c} + 1} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{(a+bx^3)^{3/4} (c+dx^3)^{7/12}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(3/4)\*(c + d\*x^3)^(7/12)), x]

[Out] (x\*(1 + (b\*x^3)/a)^(3/4)\*(1 + (d\*x^3)/c)^(1/4)\*Hypergeometric2F1[1/3, 3/4, 4/3, ((-b\*c) + a\*d)\*x^3/(a\*(c + d\*x^3))]/((a + b\*x^3)^(3/4)\*(c + d\*x^3)^(7/12))

**fricas** [F] time = 4.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^{\frac{1}{4}}(dx^3 + c)^{\frac{5}{12}}}{bdx^6 + (bc + ad)x^3 + ac}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(3/4)/(d\*x^3+c)^(7/12),x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(5/12)/(b\*d\*x^6 + (b\*c + a\*d)\*x^3 + a\*c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{7}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(3/4)/(d\*x^3+c)^(7/12),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(7/12)), x)

**maple** [F] time = 0.53, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{7}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(3/4)/(d\*x^3+c)^(7/12),x)

[Out] int(1/(b\*x^3+a)^(3/4)/(d\*x^3+c)^(7/12),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{7}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(3/4)/(d\*x^3+c)^(7/12),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(7/12)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{7}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(3/4)\*(c + d\*x^3)^(7/12)),x)

[Out] int(1/((a + b\*x^3)^(3/4)\*(c + d\*x^3)^(7/12)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{3}{4}}(c + dx^3)^{\frac{7}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(3/4)/(d*x**3+c)**(7/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(3/4)*(c + d*x**3)**(7/12)), x)
```

$$3.128 \quad \int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}} dx$$

**Optimal.** Leaf size=87

$$\frac{x(c+dx^3)^{11/12} \left(\frac{c(ax^3+bx^3)}{a(c+dx^3)}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{5/4}}$$

[Out]  $x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*\text{hypergeom}([1/3, 5/4], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/c/(b*x^3+a)^(5/4)$

**Rubi [A]** time = 0.02, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {380}

$$\frac{x(c+dx^3)^{11/12} \left(\frac{c(ax^3+bx^3)}{a(c+dx^3)}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{c(a+bx^3)^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^3)^(5/4)\*(c + d\*x^3)^(1/12)),x]

[Out]  $(x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*\text{Hypergeometric2F1}[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))]/(c*(a + b*x^3)^(5/4))$

**Rule 380**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

**Rubi steps**

$$\int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}} dx = \frac{x \left(\frac{c(ax^3+bx^3)}{a(c+dx^3)}\right)^{5/4} (c+dx^3)^{11/12} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{c(a+bx^3)^{5/4}}$$

**Mathematica [A]** time = 0.04, size = 89, normalized size = 1.02

$$\frac{x \sqrt[4]{\frac{bx^3}{a} + 1} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{a \sqrt[4]{a+bx^3} \sqrt[12]{c+dx^3} \sqrt[4]{\frac{dx^3}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^3)^(5/4)\*(c + d\*x^3)^(1/12)),x]

[Out]  $(x*(1 + (b*x^3)/a)^(1/4)*\text{Hypergeometric2F1}[1/3, 5/4, 4/3, ((-b*c) + a*d)*x^3/(a*(c + d*x^3))])/(a*(a + b*x^3)^(1/4)*(c + d*x^3)^(1/12)*(1 + (d*x^3)/c)^(1/4))$

**fricas** [F] time = 64.29, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{\frac{3}{4}} (dx^3 + c)^{\frac{11}{12}}}{b^2 dx^9 + (b^2 c + 2abd)x^6 + (2abc + a^2 d)x^3 + a^2 c}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/4)/(d\*x^3+c)^(1/12),x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(11/12)/(b^2\*d\*x^9 + (b^2\*c + 2\*a\*b\*d)\*x^6 + (2\*a\*b\*c + a^2\*d)\*x^3 + a^2\*c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{4}} (dx^3 + c)^{\frac{1}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/4)/(d\*x^3+c)^(1/12),x, algorithm="giac")

[Out] integrate(1/((b\*x^3 + a)^(5/4)\*(d\*x^3 + c)^(1/12)), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{4}} (dx^3 + c)^{\frac{1}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^3+a)^(5/4)/(d\*x^3+c)^(1/12),x)

[Out] int(1/(b\*x^3+a)^(5/4)/(d\*x^3+c)^(1/12),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{4}} (dx^3 + c)^{\frac{1}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^3+a)^(5/4)/(d\*x^3+c)^(1/12),x, algorithm="maxima")

[Out] integrate(1/((b\*x^3 + a)^(5/4)\*(d\*x^3 + c)^(1/12)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^3 + a)^{\frac{5}{4}} (dx^3 + c)^{\frac{1}{12}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^3)^(5/4)\*(c + d\*x^3)^(1/12)),x)

[Out] int(1/((a + b\*x^3)^(5/4)\*(c + d\*x^3)^(1/12)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^3)^{\frac{5}{4}} \sqrt[12]{c + dx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**3+a)**(5/4)/(d*x**3+c)**(1/12),x)
```

```
[Out] Integral(1/((a + b*x**3)**(5/4)*(c + d*x**3)**(1/12)), x)
```

$$3.129 \quad \int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx$$

**Optimal.** Leaf size=121

$$\frac{5x(c+dx^3)^{5/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{3/4} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

[Out]  $4/9*x*(d*x^3+c)^(5/12)/a/(b*x^3+a)^(3/4)+5/9*x*(c*(b*x^3+a)/a/(d*x^3+c))^(3/4)*(d*x^3+c)^(5/12)*\text{hypergeom}([1/3, 3/4], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/a/(b*x^3+a)^(3/4)$

**Rubi [A]** time = 0.04, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{5x(c+dx^3)^{5/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{3/4} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{9a(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(5/12)/(a + b\*x^3)^(7/4), x]

[Out]  $(4*x*(c+d*x^3)^(5/12))/(9*a*(a+b*x^3)^(3/4)) + (5*x*((c*(a+b*x^3))/(a*(c+d*x^3)))^(3/4)*(c+d*x^3)^(5/12)*\text{Hypergeometric2F1}[1/3, 3/4, 4/3, -((b*c-a*d)*x^3)/(a*(c+d*x^3))])/(9*a*(a+b*x^3)^(3/4))$

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(a\*n\*(p+1)), x] - Dist[(c\*q)/(a\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0]

#### Rubi steps

$$\int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx = \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}} + \frac{(5c) \int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}} dx}{9a}$$

$$= \frac{4x(c+dx^3)^{5/12}}{9a(a+bx^3)^{3/4}} + \frac{5x \left(\frac{c+bx^3}{a+dx^3}\right)^{3/4} (c+dx^3)^{5/12} {}_2F_1\left(\frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{9a(a+bx^3)^{3/4}}$$



**Mathematica** [A] time = 0.05, size = 89, normalized size = 0.74

$$\frac{x \left( \frac{bx^3}{a} + 1 \right)^{3/4} (c + dx^3)^{5/12} {}_2F_1 \left( \frac{1}{3}, \frac{7}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)} \right)}{a (a + bx^3)^{3/4} \left( \frac{dx^3}{c} + 1 \right)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(5/12)/(a + b\*x^3)^(7/4), x]

[Out] (x\*(1 + (b\*x^3)/a)^(3/4)\*(c + d\*x^3)^(5/12)\*Hypergeometric2F1[1/3, 7/4, 4/3, ((-(b\*c) + a\*d)\*x^3)/(a\*(c + d\*x^3))])/(a\*(a + b\*x^3)^(3/4)\*(1 + (d\*x^3)/c)^(3/4))

**fricas** [F] time = 2.82, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{1/4} (dx^3 + c)^{5/12}}{b^2x^6 + 2abx^3 + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(5/12)/(b\*x^3+a)^(7/4), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(5/12)/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{5/12}}{(bx^3 + a)^{7/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(5/12)/(b\*x^3+a)^(7/4), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^(5/12)/(b\*x^3 + a)^(7/4), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{5/12}}{(bx^3 + a)^{7/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^(5/12)/(b\*x^3+a)^(7/4), x)

[Out] int((d\*x^3+c)^(5/12)/(b\*x^3+a)^(7/4), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{5/12}}{(bx^3 + a)^{7/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(5/12)/(b\*x^3+a)^(7/4),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^(5/12)/(b\*x^3 + a)^(7/4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^{5/12}}{(bx^3 + a)^{7/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^(5/12)/(a + b\*x^3)^(7/4),x)

[Out] int((c + d\*x^3)^(5/12)/(a + b\*x^3)^(7/4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^3)^{5/12}}{(a + bx^3)^{7/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*(5/12)/(b\*x\*\*3+a)\*\*(7/4),x)

[Out] Integral((c + d\*x\*\*3)\*\*(5/12)/(a + b\*x\*\*3)\*\*(7/4), x)

$$3.130 \quad \int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}} dx$$

**Optimal.** Leaf size=121

$$\frac{11x(c+dx^3)^{11/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{15a(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}$$

[Out]  $4/15*x*(d*x^3+c)^(11/12)/a/(b*x^3+a)^(5/4)+11/15*x*(c*(b*x^3+a)/a/(d*x^3+c)^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4], [4/3], -(-a*d+b*c)*x^3/a/(d*x^3+c))/a/(b*x^3+a)^(5/4)$

**Rubi [A]** time = 0.03, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{11x(c+dx^3)^{11/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{15a(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(11/12)/(a + b\*x^3)^(9/4), x]

[Out]  $(4*x*(c + d*x^3)^(11/12))/(15*a*(a + b*x^3)^(5/4)) + (11*x*((c*(a + b*x^3))/(a*(c + d*x^3)))^(5/4)*(c + d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -(((b*c - a*d)*x^3)/(a*(c + d*x^3)))]/(15*a*(a + b*x^3)^(5/4))$

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(a\*n\*(p+1)), x] - Dist[(c\*q)/(a\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rule 380**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0]

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}} dx &= \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}} + \frac{(11c) \int \frac{1}{(a+bx^3)^{5/4} \sqrt{c+dx^3}} dx}{15a} \\ &= \frac{4x(c+dx^3)^{11/12}}{15a(a+bx^3)^{5/4}} + \frac{11x \left(\frac{c+bx^3}{a+dx^3}\right)^{5/4} (c+dx^3)^{11/12} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(c+dx^3)}\right)}{15a(a+bx^3)^{5/4}} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 89, normalized size = 0.74

$$\frac{x \sqrt[4]{\frac{bx^3}{a} + 1} (c + dx^3)^{11/12} {}_2F_1\left(\frac{1}{3}, \frac{9}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)}\right)}{a^2 \sqrt[4]{a + bx^3} \left(\frac{dx^3}{c} + 1\right)^{5/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(11/12)/(a + b\*x^3)^(9/4), x]

[Out] (x\*(1 + (b\*x^3)/a)^(1/4)\*(c + d\*x^3)^(11/12)\*Hypergeometric2F1[1/3, 9/4, 4/3, ((-b\*c) + a\*d)\*x^3/(a\*(c + d\*x^3))]/(a^2\*(a + b\*x^3)^(1/4)\*(1 + (d\*x^3)/c)^(5/4))

**fricas** [F] time = 64.26, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^{\frac{3}{4}}(dx^3 + c)^{\frac{11}{12}}}{b^3x^9 + 3ab^2x^6 + 3a^2bx^3 + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(11/12)/(b\*x^3+a)^(9/4), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(11/12)/(b^3\*x^9 + 3\*a\*b^2\*x^6 + 3\*a^2\*b\*x^3 + a^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{11}{12}}}{(bx^3 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(11/12)/(b\*x^3+a)^(9/4), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^(11/12)/(b\*x^3 + a)^(9/4), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{11}{12}}}{(bx^3 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^(11/12)/(b\*x^3+a)^(9/4), x)

[Out] int((d\*x^3+c)^(11/12)/(b\*x^3+a)^(9/4), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{11}{12}}}{(bx^3 + a)^{\frac{9}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(11/12)/(b\*x^3+a)^(9/4),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^(11/12)/(b\*x^3 + a)^(9/4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^{11/12}}{(bx^3 + a)^{9/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^(11/12)/(a + b\*x^3)^(9/4),x)

[Out] int((c + d\*x^3)^(11/12)/(a + b\*x^3)^(9/4), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*(11/12)/(b\*x\*\*3+a)\*\*(9/4),x)

[Out] Timed out

$$3.131 \quad \int \frac{(c+dx^3)^{17/12}}{(a+bx^3)^{11/4}} dx$$

**Optimal.** Leaf size=153

$$\frac{85cx(c+dx^3)^{5/12} \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right)}{189a^2(a+bx^3)^{3/4}} + \frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}}$$

[Out] 68/189\*c\*x\*(d\*x^3+c)^(5/12)/a^2/(b\*x^3+a)^(3/4)+4/21\*x\*(d\*x^3+c)^(17/12)/a/(b\*x^3+a)^(7/4)+85/189\*c\*x\*(c\*(b\*x^3+a)/a/(d\*x^3+c))^(3/4)\*(d\*x^3+c)^(5/12)\*hypergeom([1/3, 3/4], [4/3], -(a\*d+b\*c)\*x^3/a/(d\*x^3+c))/a^2/(b\*x^3+a)^(3/4)

**Rubi [A]** time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{85cx(c+dx^3)^{5/12} \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{3/4} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)} \right)}{189a^2(a+bx^3)^{3/4}} + \frac{68cx(c+dx^3)^{5/12}}{189a^2(a+bx^3)^{3/4}} + \frac{4x(c+dx^3)^{17/12}}{21a(a+bx^3)^{7/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(17/12)/(a + b\*x^3)^(11/4), x]

[Out] (68\*c\*x\*(c + d\*x^3)^(5/12))/(189\*a^2\*(a + b\*x^3)^(3/4)) + (4\*x\*(c + d\*x^3)^(17/12))/(21\*a\*(a + b\*x^3)^(7/4)) + (85\*c\*x\*((c\*(a + b\*x^3))/(a\*(c + d\*x^3)))^(3/4)\*(c + d\*x^3)^(5/12)\*Hypergeometric2F1[1/3, 3/4, 4/3, -((b\*c - a\*d)\*x^3)/(a\*(c + d\*x^3))])/(189\*a^2\*(a + b\*x^3)^(3/4))

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^3)^{17/12}}{(a + bx^3)^{11/4}} dx &= \frac{4x(c + dx^3)^{17/12}}{21a(a + bx^3)^{7/4}} + \frac{(17c) \int \frac{(c+dx^3)^{5/12}}{(a+bx^3)^{7/4}} dx}{21a} \\
&= \frac{68cx(c + dx^3)^{5/12}}{189a^2(a + bx^3)^{3/4}} + \frac{4x(c + dx^3)^{17/12}}{21a(a + bx^3)^{7/4}} + \frac{(85c^2) \int \frac{1}{(a+bx^3)^{3/4}(c+dx^3)^{7/12}} dx}{189a^2} \\
&= \frac{68cx(c + dx^3)^{5/12}}{189a^2(a + bx^3)^{3/4}} + \frac{4x(c + dx^3)^{17/12}}{21a(a + bx^3)^{7/4}} + \frac{85cx \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{3/4} (c + dx^3)^{5/12} {}_2F_1 \left( \frac{1}{3}, \frac{3}{4}; \frac{4}{3}; -\frac{(ad-bc)x^3}{a(dx^3+c)} \right)}{189a^2(a + bx^3)^{3/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 90, normalized size = 0.59

$$\frac{cx \left( \frac{bx^3}{a} + 1 \right)^{3/4} (c + dx^3)^{5/12} {}_2F_1 \left( \frac{1}{3}, \frac{11}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)} \right)}{a^2 (a + bx^3)^{3/4} \left( \frac{dx^3}{c} + 1 \right)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(17/12)/(a + b\*x^3)^(11/4), x]

[Out] (c\*x\*(1 + (b\*x^3)/a)^(3/4)\*(c + d\*x^3)^(5/12)\*Hypergeometric2F1[1/3, 11/4, 4/3, ((-b\*c) + a\*d)\*x^3/(a\*(c + d\*x^3))]/(a^2\*(a + b\*x^3)^(3/4)\*(1 + (d\*x^3)/c)^(3/4))

**fricas [F]** time = 2.85, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{1/4} (dx^3 + c)^{17/12}}{b^3 x^9 + 3 ab^2 x^6 + 3 a^2 bx^3 + a^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(17/12)/(b\*x^3+a)^(11/4), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(1/4)\*(d\*x^3 + c)^(17/12)/(b^3\*x^9 + 3\*a\*b^2\*x^6 + 3\*a^2\*b\*x^3 + a^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{17/12}}{(bx^3 + a)^{11/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(17/12)/(b\*x^3+a)^(11/4), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^(17/12)/(b\*x^3 + a)^(11/4), x)

**maple [F]** time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{17/12}}{(bx^3 + a)^{11/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)`

[Out] `int((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{17}{12}}}{(bx^3 + a)^{\frac{11}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^3+c)^(17/12)/(b*x^3+a)^(11/4),x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)^(17/12)/(b*x^3 + a)^(11/4), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^{17/12}}{(bx^3 + a)^{11/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4),x)`

[Out] `int((c + d*x^3)^(17/12)/(a + b*x^3)^(11/4), x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x**3+c)**(17/12)/(b*x**3+a)**(11/4),x)`

[Out] Timed out



$$3.132 \quad \int \frac{(c+dx^3)^{23/12}}{(a+bx^3)^{13/4}} dx$$

**Optimal.** Leaf size=153

$$\frac{253cx(c+dx^3)^{11/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2(a+bx^3)^{5/4}} + \frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}}$$

[Out]  $92/405*c*x*(d*x^3+c)^(11/12)/a^2/(b*x^3+a)^(5/4)+4/27*x*(d*x^3+c)^(23/12)/a/(b*x^3+a)^(9/4)+253/405*c*x*(c*(b*x^3+a)/a/(d*x^3+c))^(5/4)*(d*x^3+c)^(11/12)*hypergeom([1/3, 5/4], [4/3], -(a*d+b*c)*x^3/a/(d*x^3+c))/a^2/(b*x^3+a)^(5/4)$

**Rubi [A]** time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 380}

$$\frac{253cx(c+dx^3)^{11/12} \left(\frac{c+bx^3}{a+dx^3}\right)^{5/4} {}_2F_1\left(\frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{(bc-ad)x^3}{a(dx^3+c)}\right)}{405a^2(a+bx^3)^{5/4}} + \frac{92cx(c+dx^3)^{11/12}}{405a^2(a+bx^3)^{5/4}} + \frac{4x(c+dx^3)^{23/12}}{27a(a+bx^3)^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^(23/12)/(a + b\*x^3)^(13/4), x]

[Out]  $(92*c*x*(c+d*x^3)^(11/12))/(405*a^2*(a+b*x^3)^(5/4)) + (4*x*(c+d*x^3)^(23/12))/(27*a*(a+b*x^3)^(9/4)) + (253*c*x*((c*(a+b*x^3))/(a*(c+d*x^3)))^(5/4)*(c+d*x^3)^(11/12)*Hypergeometric2F1[1/3, 5/4, 4/3, -((b*c-a*d)*x^3)/(a*(c+d*x^3))])/(405*a^2*(a+b*x^3)^(5/4))$

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol]  
 :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^3)^{23/12}}{(a + bx^3)^{13/4}} dx &= \frac{4x(c + dx^3)^{23/12}}{27a(a + bx^3)^{9/4}} + \frac{(23c) \int \frac{(c+dx^3)^{11/12}}{(a+bx^3)^{9/4}} dx}{27a} \\ &= \frac{92cx(c + dx^3)^{11/12}}{405a^2(a + bx^3)^{5/4}} + \frac{4x(c + dx^3)^{23/12}}{27a(a + bx^3)^{9/4}} + \frac{(253c^2) \int \frac{1}{(a+bx^3)^{5/4} \sqrt[12]{c+dx^3}} dx}{405a^2} \\ &= \frac{92cx(c + dx^3)^{11/12}}{405a^2(a + bx^3)^{5/4}} + \frac{4x(c + dx^3)^{23/12}}{27a(a + bx^3)^{9/4}} + \frac{253cx \left( \frac{c(a+bx^3)}{a(c+dx^3)} \right)^{5/4} (c + dx^3)^{11/12} {}_2F_1 \left( \frac{1}{3}, \frac{5}{4}; \frac{4}{3}; -\frac{c}{a} \right)}{405a^2(a + bx^3)^{5/4}} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 90, normalized size = 0.59

$$\frac{cx \sqrt[4]{\frac{bx^3}{a} + 1} (c + dx^3)^{11/12} {}_2F_1 \left( \frac{1}{3}, \frac{13}{4}; \frac{4}{3}; \frac{(ad-bc)x^3}{a(dx^3+c)} \right)}{a^3 \sqrt[4]{a + bx^3} \left( \frac{dx^3}{c} + 1 \right)^{5/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^(23/12)/(a + b\*x^3)^(13/4), x]

[Out] (c\*x\*(1 + (b\*x^3)/a)^(1/4)\*(c + d\*x^3)^(11/12)\*Hypergeometric2F1[1/3, 13/4, 4/3, ((-b\*c) + a\*d)\*x^3/(a\*(c + d\*x^3))]/(a^3\*(a + b\*x^3)^(1/4)\*(1 + (d\*x^3)/c)^(5/4))

**fricas** [F] time = 66.28, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^3 + a)^{3/4} (dx^3 + c)^{23/12}}{b^4 x^{12} + 4ab^3 x^9 + 6a^2 b^2 x^6 + 4a^3 b x^3 + a^4}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(23/12)/(b\*x^3+a)^(13/4), x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^(3/4)\*(d\*x^3 + c)^(23/12)/(b^4\*x^12 + 4\*a\*b^3\*x^9 + 6\*a^2\*b^2\*x^6 + 4\*a^3\*b\*x^3 + a^4), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{23/12}}{(bx^3 + a)^{13/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(23/12)/(b\*x^3+a)^(13/4), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^(23/12)/(b\*x^3 + a)^(13/4), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{23/12}}{(bx^3 + a)^{13/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^(23/12)/(b\*x^3+a)^(13/4),x)

[Out] int((d\*x^3+c)^(23/12)/(b\*x^3+a)^(13/4),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^{\frac{23}{12}}}{(bx^3 + a)^{\frac{13}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^(23/12)/(b\*x^3+a)^(13/4),x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^(23/12)/(b\*x^3 + a)^(13/4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(dx^3 + c)^{23/12}}{(bx^3 + a)^{13/4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^(23/12)/(a + b\*x^3)^(13/4),x)

[Out] int((c + d\*x^3)^(23/12)/(a + b\*x^3)^(13/4), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*3+c)\*\*(23/12)/(b\*x\*\*3+a)\*\*(13/4),x)

[Out] Timed out

### 3.133 $\int (a + bx^3)^m (c + dx^3)^p dx$

**Optimal.** Leaf size=79

$$x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (c + dx^3)^p \left(\frac{dx^3}{c} + 1\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

[Out]  $x*(b*x^3+a)^m*(d*x^3+c)^p*AppellF1(1/3, -m, -p, 4/3, -b*x^3/a, -d*x^3/c)/((1+b*x^3/a)^m)/((1+d*x^3/c)^p)$

**Rubi [A]** time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (c + dx^3)^p \left(\frac{dx^3}{c} + 1\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^p,x]

[Out]  $(x*(a + b*x^3)^m*(c + d*x^3)^p*AppellF1[1/3, -m, -p, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/((1 + (b*x^3)/a)^m*(1 + (d*x^3)/c)^p)$

#### Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rubi steps

$$\begin{aligned} \int (a + bx^3)^m (c + dx^3)^p dx &= \left( (a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \right) \int \left(1 + \frac{bx^3}{a}\right)^m (c + dx^3)^p dx \\ &= \left( (a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} (c + dx^3)^p \left(1 + \frac{dx^3}{c}\right)^{-p} \right) \int \left(1 + \frac{bx^3}{a}\right)^m \left(1 + \frac{dx^3}{c}\right)^p dx \\ &= x(a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} (c + dx^3)^p \left(1 + \frac{dx^3}{c}\right)^{-p} F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \end{aligned}$$

**Mathematica [B]** time = 0.43, size = 172, normalized size = 2.18

$$\frac{4acx(a + bx^3)^m (c + dx^3)^p F_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{3x^3 \left( bcmF_1\left(\frac{4}{3}; 1 - m, -p; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) + adpF_1\left(\frac{4}{3}; -m, 1 - p; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) + 4acF_1\left(\frac{1}{3}; -m, -p; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^p,x]

[Out] (4\*a\*c\*x\*(a + b\*x^3)^m\*(c + d\*x^3)^p\*AppellF1[1/3, -m, -p, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/(4\*a\*c\*AppellF1[1/3, -m, -p, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 3\*x^3\*(b\*c\*m\*AppellF1[4/3, 1 - m, -p, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] + a\*d\*p\*AppellF1[4/3, -m, 1 - p, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F] time = 1.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3 + a\right)^m\left(dx^3 + c\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^p,x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m\*(d\*x^3 + c)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^p,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m\*(d\*x^3 + c)^p, x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c)^p,x)

[Out] int((b\*x^3+a)^m\*(d\*x^3+c)^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^p,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m\*(d\*x^3 + c)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^m (dx^3 + c)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^m\*(c + d\*x^3)^p,x)

[Out] int((a + b\*x^3)^m\*(c + d\*x^3)^p, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*p,x)

[Out] Timed out

### 3.134 $\int (a + bx^3)^2 (c + dx^3)^q dx$

**Optimal.** Leaf size=167

$$\frac{x(c + dx^3)^{q+1} \left( a^2 d^2 (9q^2 + 33q + 28) - 2abcd(3q + 7) + 4b^2 c^2 \right) {}_2F_1 \left( 1, q + \frac{4}{3}; \frac{4}{3}; -\frac{dx^3}{c} \right) + bx(c + dx^3)^{q+1} (4bc - ad)}{cd^2(3q + 4)(3q + 7) + d^2(3q + 4)(3q + 7)}$$

[Out]  $-b*(4*b*c-a*d*(10+3*q))*x*(d*x^3+c)^(1+q)/d^2/(9*q^2+33*q+28)+b*x*(b*x^3+a)*(d*x^3+c)^(1+q)/d/(7+3*q)+(4*b^2*c^2-2*a*b*c*d*(7+3*q)+a^2*d^2*(9*q^2+33*q+28))*x*(d*x^3+c)^(1+q)*\text{hypergeom}([1, 4/3+q], [4/3], -d*x^3/c)/c/d^2/(9*q^2+33*q+28)$

**Rubi [A]** time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 388, 246, 245}

$$\frac{x(c + dx^3)^q \left( \frac{dx^3}{c} + 1 \right)^{-q} \left( a^2 d^2 (9q^2 + 33q + 28) - 2abcd(3q + 7) + 4b^2 c^2 \right) {}_2F_1 \left( \frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c} \right) + bx(c + dx^3)^{q+1} (4bc - ad)}{d^2(3q + 4)(3q + 7) + d^2(3q + 4)(3q + 7)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^2\*(c + d\*x^3)^q,x]

[Out]  $-((b*(4*b*c - a*d*(10 + 3*q))*x*(c + d*x^3)^(1 + q))/(d^2*(4 + 3*q)*(7 + 3*q)) + (b*x*(a + b*x^3)*(c + d*x^3)^(1 + q))/(d*(7 + 3*q)) + ((4*b^2*c^2 - 2*a*b*c*d*(7 + 3*q) + a^2*d^2*(28 + 33*q + 9*q^2))*x*(c + d*x^3)^q*\text{Hypergeometric2F1}[1/3, -q, 4/3, -((d*x^3)/c)])/(d^2*(4 + 3*q)*(7 + 3*q)*(1 + (d*x^3)/c)^q)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^2 (c + dx^3)^q dx &= \frac{bx(a + bx^3)(c + dx^3)^{1+q}}{d(7 + 3q)} + \frac{\int (c + dx^3)^q (-a(bc - ad(7 + 3q)) - b(4bc - ad(10 + 3q))) dx}{d(7 + 3q)} \\ &= -\frac{b(4bc - ad(10 + 3q))x(c + dx^3)^{1+q}}{d^2(4 + 3q)(7 + 3q)} + \frac{bx(a + bx^3)(c + dx^3)^{1+q}}{d(7 + 3q)} + \frac{(4b^2c^2 - 2ad(4 + 3q))x(c + dx^3)^{1+q}}{d^2(4 + 3q)(7 + 3q)} \\ &= -\frac{b(4bc - ad(10 + 3q))x(c + dx^3)^{1+q}}{d^2(4 + 3q)(7 + 3q)} + \frac{bx(a + bx^3)(c + dx^3)^{1+q}}{d(7 + 3q)} + \frac{\left((4b^2c^2 - 2ad(4 + 3q))x(c + dx^3)^{1+q}\right)}{d^2(4 + 3q)(7 + 3q)} \\ &= -\frac{b(4bc - ad(10 + 3q))x(c + dx^3)^{1+q}}{d^2(4 + 3q)(7 + 3q)} + \frac{bx(a + bx^3)(c + dx^3)^{1+q}}{d(7 + 3q)} + \frac{(4b^2c^2 - 2ad(4 + 3q))x(c + dx^3)^{1+q}}{d^2(4 + 3q)(7 + 3q)} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 106, normalized size = 0.63

$$\frac{1}{14}x(c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left(14a^2 {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + bx^3 \left(7a {}_2F_1\left(\frac{4}{3}, -q; \frac{7}{3}; -\frac{dx^3}{c}\right) + 2bx^3 {}_2F_1\left(\frac{7}{3}, -q; \frac{10}{3}; -\frac{dx^3}{c}\right)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^2\*(c + d\*x^3)^q,x]

[Out] (x\*(c + d\*x^3)^q\*(14\*a^2\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)] + b\*x^3\*(7\*a\*Hypergeometric2F1[4/3, -q, 7/3, -((d\*x^3)/c)] + 2\*b\*x^3\*Hypergeometric2F1[7/3, -q, 10/3, -((d\*x^3)/c)]))/(14\*(1 + (d\*x^3)/c)^q)

**fricas [F]** time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^6 + 2abx^3 + a^2\right)(dx^3 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^q,x, algorithm="fricas")

[Out] integral((b^2\*x^6 + 2\*a\*b\*x^3 + a^2)\*(d\*x^3 + c)^q, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^2\*(d\*x^3+c)^q,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^2\*(d\*x^3 + c)^q, x)

**maple [F]** time = 0.53, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^2\*(d\*x^3+c)^q,x)

[Out] `int((b*x^3+a)^2*(d*x^3+c)^q,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^2*(d*x^3+c)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^2*(d*x^3 + c)^q, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^2 (dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^2*(c + d*x^3)^q,x)`

[Out] `int((a + b*x^3)^2*(c + d*x^3)^q, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**2*(d*x**3+c)**q,x)`

[Out] Timed out



### 3.135 $\int (a + bx^3)(c + dx^3)^q dx$

Optimal. Leaf size=84

$$\frac{bx(c + dx^3)^{q+1}}{d(3q + 4)} - \frac{x(c + dx^3)^{q+1}(bc - ad(3q + 4)) {}_2F_1\left(1, q + \frac{4}{3}; \frac{4}{3}; -\frac{dx^3}{c}\right)}{cd(3q + 4)}$$

[Out] b\*x\*(d\*x^3+c)^(1+q)/d/(4+3\*q)-(b\*c-a\*d\*(4+3\*q))\*x\*(d\*x^3+c)^(1+q)\*hypergeom([1, 4/3+q], [4/3], -d\*x^3/c)/c/d/(4+3\*q)

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {388, 246, 245}

$$x(c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left(a - \frac{bc}{3dq + 4d}\right) {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + \frac{bx(c + dx^3)^{q+1}}{d(3q + 4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)\*(c + d\*x^3)^q,x]

[Out] (b\*x\*(c + d\*x^3)^(1 + q))/(d\*(4 + 3\*q)) + ((a - (b\*c)/(4\*d + 3\*d\*q))\*x\*(c + d\*x^3)^q\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)])/(1 + (d\*x^3)/c)^q

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (a + bx^3)(c + dx^3)^q dx &= \frac{bx(c + dx^3)^{1+q}}{d(4 + 3q)} - \left(-a + \frac{bc}{4d + 3dq}\right) \int (c + dx^3)^q dx \\ &= \frac{bx(c + dx^3)^{1+q}}{d(4 + 3q)} - \left(\left(-a + \frac{bc}{4d + 3dq}\right)(c + dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q}\right) \int \left(1 + \frac{dx^3}{c}\right)^q dx \\ &= \frac{bx(c + dx^3)^{1+q}}{d(4 + 3q)} + \left(a - \frac{bc}{4d + 3dq}\right) x(c + dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q} {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 90, normalized size = 1.07

$$\frac{x(c + dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} \left( (ad(3q + 4) - bc) {}_2F_1\left(\frac{1}{3}, -q; \frac{4}{3}; -\frac{dx^3}{c}\right) + b(c + dx^3) \left(\frac{dx^3}{c} + 1\right)^q \right)}{d(3q + 4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)\*(c + d\*x^3)^q,x]

[Out] (x\*(c + d\*x^3)^q\*(b\*(c + d\*x^3)\*(1 + (d\*x^3)/c)^q + (-b\*c) + a\*d\*(4 + 3\*q))\*Hypergeometric2F1[1/3, -q, 4/3, -((d\*x^3)/c)]/(d\*(4 + 3\*q)\*(1 + (d\*x^3)/c)^q)

**fricas [F]** time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^3 + a\right)\left(dx^3 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^q,x, algorithm="fricas")

[Out] integral((b\*x^3 + a)\*(d\*x^3 + c)^q, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^q,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)\*(d\*x^3 + c)^q, x)

**maple [F]** time = 0.40, size = 0, normalized size = 0.00

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)\*(d\*x^3+c)^q,x)

[Out] int((b\*x^3+a)\*(d\*x^3+c)^q,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)\*(d\*x^3+c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)\*(d\*x^3 + c)^q, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)(dx^3 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)*(c + d*x^3)^q, x)`

[Out] `int((a + b*x^3)*(c + d*x^3)^q, x)`

**sympy [C]** time = 82.86, size = 75, normalized size = 0.89

$$\frac{ac^q x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -q \middle| \frac{dx^3 e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{bc^q x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, -q \middle| \frac{dx^3 e^{i\pi}}{c}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)*(d*x**3+c)**q, x)`

[Out] `a*c**q*x*gamma(1/3)*hyper((1/3, -q), (4/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(4/3)) + b*c**q*x**4*gamma(4/3)*hyper((4/3, -q), (7/3,), d*x**3*exp_polar(I*pi)/c)/(3*gamma(7/3))`

$$3.136 \quad \int \frac{(c+dx^3)^q}{a+bx^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 1, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a}$$

[Out] x\*(d\*x^3+c)^q\*AppellF1(1/3,1,-q,4/3,-b\*x^3/a,-d\*x^3/c)/a/((1+d\*x^3/c)^q)

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 1, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^q/(a + b\*x^3), x]

[Out] (x\*(c + d\*x^3)^q\*AppellF1[1/3, 1, -q, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)])/(a\*(1 + (d\*x^3)/c)^q)

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^3)^q}{a+bx^3} dx &= \left( (c+dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q} \right) \int \frac{\left(1 + \frac{dx^3}{c}\right)^q}{a+bx^3} dx \\ &= \frac{x(c+dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3}; 1, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a} \end{aligned}$$

**Mathematica [B]** time = 0.20, size = 162, normalized size = 2.84

$$\frac{4acx(c+dx^3)^q F_1\left(\frac{1}{3}; -q, 1; \frac{4}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right)}{(a+bx^3) \left( 3x^3 \left( adq F_1\left(\frac{4}{3}; 1-q, 1; \frac{7}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right) - bc F_1\left(\frac{4}{3}; -q, 2; \frac{7}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right) \right) + 4ac F_1\left(\frac{1}{3}; -q, 1; \frac{4}{3}; -\frac{dx^3}{c}, -\frac{bx^3}{a}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^q/(a + b\*x^3), x]

```
[Out] (4*a*c*x*(c + d*x^3)^q*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
]/((a + b*x^3)*(4*a*c*AppellF1[1/3, -q, 1, 4/3, -((d*x^3)/c), -((b*x^3)/a)
] + 3*x^3*(a*d*q*AppellF1[4/3, 1 - q, 1, 7/3, -((d*x^3)/c), -((b*x^3)/a)] -
b*c*AppellF1[4/3, -q, 2, 7/3, -((d*x^3)/c), -((b*x^3)/a)]))
```

**fricas** [F] time = 1.16, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx^3 + c)^q}{bx^3 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^3 + c)^q/(b*x^3 + a), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^3+c)^q/(b*x^3+a),x)
```

```
[Out] int((d*x^3+c)^q/(b*x^3+a),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^3+c)^q/(b*x^3+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^3 + c)^q/(b*x^3 + a), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx^3 + c)^q}{bx^3 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^3)^q/(a + b*x^3),x)
```

```
[Out] int((c + d*x^3)^q/(a + b*x^3), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a),x)
```

```
[Out] Timed out
```

$$3.137 \quad \int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2}$$

[Out]  $x*(d*x^3+c)^q*AppellF1(1/3,2,-q,4/3,-b*x^3/a,-d*x^3/c)/a^2/((1+d*x^3/c)^q)$

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(c+dx^3)^q \left(\frac{dx^3}{c} + 1\right)^{-q} F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^3)^q/(a + b\*x^3)^2,x]

[Out]  $(x*(c + d*x^3)^q*AppellF1[1/3, 2, -q, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(a^2*(1 + (d*x^3)/c)^q)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^3)^q}{(a+bx^3)^2} dx &= \left( (c+dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q} \right) \int \frac{\left(1 + \frac{dx^3}{c}\right)^q}{(a+bx^3)^2} dx \\ &= \frac{x(c+dx^3)^q \left(1 + \frac{dx^3}{c}\right)^{-q} F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{a^2} \end{aligned}$$

**Mathematica [B]** time = 0.26, size = 162, normalized size = 2.84

$$\frac{4acx(c+dx^3)^q F_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(a+bx^3)^2 \left(3x^3 \left(adqF_1\left(\frac{4}{3}; 2, 1-q; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 2bcF_1\left(\frac{4}{3}; 3, -q; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) + 4acF_1\left(\frac{1}{3}; 2, -q; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^3)^q/(a + b\*x^3)^2,x]

[Out] (4\*a\*c\*x\*(c + d\*x^3)^q\*AppellF1[1/3, 2, -q, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)]/((a + b\*x^3)^2\*(4\*a\*c\*AppellF1[1/3, 2, -q, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] + 3\*x^3\*(a\*d\*q\*AppellF1[4/3, 2, 1 - q, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 2\*b\*c\*AppellF1[4/3, 3, -q, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^3 + c)^q}{b^2x^6 + 2abx^3 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^q/(b\*x^3+a)^2,x, algorithm="fricas")

[Out] integral((d\*x^3 + c)^q/(b^2\*x^6 + 2\*a\*b\*x^3 + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^q/(b\*x^3+a)^2,x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^3+c)^q/(b\*x^3+a)^2,x)

[Out] int((d\*x^3+c)^q/(b\*x^3+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^3+c)^q/(b\*x^3+a)^2,x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^q/(b\*x^3 + a)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx^3 + c)^q}{(bx^3 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^3)^q/(a + b\*x^3)^2,x)



```
[Out] int((c + d*x^3)^q/(a + b*x^3)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**3+c)**q/(b*x**3+a)**2,x)
```

```
[Out] Timed out
```

### 3.138 $\int (a + bx^3)^m (c + dx^3)^3 dx$

**Optimal.** Leaf size=298

$$\frac{dx (a + bx^3)^{m+1} (28a^2d^2 - abcd(15m + 92) + b^2c^2(9m^2 + 60m + 118))}{b^3(3m + 4)(3m + 7)(3m + 10)} x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (28a^3d^3 - 12a^2d^2c + 3a^2cd^2 - 3ab^2cd + b^3c^2)$$

[Out]  $d*(28*a^2*d^2 - a*b*c*d*(92+15*m) + b^2*c^2*(9*m^2+60*m+118))*x*(b*x^3+a)^(1+m)/b^3/(10+3*m)/(9*m^2+33*m+28) - d*(7*a*d - b*c*(16+3*m))*x*(b*x^3+a)^(1+m)*(d*x^3+c)/b^2/(9*m^2+51*m+70) + d*x*(b*x^3+a)^(1+m)*(d*x^3+c)^2/b/(10+3*m) - (28*a^3*d^3 - 12*a^2*b*c*d^2*(10+3*m) + 3*a*b^2*c^2*d*(9*m^2+51*m+70) - b^3*c^3*(27*m^3 + 189*m^2 + 414*m + 280))*x*(b*x^3+a)^m*hypergeom([1/3, -m], [4/3], -b*x^3/a)/b^3/(10+3*m)/(9*m^2+33*m+28)/((1+b*x^3/a)^m)$

**Rubi [A]** time = 0.30, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {416, 528, 388, 246, 245}

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (-12a^2bcd^2(3m + 10) + 28a^3d^3 + 3ab^2c^2d(9m^2 + 51m + 70) - b^3c^3(27m^3 + 189m^2 + 414m + 280))}{b^3(3m + 4)(3m + 7)(3m + 10)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^3,x]

[Out]  $(d*(28*a^2*d^2 - a*b*c*d*(92 + 15*m) + b^2*c^2*(118 + 60*m + 9*m^2))*x*(a + b*x^3)^(1 + m))/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)) - (d*(7*a*d - b*c*(16 + 3*m))*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b^2*(7 + 3*m)*(10 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3)^2)/(b*(10 + 3*m)) - ((28*a^3*d^3 - 12*a^2*b*c*d^2*(10 + 3*m) + 3*a*b^2*c^2*d*(70 + 51*m + 9*m^2) - b^3*c^3*(280 + 414*m + 189*m^2 + 27*m^3))*x*(a + b*x^3)^m*Hypergeometric2F1[1/3, -m, 4/3, -(b*x^3/a)])/(b^3*(4 + 3*m)*(7 + 3*m)*(10 + 3*m)*(1 + (b*x^3/a)^m)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x]

$x] + \text{Dist}[1/(b*(n*(p + q) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*\text{Simp}[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, n, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[q, 1] \&\& \text{NeQ}[n*(p + q) + 1, 0] \&\& !\text{IGtQ}[p, 1] \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

### Rule 528

$\text{Int}[(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n), x\_Symbol] := \text{Simp}[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(b*(n*(p + q + 1) + 1)), x] + \text{Dist}[1/(b*(n*(p + q + 1) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^(q - 1)*\text{Simp}[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[q, 0] \&\& \text{NeQ}[n*(p + q + 1) + 1, 0]$

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^m (c + dx^3)^3 dx &= \frac{dx (a + bx^3)^{1+m} (c + dx^3)^2}{b(10 + 3m)} + \frac{\int (a + bx^3)^m (c + dx^3) (-c(ad - bc(10 + 3m)) - a}{b(10 + 3m)} \\ &= -\frac{d(7ad - bc(16 + 3m))x (a + bx^3)^{1+m} (c + dx^3)}{b^2(7 + 3m)(10 + 3m)} + \frac{dx (a + bx^3)^{1+m} (c + dx^3)^2}{b(10 + 3m)} \\ &= \frac{d(28a^2d^2 - abcd(92 + 15m) + b^2c^2(118 + 60m + 9m^2))x (a + bx^3)^{1+m}}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} - \frac{d(7a}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} \\ &= \frac{d(28a^2d^2 - abcd(92 + 15m) + b^2c^2(118 + 60m + 9m^2))x (a + bx^3)^{1+m}}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} - \frac{d(7a}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} \\ &= \frac{d(28a^2d^2 - abcd(92 + 15m) + b^2c^2(118 + 60m + 9m^2))x (a + bx^3)^{1+m}}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} - \frac{d(7a}{b^3(4 + 3m)(7 + 3m)(10 + 3m)} \end{aligned}$$

**Mathematica [A]** time = 5.08, size = 137, normalized size = 0.46

$$\frac{1}{140}x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left(140c^3 {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + dx^3 \left(105c^2 {}_2F_1\left(\frac{4}{3}, -m; \frac{7}{3}; -\frac{bx^3}{a}\right) + 2dx^3 \left(30c {}_2F_1\left(\frac{7}{3}, -m; \frac{10}{3}; -\frac{bx^3}{a}\right) + 7dx^3 {}_2F_1\left(\frac{10}{3}, -m; \frac{13}{3}; -\frac{bx^3}{a}\right)\right)\right)\right) / (140*(1 + (bx^3)/a)^m)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^3,x]

[Out] (x\*(a + b\*x^3)^m\*(140\*c^3\*Hypergeometric2F1[1/3, -m, 4/3, -((b\*x^3)/a)] + d\*x^3\*(105\*c^2\*Hypergeometric2F1[4/3, -m, 7/3, -((b\*x^3)/a)] + 2\*d\*x^3\*(30\*c\*Hypergeometric2F1[7/3, -m, 10/3, -((b\*x^3)/a)] + 7\*d\*x^3\*Hypergeometric2F1[10/3, -m, 13/3, -((b\*x^3)/a)])))/(140\*(1 + (b\*x^3)/a)^m)

**fricas [F]** time = 1.32, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3\right)(bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^3,x, algorithm="fricas")

[Out] integral((d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3)\*(b\*x^3 + a)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^3 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^3\*(b\*x^3 + a)^m, x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^3 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^m\*(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^3 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)^3\*(b\*x^3 + a)^m, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (bx^3 + a)^m (dx^3 + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^m\*(c + d\*x^3)^3,x)

[Out] int((a + b\*x^3)^m\*(c + d\*x^3)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m\*(d\*x\*\*3+c)\*\*3,x)

[Out] Timed out

### 3.139 $\int (a + bx^3)^m (c + dx^3)^2 dx$

**Optimal.** Leaf size=176

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (4a^2d^2 - 2abcd(3m + 7) + b^2c^2(9m^2 + 33m + 28)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) dx (a + bx^3)^m}{b^2(3m + 4)(3m + 7)}$$

[Out]  $-d*(4*a*d-b*c*(10+3*m))*x*(b*x^3+a)^(1+m)/b^2/(9*m^2+33*m+28)+d*x*(b*x^3+a)^(1+m)*(d*x^3+c)/b/(7+3*m)+(4*a^2*d^2-2*a*b*c*d*(7+3*m)+b^2*c^2*(9*m^2+33*m+28))*x*(b*x^3+a)^m*\text{hypergeom}([1/3, -m], [4/3], -b*x^3/a)/b^2/(9*m^2+33*m+28)/((1+b*x^3/a)^m)$

**Rubi [A]** time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 388, 246, 245}

$$\frac{x (a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (4a^2d^2 - 2abcd(3m + 7) + b^2c^2(9m^2 + 33m + 28)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) dx (a + bx^3)^m}{b^2(3m + 4)(3m + 7)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3)^2,x]

[Out]  $-((d*(4*a*d - b*c*(10 + 3*m))*x*(a + b*x^3)^(1 + m))/(b^2*(4 + 3*m)*(7 + 3*m)) + (d*x*(a + b*x^3)^(1 + m)*(c + d*x^3))/(b*(7 + 3*m)) + ((4*a^2*d^2 - 2*a*b*c*d*(7 + 3*m) + b^2*c^2*(28 + 33*m + 9*m^2))*x*(a + b*x^3)^m*\text{Hypergeometric2F1}[1/3, -m, 4/3, -((b*x^3)/a)]/(b^2*(4 + 3*m)*(7 + 3*m)*(1 + (b*x^3)/a)^m)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d] + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^3)^m (c + dx^3)^2 dx &= \frac{dx (a + bx^3)^{1+m} (c + dx^3)}{b(7 + 3m)} + \frac{\int (a + bx^3)^m (-c(ad - bc(7 + 3m)) - d(4ad - bc(10 + 3m))) dx}{b(7 + 3m)} \\ &= -\frac{d(4ad - bc(10 + 3m))x (a + bx^3)^{1+m}}{b^2(4 + 3m)(7 + 3m)} + \frac{dx (a + bx^3)^{1+m} (c + dx^3)}{b(7 + 3m)} + \frac{(4a^2d^2 - 2ad^2)}{b^2(4 + 3m)(7 + 3m)} \\ &= -\frac{d(4ad - bc(10 + 3m))x (a + bx^3)^{1+m}}{b^2(4 + 3m)(7 + 3m)} + \frac{dx (a + bx^3)^{1+m} (c + dx^3)}{b(7 + 3m)} + \frac{\left( (4a^2d^2 - 2ad^2) \right)}{b^2(4 + 3m)(7 + 3m)} \\ &= -\frac{d(4ad - bc(10 + 3m))x (a + bx^3)^{1+m}}{b^2(4 + 3m)(7 + 3m)} + \frac{dx (a + bx^3)^{1+m} (c + dx^3)}{b(7 + 3m)} + \frac{(4a^2d^2 - 2ad^2)}{b^2(4 + 3m)(7 + 3m)} \end{aligned}$$

**Mathematica** [A] time = 5.04, size = 106, normalized size = 0.60

$$\frac{1}{14}x(a + bx^3)^m \left( \frac{bx^3}{a} + 1 \right)^{-m} \left( 14c^2 {}_2F_1 \left( \frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a} \right) + dx^3 \left( 7c {}_2F_1 \left( \frac{4}{3}, -m; \frac{7}{3}; -\frac{bx^3}{a} \right) + 2dx^3 {}_2F_1 \left( \frac{7}{3}, -m; \frac{10}{3}; -\frac{bx^3}{a} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3)^2,x]

[Out] (x\*(a + b\*x^3)^m\*(14\*c^2\*Hypergeometric2F1[1/3, -m, 4/3, -(b\*x^3)/a]) + d\*x^3\*(7\*c\*Hypergeometric2F1[4/3, -m, 7/3, -(b\*x^3)/a] + 2\*d\*x^3\*Hypergeometric2F1[7/3, -m, 10/3, -(b\*x^3)/a]))/(14\*(1 + (b\*x^3)/a)^m)

**fricas** [F] time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2x^6 + 2cdx^3 + c^2\right)(bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^2,x, algorithm="fricas")

[Out] integral((d^2\*x^6 + 2\*c\*d\*x^3 + c^2)\*(b\*x^3 + a)^m, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^2 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((d\*x^3 + c)^2\*(b\*x^3 + a)^m, x)

**maple** [F] time = 0.54, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^2 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c)^2,x)

[Out] `int((b*x^3+a)^m*(d*x^3+c)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)^2 (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^m*(d*x^3+c)^2,x, algorithm="maxima")`

[Out] `integrate((d*x^3 + c)^2*(b*x^3 + a)^m, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^m (dx^3 + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^m*(c + d*x^3)^2,x)`

[Out] `int((a + b*x^3)^m*(c + d*x^3)^2, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m*(d*x**3+c)**2,x)`

[Out] Timed out

### 3.140 $\int (a + bx^3)^m (c + dx^3) dx$

**Optimal.** Leaf size=93

$$\frac{dx(a + bx^3)^{m+1}}{b(3m + 4)} - \frac{x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} (ad - bc(3m + 4)) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)}{b(3m + 4)}$$

[Out] d\*x\*(b\*x^3+a)^(1+m)/b/(4+3\*m)-(a\*d-b\*c\*(4+3\*m))\*x\*(b\*x^3+a)^m\*hypergeom([1/3, -m], [4/3], -b\*x^3/a)/b/(4+3\*m)/((1+b\*x^3/a)^m)

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {388, 246, 245}

$$x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left(c - \frac{ad}{3bm + 4b}\right) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + \frac{dx(a + bx^3)^{m+1}}{b(3m + 4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m\*(c + d\*x^3), x]

[Out] (d\*x\*(a + b\*x^3)^(1 + m))/(b\*(4 + 3\*m)) + ((c - (a\*d)/(4\*b + 3\*b\*m))\*x\*(a + b\*x^3)^m\*Hypergeometric2F1[1/3, -m, 4/3, -((b\*x^3)/a)])/(1 + (b\*x^3)/a)^m

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (a + bx^3)^m (c + dx^3) dx &= \frac{dx(a + bx^3)^{1+m}}{b(4 + 3m)} - \left(-c + \frac{ad}{4b + 3bm}\right) \int (a + bx^3)^m dx \\ &= \frac{dx(a + bx^3)^{1+m}}{b(4 + 3m)} - \left(\left(-c + \frac{ad}{4b + 3bm}\right) (a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m}\right) \int \left(1 + \frac{bx^3}{a}\right)^m dx \\ &= \frac{dx(a + bx^3)^{1+m}}{b(4 + 3m)} + \left(c - \frac{ad}{4b + 3bm}\right) x(a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) \end{aligned}$$



**Mathematica [A]** time = 0.05, size = 90, normalized size = 0.97

$$\frac{x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} \left( (bc(3m + 4) - ad) {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) + d(a + bx^3) \left(\frac{bx^3}{a} + 1\right)^m \right)}{b(3m + 4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^3)^m\*(c + d\*x^3), x]

[Out] (x\*(a + b\*x^3)^m\*(d\*(a + b\*x^3)\*(1 + (b\*x^3)/a)^m + (-(a\*d) + b\*c\*(4 + 3\*m))\*Hypergeometric2F1[1/3, -m, 4/3, -((b\*x^3)/a)])/(b\*(4 + 3\*m)\*(1 + (b\*x^3)/a)^m)

**fricas [F]** time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^3 + c\right)\left(bx^3 + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c), x, algorithm="fricas")

[Out] integral((d\*x^3 + c)\*(b\*x^3 + a)^m, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)(bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c), x, algorithm="giac")

[Out] integrate((d\*x^3 + c)\*(b\*x^3 + a)^m, x)

**maple [F]** time = 0.39, size = 0, normalized size = 0.00

$$\int (dx^3 + c)(bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m\*(d\*x^3+c), x)

[Out] int((b\*x^3+a)^m\*(d\*x^3+c), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^3 + c)(bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m\*(d\*x^3+c), x, algorithm="maxima")

[Out] integrate((d\*x^3 + c)\*(b\*x^3 + a)^m, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^3 + a)^m (dx^3 + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^m\*(c + d\*x^3), x)

[Out]  $\text{int}((a + b*x^3)^m*(c + d*x^3), x)$

**sympy** [C] time = 100.31, size = 75, normalized size = 0.81

$$\frac{a^m c x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -m \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)} + \frac{a^m d x^4 \Gamma\left(\frac{4}{3}\right) {}_2F_1\left(\frac{4}{3}, -m \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{7}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**3+a)**m*(d*x**3+c),x)`

[Out] `a**m*c*x*gamma(1/3)*hyper((1/3, -m), (4/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(4/3)) + a**m*d*x**4*gamma(4/3)*hyper((4/3, -m), (7/3,), b*x**3*exp_polar(I*pi)/a)/(3*gamma(7/3))`

### 3.141 $\int (a + bx^3)^m dx$

**Optimal.** Leaf size=44

$$x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)$$

[Out]  $x*(b*x^3+a)^m*\text{hypergeom}([1/3, -m], [4/3], -b*x^3/a)/((1+b*x^3/a)^m)$

**Rubi [A]** time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {246, 245}

$$x(a + bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m, x]

[Out]  $(x*(a + b*x^3)^m*\text{Hypergeometric2F1}[1/3, -m, 4/3, -((b*x^3)/a)])/(1 + (b*x^3)/a)^m$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (a + bx^3)^m dx &= \left( (a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \right) \int \left(1 + \frac{bx^3}{a}\right)^m dx \\ &= x(a + bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right) \end{aligned}$$

**Mathematica [C]** time = 0.20, size = 196, normalized size = 4.45

$$\frac{2^{-m} \left( (-1)^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x \right) \left( \frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}} \right)^{-m} \left( \frac{i \left( \frac{\sqrt[3]{b} x}{\sqrt[3]{a}} + 1 \right)}{\sqrt{3} + 3i} \right)^{-m} (a + bx^3)^m F_1 \left( m + 1; -m, -m; m + 2; -\frac{i \left( \sqrt[3]{b} x + (-1)^{2/3} \sqrt[3]{a} \right)}{\sqrt{3} \sqrt[3]{a}} \right)}{\sqrt[3]{b} (m + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m, x]

[Out]  $((-1)^{2/3}a^{1/3} + b^{1/3}x)(a + bx^3)^m \text{AppellF1}[1 + m, -m, -m, 2 + m, ((-1)^{2/3}a^{1/3} + b^{1/3}x)/(\sqrt{3}a^{1/3}), (1 + \sqrt{3}) - ((2I)b^{1/3}x/a^{1/3})/(3I + \sqrt{3})]/(2^m b^{1/3}(1 + m)((a^{1/3} + (-1)^{2/3}b^{1/3}x)/((1 + (-1)^{1/3})a^{1/3}))^m ((I(1 + (b^{1/3}x)/a^{1/3}))/ (3I + \sqrt{3}))^m)$

**fricas** [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left((bx^3 + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^m,x, algorithm="fricas")`

[Out] `integral((b*x^3 + a)^m, x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^m,x, algorithm="giac")`

[Out] `integrate((b*x^3 + a)^m, x)`

**maple** [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^3+a)^m,x)`

[Out] `int((b*x^3+a)^m,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^3+a)^m,x, algorithm="maxima")`

[Out] `integrate((b*x^3 + a)^m, x)`

**mupad** [B] time = 1.35, size = 41, normalized size = 0.93

$$\frac{x(bx^3 + a)^m {}_2F_1\left(\frac{1}{3}, -m; \frac{4}{3}; -\frac{bx^3}{a}\right)}{\left(\frac{bx^3}{a} + 1\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^3)^m,x)`

[Out] `(x*(a + b*x^3)^m*hypergeom([1/3, -m], 4/3, -(b*x^3)/a))/((b*x^3)/a + 1)^m`

sympy [C] time = 15.56, size = 34, normalized size = 0.77

$$\frac{a^m x \Gamma\left(\frac{1}{3}\right) {}_2F_1\left(\frac{1}{3}, -m \middle| \frac{bx^3 e^{i\pi}}{a}\right)}{3\Gamma\left(\frac{4}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*3+a)\*\*m,x)

[Out] a\*\*m\*x\*gamma(1/3)\*hyper((1/3, -m), (4/3,), b\*x\*\*3\*exp\_polar(I\*pi)/a)/(3\*gamma(4/3))

$$3.142 \quad \int \frac{(a+bx^3)^m}{c+dx^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c}$$

[Out]  $x*(b*x^3+a)^m*AppellF1(1/3,-m,1,4/3,-b*x^3/a,-d*x^3/c)/c/((1+b*x^3/a)^m)$

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3), x]

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c*(1 + (b*x^3)/a)^m)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^m}{c+dx^3} dx &= \left( (a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^m}{c+dx^3} dx \\ &= \frac{x(a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c} \end{aligned}$$

**Mathematica [B]** time = 0.25, size = 162, normalized size = 2.84

$$\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c+dx^3) \left( 3x^3 \left( adF_1\left(\frac{4}{3}; -m, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - bcmF_1\left(\frac{4}{3}; 1-m, 1; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right) - 4acF_1\left(\frac{1}{3}; -m, 1; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3), x]

```
[Out] (-4*a*c*x*(a + b*x^3)^m*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/c
)])/((c + d*x^3)*(-4*a*c*AppellF1[1/3, -m, 1, 4/3, -((b*x^3)/a), -((d*x^3)/
c)] + 3*x^3*(-(b*c*m*AppellF1[4/3, 1 - m, 1, 7/3, -((b*x^3)/a), -((d*x^3)/c
)]) + a*d*AppellF1[4/3, -m, 2, 7/3, -((b*x^3)/a), -((d*x^3)/c)]))
```

**fricas** [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^m}{dx^3 + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="fricas")
```

```
[Out] integral((b*x^3 + a)^m/(d*x^3 + c), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="giac")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^3+a)^m/(d*x^3+c),x)
```

```
[Out] int((b*x^3+a)^m/(d*x^3+c),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^3+a)^m/(d*x^3+c),x, algorithm="maxima")
```

```
[Out] integrate((b*x^3 + a)^m/(d*x^3 + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^m}{dx^3 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^3)^m/(c + d*x^3),x)
```

```
[Out] int((a + b*x^3)^m/(c + d*x^3), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c),x)
```

```
[Out] Timed out
```



$$3.143 \quad \int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx$$

Optimal. Leaf size=57

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2}$$

[Out]  $x*(b*x^3+a)^m*AppellF1(1/3,-m,2,4/3,-b*x^3/a,-d*x^3/c)/c^2/((1+b*x^3/a)^m)$

Rubi [A] time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3)^2,x]

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 2, 4/3, -((b*x^3)/a), -((d*x^3)/c)]/(c^2*(1 + (b*x^3)/a)^m)$

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^3)^m}{(c+dx^3)^2} dx &= \left( (a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^m}{(c+dx^3)^2} dx \\ &= \frac{x(a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^2} \end{aligned}$$

Mathematica [B] time = 0.26, size = 162, normalized size = 2.84

$$\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c+dx^3)^2 \left(-3x^3 \left(bcmF_1\left(\frac{4}{3}; 1-m, 2; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 2adF_1\left(\frac{4}{3}; -m, 3; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) - 4acF_1\left(\frac{1}{3}; -m, 2; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3)^2,x]

[Out] (-4\*a\*c\*x\*(a + b\*x^3)^m\*AppellF1[1/3, -m, 2, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)])/((c + d\*x^3)^2\*(-4\*a\*c\*AppellF1[1/3, -m, 2, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 3\*x^3\*(b\*c\*m\*AppellF1[4/3, 1 - m, 2, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 2\*a\*d\*AppellF1[4/3, -m, 3, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F] time = 1.20, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^m}{d^2x^6 + 2cdx^3 + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^2,x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m/(d^2\*x^6 + 2\*c\*d\*x^3 + c^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x)

**maple** [F] time = 0.52, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m/(d\*x^3+c)^2,x)

[Out] int((b\*x^3+a)^m/(d\*x^3+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^m/(c + d\*x^3)^2,x)

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

$$3.144 \quad \int \frac{(a+bx^3)^m}{(c+dx^3)^3} dx$$

**Optimal.** Leaf size=57

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3}$$

[Out]  $x*(b*x^3+a)^m*AppellF1(1/3, -m, 3, 4/3, -b*x^3/a, -d*x^3/c)/c^3/((1+b*x^3/a)^m)$

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^3)^m \left(\frac{bx^3}{a} + 1\right)^{-m} F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^3)^m/(c + d\*x^3)^3,x]

[Out]  $(x*(a + b*x^3)^m*AppellF1[1/3, -m, 3, 4/3, -((b*x^3)/a), -((d*x^3)/c)])/(c^3*(1 + (b*x^3)/a)^m)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^3)^m}{(c+dx^3)^3} dx &= \left( (a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} \right) \int \frac{\left(1 + \frac{bx^3}{a}\right)^m}{(c+dx^3)^3} dx \\ &= \frac{x(a+bx^3)^m \left(1 + \frac{bx^3}{a}\right)^{-m} F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{c^3} \end{aligned}$$

**Mathematica [B]** time = 0.41, size = 162, normalized size = 2.84

$$\frac{4acx(a+bx^3)^m F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}{(c+dx^3)^3 \left(-3x^3 \left(bcm F_1\left(\frac{4}{3}; 1-m, 3; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right) - 3ad F_1\left(\frac{4}{3}; -m, 4; \frac{7}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)\right) - 4ac F_1\left(\frac{1}{3}; -m, 3; \frac{4}{3}; -\frac{bx^3}{a}, -\frac{dx^3}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^3)^m/(c + d\*x^3)^3,x]

[Out] (-4\*a\*c\*x\*(a + b\*x^3)^m\*AppellF1[1/3, -m, 3, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)])/((c + d\*x^3)^3\*(-4\*a\*c\*AppellF1[1/3, -m, 3, 4/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 3\*x^3\*(b\*c\*m\*AppellF1[4/3, 1 - m, 3, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)] - 3\*a\*d\*AppellF1[4/3, -m, 4, 7/3, -((b\*x^3)/a), -((d\*x^3)/c)]))

**fricas** [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^3 + a)^m}{d^3x^9 + 3cd^2x^6 + 3c^2dx^3 + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^3,x, algorithm="fricas")

[Out] integral((b\*x^3 + a)^m/(d^3\*x^9 + 3\*c\*d^2\*x^6 + 3\*c^2\*d\*x^3 + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^3,x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3, x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^m/(d\*x^3+c)^3,x)

[Out] int((b\*x^3+a)^m/(d\*x^3+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^m/(d\*x^3+c)^3,x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^m/(d\*x^3 + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(bx^3 + a)^m}{(dx^3 + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^m/(c + d\*x^3)^3,x)

```
[Out] int((a + b*x^3)^m/(c + d*x^3)^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**m/(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

$$3.145 \quad \int (a + bx^3)^{-1 - \frac{bc}{3bc-3ad}} (c + dx^3)^{-1 + \frac{ad}{3bc-3ad}} dx$$

**Optimal.** Leaf size=53

$$\frac{x(a + bx^3)^{-\frac{bc}{3bc-3ad}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

[Out]  $x*(d*x^3+c)^{(a*d/(-3*a*d+3*b*c))}/a/c/((b*x^3+a)^{(b*c/(-3*a*d+3*b*c))})$

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 50,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.020$ , Rules used = {381}

$$\frac{x(a + bx^3)^{-\frac{bc}{3bc-3ad}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x^3)^{-1 - (b*c)/(3*b*c - 3*a*d)}*(c + d*x^3)^{-1 + (a*d)/(3*b*c - 3*a*d)}, x]$

[Out]  $(x*(c + d*x^3)^{((a*d)/(3*b*c - 3*a*d))}/(a*c*(a + b*x^3)^{((b*c)/(3*b*c - 3*a*d))})$

**Rule 381**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:\> \text{Simp}[(x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)})/(a*c), x] /;$  FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && EqQ[a\*d\*(p + 1) + b\*c\*(q + 1), 0]

**Rubi steps**

$$\int (a + bx^3)^{-1 - \frac{bc}{3bc-3ad}} (c + dx^3)^{-1 + \frac{ad}{3bc-3ad}} dx = \frac{x(a + bx^3)^{-\frac{bc}{3bc-3ad}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

**Mathematica [A]** time = 0.04, size = 52, normalized size = 0.98

$$\frac{x(a + bx^3)^{\frac{bc}{3ad-3bc}} (c + dx^3)^{\frac{ad}{3bc-3ad}}}{ac}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a + b*x^3)^{-1 - (b*c)/(3*b*c - 3*a*d)}*(c + d*x^3)^{-1 + (a*d)/(3*b*c - 3*a*d)}, x]$

[Out]  $(x*(a + b*x^3)^{((b*c)/(-3*b*c + 3*a*d))}*(c + d*x^3)^{((a*d)/(3*b*c - 3*a*d))}/(a*c)$

**fricas [A]** time = 1.32, size = 91, normalized size = 1.72

$$\frac{bdx^7 + (bc + ad)x^4 + acx}{(bx^3 + a)^{\frac{4bc-3ad}{3(bc-ad)}} (dx^3 + c)^{\frac{3bc-4ad}{3(bc-ad)}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x^3+c)^(-1+a\*d/(-3\*a\*d+3\*b\*c)),x, algorithm="fricas")

[Out] (b\*d\*x^7 + (b\*c + a\*d)\*x^4 + a\*c\*x)/((b\*x^3 + a)^(1/3\*(4\*b\*c - 3\*a\*d)/(b\*c - a\*d))\*(d\*x^3 + c)^(1/3\*(3\*b\*c - 4\*a\*d)/(b\*c - a\*d))\*a\*c)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{-\frac{bc}{3(bc-ad)}-1} (dx^3 + c)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x^3+c)^(-1+a\*d/(-3\*a\*d+3\*b\*c)),x, algorithm="giac")

[Out] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d) - 1), x)

**maple** [A] time = 0.04, size = 71, normalized size = 1.34

$$\frac{x (bx^3 + a)^{1-\frac{3ad-4bc}{3(ad-bc)}} (dx^3 + c)^{1-\frac{4ad-3bc}{3(ad-bc)}}}{ac}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^3+a)^(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x^3+c)^(-1+a\*d/(-3\*a\*d+3\*b\*c)),x)

[Out] (b\*x^3+a)^(1-1/3\*(3\*a\*d-4\*b\*c)/(a\*d-b\*c))\*(d\*x^3+c)^(1-1/3\*(4\*a\*d-3\*b\*c)/(a\*d-b\*c))/a/c\*x

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^3 + a)^{-\frac{bc}{3(bc-ad)}-1} (dx^3 + c)^{\frac{ad}{3(bc-ad)}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^3+a)^(-1-b\*c/(-3\*a\*d+3\*b\*c))\*(d\*x^3+c)^(-1+a\*d/(-3\*a\*d+3\*b\*c)),x, algorithm="maxima")

[Out] integrate((b\*x^3 + a)^(-1/3\*b\*c/(b\*c - a\*d) - 1)\*(d\*x^3 + c)^(1/3\*a\*d/(b\*c - a\*d) - 1), x)

**mupad** [B] time = 1.90, size = 131, normalized size = 2.47

$$\frac{x (bx^3 + a)^{\frac{bc}{3ad-3bc}-1} + \frac{x^4 (bx^3+a)^{\frac{bc}{3ad-3bc}-1} (ad+bc)}{ac} + \frac{bdx^7 (bx^3+a)^{\frac{bc}{3ad-3bc}-1}}{ac}}{(dx^3 + c)^{\frac{ad}{3ad-3bc}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^3)^((b\*c)/(3\*a\*d - 3\*b\*c) - 1)/(c + d\*x^3)^((a\*d)/(3\*a\*d - 3\*b\*c) + 1),x)

[Out] (x\*(a + b\*x^3)^((b\*c)/(3\*a\*d - 3\*b\*c) - 1) + (x^4\*(a + b\*x^3)^((b\*c)/(3\*a\*d - 3\*b\*c) - 1)\*(a\*d + b\*c))/(a\*c) + (b\*d\*x^7\*(a + b\*x^3)^((b\*c)/(3\*a\*d - 3\*b\*c) - 1))/(a\*c)/(c + d\*x^3)^((a\*d)/(3\*a\*d - 3\*b\*c) + 1)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**3+a)**(-1-b*c/(-3*a*d+3*b*c))*(d*x**3+c)**(-1+a*d/(-3*a*d+3*b*c)),x)
```

```
[Out] Timed out
```

$$3.146 \quad \int (a + bx^4)(c + dx^4)^4 dx$$

**Optimal.** Leaf size=94

$$\frac{1}{5}c^3x^5(4ad+bc) + \frac{2}{9}c^2dx^9(3ad+2bc) + \frac{1}{17}d^3x^{17}(ad+4bc) + \frac{2}{13}cd^2x^{13}(2ad+3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

[Out] a\*c^4\*x+1/5\*c^3\*(4\*a\*d+b\*c)\*x^5+2/9\*c^2\*d\*(3\*a\*d+2\*b\*c)\*x^9+2/13\*c\*d^2\*(2\*a\*d+3\*b\*c)\*x^13+1/17\*d^3\*(a\*d+4\*b\*c)\*x^17+1/21\*b\*d^4\*x^21

**Rubi [A]** time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{2}{9}c^2dx^9(3ad+2bc) + \frac{1}{5}c^3x^5(4ad+bc) + \frac{1}{17}d^3x^{17}(ad+4bc) + \frac{2}{13}cd^2x^{13}(2ad+3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^4,x]

[Out] a\*c^4\*x + (c^3\*(b\*c + 4\*a\*d)\*x^5)/5 + (2\*c^2\*d\*(2\*b\*c + 3\*a\*d)\*x^9)/9 + (2\*c\*d^2\*(3\*b\*c + 2\*a\*d)\*x^13)/13 + (d^3\*(4\*b\*c + a\*d)\*x^17)/17 + (b\*d^4\*x^21)/21

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)(c + dx^4)^4 dx &= \int (ac^4 + c^3(bc + 4ad)x^4 + 2c^2d(2bc + 3ad)x^8 + 2cd^2(3bc + 2ad)x^{12} + d^3(4bc + ad)x^{16} + bd^4x^{20}) dx \\ &= ac^4x + \frac{1}{5}c^3(bc + 4ad)x^5 + \frac{2}{9}c^2d(2bc + 3ad)x^9 + \frac{2}{13}cd^2(3bc + 2ad)x^{13} + \frac{1}{17}d^3(4bc + ad)x^{17} + \frac{1}{21}bd^4x^{21} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 94, normalized size = 1.00

$$\frac{1}{5}c^3x^5(4ad+bc) + \frac{2}{9}c^2dx^9(3ad+2bc) + \frac{1}{17}d^3x^{17}(ad+4bc) + \frac{2}{13}cd^2x^{13}(2ad+3bc) + ac^4x + \frac{1}{21}bd^4x^{21}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^4,x]

[Out] a\*c^4\*x + (c^3\*(b\*c + 4\*a\*d)\*x^5)/5 + (2\*c^2\*d\*(2\*b\*c + 3\*a\*d)\*x^9)/9 + (2\*c\*d^2\*(3\*b\*c + 2\*a\*d)\*x^13)/13 + (d^3\*(4\*b\*c + a\*d)\*x^17)/17 + (b\*d^4\*x^21)/21

**fricas [A]** time = 0.90, size = 98, normalized size = 1.04

$$\frac{1}{21}x^{21}d^4b + \frac{4}{17}x^{17}d^3cb + \frac{1}{17}x^{17}d^4a + \frac{6}{13}x^{13}d^2c^2b + \frac{4}{13}x^{13}d^3ca + \frac{4}{9}x^9dc^3b + \frac{2}{3}x^9d^2c^2a + \frac{1}{5}x^5c^4b + \frac{4}{5}x^5dc^3a + xc^4a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^4,x, algorithm="fricas")

[Out]  $1/21*x^{21}*d^4*b + 4/17*x^{17}*d^3*c*b + 1/17*x^{17}*d^4*a + 6/13*x^{13}*d^2*c^2*b + 4/13*x^{13}*d^3*c*a + 4/9*x^9*d*c^3*b + 2/3*x^9*d^2*c^2*a + 1/5*x^5*c^4*b + 4/5*x^5*d*c^3*a + x*c^4*a$

**giac** [A] time = 0.16, size = 98, normalized size = 1.04

$$\frac{1}{21}bd^4x^{21} + \frac{4}{17}bcd^3x^{17} + \frac{1}{17}ad^4x^{17} + \frac{6}{13}bc^2d^2x^{13} + \frac{4}{13}acd^3x^{13} + \frac{4}{9}bc^3dx^9 + \frac{2}{3}ac^2d^2x^9 + \frac{1}{5}bc^4x^5 + \frac{4}{5}ac^3dx^5 + ac^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^4,x, algorithm="giac")

[Out]  $1/21*b*d^4*x^{21} + 4/17*b*c*d^3*x^{17} + 1/17*a*d^4*x^{17} + 6/13*b*c^2*d^2*x^{13} + 4/13*a*c*d^3*x^{13} + 4/9*b*c^3*d*x^9 + 2/3*a*c^2*d^2*x^9 + 1/5*b*c^4*x^5 + 4/5*a*c^3*d*x^5 + a*c^4*x$

**maple** [A] time = 0.04, size = 97, normalized size = 1.03

$$\frac{bd^4x^{21}}{21} + \frac{(ad^4 + 4bcd^3)x^{17}}{17} + \frac{(4acd^3 + 6c^2d^2b)x^{13}}{13} + \frac{(6ac^2d^2 + 4c^3db)x^9}{9} + ac^4x + \frac{(4ac^3d + bc^4)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^4,x)

[Out]  $1/21*b*d^4*x^{21} + 1/17*(a*d^4 + 4*b*c*d^3)*x^{17} + 1/13*(4*a*c*d^3 + 6*b*c^2*d^2)*x^{13} + 1/9*(6*a*c^2*d^2 + 4*b*c^3*d)*x^9 + 1/5*(4*a*c^3*d + b*c^4)*x^5 + a*c^4*x$

**maxima** [A] time = 0.65, size = 96, normalized size = 1.02

$$\frac{1}{21}bd^4x^{21} + \frac{1}{17}(4bcd^3 + ad^4)x^{17} + \frac{2}{13}(3bc^2d^2 + 2acd^3)x^{13} + \frac{2}{9}(2bc^3d + 3ac^2d^2)x^9 + ac^4x + \frac{1}{5}(bc^4 + 4ac^3d)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^4,x, algorithm="maxima")

[Out]  $1/21*b*d^4*x^{21} + 1/17*(4*b*c*d^3 + a*d^4)*x^{17} + 2/13*(3*b*c^2*d^2 + 2*a*c*d^3)*x^{13} + 2/9*(2*b*c^3*d + 3*a*c^2*d^2)*x^9 + a*c^4*x + 1/5*(b*c^4 + 4*a*c^3*d)*x^5$

**mupad** [B] time = 1.30, size = 88, normalized size = 0.94

$$x^5 \left( \frac{bc^4}{5} + \frac{4ad^3c}{5} \right) + x^{17} \left( \frac{ad^4}{17} + \frac{4bcd^3}{17} \right) + \frac{bd^4x^{21}}{21} + ac^4x + \frac{2c^2dx^9(3ad + 2bc)}{9} + \frac{2cd^2x^{13}(2ad + 3bc)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)\*(c + d\*x^4)^4,x)

[Out]  $x^5*((b*c^4)/5 + (4*a*c^3*d)/5) + x^{17}*((a*d^4)/17 + (4*b*c*d^3)/17) + (b*d^4*x^{21})/21 + a*c^4*x + (2*c^2*d*x^9*(3*a*d + 2*b*c))/9 + (2*c*d^2*x^{13}*(2*a*d + 3*b*c))/13$

**sympy** [A] time = 0.09, size = 107, normalized size = 1.14

$$ac^4x + \frac{bd^4x^{21}}{21} + x^{17} \left( \frac{ad^4}{17} + \frac{4bcd^3}{17} \right) + x^{13} \left( \frac{4acd^3}{13} + \frac{6bc^2d^2}{13} \right) + x^9 \left( \frac{2ac^2d^2}{3} + \frac{4bc^3d}{9} \right) + x^5 \left( \frac{4ac^3d}{5} + \frac{bc^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*4,x)

[Out]  $a*c**4*x + b*d**4*x**21/21 + x**17*(a*d**4/17 + 4*b*c*d**3/17) + x**13*(4*a*c*d**3/13 + 6*b*c**2*d**2/13) + x**9*(2*a*c**2*d**2/3 + 4*b*c**3*d/9) + x**5*(4*a*c**3*d/5 + b*c**4/5)$

$$3.147 \quad \int (a + bx^4)(c + dx^4)^3 dx$$

**Optimal.** Leaf size=70

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

[Out]  $a*c^3*x+1/5*c^2*(3*a*d+b*c)*x^5+1/3*c*d*(a*d+b*c)*x^9+1/13*d^2*(a*d+3*b*c)*x^{13}+1/17*b*d^3*x^{17}$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^3,x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c + a*d)*x^{13})/13 + (b*d^3*x^{17})/17$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)(c + dx^4)^3 dx &= \int (ac^3 + c^2(bc + 3ad)x^4 + 3cd(bc + ad)x^8 + d^2(3bc + ad)x^{12} + bd^3x^{16}) dx \\ &= ac^3x + \frac{1}{5}c^2(bc + 3ad)x^5 + \frac{1}{3}cd(bc + ad)x^9 + \frac{1}{13}d^2(3bc + ad)x^{13} + \frac{1}{17}bd^3x^{17} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 70, normalized size = 1.00

$$\frac{1}{5}c^2x^5(3ad + bc) + \frac{1}{13}d^2x^{13}(ad + 3bc) + \frac{1}{3}cdx^9(ad + bc) + ac^3x + \frac{1}{17}bd^3x^{17}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^3,x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^5)/5 + (c*d*(b*c + a*d)*x^9)/3 + (d^2*(3*b*c + a*d)*x^{13})/13 + (b*d^3*x^{17})/17$

**fricas [A]** time = 0.96, size = 74, normalized size = 1.06

$$\frac{1}{17}x^{17}d^3b + \frac{3}{13}x^{13}d^2cb + \frac{1}{13}x^{13}d^3a + \frac{1}{3}x^9dc^2b + \frac{1}{3}x^9d^2ca + \frac{1}{5}x^5c^3b + \frac{3}{5}x^5dc^2a + xc^3a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^3,x, algorithm="fricas")

[Out]  $1/17*x^{17}*d^3*b + 3/13*x^{13}*d^2*c*b + 1/13*x^{13}*d^3*a + 1/3*x^9*d*c^2*b + 1/3*x^9*d^2*c*a + 1/5*x^5*c^3*b + 3/5*x^5*d*c^2*a + x*c^3*a$

**giac** [A] time = 0.15, size = 74, normalized size = 1.06

$$\frac{1}{17}bd^3x^{17} + \frac{3}{13}bcd^2x^{13} + \frac{1}{13}ad^3x^{13} + \frac{1}{3}bc^2dx^9 + \frac{1}{3}acd^2x^9 + \frac{1}{5}bc^3x^5 + \frac{3}{5}ac^2dx^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^3,x, algorithm="giac")

[Out] 1/17\*b\*d^3\*x^17 + 3/13\*b\*c\*d^2\*x^13 + 1/13\*a\*d^3\*x^13 + 1/3\*b\*c^2\*d\*x^9 + 1/3\*a\*c\*d^2\*x^9 + 1/5\*b\*c^3\*x^5 + 3/5\*a\*c^2\*d\*x^5 + a\*c^3\*x

**maple** [A] time = 0.04, size = 73, normalized size = 1.04

$$\frac{bd^3x^{17}}{17} + \frac{(ad^3 + 3bcd^2)x^{13}}{13} + \frac{(3acd^2 + 3bc^2d)x^9}{9} + ac^3x + \frac{(3ac^2d + bc^3)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^3,x)

[Out] 1/17\*b\*d^3\*x^17+1/13\*(a\*d^3+3\*b\*c\*d^2)\*x^13+1/9\*(3\*a\*c\*d^2+3\*b\*c^2\*d)\*x^9+1/5\*(3\*a\*c^2\*d+b\*c^3)\*x^5+a\*c^3\*x

**maxima** [A] time = 0.65, size = 70, normalized size = 1.00

$$\frac{1}{17}bd^3x^{17} + \frac{1}{13}(3bcd^2 + ad^3)x^{13} + \frac{1}{3}(bc^2d + acd^2)x^9 + \frac{1}{5}(bc^3 + 3ac^2d)x^5 + ac^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^3,x, algorithm="maxima")

[Out] 1/17\*b\*d^3\*x^17 + 1/13\*(3\*b\*c\*d^2 + a\*d^3)\*x^13 + 1/3\*(b\*c^2\*d + a\*c\*d^2)\*x^9 + 1/5\*(b\*c^3 + 3\*a\*c^2\*d)\*x^5 + a\*c^3\*x

**mupad** [B] time = 1.24, size = 66, normalized size = 0.94

$$x^5 \left( \frac{bc^3}{5} + \frac{3adc^2}{5} \right) + x^{13} \left( \frac{ad^3}{13} + \frac{3bcd^2}{13} \right) + \frac{bd^3x^{17}}{17} + ac^3x + \frac{cdx^9(ad+bc)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)\*(c + d\*x^4)^3,x)

[Out] x^5\*((b\*c^3)/5 + (3\*a\*c^2\*d)/5) + x^13\*((a\*d^3)/13 + (3\*b\*c\*d^2)/13) + (b\*d^3\*x^17)/17 + a\*c^3\*x + (c\*d\*x^9\*(a\*d + b\*c))/3

**sympy** [A] time = 0.08, size = 76, normalized size = 1.09

$$ac^3x + \frac{bd^3x^{17}}{17} + x^{13} \left( \frac{ad^3}{13} + \frac{3bcd^2}{13} \right) + x^9 \left( \frac{acd^2}{3} + \frac{bc^2d}{3} \right) + x^5 \left( \frac{3ac^2d}{5} + \frac{bc^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*3,x)

[Out] a\*c\*\*3\*x + b\*d\*\*3\*x\*\*17/17 + x\*\*13\*(a\*d\*\*3/13 + 3\*b\*c\*d\*\*2/13) + x\*\*9\*(a\*c\*d\*\*2/3 + b\*c\*\*2\*d/3) + x\*\*5\*(3\*a\*c\*\*2\*d/5 + b\*c\*\*3/5)

### 3.148 $\int (a + bx^4)(c + dx^4)^2 dx$

**Optimal.** Leaf size=50

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

[Out]  $a*c^2*x + 1/5*c*(2*a*d+b*c)*x^5 + 1/9*d*(a*d+2*b*c)*x^9 + 1/13*b*d^2*x^{13}$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^2,x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^{13})/13$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)(c + dx^4)^2 dx &= \int (ac^2 + c(bc + 2ad)x^4 + d(2bc + ad)x^8 + bd^2x^{12}) dx \\ &= ac^2x + \frac{1}{5}c(bc + 2ad)x^5 + \frac{1}{9}d(2bc + ad)x^9 + \frac{1}{13}bd^2x^{13} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 1.00

$$\frac{1}{9}dx^9(ad + 2bc) + \frac{1}{5}cx^5(2ad + bc) + ac^2x + \frac{1}{13}bd^2x^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^2,x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^5)/5 + (d*(2*b*c + a*d)*x^9)/9 + (b*d^2*x^{13})/13$

**fricas [A]** time = 0.77, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}d^2b + \frac{2}{9}x^9dcb + \frac{1}{9}x^9d^2a + \frac{1}{5}x^5c^2b + \frac{2}{5}x^5dca + xc^2a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^2,x, algorithm="fricas")

[Out]  $1/13*x^{13}*d^2*b + 2/9*x^9*d*c*b + 1/9*x^9*d^2*a + 1/5*x^5*c^2*b + 2/5*x^5*d*c*a + x*c^2*a$

**giac** [A] time = 0.16, size = 50, normalized size = 1.00

$$\frac{1}{13}bd^2x^{13} + \frac{2}{9}bcdx^9 + \frac{1}{9}ad^2x^9 + \frac{1}{5}bc^2x^5 + \frac{2}{5}acdx^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^2,x, algorithm="giac")

[Out] 1/13\*b\*d^2\*x^13 + 2/9\*b\*c\*d\*x^9 + 1/9\*a\*d^2\*x^9 + 1/5\*b\*c^2\*x^5 + 2/5\*a\*c\*d\*x^5 + a\*c^2\*x

**maple** [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{bd^2x^{13}}{13} + \frac{(ad^2 + 2bcd)x^9}{9} + \frac{(2acd + bc^2)x^5}{5} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^2,x)

[Out] 1/13\*b\*d^2\*x^13+1/9\*(a\*d^2+2\*b\*c\*d)\*x^9+1/5\*(2\*a\*c\*d+b\*c^2)\*x^5+a\*c^2\*x

**maxima** [A] time = 0.60, size = 48, normalized size = 0.96

$$\frac{1}{13}bd^2x^{13} + \frac{1}{9}(2bcd + ad^2)x^9 + \frac{1}{5}(bc^2 + 2acd)x^5 + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^2,x, algorithm="maxima")

[Out] 1/13\*b\*d^2\*x^13 + 1/9\*(2\*b\*c\*d + a\*d^2)\*x^9 + 1/5\*(b\*c^2 + 2\*a\*c\*d)\*x^5 + a\*c^2\*x

**mupad** [B] time = 0.05, size = 48, normalized size = 0.96

$$x^5 \left( \frac{bc^2}{5} + \frac{2adc}{5} \right) + x^9 \left( \frac{ad^2}{9} + \frac{2bcd}{9} \right) + \frac{bd^2x^{13}}{13} + ac^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)\*(c + d\*x^4)^2,x)

[Out] x^5\*((b\*c^2)/5 + (2\*a\*c\*d)/5) + x^9\*((a\*d^2)/9 + (2\*b\*c\*d)/9) + (b\*d^2\*x^13)/13 + a\*c^2\*x

**sympy** [A] time = 0.08, size = 53, normalized size = 1.06

$$ac^2x + \frac{bd^2x^{13}}{13} + x^9 \left( \frac{ad^2}{9} + \frac{2bcd}{9} \right) + x^5 \left( \frac{2acd}{5} + \frac{bc^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c)\*\*2,x)

[Out] a\*c\*\*2\*x + b\*d\*\*2\*x\*\*13/13 + x\*\*9\*(a\*d\*\*2/9 + 2\*b\*c\*d/9) + x\*\*5\*(2\*a\*c\*d/5 + b\*c\*\*2/5)

### 3.149 $\int (a + bx^4)(c + dx^4) dx$

**Optimal.** Leaf size=28

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

[Out] a\*c\*x+1/5\*(a\*d+b\*c)\*x^5+1/9\*b\*d\*x^9

**Rubi [A]** time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {373}

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4), x]

[Out] a\*c\*x + ((b\*c + a\*d)\*x^5)/5 + (b\*d\*x^9)/9

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)(c + dx^4) dx &= \int (ac + (bc + ad)x^4 + bdx^8) dx \\ &= acx + \frac{1}{5}(bc + ad)x^5 + \frac{1}{9}bdx^9 \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 28, normalized size = 1.00

$$\frac{1}{5}x^5(ad + bc) + acx + \frac{1}{9}bdx^9$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4), x]

[Out] a\*c\*x + ((b\*c + a\*d)\*x^5)/5 + (b\*d\*x^9)/9

**fricas [A]** time = 0.84, size = 26, normalized size = 0.93

$$\frac{1}{9}x^9db + \frac{1}{5}x^5cb + \frac{1}{5}x^5da + xca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c), x, algorithm="fricas")

[Out] 1/9\*x^9\*d\*b + 1/5\*x^5\*c\*b + 1/5\*x^5\*d\*a + x\*c\*a

**giac [A]** time = 0.15, size = 26, normalized size = 0.93

$$\frac{1}{9}bdx^9 + \frac{1}{5}bcx^5 + \frac{1}{5}adx^5 + acx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c),x, algorithm="giac")

[Out] 1/9\*b\*d\*x^9 + 1/5\*b\*c\*x^5 + 1/5\*a\*d\*x^5 + a\*c\*x

**maple** [A] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bdx^9}{9} + \frac{(ad+bc)x^5}{5} + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c),x)

[Out] a\*c\*x+1/5\*(a\*d+b\*c)\*x^5+1/9\*b\*d\*x^9

**maxima** [A] time = 0.49, size = 24, normalized size = 0.86

$$\frac{1}{9}bdx^9 + \frac{1}{5}(bc+ad)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c),x, algorithm="maxima")

[Out] 1/9\*b\*d\*x^9 + 1/5\*(b\*c + a\*d)\*x^5 + a\*c\*x

**mupad** [B] time = 0.04, size = 25, normalized size = 0.89

$$\frac{bdx^9}{9} + \left(\frac{ad}{5} + \frac{bc}{5}\right)x^5 + acx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)\*(c + d\*x^4),x)

[Out] x^5\*((a\*d)/5 + (b\*c)/5) + a\*c\*x + (b\*d\*x^9)/9

**sympy** [A] time = 0.06, size = 26, normalized size = 0.93

$$acx + \frac{bdx^9}{9} + x^5\left(\frac{ad}{5} + \frac{bc}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*(d\*x\*\*4+c),x)

[Out] a\*c\*x + b\*d\*x\*\*9/9 + x\*\*5\*(a\*d/5 + b\*c/5)

$$3.150 \quad \int \frac{a+bx^4}{c+dx^4} dx$$

**Optimal.** Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{4}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}$$

[Out]  $b*x/d-1/4*(-a*d+b*c)*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(3/4)}/d^{(5/4)*2^{(1/2)}}-1/4*(-a*d+b*c)*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(3/4)}/d^{(5/4)*2^{(1/2)}}+1/8*(-a*d+b*c)*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(3/4)}/d^{(5/4)*2^{(1/2)}}-1/8*(-a*d+b*c)*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(3/4)}/d^{(5/4)*2^{(1/2)}}$

**Rubi [A]** time = 0.15, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2\right)}{4\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}}{4}\right)}{2\sqrt{2}c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4), x]

[Out]  $(b*x)/d + ((b*c - a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*\text{Sqrt}[2]*c^{(3/4)}*d^{(5/4)}) - ((b*c - a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*\text{Sqrt}[2]*c^{(3/4)}*d^{(5/4)}) + ((b*c - a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(4*\text{Sqrt}[2]*c^{(3/4)}*d^{(5/4)}) - ((b*c - a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(4*\text{Sqrt}[2]*c^{(3/4)}*d^{(5/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[Rt[-b, 2]\*x]/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{a + bx^4}{c + dx^4} dx &= \frac{bx}{d} - \frac{(bc - ad) \int \frac{1}{c + dx^4} dx}{d} \\ &= \frac{bx}{d} - \frac{(bc - ad) \int \frac{\sqrt{c} - \sqrt{d}x^2}{c + dx^4} dx}{2\sqrt{c}d} - \frac{(bc - ad) \int \frac{\sqrt{c} + \sqrt{d}x^2}{c + dx^4} dx}{2\sqrt{c}d} \\ &= \frac{bx}{d} - \frac{(bc - ad) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{3/2}} - \frac{(bc - ad) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{3/2}} + \frac{(bc - ad) \int \frac{\frac{\sqrt{2}\sqrt[4]{c}}{\sqrt[4]{d}} + 2x}{-\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} - x^2} dx}{4\sqrt{2}c^{3/4}d^{5/4}} \\ &= \frac{bx}{d} + \frac{(bc - ad) \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc - ad) \log(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2}c^{3/4}d^{5/4}} \\ &= \frac{bx}{d} + \frac{(bc - ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} - \frac{(bc - ad) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{5/4}} + \frac{(bc - ad) \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2}c^{3/4}d^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.18, size = 196, normalized size = 0.88

$$\frac{\sqrt{2}(bc - ad) \log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) - \sqrt{2}(bc - ad) \log(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) + 2\sqrt{2}(bc - ad) \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{8c^{3/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)/(c + d\*x^4), x]

[Out] (8\*b\*c^(3/4)\*d^(1/4)\*x + 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] - Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2]/(8\*c^(3/4)\*d^(5/4))

**fricas [B]** time = 1.32, size = 639, normalized size = 2.87

$$4d \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5} \right)^{\frac{1}{4}} \arctan \left( \frac{c^2d^4x \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5} \right)^{\frac{3}{4}} - c^2d^4 \sqrt{\frac{c^2d^2 \sqrt{-\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{c^3d^5}}}{b^2}}}{b^3c^3 - 3ab^2c^2d + \dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c),x, algorithm="fricas")

[Out]  $\frac{1}{4} * (4*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(1/4)} * \arctan((c^2*d^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(3/4)} - c^2*d^4*\sqrt{(c^2*d^2*\sqrt{-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5)} + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(3/4)})/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)) + d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(1/4)} * \log(c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(1/4)} - (b*c - a*d)*x) - d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(1/4)} * \log(-c*d*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(c^3*d^5))^{(1/4)} - (b*c - a*d)*x) + 4*b*x)/d$

**giac [A]** time = 0.16, size = 245, normalized size = 1.10

$$\frac{bx}{d} \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4cd^2} - \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc - (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4cd^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c),x, algorithm="giac")

[Out]  $b*x/d - 1/4*\sqrt{2}*((c*d^3)^{(1/4)}*b*c - (c*d^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(c*d^2) - 1/4*\sqrt{2}*((c*d^3)^{(1/4)}*b*c - (c*d^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(c*d^2) - 1/8*\sqrt{2}*((c*d^3)^{(1/4)}*b*c - (c*d^3)^{(1/4)}*a*d)*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{2}*(c/d))/(c*d^2) + 1/8*\sqrt{2}*((c*d^3)^{(1/4)}*b*c - (c*d^3)^{(1/4)}*a*d)*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{2}*(c/d))/(c*d^2)$

**maple [A]** time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1 \right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1 \right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} a \ln \left( \frac{x^2 + \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}} \right)}{8c} + \frac{bx}{d} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)/(d\*x^4+c),x)

[Out]  $b/d*x + 1/4*(c/d)^{(1/4)}/c^2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*a-1/4/d*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*b+1/8*(c/d)^{(1/4)}/c^2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*x*2^{(1/2)}+(c/d)^{(1/2)))/(x^2-(c/d)^{(1/4)}*x*2^{(1/2)}+(c/$

$$d^{(1/2)}) * a - 1/8/d * (c/d)^{(1/4)} * 2^{(1/2)} * \ln((x^2 + (c/d)^{(1/4)} * x * 2^{(1/2)} + (c/d)^{(1/2)}) / (x^2 - (c/d)^{(1/4)} * x * 2^{(1/2)} + (c/d)^{(1/2)})) * b + 1/4 * (c/d)^{(1/4)} / c * 2^{(1/2)} * \arctan(2^{(1/2)} / (c/d)^{(1/4)} * x + 1) * a - 1/4/d * (c/d)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (c/d)^{(1/4)} * x + 1) * b$$

**maxima** [A] time = 1.28, size = 212, normalized size = 0.95

$$\frac{bx}{d} - \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{d}x^2+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}x+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c),x, algorithm="maxima")

[Out]  $b*x/d - 1/8*(2*\sqrt{2}*(b*c - a*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x + \sqrt{2} * c^{(1/4)}*d^{(1/4)})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}})) + 2*\sqrt{2}*(b*c - a*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x - \sqrt{2} * c^{(1/4)}*d^{(1/4)})/\sqrt{\sqrt{c}*\sqrt{d}})/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}})) + \sqrt{2}*(b*c - a*d)*\log(\sqrt{d}*x^2 + \sqrt{2} * c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/(c^{(3/4)}*d^{(1/4)}) - \sqrt{2}*(b*c - a*d)*\log(\sqrt{d}*x^2 - \sqrt{2} * c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/(c^{(3/4)}*d^{(1/4)})/d$

**mupad** [B] time = 1.48, size = 720, normalized size = 3.23

$$\frac{bx}{d} - \frac{\operatorname{atan}\left(\frac{\left(x(4a^2d^3-8abcd^2+4b^2c^2d)-\frac{(16b^2d^2-16acd^3)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)1i}{4(-c)^{3/4}d^{5/4}} + \frac{\left(x(4a^2d^3-8abcd^2+4b^2c^2d)+\frac{(16b^2d^2-16acd^3)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)1i}{4(-c)^{3/4}d^{5/4}}}{\frac{\left(x(4a^2d^3-8abcd^2+4b^2c^2d)-\frac{(16b^2d^2-16acd^3)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}} - \frac{\left(x(4a^2d^3-8abcd^2+4b^2c^2d)+\frac{(16b^2d^2-16acd^3)(ad-bc)}{4(-c)^{3/4}d^{5/4}}\right)(ad-bc)}{4(-c)^{3/4}d^{5/4}}}\right)}{2(-c)^{3/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)/(c + d\*x^4),x)

[Out]  $(b*x)/d - \left(\operatorname{atan}\left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)-\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) + \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)+\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) - \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)-\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(2(-c)^{3/4}d^{5/4}\right) - \operatorname{atan}\left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)-\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) + \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)+\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) - \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)-\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) + \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)+\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(4(-c)^{3/4}d^{5/4}\right) - \left(\frac{\left(x(4a^2d^3+4b^2c^2d-8a*b*c*d^2)-\left(\frac{16b^2c^2d^2-16a*c*d^3}{4(-c)^{3/4}d^{5/4}}\right)*(a*d-b*c)\right)}{4(-c)^{3/4}d^{5/4}}\right)*1i\right)/\left(2(-c)^{3/4}d^{5/4}\right)$

**sympy** [A] time = 0.66, size = 87, normalized size = 0.39

$$\frac{bx}{d} + \operatorname{RootSum}\left(256t^4c^3d^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left(t \mapsto t \log\left(\frac{4tcd}{ad-bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)/(d*x**4+c),x)
```

```
[Out] b*x/d + RootSum(256*_t**4*c**3*d**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*  
b**2*c**2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(4*_t*c*d/(a  
*d - b*c) + x)))
```

$$3.151 \quad \int \frac{a+bx^4}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=245

$$\frac{(3ad + bc) \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{5/4}} + \frac{(3ad + bc) \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{5/4}} - \frac{(3ad + bc) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}\right)}{8\sqrt{2} c^{7/4} d^{5/4}}$$

[Out]  $-1/4*(-a*d+b*c)*x/c/d/(d*x^4+c)+1/16*(3*a*d+b*c)*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/d^{(5/4)}*2^{(1/2)}+1/16*(3*a*d+b*c)*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/d^{(5/4)}*2^{(1/2)}-1/32*(3*a*d+b*c)*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/d^{(5/4)}*2^{(1/2)}+1/32*(3*a*d+b*c)*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/d^{(5/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(3ad + bc) \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{5/4}} + \frac{(3ad + bc) \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{5/4}} - \frac{(3ad + bc) \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}\right)}{8\sqrt{2} c^{7/4} d^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4)^2, x]

[Out]  $-((b*c - a*d)*x)/(4*c*d*(c + d*x^4)) - ((b*c + 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + ((b*c + 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) - ((b*c + 3*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)}) + ((b*c + 3*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(5/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] :> Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] :> With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^4}{(c + dx^4)^2} dx &= -\frac{(bc - ad)x}{4cd(c + dx^4)} + \frac{(bc + 3ad) \int \frac{1}{c+dx^4} dx}{4cd} \\ &= -\frac{(bc - ad)x}{4cd(c + dx^4)} + \frac{(bc + 3ad) \int \frac{\sqrt{c} - \sqrt{d}x^2}{c+dx^4} dx}{8c^{3/2}d} + \frac{(bc + 3ad) \int \frac{\sqrt{c} + \sqrt{d}x^2}{c+dx^4} dx}{8c^{3/2}d} \\ &= -\frac{(bc - ad)x}{4cd(c + dx^4)} + \frac{(bc + 3ad) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{16c^{3/2}d^{3/2}} + \frac{(bc + 3ad) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{16c^{3/2}d^{3/2}} - \frac{(bc + 3ad)}{16c^{3/2}d^{3/2}} \\ &= -\frac{(bc - ad)x}{4cd(c + dx^4)} - \frac{(bc + 3ad) \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{16\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc + 3ad) \log(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{16\sqrt{2}c^{7/4}d^{5/4}} \\ &= -\frac{(bc - ad)x}{4cd(c + dx^4)} - \frac{(bc + 3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} + \frac{(bc + 3ad) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{8\sqrt{2}c^{7/4}d^{5/4}} - \frac{(bc + 3ad)}{16\sqrt{2}c^{7/4}d^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 212, normalized size = 0.87

$$\frac{-\frac{8c^{3/4}\sqrt[4]{d}x(bc-ad)}{c+dx^4} - \sqrt{2}(3ad+bc)\log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) + \sqrt{2}(3ad+bc)\log(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2)}{32c^{7/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)/(c + d\*x^4)^2, x]

[Out] ((-8\*c^(3/4)\*d^(1/4)\*(b\*c - a\*d)\*x)/(c + d\*x^4) - 2\*Sqrt[2]\*(b\*c + 3\*a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 2\*Sqrt[2]\*(b\*c + 3\*a\*d)\*ArcTan[1 +



$(\sqrt{2} * d^{(1/4)} * x) / c^{(1/4)} - \sqrt{2} * (b * c + 3 * a * d) * \text{Log}[\sqrt{c} - \sqrt{2} * c^{(1/4)} * d^{(1/4)} * x + \sqrt{d} * x^2] + \sqrt{2} * (b * c + 3 * a * d) * \text{Log}[\sqrt{c} + \sqrt{2} * c^{(1/4)} * d^{(1/4)} * x + \sqrt{d} * x^2] / (32 * c^{(7/4)} * d^{(5/4)})$

**fricas [B]** time = 1.19, size = 711, normalized size = 2.90

$$4 (cd^2 x^4 + c^2 d) \left( -\frac{b^4 c^4 + 12 ab^3 c^3 d + 54 a^2 b^2 c^2 d^2 + 108 a^3 b c d^3 + 81 a^4 d^4}{c^7 d^5} \right)^{\frac{1}{4}} \arctan \left( -\frac{c^5 d^4 x \left( -\frac{b^4 c^4 + 12 ab^3 c^3 d + 54 a^2 b^2 c^2 d^2 + 108 a^3 b c d^3 + 81 a^4 d^4}{c^7 d^5} \right)}{c^5 d^4 x \left( -\frac{b^4 c^4 + 12 ab^3 c^3 d + 54 a^2 b^2 c^2 d^2 + 108 a^3 b c d^3 + 81 a^4 d^4}{c^7 d^5} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out]  $\frac{1}{16} * (4 * (c * d^2 * x^4 + c^2 * d) * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(1/4)} * \arctan(- (c^5 * d^4 * x * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5)) + (b^2 * c^2 + 6 * a * b * c * d + 9 * a^2 * d^2) * x^2) / (b^2 * c^2 + 6 * a * b * c * d + 9 * a^2 * d^2)) * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(3/4)} / (b^3 * c^3 + 9 * a * b^2 * c^2 * d + 27 * a^2 * b * c * d^2 + 27 * a^3 * d^3)) + (c * d^2 * x^4 + c^2 * d) * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(1/4)} * \log(c^2 * d * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(1/4)} + (b * c + 3 * a * d) * x) - (c * d^2 * x^4 + c^2 * d) * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(1/4)} * \log(- c^2 * d * (- (b^4 * c^4 + 12 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 108 * a^3 * b * c * d^3 + 81 * a^4 * d^4) / (c^7 * d^5))^{(1/4)} + (b * c + 3 * a * d) * x) - 4 * (b * c - a * d) * x) / (c * d^2 * x^4 + c^2 * d)$

**giac [A]** time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc + 3 (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{16 c^2 d^2} + \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc + 3 (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{16 c^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c)^2,x, algorithm="giac")

[Out]  $\frac{1}{16} * \sqrt{2} * ((c * d^3)^{(1/4)} * b * c + 3 * (c * d^3)^{(1/4)} * a * d) * \arctan(1/2 * \sqrt{2} * (2 * x + \sqrt{2} * (c/d)^{(1/4)}) / (c/d)^{(1/4)}) / (c^2 * d^2) + 1/16 * \sqrt{2} * ((c * d^3)^{(1/4)} * b * c + 3 * (c * d^3)^{(1/4)} * a * d) * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (c/d)^{(1/4)}) / (c/d)^{(1/4)}) / (c^2 * d^2) + 1/32 * \sqrt{2} * ((c * d^3)^{(1/4)} * b * c + 3 * (c * d^3)^{(1/4)} * a * d) * \log(x^2 + \sqrt{2} * x * (c/d)^{(1/4)} + \sqrt{c/d}) / (c^2 * d^2) - 1/32 * \sqrt{2} * ((c * d^3)^{(1/4)} * b * c + 3 * (c * d^3)^{(1/4)} * a * d) * \log(x^2 - \sqrt{2} * x * (c/d)^{(1/4)} + \sqrt{c/d}) / (c^2 * d^2) - 1/4 * (b * c * x - a * d * x) / ((d * x^4 + c) * c * d)$

**maple [A]** time = 0.05, size = 295, normalized size = 1.20

$$\frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} - 1 \right)}{16 c^2} + \frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} + 1 \right)}{16 c^2} + \frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \ln \left( \frac{x^2 + \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}} \right)}{32 c^2} + \frac{\left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} b x}{16 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)/(d*x^4+c)^2,x)`

[Out]  $\frac{1}{4} \frac{(a*d-b*c)}{d/c*x/(d*x^4+c)} + \frac{3}{16} \frac{c^2*(c/d)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(c/d)^{(1/4)*x+1)*a+1/16/c/d*(c/d)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(c/d)^{(1/4)*x+1)*b+3/16/c^2*(c/d)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(c/d)^{(1/4)*x-1)*a+1/16/c/d*(c/d)^{(1/4)*2^{(1/2)}}*\arctan(2^{(1/2)}/(c/d)^{(1/4)*x-1)*b+3/32/c^2*(c/d)^{(1/4)*2^{(1/2)}}*\ln((x^2+(c/d)^{(1/4)*2^{(1/2)}}*x+(c/d)^{(1/2)))/(x^2-(c/d)^{(1/4)*2^{(1/2)}}*x+(c/d)^{(1/2))}}{32cd}$

**maxima** [A] time = 1.16, size = 236, normalized size = 0.96

$$\frac{(bc-ad)x}{4(cd^2x^4+c^2d)} + \frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x+\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(bc+3ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x-\sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{\sqrt{2}(bc+3ad)\log\left(\sqrt{d}x^2+\sqrt{c}\right)}{c^{\frac{3}{4}}d^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+a)/(d*x^4+c)^2,x, algorithm="maxima")`

[Out]  $-\frac{1}{4} \frac{(b*c - a*d)*x/(c*d^2*x^4 + c^2*d)}{c^2} + \frac{1}{32} \frac{(2*\sqrt{2}*(b*c + 3*a*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x + \sqrt{2}*c^{1/4}*d^{1/4})/\sqrt{\sqrt{c}*\sqrt{d}})))/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}}) + 2*\sqrt{2}*(b*c + 3*a*d)*\arctan(1/2*\sqrt{2}*(2*\sqrt{d}*x - \sqrt{2}*c^{1/4}*d^{1/4})/\sqrt{\sqrt{c}*\sqrt{d}}))/(\sqrt{c}*\sqrt{\sqrt{c}*\sqrt{d}}) + \sqrt{2}*(b*c + 3*a*d)*\log(\sqrt{d}*x^2 + \sqrt{2}*c^{1/4}*d^{1/4}*x + \sqrt{c}))/c^{3/4}*d^{1/4} - \sqrt{2}*(b*c + 3*a*d)*\log(\sqrt{d}*x^2 - \sqrt{2}*c^{1/4}*d^{1/4}*x + \sqrt{c}))/c^{3/4}*d^{1/4}}{c*d}$

**mupad** [B] time = 1.52, size = 740, normalized size = 3.02

$$\operatorname{atan}\left(\frac{\left(\frac{x(9a^2d^3+6abcd^2+b^2c^2d)}{4c^2} - \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}} + \frac{\left(\frac{x(9a^2d^3+6abcd^2+b^2c^2d)}{4c^2} + \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}\right) \frac{(3ad+bc)1i}{8(-c)^{7/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^4)/(c + d*x^4)^2,x)`

[Out]  $\operatorname{atan}\left(\frac{\left(\frac{x(9a^2d^3+b^2c^2d+6a*b*c*d^2)}{4c^2} - \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}} + \frac{\left(\frac{x(9a^2d^3+b^2c^2d+6a*b*c*d^2)}{4c^2} + \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}\right) - \frac{\left(\frac{x(9a^2d^3+b^2c^2d+6a*b*c*d^2)}{4c^2} - \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}} + \frac{\left(\frac{x(9a^2d^3+b^2c^2d+6a*b*c*d^2)}{4c^2} + \frac{(3ad+bc)(12ad^3+4bcd^2)1i}{16(-c)^{7/4}d^{5/4}}\right)(3ad+bc)}{16(-c)^{7/4}d^{5/4}}\right) \frac{(3ad+bc)1i}{8(-c)^{7/4}d^{5/4}}$

$$\begin{aligned} & c)^{(7/4)}d^{(5/4)})*(3*a*d + b*c)*1i)/(16*(-c)^{(7/4)}*d^{(5/4)}) - (((x*(9*a^2*d^3 + b^2*c^2*d + 6*a*b*c*d^2))/(4*c^2) + ((3*a*d + b*c)*(12*a*d^3 + 4*b*c*d^2)*1i)/(16*(-c)^{(7/4)}*d^{(5/4)}))*(3*a*d + b*c)*1i)/(16*(-c)^{(7/4)}*d^{(5/4)})) \\ & ))*(3*a*d + b*c))/(8*(-c)^{(7/4)}*d^{(5/4)}) + (x*(a*d - b*c))/(4*c*d*(c + d*x^4)) \end{aligned}$$

**sympy [A]** time = 0.85, size = 112, normalized size = 0.46

$$\frac{x(ad - bc)}{4c^2d + 4cd^2x^4} + \text{RootSum}\left(65536t^4c^7d^5 + 81a^4d^4 + 108a^3bcd^3 + 54a^2b^2c^2d^2 + 12ab^3c^3d + b^4c^4, \left(t \mapsto t \log\left(\frac{1}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*2,x)

[Out] x\*(a\*d - b\*c)/(4\*c\*\*2\*d + 4\*c\*d\*\*2\*x\*\*4) + RootSum(65536\*\_t\*\*4\*c\*\*7\*d\*\*5 + 81\*a\*\*4\*d\*\*4 + 108\*a\*\*3\*b\*c\*d\*\*3 + 54\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 + 12\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4, Lambda(\_t, \_t\*log(16\*\_t\*c\*\*2\*d/(3\*a\*d + b\*c) + x)))

$$3.152 \quad \int \frac{a+bx^4}{(c+dx^4)^3} dx$$

**Optimal.** Leaf size=273

$$\frac{3(7ad+bc)\log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(7ad+bc)\log(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} - \frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x-\sqrt{c}-\sqrt{d}x^2}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}$$

[Out]  $-1/8*(-a*d+b*c)*x/c/d/(d*x^4+c)^2+1/32*(7*a*d+b*c)*x/c^2/d/(d*x^4+c)+3/128*(7*a*d+b*c)*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(11/4)}/d^{(5/4)}*2^{(1/2)}+3/128*(7*a*d+b*c)*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(11/4)}/d^{(5/4)}*2^{(1/2)}-3/256*(7*a*d+b*c)*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(11/4)}/d^{(5/4)}*2^{(1/2)}+3/256*(7*a*d+b*c)*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(11/4)}/d^{(5/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.17, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {385, 199, 211, 1165, 628, 1162, 617, 204}

$$\frac{3(7ad+bc)\log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(7ad+bc)\log(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} - \frac{3(7ad+bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x+\sqrt{c}+\sqrt{d}x^2}{\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x-\sqrt{c}-\sqrt{d}x^2}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)/(c + d\*x^4)^3, x]

[Out]  $-\frac{(b*c - a*d)*x}{(8*c*d*(c + d*x^4)^2)} + \frac{(b*c + 7*a*d)*x}{(32*c^2*d*(c + d*x^4))} - \frac{(3*(b*c + 7*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)})]}{(64*\text{Sqrt}[2]*c^{(11/4)}*d^{(5/4)})} + \frac{(3*(b*c + 7*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)})]}{(64*\text{Sqrt}[2]*c^{(11/4)}*d^{(5/4)})} - \frac{(3*(b*c + 7*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])}{(128*\text{Sqrt}[2]*c^{(11/4)}*d^{(5/4)})} + \frac{(3*(b*c + 7*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])}{(128*\text{Sqrt}[2]*c^{(11/4)}*d^{(5/4)})}$

#### Rule 199

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2\*p] || (n == 2 && IntegerQ[4\*p]) || (n == 2 && IntegerQ[3\*p]) || Denominator[p + 1/n] < Denominator[p])

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^4}{(c + dx^4)^3} dx &= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad) \int \frac{1}{(c+dx^4)^2} dx}{8cd} \\
&= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad)x}{32c^2d(c + dx^4)} + \frac{(3(bc + 7ad)) \int \frac{1}{c+dx^4} dx}{32c^2d} \\
&= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad)x}{32c^2d(c + dx^4)} + \frac{(3(bc + 7ad)) \int \frac{\sqrt{c} - \sqrt{d}x^2}{c+dx^4} dx}{64c^{5/2}d} + \frac{(3(bc + 7ad)) \int \frac{\sqrt{c} + \sqrt{d}x^2}{c+dx^4} dx}{64c^{5/2}d} \\
&= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad)x}{32c^2d(c + dx^4)} + \frac{(3(bc + 7ad)) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{128c^{5/2}d^{3/2}} + \frac{(3(bc + 7ad)) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{128c^{5/2}d^{3/2}} \\
&= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad)x}{32c^2d(c + dx^4)} - \frac{3(bc + 7ad) \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc + 7ad) \log(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{128\sqrt{2}c^{11/4}d^{5/4}} \\
&= -\frac{(bc - ad)x}{8cd(c + dx^4)^2} + \frac{(bc + 7ad)x}{32c^2d(c + dx^4)} - \frac{3(bc + 7ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}} + \frac{3(bc + 7ad) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{64\sqrt{2}c^{11/4}d^{5/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 243, normalized size = 0.89

$$\frac{-\frac{32c^{7/4}\sqrt[4]{d}x(bc-ad)}{(c+dx^4)^2} + \frac{8c^{3/4}\sqrt[4]{d}x(7ad+bc)}{c+dx^4} - 3\sqrt{2}(7ad+bc)\log\left(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right) + 3\sqrt{2}(7ad+bc)\log\left(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2\right)}{256c^{11/4}d^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)/(c + d\*x^4)^3,x]

[Out] ((-32\*c^(7/4)\*d^(1/4)\*(b\*c - a\*d)\*x)/(c + d\*x^4)^2 + (8\*c^(3/4)\*d^(1/4)\*(b\*c + 7\*a\*d)\*x)/(c + d\*x^4) - 6\*Sqrt[2]\*(b\*c + 7\*a\*d)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 6\*Sqrt[2]\*(b\*c + 7\*a\*d)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - 3\*Sqrt[2]\*(b\*c + 7\*a\*d)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] + 3\*Sqrt[2]\*(b\*c + 7\*a\*d)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(256\*c^(11/4)\*d^(5/4))

**fricas [B]** time = 1.28, size = 787, normalized size = 2.88

$$4(bcd + 7ad^2)x^5 + 12(c^2d^3x^8 + 2c^3d^2x^4 + c^4d) \left( -\frac{b^4c^4 + 28ab^3c^3d + 294a^2b^2c^2d^2 + 1372a^3bcd^3 + 2401a^4d^4}{c^{11}d^5} \right)^{\frac{1}{4}} \arctan \left( \frac{c^8d^4x^4 - \dots}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c)^3,x, algorithm="fricas")

[Out] 1/128\*(4\*(b\*c\*d + 7\*a\*d^2)\*x^5 + 12\*(c^2\*d^3\*x^8 + 2\*c^3\*d^2\*x^4 + c^4\*d)\*(-b^4\*c^4 + 28\*a\*b^3\*c^3\*d + 294\*a^2\*b^2\*c^2\*d^2 + 1372\*a^3\*b\*c\*d^3 + 2401\*a^4\*d^4)

$$\begin{aligned} & a^4 d^4 / (c^{11} d^5)^{1/4} \arctan(- (c^8 d^4 x x - (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{3/4} - \\ & c^8 d^4 \sqrt{(c^6 d^2 \sqrt{-(b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5)} + (b^2 c^2 + 14 a b c d + 49 a^2 d^2) x^2) / (b^2 c^2 + 14 a b c d + 49 a^2 d^2)) * (- (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{3/4}) / (b^3 c^3 + 21 a b^2 c^2 d + 147 a^2 b c d^2 + 343 a^3 d^3) + 3 * (c^2 d^3 x^8 + 2 c^3 d^2 x^4 + c^4 d) * (- (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{1/4} * \log(3 c^3 d * (- (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{1/4} + 3 * (b c + 7 a d) * x) - 3 * (c^2 d^3 x^8 + 2 c^3 d^2 x^4 + c^4 d) * (- (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{1/4} * \log(- 3 c^3 d * (- (b^4 c^4 + 28 a b^3 c^3 d + 294 a^2 b^2 c^2 d^2 + 1372 a^3 b c d^3 + 2401 a^4 d^4) / (c^{11} d^5))^{1/4} + 3 * (b c + 7 a d) * x) - 4 * (3 b c^2 - 11 a c d) * x) / (c^2 d^3 x^8 + 2 c^3 d^2 x^4 + c^4 d) \end{aligned}$$

**giac [A]** time = 0.19, size = 286, normalized size = 1.05

$$\frac{3 \sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc + 7 (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{128 c^3 d^2} + \frac{3 \sqrt{2} \left( (cd^3)^{\frac{1}{4}} bc + 7 (cd^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{128 c^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)/(d\*x^4+c)^3,x, algorithm="giac")

[Out] 3/128\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^2) + 3/128\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^2) + 3/256\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*log(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^2) - 3/256\*sqrt(2)\*((c\*d^3)^(1/4)\*b\*c + 7\*(c\*d^3)^(1/4)\*a\*d)\*log(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^2) + 1/32\*(b\*c\*d\*x^5 + 7\*a\*d^2\*x^5 - 3\*b\*c^2\*x + 11\*a\*c\*d\*x)/((d\*x^4 + c)^2\*c^2\*d)

**maple [A]** time = 0.05, size = 314, normalized size = 1.15

$$\frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} - 1 \right)}{128 c^3} + \frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} + 1 \right)}{128 c^3} + \frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a \ln \left( \frac{x^2 + \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}} \right)}{256 c^3} + \frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}}}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)/(d\*x^4+c)^3,x)

[Out] (1/32\*(7\*a\*d+b\*c)/c^2\*x^5+1/32\*(11\*a\*d-3\*b\*c)/c/d\*x)/(d\*x^4+c)^2+21/128/c^3\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a+3/128/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*b+21/256/c^3\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2)))\*a+3/256/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2)))\*b+21/128/c^3\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a+3/128/c^2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*b

**maxima** [A] time = 1.21, size = 271, normalized size = 0.99

$$\frac{(bcd + 7ad^2)x^5 - (3bc^2 - 11acd)x}{32(c^2d^3x^8 + 2c^3d^2x^4 + c^4d)} + \frac{3 \left( \frac{2\sqrt{2}(bc+7ad) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x + \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} \right) + \frac{2\sqrt{2}(bc+7ad) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x - \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} \right)}{256c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*((b*c*d + 7*a*d^2)*x^5 - (3*b*c^2 - 11*a*c*d)*x)/(c^2*d^3*x^8 + 2*c^3*d^2*x^4 + c^4*d) + 3/256*(2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*(b*c + 7*a*d)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(b*c + 7*a*d)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/(c^2*d)
```

**mupad** [B] time = 1.58, size = 762, normalized size = 2.79

$$\frac{\frac{x^5(7ad+bc)}{32c^2} + \frac{x(11ad-3bc)}{32cd}}{c^2 + 2cdx^4 + d^2x^8} - \frac{\operatorname{atan}\left(\frac{\left(\frac{9x(49a^2d^3+14abcd^2+b^2c^2d)}{256c^4} - \frac{9(7ad+bc)(7ad^3+bc^2d)}{256(-c)^{15/4}d^{5/4}}\right)(7ad+bc)^{3i}}{128(-c)^{11/4}d^{5/4}} + \frac{\left(\frac{9x(49a^2d^3+14abcd^2+b^2c^2d)}{256c^4} + \frac{9(7ad+bc)(7ad^3+bc^2d)}{256(-c)^{15/4}d^{5/4}}\right)(7ad+bc)^{-3i}}{128(-c)^{11/4}d^{5/4}}\right)}{64(-c)^{11/4}d^{5/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)/(c + d*x^4)^3,x)
```

```
[Out] ((x^5*(7*a*d + b*c))/(32*c^2) + (x*(11*a*d - 3*b*c))/(32*c*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) + (((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)))/((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)) - (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + (9*(7*a*d + b*c)*(7*a*d^3 + b*c*d^2))/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c)*3i)/(64*(-c)^(11/4)*d^(5/4)) - (3*atan((((3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)) + (3*((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4)))/((((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) - ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c)*3i)/(128*(-c)^(11/4)*d^(5/4)) - (((9*x*(49*a^2*d^3 + b^2*c^2*d + 14*a*b*c*d^2))/(256*c^4) + ((7*a*d + b*c)*(7*a*d^3 + b*c*d^2)*9i)/(256*(-c)^(15/4)*d^(5/4)))*(7*a*d + b*c))/(128*(-c)^(11/4)*d^(5/4))))*(7*a*d + b*c))/(64*(-c)^(11/4)*d^(5/4))
```



sympy [A] time = 1.03, size = 151, normalized size = 0.55

$$\frac{x^5(7ad^2 + bcd) + x(11acd - 3bc^2)}{32c^4d + 64c^3d^2x^4 + 32c^2d^3x^8} + \text{RootSum}\left(268435456t^4c^{11}d^5 + 194481a^4d^4 + 111132a^3bcd^3 + 23814a^2b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*3,x)

[Out] (x\*\*5\*(7\*a\*d\*\*2 + b\*c\*d) + x\*(11\*a\*c\*d - 3\*b\*c\*\*2))/(32\*c\*\*4\*d + 64\*c\*\*3\*d\*\*2\*x\*\*4 + 32\*c\*\*2\*d\*\*3\*x\*\*8) + RootSum(268435456\*\_t\*\*4\*c\*\*11\*d\*\*5 + 194481\*a\*\*4\*d\*\*4 + 111132\*a\*\*3\*b\*c\*d\*\*3 + 23814\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 + 2268\*a\*b\*\*3\*c\*\*3\*d + 81\*b\*\*4\*c\*\*4, Lambda(\_t, \_t\*log(128\*\_t\*c\*\*3\*d/(21\*a\*d + 3\*b\*c) + x)))

$$3.153 \quad \int (a + bx^4)^2 (c + dx^4)^4 dx$$

**Optimal.** Leaf size=154

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5$$

[Out] a^2\*c^4\*x+2/5\*a\*c^3\*(2\*a\*d+b\*c)\*x^5+1/9\*c^2\*(6\*a^2\*d^2+8\*a\*b\*c\*d+b^2\*c^2)\*x^9+4/13\*c\*d\*(a^2\*d^2+3\*a\*b\*c\*d+b^2\*c^2)\*x^13+1/17\*d^2\*(a^2\*d^2+8\*a\*b\*c\*d+6\*b^2\*c^2)\*x^17+2/21\*b\*d^3\*(a\*d+2\*b\*c)\*x^21+1/25\*b^2\*d^4\*x^25

**Rubi [A]** time = 0.11, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^4,x]

[Out] a^2\*c^4\*x + (2\*a\*c^3\*(b\*c + 2\*a\*d)\*x^5)/5 + (c^2\*(b^2\*c^2 + 8\*a\*b\*c\*d + 6\*a^2\*d^2)\*x^9)/9 + (4\*c\*d\*(b^2\*c^2 + 3\*a\*b\*c\*d + a^2\*d^2)\*x^13)/13 + (d^2\*(6\*b^2\*c^2 + 8\*a\*b\*c\*d + a^2\*d^2)\*x^17)/17 + (2\*b\*d^3\*(2\*b\*c + a\*d)\*x^21)/21 + (b^2\*d^4\*x^25)/25

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)^2 (c + dx^4)^4 dx &= \int (a^2c^4 + 2ac^3(bc + 2ad)x^4 + c^2(b^2c^2 + 8abcd + 6a^2d^2)x^8 + 4cd(b^2c^2 + 3abcd + a^2d^2)x^{12} + d^2(6b^2c^2 + 8abcd + a^2d^2)x^{16} + 2b^2cd^3(2b^2c^2 + 3abcd + a^2d^2)x^{20} + b^2d^4x^{24}) dx \\ &= a^2c^4x + \frac{2}{5}ac^3(bc + 2ad)x^5 + \frac{1}{9}c^2(b^2c^2 + 8abcd + 6a^2d^2)x^9 + \frac{4}{13}cd(b^2c^2 + 3abcd + a^2d^2)x^{13} + \frac{2}{21}b^2cd^3(2b^2c^2 + 3abcd + a^2d^2)x^{17} + \frac{1}{25}b^2d^4x^{25} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 154, normalized size = 1.00

$$\frac{1}{17}d^2x^{17}(a^2d^2 + 8abcd + 6b^2c^2) + \frac{4}{13}cdx^{13}(a^2d^2 + 3abcd + b^2c^2) + \frac{1}{9}c^2x^9(6a^2d^2 + 8abcd + b^2c^2) + a^2c^4x + \frac{2}{5}ac^3x^5$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^4,x]

[Out] a^2\*c^4\*x + (2\*a\*c^3\*(b\*c + 2\*a\*d)\*x^5)/5 + (c^2\*(b^2\*c^2 + 8\*a\*b\*c\*d + 6\*a^2\*d^2)\*x^9)/9 + (4\*c\*d\*(b^2\*c^2 + 3\*a\*b\*c\*d + a^2\*d^2)\*x^13)/13 + (d^2\*(6\*b^2\*c^2 + 8\*a\*b\*c\*d + a^2\*d^2)\*x^17)/17 + (2\*b\*d^3\*(2\*b\*c + a\*d)\*x^21)/21 + (b^2\*d^4\*x^25)/25

**fricas [A]** time = 0.61, size = 173, normalized size = 1.12

$$\frac{1}{25}x^{25}d^4b^2 + \frac{4}{21}x^{21}d^3cb^2 + \frac{2}{21}x^{21}d^4ba + \frac{6}{17}x^{17}d^2c^2b^2 + \frac{8}{17}x^{17}d^3cba + \frac{1}{17}x^{17}d^4a^2 + \frac{4}{13}x^{13}dc^3b^2 + \frac{12}{13}x^{13}d^2c^2ba + \frac{4}{13}x^{13}d^3c^2ba + \frac{2}{5}ac^3x^5 + a^2c^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^4,x, algorithm="fricas")

[Out]  $\frac{1}{25}x^{25}d^4b^2 + \frac{4}{21}x^{21}d^3c^*b^2 + \frac{2}{21}x^{21}d^4b^*a + \frac{6}{17}x^{17}d^2c^2b^2 + \frac{8}{17}x^{17}d^3c^*b^*a + \frac{1}{17}x^{17}d^4a^2 + \frac{4}{13}x^{13}d^3c^3b^2 + \frac{12}{13}x^{13}d^2c^2b^*a + \frac{4}{13}x^{13}d^3c^*a^2 + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9d^3c^3b^*a + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4b^*a + \frac{4}{5}x^5d^3c^3a^2 + x^c^4a^2$

**giac** [A] time = 0.15, size = 173, normalized size = 1.12

$$\frac{1}{25}b^2d^4x^{25} + \frac{4}{21}b^2cd^3x^{21} + \frac{2}{21}abd^4x^{21} + \frac{6}{17}b^2c^2d^2x^{17} + \frac{8}{17}abcd^3x^{17} + \frac{1}{17}a^2d^4x^{17} + \frac{4}{13}b^2c^3dx^{13} + \frac{12}{13}abc^2d^2x^{13} + \frac{4}{13}a^2c^3d^2x^{13} + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9d^3c^3b^*a + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4b^*a + \frac{4}{5}x^5d^3c^3a^2 + x^c^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^4,x, algorithm="giac")

[Out]  $\frac{1}{25}b^2d^4x^{25} + \frac{4}{21}b^2c^*d^3x^{21} + \frac{2}{21}a^*b^*d^4x^{21} + \frac{6}{17}b^2c^2d^2x^{17} + \frac{8}{17}a^*b^*c^*d^3x^{17} + \frac{1}{17}a^2d^4x^{17} + \frac{4}{13}b^2c^3d^2x^{13} + \frac{12}{13}a^*b^*c^2d^2x^{13} + \frac{4}{13}a^2c^3d^2x^{13} + \frac{1}{9}b^2c^4x^9 + \frac{8}{9}a^*b^*c^3d^3x^9 + \frac{2}{3}a^2c^2d^2x^9 + \frac{2}{5}a^*b^*c^4x^5 + \frac{4}{5}a^2c^3d^3x^5 + a^2c^4x^4$

**maple** [A] time = 0.04, size = 163, normalized size = 1.06

$$\frac{b^2d^4x^{25}}{25} + \frac{(2abd^4 + 4b^2cd^3)x^{21}}{21} + \frac{(a^2d^4 + 8abcd^3 + 6b^2c^2d^2)x^{17}}{17} + \frac{(4a^2cd^3 + 12abc^2d^2 + 4b^2c^3d)x^{13}}{13} + \frac{(6a^2c^3d^2 + 4a^2c^3d^2)x^{13}}{13} + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9d^3c^3b^*a + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4b^*a + \frac{4}{5}x^5d^3c^3a^2 + x^c^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^4,x)

[Out]  $\frac{1}{25}b^2d^4x^{25} + \frac{1}{21}(2a^*b^*d^4 + 4b^2c^*d^3)x^{21} + \frac{1}{17}(a^2d^4 + 8a^*b^*c^*d^3 + 6b^2c^2d^2)x^{17} + \frac{1}{13}(4a^2c^3d^3 + 12a^*b^*c^2d^2 + 4b^2c^3d)x^{13} + \frac{1}{9}(6a^2c^2d^2 + 8a^*b^*c^3d + b^2c^4)x^9 + \frac{1}{5}(4a^2c^3d + 2a^*b^*c^4)x^5 + a^2c^4x^4$

**maxima** [A] time = 0.55, size = 158, normalized size = 1.03

$$\frac{1}{25}b^2d^4x^{25} + \frac{2}{21}(2b^2cd^3 + abd^4)x^{21} + \frac{1}{17}(6b^2c^2d^2 + 8abcd^3 + a^2d^4)x^{17} + \frac{4}{13}(b^2c^3d + 3abc^2d^2 + a^2cd^3)x^{13} + \frac{1}{9}x^9c^4b^2 + \frac{8}{9}x^9d^3c^3b^*a + \frac{2}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^4b^*a + \frac{4}{5}x^5d^3c^3a^2 + x^c^4a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^4,x, algorithm="maxima")

[Out]  $\frac{1}{25}b^2d^4x^{25} + \frac{2}{21}(2b^2c^*d^3 + a^*b^*d^4)x^{21} + \frac{1}{17}(6b^2c^2d^2 + 8a^*b^*c^*d^3 + a^2d^4)x^{17} + \frac{4}{13}(b^2c^3d + 3a^*b^*c^2d^2 + a^2c^*d^3)x^{13} + \frac{1}{9}(b^2c^4 + 8a^*b^*c^3d + 6a^2c^2d^2)x^9 + a^2c^4x^4 + \frac{2}{5}(a^*b^*c^4 + 2a^2c^3d)x^5$

**mupad** [B] time = 0.07, size = 146, normalized size = 0.95

$$x^9 \left( \frac{2a^2c^2d^2}{3} + \frac{8abc^3d}{9} + \frac{b^2c^4}{9} \right) + x^{17} \left( \frac{a^2d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2c^2d^2}{17} \right) + a^2c^4x^4 + \frac{b^2d^4x^{25}}{25} + \frac{2ac^3x^5(2ad + b^2c^3)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^2\*(c + d\*x^4)^4,x)

[Out]  $x^9 \left( \frac{b^2c^4}{9} + \frac{2a^2c^2d^2}{3} + \frac{8a^*b^*c^3d}{9} \right) + x^{17} \left( \frac{a^2d^4}{17} + \frac{8a^*b^*c^3d}{17} + \frac{6b^2c^2d^2}{17} \right) + a^2c^4x^4 + \frac{b^2d^4x^{25}}{25} + \frac{2a^*c^3x^5(2ad + b^2c^3)}{5} + \frac{2b^2d^3x^{21}(ad + 2b^2c^3)}{21} + \frac{4c^*d^3x^{13}(a^2d^2 + b^2c^2 + 3a^*b^*c^*d)}{13}$

sympy [A] time = 0.12, size = 185, normalized size = 1.20

$$a^2c^4x + \frac{b^2d^4x^{25}}{25} + x^{21} \left( \frac{2abd^4}{21} + \frac{4b^2cd^3}{21} \right) + x^{17} \left( \frac{a^2d^4}{17} + \frac{8abcd^3}{17} + \frac{6b^2c^2d^2}{17} \right) + x^{13} \left( \frac{4a^2cd^3}{13} + \frac{12abc^2d^2}{13} + \frac{4b^2c^3d}{13} \right) + x^9 \left( \frac{2a^2c^2d^2}{9} + \frac{8abc^3d}{9} + \frac{b^2c^4}{9} \right) + x^5 \left( \frac{4a^2c^3d}{5} + \frac{2abc^4}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*4,x)

[Out] a\*\*2\*c\*\*4\*x + b\*\*2\*d\*\*4\*x\*\*25/25 + x\*\*21\*(2\*a\*b\*d\*\*4/21 + 4\*b\*\*2\*c\*d\*\*3/21) + x\*\*17\*(a\*\*2\*d\*\*4/17 + 8\*a\*b\*c\*d\*\*3/17 + 6\*b\*\*2\*c\*\*2\*d\*\*2/17) + x\*\*13\*(4\*a\*\*2\*c\*d\*\*3/13 + 12\*a\*b\*c\*\*2\*d\*\*2/13 + 4\*b\*\*2\*c\*\*3\*d/13) + x\*\*9\*(2\*a\*\*2\*c\*\*2\*d\*\*2/9 + 8\*a\*b\*c\*\*3\*d/9 + b\*\*2\*c\*\*4/9) + x\*\*5\*(4\*a\*\*2\*c\*\*3\*d/5 + 2\*a\*b\*c\*\*4/5)

$$3.154 \quad \int (a + bx^4)^2 (c + dx^4)^3 dx$$

**Optimal.** Leaf size=122

$$\frac{1}{13}dx^{13}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad+2bc) + \frac{1}{17}bd^2x^{17}(2ad+3bc)$$

[Out] a^2\*c^3\*x+1/5\*a\*c^2\*(3\*a\*d+2\*b\*c)\*x^5+1/9\*c\*(3\*a^2\*d^2+6\*a\*b\*c\*d+b^2\*c^2)\*x^9+1/13\*d\*(a^2\*d^2+6\*a\*b\*c\*d+3\*b^2\*c^2)\*x^13+1/17\*b\*d^2\*(2\*a\*d+3\*b\*c)\*x^17+1/21\*b^2\*d^3\*x^21

**Rubi [A]** time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{1}{13}dx^{13}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad+2bc) + \frac{1}{17}bd^2x^{17}(2ad+3bc)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^3,x]

[Out] a^2\*c^3\*x + (a\*c^2\*(2\*b\*c + 3\*a\*d)\*x^5)/5 + (c\*(b^2\*c^2 + 6\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^9)/9 + (d\*(3\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^13)/13 + (b\*d^2\*(3\*b\*c + 2\*a\*d)\*x^17)/17 + (b^2\*d^3\*x^21)/21

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)^2 (c + dx^4)^3 dx &= \int (a^2c^3 + ac^2(2bc + 3ad)x^4 + c(b^2c^2 + 6abcd + 3a^2d^2)x^8 + d(3b^2c^2 + 6abcd + 3a^2d^2)x^{12} + d^2(3b^2c^2 + 6abcd + 3a^2d^2)x^{16} + d^3b^2c^2)x^{20} dx \\ &= a^2c^3x + \frac{1}{5}ac^2(2bc + 3ad)x^5 + \frac{1}{9}c(b^2c^2 + 6abcd + 3a^2d^2)x^9 + \frac{1}{13}d(3b^2c^2 + 6abcd + 3a^2d^2)x^{13} + \frac{1}{17}d^2(3b^2c^2 + 6abcd + 3a^2d^2)x^{17} + \frac{1}{21}d^3b^2c^2x^{21} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 122, normalized size = 1.00

$$\frac{1}{13}dx^{13}(a^2d^2 + 6abcd + 3b^2c^2) + \frac{1}{9}cx^9(3a^2d^2 + 6abcd + b^2c^2) + a^2c^3x + \frac{1}{5}ac^2x^5(3ad+2bc) + \frac{1}{17}bd^2x^{17}(2ad+3bc)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^3,x]

[Out] a^2\*c^3\*x + (a\*c^2\*(2\*b\*c + 3\*a\*d)\*x^5)/5 + (c\*(b^2\*c^2 + 6\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^9)/9 + (d\*(3\*b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^13)/13 + (b\*d^2\*(3\*b\*c + 2\*a\*d)\*x^17)/17 + (b^2\*d^3\*x^21)/21

**fricas [A]** time = 0.73, size = 132, normalized size = 1.08

$$\frac{1}{21}x^{21}d^3b^2 + \frac{3}{17}x^{17}d^2cb^2 + \frac{2}{17}x^{17}d^3ba + \frac{3}{13}x^{13}dc^2b^2 + \frac{6}{13}x^{13}d^2cba + \frac{1}{13}x^{13}d^3a^2 + \frac{1}{9}x^9c^3b^2 + \frac{2}{3}x^9dc^2ba + \frac{1}{3}x^9d^2ca^2 + \frac{2}{5}x^5ac^2(2bc + 3ad) + a^2c^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^3,x, algorithm="fricas")

[Out]  $\frac{1}{21}x^{21}d^3b^2 + \frac{3}{17}x^{17}d^2c^2b^2 + \frac{2}{17}x^{17}d^3b^2a + \frac{3}{13}x^{13}d^2c^2b^2 + \frac{6}{13}x^{13}d^2c^2b^2a + \frac{1}{13}x^{13}d^3a^2 + \frac{1}{9}x^9c^3b^2 + \frac{2}{3}x^9d^2c^2b^2a + \frac{1}{3}x^9d^2c^2a^2 + \frac{2}{5}x^5c^3b^2a + \frac{3}{5}x^5d^2c^2a^2 + x^3a^2$

**giac** [A] time = 0.15, size = 132, normalized size = 1.08

$$\frac{1}{21}b^2d^3x^{21} + \frac{3}{17}b^2cd^2x^{17} + \frac{2}{17}abd^3x^{17} + \frac{3}{13}b^2c^2dx^{13} + \frac{6}{13}abcd^2x^{13} + \frac{1}{13}a^2d^3x^{13} + \frac{1}{9}b^2c^3x^9 + \frac{2}{3}abc^2dx^9 + \frac{1}{3}a^2cd^2x^9 + \frac{2}{5}a^2c^3x^5 + \frac{3}{5}a^2c^2d^2x^5 + a^2c^3x + \frac{3a^2c^2d^2}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^3,x, algorithm="giac")

[Out]  $\frac{1}{21}b^2d^3x^{21} + \frac{3}{17}b^2c^2d^2x^{17} + \frac{2}{17}a^2b^2d^3x^{17} + \frac{3}{13}b^2c^2d^2x^{13} + \frac{6}{13}a^2b^2c^2d^2x^{13} + \frac{1}{13}a^2d^3x^{13} + \frac{1}{9}b^2c^3x^9 + \frac{2}{3}a^2b^2c^2d^2x^9 + \frac{1}{3}a^2c^2d^2x^9 + \frac{2}{5}a^2b^2c^3x^5 + \frac{3}{5}a^2c^2d^2x^5 + a^2c^3x$

**maple** [A] time = 0.04, size = 125, normalized size = 1.02

$$\frac{b^2d^3x^{21}}{21} + \frac{(2abd^3 + 3b^2cd^2)x^{17}}{17} + \frac{(a^2d^3 + 6abcd^2 + 3b^2c^2d)x^{13}}{13} + \frac{(3a^2cd^2 + 6abc^2d + b^2c^3)x^9}{9} + a^2c^3x + \frac{(3a^2c^2d + 6abc^2d + b^2c^3)x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^3,x)

[Out]  $\frac{1}{21}b^2d^3x^{21} + \frac{1}{17}(2a^2b^2d^3 + 3b^2c^2d^2)x^{17} + \frac{1}{13}(a^2d^3 + 6abcd^2 + 3b^2c^2d)x^{13} + \frac{1}{9}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^9 + \frac{1}{5}(3a^2c^2d + 6abc^2d + b^2c^3)x^5 + a^2c^3x$

**maxima** [A] time = 0.54, size = 124, normalized size = 1.02

$$\frac{1}{21}b^2d^3x^{21} + \frac{1}{17}(3b^2cd^2 + 2abd^3)x^{17} + \frac{1}{13}(3b^2c^2d + 6abcd^2 + a^2d^3)x^{13} + \frac{1}{9}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^9 + a^2c^3x + \frac{3a^2c^2d + 6abc^2d + b^2c^3}{5}x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^3,x, algorithm="maxima")

[Out]  $\frac{1}{21}b^2d^3x^{21} + \frac{1}{17}(3b^2c^2d^2 + 2a^2b^2d^3)x^{17} + \frac{1}{13}(3b^2c^2d^2 + 6a^2b^2c^2d^2 + a^2d^3)x^{13} + \frac{1}{9}(b^2c^3 + 6abc^2d + 3a^2cd^2)x^9 + a^2c^3x + \frac{1}{5}(2a^2b^2c^3 + 3a^2c^2d^2)x^5$

**mupad** [B] time = 1.30, size = 116, normalized size = 0.95

$$x^9 \left( \frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9} \right) + x^{13} \left( \frac{a^2d^3}{13} + \frac{6abcd^2}{13} + \frac{3b^2c^2d}{13} \right) + a^2c^3x + \frac{b^2d^3x^{21}}{21} + \frac{a^2c^2x^5(3ad + 2bc)}{5} + \frac{b^2c^3x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^2\*(c + d\*x^4)^3,x)

[Out]  $x^9 \left( \frac{b^2c^3}{9} + \frac{a^2cd^2}{3} + \frac{2a^2b^2c^2d}{3} \right) + x^{13} \left( \frac{a^2d^3}{13} + \frac{3b^2c^2d}{13} + \frac{6a^2b^2c^2d}{13} \right) + a^2c^3x + \frac{b^2d^3x^{21}}{21} + \frac{a^2c^2x^5(3ad + 2bc)}{5} + \frac{b^2c^3x^5}{5}$

**sympy** [A] time = 0.10, size = 139, normalized size = 1.14

$$a^2c^3x + \frac{b^2d^3x^{21}}{21} + x^{17} \left( \frac{2abd^3}{17} + \frac{3b^2cd^2}{17} \right) + x^{13} \left( \frac{a^2d^3}{13} + \frac{6abcd^2}{13} + \frac{3b^2c^2d}{13} \right) + x^9 \left( \frac{a^2cd^2}{3} + \frac{2abc^2d}{3} + \frac{b^2c^3}{9} \right) + x^5 \left( \frac{3a^2c^2d + 6abc^2d + b^2c^3}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**2*(d*x**4+c)**3,x)
```

```
[Out] a**2*c**3*x + b**2*d**3*x**21/21 + x**17*(2*a*b*d**3/17 + 3*b**2*c*d**2/17)
+ x**13*(a**2*d**3/13 + 6*a*b*c*d**2/13 + 3*b**2*c**2*d/13) + x**9*(a**2*c
*d**2/3 + 2*a*b*c**2*d/3 + b**2*c**3/9) + x**5*(3*a**2*c**2*d/5 + 2*a*b*c**
3/5)
```

$$3.155 \quad \int (a + bx^4)^2 (c + dx^4)^2 dx$$

**Optimal.** Leaf size=82

$$\frac{1}{9}x^9(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

[Out]  $a^2c^2x + 2/5acx^5(ad + bc) + 1/9(a^2d^2 + 4abcd + b^2c^2)x^9 + 2/13bdx^{13}(ad + bc) + 1/17b^2d^2x^{17}$

**Rubi [A]** time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{1}{9}x^9(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^2,x]

[Out]  $a^2c^2x + (2ac(bc + ad)x^5)/5 + ((b^2c^2 + 4abcd + a^2d^2)x^9)/9 + (2bd(bc + ad)x^{13})/13 + (b^2d^2x^{17})/17$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)^2 (c + dx^4)^2 dx &= \int (a^2c^2 + 2ac(bc + ad)x^4 + (b^2c^2 + 4abcd + a^2d^2)x^8 + 2bd(bc + ad)x^{12} + b^2d^2x^{16}) \\ &= a^2c^2x + \frac{2}{5}ac(bc + ad)x^5 + \frac{1}{9}(b^2c^2 + 4abcd + a^2d^2)x^9 + \frac{2}{13}bd(bc + ad)x^{13} + \frac{1}{17}b^2d^2x^{17} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 82, normalized size = 1.00

$$\frac{1}{9}x^9(a^2d^2 + 4abcd + b^2c^2) + a^2c^2x + \frac{2}{13}bdx^{13}(ad + bc) + \frac{2}{5}acx^5(ad + bc) + \frac{1}{17}b^2d^2x^{17}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^2,x]

[Out]  $a^2c^2x + (2ac(bc + ad)x^5)/5 + ((b^2c^2 + 4abcd + a^2d^2)x^9)/9 + (2bd(bc + ad)x^{13})/13 + (b^2d^2x^{17})/17$

**fricas [A]** time = 0.99, size = 91, normalized size = 1.11

$$\frac{1}{17}x^{17}d^2b^2 + \frac{2}{13}x^{13}dcb^2 + \frac{2}{13}x^{13}d^2ba + \frac{1}{9}x^9c^2b^2 + \frac{4}{9}x^9dcba + \frac{1}{9}x^9d^2a^2 + \frac{2}{5}x^5c^2ba + \frac{2}{5}x^5dca^2 + xc^2a^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^2,x, algorithm="fricas")

[Out]  $1/17*x^{17}*d^2*b^2 + 2/13*x^{13}*d*c*b^2 + 2/13*x^{13}*d^2*b*a + 1/9*x^9*c^2*b^2 + 4/9*x^9*d*c*b*a + 1/9*x^9*d^2*a^2 + 2/5*x^5*c^2*b*a + 2/5*x^5*d*c*a^2 + x*c^2*a^2$



**giac** [A] time = 0.17, size = 91, normalized size = 1.11

$$\frac{1}{17} b^2 d^2 x^{17} + \frac{2}{13} b^2 c d x^{13} + \frac{2}{13} a b d^2 x^{13} + \frac{1}{9} b^2 c^2 x^9 + \frac{4}{9} a b c d x^9 + \frac{1}{9} a^2 d^2 x^9 + \frac{2}{5} a b c^2 x^5 + \frac{2}{5} a^2 c d x^5 + a^2 c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^2,x, algorithm="giac")

[Out] 1/17\*b^2\*d^2\*x^17 + 2/13\*b^2\*c\*d\*x^13 + 2/13\*a\*b\*d^2\*x^13 + 1/9\*b^2\*c^2\*x^9 + 4/9\*a\*b\*c\*d\*x^9 + 1/9\*a^2\*d^2\*x^9 + 2/5\*a\*b\*c^2\*x^5 + 2/5\*a^2\*c\*d\*x^5 + a^2\*c^2\*x

**maple** [A] time = 0.04, size = 87, normalized size = 1.06

$$\frac{b^2 d^2 x^{17}}{17} + \frac{(2 a b d^2 + 2 b^2 c d) x^{13}}{13} + \frac{(a^2 d^2 + 4 a b c d + b^2 c^2) x^9}{9} + a^2 c^2 x + \frac{(2 a^2 c d + 2 a b c^2) x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^2,x)

[Out] 1/17\*b^2\*d^2\*x^17+1/13\*(2\*a\*b\*d^2+2\*b^2\*c\*d)\*x^13+1/9\*(a^2\*d^2+4\*a\*b\*c\*d+b^2\*c^2)\*x^9+1/5\*(2\*a^2\*c\*d+2\*a\*b\*c^2)\*x^5+a^2\*c^2\*x

**maxima** [A] time = 0.69, size = 82, normalized size = 1.00

$$\frac{1}{17} b^2 d^2 x^{17} + \frac{2}{13} (b^2 c d + a b d^2) x^{13} + \frac{1}{9} (b^2 c^2 + 4 a b c d + a^2 d^2) x^9 + \frac{2}{5} (a b c^2 + a^2 c d) x^5 + a^2 c^2 x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^2,x, algorithm="maxima")

[Out] 1/17\*b^2\*d^2\*x^17 + 2/13\*(b^2\*c\*d + a\*b\*d^2)\*x^13 + 1/9\*(b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*x^9 + 2/5\*(a\*b\*c^2 + a^2\*c\*d)\*x^5 + a^2\*c^2\*x

**mupad** [B] time = 0.05, size = 75, normalized size = 0.91

$$x^9 \left( \frac{a^2 d^2}{9} + \frac{4 a b c d}{9} + \frac{b^2 c^2}{9} \right) + a^2 c^2 x + \frac{b^2 d^2 x^{17}}{17} + \frac{2 a c x^5 (a d + b c)}{5} + \frac{2 b d x^{13} (a d + b c)}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^2\*(c + d\*x^4)^2,x)

[Out] x^9\*((a^2\*d^2)/9 + (b^2\*c^2)/9 + (4\*a\*b\*c\*d)/9) + a^2\*c^2\*x + (b^2\*d^2\*x^17)/17 + (2\*a\*c\*x^5\*(a\*d + b\*c))/5 + (2\*b\*d\*x^13\*(a\*d + b\*c))/13

**sympy** [A] time = 0.09, size = 97, normalized size = 1.18

$$a^2 c^2 x + \frac{b^2 d^2 x^{17}}{17} + x^{13} \left( \frac{2 a b d^2}{13} + \frac{2 b^2 c d}{13} \right) + x^9 \left( \frac{a^2 d^2}{9} + \frac{4 a b c d}{9} + \frac{b^2 c^2}{9} \right) + x^5 \left( \frac{2 a^2 c d}{5} + \frac{2 a b c^2}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c)\*\*2,x)

[Out] a\*\*2\*c\*\*2\*x + b\*\*2\*d\*\*2\*x\*\*17/17 + x\*\*13\*(2\*a\*b\*d\*\*2/13 + 2\*b\*\*2\*c\*d/13) + x\*\*9\*(a\*\*2\*d\*\*2/9 + 4\*a\*b\*c\*d/9 + b\*\*2\*c\*\*2/9) + x\*\*5\*(2\*a\*\*2\*c\*d/5 + 2\*a\*b\*c\*\*2/5)

### 3.156 $\int (a + bx^4)^2 (c + dx^4) dx$

**Optimal.** Leaf size=50

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

[Out]  $a^2c*x + 1/5*a*(a*d + 2*b*c)*x^5 + 1/9*b*(2*a*d + b*c)*x^9 + 1/13*b^2*d*x^{13}$

**Rubi [A]** time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^{13})/13$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^4)^2 (c + dx^4) dx &= \int (a^2c + a(2bc + ad)x^4 + b(bc + 2ad)x^8 + b^2dx^{12}) dx \\ &= a^2cx + \frac{1}{5}a(2bc + ad)x^5 + \frac{1}{9}b(bc + 2ad)x^9 + \frac{1}{13}b^2dx^{13} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 50, normalized size = 1.00

$$a^2cx + \frac{1}{9}bx^9(2ad + bc) + \frac{1}{5}ax^5(ad + 2bc) + \frac{1}{13}b^2dx^{13}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^5)/5 + (b*(b*c + 2*a*d)*x^9)/9 + (b^2*d*x^{13})/13$

**fricas [A]** time = 0.81, size = 50, normalized size = 1.00

$$\frac{1}{13}x^{13}db^2 + \frac{1}{9}x^9cb^2 + \frac{2}{9}x^9dba + \frac{2}{5}x^5cba + \frac{1}{5}x^5da^2 + xca^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c), x, algorithm="fricas")

[Out]  $1/13*x^{13}*d*b^2 + 1/9*x^9*c*b^2 + 2/9*x^9*d*b*a + 2/5*x^5*c*b*a + 1/5*x^5*d*a^2 + x*c*a^2$

**giac** [A] time = 0.15, size = 50, normalized size = 1.00

$$\frac{1}{13} b^2 dx^{13} + \frac{1}{9} b^2 cx^9 + \frac{2}{9} abdx^9 + \frac{2}{5} abcx^5 + \frac{1}{5} a^2 dx^5 + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c), x, algorithm="giac")

[Out] 1/13\*b^2\*d\*x^13 + 1/9\*b^2\*c\*x^9 + 2/9\*a\*b\*d\*x^9 + 2/5\*a\*b\*c\*x^5 + 1/5\*a^2\*d\*x^5 + a^2\*c\*x

**maple** [A] time = 0.04, size = 49, normalized size = 0.98

$$\frac{b^2 d x^{13}}{13} + \frac{(2abd + b^2 c) x^9}{9} + \frac{(a^2 d + 2abc) x^5}{5} + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c), x)

[Out] 1/13\*b^2\*d\*x^13+1/9\*(2\*a\*b\*d+b^2\*c)\*x^9+1/5\*(a^2\*d+2\*a\*b\*c)\*x^5+a^2\*c\*x

**maxima** [A] time = 0.48, size = 48, normalized size = 0.96

$$\frac{1}{13} b^2 dx^{13} + \frac{1}{9} (b^2 c + 2abd) x^9 + \frac{1}{5} (2abc + a^2 d) x^5 + a^2 cx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c), x, algorithm="maxima")

[Out] 1/13\*b^2\*d\*x^13 + 1/9\*(b^2\*c + 2\*a\*b\*d)\*x^9 + 1/5\*(2\*a\*b\*c + a^2\*d)\*x^5 + a^2\*c\*x

**mupad** [B] time = 0.04, size = 48, normalized size = 0.96

$$x^5 \left( \frac{d a^2}{5} + \frac{2 b c a}{5} \right) + x^9 \left( \frac{c b^2}{9} + \frac{2 a d b}{9} \right) + \frac{b^2 d x^{13}}{13} + a^2 c x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^2\*(c + d\*x^4), x)

[Out] x^5\*((a^2\*d)/5 + (2\*a\*b\*c)/5) + x^9\*((b^2\*c)/9 + (2\*a\*b\*d)/9) + (b^2\*d\*x^13)/13 + a^2\*c\*x

**sympy** [A] time = 0.08, size = 53, normalized size = 1.06

$$a^2 cx + \frac{b^2 dx^{13}}{13} + x^9 \left( \frac{2abd}{9} + \frac{b^2 c}{9} \right) + x^5 \left( \frac{a^2 d}{5} + \frac{2abc}{5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2\*(d\*x\*\*4+c), x)

[Out] a\*\*2\*c\*x + b\*\*2\*d\*x\*\*13/13 + x\*\*9\*(2\*a\*b\*d/9 + b\*\*2\*c/9) + x\*\*5\*(a\*\*2\*d/5 + 2\*a\*b\*c/5)

$$3.157 \quad \int \frac{(a+bx^4)^2}{c+dx^4} dx$$

**Optimal.** Leaf size=253

$$\frac{(bc-ad)^2 \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4} d^{9/4}} + \frac{(bc-ad)^2 \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4} d^{9/4}} - \frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}{2\sqrt{2} c^{3/4} d^{9/4}}\right)}{2\sqrt{2} c^{3/4} d^{9/4}}$$

[Out]  $-b*(-2*a*d+b*c)*x/d^2+1/5*b^2*x^5/d+1/4*(-a*d+b*c)^2*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(3/4)}/d^{(9/4)}*2^{(1/2)}+1/4*(-a*d+b*c)^2*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(3/4)}/d^{(9/4)}*2^{(1/2)}-1/8*(-a*d+b*c)^2*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(3/4)}/d^{(9/4)}*2^{(1/2)}+1/8*(-a*d+b*c)^2*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(3/4)}/d^{(9/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^2 \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4} d^{9/4}} + \frac{(bc-ad)^2 \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{4\sqrt{2} c^{3/4} d^{9/4}} - \frac{(bc-ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2}{2\sqrt{2} c^{3/4} d^{9/4}}\right)}{2\sqrt{2} c^{3/4} d^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2/(c + d\*x^4), x]

[Out]  $-((b*(b*c - 2*a*d)*x)/d^2) + (b^2*x^5)/(5*d) - ((b*c - a*d)^2*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*\text{Sqrt}[2]*c^{(3/4)}*d^{(9/4)}) + ((b*c - a*d)^2*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(2*\text{Sqrt}[2]*c^{(3/4)}*d^{(9/4)}) - ((b*c - a*d)^2*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(4*\text{Sqrt}[2]*c^{(3/4)}*d^{(9/4)}) + ((b*c - a*d)^2*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(4*\text{Sqrt}[2]*c^{(3/4)}*d^{(9/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\{(d_) + (e_)(x_)\}/\{(a_) + (b_)(x_) + (c_)(x_)^2\}, x\_Symbol] \ :> \ \text{Simp}[\{d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]\}/b, x] \ /; \ \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2 \cdot c \cdot d - b \cdot e, 0]$

### Rule 1162

$\text{Int}[\{(d_) + (e_)(x_)^2\}/\{(a_) + (c_)(x_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e + q \cdot x + x^2, x], x] + \text{Dist}[e/(2 \cdot c), \text{Int}[1/\text{Simp}[d/e - q \cdot x + x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{PosQ}[d \cdot e]$

### Rule 1165

$\text{Int}[\{(d_) + (e_)(x_)^2\}/\{(a_) + (c_)(x_)^4\}, x\_Symbol] \ :> \ \text{With}[\{q = \text{Rt}[-(2 \cdot d)/e, 2]\}, \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q - 2 \cdot x)/\text{Simp}[d/e + q \cdot x - x^2, x], x] + \text{Dist}[e/(2 \cdot c \cdot q), \text{Int}[(q + 2 \cdot x)/\text{Simp}[d/e - q \cdot x - x^2, x], x], x] \ /; \ \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c \cdot d^2 - a \cdot e^2, 0] \ \&\& \ \text{NegQ}[d \cdot e]$

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^4)^2}{c + dx^4} dx &= \int \left( -\frac{b(bc - 2ad)}{d^2} + \frac{b^2x^4}{d} + \frac{b^2c^2 - 2abcd + a^2d^2}{d^2(c + dx^4)} \right) dx \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^5}{5d} + \frac{(bc - ad)^2 \int \frac{1}{c + dx^4} dx}{d^2} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^5}{5d} + \frac{(bc - ad)^2 \int \frac{\sqrt{c} - \sqrt{d}x^2}{c + dx^4} dx}{2\sqrt{c}d^2} + \frac{(bc - ad)^2 \int \frac{\sqrt{c} + \sqrt{d}x^2}{c + dx^4} dx}{2\sqrt{c}d^2} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^5}{5d} + \frac{(bc - ad)^2 \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{5/2}} + \frac{(bc - ad)^2 \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} + \frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{d}} + x^2} dx}{4\sqrt{c}d^{5/2}} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^5}{5d} - \frac{(bc - ad)^2 \log(\sqrt{c} - \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc - ad)^2 \log(\sqrt{c} + \sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{d}x^2)}{4\sqrt{2}c^{3/4}d^{9/4}} \\ &= -\frac{b(bc - 2ad)x}{d^2} + \frac{b^2x^5}{5d} - \frac{(bc - ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} + \frac{(bc - ad)^2 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)}{2\sqrt{2}c^{3/4}d^{9/4}} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 231, normalized size = 0.91

$$-40bc^{3/4}\sqrt[4]{d}x(bc - 2ad) - 5\sqrt{2}(bc - ad)^2 \log(-\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2) + 5\sqrt{2}(bc - ad)^2 \log(\sqrt{2}\sqrt[4]{c}\sqrt[4]{d}x + \sqrt{c} + \sqrt{d}x^2)$$

40c<sup>3/4</sup>

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2/(c + d\*x^4), x]

[Out] (-40\*b\*c^(3/4)\*d^(1/4)\*(b\*c - 2\*a\*d)\*x + 8\*b^2\*c^(3/4)\*d^(5/4)\*x^5 - 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 10\*Sqrt[2]\*(b\*c

$$c - a*d)^2 * \text{ArcTan}\left[1 + \frac{\sqrt{2} * d^{1/4} * x}{c^{1/4}}\right] - 5 * \sqrt{2} * (b*c - a*d)^2 * \text{Log}\left[\frac{\sqrt{c} - \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{d} * x^2}{\sqrt{c} + \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{d} * x^2}\right] / (40 * c^{3/4} * d^{9/4})$$

**fricas [B]** time = 1.13, size = 1239, normalized size = 4.90

$$4 b^2 d x^5 + 20 d^2 \left( -\frac{b^8 c^8 - 8 a b^7 c^7 d + 28 a^2 b^6 c^6 d^2 - 56 a^3 b^5 c^5 d^3 + 70 a^4 b^4 c^4 d^4 - 56 a^5 b^3 c^3 d^5 + 28 a^6 b^2 c^2 d^6 - 8 a^7 b c d^7 + a^8 d^8}{c^3 d^9} \right)^{\frac{1}{4}} \arctan \left( -\frac{c^2 d^7 x \left( -\frac{b^8 c^8}{c^3 d^9} \right)}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c),x, algorithm="fricas")

[Out]  $\frac{1}{20} * (4 * b^2 * d * x^5 + 20 * d^2 * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{1/4} * \arctan(- (c^2 * d^7 * x * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{3/4} - c^2 * d^7 * \sqrt{(c^2 * d^4 * \sqrt{(c^2 * d^4 * \sqrt{(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))} + (b^4 * c^4 - 4 * a * b^3 * c^3 * d + 6 * a^2 * b^2 * c^2 * d^2 - 4 * a^3 * b * c * d^3 + a^4 * d^4) * x^2) / (b^4 * c^4 - 4 * a * b^3 * c^3 * d + 6 * a^2 * b^2 * c^2 * d^2 - 4 * a^3 * b * c * d^3 + a^4 * d^4)) * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{3/4} / (b^6 * c^6 - 6 * a * b^5 * c^5 * d + 15 * a^2 * b^4 * c^4 * d^2 - 20 * a^3 * b^3 * c^3 * d^3 + 15 * a^4 * b^2 * c^2 * d^4 - 6 * a^5 * b * c * d^5 + a^6 * d^6)) + 5 * d^2 * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{1/4} * \log(c * d^2 * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{1/4} + (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) * x) - 5 * d^2 * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{1/4} * \log(-c * d^2 * (-(b^8 * c^8 - 8 * a * b^7 * c^7 * d + 28 * a^2 * b^6 * c^6 * d^2 - 56 * a^3 * b^5 * c^5 * d^3 + 70 * a^4 * b^4 * c^4 * d^4 - 56 * a^5 * b^3 * c^3 * d^5 + 28 * a^6 * b^2 * c^2 * d^6 - 8 * a^7 * b * c * d^7 + a^8 * d^8) / (c^3 * d^9))^{1/4} + (b^2 * c^2 - 2 * a * b * c * d + a^2 * d^2) * x) - 20 * (b^2 * c^2 - 2 * a * b * c * d) * x) / d^2$

**giac [A]** time = 0.20, size = 353, normalized size = 1.40

$$\frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4 cd^3} + \frac{\sqrt{2} \left( (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd + (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{4 cd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c),x, algorithm="giac")

[Out]  $\frac{1}{4} * \sqrt{2} * ((c*d^3)^{1/4} * b^2 * c^2 - 2 * (c*d^3)^{1/4} * a * b * c * d + (c*d^3)^{1/4} * a^2 * d^2) * \arctan(1/2 * \sqrt{2} * (2 * x + \sqrt{2} * (c/d)^{1/4}) / (c/d)^{1/4}) / (c*d^3) + 1/4 * \sqrt{2} * ((c*d^3)^{1/4} * b^2 * c^2 - 2 * (c*d^3)^{1/4} * a * b * c * d + (c*d^3)^{1/4} * a^2 * d^2) * \arctan(1/2 * \sqrt{2} * (2 * x - \sqrt{2} * (c/d)^{1/4}) / (c/d)^{1/4}) / (c*d^3)$

$$\begin{aligned} &)/(c*d^3) + 1/8*\sqrt{2}*((c*d^3)^{(1/4)}*b^2*c^2 - 2*(c*d^3)^{(1/4)}*a*b*c*d + \\ &(c*d^3)^{(1/4)}*a^2*d^2)*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(c*d^3) \\ &- 1/8*\sqrt{2}*((c*d^3)^{(1/4)}*b^2*c^2 - 2*(c*d^3)^{(1/4)}*a*b*c*d + (c*d^3)^{(1/4)} \\ &(1/4)*a^2*d^2)*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(c*d^3) + 1/5*(b \\ &^2*d^4*x^5 - 5*b^2*c*d^3*x + 10*a*b*d^4*x)/d^5 \end{aligned}$$

**maple [B]** time = 0.05, size = 436, normalized size = 1.72

$$\frac{b^2x^5}{5d} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}-1\right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}a^2\arctan\left(\frac{\sqrt{2}x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}+1\right)}{4c} + \frac{\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}a^2\ln\left(\frac{x^2+\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}{x^2-\left(\frac{c}{d}\right)^{\frac{1}{4}}\sqrt{2}x+\sqrt{\frac{c}{d}}}\right)}{8c} + \frac{2abx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2/(d\*x^4+c), x)

[Out] 1/5\*b^2\*x^5/d+2\*b/d\*a\*x-b^2/d^2\*c\*x+1/8\*(c/d)^(1/4)/c\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2)))\*a^2-1/4/d\*(c/d)^(1/4)\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2)))\*a\*b+1/8/d^2\*(c/d)^(1/4)\*c\*2^(1/2)\*ln((x^2+(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)\*2^(1/2)\*x+(c/d)^(1/2)))\*b^2+1/4\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a^2-1/2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*a\*b+1/4/d^2\*(c/d)^(1/4)\*c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x-1)\*b^2+1/4\*(c/d)^(1/4)/c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a^2-1/2/d\*(c/d)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*a\*b+1/4/d^2\*(c/d)^(1/4)\*c\*2^(1/2)\*arctan(2^(1/2)/(c/d)^(1/4)\*x+1)\*b^2

**maxima [A]** time = 1.29, size = 286, normalized size = 1.13

$$\frac{b^2dx^5 - 5(b^2c - 2abd)x}{5d^2} + \frac{2\sqrt{2}(b^2c^2 - 2abcd + a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x + \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}} + \frac{2\sqrt{2}(b^2c^2 - 2abcd + a^2d^2)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{d}x - \sqrt{2}c^{\frac{1}{4}}d^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c), x, algorithm="maxima")

[Out] 1/5\*(b^2\*d\*x^5 - 5\*(b^2\*c - 2\*a\*b\*d)\*x)/d^2 + 1/8\*(2\*sqrt(2)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(d)\*x + sqrt(2)\*c^(1/4)\*d^(1/4))/sqrt(sqrt(c)\*sqrt(d)))/sqrt(c)\*sqrt(sqrt(c)\*sqrt(d)) + 2\*sqrt(2)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(d)\*x - sqrt(2)\*c^(1/4)\*d^(1/4))/sqrt(sqrt(c)\*sqrt(d)))/sqrt(c)\*sqrt(sqrt(c)\*sqrt(d)) + sqrt(2)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log(sqrt(d)\*x^2 + sqrt(2)\*c^(1/4)\*d^(1/4)\*x + sqrt(c))/(c^(3/4)\*d^(1/4)) - sqrt(2)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*log(sqrt(d)\*x^2 - sqrt(2)\*c^(1/4)\*d^(1/4)\*x + sqrt(c))/(c^(3/4)\*d^(1/4))/d^2

**mupad [B]** time = 1.48, size = 1081, normalized size = 4.27

$$\frac{b^2x^5}{5d} - x\left(\frac{b^2c}{d^2} - \frac{2ab}{d}\right) + \frac{\operatorname{atan}\left(\frac{\frac{(ad-bc)^2\left(x\frac{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d} - \frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)}{4(-c)^{3/4}d^{9/4}}\right)}{(-c)^{3/4}d^{9/4}}}{\frac{(ad-bc)^2\left(x\frac{(a^4d^4-4a^3bcd^3+6a^2b^2c^2d^2-4ab^3c^3d+b^4c^4)}{d} - \frac{(ad-bc)^2(4a^2cd^3-8abc^2d^2+4b^2c^3d)}{4(-c)^{3/4}d^{9/4}}\right)}{(-c)^{3/4}d^{9/4}}}\right)}{2(-c)^{3/4}d^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^2/(c + d\*x^4),x)

[Out] (b^2\*x^5)/(5\*d) - x\*((b^2\*c)/d^2 - (2\*a\*b)/d) + (atan((((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d - ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2))/(4\*(-c)^(3/4)\*d^(9/4))))\*1i)/((-c)^(3/4)\*d^(9/4)) + ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d + ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2))/(4\*(-c)^(3/4)\*d^(9/4))))\*1i)/((-c)^(3/4)\*d^(9/4)))/(((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d - ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2))/(4\*(-c)^(3/4)\*d^(9/4)))))/((-c)^(3/4)\*d^(9/4)) - ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d + ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2))/(4\*(-c)^(3/4)\*d^(9/4)))))/((-c)^(3/4)\*d^(9/4)))\*1i)/(2\*(-c)^(3/4)\*d^(9/4)) + (atan((((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d - ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2)\*1i)/(4\*(-c)^(3/4)\*d^(9/4)))))/((-c)^(3/4)\*d^(9/4)) + ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d + ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2)\*1i)/(4\*(-c)^(3/4)\*d^(9/4)))))/((-c)^(3/4)\*d^(9/4)))/(((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d - ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2)\*1i)/(4\*(-c)^(3/4)\*d^(9/4)))))/((-c)^(3/4)\*d^(9/4)) - ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/d + ((a\*d - b\*c)^2\*(4\*a^2\*c\*d^3 + 4\*b^2\*c^3\*d - 8\*a\*b\*c^2\*d^2)\*1i)/(4\*(-c)^(3/4)\*d^(9/4))))\*1i)/((-c)^(3/4)\*d^(9/4)))\*1i)/((-c)^(3/4)\*d^(9/4)) - ((a\*d - b\*c)^2)/((2\*(-c)^(3/4)\*d^(9/4))

**sympy** [A] time = 1.12, size = 187, normalized size = 0.74

$$\frac{b^2x^5}{5d} + x \left( \frac{2ab}{d} - \frac{b^2c}{d^2} \right) + \text{RootSum} \left( 256t^4c^3d^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c),x)

[Out] b\*\*2\*x\*\*5/(5\*d) + x\*(2\*a\*b/d - b\*\*2\*c/d\*\*2) + RootSum(256\*\_t\*\*4\*c\*\*3\*d\*\*9 + a\*\*8\*d\*\*8 - 8\*a\*\*7\*b\*c\*d\*\*7 + 28\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 - 56\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 70\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 - 56\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 + 28\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 - 8\*a\*b\*\*7\*c\*\*7\*d + b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(4\*\_t\*c\*d\*\*2/(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2) + x)))



$$3.158 \quad \int \frac{(a+bx^4)^2}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=291

$$\frac{(bc-ad)(3ad+5bc) \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{9/4}} - \frac{(bc-ad)(3ad+5bc) \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{9/4}}$$

[Out]  $b^2x/d^2+1/4*(-a*d+b*c)^2*x/c/d^2/(d*x^4+c)-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/d^{(9/4)}*2^{(1/2)}-1/16*(-a*d+b*c)*(3*a*d+5*b*c)*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/d^{(9/4)}*2^{(1/2)}+1/32*(-a*d+b*c)*(3*a*d+5*b*c)*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/d^{(9/4)}*2^{(1/2)}-1/32*(-a*d+b*c)*(3*a*d+5*b*c)*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/d^{(9/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.37, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(3ad+5bc) \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{9/4}} - \frac{(bc-ad)(3ad+5bc) \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{16\sqrt{2} c^{7/4} d^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2/(c + d\*x^4)^2, x]

[Out]  $(b^2*x)/d^2 + ((b*c - a*d)^2*x)/(4*c*d^2*(c + d*x^4)) + ((b*c - a*d)*(5*b*c + 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(9/4)}) - ((b*c - a*d)*(5*b*c + 3*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*\text{Sqrt}[2]*c^{(7/4)}*d^{(9/4)}) + ((b*c - a*d)*(5*b*c + 3*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(9/4)}) - ((b*c - a*d)*(5*b*c + 3*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{(1/4)}*d^{(1/4)}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{(7/4)}*d^{(9/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :=> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :=> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :=> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :=> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^4)^2}{(c + dx^4)^2} dx &= \int \left( \frac{b^2}{d^2} - \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^4}{d^2(c + dx^4)^2} \right) dx \\
&= \frac{b^2x}{d^2} - \frac{\int \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^4}{(c + dx^4)^2} dx}{d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{4cd^2(c + dx^4)} - \frac{((bc - ad)(5bc + 3ad)) \int \frac{1}{c + dx^4} dx}{4cd^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{4cd^2(c + dx^4)} - \frac{((bc - ad)(5bc + 3ad)) \int \frac{\sqrt{c} - \sqrt{d}x^2}{c + dx^4} dx}{8c^{3/2}d^2} - \frac{((bc - ad)(5bc + 3ad)) \int \frac{\sqrt{c}}{c + dx^4} dx}{8c^{3/2}d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{4cd^2(c + dx^4)} - \frac{((bc - ad)(5bc + 3ad)) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2}{\sqrt{d}}} dx}{16c^{3/2}d^{5/2}} - \frac{((bc - ad)(5bc + 3ad)) \int \frac{\sqrt{c}}{c + dx^4} dx}{16c^{3/2}d^2} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{4cd^2(c + dx^4)} + \frac{(bc - ad)(5bc + 3ad) \log(\sqrt{c} - \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{16\sqrt{2} c^{7/4} d^{9/4}} - \frac{(bc - ad)(5bc + 3ad) \log(\sqrt{c} + \sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{d} x^2)}{16\sqrt{2} c^{7/4} d^{9/4}} \\
&= \frac{b^2x}{d^2} + \frac{(bc - ad)^2x}{4cd^2(c + dx^4)} + \frac{(bc - ad)(5bc + 3ad) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{8\sqrt{2} c^{7/4} d^{9/4}} - \frac{(bc - ad)(5bc + 3ad) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{8\sqrt{2} c^{7/4} d^{9/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.22, size = 298, normalized size = 1.02

$$\frac{\sqrt{2}(-3a^2d^2 - 2abcd + 5b^2c^2) \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{c^{7/4}} - \frac{\sqrt{2}(-3a^2d^2 - 2abcd + 5b^2c^2) \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right)}{c^{7/4}} + \frac{2\sqrt{2}(-3a^2d^2 - 2abcd)}{32d^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2/(c + d\*x^4)^2,x]

[Out] (32\*b^2\*d^(1/4)\*x + (8\*d^(1/4)\*(b\*c - a\*d)^2\*x)/(c\*(c + d\*x^4)) + (2\*sqrt[2] \* (5\*b^2\*c^2 - 2\*a\*b\*c\*d - 3\*a^2\*d^2)\*ArcTan[1 - (sqrt[2]\*d^(1/4)\*x)/c^(1/4)]) / c^(7/4) - (2\*sqrt[2] \* (5\*b^2\*c^2 - 2\*a\*b\*c\*d - 3\*a^2\*d^2)\*ArcTan[1 + (sqrt[2]\*d^(1/4)\*x)/c^(1/4)]) / c^(7/4) + (sqrt[2] \* (5\*b^2\*c^2 - 2\*a\*b\*c\*d - 3\*a^2\*d^2)\*Log[sqrt[c] - sqrt[2]\*c^(1/4)\*d^(1/4)\*x + sqrt[d]\*x^2]) / c^(7/4) - (sqrt[2] \* (5\*b^2\*c^2 - 2\*a\*b\*c\*d - 3\*a^2\*d^2)\*Log[sqrt[c] + sqrt[2]\*c^(1/4)\*d^(1/4)\*x + sqrt[d]\*x^2]) / c^(7/4)) / (32\*d^(9/4))

**fricas [B]** time = 1.27, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] 1/16\*(16\*b^2\*c\*d\*x^5 + 4\*(c\*d^3\*x^4 + c^2\*d^2)\*(-625\*b^8\*c^8 - 1000\*a\*b^7\*c^7\*d - 900\*a^2\*b^6\*c^6\*d^2 + 1640\*a^3\*b^5\*c^5\*d^3 + 646\*a^4\*b^4\*c^4\*d^4 - 984\*a^5\*b^3\*c^3\*d^5 - 324\*a^6\*b^2\*c^2\*d^6 + 216\*a^7\*b\*c\*d^7 + 81\*a^8\*d^8)/(c^7\*d^9))^(1/4)\*arctan((c^5\*d^7\*x\*(-625\*b^8\*c^8 - 1000\*a\*b^7\*c^7\*d - 900\*a^2\*b^6\*c^6\*d^2 + 1640\*a^3\*b^5\*c^5\*d^3 + 646\*a^4\*b^4\*c^4\*d^4 - 984\*a^5\*b^3\*c^3\*d^5 - 324\*a^6\*b^2\*c^2\*d^6 + 216\*a^7\*b\*c\*d^7 + 81\*a^8\*d^8)/(c^7\*d^9))^(1/4) + sqrt[d]\*x^2)/sqrt[c] + sqrt[2]\*c^(1/4)\*d^(1/4)\*x + sqrt[d]\*x^2)) / (32\*d^(9/4))

$$\begin{aligned} & \left( c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right)^{3/4} - c^5 d^7 \sqrt{\left( c^4 d^4 \sqrt{-\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right)} \right) + \left( 25 b^4 c^4 - 20 a b^3 c^3 d - 26 a^2 b^2 c^2 d^2 + 12 a^3 b c d^3 + 9 a^4 d^4 \right) * x^2 / \left( 25 b^4 c^4 - 20 a b^3 c^3 d - 26 a^2 b^2 c^2 d^2 + 12 a^3 b c d^3 + 9 a^4 d^4 \right) * \left( -\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right) \right)^{3/4} / \left( 125 b^6 c^6 - 150 a b^5 c^5 d - 165 a^2 b^4 c^4 d^2 + 172 a^3 b^3 c^3 d^3 + 99 a^4 b^2 c^2 d^4 - 54 a^5 b c d^5 - 27 a^6 d^6 \right) + \left( c d^3 x^4 + c^2 d^2 \right) * \left( -\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right) \right)^{1/4} * \log \left( c^2 d^2 * \left( -\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right) \right)^{1/4} - \left( 5 b^2 c^2 - 2 a b c d - 3 a^2 d^2 \right) * x \right) - \left( c d^3 x^4 + c^2 d^2 \right) * \left( -\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right) \right)^{1/4} * \log \left( -c^2 d^2 * \left( -\left( 625 b^8 c^8 - 1000 a b^7 c^7 d - 900 a^2 b^6 c^6 d^2 + 1640 a^3 b^5 c^5 d^3 + 646 a^4 b^4 c^4 d^4 - 984 a^5 b^3 c^3 d^5 - 324 a^6 b^2 c^2 d^6 + 216 a^7 b c d^7 + 81 a^8 d^8 \right) / \left( c^7 d^9 \right) \right)^{1/4} - \left( 5 b^2 c^2 - 2 a b c d - 3 a^2 d^2 \right) * x \right) + 4 * \left( 5 b^2 c^2 - 2 a b c d + a^2 d^2 \right) * x \right) / \left( c d^3 x^4 + c^2 d^2 \right) \end{aligned}$$

**giac** [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{b^2 x}{d^2} - \frac{\sqrt{2} \left( 5 (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd - 3 (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{16 c^2 d^3} - \frac{\sqrt{2} \left( 5 (cd^3)^{\frac{1}{4}} b^2 c^2 - 2 (cd^3)^{\frac{1}{4}} abcd - 3 (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{16 c^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c)^2,x, algorithm="giac")

[Out] 
$$\begin{aligned} & b^2 x / d^2 - 1/16 \sqrt{2} * \left( 5 * \left( c d^3 \right)^{1/4} * b^2 c^2 - 2 * \left( c d^3 \right)^{1/4} * a b c d - 3 * \left( c d^3 \right)^{1/4} * a^2 d^2 \right) * \arctan \left( 1/2 \sqrt{2} * \left( 2 * x + \sqrt{2} * \left( c / d \right)^{1/4} \right) / \left( c / d \right)^{1/4} \right) / \left( c^2 d^3 \right) - 1/16 \sqrt{2} * \left( 5 * \left( c d^3 \right)^{1/4} * b^2 c^2 - 2 * \left( c d^3 \right)^{1/4} * a b c d - 3 * \left( c d^3 \right)^{1/4} * a^2 d^2 \right) * \arctan \left( 1/2 \sqrt{2} * \left( 2 * x - \sqrt{2} * \left( c / d \right)^{1/4} \right) / \left( c / d \right)^{1/4} \right) / \left( c^2 d^3 \right) - 1/32 \sqrt{2} * \left( 5 * \left( c d^3 \right)^{1/4} * b^2 c^2 - 2 * \left( c d^3 \right)^{1/4} * a b c d - 3 * \left( c d^3 \right)^{1/4} * a^2 d^2 \right) * \log \left( x^2 + \sqrt{2} * x * \left( c / d \right)^{1/4} + \sqrt{c / d} \right) / \left( c^2 d^3 \right) + 1/32 \sqrt{2} * \left( 5 * \left( c d^3 \right)^{1/4} * b^2 c^2 - 2 * \left( c d^3 \right)^{1/4} * a b c d - 3 * \left( c d^3 \right)^{1/4} * a^2 d^2 \right) * \log \left( x^2 - \sqrt{2} * x * \left( c / d \right)^{1/4} + \sqrt{c / d} \right) / \left( c^2 d^3 \right) + 1/4 * \left( b^2 c^2 x - 2 a b c d x + a^2 d^2 x \right) / \left( \left( d x^4 + c \right) * c d^2 \right) \end{aligned}$$

**maple** [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{a^2 x}{4 \left( d x^4 + c \right) c} - \frac{a b x}{2 \left( d x^4 + c \right) d} + \frac{b^2 c x}{4 \left( d x^4 + c \right) d^2} + \frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} - 1 \right)}{16 c^2} + \frac{3 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} + 1 \right)}{16 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2/(d\*x^4+c)^2,x)

[Out] 
$$b^2/d^2*x+1/4/c*x/(d*x^4+c)*a^2-1/2/d*x/(d*x^4+c)*a*b+1/4/d^2*c*x/(d*x^4+c)*b^2+3/16/c^2*(c/d)^{1/4}*2^{1/2}*arctan(2^{1/2}/(c/d)^{1/4}*x-1)*a^2+1/8/d$$



$$\begin{aligned}
& ) + ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)* \\
& 1i)/(16*(-c)^{(7/4)}*d^{(9/4)}))*(a*d - b*c)*(3*a*d + 5*b*c))/(16*(-c)^{(7/4)}*d^{(9/4)})) \\
& )/(((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d \\
& + 12*a^3*b*c*d^3))/(4*c^2*d) - ((a*d - b*c)*(3*a*d + 5*b*c)*(12*a^2*d^3 - \\
& 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^{(7/4)}*d^{(9/4)}))*(a*d - b*c)*(3*a*d \\
& + 5*b*c)*1i)/(16*(-c)^{(7/4)}*d^{(9/4)}) - (((x*(9*a^4*d^4 + 25*b^4*c^4 - 26*a \\
& ^2*b^2*c^2*d^2 - 20*a*b^3*c^3*d + 12*a^3*b*c*d^3))/(4*c^2*d) + ((a*d - b*c) \\
& *(3*a*d + 5*b*c)*(12*a^2*d^3 - 20*b^2*c^2*d + 8*a*b*c*d^2)*1i)/(16*(-c)^{(7/4)}*d^{(9/4)})) \\
& )*(a*d - b*c)*(3*a*d + 5*b*c)*1i)/(16*(-c)^{(7/4)}*d^{(9/4)})))*(a*d \\
& - b*c)*(3*a*d + 5*b*c))/(8*(-c)^{(7/4)}*d^{(9/4)})
\end{aligned}$$

**sympy [A]** time = 1.98, size = 219, normalized size = 0.75

$$\frac{b^2x}{d^2} + \frac{x(a^2d^2 - 2abcd + b^2c^2)}{4c^2d^2 + 4cd^3x^4} + \text{RootSum}\left(65536t^4c^7d^9 + 81a^8d^8 + 216a^7bcd^7 - 324a^6b^2c^2d^6 - 984a^5b^3c^3d^5 + 64\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*2,x)

[Out] b\*\*2\*x/d\*\*2 + x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(4\*c\*\*2\*d\*\*2 + 4\*c\*d\*\*3\*x\*\*4) + RootSum(65536\*\_t\*\*4\*c\*\*7\*d\*\*9 + 81\*a\*\*8\*d\*\*8 + 216\*a\*\*7\*b\*c\*d\*\*7 - 324\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 - 984\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 646\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 + 1640\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 - 900\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 - 1000\*a\*b\*\*7\*c\*\*7\*d + 625\*b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(16\*\_t\*c\*\*2\*d\*\*2/(3\*a\*\*2\*d\*\*2 + 2\*a\*b\*c\*d - 5\*b\*\*2\*c\*\*2) + x)))

$$3.159 \quad \int \frac{(a+bx^4)^2}{(c+dx^4)^3} dx$$

**Optimal.** Leaf size=349

$$\frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{128\sqrt{2} c^{11/4} d^{9/4}} + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{128\sqrt{2} c^{11/4} d^{9/4}}$$

[Out]  $-1/8*(-a*d+b*c)*x*(b*x^4+a)/c/d/(d*x^4+c)^2-1/32*(-a*d+b*c)*(7*a*d+5*b*c)*x/c^2/d^2/(d*x^4+c)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*\arctan(-1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)+1/128*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*\arctan(1+d^(1/4)*x*2^(1/2)/c^(1/4))/c^(11/4)/d^(9/4)*2^(1/2)-1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*\ln(-c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)+1/256*(21*a^2*d^2+6*a*b*c*d+5*b^2*c^2)*\ln(c^(1/4)*d^(1/4)*x*2^(1/2)+c^(1/2)+x^2*d^(1/2))/c^(11/4)/d^(9/4)*2^(1/2)$

**Rubi [A]** time = 0.27, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {413, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{128\sqrt{2} c^{11/4} d^{9/4}} + \frac{(21a^2d^2 + 6abcd + 5b^2c^2) \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{128\sqrt{2} c^{11/4} d^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2/(c + d\*x^4)^3, x]

[Out]  $-((b*c - a*d)*x*(a + b*x^4))/(8*c*d*(c + d*x^4)^2) - ((b*c - a*d)*(5*b*c + 7*a*d)*x)/(32*c^2*d^2*(c + d*x^4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\text{ArcTan}[1 - (\text{Sqrt}[2]*d^(1/4)*x)/c^(1/4)])/(64*\text{Sqrt}[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\text{ArcTan}[1 + (\text{Sqrt}[2]*d^(1/4)*x)/c^(1/4)])/(64*\text{Sqrt}[2]*c^(11/4)*d^(9/4)) - ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^(1/4)*d^(1/4)*x + \text{Sqrt}[d]*x^2])/(128*\text{Sqrt}[2]*c^(11/4)*d^(9/4)) + ((5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^(1/4)*d^(1/4)*x + \text{Sqrt}[d]*x^2])/(128*\text{Sqrt}[2]*c^(11/4)*d^(9/4))$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n +

p, 0])

### Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps



$$\begin{aligned}
\int \frac{(a + bx^4)^2}{(c + dx^4)^3} dx &= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} + \frac{\int \frac{a(bc+7ad)+b(5bc+3ad)x^4}{(c+dx^4)^2} dx}{8cd} \\
&= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} - \frac{(bc - ad)(5bc + 7ad)x}{32c^2d^2(c + dx^4)} + \frac{(5b^2c^2 + 6abcd + 21a^2d^2) \int \frac{1}{c+dx^4} dx}{32c^2d^2} \\
&= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} - \frac{(bc - ad)(5bc + 7ad)x}{32c^2d^2(c + dx^4)} + \frac{(5b^2c^2 + 6abcd + 21a^2d^2) \int \frac{\sqrt{c} - \sqrt{d}x^2}{c+dx^4} dx}{64c^{5/2}d^2} \\
&= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} - \frac{(bc - ad)(5bc + 7ad)x}{32c^2d^2(c + dx^4)} + \frac{(5b^2c^2 + 6abcd + 21a^2d^2) \int \frac{1}{\frac{\sqrt{c}}{\sqrt{d}} - \frac{\sqrt{d}}{\sqrt{c}}x^2} dx}{128c^{5/2}d^{5/2}} \\
&= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} - \frac{(bc - ad)(5bc + 7ad)x}{32c^2d^2(c + dx^4)} - \frac{(5b^2c^2 + 6abcd + 21a^2d^2) \log(\sqrt{c} - \sqrt{d})}{128\sqrt{2}c^{11/4}d^{9/4}} \\
&= -\frac{(bc - ad)x(a + bx^4)}{8cd(c + dx^4)^2} - \frac{(bc - ad)(5bc + 7ad)x}{32c^2d^2(c + dx^4)} - \frac{(5b^2c^2 + 6abcd + 21a^2d^2) \tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt{d}}\right)}{64\sqrt{2}c^{11/4}d^{9/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 319, normalized size = 0.91

$$-\sqrt{2} (21a^2d^2 + 6abcd + 5b^2c^2) \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2) + \sqrt{2} (21a^2d^2 + 6abcd + 5b^2c^2) \log(\sqrt{2} \sqrt[4]{c})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2/(c + d\*x^4)^3,x]

[Out] ((32\*c^(7/4)\*d^(1/4)\*(b\*c - a\*d)^2\*x)/(c + d\*x^4)^2 - (8\*c^(3/4)\*d^(1/4)\*(9\*b^2\*c^2 - 2\*a\*b\*c\*d - 7\*a^2\*d^2)\*x)/(c + d\*x^4) - 2\*Sqrt[2]\*(5\*b^2\*c^2 + 6\*a\*b\*c\*d + 21\*a^2\*d^2)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 2\*Sqrt[2]\*(5\*b^2\*c^2 + 6\*a\*b\*c\*d + 21\*a^2\*d^2)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - Sqrt[2]\*(5\*b^2\*c^2 + 6\*a\*b\*c\*d + 21\*a^2\*d^2)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] + Sqrt[2]\*(5\*b^2\*c^2 + 6\*a\*b\*c\*d + 21\*a^2\*d^2)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(256\*c^(11/4)\*d^(9/4))

**fricas [B]** time = 1.40, size = 1411, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c)^3,x, algorithm="fricas")

[Out] -1/128\*(4\*(9\*b^2\*c^2\*d - 2\*a\*b\*c\*d^2 - 7\*a^2\*d^3)\*x^5 - 4\*(c^2\*d^4\*x^8 + 2\*c^3\*d^3\*x^4 + c^4\*d^2)\*(-(625\*b^8\*c^8 + 3000\*a\*b^7\*c^7\*d + 15900\*a^2\*b^6\*c^6\*d^2 + 42120\*a^3\*b^5\*c^5\*d^3 + 112806\*a^4\*b^4\*c^4\*d^4 + 176904\*a^5\*b^3\*c^3\*d^5 + 280476\*a^6\*b^2\*c^2\*d^6 + 222264\*a^7\*b\*c\*d^7 + 194481\*a^8\*d^8))/(c^11\*d^9)^(1/4)\*arctan(-(c^8\*d^7\*x\*(-(625\*b^8\*c^8 + 3000\*a\*b^7\*c^7\*d + 15900\*a^2\*b^6\*c^6\*d^2 + 42120\*a^3\*b^5\*c^5\*d^3 + 112806\*a^4\*b^4\*c^4\*d^4 + 176904\*a^5

$$\begin{aligned} & *b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8 \\ & )/(c^{11}*d^9))^{(3/4)} - c^8*d^7*\sqrt{((c^6*d^4*\sqrt{-(625*b^8*c^8 + 3000*a*b^7 \\ & *c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4*b^4*c^4 \\ & *d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7*b*c*d^7 \\ & + 194481*a^8*d^8)/(c^{11}*d^9))} + (25*b^4*c^4 + 60*a*b^3*c^3*d + 246*a^2*b^2 \\ & *c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4)*x^2)/(25*b^4*c^4 + 60*a*b^3*c^3*d \\ & + 246*a^2*b^2*c^2*d^2 + 252*a^3*b*c*d^3 + 441*a^4*d^4))*(-(625*b^8*c^8 + 3 \\ & 000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^5*d^3 + 112806*a^4 \\ & *b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^2*d^6 + 222264*a^7 \\ & *b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{(3/4)})/(125*b^6*c^6 + 450*a*b^5*c^5 \\ & *d + 2115*a^2*b^4*c^4*d^2 + 3996*a^3*b^3*c^3*d^3 + 8883*a^4*b^2*c^2*d^4 + 7 \\ & 938*a^5*b*c*d^5 + 9261*a^6*d^6)) - (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)* \\ & (-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^ \\ & ^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^ \\ & ^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{(1/4)}*\log(c^3*d^2 \\ & *(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^ \\ & ^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^ \\ & ^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{(1/4)} + (5*b^2*c \\ & ^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + (c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2)* \\ & (-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^ \\ & ^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^ \\ & ^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{(1/4)}*\log(-c^3*d^2 \\ & *(-(625*b^8*c^8 + 3000*a*b^7*c^7*d + 15900*a^2*b^6*c^6*d^2 + 42120*a^3*b^5*c^ \\ & ^5*d^3 + 112806*a^4*b^4*c^4*d^4 + 176904*a^5*b^3*c^3*d^5 + 280476*a^6*b^2*c^ \\ & ^2*d^6 + 222264*a^7*b*c*d^7 + 194481*a^8*d^8)/(c^{11}*d^9))^{(1/4)} + (5*b^2*c \\ & ^2 + 6*a*b*c*d + 21*a^2*d^2)*x) + 4*(5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2 \\ & )x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) \end{aligned}$$

**giac [A]** time = 0.21, size = 407, normalized size = 1.17

$$\frac{\sqrt{2} \left( 5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{128 c^3 d^3} + \frac{\sqrt{2} \left( 5 (cd^3)^{\frac{1}{4}} b^2 c^2 + 6 (cd^3)^{\frac{1}{4}} abcd + 21 (cd^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{c}{d} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{c}{d} \right)^{\frac{1}{4}}} \right)}{128 c^3 d^3} + \frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \ln \left( \frac{x^2 + \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}} \right)}{256 c^3} + 3 \left( \frac{c}{d} \right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2/(d\*x^4+c)^3,x, algorithm="giac")

[Out] 1/128\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^3) + 1/128\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(c/d)^(1/4))/(c/d)^(1/4))/(c^3\*d^3) + 1/256\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*log(x^2 + sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^3) - 1/256\*sqrt(2)\*(5\*(c\*d^3)^(1/4)\*b^2\*c^2 + 6\*(c\*d^3)^(1/4)\*a\*b\*c\*d + 21\*(c\*d^3)^(1/4)\*a^2\*d^2)\*log(x^2 - sqrt(2)\*x\*(c/d)^(1/4) + sqrt(c/d))/(c^3\*d^3) - 1/32\*(9\*b^2\*c^2\*d\*x^5 - 2\*a\*b\*c\*d^2\*x^5 - 7\*a^2\*d^3\*x^5 + 5\*b^2\*c^3\*x + 6\*a\*b\*c^2\*d\*x - 11\*a^2\*c\*d^2\*x)/((d\*x^4 + c)^2\*c^2\*d^2)

**maple [A]** time = 0.06, size = 499, normalized size = 1.43

$$\frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} - 1 \right)}{128 c^3} + \frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \arctan \left( \frac{\sqrt{2} x}{\left( \frac{c}{d} \right)^{\frac{1}{4}}} + 1 \right)}{128 c^3} + \frac{21 \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} a^2 \ln \left( \frac{x^2 + \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left( \frac{c}{d} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}} \right)}{256 c^3} + 3 \left( \frac{c}{d} \right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^2/(d*x^4+c)^3,x)
```

```
[Out] (1/32*(7*a^2*d^2+2*a*b*c*d-9*b^2*c^2)/c^2/d*x^5+1/32*(11*a^2*d^2-6*a*b*c*d-5*b^2*c^2)/d^2/c*x)/(d*x^4+c)^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x-1)*b^2+21/256/c^3*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a^2+3/128/c^2/d*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*a*b+5/256/c/d^2*(c/d)^(1/4)*2^(1/2)*ln((x^2+(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2))/(x^2-(c/d)^(1/4)*2^(1/2)*x+(c/d)^(1/2)))*b^2+21/128/c^3*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a^2+3/64/c^2/d*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*a*b+5/128/c/d^2*(c/d)^(1/4)*2^(1/2)*arctan(2^(1/2)/(c/d)^(1/4)*x+1)*b^2
```

**maxima [A]** time = 1.22, size = 361, normalized size = 1.03

$$\frac{(9b^2c^2d - 2abcd^2 - 7a^2d^3)x^5 + (5b^2c^3 + 6abc^2d - 11a^2cd^2)x}{32(c^2d^4x^8 + 2c^3d^3x^4 + c^4d^2)} + \frac{2\sqrt{2}(5b^2c^2 + 6abcd + 21a^2d^2) \arctan\left(\frac{\sqrt{2}(2\sqrt{d}x + \sqrt{2}c^{1/4}d^{1/4})}{2\sqrt{\sqrt{c}\sqrt{d}}}\right)}{\sqrt{c}\sqrt{\sqrt{c}\sqrt{d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^2/(d*x^4+c)^3,x, algorithm="maxima")
```

```
[Out] -1/32*((9*b^2*c^2*d - 2*a*b*c*d^2 - 7*a^2*d^3)*x^5 + (5*b^2*c^3 + 6*a*b*c^2*d - 11*a^2*c*d^2)*x)/(c^2*d^4*x^8 + 2*c^3*d^3*x^4 + c^4*d^2) + 1/256*(2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x + sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(c)*sqrt(sqrt(c)*sqrt(d)) + 2*sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*arctan(1/2*sqrt(2)*(2*sqrt(d)*x - sqrt(2)*c^(1/4)*d^(1/4))/sqrt(sqrt(c)*sqrt(d)))/sqrt(c)*sqrt(sqrt(c)*sqrt(d)) + sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 + sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)) - sqrt(2)*(5*b^2*c^2 + 6*a*b*c*d + 21*a^2*d^2)*log(sqrt(d)*x^2 - sqrt(2)*c^(1/4)*d^(1/4)*x + sqrt(c))/(c^(3/4)*d^(1/4)))/c^2*d^2
```

**mupad [B]** time = 1.66, size = 1401, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^2/(c + d*x^4)^3,x)
```

```
[Out] -((x*(5*b^2*c^2 - 11*a^2*d^2 + 6*a*b*c*d))/(32*c*d^2) - (x^5*(7*a^2*d^2 - 9*b^2*c^2 + 2*a*b*c*d))/(32*c^2*d))/(c^2 + d^2*x^8 + 2*c*d*x^4) - (atan((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)) - (((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*1i)/(128*(-c)^(11/4)*d^(9/4)))/((((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) - (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) + (((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4)) + (((21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d)*(21*a^2*d^3 + 5*b^2*c^2*d + 6*a*b*c*d^2))/(256*(-c)^(15/4)*d^(9/4)) + (x*(441*a^4*d^4 + 25*b^4*c^4 + 246*a^2*b^2*c^2*d^2 + 60*a*b^3*c^3*d + 252*a^3*b*c*d^3))/(256*c^4*d))*(21*a^2*d^2 + 5*b^2*c^2 + 6*a*b*c*d))/(128*(-c)^(11/4)*d^(9/4))
```

$$\frac{246a^2b^2c^2d^2 + 60ab^3c^3d + 252a^3b^2cd^3}{(256c^4d)} \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) / (128(-c)^{11/4}d^{9/4}) \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) \cdot i / (64(-c)^{11/4}d^{9/4}) - \operatorname{atan}\left(\frac{(21a^2d^2 + 5b^2c^2 + 6ab^2cd) \cdot (21a^2d^3 + 5b^2c^2d + 6ab^2cd^2) \cdot i}{256(-c)^{15/4}d^{9/4}}\right) - \frac{(441a^4d^4 + 25b^4c^4 + 246a^2b^2c^2d^2 + 60ab^3c^3d + 252a^3b^2cd^3)}{(256c^4d)} \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) / (128(-c)^{11/4}d^{9/4}) - \frac{(21a^2d^2 + 5b^2c^2 + 6ab^2cd) \cdot (21a^2d^3 + 5b^2c^2d + 6ab^2cd^2) \cdot i}{256(-c)^{15/4}d^{9/4}} + \frac{(441a^4d^4 + 25b^4c^4 + 246a^2b^2c^2d^2 + 60ab^3c^3d + 252a^3b^2cd^3)}{(256c^4d)} \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) / (128(-c)^{11/4}d^{9/4}) + \frac{(21a^2d^2 + 5b^2c^2 + 6ab^2cd) \cdot (21a^2d^3 + 5b^2c^2d + 6ab^2cd^2) \cdot i}{256(-c)^{15/4}d^{9/4}} + \frac{(441a^4d^4 + 25b^4c^4 + 246a^2b^2c^2d^2 + 60ab^3c^3d + 252a^3b^2cd^3)}{(256c^4d)} \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) / (128(-c)^{11/4}d^{9/4}) + \frac{(21a^2d^2 + 5b^2c^2 + 6ab^2cd) \cdot (21a^2d^3 + 5b^2c^2d + 6ab^2cd^2) \cdot i}{256(-c)^{15/4}d^{9/4}} + \frac{(441a^4d^4 + 25b^4c^4 + 246a^2b^2c^2d^2 + 60ab^3c^3d + 252a^3b^2cd^3)}{(256c^4d)} \cdot (21a^2d^2 + 5b^2c^2 + 6ab^2cd) / (128(-c)^{11/4}d^{9/4}) \cdot i / (64(-c)^{11/4}d^{9/4})$$

**sympy [A]** time = 5.85, size = 264, normalized size = 0.76

$$\frac{x^5(7a^2d^3 + 2abcd^2 - 9b^2c^2d) + x(11a^2cd^2 - 6abc^2d - 5b^2c^3)}{32c^4d^2 + 64c^3d^3x^4 + 32c^2d^4x^8} + \operatorname{RootSum}\left(268435456t^4c^{11}d^9 + 194481a^8d^8 + 222264a^7b^2cd^7 + 280476a^6b^2c^2d^6 + 176904a^5b^3c^3d^5 + 112806a^4b^4c^4d^4 + 42120a^3b^5c^5d^3 + 15900a^2b^6c^6d^2 + 3000ab^7c^7d + 625b^8c^8, \operatorname{Lambda}(t, t \cdot \log(128t^3cd^2 / (21a^2d^2 + 6ab^2cd + 5b^2c^2) + x))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*3,x)

[Out] (x\*\*5\*(7\*a\*\*2\*d\*\*3 + 2\*a\*b\*c\*d\*\*2 - 9\*b\*\*2\*c\*\*2\*d) + x\*(11\*a\*\*2\*c\*d\*\*2 - 6\*a\*b\*c\*\*2\*d - 5\*b\*\*2\*c\*\*3))/(32\*c\*\*4\*d\*\*2 + 64\*c\*\*3\*d\*\*3\*x\*\*4 + 32\*c\*\*2\*d\*\*4\*x\*\*8) + RootSum(268435456\*\_t\*\*4\*c\*\*11\*d\*\*9 + 194481\*a\*\*8\*d\*\*8 + 222264\*a\*\*7\*b\*c\*d\*\*7 + 280476\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 + 176904\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 112806\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 + 42120\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 + 15900\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 + 3000\*a\*b\*\*7\*c\*\*7\*d + 625\*b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(128\*\_t\*c\*\*3\*d\*\*2/(21\*a\*\*2\*d\*\*2 + 6\*a\*b\*c\*d + 5\*b\*\*2\*c\*\*2) + x)))

$$3.160 \quad \int \frac{(c+dx^4)^4}{a+bx^4} dx$$

**Optimal.** Leaf size=332

$$\frac{(bc-ad)^4 \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{17/4}} + \frac{(bc-ad)^4 \log(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{17/4}} - \frac{(bc-ad)^4 \tan^{-1}}{2\sqrt{2} a^{3/4} b^{17/4}}$$

[Out]  $d*(-a*d+2*b*c)*(a^2*d^2-2*a*b*c*d+2*b^2*c^2)*x/b^4+1/5*d^2*(a^2*d^2-4*a*b*c*d+6*b^2*c^2)*x^5/b^3+1/9*d^3*(-a*d+4*b*c)*x^9/b^2+1/13*d^4*x^{13}/b+1/4*(-a*d+b*c)^4*\arctan(-1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(17/4)}*2^{(1/2)}+1/4*(-a*d+b*c)^4*\arctan(1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(17/4)}*2^{(1/2)}-1/8*(-a*d+b*c)^4*\ln(-a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/b^{(17/4)}*2^{(1/2)}+1/8*(-a*d+b*c)^4*\ln(a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/b^{(17/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.27, antiderivative size = 332, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2 x^5 (a^2 d^2 - 4abcd + 6b^2 c^2)}{5b^3} + \frac{dx(2bc - ad)(a^2 d^2 - 2abcd + 2b^2 c^2)}{b^4} - \frac{(bc - ad)^4 \log(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4} b^{17/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^4/(a + b\*x^4), x]

[Out]  $(d*(2*b*c - a*d)*(2*b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x)/b^4 + (d^2*(6*b^2*c^2 - 4*a*b*c*d + a^2*d^2)*x^5)/(5*b^3) + (d^3*(4*b*c - a*d)*x^9)/(9*b^2) + (d^4*x^{13})/(13*b) - ((b*c - a*d)^4*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*b^{(17/4)}) + ((b*c - a*d)^4*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*b^{(17/4)}) - ((b*c - a*d)^4*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*b^{(17/4)}) + ((b*c - a*d)^4*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*b^{(17/4)})$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

**Rule 390**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

**Rule 617**

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^4)^4}{a + bx^4} dx &= \int \left( \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^4}{b^3} + \frac{d^3(4bc - ad)x^8}{b^2} + \frac{d^4x^{12}}{b} \right) dx \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^5}{5b^3} + \frac{d^3(4bc - ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^5}{5b^3} + \frac{d^3(4bc - ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^5}{5b^3} + \frac{d^3(4bc - ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} \\ &= \frac{d(2bc - ad)(2b^2c^2 - 2abcd + a^2d^2)x}{b^4} + \frac{d^2(6b^2c^2 - 4abcd + a^2d^2)x^5}{5b^3} + \frac{d^3(4bc - ad)x^9}{9b^2} + \frac{d^4x^{13}}{13b} \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 322, normalized size = 0.97

$$\frac{585\sqrt{2}(bc-ad)^4 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{a^{3/4}} + \frac{585\sqrt{2}(bc-ad)^4 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{a^{3/4}} - \frac{1170\sqrt{2}(bc-ad)^4 \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{a^{3/4}} + \dots$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^4/(a + b\*x^4),x]

[Out] (4680\*b^(1/4)\*d\*(4\*b^3\*c^3 - 6\*a\*b^2\*c^2\*d + 4\*a^2\*b\*c\*d^2 - a^3\*d^3)\*x + 9\*36\*b^(5/4)\*d^2\*(6\*b^2\*c^2 - 4\*a\*b\*c\*d + a^2\*d^2)\*x^5 + 520\*b^(9/4)\*d^3\*(4\*b\*c - a\*d)\*x^9 + 360\*b^(13/4)\*d^4\*x^13 - (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(3/4) + (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(3/4) - (585\*Sqrt[2]\*(b\*c - a\*d)^4\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(3/4) + (585\*Sqrt[2]\*(b\*c - a\*d)^4\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(3/4))/(4680\*b^(17/4))

**fricas** [B] time = 1.31, size = 2477, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^4/(b\*x^4+a),x, algorithm="fricas")

[Out] 1/2340\*(180\*b^3\*d^4\*x^13 + 260\*(4\*b^3\*c\*d^3 - a\*b^2\*d^4)\*x^9 + 468\*(6\*b^3\*c^2\*d^2 - 4\*a\*b^2\*c\*d^3 + a^2\*b\*d^4)\*x^5 + 2340\*b^4\*(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17))^(1/4) \*arctan(-(a^2\*b^13\*x\*(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17))^(3/4) - a^2\*b^13\*sqrt((a^2\*b^8\*sqrt(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17)) + (b^8\*c^8 - 8\*a\*b^7\*c^7\*d + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a^7\*b\*c\*d^7 + a^8\*d^8)\*x^2)/(b^8\*c^8 - 8\*a\*b^7\*c^7\*d + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a^7\*b\*c\*d^7 + a^8\*d^8))\*(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17))^(1/4)\*log(a\*b^4\*(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17))^(1/4)\*log(a\*b^4\*(-(b^16\*c^16 - 16\*a\*b^15\*c^15\*d + 120\*a^2\*b^14\*c^14\*d^2 - 560\*a^3\*b^13\*c^13\*d^3 + 1820\*a^4\*b^12\*c^12\*d^4 - 4368\*a^5\*b^11\*c^11\*d^5 + 8008\*a^6\*b^10\*c^10\*d^6 - 11440\*a^7\*b^9\*c^9\*d^7 + 12870\*a^8\*b^8\*c^8\*d^8 - 11440\*a^9\*b^7\*c^7\*d^9 + 8008\*a^10\*b^6\*c^6\*d^10 - 4368\*a^11\*b^5\*c^5\*d^11 + 1820\*a^12\*b^4\*c^4\*d^12 - 560\*a^13\*b^3\*c^3\*d^13 + 120\*a^14\*b^2\*c^2\*d^14 - 16\*a^15\*b\*c\*d^15 + a^16\*d^16)/(a^3\*b^17))^(1/4))

$$\begin{aligned} & *d^{16}/(a^3*b^{17})^{1/4} + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4 \\ & *a^3*b*c*d^3 + a^4*d^4)*x) - 585*b^4*(-(b^{16}*c^{16} - 16*a*b^{15}*c^{15}*d + 120* \\ & a^2*b^{14}*c^{14}*d^2 - 560*a^3*b^{13}*c^{13}*d^3 + 1820*a^4*b^{12}*c^{12}*d^4 - 4368*a \\ & ^5*b^{11}*c^{11}*d^5 + 8008*a^6*b^{10}*c^{10}*d^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a \\ & ^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 + 8008*a^{10}*b^6*c^6*d^{10} - 4368*a^{11} \\ & *b^5*c^5*d^{11} + 1820*a^{12}*b^4*c^4*d^{12} - 560*a^{13}*b^3*c^3*d^{13} + 120*a^{14}*b \\ & ^2*c^2*d^{14} - 16*a^{15}*b*c*d^{15} + a^{16}*d^{16})/(a^3*b^{17})^{1/4} * \log(-a*b^4*(- \\ & (b^{16}*c^{16} - 16*a*b^{15}*c^{15}*d + 120*a^2*b^{14}*c^{14}*d^2 - 560*a^3*b^{13}*c^{13}*d \\ & ^3 + 1820*a^4*b^{12}*c^{12}*d^4 - 4368*a^5*b^{11}*c^{11}*d^5 + 8008*a^6*b^{10}*c^{10}*d \\ & ^6 - 11440*a^7*b^9*c^9*d^7 + 12870*a^8*b^8*c^8*d^8 - 11440*a^9*b^7*c^7*d^9 \\ & + 8008*a^{10}*b^6*c^6*d^{10} - 4368*a^{11}*b^5*c^5*d^{11} + 1820*a^{12}*b^4*c^4*d^{12} \\ & - 560*a^{13}*b^3*c^3*d^{13} + 120*a^{14}*b^2*c^2*d^{14} - 16*a^{15}*b*c*d^{15} + a^{16}*d \\ & ^{16})/(a^3*b^{17})^{1/4} + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a \\ & ^3*b*c*d^3 + a^4*d^4)*x) + 2340*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^3 - \\ & a^3*d^4)*x)/b^4 \end{aligned}$$

**giac [B]** time = 0.48, size = 617, normalized size = 1.86

$$\frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^4 c^4 - 4 (ab^3)^{\frac{1}{4}} ab^3 c^3 d + 6 (ab^3)^{\frac{1}{4}} a^2 b^2 c^2 d^2 - 4 (ab^3)^{\frac{1}{4}} a^3 b c d^3 + (ab^3)^{\frac{1}{4}} a^4 d^4 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4 ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^4/(b\*x^4+a),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^5) + 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^5) + 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*log(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^5) - 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^4\*c^4 - 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d + 6\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 - 4\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 + (a\*b^3)^(1/4)\*a^4\*d^4)\*log(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^5) + 1/585\*(45\*b^12\*d^4\*x^13 + 260\*b^12\*c\*d^3\*x^9 - 65\*a\*b^11\*d^4\*x^9 + 702\*b^12\*c^2\*d^2\*x^5 - 468\*a\*b^11\*c\*d^3\*x^5 + 117\*a^2\*b^10\*d^4\*x^5 + 2340\*b^12\*c^3\*d\*x - 3510\*a\*b^11\*c^2\*d^2\*x + 2340\*a^2\*b^10\*c\*d^3\*x - 585\*a^3\*b^9\*d^4\*x)/b^13

**maple [B]** time = 0.05, size = 837, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^4/(b\*x^4+a),x)

[Out] -4/5\*d^3/b^2\*x^5\*a\*c+4\*d^3/b^3\*a^2\*c\*x-6\*d^2/b^2\*a\*c^2\*x+1/4\*(a/b)^(1/4)/a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^4+1/8\*(a/b)^(1/4)/a^2^(1/2)\*ln((x^2+(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2))) \*c^4+1/4\*(a/b)^(1/4)/a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^4+1/13\*d^4\*x^13/b+3/4/b^2\*(a/b)^(1/4)\*a^2^(1/2)\*ln((x^2+(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*x\*2^(1/2)+(a/b)^(1/2))) \*c^2\*d^2-1/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c\*d^3+3/2/b^2\*(a/b)^(1/4)\*a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^2\*d^2-1/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c\*d^3+3/2/b^2\*(a/b)^(1/4)\*a^2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^2\*d^2-1/2/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*ln((x^2+(



$$\frac{a/b^{1/4} * x^2 + (a/b)^{1/2}}{(x^2 - (a/b)^{1/4} * x^2 + (a/b)^{1/2})} * c^3 * d^3 + \frac{6}{5} * \frac{d^2}{b} * x^5 * c^2 - \frac{d^4}{b^4} * a^3 * x^4 + \frac{d}{b} * c^3 * x - \frac{1}{9} * \frac{d^4}{b^2} * x^9 * a + \frac{4}{9} * \frac{d^3}{b} * x^9 * c + \frac{1}{5} * \frac{d^4}{b^3} * x^5 * a^2 + \frac{1}{4} * \frac{d^4}{b^4} * (a/b)^{1/4} * a^3 * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x - 1) * d^4 - \frac{1}{b} * (a/b)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x - 1) * c^3 * d + \frac{1}{8} * \frac{d^4}{b^4} * (a/b)^{1/4} * a^3 * 2^{1/2} * \ln((x^2 + (a/b)^{1/4} * x^2 + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} * x^2 + (a/b)^{1/2})) * d^4 - \frac{1}{2} * \frac{d^4}{b} * (a/b)^{1/4} * 2^{1/2} * \ln((x^2 + (a/b)^{1/4} * x^2 + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} * x^2 + (a/b)^{1/2})) * c^3 * d + \frac{1}{4} * \frac{d^4}{b^4} * (a/b)^{1/4} * a^3 * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x + 1) * d^4 - \frac{1}{b} * (a/b)^{1/4} * 2^{1/2} * \arctan(2^{1/2} / (a/b)^{1/4} * x + 1) * c^3 * d$$

**maxima** [A] time = 1.44, size = 489, normalized size = 1.47

$$\frac{45 b^3 d^4 x^{13} + 65 (4 b^3 c d^3 - a b^2 d^4) x^9 + 117 (6 b^3 c^2 d^2 - 4 a b^2 c d^3 + a^2 b d^4) x^5 + 585 (4 b^3 c^3 d - 6 a b^2 c^2 d^2 + 4 a^2 b c d^3 - 4 a^3 d^4) x + 585 b^4}{585 b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^4/(b\*x^4+a),x, algorithm="maxima")

[Out] 1/585\*(45\*b^3\*d^4\*x^13 + 65\*(4\*b^3\*c\*d^3 - a\*b^2\*d^4)\*x^9 + 117\*(6\*b^3\*c^2\*d^2 - 4\*a\*b^2\*c\*d^3 + a^2\*b\*d^4)\*x^5 + 585\*(4\*b^3\*c^3\*d - 6\*a\*b^2\*c^2\*d^2 + 4\*a^2\*b\*c\*d^3 - a^3\*d^4)\*x)/b^4 + 1/8\*(2\*sqrt(2)\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x + sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + 2\*sqrt(2)\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + sqrt(2)\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*log(sqrt(b)\*x^2 + sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4)) - sqrt(2)\*(b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*log(sqrt(b)\*x^2 - sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4))/b^4

**mupad** [B] time = 1.51, size = 1822, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^4/(a + b\*x^4),x)

[Out] x\*((4\*c^3\*d)/b - (a\*((a\*(a\*d^4)/b^2 - (4\*c\*d^3)/b))/b + (6\*c^2\*d^2)/b)/b - x^9\*((a\*d^4)/(9\*b^2) - (4\*c\*d^3)/(9\*b)) + x^5\*((a\*(a\*d^4)/b^2 - (4\*c\*d^3)/b)/(5\*b) + (6\*c^2\*d^2)/(5\*b)) + (d^4\*x^13)/(13\*b) + (atan((((4\*x\*(a^8\*d^8 + b^8\*c^8 + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a\*b^7\*c^7\*d - 8\*a^7\*b\*c\*d^7)))/b^5 - (4\*(a\*d - b\*c)^4\*(a^5\*d^4 + a\*b^4\*c^4 - 4\*a^2\*b^3\*c^3\*d + 6\*a^3\*b^2\*c^2\*d^2 - 4\*a^4\*b\*c\*d^3))/((-a)^(3/4)\*b^(21/4)))\*(a\*d - b\*c)^4\*1i)/(4\*(-a)^(3/4)\*b^(17/4)) + (((4\*x\*(a^8\*d^8 + b^8\*c^8 + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a\*b^7\*c^7\*d - 8\*a^7\*b\*c\*d^7))/b^5 + (4\*(a\*d - b\*c)^4\*(a^5\*d^4 + a\*b^4\*c^4 - 4\*a^2\*b^3\*c^3\*d + 6\*a^3\*b^2\*c^2\*d^2 - 4\*a^4\*b\*c\*d^3))/((-a)^(3/4)\*b^(21/4)))\*(a\*d - b\*c)^4\*1i)/(4\*(-a)^(3/4)\*b^(17/4)))/(((((4\*x\*(a^8\*d^8 + b^8\*c^8 + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a\*b^7\*c^7\*d - 8\*a^7\*b\*c\*d^7))/b^5 - (4\*(a\*d - b\*c)^4\*(a^5\*d^4 + a\*b^4\*c^4 - 4\*a^2\*b^3\*c^3\*d + 6\*a^3\*b^2\*c^2\*d^2 - 4\*a^4\*b\*c\*d^3))/((-a)^(3/4)\*b^(21/4)))\*(a\*d - b\*c)^4)/(4\*(-a)^(3/4)\*b^(17/4)) - (((4\*x\*(a^8\*d^8 + b^8\*c^8 + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a\*b^7\*c^7\*d - 8\*a^7\*b\*c\*d^7))/b^5 - (4\*(a\*d - b\*c)^4\*(a^5\*d^4 + a\*b^4\*c^4 - 4\*a^2\*b^3\*c^3\*d + 6\*a^3\*b^2\*c^2\*d^2 - 4\*a^4\*b\*c\*d^3))/((-a)^(3/4)\*b^(21/4)))\*(a\*d - b\*c)^4)/(4\*(-a)^(3/4)\*b^(17/4)) - (((4\*x\*(a^8\*d^8 + b^8\*c^8 + 28\*a^2\*b^6\*c^6\*d^2 - 56\*a^3\*b^5\*c^5\*d^3 + 70\*a^4\*b^4\*c^4\*d^4 - 56\*a^5\*b^3\*c^3\*d^5 + 28\*a^6\*b^2\*c^2\*d^6 - 8\*a\*b^7\*c^7\*d - 8\*a^7\*b\*c\*d^7))/b^5 - (4\*(a\*d - b\*c)^4\*(a^5\*d^4 + a\*b^4\*c^4 - 4\*a^2\*b^3\*c^3\*d + 6\*a^3\*b^2\*c^2\*d^2 - 4\*a^4\*b\*c\*d^3))/((-a)^(3/4)\*b^(21/4)))\*(a\*d - b\*c)^4)/(4\*(-a)^(3/4)\*b^(17/4))

```

*d - 8*a^7*b*c*d^7))/b^5 + (4*(a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^
3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3))/((-a)^(3/4)*b^(21/4))*(a*d -
b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))*(a*d - b*c)^4*1i)/(2*(-a)^(3/4)*b^(17/4)
) + (atan((((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*
d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^
7*c^7*d - 8*a^7*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2
*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*
(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)) + (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2
*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5
+ 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^
4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^
3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4)/(4*(-a)^(3/4)*b^(17/4)))/(((4
*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2 - 56*a^3*b^5*c^5*d^3 + 70*a^4*b^
4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7
*b*c*d^7))/b^5 - ((a*d - b*c)^4*(a^5*d^4 + a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*
a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)^(3/4)*b^(21/4)))*(a*d - b*c)^4*1
i)/(4*(-a)^(3/4)*b^(17/4)) - (((4*x*(a^8*d^8 + b^8*c^8 + 28*a^2*b^6*c^6*d^2
- 56*a^3*b^5*c^5*d^3 + 70*a^4*b^4*c^4*d^4 - 56*a^5*b^3*c^3*d^5 + 28*a^6*b^
2*c^2*d^6 - 8*a*b^7*c^7*d - 8*a^7*b*c*d^7))/b^5 + ((a*d - b*c)^4*(a^5*d^4 +
a*b^4*c^4 - 4*a^2*b^3*c^3*d + 6*a^3*b^2*c^2*d^2 - 4*a^4*b*c*d^3)*4i)/((-a)
^(3/4)*b^(21/4)))*(a*d - b*c)^4*1i)/(4*(-a)^(3/4)*b^(17/4)))*(a*d - b*c)^4
)/(2*(-a)^(3/4)*b^(17/4))

```

**sympy [A]** time = 3.59, size = 435, normalized size = 1.31

$$x^9 \left( -\frac{ad^4}{9b^2} + \frac{4cd^3}{9b} \right) + x^5 \left( \frac{a^2d^4}{5b^3} - \frac{4acd^3}{5b^2} + \frac{6c^2d^2}{5b} \right) + x \left( -\frac{a^3d^4}{b^4} + \frac{4a^2cd^3}{b^3} - \frac{6ac^2d^2}{b^2} + \frac{4c^3d}{b} \right) + \text{RootSum} \left( 256t^4 a^3 b^{17} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*4/(b\*x\*\*4+a), x)

```

[Out] x**9*(-a*d**4/(9*b**2) + 4*c*d**3/(9*b)) + x**5*(a**2*d**4/(5*b**3) - 4*a*c
*d**3/(5*b**2) + 6*c**2*d**2/(5*b)) + x*(-a**3*d**4/b**4 + 4*a**2*c*d**3/b
**3 - 6*a*c**2*d**2/b**2 + 4*c**3*d/b) + RootSum(256*_t**4*a**3*b**17 + a**1
6*d**16 - 16*a**15*b*c*d**15 + 120*a**14*b**2*c**2*d**14 - 560*a**13*b**3*c
**3*d**13 + 1820*a**12*b**4*c**4*d**12 - 4368*a**11*b**5*c**5*d**11 + 8008*
a**10*b**6*c**6*d**10 - 11440*a**9*b**7*c**7*d**9 + 12870*a**8*b**8*c**8*d
**8 - 11440*a**7*b**9*c**9*d**7 + 8008*a**6*b**10*c**10*d**6 - 4368*a**5*b**
11*c**11*d**5 + 1820*a**4*b**12*c**12*d**4 - 560*a**3*b**13*c**13*d**3 + 12
0*a**2*b**14*c**14*d**2 - 16*a*b**15*c**15*d + b**16*c**16, Lambda(_t, _t*log(4*_t*a*b**4/(a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**2*d**2 - 4*a*b
**3*c**3*d + b**4*c**4) + x))) + d**4*x**13/(13*b)

```

$$3.161 \quad \int \frac{(c+dx^4)^3}{a+bx^4} dx$$

**Optimal.** Leaf size=288

$$\frac{(bc-ad)^3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{13/4}} - \frac{(bc-ad)^3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2}{\sqrt{2} a^{3/4} b^{13/4}}\right)}{2\sqrt{2} a^{3/4} b^{13/4}}$$

[Out] d\*(a^2\*d^2-3\*a\*b\*c\*d+3\*b^2\*c^2)\*x/b^3+1/5\*d^2\*(-a\*d+3\*b\*c)\*x^5/b^2+1/9\*d^3\*x^9/b+1/4\*(-a\*d+b\*c)^3\*arctan(-1+b^(1/4)\*x\*2^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)\*2^(1/2)+1/4\*(-a\*d+b\*c)^3\*arctan(1+b^(1/4)\*x\*2^(1/2)/a^(1/4))/a^(3/4)/b^(13/4)\*2^(1/2)-1/8\*(-a\*d+b\*c)^3\*ln(-a^(1/4)\*b^(1/4)\*x\*2^(1/2)+a^(1/2)+x^2\*b^(1/2))/a^(3/4)/b^(13/4)\*2^(1/2)+1/8\*(-a\*d+b\*c)^3\*ln(a^(1/4)\*b^(1/4)\*x\*2^(1/2)+a^(1/2)+x^2\*b^(1/2))/a^(3/4)/b^(13/4)\*2^(1/2)

**Rubi [A]** time = 0.22, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{dx(a^2d^2 - 3abcd + 3b^2c^2)}{b^3} - \frac{(bc-ad)^3 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{13/4}} + \frac{(bc-ad)^3 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{13/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^3/(a + b\*x^4), x]

[Out] (d\*(3\*b^2\*c^2 - 3\*a\*b\*c\*d + a^2\*d^2)\*x)/b^3 + (d^2\*(3\*b\*c - a\*d)\*x^5)/(5\*b^2) + (d^3\*x^9)/(9\*b) - ((b\*c - a\*d)^3\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/((2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/((2\*Sqrt[2]\*a^(3/4)\*b^(13/4)) - ((b\*c - a\*d)^3\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(13/4)) + ((b\*c - a\*d)^3\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(13/4))

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

**Rule 390**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

**Rule 617**

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^4)^3}{a + bx^4} dx &= \int \left( \frac{d(3b^2c^2 - 3abcd + a^2d^2)}{b^3} + \frac{d^2(3bc - ad)x^4}{b^2} + \frac{d^3x^8}{b} + \frac{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3}{b^3(a + bx^4)} \right) dx \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc - ad)^3 \int \frac{1}{a + bx^4} dx}{b^3} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc - ad)^3 \int \frac{\sqrt{a} - \sqrt{b}x^2}{a + bx^4} dx}{2\sqrt{a}b^3} + \frac{(bc - ad)^3}{2} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} + \frac{(bc - ad)^3 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{7/2}} + \frac{(bc - ad)^3}{2} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc - ad)^3 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a})}{4\sqrt{2}a^{3/4}b^{13/4}} \\ &= \frac{d(3b^2c^2 - 3abcd + a^2d^2)x}{b^3} + \frac{d^2(3bc - ad)x^5}{5b^2} + \frac{d^3x^9}{9b} - \frac{(bc - ad)^3 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{13/4}} + \frac{(bc - ad)^3}{2} \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 271, normalized size = 0.94

$$-72a^{3/4}b^{5/4}d^2x^5(ad - 3bc) + 40a^{3/4}b^{9/4}d^3x^9 + 360a^{3/4}\sqrt[4]{b} dx (a^2d^2 - 3abcd + 3b^2c^2) - 45\sqrt{2}(bc - ad)^3 \log(-\sqrt{2}\sqrt[4]{a}x + \sqrt{a})$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x^4)^3/(a + b*x^4),x]
```

```
[Out] (360*a^(3/4)*b^(1/4)*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x - 72*a^(3/4)*b^(5/4)*d^2*(-3*b*c + a*d)*x^5 + 40*a^(3/4)*b^(9/4)*d^3*x^9 - 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 - (Sqrt[2]*b^(1/4)*x)/a^(1/4)] + 90*Sqrt[2]*(b*c - a*d)^3*ArcTan[1 + (Sqrt[2]*b^(1/4)*x)/a^(1/4)] - 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2] + 45*Sqrt[2]*(b*c - a*d)^3*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*b^(1/4)*x + Sqrt[b]*x^2])/(360*a^(3/4)*b^(13/4))
```

```
fricas [B] time = 1.16, size = 1855, normalized size = 6.44
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^3/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] 1/180*(20*b^2*d^3*x^9 + 36*(3*b^2*c*d^2 - a*b*d^3)*x^5 - 180*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*arctan((a^2*b^10*x*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4) - a^2*b^10*sqrt((a^2*b^6*sqrt(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13)) + (b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*x^2)/(b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6))*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(3/4))/(b^9*c^9 - 9*a*b^8*c^8*d + 36*a^2*b^7*c^7*d^2 - 84*a^3*b^6*c^6*d^3 + 126*a^4*b^5*c^5*d^4 - 126*a^5*b^4*c^4*d^5 + 84*a^6*b^3*c^3*d^6 - 36*a^7*b^2*c^2*d^7 + 9*a^8*b*c*d^8 - a^9*d^9)) - 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4)*log(a*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 45*b^3*(-(b^12*c^12 - 12*a*b^11*c^11*d + 66*a^2*b^10*c^10*d^2 - 220*a^3*b^9*c^9*d^3 + 495*a^4*b^8*c^8*d^4 - 792*a^5*b^7*c^7*d^5 + 924*a^6*b^6*c^6*d^6 - 792*a^7*b^5*c^5*d^7 + 495*a^8*b^4*c^4*d^8 - 220*a^9*b^3*c^3*d^9 + 66*a^10*b^2*c^2*d^10 - 12*a^11*b*c*d^11 + a^12*d^12)/(a^3*b^13))^(1/4) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*x) + 180*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*x)/b^3
```

**giac [B]** time = 0.17, size = 481, normalized size = 1.67

$$\frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 - 3 (ab^3)^{\frac{1}{4}} ab^2 c^2 d + 3 (ab^3)^{\frac{1}{4}} a^2 b c d^2 - (ab^3)^{\frac{1}{4}} a^3 d^3 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right) + \sqrt{2} \left( (ab^3)^{\frac{1}{4}} b^3 c^3 - \dots \right)}{4 ab^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^3/(b\*x^4+a),x, algorithm="giac")

[Out] 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^3\*c^3 - 3\*(a\*b^3)^(1/4)\*a\*b^2\*c^2\*d + 3\*(a\*b^3)^(1/4)\*a^2\*b\*c\*d^2 - (a\*b^3)^(1/4)\*a^3\*d^3)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^4) + 1/4\*sqrt(2)\*((a\*b^3)^(1/4)\*b^3\*c^3 - 3\*(a\*b^3)^(1/4)\*a\*b^2\*c^2\*d + 3\*(a\*b^3)^(1/4)\*a^2\*b\*c\*d^2 - (a\*b^3)^(1/4)\*a^3\*d^3)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a\*b^4) + 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^3\*c^3 - 3\*(a\*b^3)^(1/4)\*a\*b^2\*c^2\*d + 3\*(a\*b^3)^(1/4)\*a^2\*b\*c\*d^2 - (a\*b^3)^(1/4)\*a^3\*d^3)\*log(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^4) - 1/8\*sqrt(2)\*((a\*b^3)^(1/4)\*b^3\*c^3 - 3\*(a\*b^3)^(1/4)\*a\*b^2\*c^2\*d + 3\*(a\*b^3)^(1/4)\*a^2\*b\*c\*d^2 - (a\*b^3)^(1/4)\*a^3\*d^3)\*log(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a\*b^4) + 1/45\*(5\*b^8\*d^3\*x^9 + 27\*b^8\*c\*d^2\*x^5 - 9\*a\*b^7\*d^3\*x^5 + 135\*b^8\*c^2\*d\*x - 135\*a\*b^7\*c\*d^2\*x + 45\*a^2\*b^6\*d^3\*x)/b^9

**maple [B]** time = 0.05, size = 627, normalized size = 2.18

$$\frac{d^3 x^9}{9b} - \frac{a d^3 x^5}{5b^2} + \frac{3c d^2 x^5}{5b} + \frac{a^2 d^3 x}{b^3} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a^2 d^3 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{4b^3} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a^2 d^3 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{4b^3} - \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a^2 d^3}{4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^3/(b\*x^4+a),x)

[Out] 1/9\*d^3\*x^9/b-1/5\*d^3/b^2\*x^5\*a+3/5\*d^2/b\*x^5\*c+d^3/b^3\*a^2\*x-3\*d^2/b^2\*a\*c\*x+3\*d/b\*c^2\*x-1/8/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*d^3+3/8/b^2\*(a/b)^(1/4)\*a\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c\*d^2-3/8/b\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c^2\*d+1/8\*(a/b)^(1/4)/a\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c^3-1/4/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*d^3+3/4/b^2\*(a/b)^(1/4)\*a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^2\*d+1/4\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^3-1/4/b^3\*(a/b)^(1/4)\*a^2\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*d^3+3/4/b^2\*(a/b)^(1/4)\*a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^2\*d+1/4\*(a/b)^(1/4)/a\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^3

**maxima [A]** time = 1.21, size = 385, normalized size = 1.34

$$\frac{5 b^2 d^3 x^9 + 9 (3 b^2 c d^2 - a b d^3) x^5 + 45 (3 b^2 c^2 d - 3 a b c d^2 + a^2 d^3) x}{45 b^3} + \frac{2 \sqrt{2} (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) \arctan \left( \frac{\sqrt{2} (2 \sqrt{b} x + \sqrt{2} a)}{2 \sqrt{\sqrt{a} \sqrt{b}}} \right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^3/(b\*x^4+a),x, algorithm="maxima")

[Out]  $\frac{1}{45}(5b^2d^3x^9 + 9(3b^2cd^2 - abd^3)x^5 + 45(3b^2c^2d - 3ab^2cd^2 + a^2d^3)x)/b^3 + \frac{1}{8}(2\sqrt{2})(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3)\arctan(1/2\sqrt{2})(2\sqrt{2}(b)x + \sqrt{2}a^{1/4}b^{1/4})/\sqrt{\sqrt{a}\sqrt{b}})/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}) + 2\sqrt{2}(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3)\arctan(1/2\sqrt{2})(2\sqrt{2}(b)x - \sqrt{2}a^{1/4}b^{1/4})/\sqrt{\sqrt{a}\sqrt{b}})/(\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}) + \sqrt{2}(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3)\log(\sqrt{b}x^2 + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a})/(a^{3/4}b^{1/4}) - \sqrt{2}(b^3c^3 - 3ab^2c^2d + 3a^2b^2cd^2 - a^3d^3)\log(\sqrt{b}x^2 - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a})/(a^{3/4}b^{1/4})/b^3$

**mupad [B]** time = 1.49, size = 1433, normalized size = 4.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^3/(a + b\*x^4),x)

[Out]  $x((3c^2d)/b + (a((ad^3)/b^2 - (3cd^2)/b))/b - x^5((ad^3)/(5b^2) - (3cd^2)/(5b)) + (d^3x^9)/(9b) - (\operatorname{atan}(\frac{(x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} - ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3i)/((-a)^{3/4}b^{13/4}) + ((x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} + ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3i)/((-a)^{3/4}b^{13/4})/(((x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} - ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3)/((-a)^{3/4}b^{13/4})) * (ad - bc)^3i)/(2(-a)^{3/4}b^{13/4}) - (\operatorname{atan}(\frac{(x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} - ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3i)/((-a)^{3/4}b^{13/4}) - ((x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} + ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3i)/((-a)^{3/4}b^{13/4})) * (ad - bc)^3i)/((-a)^{3/4}b^{13/4}) - ((x(a^6d^6 + b^6c^6 + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d - 6a^5b^5c^5d)}{b^3} - ((ad - bc)^3(4a^4d^3 - 4ab^3c^3 + 12a^2b^2c^2d - 12a^3b^3cd^2))/(4(-a)^{3/4}b^{13/4}))(ad - bc)^3i)/((-a)^{3/4}b^{13/4})) * (ad - bc)^3i)/(2(-a)^{3/4}b^{13/4})$

**sympy [A]** time = 1.70, size = 303, normalized size = 1.05

$$x^5 \left( -\frac{ad^3}{5b^2} + \frac{3cd^2}{5b} \right) + x \left( \frac{a^2d^3}{b^3} - \frac{3acd^2}{b^2} + \frac{3c^2d}{b} \right) + \operatorname{RootSum} \left( 256t^4a^3b^{13} + a^{12}d^{12} - 12a^{11}bcd^{11} + 66a^{10}b^2c^2d^{10} - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*3/(b\*x\*\*4+a),x)

```
[Out] x**5*(-a*d**3/(5*b**2) + 3*c*d**2/(5*b)) + x*(a**2*d**3/b**3 - 3*a*c*d**2/b
**2 + 3*c**2*d/b) + RootSum(256*_t**4*a**3*b**13 + a**12*d**12 - 12*a**11*b
*c*d**11 + 66*a**10*b**2*c**2*d**10 - 220*a**9*b**3*c**3*d**9 + 495*a**8*b*
*4*c**4*d**8 - 792*a**7*b**5*c**5*d**7 + 924*a**6*b**6*c**6*d**6 - 792*a**5
*b**7*c**7*d**5 + 495*a**4*b**8*c**8*d**4 - 220*a**3*b**9*c**9*d**3 + 66*a*
*2*b**10*c**10*d**2 - 12*a*b**11*c**11*d + b**12*c**12, Lambda(_t, _t*log(-
4*_t*a*b**3/(a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3) + x
)) + d**3*x**9/(9*b))
```



$$3.162 \quad \int \frac{(c+dx^4)^2}{a+bx^4} dx$$

**Optimal.** Leaf size=253

$$\frac{(bc-ad)^2 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{9/4}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2}{2\sqrt{2} a^{3/4} b^{9/4}}\right)}{2\sqrt{2} a^{3/4} b^{9/4}}$$

[Out] d\*(-a\*d+2\*b\*c)\*x/b^2+1/5\*d^2\*x^5/b+1/4\*(-a\*d+b\*c)^2\*arctan(-1+b^(1/4)\*x\*2^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)\*2^(1/2)+1/4\*(-a\*d+b\*c)^2\*arctan(1+b^(1/4)\*x\*2^(1/2)/a^(1/4))/a^(3/4)/b^(9/4)\*2^(1/2)-1/8\*(-a\*d+b\*c)^2\*ln(-a^(1/4)\*b^(1/4)\*x\*2^(1/2)+a^(1/2)+x^2\*b^(1/2))/a^(3/4)/b^(9/4)\*2^(1/2)+1/8\*(-a\*d+b\*c)^2\*ln(a^(1/4)\*b^(1/4)\*x\*2^(1/2)+a^(1/2)+x^2\*b^(1/2))/a^(3/4)/b^(9/4)\*2^(1/2)

**Rubi [A]** time = 0.19, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {390, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)^2 \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{9/4}} + \frac{(bc-ad)^2 \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4} b^{9/4}} - \frac{(bc-ad)^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2}{2\sqrt{2} a^{3/4} b^{9/4}}\right)}{2\sqrt{2} a^{3/4} b^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^2/(a + b\*x^4), x]

[Out] (d\*(2\*b\*c - a\*d)\*x)/b^2 + (d^2\*x^5)/(5\*b) - ((b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/(2\*Sqrt[2]\*a^(3/4)\*b^(9/4)) - ((b\*c - a\*d)^2\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4)) + ((b\*c - a\*d)^2\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(4\*Sqrt[2]\*a^(3/4)\*b^(9/4))

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 390

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[PolynomialDivide[(a + b\*x^n)^p, (c + d\*x^n)^(-q), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] :> With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; Free

$Q[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4ac, 0]$

### Rule 628

$\text{Int}[\frac{(d_.) + (e_.)x}{(a_.) + (b_.)x + (c_.)x^2}, x\_Symbol] \rightarrow \text{Simp}[\frac{d \cdot \text{Log}[\text{RemoveContent}[a + bx + cx^2, x]]}{b}, x] \ /; \text{FreeQ}[\{a, b, c, d, e\}, x] \ \&\& \ \text{EqQ}[2cd - be, 0]$

### Rule 1162

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2d)/e, 2]\}, \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e + qx + x^2, x], x] + \text{Dist}[e/(2c), \text{Int}[1/\text{Simp}[d/e - qx + x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{PosQ}[d^2e]$

### Rule 1165

$\text{Int}[\frac{(d_.) + (e_.)x^2}{(a_.) + (c_.)x^4}, x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2d)/e, 2]\}, \text{Dist}[e/(2cq), \text{Int}[(q - 2x)/\text{Simp}[d/e + qx - x^2, x], x] + \text{Dist}[e/(2cq), \text{Int}[(q + 2x)/\text{Simp}[d/e - qx - x^2, x], x], x]] \ /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c^2d^2 - a^2e^2, 0] \ \&\& \ \text{NegQ}[d^2e]$

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^4)^2}{a + bx^4} dx &= \int \left( \frac{d(2bc - ad)}{b^2} + \frac{d^2x^4}{b} + \frac{b^2c^2 - 2abcd + a^2d^2}{b^2(a + bx^4)} \right) dx \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc - ad)^2 \int \frac{1}{a + bx^4} dx}{b^2} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc - ad)^2 \int \frac{\sqrt{a} - \sqrt{b}x^2}{a + bx^4} dx}{2\sqrt{a}b^2} + \frac{(bc - ad)^2 \int \frac{\sqrt{a} + \sqrt{b}x^2}{a + bx^4} dx}{2\sqrt{a}b^2} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^5}{5b} + \frac{(bc - ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{5/2}} + \frac{(bc - ad)^2 \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{5/2}} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc - ad)^2 \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc - ad)^2 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{9/4}} \\ &= \frac{d(2bc - ad)x}{b^2} + \frac{d^2x^5}{5b} - \frac{(bc - ad)^2 \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} + \frac{(bc - ad)^2 \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{9/4}} - \frac{(bc - ad)^2}{40a^{3/4}b^{9/4}} \end{aligned}$$

**Mathematica** [A] time = 0.13, size = 231, normalized size = 0.91

$$8a^{3/4}b^{5/4}d^2x^5 - 40a^{3/4}\sqrt[4]{b}dx(ad - 2bc) - 5\sqrt{2}(bc - ad)^2 \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) + 5\sqrt{2}(bc - ad)^2 \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2) - \frac{(bc - ad)^2}{40a^{3/4}b^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^2/(a + b\*x^4), x]

[Out] (-40\*a^(3/4)\*b^(1/4)\*d\*(-2\*b\*c + a\*d)\*x + 8\*a^(3/4)\*b^(5/4)\*d^2\*x^5 - 10\*Sqrt[2]\*(b\*c - a\*d)^2\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + 10\*Sqrt[2]\*(b

$(b^2c - a^2d)^2 \text{ArcTan}\left[1 + \frac{\sqrt{2} b^{1/4} x}{a^{1/4}}\right] - 5\sqrt{2} (b^2c - a^2d)^2 \text{Log}\left[\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2\right] + 5\sqrt{2} (b^2c - a^2d)^2 \text{Log}\left[\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2\right]\right) / (40 a^{3/4} b^{9/4})$

**fricas** [B] time = 1.12, size = 1240, normalized size = 4.90

$$4bd^2x^5 + 20b^2 \left( -\frac{b^8c^8 - 8ab^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8}{a^3b^9} \right)^{1/4} \arctan \left( -\frac{a^2b^7x}{\dots} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a),x, algorithm="fricas")

[Out]  $\frac{1}{20} (4b^2d^2x^5 + 20b^2(-b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9))^{1/4} \arctan\left(\frac{-a^2b^7x}{(b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9)}\right)^{1/4} - \frac{a^2b^7x \sqrt{(a^2b^4 \sqrt{(b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9)) + (b^4c^4 - 4a^2b^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^2c^2d^2 - 4a^3b^2c^2d^2 + a^4d^4)x^2}}{(b^4c^4 - 4a^2b^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^2c^2d^2 - 4a^3b^2c^2d^2 + a^4d^4)} \left( -\frac{b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8}{(a^3b^9)} \right)^{3/4} - \frac{a^2b^7 \sqrt{(a^2b^4 \sqrt{(b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9)) + (b^4c^4 - 4a^2b^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^2c^2d^2 - 4a^3b^2c^2d^2 + a^4d^4)x^2}}{(b^4c^4 - 4a^2b^3c^3d + 6a^2b^2c^2d^2 - 4a^3b^2c^2d^2 - 4a^3b^2c^2d^2 + a^4d^4)} \left( -\frac{b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8}{(a^3b^9)} \right)^{3/4} \right) / (b^6c^6 - 6a^2b^5c^5d + 15a^2b^4c^4d^2 - 20a^3b^3c^3d^3 + 15a^4b^2c^2d^4 - 6a^5b^2c^2d^4 - 6a^5b^2c^2d^4 + a^6d^6) + 5b^2(-b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9))^{1/4} \log(a^2b^2(-b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9))^{1/4} + (b^2c^2 - 2a^2b^2c^2d^2 + a^2d^2)x - 5b^2(-b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9))^{1/4} \log(-a^2b^2(-b^8c^8 - 8a^2b^7c^7d + 28a^2b^6c^6d^2 - 56a^3b^5c^5d^3 + 70a^4b^4c^4d^4 - 56a^5b^3c^3d^5 + 28a^6b^2c^2d^6 - 8a^7bcd^7 + a^8d^8)/(a^3b^9))^{1/4} + (b^2c^2 - 2a^2b^2c^2d^2 + a^2d^2)x + 20(2b^2c^2d^2 - a^2d^2)x/b^2$

**giac** [A] time = 0.18, size = 353, normalized size = 1.40

$$\frac{\sqrt{2} \left( (ab^3)^{1/4} b^2c^2 - 2(ab^3)^{1/4} abcd + (ab^3)^{1/4} a^2d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{1/4} \right)}{2 \left( \frac{a}{b} \right)^{1/4}} \right) + \sqrt{2} \left( (ab^3)^{1/4} b^2c^2 - 2(ab^3)^{1/4} abcd + \dots \right)}{4ab^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a),x, algorithm="giac")

[Out]  $\frac{1}{4} \sqrt{2} ((ab^3)^{1/4} b^2c^2 - 2(ab^3)^{1/4} abcd + (ab^3)^{1/4} a^2d^2) \arctan\left(\frac{1}{2} \sqrt{2} (2x + \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}\right) / (ab^3)^{1/4} + \frac{1}{4} \sqrt{2} ((ab^3)^{1/4} b^2c^2 - 2(ab^3)^{1/4} abcd + (ab^3)^{1/4} a^2d^2) \arctan\left(\frac{1}{2} \sqrt{2} (2x - \sqrt{2} (a/b)^{1/4}) / (a/b)^{1/4}\right) / (ab^3)^{1/4}$

$$\frac{1}{(a \cdot b^3)} + \frac{1}{8} \sqrt{2} \cdot \left( (a \cdot b^3)^{1/4} \cdot b^2 \cdot c^2 - 2 \cdot (a \cdot b^3)^{1/4} \cdot a \cdot b \cdot c \cdot d + (a \cdot b^3)^{1/4} \cdot a^2 \cdot d^2 \right) \cdot \log(x^2 + \sqrt{2} \cdot x \cdot (a/b)^{1/4} + \sqrt{a/b}) / (a \cdot b^3) - \frac{1}{8} \sqrt{2} \cdot \left( (a \cdot b^3)^{1/4} \cdot b^2 \cdot c^2 - 2 \cdot (a \cdot b^3)^{1/4} \cdot a \cdot b \cdot c \cdot d + (a \cdot b^3)^{1/4} \cdot a^2 \cdot d^2 \right) \cdot \log(x^2 - \sqrt{2} \cdot x \cdot (a/b)^{1/4} + \sqrt{a/b}) / (a \cdot b^3) + \frac{1}{5} \cdot (b^4 \cdot d^2 \cdot x^5 + 10 \cdot b^4 \cdot c \cdot d \cdot x - 5 \cdot a \cdot b^3 \cdot d^2 \cdot x) / b^5$$

**maple [B]** time = 0.05, size = 436, normalized size = 1.72

$$\frac{d^2 x^5}{5b} - \frac{a d^2 x}{b^2} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{4b^2} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right)}{4b^2} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a d^2 \ln\left(\frac{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}\right)}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^2/(b\*x^4+a), x)

[Out]  $\frac{1}{5} d^2 x^5 / b - d^2 / b^2 a x + 2 d / b c x + 1/8 b^2 (a/b)^{1/4} a^2 (1/2) \ln((x^2 + (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2)) / (x^2 - (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2))) * d^2 - 1/4 b (a/b)^{1/4} a^2 (1/2) \ln((x^2 + (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2)) / (x^2 - (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2))) * c d + 1/8 (a/b)^{1/4} / a^2 (1/2) \ln((x^2 + (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2)) / (x^2 - (a/b)^{1/4} a^2 (1/2) x + (a/b)^{1/4} a^2 (1/2))) * c^2 + 1/4 b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x - 1) * d^2 - 1/2 b (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x - 1) * c d + 1/4 (a/b)^{1/4} / a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x - 1) * c^2 + 1/4 b^2 (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x + 1) * d^2 - 1/2 b (a/b)^{1/4} a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x + 1) * c d + 1/4 (a/b)^{1/4} / a^2 (1/2) \arctan(2^{1/2} / (a/b)^{1/4} x + 1) * c^2$

**maxima [A]** time = 1.09, size = 287, normalized size = 1.13

$$\frac{b d^2 x^5 + 5(2 b c d - a d^2) x}{5 b^2} + \frac{2 \sqrt{2} (b^2 c^2 - 2 a b c d + a^2 d^2) \arctan\left(\frac{\sqrt{2} (2 \sqrt{b} x + \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}})}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}} + \frac{2 \sqrt{2} (b^2 c^2 - 2 a b c d + a^2 d^2) \arctan\left(\frac{\sqrt{2} (2 \sqrt{b} x - \sqrt{2} a^{\frac{1}{4}} b^{\frac{1}{4}})}{2 \sqrt{\sqrt{a} \sqrt{b}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a), x, algorithm="maxima")

[Out]  $\frac{1}{5} (b \cdot d^2 \cdot x^5 + 5 \cdot (2 \cdot b \cdot c \cdot d - a \cdot d^2) \cdot x) / b^2 + \frac{1}{8} (2 \cdot \sqrt{2} \cdot (b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d + a^2 \cdot d^2) \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (2 \cdot \sqrt{2} \cdot \sqrt{b} \cdot x + \sqrt{2} \cdot a^{1/4} \cdot b^{1/4})) / \sqrt{a} \cdot \sqrt{b}) / (\sqrt{a} \cdot \sqrt{b}) + 2 \cdot \sqrt{2} \cdot (b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d + a^2 \cdot d^2) \cdot \arctan(1/2 \cdot \sqrt{2} \cdot (2 \cdot \sqrt{2} \cdot \sqrt{b} \cdot x - \sqrt{2} \cdot a^{1/4} \cdot b^{1/4})) / \sqrt{a} \cdot \sqrt{b}) / (\sqrt{a} \cdot \sqrt{b}) + \sqrt{2} \cdot (b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d + a^2 \cdot d^2) \cdot \log(\sqrt{b} \cdot x^2 + \sqrt{2} \cdot a^{1/4} \cdot b^{1/4} \cdot x + \sqrt{a}) / (a^{3/4} \cdot b^{1/4}) - \sqrt{2} \cdot (b^2 \cdot c^2 - 2 \cdot a \cdot b \cdot c \cdot d + a^2 \cdot d^2) \cdot \log(\sqrt{b} \cdot x^2 - \sqrt{2} \cdot a^{1/4} \cdot b^{1/4} \cdot x + \sqrt{a}) / (a^{3/4} \cdot b^{1/4}) / b^2$

**mupad [B]** time = 1.47, size = 1081, normalized size = 4.27

$$\frac{d^2 x^5}{5b} - x \left( \frac{a d^2}{b^2} - \frac{2 c d}{b} \right) + \frac{\operatorname{atan}\left(\frac{(a d - b c)^2 \left( \frac{x (a^4 d^4 - 4 a^3 b c d^3 + 6 a^2 b^2 c^2 d^2 - 4 a b^3 c^3 d + b^4 c^4)}{b} - \frac{(a d - b c)^2 (4 a^3 b d^2 - 8 a^2 b^2 c d + 4 a b^3 c^2)}{4 (-a)^{3/4} b^{9/4}} \right)}{(-a)^{3/4} b^{9/4}}\right)}{2 (-a)^{3/4} b^{9/4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^2/(a + b\*x^4),x)

[Out] (d^2\*x^5)/(5\*b) - x\*((a\*d^2)/b^2 - (2\*c\*d)/b) + (atan((((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b - ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d))/(4\*(-a)^(3/4)\*b^(9/4))))\*1i)/((-a)^(3/4)\*b^(9/4)) + ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b + ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d))/(4\*(-a)^(3/4)\*b^(9/4))))\*1i)/((-a)^(3/4)\*b^(9/4)))/(((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b - ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d))/(4\*(-a)^(3/4)\*b^(9/4)))))/((-a)^(3/4)\*b^(9/4)) - ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b + ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d))/(4\*(-a)^(3/4)\*b^(9/4)))))/((-a)^(3/4)\*b^(9/4)))\*((a\*d - b\*c)^2\*1i)/(2\*(-a)^(3/4)\*b^(9/4)) + (atan((((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b - ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d)\*1i))/(4\*(-a)^(3/4)\*b^(9/4)))))/((-a)^(3/4)\*b^(9/4)) + ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b + ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d)\*1i))/(4\*(-a)^(3/4)\*b^(9/4)))))/((-a)^(3/4)\*b^(9/4)))/(((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b - ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d)\*1i))/(4\*(-a)^(3/4)\*b^(9/4))))\*1i)/((-a)^(3/4)\*b^(9/4)) - ((a\*d - b\*c)^2\*((x\*(a^4\*d^4 + b^4\*c^4 + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a\*b^3\*c^3\*d - 4\*a^3\*b\*c\*d^3))/b + ((a\*d - b\*c)^2\*(4\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 8\*a^2\*b^2\*c\*d)\*1i))/(4\*(-a)^(3/4)\*b^(9/4))))\*1i)/((-a)^(3/4)\*b^(9/4)))\*((a\*d - b\*c)^2)/(2\*(-a)^(3/4)\*b^(9/4))

**sympy** [A] time = 1.08, size = 187, normalized size = 0.74

$$x \left( -\frac{ad^2}{b^2} + \frac{2cd}{b} \right) + \text{RootSum} \left( 256t^4a^3b^9 + a^8d^8 - 8a^7bcd^7 + 28a^6b^2c^2d^6 - 56a^5b^3c^3d^5 + 70a^4b^4c^4d^4 - 56a^3b^5c^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*2/(b\*x\*\*4+a),x)

[Out] x\*(-a\*d\*\*2/b\*\*2 + 2\*c\*d/b) + RootSum(256\*\_t\*\*4\*a\*\*3\*b\*\*9 + a\*\*8\*d\*\*8 - 8\*a\*\*7\*b\*c\*d\*\*7 + 28\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 - 56\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 70\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 - 56\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 + 28\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 - 8\*a\*b\*\*7\*c\*\*7\*d + b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(4\*\_t\*a\*b\*\*2/(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2) + x))) + d\*\*2\*x\*\*5/(5\*b)

### 3.163 $\int \frac{c+dx^4}{a+bx^4} dx$

**Optimal.** Leaf size=223

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

[Out]  $d*x/b+1/4*(-a*d+b*c)*\arctan(-1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(5/4)}*2^{(1/2)}+1/4*(-a*d+b*c)*\arctan(1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/b^{(5/4)}*2^{(1/2)}-1/8*(-a*d+b*c)*\ln(-a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/b^{(5/4)}*2^{(1/2)}+1/8*(-a*d+b*c)*\ln(a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/b^{(5/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {388, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc-ad)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{4\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc-ad)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a}+\sqrt{b}x^2}\right)}{2\sqrt{2}a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)/(a + b\*x^4), x]

[Out]  $(d*x)/b - ((b*c - a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*b^{(5/4)}) + ((b*c - a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*\text{Sqrt}[2]*a^{(3/4)}*b^{(5/4)}) - ((b*c - a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*b^{(5/4)}) + ((b*c - a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(4*\text{Sqrt}[2]*a^{(3/4)}*b^{(5/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

Rule 628

Int[((d\_) + (e\_.)\*(x\_))/((a\_.) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

Rule 1162

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

Rule 1165

Int[((d\_) + (e\_.)\*(x\_)^2)/((a\_) + (c\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

Rubi steps

$$\begin{aligned} \int \frac{c + dx^4}{a + bx^4} dx &= \frac{dx}{b} - \frac{(-bc + ad) \int \frac{1}{a+bx^4} dx}{b} \\ &= \frac{dx}{b} + \frac{(bc - ad) \int \frac{\sqrt{a} - \sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}b} + \frac{(bc - ad) \int \frac{\sqrt{a} + \sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}b} \\ &= \frac{dx}{b} + \frac{(bc - ad) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{3/2}} + \frac{(bc - ad) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}b^{3/2}} - \frac{(bc - ad) \int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2}a^{3/4}b^{5/4}} \\ &= \frac{dx}{b} - \frac{(bc - ad) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc - ad) \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{5/4}} \\ &= \frac{dx}{b} - \frac{(bc - ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} + \frac{(bc - ad) \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}b^{5/4}} - \frac{(bc - ad) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}b^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.15, size = 196, normalized size = 0.88

$$\frac{8a^{3/4}\sqrt[4]{b} dx - \sqrt{2}(bc - ad) \log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) + \sqrt{2}(bc - ad) \log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{8a^{3/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)/(a + b\*x^4), x]

[Out] (8\*a^(3/4)\*b^(1/4)\*d\*x - 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + 2\*Sqrt[2]\*(b\*c - a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] - Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2] + Sqrt[2]\*(b\*c - a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/(8\*a^(3/4)\*b^(5/4))

**fricas** [B] time = 1.32, size = 639, normalized size = 2.87

$$4b \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{a^3b^5} \right)^{\frac{1}{4}} \arctan \left( \frac{a^2b^4x \left( -\frac{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}{a^3b^5} \right)^{\frac{3}{4}} - a^2b^4 \sqrt{\frac{a^2b^2 \sqrt{b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4}}{a^3b^5}}}{b^3c^3 - 3ab^2c^2d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a),x, algorithm="fricas")

[Out]  $-1/4*(4*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)}*\arctan((a^2*b^4*x*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(3/4)} - a^2*b^4*\sqrt{(a^2*b^2*\sqrt{-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5)} + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*x^2)/(b^2*c^2 - 2*a*b*c*d + a^2*d^2))*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(3/4)})/(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)) + b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)}*\log(a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} - (b*c - a*d)*x) - b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)}*\log(-a*b*(-(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)/(a^3*b^5))^{(1/4)} - (b*c - a*d)*x) - 4*d*x)/b$

**giac** [A] time = 0.17, size = 245, normalized size = 1.10

$$\frac{dx}{b} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^2} + \frac{\sqrt{2} \left( (ab^3)^{\frac{1}{4}} bc - (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{4ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a),x, algorithm="giac")

[Out]  $d*x/b + 1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b*c - (a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^2) + 1/4*\sqrt{2}*((a*b^3)^{(1/4)}*b*c - (a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a*b^2) + 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b*c - (a*b^3)^{(1/4)}*a*d)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{2}*(a/b)^{(1/4)})/(a*b^2) - 1/8*\sqrt{2}*((a*b^3)^{(1/4)}*b*c - (a*b^3)^{(1/4)}*a*d)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{2}*(a/b)^{(1/4)})/(a*b^2)$

**maple** [A] time = 0.05, size = 266, normalized size = 1.19

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan \left( \frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1 \right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \arctan \left( \frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1 \right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} c \ln \left( \frac{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}} \right)}{8a} + \frac{dx}{b} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1 \right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1 \right)}{4a} + \frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} d \ln \left( \frac{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}} \right)}{8a} + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)/(b\*x^4+a),x)

[Out]  $1/b*d*x - 1/4/b*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*d + 1/4*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c - 1/8/b*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))$



$$(a/b)^{(1/2)}) * d + 1/8 * (a/b)^{(1/4)} / a * 2^{(1/2)} * \ln((x^2 + (a/b)^{(1/4)} * 2^{(1/2)} * x + (a/b)^{(1/2)}) / (x^2 - (a/b)^{(1/4)} * 2^{(1/2)} * x + (a/b)^{(1/2)})) * c - 1/4 * b * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * d + 1/4 * (a/b)^{(1/4)} / a * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x + 1) * c$$

**maxima** [A] time = 1.13, size = 212, normalized size = 0.95

$$\frac{dx}{b} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx}+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(bc-ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx}-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{\frac{3}{a^{\frac{3}{4}}b^{\frac{1}{4}}}} - \frac{\sqrt{2}(bc-ad)\log\left(\sqrt{b}x^2-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{\frac{3}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a),x, algorithm="maxima")

[Out] d\*x/b + 1/8\*(2\*sqrt(2)\*(b\*c - a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x + sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + 2\*sqrt(2)\*(b\*c - a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + sqrt(2)\*(b\*c - a\*d)\*log(sqrt(b)\*x^2 + sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4)) - sqrt(2)\*(b\*c - a\*d)\*log(sqrt(b)\*x^2 - sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4))/b

**mupad** [B] time = 0.22, size = 720, normalized size = 3.23

$$\frac{dx}{b} \operatorname{atan} \left( \frac{\left( \frac{x(4a^2bd^2-8ab^2cd+4b^3c^2) - \frac{(16a^2b^2d-16ab^3c)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}{4(-a)^{3/4}b^{5/4}} \right)^{(ad-bc)1i} + \left( \frac{x(4a^2bd^2-8ab^2cd+4b^3c^2) + \frac{(16a^2b^2d-16ab^3c)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}{4(-a)^{3/4}b^{5/4}} \right)^{(ad-bc)1i}}{\left( \frac{x(4a^2bd^2-8ab^2cd+4b^3c^2) - \frac{(16a^2b^2d-16ab^3c)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}{4(-a)^{3/4}b^{5/4}} \right)^{(ad-bc)} \cdot \left( \frac{x(4a^2bd^2-8ab^2cd+4b^3c^2) + \frac{(16a^2b^2d-16ab^3c)(ad-bc)}{4(-a)^{3/4}b^{5/4}}}{4(-a)^{3/4}b^{5/4}} \right)^{(ad-bc)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)/(a + b\*x^4),x)

[Out] (d\*x)/b - (atan((((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4)) + ((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) + ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4)))/((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4)) - ((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) + ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4)))/((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c)\*1i)/(2\*(-a)^(3/4)\*b^(5/4)) - (atan((((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4)) + ((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) + ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c))/(4\*(-a)^(3/4)\*b^(5/4)))/((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4)) - ((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) + ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4)))/((x\*(4\*b^3\*c^2 + 4\*a^2\*b\*d^2 - 8\*a\*b^2\*c\*d) - ((16\*a^2\*b^2\*d - 16\*a\*b^3\*c)\*(a\*d - b\*c)\*1i)/(4\*(-a)^(3/4)\*b^(5/4))))\*(a\*d - b\*c))/(2\*(-a)^(3/4)\*b^(5/4))

**sympy** [A] time = 0.61, size = 87, normalized size = 0.39

$$\operatorname{RootSum} \left( 256t^4a^3b^5 + a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4, \left( t \mapsto t \log \left( -\frac{4tab}{ad-bc} + x \right) \right) \right) + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)/(b*x**4+a),x)
```

```
[Out] RootSum(256*_t**4*a**3*b**5 + a**4*d**4 - 4*a**3*b*c*d**3 + 6*a**2*b**2*c**  
2*d**2 - 4*a*b**3*c**3*d + b**4*c**4, Lambda(_t, _t*log(-4*_t*a*b/(a*d - b*  
c) + x))) + d*x/b
```

$$3.164 \quad \int \frac{1}{(a+bx^4)(c+dx^4)} dx$$

**Optimal.** Leaf size=449

$$\frac{b^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4}}{2}$$

[Out]  $\frac{1}{4} b^{3/4} \arctan(-1 + b^{1/4} x^2 / a^{1/4}) / a^{3/4} / (-a d + b^2 c)^{1/2} + \frac{1}{4} b^{3/4} \arctan(1 + b^{1/4} x^2 / a^{1/4}) / a^{3/4} / (-a d + b^2 c)^{1/2} - \frac{1}{4} d^{3/4} \arctan(-1 + d^{1/4} x^2 / c^{1/4}) / c^{3/4} / (-a d + b^2 c)^{1/2} - \frac{1}{4} d^{3/4} \arctan(1 + d^{1/4} x^2 / c^{1/4}) / c^{3/4} / (-a d + b^2 c)^{1/2} - \frac{1}{8} b^{3/4} \ln(-a^{1/4} b^{1/4} x^2 + a^{1/2} + x^2 b^{1/2}) / a^{3/4} / (-a d + b^2 c)^{1/2} + \frac{1}{8} b^{3/4} \ln(a^{1/4} b^{1/4} x^2 + a^{1/2} + x^2 b^{1/2}) / a^{3/4} / (-a d + b^2 c)^{1/2} + \frac{1}{8} d^{3/4} \ln(-c^{1/4} d^{1/4} x^2 + c^{1/2} + x^2 d^{1/2}) / c^{3/4} / (-a d + b^2 c)^{1/2} - \frac{1}{8} d^{3/4} \ln(c^{1/4} d^{1/4} x^2 + c^{1/2} + x^2 d^{1/2}) / c^{3/4} / (-a d + b^2 c)^{1/2}$

**Rubi [A]** time = 0.27, antiderivative size = 449, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$ , Rules used = {391, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)} - \frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)} + \frac{b^{3/4}}{2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)\*(c + d\*x^4)), x]

[Out]  $-(b^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] b^{1/4} x) / a^{1/4}]) / (2 \text{Sqrt}[2] a^{3/4} (b^2 c - a d)) + (b^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] b^{1/4} x) / a^{1/4}]) / (2 \text{Sqrt}[2] a^{3/4} (b^2 c - a d)) + (d^{3/4} \text{ArcTan}[1 - (\text{Sqrt}[2] d^{1/4} x) / c^{1/4}]) / (2 \text{Sqrt}[2] c^{3/4} (b^2 c - a d)) - (d^{3/4} \text{ArcTan}[1 + (\text{Sqrt}[2] d^{1/4} x) / c^{1/4}]) / (2 \text{Sqrt}[2] c^{3/4} (b^2 c - a d)) - (b^{3/4} \text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2] a^{1/4} b^{1/4} x + \text{Sqrt}[b] x^2]) / (4 \text{Sqrt}[2] a^{3/4} (b^2 c - a d)) + (b^{3/4} \text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2] a^{1/4} b^{1/4} x + \text{Sqrt}[b] x^2]) / (4 \text{Sqrt}[2] a^{3/4} (b^2 c - a d)) + (d^{3/4} \text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2] c^{1/4} d^{1/4} x + \text{Sqrt}[d] x^2]) / (4 \text{Sqrt}[2] c^{3/4} (b^2 c - a d)) - (d^{3/4} \text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2] c^{1/4} d^{1/4} x + \text{Sqrt}[d] x^2]) / (4 \text{Sqrt}[2] c^{3/4} (b^2 c - a d))$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

**Rule 391**

Int[1/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c +

$d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

### Rule 617

$\text{Int}[(a_ + (b_)*(x_ ) + (c_)*(x_ )^2)^{-1}, x\_Symbol] \text{:>} \text{With}[\{q = 1 - 4*S$   
 $\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b$   
 $], x] /; \text{RationalQ}[q] \ \&\& \ (\text{EqQ}[q^2, 1] \ || \ !\text{RationalQ}[b^2 - 4*a*c]) /; \text{Free}$   
 $\text{Q}[\{a, b, c\}, x] \ \&\& \ \text{NeQ}[b^2 - 4*a*c, 0]$

### Rule 628

$\text{Int}[(d_ ) + (e_)*(x_ )]/((a_ ) + (b_)*(x_ ) + (c_)*(x_ )^2), x\_Symbol] \text{:>} S$   
 $\text{imp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d,$   
 $e\}, x] \ \&\& \ \text{EqQ}[2*c*d - b*e, 0]$

### Rule 1162

$\text{Int}[(d_ ) + (e_)*(x_ )^2]/((a_ ) + (c_)*(x_ )^4), x\_Symbol] \text{:>} \text{With}[\{q = \text{Rt}[($   
 $2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e$   
 $/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \ \&$   
 $\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[d*e]$

### Rule 1165

$\text{Int}[(d_ ) + (e_)*(x_ )^2]/((a_ ) + (c_)*(x_ )^4), x\_Symbol] \text{:>} \text{With}[\{q = \text{Rt}[($   
 $-2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x],$   
 $x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{Fre}$   
 $e\text{Q}[\{a, c, d, e\}, x] \ \&\& \ \text{EqQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{NegQ}[d*e]$

### Rubi steps

$$\int \frac{1}{(a + bx^4)(c + dx^4)} dx = \frac{b \int \frac{1}{a+bx^4} dx}{bc - ad} - \frac{d \int \frac{1}{c+dx^4} dx}{bc - ad}$$

$$= \frac{b \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc - ad)} + \frac{b \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc - ad)} - \frac{d \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc - ad)} - \frac{d \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc - ad)}$$

$$= \frac{\sqrt{b} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc - ad)} + \frac{\sqrt{b} \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc - ad)} - \frac{b^{3/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} + 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} a^{3/4}(bc - ad)} - \frac{b^{3/4} \int \frac{\frac{\sqrt{2} \sqrt[4]{a}}{\sqrt[4]{b}} - 2x}{-\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} - x^2} dx}{4\sqrt{2} a^{3/4}(bc - ad)}$$

$$= -\frac{b^{3/4} \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4}(bc - ad)} + \frac{b^{3/4} \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{b} x^2)}{4\sqrt{2} a^{3/4}(bc - ad)} + \frac{d^{3/4} \log(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{2\sqrt{2} c^{3/4}(bc - ad)} - \frac{d^{3/4} \log(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2)}{2\sqrt{2} c^{3/4}(bc - ad)}$$

$$= -\frac{b^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc - ad)} + \frac{b^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc - ad)} + \frac{d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc - ad)} - \frac{d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)}{2\sqrt{2} c^{3/4}(bc - ad)}$$

**Mathematica [A]** time = 0.14, size = 340, normalized size = 0.76

$$a^{3/4} d^{3/4} \log\left(-\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right) - a^{3/4} d^{3/4} \log\left(\sqrt{2} \sqrt[4]{c} \sqrt[4]{d} x + \sqrt{c} + \sqrt{d} x^2\right) + 2a^{3/4} d^{3/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right) - 2a^{3/4} d^{3/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{d} x}{\sqrt[4]{c}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)\*(c + d\*x^4)),x]

[Out]  $(-2*b^{3/4}*c^{3/4}*ArcTan[1 - (Sqrt[2]*b^{1/4}*x)/a^{1/4}] + 2*b^{3/4}*c^{3/4}*ArcTan[1 + (Sqrt[2]*b^{1/4}*x)/a^{1/4}] + 2*a^{3/4}*d^{3/4}*ArcTan[1 - (Sqrt[2]*d^{1/4}*x)/c^{1/4}] - 2*a^{3/4}*d^{3/4}*ArcTan[1 + (Sqrt[2]*d^{1/4}*x)/c^{1/4}] - b^{3/4}*c^{3/4}*Log[Sqrt[a] - Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2] + b^{3/4}*c^{3/4}*Log[Sqrt[a] + Sqrt[2]*a^{1/4}*b^{1/4}*x + Sqrt[b]*x^2] + a^{3/4}*d^{3/4}*Log[Sqrt[c] - Sqrt[2]*c^{1/4}*d^{1/4}*x + Sqrt[d]*x^2] - a^{3/4}*d^{3/4}*Log[Sqrt[c] + Sqrt[2]*c^{1/4}*d^{1/4}*x + Sqrt[d]*x^2])/(4*Sqrt[2]*a^{3/4}*c^{3/4}*(b*c - a*d))$

**fricas** [B] time = 1.36, size = 1356, normalized size = 3.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c),x, algorithm="fricas")

[Out]  $(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4}*\arctan(((a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)))^{3/4}*x - (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - a^5*d^3)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)))^{3/4}*\sqrt{(b^2*x^2 + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*\sqrt{-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))}/b^2))/b^2) + (-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}*\arctan(((b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))^{3/4}*x - (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*c^2*d^3)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4)))^{3/4}*\sqrt{(d^2*x^2 + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*\sqrt{-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))}/d^2))/d^2) + 1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4}*\log(b*x + (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4})) - 1/4*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4}*\log(b*x - (a*b*c - a^2*d)*(-b^3/(a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4))^{1/4})) - 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}*\log(d*x + (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4})) + 1/4*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}*\log(d*x - (b*c^2 - a*c*d)*(-d^3/(b^4*c^7 - 4*a*b^3*c^6*d + 6*a^2*b^2*c^5*d^2 - 4*a^3*b*c^4*d^3 + a^4*c^3*d^4))^{1/4}))$

**giac** [A] time = 0.21, size = 437, normalized size = 0.97

$$\frac{(ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}abc - \sqrt{2}a^2d)} + \frac{(ab^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}abc - \sqrt{2}a^2d)} - \frac{(cd^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}bc^2 - \sqrt{2}acd)} - \frac{(cd^3)^{\frac{1}{4}} \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2(\sqrt{2}bc^2 - \sqrt{2}acd)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c),x, algorithm="giac")

[Out]  $1/2*(a*b^3)^{1/4}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{1/4}))/((a/b)^{1/4}*\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) + 1/2*(a*b^3)^{1/4}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{1/4}))/((a/b)^{1/4}*\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - 1$

$$\frac{1}{2}*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) - 1/2*(c*d^3)^{(1/4)}*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) + 1/4*(a*b^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - 1/4*(a*b^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b*c - \sqrt{2}*a^2*d) - 1/4*(c*d^3)^{(1/4)}*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d) + 1/4*(c*d^3)^{(1/4)}*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b*c^2 - \sqrt{2}*a*c*d)$$

**maple [A]** time = 0.06, size = 320, normalized size = 0.71

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right) - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} + 1\right) - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} b \ln\left(\frac{x^2 + \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 - \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}\right) + \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right) - \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} + 1\right) - \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2 + \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}\right) + \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} d \ln\left(\frac{x^2 + \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}{x^2 - \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{c}{d}}}\right)}{4(ad - bc)a - 4(ad - bc)a - 8(ad - bc)a + 4(ad - bc)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)/(d\*x^4+c), x)

[Out]  $\frac{1}{8}*\frac{d}{(a*d-b*c)}*(c/d)^{(1/4)}/c*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))+1/4*d/(a*d-b*c)*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)+1/4*d/(a*d-b*c)*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)-1/8*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))-1/4*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)-1/4*b/(a*d-b*c)*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

**maxima [A]** time = 1.43, size = 365, normalized size = 0.81

$$\frac{2\sqrt{2}b \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{a}\sqrt{b}} + \frac{2\sqrt{2}b \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{a}\sqrt{b}} + \frac{\sqrt{2}b^{\frac{3}{4}} \log\left(\sqrt{b}x^2 + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}} - \frac{\sqrt{2}b^{\frac{3}{4}} \log\left(\sqrt{b}x^2 - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x + \sqrt{a}\right)}{a^{\frac{3}{4}}}{8(bc - ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c), x, algorithm="maxima")

[Out]  $\frac{1}{8}*(2*\sqrt{2}*b*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*b*x + \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/(\sqrt{a}*\sqrt{a}*\sqrt{b}) + 2*\sqrt{2}*b*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*b*x - \sqrt{2}*a^{(1/4)}*b^{(1/4)})/\sqrt{a}*\sqrt{b}))/(\sqrt{a}*\sqrt{a}*\sqrt{b}) + \sqrt{2}*b^{(3/4)}*\log(\sqrt{b}*x^2 + \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/a^{(3/4)} - \sqrt{2}*b^{(3/4)}*\log(\sqrt{b}*x^2 - \sqrt{2}*a^{(1/4)}*b^{(1/4)}*x + \sqrt{a})/a^{(3/4)})/(b*c - a*d) - 1/8*(2*\sqrt{2})*d*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*d*x + \sqrt{2}*c^{(1/4)}*d^{(1/4)})/\sqrt{c}*\sqrt{d}))/(\sqrt{c}*\sqrt{c}*\sqrt{d}) + 2*\sqrt{2}*d*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*d*x - \sqrt{2}*c^{(1/4)}*d^{(1/4)})/\sqrt{c}*\sqrt{d}))/(\sqrt{c}*\sqrt{c}*\sqrt{d}) + \sqrt{2}*d^{(3/4)}*\log(\sqrt{d}*x^2 + \sqrt{2}*c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/c^{(3/4)} - \sqrt{2}*d^{(3/4)}*\log(\sqrt{d}*x^2 - \sqrt{2}*c^{(1/4)}*d^{(1/4)}*x + \sqrt{c})/c^{(3/4)})/(b*c - a*d)$

**mapad [B]** time = 2.76, size = 6153, normalized size = 13.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)\*(c + d\*x^4)), x)



$$\begin{aligned}
& (3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge \\
& (1/4) * i_1 - ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * (4096a^8b^4c^4d^4 + 4096a^8b^4c^4d^4 - 20480a^2b^10c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^10) - x*(1024a^7b^4d^11 + 1024b^11c^7d^4 - 4096a^10b^10c^6d^5 - 4096a^6b^5c^5d^10 + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7c^3d^8 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32a^2b^7c^2d^7) - 8b^7d^7x)*(-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * i_1) / (((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * (4096a^8b^4c^4d^4 + 4096a^8b^4c^4d^4 - 20480a^2b^10c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^10) + x*(1024a^7b^4d^11 + 1024b^11c^7d^4 - 4096a^10b^10c^6d^5 - 4096a^6b^5c^5d^10 + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32a^2b^7c^2d^7) + 8b^7d^7x)*(-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) + ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * (4096a^8b^4c^4d^4 + 4096a^8b^4c^4d^4 - 20480a^2b^10c^7d^5 + 36864a^3b^9c^6d^6 - 20480a^4b^8c^5d^7 - 20480a^5b^7c^4d^8 + 36864a^6b^6c^3d^9 - 20480a^7b^5c^2d^10) - x*(1024a^7b^4d^11 + 1024b^11c^7d^4 - 4096a^10b^10c^6d^5 - 4096a^6b^5c^5d^10 + 6144a^2b^9c^5d^6 - 3072a^3b^8c^4d^7 - 3072a^4b^7c^3d^8 + 6144a^5b^6c^2d^9)) - 16a^2b^6d^8 - 16b^8c^2d^6 + 32a^2b^7c^2d^7) - 8b^7d^7x)*(-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4))) * (-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * 2i - 2*atan((b^3d^3x - (128b^10c^7x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) - (128a^7b^3d^7x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) + (384a^3b^7c^4d^3x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) - (768a^5b^5c^2d^5x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) + (512a^6b^9c^6d^6x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) + (512a^6b^4c^6d^6x)/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) / ((-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) * ((b^3*(512a^6b^7c^8 + 512a^8c^8d^7 - 2560a^2b^6c^7d - 2560a^7b^6c^2d^6 + 4608a^3b^5c^6d^2 - 2560a^4b^4c^5d^3 - 2560a^5b^3c^4d^4 + 4608a^6b^2c^3d^5)) / (256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) + 2a^2b^2d^4 + 2b^4c^2d^2 - 4a^2b^3c^3d^3))) * (-b^3/(256a^7d^4 + 256a^3b^4c^4 - 1024a^4b^3c^3d + 1536a^5b^2c^2d^2 - 1024a^6b^2c^2d^2 - 1024a^6b^2c^2d^2) \wedge (1/4) - 2*atan((b^3d^3x - (128a^7d^10x)/(256b^4c^7 + 256a^4c^3d^4 - 1024a^3b^c^4d^3 +
\end{aligned}$$



$$\begin{aligned}
& 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (128*b^7*c^7*d^3*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (768*a^2*b^5*c^5*d^5*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^3*b^4*c^4*d^6*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (384*a^4*b^3*c^3*d^7*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) - (768*a^5*b^2*c^2*d^8*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a^6*b*c*d^9*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + (512*a*b^6*c^6*d^4*x)/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))/((-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4))*((d^3*(512*a*b^7*c^8 + 512*a^8*c*d^7 - 2560*a^2*b^6*c^7*d - 2560*a^7*b*c^2*d^6 + 4608*a^3*b^5*c^6*d^2 - 2560*a^4*b^4*c^5*d^3 - 2560*a^5*b^3*c^4*d^4 + 4608*a^6*b^2*c^3*d^5))/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d) + 2*a^2*b^2*d^4 + 2*b^4*c^2*d^2 - 4*a*b^3*c*d^3))*(-d^3/(256*b^4*c^7 + 256*a^4*c^3*d^4 - 1024*a^3*b*c^4*d^3 + 1536*a^2*b^2*c^5*d^2 - 1024*a*b^3*c^6*d))^(1/4)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)/(d\*x\*\*4+c), x)

[Out] Timed out

$$3.165 \quad \int \frac{1}{(a+bx^4)(c+dx^4)^2} dx$$

**Optimal.** Leaf size=513

$$\frac{b^{7/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)^2} - \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)^2}$$

[Out]  $-1/4*d*x/c/(-a*d+b*c)/(d*x^4+c)+1/4*b^{(7/4)}*\arctan(-1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/(-a*d+b*c)^2*2^{(1/2)}+1/4*b^{(7/4)}*\arctan(1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(3/4)}/(-a*d+b*c)^2*2^{(1/2)}-1/16*d^{(3/4)}*(-3*a*d+7*b*c)*\arctan(-1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/(-a*d+b*c)^2*2^{(1/2)}-1/16*d^{(3/4)}*(-3*a*d+7*b*c)*\arctan(1+d^{(1/4)}*x*2^{(1/2)}/c^{(1/4)})/c^{(7/4)}/(-a*d+b*c)^2*2^{(1/2)}-1/8*b^{(7/4)}*\ln(-a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/(-a*d+b*c)^2*2^{(1/2)}+1/8*b^{(7/4)}*\ln(a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(3/4)}/(-a*d+b*c)^2*2^{(1/2)}+1/32*d^{(3/4)}*(-3*a*d+7*b*c)*\ln(-c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/(-a*d+b*c)^2*2^{(1/2)}-1/32*d^{(3/4)}*(-3*a*d+7*b*c)*\ln(c^{(1/4)}*d^{(1/4)}*x*2^{(1/2)}+c^{(1/2)}+x^2*d^{(1/2)})/c^{(7/4)}/(-a*d+b*c)^2*2^{(1/2)}$

**Rubi [A]** time = 0.42, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4} \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{4\sqrt{2} a^{3/4}(bc-ad)^2} - \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b} x}{\sqrt[4]{a}}\right)}{2\sqrt{2} a^{3/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x)/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^{(7/4)}*ArcTan[1 - (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*Sqrt[2]*a^{(3/4)}*(b*c - a*d)^2) + (b^{(7/4)}*ArcTan[1 + (Sqrt[2]*b^{(1/4)}*x)/a^{(1/4)}])/(2*Sqrt[2]*a^{(3/4)}*(b*c - a*d)^2) + (d^{(3/4)}*(7*b*c - 3*a*d)*ArcTan[1 - (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*Sqrt[2]*c^{(7/4)}*(b*c - a*d)^2) - (d^{(3/4)}*(7*b*c - 3*a*d)*ArcTan[1 + (Sqrt[2]*d^{(1/4)}*x)/c^{(1/4)}])/(8*Sqrt[2]*c^{(7/4)}*(b*c - a*d)^2) - (b^{(7/4)}*Log[Sqrt[a] - Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{(3/4)}*(b*c - a*d)^2) + (b^{(7/4)}*Log[Sqrt[a] + Sqrt[2]*a^{(1/4)}*b^{(1/4)}*x + Sqrt[b]*x^2])/(4*Sqrt[2]*a^{(3/4)}*(b*c - a*d)^2) + (d^{(3/4)}*(7*b*c - 3*a*d)*Log[Sqrt[c] - Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^{(7/4)}*(b*c - a*d)^2) - (d^{(3/4)}*(7*b*c - 3*a*d)*Log[Sqrt[c] + Sqrt[2]*c^{(1/4)}*d^{(1/4)}*x + Sqrt[d]*x^2])/(16*Sqrt[2]*c^{(7/4)}*(b*c - a*d)^2)$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)(c+dx^4)^2} dx &= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-3bdx^4}{(a+bx^4)(c+dx^4)} dx}{4c(bc-ad)} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2 \int \frac{1}{a+bx^4} dx}{(bc-ad)^2} - \frac{(d(7bc-3ad)) \int \frac{1}{c+dx^4} dx}{4c(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^2 \int \frac{\sqrt{a}-\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc-ad)^2} + \frac{b^2 \int \frac{\sqrt{a}+\sqrt{b}x^2}{a+bx^4} dx}{2\sqrt{a}(bc-ad)^2} - \frac{(d(7bc-3ad)) \int \frac{1}{c+dx^4} dx}{8c^{3/2}(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} + \frac{b^{3/2} \int \frac{1}{\frac{\sqrt{a}-\sqrt{2}\sqrt[4]{a}x}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc-ad)^2} + \frac{b^{3/2} \int \frac{1}{\frac{\sqrt{a}+\sqrt{2}\sqrt[4]{a}x}{\sqrt{b}} + \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{4\sqrt{a}(bc-ad)^2} - \frac{b^{7/4} \int \frac{1}{c+dx^4} dx}{4\sqrt{2}a^{3/4}(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4} \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \log(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{b}x^2)}{4\sqrt{2}a^{3/4}(bc-ad)^2} \\
&= -\frac{dx}{4c(bc-ad)(c+dx^4)} - \frac{b^{7/4} \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{b^{7/4} \tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{2\sqrt{2}a^{3/4}(bc-ad)^2} + \frac{d^{3/4}(7bc-3ad)}{8c^{3/2}(bc-ad)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.33, size = 498, normalized size = 0.97

$$8a^{3/4}c^{3/4}dx(ad-bc) - 2\sqrt{2}a^{3/4}d^{3/4}(c+dx^4)(3ad-7bc)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right) + 2\sqrt{2}a^{3/4}d^{3/4}(c+dx^4)(3ad-7bc)\tan^{-1}\left(1 + \frac{\sqrt{2}\sqrt[4]{d}x}{\sqrt[4]{c}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)\*(c + d\*x^4)^2), x]

[Out]  $(8a^{3/4}c^{3/4}d(-bc+ad)x - 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\text{ArcTan}\left[1 - \frac{\sqrt{2}b^{1/4}x}{a^{1/4}}\right] + 8\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\text{ArcTan}\left[1 + \frac{\sqrt{2}b^{1/4}x}{a^{1/4}}\right] - 2\sqrt{2}a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\text{ArcTan}\left[1 - \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right] + 2\sqrt{2}a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\text{ArcTan}\left[1 + \frac{\sqrt{2}d^{1/4}x}{c^{1/4}}\right] - 4\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\text{Log}\left[\sqrt{a} - \sqrt{2}b^{1/4}x + \sqrt{b}x^2\right] + 4\sqrt{2}b^{7/4}c^{7/4}(c+dx^4)\text{Log}\left[\sqrt{a} + \sqrt{2}b^{1/4}x + \sqrt{b}x^2\right] + \sqrt{2}a^{3/4}d^{3/4}(7bc-3ad)(c+dx^4)\text{Log}\left[\sqrt{c} - \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{d}x^2\right] + \sqrt{2}a^{3/4}d^{3/4}(-7bc+3ad)(c+dx^4)\text{Log}\left[\sqrt{c} + \sqrt{2}c^{1/4}d^{1/4}x + \sqrt{d}x^2\right]) / (32a^{3/4}c^{7/4}(bc-ad)^2(c+dx^4))$

**fricas [B]** time = 46.36, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out]  $1/16*(4*((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)*(-(2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 35*a^6*b^2*c^9*d^6 - 28*a^7*b*c^8*d^7 + 14*a^8*b^2*c^7*d^8 - 8*a^9*b*c^6*d^9 + 4*a^10*b^2*c^5*d^10 - 4*a^11*b*c^4*d^11 + 2*a^12*b^2*c^3*d^12 - 2*a^13*b*c^2*d^13 + a^14*b^2*d^14))$

$$\begin{aligned}
& ^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8) \wedge (1/4) * \arctan((b^6c^{11} - 6a^5b^3c^{10}d + 15a^2b^4c^9d^2 - 20a^3b^3c^8d^3 + 15a^4b^2c^7d^4 - 6a^5b^1c^6d^5 + a^6c^5d^6) * x - (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8)) \wedge (3/4) - (b^6c^{11} - 6a^5b^3c^{10}d + 15a^2b^4c^9d^2 - 20a^3b^3c^8d^3 + 15a^4b^2c^7d^4 - 6a^5b^1c^6d^5 + a^6c^5d^6) * (- (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8)) \wedge (3/4) * \sqrt{((49b^2c^2d^2 - 42a^3b^1c^1d^3 + 9a^2d^4) * x^2 + (b^4c^8 - 4a^3b^3c^7d + 6a^2b^2c^6d^2 - 4a^1b^1c^5d^3 + a^4c^4d^4) * \sqrt{- (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8))} / (49b^2c^2d^2 - 42a^3b^1c^1d^3 + 9a^2d^4)) / (343b^3c^3d^2 - 441a^4b^2c^2d^3 + 189a^2b^1c^1d^4 - 27a^3d^5)) + 16 * (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (1/4) * ((b^2c^2d - a^2c^2d) * x^4 + b^3c^3 - a^2c^2d) * \arctan(-((a^2b^6c^6 - 6a^3b^5c^5d + 15a^4b^4c^4d^2 - 20a^5b^3c^3d^3 + 15a^6b^2c^2d^4 - 6a^7b^1c^1d^5 + a^8d^6) * (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8))) \wedge (3/4) * x - (a^2b^6c^6 - 6a^3b^5c^5d + 15a^4b^4c^4d^2 - 20a^5b^3c^3d^3 + 15a^6b^2c^2d^4 - 6a^7b^1c^1d^5 + a^8d^6) * (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (3/4) * \sqrt{(b^4x^2 + (a^2b^4c^4 - 4a^3b^3c^3d + 6a^4b^2c^2d^2 - 4a^5b^1c^1d^3 + a^6d^4) * \sqrt{-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8))} / b^4) / b^5) + 4 * (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (1/4) * ((b^2c^2d - a^2c^2d) * x^4 + b^3c^3 - a^2c^2d) * \log(b^2 * x + (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (1/4) * (a^2b^2c^2 - 2a^2b^1c^1d + a^3d^2)) - 4 * (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (1/4) * ((b^2c^2d - a^2c^2d) * x^4 + b^3c^3 - a^2c^2d) * \log(b^2 * x - (-b^7 / (a^3b^8c^8 - 8a^4b^7c^7d + 28a^5b^6c^6d^2 - 56a^6b^5c^5d^3 + 70a^7b^4c^4d^4 - 56a^8b^3c^3d^5 + 28a^9b^2c^2d^6 - 8a^10b^1c^1d^7 + a^11d^8)) \wedge (1/4) * (a^2b^2c^2 - 2a^2b^1c^1d + a^3d^2)) + ((b^2c^2d - a^2c^2d) * x^4 + b^3c^3 - a^2c^2d) * (- (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8)) \wedge (1/4) * \log(- (7 * b^3c^3d - 3 * a^3d^2) * x + (b^2c^4 - 2 * a^2b^1c^1d + a^2c^2d^2) * (- (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8)) \wedge (1/4)) - ((b^2c^2d - a^2c^2d) * x^4 + b^3c^3 - a^2c^2d) * (- (2401b^4c^4d^3 - 4116a^5b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^1c^1d^6 + 81a^4d^7) / (b^8c^{15} - 8a^5b^7c^{14}d + 28a^2b^6c^{13}d^2 - 56a^3b^5c^{12}d^3 + 70a^4b^4c^{11}d^4 - 56a^5b^3c^{10}d^5 + 28a^6b^2c^9d^6 - 8a^7b^1c^8d^7 + a^8c^7d^8)) \wedge (1/4) * 1
\end{aligned}$$

$\log(- (7*b*c*d - 3*a*d^2)*x - (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*(- (2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 + 81*a^4*d^7)/(b^8*c^15 - 8*a*b^7*c^14*d + 28*a^2*b^6*c^13*d^2 - 56*a^3*b^5*c^12*d^3 + 70*a^4*b^4*c^11*d^4 - 56*a^5*b^3*c^10*d^5 + 28*a^6*b^2*c^9*d^6 - 8*a^7*b*c^8*d^7 + a^8*c^7*d^8))^{(1/4)}) - 4*d*x)/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d)$

**giac** [A] time = 0.19, size = 667, normalized size = 1.30

$$\frac{(ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right) + (ab^3)^{\frac{1}{4}} b \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right) + (ab^3)^{\frac{1}{4}} b \log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{b}}\right)}{2\left(\sqrt{2}ab^2c^2 - 2\sqrt{2}a^2bcd + \sqrt{2}a^3d^2\right) + 2\left(\sqrt{2}ab^2c^2 - 2\sqrt{2}a^2bcd + \sqrt{2}a^3d^2\right) + 4\left(\sqrt{2}ab^2c^2 - 2\sqrt{2}a^2bcd + \sqrt{2}a^3d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c)^2,x, algorithm="giac")

[Out]  $\frac{1}{2}*(a*b^3)^{(1/4)}*b*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a/b)^{(1/4)}/(\sqrt{2}*a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) + \frac{1}{2}*(a*b^3)^{(1/4)}*b*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a/b)^{(1/4)}/(\sqrt{2}*a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) + \frac{1}{4}*(a*b^3)^{(1/4)}*b*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) - \frac{1}{4}*(a*b^3)^{(1/4)}*b*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a*b^2*c^2 - 2*\sqrt{2}*a^2*b*c*d + \sqrt{2}*a^3*d^2) - \frac{1}{8}*(7*(c*d^3)^{(1/4)}*b*c - 3*(c*d^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(c/d)^{(1/4)}/(\sqrt{2}*b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - \frac{1}{8}*(7*(c*d^3)^{(1/4)}*b*c - 3*(c*d^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(c/d)^{(1/4)}/(\sqrt{2}*b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - \frac{1}{16}*(7*(c*d^3)^{(1/4)}*b*c - 3*(c*d^3)^{(1/4)}*a*d)*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) + \frac{1}{16}*(7*(c*d^3)^{(1/4)}*b*c - 3*(c*d^3)^{(1/4)}*a*d)*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^4 - 2*\sqrt{2}*a*b*c^3*d + \sqrt{2}*a^2*c^2*d^2) - \frac{1}{4}*d*x/((d*x^4 + c)*(b*c^2 - a*c*d))$

**maple** [A] time = 0.06, size = 550, normalized size = 1.07

$$\frac{a d^2 x}{4(ad - bc)^2 (d x^4 + c) c} - \frac{b d x}{4(ad - bc)^2 (d x^4 + c)} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} a d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}} - 1\right)}{16(ad - bc)^2 c^2} + \frac{3\left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2} a d^2 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{16(ad - bc)^2 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)/(d\*x^4+c)^2,x)

[Out]  $\frac{1}{4}*d^2/(a*d-b*c)^2/c*x/(d*x^4+c)*a^{-1/4}*d/(a*d-b*c)^2*x/(d*x^4+c)*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*a^{-7/16}*d/(a*d-b*c)^2/c*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)*b+3/32*d^2/(a*d-b*c)^2/c^2*(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})))*a^{-7/32}*d/(a*d-b*c)^2/c*(c/d)^{(1/4)}*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})))*b+3/16*d^2/(a*d-b*c)^2/c^2*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)*a^{-7/16}*d/(a*d-b*c)^2/c*(c/d)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)*b+1/8*b^2/(a*d-b*c)^2*(a/b)^{(1/4)}/a*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})))+1/4*b^2/(a*d-b*c)^2*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)+1/4*b^2/(a*d-b*c)^2*(a/b)^{(1/4)}/a*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)$

**maxima** [A] time = 1.57, size = 481, normalized size = 0.94

$$\frac{dx}{4((bc^2d - acd^2)x^4 + bc^3 - ac^2d)} + \frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx} + \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx} - \sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}b^{\frac{7}{4}} \log\left(\sqrt{b}\right)}{8(b^2c^2 - 2abcd + a^2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out]  $-1/4*d*x/((b*c^2*d - a*c*d^2)*x^4 + b*c^3 - a*c^2*d) + 1/8*(2*\sqrt{2}*b^2*a$   
 $\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*b*x + \sqrt{2}*a^{1/4}*b^{1/4})/\sqrt{a*\sqrt{b}})/(\sqrt{a}*\sqrt{a*\sqrt{b}}) + 2*\sqrt{2}*b^2*\arctan(1/2*\sqrt{2}*(2$   
 $*\sqrt{2}*b*x - \sqrt{2}*a^{1/4}*b^{1/4})/\sqrt{a*\sqrt{b}})/(\sqrt{a}*\sqrt{a*\sqrt{b}}) + \sqrt{2}*b^{7/4}*\log(\sqrt{b}*x^2 + \sqrt{2}*a^{1/4}*b^{1/4}$   
 $*x + \sqrt{a})/a^{3/4} - \sqrt{2}*b^{7/4}*\log(\sqrt{b}*x^2 - \sqrt{2}*a^{1/4}$   
 $*b^{1/4}*x + \sqrt{a})/a^{3/4})/(b^2*c^2 - 2*a*b*c*d + a^2*d^2) - 1/32*(2*\sqrt{2}$   
 $*\sqrt{2}*(7*b*c*d - 3*a*d^2)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*d*x + \sqrt{2}*c^{1/4}$   
 $*d^{1/4})/\sqrt{c*\sqrt{d}})/(\sqrt{c}*\sqrt{c*\sqrt{d}}) + 2*\sqrt{2}$   
 $*(7*b*c*d - 3*a*d^2)*\arctan(1/2*\sqrt{2}*(2*\sqrt{2}*d*x - \sqrt{2}*c^{1/4}$   
 $*d^{1/4})/\sqrt{c*\sqrt{d}})/(\sqrt{c}*\sqrt{c*\sqrt{d}}) + \sqrt{2}*(7*b$   
 $*c*d - 3*a*d^2)*\log(\sqrt{d}*x^2 + \sqrt{2}*c^{1/4}*d^{1/4}*x + \sqrt{c})/(c^{3/4}$   
 $*d^{1/4}) - \sqrt{2}*(7*b*c*d - 3*a*d^2)*\log(\sqrt{d}*x^2 - \sqrt{2}*c^{1/4}$   
 $*d^{1/4}*x + \sqrt{c})/(c^{3/4}*d^{1/4})/(b^2*c^3 - 2*a*b*c^2*d + a^2*c*d$   
 $^2)$

**mupad** [B] time = 4.00, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)\*(c + d\*x^4)^2),x)

[Out]  $2*\operatorname{atan}\left(\frac{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}\right)^{1/4} * \left(\frac{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}\right)^{3/4} * \left(\frac{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}{(-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d)}\right)^{1/4} * (28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3$

$$\begin{aligned}
& *a*b^2*c^6*d) - (x*(65536*b^{17}*c^{15}*d^4 - 524288*a*b^{16}*c^{14}*d^5 + 1835008* \\
& a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}*d^7 + 2809856*a^4*b^{13}*c^{11}*d^8 + \\
& 3362816*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6*b^{11}*c^9*d^{10} + 24190976*a^7*b^{10} \\
& *c^8*d^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17833984*a^9*b^8*c^6*d^{13} - 8486912 \\
& *a^{10}*b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4*d^{15} - 466944*a^{12}*b^5*c^3*d^{16} + \\
& 36864*a^{13}*b^4*c^2*d^{17})*1i)/(64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 \\
& + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) \\
& *1i) + (x*(81*a^4*b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 \\
& - 756*a^3*b^{10}*c*d^{10} + 2790*a^2*b^{11}*c^2*d^9))/(64*(b^6*c^{10} + a^6*c^4*d^6 \\
& - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 \\
& - 6*a*b^5*c^9*d)) - ((-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + \\
& 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - \\
& 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + \\
& 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - \\
& 524288*a*b^7*c^{14}*d))^{(1/4)}*((-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + \\
& 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - \\
& 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + \\
& 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - \\
& 524288*a*b^7*c^{14}*d))^{(1/4)}*(((81*a^4*b^7*d^{10})/16 + 28*b^{11}*c^4*d^6 - (2145*a*b^{10}*c^3*d^7)/16 - \\
& (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2 \\
& *b*c^5*d^2 - 3*a*b^2*c^6*d) + ((-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3 \\
& *c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536* \\
& a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3 \\
& *b^5*c^{12}*d^3 + 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 18350 \\
& 08*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(3/4)}*(((81*a^4*d^7 + 2401*b^4 \\
& *c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65 \\
& 536*b^8*c^{15} + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^ \\
& ^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5 \\
& *b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(1/4)}*(2867 \\
& 2*a^2*b^{13}*c^{13}*d^5 - 4096*a*b^{14}*c^{14}*d^4 - 78848*a^3*b^{12}*c^{12}*d^6 + 9011 \\
& 2*a^4*b^{11}*c^{11}*d^7 + 28672*a^5*b^{10}*c^{10}*d^8 - 229376*a^6*b^9*c^9*d^9 + 32 \\
& 9728*a^7*b^8*c^8*d^{10} - 253952*a^8*b^7*c^7*d^{11} + 114688*a^9*b^6*c^6*d^{12} - \\
& 28672*a^{10}*b^5*c^5*d^{13} + 3072*a^{11}*b^4*c^4*d^{14}))/((b^3*c^7 - a^3*c^4*d^3 \\
& + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^{17}*c^{15}*d^4 - 524288*a*b^{16} \\
& *c^{14}*d^5 + 1835008*a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}*d^7 + 280985 \\
& 6*a^4*b^{13}*c^{11}*d^8 + 3362816*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6*b^{11}*c^9*d^{10} \\
& + 24190976*a^7*b^{10}*c^8*d^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17833984*a^9*b^8 \\
& *c^6*d^{13} - 8486912*a^{10}*b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4*d^{15} - 46694 \\
& 4*a^{12}*b^5*c^3*d^{16} + 36864*a^{13}*b^4*c^2*d^{17})*1i)/(64*(b^6*c^{10} + a^6*c^4*d^6 \\
& - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - \\
& 6*a*b^5*c^9*d)))*1i) - (x*(81*a^4*b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12} \\
& *c^3*d^8 - 756*a^3*b^{10}*c*d^{10} + 2790*a^2*b^{11}*c^2*d^9))/(64*(b^6*c^{10} + a^6*c^4*d^6 \\
& - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - \\
& 6*a*b^5*c^9*d)))/(((81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2 \\
& *b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - 524288*a^7 \\
& *b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + 4587520*a^4 \\
& *b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7 \\
& *c^{14}*d))^{(1/4)}*(((81*a^4*b^7*d^{10})/16 + 28*b^{11}*c^4*d^6 - (2145*a*b^{10}*c^3*d^7)/16 - \\
& (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*1i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2 \\
& *b*c^5*d^2 - 3*a*b^2*c^6*d) + ((-81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + \\
& 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - \\
& 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + \\
& 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5
\end{aligned}$$



$$\begin{aligned}
& *b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(3/4)}*(((- ( \\
& 81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - \\
& 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 \\
& + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + 4587520*a^4*b^4*c \\
& ^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7 \\
& *c^{14}*d))^{(1/4)}*(28672*a^2*b^{13}*c^{13}*d^5 - 4096*a*b^{14}*c^{14}*d^4 - 78848*a^3 \\
& *b^{12}*c^{12}*d^6 + 90112*a^4*b^{11}*c^{11}*d^7 + 28672*a^5*b^{10}*c^{10}*d^8 - 229376 \\
& *a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^{10} - 253952*a^8*b^7*c^7*d^{11} + 1146 \\
& 88*a^9*b^6*c^6*d^{12} - 28672*a^{10}*b^5*c^5*d^{13} + 3072*a^{11}*b^4*c^4*d^{14}))/ (b \\
& ^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^{17}*c^ \\
& 15*d^4 - 524288*a*b^{16}*c^{14}*d^5 + 1835008*a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b \\
& ^{14}*c^{12}*d^7 + 2809856*a^4*b^{13}*c^{11}*d^8 + 3362816*a^5*b^{12}*c^{10}*d^9 - 1451 \\
& 6224*a^6*b^{11}*c^9*d^{10} + 24190976*a^7*b^{10}*c^8*d^{11} - 25280512*a^8*b^9*c^7* \\
& d^{12} + 17833984*a^9*b^8*c^6*d^{13} - 8486912*a^{10}*b^7*c^5*d^{14} + 2609152*a^{11} \\
& *b^6*c^4*d^{15} - 466944*a^{12}*b^5*c^3*d^{16} + 36864*a^{13}*b^4*c^2*d^{17})*i)/(64 \\
& *(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^ \\
& 3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*i + (x*(81*a^4*b^9*d \\
& ^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 - 756*a^3*b^{10}*c*d^{10} + 2790* \\
& a^2*b^{11}*c^2*d^9)*i)/(64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^ \\
& 2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) \\
& + ((- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2* \\
& d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^ \\
& ^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + 4587520*a^4* \\
& b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288* \\
& a*b^7*c^{14}*d))^{(1/4)}*(((- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 \\
& + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7* \\
& d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{1 \\
& 2}*d^3 + 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b \\
& ^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(1/4)}*((((81*a^4*b^7*d^{10})/16 + 28*b^{11}* \\
& c^4*d^6 - (2145*a*b^{10}*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9 \\
& *c^2*d^8)/16)*i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) \\
& + ((- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2 \\
& *d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^7*d^8 - 524288*a^7*b* \\
& c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^{12}*d^3 + 4587520*a^4 \\
& *b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288 \\
& *a*b^7*c^{14}*d))^{(3/4)}*(((- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d \\
& ^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^8*c^ \\
& 7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3*b^5*c^ \\
& ^{12}*d^3 + 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 1835008*a^6 \\
& *b^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(1/4)}*(28672*a^2*b^{13}*c^{13}*d^5 - 4096* \\
& a*b^{14}*c^{14}*d^4 - 78848*a^3*b^{12}*c^{12}*d^6 + 90112*a^4*b^{11}*c^{11}*d^7 + 28672 \\
& *a^5*b^{10}*c^{10}*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^{10} - 253 \\
& 952*a^8*b^7*c^7*d^{11} + 114688*a^9*b^6*c^6*d^{12} - 28672*a^{10}*b^5*c^5*d^{13} + \\
& 3072*a^{11}*b^4*c^4*d^{14}))/ (b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2 \\
& *c^6*d) + (x*(65536*b^{17}*c^{15}*d^4 - 524288*a*b^{16}*c^{14}*d^5 + 1835008*a^2*b^ \\
& 15*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}*d^7 + 2809856*a^4*b^{13}*c^{11}*d^8 + 33628 \\
& 16*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6*b^{11}*c^9*d^{10} + 24190976*a^7*b^{10}*c^8*d \\
& ^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17833984*a^9*b^8*c^6*d^{13} - 8486912*a^{10}* \\
& b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4*d^{15} - 466944*a^{12}*b^5*c^3*d^{16} + 36864 \\
& *a^{13}*b^4*c^2*d^{17})*i)/(64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15* \\
& a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) \\
& ) * i) * i - (x*(81*a^4*b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 - \\
& 756*a^3*b^{10}*c*d^{10} + 2790*a^2*b^{11}*c^2*d^9)*i)/(64*(b^6*c^{10} + a^6*c^4*d^ \\
& 6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^ \\
& ^6*d^4 - 6*a*b^5*c^9*d))))) * (- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3* \\
& c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^{15} + 65536*a^ \\
& ^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^{13}*d^2 - 3670016*a^3* \\
& b^5*c^{12}*d^3 + 4587520*a^4*b^4*c^{11}*d^4 - 3670016*a^5*b^3*c^{10}*d^5 + 183500 \\
& 8*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^{14}*d))^{(1/4)} - \operatorname{atan}((( - (81*a^4*d^7 + 240
\end{aligned}$$

$$\begin{aligned}
& 1*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6 \\
& )/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2* \\
& b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 367001 \\
& 6*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(1/4)*} \\
& ((- (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - \\
& 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + \\
& 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - \\
& 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(1/4)*} \\
& (((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - (2145*a*b^10 \\
& *c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/(b^3*c^7 \\
& - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (- (81*a^4*d^7 + 2401*b^4 \\
& *c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(6 \\
& 5536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6* \\
& c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5 \\
& *b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(3/4)*}((( - \\
& (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - \\
& 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 \\
& + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11 \\
& *d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(1/4)*} \\
& (28672*a^2*b^13*c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3 \\
& *b^12*c^12*d^6 + 90112*a^4*b^11*c^11*d^7 + 28672*a^5*b^10*c^10*d^8 - 22937 \\
& 6*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - 253952*a^8*b^7*c^7*d^11 + 114 \\
& 688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + 3072*a^11*b^4*c^4*d^14))/( \\
& b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^17*c^ \\
& ^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^6 - 3469312*a^3* \\
& b^14*c^12*d^7 + 2809856*a^4*b^13*c^11*d^8 + 3362816*a^5*b^12*c^10*d^9 - 145 \\
& 16224*a^6*b^11*c^9*d^10 + 24190976*a^7*b^10*c^8*d^11 - 25280512*a^8*b^9*c^7 \\
& *d^12 + 17833984*a^9*b^8*c^6*d^13 - 8486912*a^10*b^7*c^5*d^14 + 2609152*a^1 \\
& 1*b^6*c^4*d^15 - 466944*a^12*b^5*c^3*d^16 + 36864*a^13*b^4*c^2*d^17))/(64*( \\
& b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3* \\
& c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*1i - (x*(81*a^4*b^9*d^11 + \\
& 3185*b^13*c^4*d^7 - 4788*a*b^12*c^3*d^8 - 756*a^3*b^10*c*d^10 + 2790*a^2*b^ \\
& ^11*c^2*d^9)*1i)/(64*(b^6*c^10 + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4 \\
& *c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) - (- ( \\
& 81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - \\
& 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 \\
& + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11 \\
& *d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7 \\
& *c^14*d))^{(1/4)*}((( - (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 26 \\
& 46*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - \\
& 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 \\
& + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9 \\
& *d^6 - 524288*a*b^7*c^14*d))^{(1/4)*}(((81*a^4*b^7*d^10)/16 + 28*b^11*c^4*d^6 - \\
& (2145*a*b^10*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)/( \\
& b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (- (81*a^4*d^7 + \\
& 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - 756*a^3*b*c*d^6) \\
& )/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 + 1835008*a^2*b^6 \\
& *c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11*d^4 - 3670016*a^5 \\
& *b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(3/4)*}((( - \\
& (81*a^4*d^7 + 2401*b^4*c^4*d^3 - 4116*a*b^3*c^3*d^4 + 2646*a^2*b^2*c^2*d^5 - \\
& 756*a^3*b*c*d^6)/(65536*b^8*c^15 + 65536*a^8*c^7*d^8 - 524288*a^7*b*c^8*d^7 \\
& + 1835008*a^2*b^6*c^13*d^2 - 3670016*a^3*b^5*c^12*d^3 + 4587520*a^4*b^4*c^11 \\
& *d^4 - 3670016*a^5*b^3*c^10*d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14 \\
& *d^5 + 1835008*a^6*b^2*c^9*d^6 - 524288*a*b^7*c^14*d))^{(1/4)*} (28672*a^2*b^13 \\
& *c^13*d^5 - 4096*a*b^14*c^14*d^4 - 78848*a^3*b^12*c^12*d^6 + 90112*a^4*b^11*c^11 \\
& *d^7 + 28672*a^5*b^10*c^10*d^8 - 229376*a^6*b^9*c^9*d^9 + 329728*a^7*b^8*c^8*d^10 - \\
& 253952*a^8*b^7*c^7*d^11 + 114688*a^9*b^6*c^6*d^12 - 28672*a^10*b^5*c^5*d^13 + \\
& 3072*a^11*b^4*c^4*d^14))/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + \\
& (x*(65536*b^17*c^15*d^4 - 524288*a*b^16*c^14*d^5 + 1835008*a^2*b^15*c^13*d^
\end{aligned}$$

$$\begin{aligned}
& 6 - 3469312a^3b^{14}c^{12}d^7 + 2809856a^4b^{13}c^{11}d^8 + 3362816a^5b^{12}c^{10}d^9 - 14516224a^6b^{11}c^9d^{10} + 24190976a^7b^{10}c^8d^{11} - 25280512a^8b^9c^7d^{12} + 17833984a^9b^8c^6d^{13} - 8486912a^{10}b^7c^5d^{14} + 2609152a^{11}b^6c^4d^{15} - 466944a^{12}b^5c^3d^{16} + 36864a^{13}b^4c^2d^{17}) / (64(b^6c^{10} + a^6c^4d^6 - 6a^5b^3c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)) * i + (x(81a^4b^9d^{11} + 3185b^{13}c^4d^7 - 4788a^3b^{12}c^3d^8 - 756a^3b^{10}c^2d^{10} + 2790a^2b^{11}c^2d^9) * i) / (64(b^6c^{10} + a^6c^4d^6 - 6a^5b^3c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)) / ((- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{1/4} * ((- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{1/4} * ((81a^4b^7d^{10}) / 16 + 28b^{11}c^4d^6 - (2145a^3b^{10}c^3d^7) / 16 - (675a^3b^8c^9d^9) / 16 + (1971a^2b^9c^2d^8) / 16) / (b^3c^7 - a^3c^4d^3 + 3a^2b^3c^5d^2 - 3a^3b^2c^6d) + (- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{1/4} * (28672a^2b^{13}c^{13}d^5 - 4096a^3b^{14}c^{14}d^4 - 78848a^3b^{12}c^{12}d^6 + 90112a^4b^{11}c^{11}d^7 + 28672a^5b^{10}c^{10}d^8 - 229376a^6b^9c^9d^9 + 329728a^7b^8c^8d^{10} - 253952a^8b^7c^7d^{11} + 114688a^9b^6c^6d^{12} - 28672a^{10}b^5c^5d^{13} + 3072a^{11}b^4c^4d^{14}) / (b^3c^7 - a^3c^4d^3 + 3a^2b^3c^5d^2 - 3a^3b^2c^6d) - (x(65536b^{17}c^{15}d^4 - 524288a^3b^{16}c^{14}d^5 + 1835008a^2b^{15}c^{13}d^6 - 3469312a^3b^{14}c^{12}d^7 + 2809856a^4b^{13}c^{11}d^8 + 3362816a^5b^{12}c^{10}d^9 - 14516224a^6b^{11}c^9d^{10} + 24190976a^7b^{10}c^8d^{11} - 25280512a^8b^9c^7d^{12} + 17833984a^9b^8c^6d^{13} - 8486912a^{10}b^7c^5d^{14} + 2609152a^{11}b^6c^4d^{15} - 466944a^{12}b^5c^3d^{16} + 36864a^{13}b^4c^2d^{17}) / (64(b^6c^{10} + a^6c^4d^6 - 6a^5b^3c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d))) - (x(81a^4b^9d^{11} + 3185b^{13}c^4d^7 - 4788a^3b^{12}c^3d^8 - 756a^3b^{10}c^2d^{10} + 2790a^2b^{11}c^2d^9) / (64(b^6c^{10} + a^6c^4d^6 - 6a^5b^3c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d))) + (- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{1/4} * ((- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{1/4} * ((81a^4b^7d^{10}) / 16 + 28b^{11}c^4d^6 - (2145a^3b^{10}c^3d^7) / 16 - (675a^3b^8c^9d^9) / 16 + (1971a^2b^9c^2d^8) / 16) / (b^3c^7 - a^3c^4d^3 + 3a^2b^3c^5d^2 - 3a^3b^2c^6d) + (- (81a^4d^7 + 2401b^4c^4d^3 - 4116a^3b^3c^3d^4 + 2646a^2b^2c^2d^5 - 756a^3b^3c^3d^6) / (65536b^8c^{15} + 65536a^8c^7d^8 - 524288a^7b^3c^8d^7 + 1835008a^2b^6c^{13}d^2 - 3670016a^3b^5c^{12}d^3 + 4587520a^4b^4c^{11}d^4 - 3670016a^5b^3c^{10}d^5 + 1835008a^6b^2c^9d^6 - 524288a^7b^3c^{14}d))^{3/4} * (((- (81a^4d^7 + 2401b^4c^4d^3 -
\end{aligned}$$



$$\begin{aligned}
& *c*d^7)^{(3/4)} * (((-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d \\
& + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 1 \\
& 4336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} * (28 \\
& 672*a^2*b^{13}*c^{13}*d^5 - 4096*a*b^{14}*c^{14}*d^4 - 78848*a^3*b^{12}*c^{12}*d^6 + 90 \\
& 112*a^4*b^{11}*c^{11}*d^7 + 28672*a^5*b^{10}*c^{10}*d^8 - 229376*a^6*b^9*c^9*d^9 + \\
& 329728*a^7*b^8*c^8*d^{10} - 253952*a^8*b^7*c^7*d^{11} + 114688*a^9*b^6*c^6*d^{12} \\
& - 28672*a^{10}*b^5*c^5*d^{13} + 3072*a^{11}*b^4*c^4*d^{14}))/ (b^3*c^7 - a^3*c^4*d^ \\
& 3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^{17}*c^{15}*d^4 - 524288*a*b \\
& ^{16}*c^{14}*d^5 + 1835008*a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}*d^7 + 2809 \\
& 856*a^4*b^{13}*c^{11}*d^8 + 3362816*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6*b^{11}*c^9*d \\
& ^{10} + 24190976*a^7*b^{10}*c^8*d^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17833984*a^9 \\
& *b^8*c^6*d^{13} - 8486912*a^{10}*b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4*d^{15} - 466 \\
& 944*a^{12}*b^5*c^3*d^{16} + 36864*a^{13}*b^4*c^2*d^{17}))/ (64*(b^6*c^{10} + a^6*c^4*d \\
& ^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2 \\
& *c^6*d^4 - 6*a*b^5*c^9*d)) + ((81*a^4*b^7*d^{10})/16 + 28*b^{11}*c^4*d^6 - (21 \\
& 45*a*b^{10}*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16) \\
& / (b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) * i + (x*(81*a^4 \\
& *b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 - 756*a^3*b^{10}*c*d^{10} + \\
& 2790*a^2*b^{11}*c^2*d^9) * i) / (64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + \\
& 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9 \\
& *d)))/ ((-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a \\
& ^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8* \\
& b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} * ((-b^7/(256* \\
& a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14 \\
& 336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168* \\
& a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} * ((-b^7/(256*a^{11}*d^8 + 256*a^3* \\
& b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 \\
& + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2 \\
& 048*a^{10}*b*c*d^7))^{(3/4)} * (((-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4 \\
& *b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c \\
& ^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7)) \\
& ^{(1/4)} * (28672*a^2*b^{13}*c^{13}*d^5 - 4096*a*b^{14}*c^{14}*d^4 - 78848*a^3*b^{12}*c^{1 \\
& 2*d^6 + 90112*a^4*b^{11}*c^{11}*d^7 + 28672*a^5*b^{10}*c^{10}*d^8 - 229376*a^6*b^9* \\
& c^9*d^9 + 329728*a^7*b^8*c^8*d^{10} - 253952*a^8*b^7*c^7*d^{11} + 114688*a^9*b^ \\
& 6*c^6*d^{12} - 28672*a^{10}*b^5*c^5*d^{13} + 3072*a^{11}*b^4*c^4*d^{14}))/ (b^3*c^7 - \\
& a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x*(65536*b^{17}*c^{15}*d^4 - \\
& 524288*a*b^{16}*c^{14}*d^5 + 1835008*a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}* \\
& d^7 + 2809856*a^4*b^{13}*c^{11}*d^8 + 3362816*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6* \\
& b^{11}*c^9*d^{10} + 24190976*a^7*b^{10}*c^8*d^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17 \\
& 833984*a^9*b^8*c^6*d^{13} - 8486912*a^{10}*b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4* \\
& d^{15} - 466944*a^{12}*b^5*c^3*d^{16} + 36864*a^{13}*b^4*c^2*d^{17}))/ (64*(b^6*c^{10} + \\
& a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + \\
& 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)) + ((81*a^4*b^7*d^{10})/16 + 28*b^{11}*c^4 \\
& *d^6 - (2145*a*b^{10}*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^ \\
& 2*d^8)/16) / (b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) - (x* \\
& (81*a^4*b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 - 756*a^3*b^{10}*c \\
& *d^{10} + 2790*a^2*b^{11}*c^2*d^9))/ (64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d \\
& ^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5 \\
& *c^9*d)) + (-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 71 \\
& 68*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336* \\
& a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} * ((-b^7/( \\
& 256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 \\
& - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7 \\
& 168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} * ((-b^7/(256*a^{11}*d^8 + 256* \\
& a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5 \\
& *d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 \\
& - 2048*a^{10}*b*c*d^7))^{(3/4)} * (((-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048 \\
& *a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b \\
& ^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d
\end{aligned}$$

$$\begin{aligned} & \left. \left. \left. \left. \left. \left( \frac{28672a^2b^{13}c^{13}d^5 - 4096a^5b^{14}c^{14}d^4 - 78848a^3b^{12}c^{12}d^6 + 90112a^4b^{11}c^{11}d^7 + 28672a^5b^{10}c^{10}d^8 - 229376a^6b^9c^9d^9 + 329728a^7b^8c^8d^{10} - 253952a^8b^7c^7d^{11} + 114688a^9b^6c^6d^{12} - 28672a^{10}b^5c^5d^{13} + 3072a^{11}b^4c^4d^{14}}{b^3c^7 - a^3c^4d^3 + 3a^2b^5c^5d^2 - 3a^2b^2c^6d} \right) \right) \right) \right) \right) \\ & + \left( \frac{x(65536b^{17}c^{15}d^4 - 524288a^2b^{16}c^{14}d^5 + 1835008a^2b^{15}c^{13}d^6 - 3469312a^3b^{14}c^{12}d^7 + 2809856a^4b^{13}c^{11}d^8 + 3362816a^5b^{12}c^{10}d^9 - 14516224a^6b^{11}c^9d^{10} + 24190976a^7b^{10}c^8d^{11} - 25280512a^8b^9c^7d^{12} + 17833984a^9b^8c^6d^{13} - 8486912a^{10}b^7c^5d^{14} + 2609152a^{11}b^6c^4d^{15} - 466944a^{12}b^5c^3d^{16} + 36864a^{13}b^4c^2d^{17}}{64(b^6c^{10} + a^6c^4d^6 - 6a^5b^5c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)} \right) \right) \\ & + \left( \frac{81a^4b^7d^{10}}{16} + 28b^{11}c^4d^6 - \frac{2145a^2b^{10}c^3d^7}{16} - \frac{675a^3b^8c^9d^9}{16} + \frac{1971a^2b^9c^2d^8}{16} \right) / (b^3c^7 - a^3c^4d^3 + 3a^2b^5c^5d^2 - 3a^2b^2c^6d) + \left( \frac{x(81a^4b^9d^{11} + 3185b^{13}c^4d^7 - 4788a^2b^{12}c^3d^8 - 756a^3b^{10}c^2d^{10} + 2790a^2b^{11}c^2d^9)}{64(b^6c^{10} + a^6c^4d^6 - 6a^5b^5c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)} \right) \\ & \left. \left. \left. \left. \left. \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \right) \right) \right) \right) \\ & + 2 \operatorname{atan} \left( \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \right) \\ & \left. \left. \left. \left. \left. \left( \frac{28672a^2b^{13}c^{13}d^5 - 4096a^5b^{14}c^{14}d^4 - 78848a^3b^{12}c^{12}d^6 + 90112a^4b^{11}c^{11}d^7 + 28672a^5b^{10}c^{10}d^8 - 229376a^6b^9c^9d^9 + 329728a^7b^8c^8d^{10} - 253952a^8b^7c^7d^{11} + 114688a^9b^6c^6d^{12} - 28672a^{10}b^5c^5d^{13} + 3072a^{11}b^4c^4d^{14}}{b^3c^7 - a^3c^4d^3 + 3a^2b^5c^5d^2 - 3a^2b^2c^6d} \right) \right) \right) \right) \right) \right) \\ & - \left( \frac{x(65536b^{17}c^{15}d^4 - 524288a^2b^{16}c^{14}d^5 + 1835008a^2b^{15}c^{13}d^6 - 3469312a^3b^{14}c^{12}d^7 + 2809856a^4b^{13}c^{11}d^8 + 3362816a^5b^{12}c^{10}d^9 - 14516224a^6b^{11}c^9d^{10} + 24190976a^7b^{10}c^8d^{11} - 25280512a^8b^9c^7d^{12} + 17833984a^9b^8c^6d^{13} - 8486912a^{10}b^7c^5d^{14} + 2609152a^{11}b^6c^4d^{15} - 466944a^{12}b^5c^3d^{16} + 36864a^{13}b^4c^2d^{17})}{64(b^6c^{10} + a^6c^4d^6 - 6a^5b^5c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)} \right) \\ & + \left( \frac{81a^4b^7d^{10}}{16} + 28b^{11}c^4d^6 - \frac{2145a^2b^{10}c^3d^7}{16} - \frac{675a^3b^8c^9d^9}{16} + \frac{1971a^2b^9c^2d^8}{16} \right) / (b^3c^7 - a^3c^4d^3 + 3a^2b^5c^5d^2 - 3a^2b^2c^6d) + \left( \frac{x(81a^4b^9d^{11} + 3185b^{13}c^4d^7 - 4788a^2b^{12}c^3d^8 - 756a^3b^{10}c^2d^{10} + 2790a^2b^{11}c^2d^9)}{64(b^6c^{10} + a^6c^4d^6 - 6a^5b^5c^5d^5 + 15a^2b^4c^8d^2 - 20a^3b^3c^7d^3 + 15a^4b^2c^6d^4 - 6a^5b^5c^9d)} \right) \\ & - \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \\ & \left. \left. \left. \left. \left. \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \right) \right) \right) \right) \\ & + \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \\ & + \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \\ & + \left( \frac{-b^7(256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7)}{256a^{11}d^8 + 256a^3b^8c^8 - 2048a^4b^7c^7d + 7168a^5b^6c^6d^2 - 14336a^6b^5c^5d^3 + 17920a^7b^4c^4d^4 - 14336a^8b^3c^3d^5 + 7168a^9b^2c^2d^6 - 2048a^{10}b^2c^2d^7} \right) \right) \end{aligned}$$



$$8*c^8*d^{10} - 253952*a^8*b^7*c^7*d^{11} + 114688*a^9*b^6*c^6*d^{12} - 28672*a^{10}*b^5*c^5*d^{13} + 3072*a^{11}*b^4*c^4*d^{14})/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d) + (x*(65536*b^{17}*c^{15}*d^4 - 524288*a*b^{16}*c^{14}*d^5 + 1835008*a^2*b^{15}*c^{13}*d^6 - 3469312*a^3*b^{14}*c^{12}*d^7 + 2809856*a^4*b^{13}*c^{11}*d^8 + 3362816*a^5*b^{12}*c^{10}*d^9 - 14516224*a^6*b^{11}*c^9*d^{10} + 24190976*a^7*b^{10}*c^8*d^{11} - 25280512*a^8*b^9*c^7*d^{12} + 17833984*a^9*b^8*c^6*d^{13} - 8486912*a^{10}*b^7*c^5*d^{14} + 2609152*a^{11}*b^6*c^4*d^{15} - 466944*a^{12}*b^5*c^3*d^{16} + 36864*a^{13}*b^4*c^2*d^{17})*i)/(64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d)))*i + (((81*a^4*b^7*d^{10})/16 + 28*b^{11}*c^4*d^6 - (2145*a*b^{10}*c^3*d^7)/16 - (675*a^3*b^8*c*d^9)/16 + (1971*a^2*b^9*c^2*d^8)/16)*i)/(b^3*c^7 - a^3*c^4*d^3 + 3*a^2*b*c^5*d^2 - 3*a*b^2*c^6*d))*i - (x*(81*a^4*b^9*d^{11} + 3185*b^{13}*c^4*d^7 - 4788*a*b^{12}*c^3*d^8 - 756*a^3*b^{10}*c*d^{10} + 2790*a^2*b^{11}*c^2*d^9)*i)/(64*(b^6*c^{10} + a^6*c^4*d^6 - 6*a^5*b*c^5*d^5 + 15*a^2*b^4*c^8*d^2 - 20*a^3*b^3*c^7*d^3 + 15*a^4*b^2*c^6*d^4 - 6*a*b^5*c^9*d))))*(-b^7/(256*a^{11}*d^8 + 256*a^3*b^8*c^8 - 2048*a^4*b^7*c^7*d + 7168*a^5*b^6*c^6*d^2 - 14336*a^6*b^5*c^5*d^3 + 17920*a^7*b^4*c^4*d^4 - 14336*a^8*b^3*c^3*d^5 + 7168*a^9*b^2*c^2*d^6 - 2048*a^{10}*b*c*d^7))^{(1/4)} + (d*x)/(4*c*(c + d*x^4)*(a*d - b*c))$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out



$$3.166 \quad \int \frac{(c+dx^4)^5}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=407

$$\frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}} + \frac{(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}$$

[Out]  $d^2(-4a^3d^3+15a^2b^2cd^2-20ab^2c^2d+10b^3c^3)x/b^5+1/5d^3(3a^2d^2-10ab^2cd+10b^2c^2)x^5/b^4+1/9d^4(-2ad+5b^2c)x^9/b^3+1/13d^5x^{13}/b^2+1/4(-ad+bc)^5x/a/b^5/(bx^4+a)+1/16(-ad+bc)^4(17ad+3b^2c)\arctan(-1+b^{1/4}x^2^{1/2}/a^{1/4})/a^{7/4}/b^{21/4}+1/16(-ad+bc)^4(17ad+3b^2c)\arctan(1+b^{1/4}x^2^{1/2}/a^{1/4})/a^{7/4}/b^{21/4}+1/32(-ad+bc)^4(17ad+3b^2c)\ln(-a^{1/4}b^{1/4}x^2^{1/2}+a^{1/2}+x^2b^{1/2})/a^{7/4}/b^{21/4}+1/32(-ad+bc)^4(17ad+3b^2c)\ln(a^{1/4}b^{1/4}x^2^{1/2}+a^{1/2}+x^2b^{1/2})/a^{7/4}/b^{21/4}+1/2$

**Rubi [A]** time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^3x^5(3a^2d^2-10abcd+10b^2c^2)}{5b^4} + \frac{d^2x(15a^2bcd^2-4a^3d^3-20ab^2c^2d+10b^3c^3)}{b^5} - \frac{(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{21/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^5/(a + b\*x^4)^2,x]

[Out]  $(d^2(10b^3c^3-20ab^2c^2d+15a^2b^2cd^2-4a^3d^3)x)/b^5+(d^3(10b^2c^2-10ab^2cd+3a^2d^2)x^5)/(5b^4)+(d^4(5b^2c-2ad)x^9)/(9b^3)+(d^5x^{13})/(13b^2)+((b^2c-ad)^5x)/(4a^5b^5(a+b^2x^4))-((b^2c-ad)^4(3b^2c+17ad)*\text{ArcTan}[1-(\text{Sqrt}[2]*b^{1/4}x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{21/4})+((b^2c-ad)^4(3b^2c+17ad)*\text{ArcTan}[1+(\text{Sqrt}[2]*b^{1/4}x)/a^{1/4}])/(8*\text{Sqrt}[2]*a^{7/4}*b^{21/4})-((b^2c-ad)^4(3b^2c+17ad)*\text{Log}[\text{Sqrt}[a]-\text{Sqrt}[2]*a^{1/4}*b^{1/4}x+\text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{21/4})+((b^2c-ad)^4(3b^2c+17ad)*\text{Log}[\text{Sqrt}[a]+\text{Sqrt}[2]*a^{1/4}*b^{1/4}x+\text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*b^{21/4})$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

**Rule 385**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
p, 0])
```

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^4)^5}{(a + bx^4)^2} dx &= \int \left( \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3)}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^4}{b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^8}{b^3} \right) dx \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^5}{5b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^9}{9b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^5}{5b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^9}{9b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^5}{5b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^9}{9b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^5}{5b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^9}{9b^3} \\
&= \frac{d^2 (10b^3c^3 - 20ab^2c^2d + 15a^2bcd^2 - 4a^3d^3) x}{b^5} + \frac{d^3 (10b^2c^2 - 10abcd + 3a^2d^2) x^5}{5b^4} + \frac{d^4 (5bc^2 - 5acd + a^2d^2) x^9}{9b^3}
\end{aligned}$$

**Mathematica [A]** time = 0.45, size = 391, normalized size = 0.96

$$-\frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{585\sqrt{2}(bc-ad)^4(17ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} - \frac{1170\sqrt{2}(bc-ad)^4}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^5/(a + b\*x^4)^2,x]

[Out] (18720\*b^(1/4)\*d^2\*(10\*b^3\*c^3 - 20\*a\*b^2\*c^2\*d + 15\*a^2\*b\*c\*d^2 - 4\*a^3\*d^3)\*x + 3744\*b^(5/4)\*d^3\*(10\*b^2\*c^2 - 10\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^5 + 2080\*b^(9/4)\*d^4\*(5\*b\*c - 2\*a\*d)\*x^9 + 1440\*b^(13/4)\*d^5\*x^13 + (4680\*b^(1/4)\*(b\*c - a\*d)^5\*x)/(a\*(a + b\*x^4)) - (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (1170\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) - (585\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4) + (585\*Sqrt[2]\*(b\*c - a\*d)^4\*(3\*b\*c + 17\*a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4))/(18720\*b^(21/4))

**fricas [B]** time = 1.01, size = 3222, normalized size = 7.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^5/(b\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/9360\*(720\*a\*b^4\*d^5\*x^17 + 80\*(65\*a\*b^4\*c\*d^4 - 17\*a^2\*b^3\*d^5)\*x^13 + 208\*(90\*a\*b^4\*c^2\*d^3 - 65\*a^2\*b^3\*c\*d^4 + 17\*a^3\*b^2\*d^5)\*x^9 + 1872\*(50\*a\*b

$$\begin{aligned}
& ^4c^3d^2 - 90a^2b^3c^2d^3 + 65a^3b^2c^2d^4 - 17a^4b^2d^5) x^5 + 23 \\
& 40(a^6b^4x^4 + a^2b^5) * (- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5 \\
& b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 50 \\
& 0110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 5482 \\
& 31440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 90948 \\
& 30a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20}) / (a^7b^{21}) \\
& ^{(1/4)} * \arctan(- (a^5b^{16}x * (- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5 \\
& b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 54 \\
& 8231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 909 \\
& 4830a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20}) / (a^7b^{21} \\
& ))^{(3/4)} - a^5b^{16} * \sqrt{(a^4b^{10} * \sqrt{(- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5 \\
& b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 1748735 \\
& 56a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 548231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 26604 \\
& 0144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20} \\
& 0) / (a^7b^{21})) + (9b^{10}c^{10} + 30a^9b^9c^9d - 275a^2b^8c^8d^2 + 40a^3b^7c^7d^3 + 3010a^4b^6c^6d^4 - 9548a^5b^5c^5d^5 + 14770a^6b^4c^4d^6 - 13400a^7b^3c^3d^7 + 7285a^8b^2c^2d^8 - 2210a^9b^1c^1d^9 \\
& + 289a^{10}d^{10}) * x^2) / (9b^{10}c^{10} + 30a^9b^9c^9d - 275a^2b^8c^8d^2 + 40a^3b^7c^7d^3 + 3010a^4b^6c^6d^4 - 9548a^5b^5c^5d^5 + 14770a^6b^4c^4d^6 - 13400a^7b^3c^3d^7 + 7285a^8b^2c^2d^8 - 2210a^9b^1c^1d^9 \\
& + 289a^{10}d^{10})) * (- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5 \\
& b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 5482 \\
& 31440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 90948 \\
& 30a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20}) / (a^7b^{21}) \\
& ^{(3/4)} / (27b^{15}c^{15} + 135a^14b^{14}c^{14}d - 1125a^2b^{13}c^{13}d^2 - 1945a^3b^{12}c^{12}d^3 + 25095a^4b^{11}c^{11}d^4 - 42141a^5b^{10}c^{10}d^5 - 1319 \\
& 45a^6b^9c^9d^6 + 774675a^7b^8c^8d^7 - 1837935a^8b^7c^7d^8 + 270 \\
& 0885a^9b^6c^6d^9 - 2702799a^{10}b^5c^5d^{10} + 1889685a^{11}b^4c^4d^{11} - 914675a^{12}b^3c^3d^{12} + 293505a^{13}b^2c^2d^{13} - 56355a^{14}b^1c^1d^{14} + 4913a^{15}d^{15})) + 585(a^6b^4x^4 + a^2b^5) * (- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 548231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20}) / (a^7b^{21}) \\
& ^{(1/4)} * \log(a^2b^5 * (- (81b^{20}c^{20} + 540a^19b^19c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 + 132205a^4b^{16}c^{16}d^4 - 13264a^5b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 548231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^1c^1d^{19} + 83521a^{20}d^{20}) / (a^7b^{21}))
\end{aligned}$$

$$\begin{aligned} & ^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^3cd^{19} + 83521a^{20}d^{20} / (a^7b^{21})^{1/4} + (3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 \\ & + 90a^3b^2c^2d^3 - 65a^4b^3cd^4 + 17a^5d^5)x) - 585(a^2b^6x^4 + a^2b^5) \cdot (- (81b^{20}c^{20} + 540a^2b^{19}c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 \\ & + 132205a^4b^{16}c^{16}d^4 - 13264a^5b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 \\ & + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 548231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} \\ & + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^3cd^{19} + 83521a^{20}d^{20}) / (a^7b^{21})^{1/4} \cdot \log(-a^2b^5 \cdot (- (81b^{20}c^{20} + 540a^2b^{19}c^{19}d - 4050a^2b^{18}c^{18}d^2 - 15780a^3b^{17}c^{17}d^3 \\ & + 132205a^4b^{16}c^{16}d^4 - 13264a^5b^{15}c^{15}d^5 - 1960920a^6b^{14}c^{14}d^6 + 6137200a^7b^{13}c^{13}d^7 - 500110a^8b^{12}c^{12}d^8 - 48530040a^9b^{11}c^{11}d^9 \\ & + 174873556a^{10}b^{10}c^{10}d^{10} - 360900280a^{11}b^9c^9d^{11} + 517559250a^{12}b^8c^8d^{12} - 548231440a^{13}b^7c^7d^{13} + 438700840a^{14}b^6c^6d^{14} - 266040144a^{15}b^5c^5d^{15} \\ & + 120836285a^{16}b^4c^4d^{16} - 39944900a^{17}b^3c^3d^{17} + 9094830a^{18}b^2c^2d^{18} - 1277380a^{19}b^3cd^{19} + 83521a^{20}d^{20}) / (a^7b^{21})^{1/4} + (3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^3cd^4 + 17a^5d^5)x) \\ & + 2340(b^5c^5 - 5a^2b^4c^4d + 50a^2b^3c^3d^2 - 90a^3b^2c^2d^3 + 65a^4b^3cd^4 - 17a^5d^5)x) / (a^2b^6x^4 + a^2b^5) \end{aligned}$$

**giac [B]** time = 0.18, size = 798, normalized size = 1.96

$$\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} b^5 c^5 + 5 (ab^3)^{\frac{1}{4}} ab^4 c^4 d - 50 (ab^3)^{\frac{1}{4}} a^2 b^3 c^3 d^2 + 90 (ab^3)^{\frac{1}{4}} a^3 b^2 c^2 d^3 - 65 (ab^3)^{\frac{1}{4}} a^4 b c d^4 + 17 (ab^3)^{\frac{1}{4}} a^5 d^5 \right) / (16 a^2 b^6)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^5/(b\*x^4+a)^2,x, algorithm="giac")

[Out] 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^5\*c^5 + 5\*(a\*b^3)^(1/4)\*a\*b^4\*c^4\*d - 50\*(a\*b^3)^(1/4)\*a^2\*b^3\*c^3\*d^2 + 90\*(a\*b^3)^(1/4)\*a^3\*b^2\*c^2\*d^3 - 65\*(a\*b^3)^(1/4)\*a^4\*b\*c\*d^4 + 17\*(a\*b^3)^(1/4)\*a^5\*d^5)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^6) + 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^5\*c^5 + 5\*(a\*b^3)^(1/4)\*a\*b^4\*c^4\*d - 50\*(a\*b^3)^(1/4)\*a^2\*b^3\*c^3\*d^2 + 90\*(a\*b^3)^(1/4)\*a^3\*b^2\*c^2\*d^3 - 65\*(a\*b^3)^(1/4)\*a^4\*b\*c\*d^4 + 17\*(a\*b^3)^(1/4)\*a^5\*d^5)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^6) + 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^5\*c^5 + 5\*(a\*b^3)^(1/4)\*a\*b^4\*c^4\*d - 50\*(a\*b^3)^(1/4)\*a^2\*b^3\*c^3\*d^2 + 90\*(a\*b^3)^(1/4)\*a^3\*b^2\*c^2\*d^3 - 65\*(a\*b^3)^(1/4)\*a^4\*b\*c\*d^4 + 17\*(a\*b^3)^(1/4)\*a^5\*d^5)\*log(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^6) - 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^5\*c^5 + 5\*(a\*b^3)^(1/4)\*a\*b^4\*c^4\*d - 50\*(a\*b^3)^(1/4)\*a^2\*b^3\*c^3\*d^2 + 90\*(a\*b^3)^(1/4)\*a^3\*b^2\*c^2\*d^3 - 65\*(a\*b^3)^(1/4)\*a^4\*b\*c\*d^4 + 17\*(a\*b^3)^(1/4)\*a^5\*d^5)\*log(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^6) + 1/4\*(b^5\*c^5\*x - 5\*a^2\*b^4\*c^4\*d\*x + 10\*a^2\*b^3\*c^3\*d^2\*x - 10\*a^3\*b^2\*c^2\*d^3\*x + 5\*a^4\*b\*c\*d^4\*x - a^5\*d^5\*x)/((b\*x^4 + a)\*a\*b^5) + 1/585\*(45\*b^24\*d^5\*x^13 + 325\*b^24\*c\*d^4\*x^9 - 130\*a\*b^23\*d^5\*x^9 + 1170\*b^24\*c^2\*d^3\*x^5 - 1170\*a\*b^23\*c\*d^4\*x^5 + 351\*a^2\*b^22\*d^5\*x^5 + 5850\*b^24\*c^3\*d^2\*x - 11700\*a\*b^23\*c^2\*d^3\*x + 8775\*a^2\*b^22\*c\*d^4\*x - 2340\*a^3\*b^21\*d^5\*x)/b^26

**maple [B]** time = 0.06, size = 1118, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^4+c)^5/(b*x^4+a)^2,x)`

[Out] 
$$\frac{5/32/b/a*(a/b)^{1/4}*2^{1/2}*\ln((x^2+(a/b)^{1/4}*2^{1/2}*x+(a/b)^{1/2}))/((x^2-(a/b)^{1/4}*2^{1/2}*x+(a/b)^{1/2}))}{4(ab^6x^4+a^2b^5)} + \frac{45b^3d^5x^{13} + 65(5b^3cd^4 - 2ab^2d^5)x^9 + 117(10b^3c^2d^3 - 10a^2b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)x^5 + 585(10b^3c^3d^2 - 20a^2b^2c^2d^3 + 15a^2b^2c^2d^4 - 4a^3d^5)x^3 + 17a^5d^5}{b^5} + \frac{1/32*(2*\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{2*\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\log(\sqrt{b}*x^2 + \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})} - \frac{\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\log(\sqrt{b}*x^2 - \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})}}{a^2*c*x - 20*a/b^3*c^2*d^3*x}$$

**maxima** [A] time = 1.45, size = 644, normalized size = 1.58

$$\frac{(b^5c^5 - 5ab^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)x}{4(ab^6x^4 + a^2b^5)} + \frac{45b^3d^5x^{13} + 65(5b^3cd^4 - 2ab^2d^5)x^9 + 117(10b^3c^2d^3 - 10a^2b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)x^5 + 585(10b^3c^3d^2 - 20a^2b^2c^2d^3 + 15a^2b^2c^2d^4 - 4a^3d^5)x^3 + 17a^5d^5}{b^5} + \frac{1/32*(2*\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{2*\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\log(\sqrt{b}*x^2 + \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})} - \frac{\sqrt{2}*(3b^5c^5 + 5a^2b^4c^4d - 50a^2b^3c^3d^2 + 90a^3b^2c^2d^3 - 65a^4b^2c^2d^4 + 17a^5d^5)*\log(\sqrt{b}*x^2 - \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})}}{a^2*c*x - 20*a/b^3*c^2*d^3*x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^4+c)^5/(b*x^4+a)^2,x, algorithm="maxima")`

[Out] 
$$\frac{1/4*(b^5c^5 - 5a^2b^4c^4d + 10a^2b^3c^3d^2 - 10a^3b^2c^2d^3 + 5a^4bcd^4 - a^5d^5)*x/(a*b^6*x^4 + a^2*b^5) + 1/585*(45*b^3*d^5*x^{13} + 65*(5*b^3*c*d^4 - 2*a*b^2*d^5)*x^9 + 117*(10*b^3*c^2*d^3 - 10*a*b^2*c*d^4 + 3*a^2*b*d^5)*x^5 + 585*(10*b^3*c^3*d^2 - 20*a*b^2*c^2*d^3 + 15*a^2*b^2*c^2*d^4 - 4*a^3*d^5)*x^3 + 17*a^5*d^5}{b^5} + \frac{1/32*(2*\sqrt{2}*(3*b^5*c^5 + 5*a^2*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b^2*c^2*d^4 + 17*a^5*d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x + \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{2*\sqrt{2}*(3*b^5*c^5 + 5*a^2*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b^2*c^2*d^4 + 17*a^5*d^5)*\arctan(1/2*\sqrt{2}*(2*\sqrt{b}*x - \sqrt{2})*a^{1/4}*b^{1/4}))/\sqrt{a*\sqrt{b}}}{\sqrt{a*\sqrt{a*\sqrt{b}}}} + \frac{\sqrt{2}*(3*b^5*c^5 + 5*a^2*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b^2*c^2*d^4 + 17*a^5*d^5)*\log(\sqrt{b}*x^2 + \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})} - \frac{\sqrt{2}*(3*b^5*c^5 + 5*a^2*b^4*c^4*d - 50*a^2*b^3*c^3*d^2 + 90*a^3*b^2*c^2*d^3 - 65*a^4*b^2*c^2*d^4 + 17*a^5*d^5)*\log(\sqrt{b}*x^2 - \sqrt{2})*a^{1/4}*b^{1/4}*x + \sqrt{a}}{(a^{3/4}*b^{1/4})}}{a^2*c*x - 20*a/b^3*c^2*d^3*x}$$

**mupad** [B] time = 1.71, size = 2490, normalized size = 6.12

result too large to display



```
3*b*c)*1i)/(16*(-a)^(7/4)*b^(21/4)))*(a*d - b*c)^4*(17*a*d + 3*b*c))/(8*(-a)^(7/4)*b^(21/4))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**5/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```



$$3.167 \quad \int \frac{(c+dx^4)^4}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=357

$$\frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$

[Out]  $d^2*(3*a^2*d^2-8*a*b*c*d+6*b^2*c^2)*x/b^4+2/5*d^3*(-a*d+2*b*c)*x^5/b^3+1/9*d^4*x^9/b^2+1/4*(-a*d+b*c)^4*x/a/b^4/(b*x^4+a)+1/16*(-a*d+b*c)^3*(13*a*d+3*b*c)*\arctan(-1+b^(1/4)*x^2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)+1/16*(-a*d+b*c)^3*(13*a*d+3*b*c)*\arctan(1+b^(1/4)*x^2^(1/2)/a^(1/4))/a^(7/4)/b^(17/4)*2^(1/2)-1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*\ln(-a^(1/4)*b^(1/4)*x^2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)+1/32*(-a*d+b*c)^3*(13*a*d+3*b*c)*\ln(a^(1/4)*b^(1/4)*x^2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(17/4)*2^(1/2)$

**Rubi [A]** time = 0.37, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{d^2x(3a^2d^2-8abcd+6b^2c^2)}{b^4} - \frac{(bc-ad)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}} + \frac{(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{17/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^4/(a + b\*x^4)^2, x]

[Out]  $(d^2*(6*b^2*c^2-8*a*b*c*d+3*a^2*d^2)*x)/b^4+(2*d^3*(2*b*c-a*d)*x^5)/(5*b^3)+(d^4*x^9)/(9*b^2)+((b*c-a*d)^4*x)/(4*a*b^4*(a+b*x^4))-((b*c-a*d)^3*(3*b*c+13*a*d)*\text{ArcTan}[1-(\text{Sqrt}[2]*b^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*b^(17/4))+((b*c-a*d)^3*(3*b*c+13*a*d)*\text{ArcTan}[1+(\text{Sqrt}[2]*b^(1/4)*x)/a^(1/4)])/(8*\text{Sqrt}[2]*a^(7/4)*b^(17/4))-((b*c-a*d)^3*(3*b*c+13*a*d)*\text{Log}[\text{Sqrt}[a]-\text{Sqrt}[2]*a^(1/4)*b^(1/4)*x+\text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*b^(17/4))+((b*c-a*d)^3*(3*b*c+13*a*d)*\text{Log}[\text{Sqrt}[a]+\text{Sqrt}[2]*a^(1/4)*b^(1/4)*x+\text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^(7/4)*b^(17/4))$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n +

p, 0])

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^4)^4}{(a + bx^4)^2} dx &= \int \left( \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)}{b^4} + \frac{2d^3(2bc - ad)x^4}{b^3} + \frac{d^4x^8}{b^2} + \frac{(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^4}{b^4(a + bx^4)^2} \right) dx \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{\int \frac{(bc - ad)^3(bc + 3ad) + 4bd(bc - ad)^3x^4}{(a + bx^4)^2} dx}{b^4} \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc - ad)^4x}{4ab^4(a + bx^4)} + \frac{(bc - ad)^3(3b^2c - ad^2)}{4ab^4(a + bx^4)} \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc - ad)^4x}{4ab^4(a + bx^4)} + \frac{(bc - ad)^3(3b^2c - ad^2)}{4ab^4(a + bx^4)} \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc - ad)^4x}{4ab^4(a + bx^4)} + \frac{(bc - ad)^3(3b^2c - ad^2)}{4ab^4(a + bx^4)} \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc - ad)^4x}{4ab^4(a + bx^4)} + \frac{(bc - ad)^3(3b^2c - ad^2)}{4ab^4(a + bx^4)} \\
&= \frac{d^2(6b^2c^2 - 8abcd + 3a^2d^2)x}{b^4} + \frac{2d^3(2bc - ad)x^5}{5b^3} + \frac{d^4x^9}{9b^2} + \frac{(bc - ad)^4x}{4ab^4(a + bx^4)} + \frac{(bc - ad)^3(3b^2c - ad^2)}{4ab^4(a + bx^4)}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 341, normalized size = 0.96

$$\frac{45\sqrt{2}(ad-bc)^3(13ad+3bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{45\sqrt{2}(bc-ad)^3(13ad+3bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{a^{7/4}} + \frac{90\sqrt{2}(ad-bc)^3(13ad+3bc)}{a^{7/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^4/(a + b\*x^4)^2,x]

[Out] (1440\*b^(1/4)\*d^2\*(6\*b^2\*c^2 - 8\*a\*b\*c\*d + 3\*a^2\*d^2)\*x + 576\*b^(5/4)\*d^3\*(2\*b\*c - a\*d)\*x^5 + 160\*b^(9/4)\*d^4\*x^9 + (360\*b^(1/4)\*(b\*c - a\*d)^4\*x)/(a\*(a + b\*x^4)) + (90\*Sqrt[2]\*(-(b\*c) + a\*d)^3\*(3\*b\*c + 13\*a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (90\*Sqrt[2]\*(b\*c - a\*d)^3\*(3\*b\*c + 13\*a\*d)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (45\*Sqrt[2]\*(-(b\*c) + a\*d)^3\*(3\*b\*c + 13\*a\*d)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4) + (45\*Sqrt[2]\*(b\*c - a\*d)^3\*(3\*b\*c + 13\*a\*d)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4)/(1440\*b^(17/4))

**fricas [B]** time = 1.04, size = 2580, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^4/(b\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/720\*(80\*a\*b^3\*d^4\*x^13 + 16\*(36\*a\*b^3\*c\*d^3 - 13\*a^2\*b^2\*d^4)\*x^9 + 144\*(30\*a\*b^3\*c^2\*d^2 - 36\*a^2\*b^2\*c\*d^3 + 13\*a^3\*b\*d^4)\*x^5 - 180\*(a\*b^5\*x^4 + a^2\*b^4)\*(-(81\*b^16\*c^16 + 432\*a\*b^15\*c^15\*d - 2376\*a^2\*b^14\*c^14\*d^2 - 8304\*a^3\*b^13\*c^13\*d^3 + 45724\*a^4\*b^12\*c^12\*d^4 + 20400\*a^5\*b^11\*c^11\*d^5 - 4

$$\begin{aligned}
& 34808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - \\
& 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} \\
& * \arctan((a^5b^{13}x * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17}))^{3/4} - a^5b^{13}\sqrt{(a^4b^8\sqrt{-(81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17}))} + (9b^8c^8 + 24a*b^7c^7d - 164a^2b^6c^6d^2 - 24a^3b^5c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 - 936a^7b^1c^1d^7 + 169a^8d^8) * x^2) / (9b^8c^8 + 24a*b^7c^7d - 164a^2b^6c^6d^2 - 24a^3b^5c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 - 936a^7b^1c^1d^7 + 169a^8d^8)) * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{3/4} / (27b^{12}c^{12} + 108a*b^{11}c^{11}d - 666a^2b^{10}c^{10}d^2 - 1124a^3b^9c^9d^3 + 8901a^4b^8c^8d^4 - 7848a^5b^7c^7d^5 - 34860a^6b^6c^6d^6 + 113688a^7b^5c^5d^7 - 161451a^8b^4c^4d^8 + 132924a^9b^3c^3d^9 - 65754a^{10}b^2c^2d^{10} + 18252a^{11}b^1c^1d^{11} - 2197a^{12}d^{12})) - 45*(a*b^5x^4 + a^2b^4) * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} * \log(a^2b^4 * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} * \log(-a^2b^4 * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} * \log(-a^2b^4 * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} - (3b^4c^4 + 4a*b^3c^3d - 30a^2b^2c^2d^2 + 36a^3b^1c^1d^3 - 13a^4d^4) * x) + 45*(a*b^5x^4 + a^2b^4) * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} * \log(-a^2b^4 * (-81b^{16}c^{16} + 432a*b^{15}c^{15}d - 2376a^2b^{14}c^{14}d^2 - 8304a^3b^{13}c^{13}d^3 + 45724a^4b^{12}c^{12}d^4 + 20400a^5b^{11}c^{11}d^5 - 434808a^6b^{10}c^{10}d^6 + 772112a^7b^9c^9d^7 + 617958a^8b^8c^8d^8 - 4810608a^9b^7c^7d^9 + 9723912a^{10}b^6c^6d^{10} - 11486160a^{11}b^5c^5d^{11} + 8923164a^{12}b^4c^4d^{12} - 4651504a^{13}b^3c^3d^{13} + 1577784a^{14}b^2c^2d^{14} - 316368a^{15}b^1c^1d^{15} + 28561a^{16}d^{16}) / (a^7b^{17})^{1/4} - (3b^4c^4 + 4a*b^3c^3d - 30a^2b^2c^2d^2 + 36a^3b^1c^1d^3 - 13a^4d^4) * x) + 180*(b^4c^4 - 4a*b^3c^3d + 30a^2b^2c^2d^2 - 36a^3b^1c^1d^3 + 13a^4d^4) * x) / (a*b^5
\end{aligned}$$

$x^4 + a^2b^4$ )

**giac [B]** time = 0.17, size = 642, normalized size = 1.80

$$\frac{\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} b^4 c^4 + 4 (ab^3)^{\frac{1}{4}} ab^3 c^3 d - 30 (ab^3)^{\frac{1}{4}} a^2 b^2 c^2 d^2 + 36 (ab^3)^{\frac{1}{4}} a^3 b c d^3 - 13 (ab^3)^{\frac{1}{4}} a^4 d^4 \right) \arctan \left( \frac{\sqrt{2} \left( 2 (ab^3)^{\frac{1}{4}} b^4 c^4 + 4 (ab^3)^{\frac{1}{4}} ab^3 c^3 d - 30 (ab^3)^{\frac{1}{4}} a^2 b^2 c^2 d^2 + 36 (ab^3)^{\frac{1}{4}} a^3 b c d^3 - 13 (ab^3)^{\frac{1}{4}} a^4 d^4 \right)}{16 a^2 b^5} \right)}{16 a^2 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^4/(b\*x^4+a)^2,x, algorithm="giac")

[Out] 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^4\*c^4 + 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d - 30\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 + 36\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 - 13\*(a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x + sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^5) + 1/16\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^4\*c^4 + 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d - 30\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 + 36\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 - 13\*(a\*b^3)^(1/4)\*a^4\*d^4)\*arctan(1/2\*sqrt(2)\*(2\*x - sqrt(2)\*(a/b)^(1/4))/(a/b)^(1/4))/(a^2\*b^5) + 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^4\*c^4 + 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d - 30\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 + 36\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 - 13\*(a\*b^3)^(1/4)\*a^4\*d^4)\*log(x^2 + sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^5) - 1/32\*sqrt(2)\*(3\*(a\*b^3)^(1/4)\*b^4\*c^4 + 4\*(a\*b^3)^(1/4)\*a\*b^3\*c^3\*d - 30\*(a\*b^3)^(1/4)\*a^2\*b^2\*c^2\*d^2 + 36\*(a\*b^3)^(1/4)\*a^3\*b\*c\*d^3 - 13\*(a\*b^3)^(1/4)\*a^4\*d^4)\*log(x^2 - sqrt(2)\*x\*(a/b)^(1/4) + sqrt(a/b))/(a^2\*b^5) + 1/4\*(b^4\*c^4\*x - 4\*a\*b^3\*c^3\*d\*x + 6\*a^2\*b^2\*c^2\*d^2\*x - 4\*a^3\*b\*c\*d^3\*x + a^4\*d^4\*x)/((b\*x^4 + a)\*a\*b^4) + 1/45\*(5\*b^16\*d^4\*x^9 + 36\*b^16\*c\*d^3\*x^5 - 18\*a\*b^15\*d^4\*x^5 + 270\*b^16\*c^2\*d^2\*x - 360\*a\*b^15\*c\*d^3\*x + 135\*a^2\*b^14\*d^4\*x)/b^18

**maple [B]** time = 0.06, size = 885, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^4/(b\*x^4+a)^2,x)

[Out] 1/4/b/a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^3\*d+9/8/b^3\*a\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c\*d^3+1/8/b/a\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c^3\*d+9/4/b^3\*a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c\*d^3+1/4/b/a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^3\*d+9/4/b^3\*a\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c\*d^3+1/9\*d^4\*x^9/b^2+1/4/a\*x/(b\*x^4+a)\*c^4-2/5\*d^4/b^3\*x^5\*a+4/5\*d^3/b^2\*x^5\*c+6/b^2\*c^2\*d^2\*x+3\*a^2/b^4\*d^4\*x-13/16/b^4\*a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*d^4-15/8/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^2\*d^2-1/b^3\*a^2\*x/(b\*x^4+a)\*c\*d^3+3/2/b^2\*a\*x/(b\*x^4+a)\*c^2\*d^2-13/16/b^4\*a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*d^4-15/8/b^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^2\*d^2-13/32/b^4\*a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*d^4-15/16/b^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c^2\*d^2+1/4/b^4\*a^3\*x/(b\*x^4+a)\*d^4-1/b\*x/(b\*x^4+a)\*c^3\*d+3/16/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c^4+3/32/a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c^4-8\*d^3/b^3\*a\*c\*x+3/16/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c^4



$$c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 - 936a^7b^7c^7d^6 - 936a^7b^7c^7d^7) / (4a^2b^5) - ((ad - bc)^3(13ad + 3bc) * (3b^4c^4 - 13a^4d^4 - 30a^2b^2c^2d^2 + 4a^3b^3c^3d + 36a^3b^3c^3d^3) * i) / (4(-a)^{(7/4)}b^{(21/4)}) * (ad - bc)^3(13ad + 3bc) / (16(-a)^{(7/4)}b^{(17/4)}) + (((x*(169a^8d^8 + 9b^8c^8 - 164a^2b^6c^6d^2 - 24a^3b^5c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 - 936a^7b^7c^7d^6 + 24a^7b^7c^7d^7) / (4a^2b^5) - ((ad - bc)^3(13ad + 3bc) * (3b^4c^4 - 13a^4d^4 - 30a^2b^2c^2d^2 + 4a^3b^3c^3d + 36a^3b^3c^3d^3) * i) / (4(-a)^{(7/4)}b^{(21/4)}) * (ad - bc)^3(13ad + 3bc) / (16(-a)^{(7/4)}b^{(17/4)}) - (((x*(169a^8d^8 + 9b^8c^8 - 164a^2b^6c^6d^2 - 24a^3b^5c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 - 936a^7b^7c^7d^6 + 24a^7b^7c^7d^7) / (4a^2b^5) - ((ad - bc)^3(13ad + 3bc) * (3b^4c^4 - 13a^4d^4 - 30a^2b^2c^2d^2 + 4a^3b^3c^3d + 36a^3b^3c^3d^3) * i) / (4(-a)^{(7/4)}b^{(21/4)}) * (ad - bc)^3(13ad + 3bc) * i) / (16(-a)^{(7/4)}b^{(17/4)}) - (((x*(169a^8d^8 + 9b^8c^8 - 164a^2b^6c^6d^2 - 24a^3b^5c^5d^3 + 1110a^4b^4c^4d^4 - 2264a^5b^3c^3d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 + 2076a^6b^2c^2d^6 + 24a^7b^7c^7d - 936a^7b^7c^7d^2 - 936a^7b^7c^7d^3 - 936a^7b^7c^7d^4 - 936a^7b^7c^7d^5 - 936a^7b^7c^7d^6 + 24a^7b^7c^7d^7) / (4a^2b^5) - ((ad - bc)^3(13ad + 3bc) * (3b^4c^4 - 13a^4d^4 - 30a^2b^2c^2d^2 + 4a^3b^3c^3d + 36a^3b^3c^3d^3) * i) / (4(-a)^{(7/4)}b^{(21/4)}) * (ad - bc)^3(13ad + 3bc) * i) / (16(-a)^{(7/4)}b^{(17/4)})) * (ad - bc)^3(13ad + 3bc) / (8(-a)^{(7/4)}b^{(17/4)})$$

**sympy [A]** time = 47.54, size = 471, normalized size = 1.32

$$x^5 \left( -\frac{2ad^4}{5b^3} + \frac{4cd^3}{5b^2} \right) + x \left( \frac{3a^2d^4}{b^4} - \frac{8acd^3}{b^3} + \frac{6c^2d^2}{b^2} \right) + \frac{x(a^4d^4 - 4a^3bcd^3 + 6a^2b^2c^2d^2 - 4ab^3c^3d + b^4c^4)}{4a^2b^4 + 4ab^5x^4} + \text{RootSum}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*4/(b\*x\*\*4+a)\*\*2,x)

[Out] x\*\*5\*(-2\*a\*d\*\*4/(5\*b\*\*3) + 4\*c\*d\*\*3/(5\*b\*\*2)) + x\*(3\*a\*\*2\*d\*\*4/b\*\*4 - 8\*a\*c\*d\*\*3/b\*\*3 + 6\*c\*\*2\*d\*\*2/b\*\*2) + x\*(a\*\*4\*d\*\*4 - 4\*a\*\*3\*b\*c\*d\*\*3 + 6\*a\*\*2\*b\*c\*\*2\*d\*\*2 - 4\*a\*b\*\*3\*c\*\*3\*d + b\*\*4\*c\*\*4)/(4\*a\*\*2\*b\*\*4 + 4\*a\*b\*\*5\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*17 + 28561\*a\*\*16\*d\*\*16 - 316368\*a\*\*15\*b\*c\*d\*\*15 + 1577784\*a\*\*14\*b\*\*2\*c\*\*2\*d\*\*14 - 4651504\*a\*\*13\*b\*\*3\*c\*\*3\*d\*\*13 + 8923164\*a\*\*12\*b\*\*4\*c\*\*4\*d\*\*12 - 11486160\*a\*\*11\*b\*\*5\*c\*\*5\*d\*\*11 + 9723912\*a\*\*10\*b\*\*6\*c\*\*6\*d\*\*10 - 4810608\*a\*\*9\*b\*\*7\*c\*\*7\*d\*\*9 + 617958\*a\*\*8\*b\*\*8\*c\*\*8\*d\*\*8 + 772112\*a\*\*7\*b\*\*9\*c\*\*9\*d\*\*7 - 434808\*a\*\*6\*b\*\*10\*c\*\*10\*d\*\*6 + 20400\*a\*\*5\*b\*\*11\*c\*\*11\*d\*\*5 + 45724\*a\*\*4\*b\*\*12\*c\*\*12\*d\*\*4 - 8304\*a\*\*3\*b\*\*13\*c\*\*13\*d\*\*3 - 2376\*a\*\*2\*b\*\*14\*c\*\*14\*d\*\*2 + 432\*a\*b\*\*15\*c\*\*15\*d + 81\*b\*\*16\*c\*\*16, Lambda(\_t, \_t\*log(-16\*\_t\*a\*\*2\*b\*\*4/(13\*a\*\*4\*d\*\*4 - 36\*a\*\*3\*b\*c\*d\*\*3 + 30\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 - 4\*a\*b\*\*3\*c\*\*3\*d - 3\*b\*\*4\*c\*\*4) + x))) + d\*\*4\*x\*\*9/(9\*b\*\*2)

$$3.168 \quad \int \frac{(c+dx^4)^3}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=317

$$\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

[Out]  $d^2*(-2*a*d+3*b*c)*x/b^3+1/5*d^3*x^5/b^2+1/4*(-a*d+b*c)^3*x/a/b^3/(b*x^4+a)$   
 $+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*\arctan(-1+b^(1/4)*x*2^(1/2)/a^(1/4))/a^(7/4)$   
 $/b^(13/4)*2^(1/2)+3/16*(-a*d+b*c)^2*(3*a*d+b*c)*\arctan(1+b^(1/4)*x*2^(1/2)/$   
 $a^(1/4))/a^(7/4)/b^(13/4)*2^(1/2)-3/32*(-a*d+b*c)^2*(3*a*d+b*c)*\ln(-a^(1/4)$   
 $*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/4)/b^(13/4)*2^(1/2)+3/32*(-a*d$   
 $+b*c)^2*(3*a*d+b*c)*\ln(a^(1/4)*b^(1/4)*x*2^(1/2)+a^(1/2)+x^2*b^(1/2))/a^(7/$   
 $4)/b^(13/4)*2^(1/2)$

**Rubi [A]** time = 0.32, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{3(bc-ad)^2(3ad+bc)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc-ad)^2(3ad+bc)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2\right)}{16\sqrt{2}a^{7/4}b^{13/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^3/(a + b\*x^4)^2, x]

[Out]  $(d^2*(3*b*c - 2*a*d)*x)/b^3 + (d^3*x^5)/(5*b^2) + ((b*c - a*d)^3*x)/(4*a*b^3$   
 $(a + b*x^4)) - (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^(1/4)$   
 $*x)/a^(1/4)]/(8*\text{Sqrt}[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)$   
 $*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^(1/4)*x)/a^(1/4)]/(8*\text{Sqrt}[2]*a^(7/4)*b^(13/4)) - (3$   
 $(b*c - a*d)^2*(b*c + 3*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^(1/4)*b^(1/4)*x + \text{Sqrt}$   
 $[b]*x^2)]/(16*\text{Sqrt}[2]*a^(7/4)*b^(13/4)) + (3*(b*c - a*d)^2*(b*c + 3*a*d)*\text{Lo}$   
 $\text{g}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^(1/4)*b^(1/4)*x + \text{Sqrt}[b]*x^2)]/(16*\text{Sqrt}[2]*a^(7/4)*b$   
 $^(13/4))$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n +



p, 0])

### Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

### Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

### Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

### Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

### Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^4)^3}{(a + bx^4)^2} dx &= \int \left( \frac{d^2(3bc - 2ad)}{b^3} + \frac{d^3x^4}{b^2} + \frac{(bc - ad)^2(bc + 2ad) + 3bd(bc - ad)^2x^4}{b^3(a + bx^4)^2} \right) dx \\
&= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{\int \frac{(bc - ad)^2(bc + 2ad) + 3bd(bc - ad)^2x^4}{(a + bx^4)^2} dx}{b^3} \\
&= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc - ad)^3x}{4ab^3(a + bx^4)} + \frac{(3(bc - ad)^2(bc + 3ad)) \int \frac{1}{a + bx^4} dx}{4ab^3} \\
&= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc - ad)^3x}{4ab^3(a + bx^4)} + \frac{(3(bc - ad)^2(bc + 3ad)) \int \frac{\sqrt{a} - \sqrt{b}x^2}{a + bx^4} dx}{8a^{3/2}b^3} + \frac{(3(bc - ad)^2(bc + 3ad)) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{16a^{3/2}b^{7/2}} + \frac{(3(bc - ad)^2(bc + 3ad)) \log(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a + \sqrt{b}x^2})}{16\sqrt{2}a^{7/4}b^{13/4}} \\
&= \frac{d^2(3bc - 2ad)x}{b^3} + \frac{d^3x^5}{5b^2} + \frac{(bc - ad)^3x}{4ab^3(a + bx^4)} - \frac{3(bc - ad)^2(bc + 3ad) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{13/4}} + \frac{3(bc - ad)^2(bc + 3ad)}{160b^{13/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.25, size = 301, normalized size = 0.95

$$\frac{15\sqrt{2}(bc - ad)^2(3ad + bc) \log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} + \frac{15\sqrt{2}(bc - ad)^2(3ad + bc) \log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} - \frac{30\sqrt{2}(bc - ad)^2(3ad + bc) \tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{a^{7/4}}$$


---

160b<sup>13/4</sup>

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^3/(a + b\*x^4)^2,x]

[Out] (160\*b^(1/4)\*d^2\*(3\*b\*c - 2\*a\*d)\*x + 32\*b^(5/4)\*d^3\*x^5 + (40\*b^(1/4)\*(b\*c - a\*d)^3\*x)/(a\*(a + b\*x^4)) - (30\*sqrt[2]\*(b\*c - a\*d)^2\*(b\*c + 3\*a\*d)\*ArcTan[1 - (sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (30\*sqrt[2]\*(b\*c - a\*d)^2\*(b\*c + 3\*a\*d)\*ArcTan[1 + (sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) - (15\*sqrt[2]\*(b\*c - a\*d)^2\*(b\*c + 3\*a\*d)\*Log[sqrt[a] - sqrt[2]\*a^(1/4)\*b^(1/4)\*x + sqrt[b]\*x^2])/a^(7/4) + (15\*sqrt[2]\*(b\*c - a\*d)^2\*(b\*c + 3\*a\*d)\*Log[sqrt[a] + sqrt[2]\*a^(1/4)\*b^(1/4)\*x + sqrt[b]\*x^2])/a^(7/4))/(160\*b^(13/4))

**fricas [B]** time = 0.90, size = 1938, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^3/(b\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/80\*(16\*a\*b^2\*d^3\*x^9 + 48\*(5\*a\*b^2\*c\*d^2 - 3\*a^2\*b\*d^3)\*x^5 + 60\*(a\*b^4\*x^4 + a^2\*b^3)\*(-b^12\*c^12 + 4\*a\*b^11\*c^11\*d - 14\*a^2\*b^10\*c^10\*d^2 - 44\*a^3\*b^9\*c^9\*d^3 + 127\*a^4\*b^8\*c^8\*d^4 + 136\*a^5\*b^7\*c^7\*d^5 - 644\*a^6\*b^6\*c^6\*d^6 + 328\*a^7\*b^5\*c^5\*d^7 + 1039\*a^8\*b^4\*c^4\*d^8 - 1932\*a^9\*b^3\*c^3\*d^9 + 1458\*a^10\*b^2\*c^2\*d^10 - 540\*a^11\*b\*c\*d^11 + 81\*a^12\*d^12)/(a^7\*b^13)^(1/4)

) $\arctan(-(\frac{a^5 b^{10} x (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12})}{(a^7 b^{13})^{3/4}} - a^5 b^{10} \sqrt{(a^4 b^6 \sqrt{(-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12})} + (b^6 c^6 + 2 a^2 b^5 c^5 d - 9 a^2 b^4 c^4 d^2 - 4 a^3 b^3 c^3 d^3 + 31 a^4 b^2 c^2 d^4 - 30 a^5 b c d^5 + 9 a^6 d^6) x^2)}{(b^6 c^6 + 2 a^2 b^5 c^5 d - 9 a^2 b^4 c^4 d^2 - 4 a^3 b^3 c^3 d^3 + 31 a^4 b^2 c^2 d^4 - 30 a^5 b c d^5 + 9 a^6 d^6))} (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{3/4} / (b^9 c^9 + 3 a^2 b^8 c^8 d - 12 a^2 b^7 c^7 d^2 - 20 a^3 b^6 c^6 d^3 + 78 a^4 b^5 c^5 d^4 - 6 a^5 b^4 c^4 d^5 - 188 a^6 b^3 c^3 d^6 + 252 a^7 b^2 c^2 d^7 - 135 a^8 b c d^8 + 27 a^9 d^9)) + 15 (a^2 b^4 x^4 + a^2 b^3) (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{1/4} \log(3 a^2 b^3 (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{1/4} + 3 (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) x) - 15 (a^2 b^4 x^4 + a^2 b^3) (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{1/4} \log(-3 a^2 b^3 (-b^{12} c^{12} + 4 a^3 b^{11} c^{11} d - 14 a^2 b^{10} c^{10} d^2 - 44 a^3 b^9 c^9 d^3 + 127 a^4 b^8 c^8 d^4 + 136 a^5 b^7 c^7 d^5 - 644 a^6 b^6 c^6 d^6 + 328 a^7 b^5 c^5 d^7 + 1039 a^8 b^4 c^4 d^8 - 1932 a^9 b^3 c^3 d^9 + 1458 a^{10} b^2 c^2 d^{10} - 540 a^{11} b c d^{11} + 81 a^{12} d^{12}) / (a^7 b^{13})^{1/4} + 3 (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) x) + 20 (b^3 c^3 - 3 a b^2 c^2 d + 15 a^2 b c d^2 - 9 a^3 d^3) x) / (a^2 b^4 x^4 + a^2 b^3)$

**giac** [A] time = 0.18, size = 496, normalized size = 1.56

$$\frac{3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 + \left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right) + 3\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}\right)}{16a^2b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^3/(b\*x^4+a)^2,x, algorithm="giac")

[Out]  $\frac{3}{16}\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 + \left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)\right) + \frac{3}{16}\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 + \left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\arctan\left(\frac{1}{2}\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right)\right) + \frac{3}{32}\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 + \left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2 + \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right) - \frac{3}{32}\sqrt{2}\left(\left(ab^3\right)^{\frac{1}{4}}b^3c^3 + \left(ab^3\right)^{\frac{1}{4}}ab^2c^2d - 5\left(ab^3\right)^{\frac{1}{4}}a^2bcd^2 + 3\left(ab^3\right)^{\frac{1}{4}}a^3d^3\right)\log\left(x^2 - \sqrt{2}x\left(\frac{a}{b}\right)^{\frac{1}{4}} + \sqrt{2}\left(\frac{a}{b}\right)^{\frac{1}{4}}\right) + \frac{1}{4}\left(b^3c^3 - 3ab^2c^2d + 15a^2bcd^2 - 9a^3d^3\right)x$

$$\frac{c^3 x^3 - 3 a b^2 c^2 d x + 3 a^2 b c d^2 x - a^3 d^3 x}{(b x^4 + a) a b^3} + \frac{1}{5} \frac{(b^8 d^3 x^5 + 15 b^8 c d^2 x - 10 a b^7 d^3 x)}{b^{10}}$$

**maple [B]** time = 0.06, size = 669, normalized size = 2.11

$$\frac{d^3 x^5}{5 b^2} - \frac{a^2 d^3 x}{4 (b x^4 + a) b^3} + \frac{3 a c d^2 x}{4 (b x^4 + a) b^2} + \frac{c^3 x}{4 (b x^4 + a) a} - \frac{3 c^2 d x}{4 (b x^4 + a) b} - \frac{2 a d^3 x}{b^3} + \frac{9 \left(\frac{a}{b}\right)^{\frac{1}{4}} \sqrt{2} a d^3 \arctan\left(\frac{\sqrt{2} x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}} - 1\right)}{16 b^3} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^3/(b\*x^4+a)^2,x)

[Out]  $\frac{1}{5} d^3 x^5 / b^2 - 2 a / b^3 d^3 x + 3 / b^2 c d^2 x - 1 / 4 b^3 a^2 x / (b x^4 + a) d^3 + 3 / 4 b^2 a x / (b x^4 + a) c d^2 - 3 / 4 b x / (b x^4 + a) c^2 d + 1 / 4 a x / (b x^4 + a) c^3 + 9 / 16 b^3 a (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1) d^3 - 15 / 16 b^2 (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1) c d^2 + 3 / 16 b / a (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1) c^2 d + 3 / 16 a^2 (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x - 1) c^3 + 9 / 32 b^3 a (a/b)^{1/4} 2^{1/2} \ln((x^2 + (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2})) d^3 - 15 / 32 b^2 (a/b)^{1/4} 2^{1/2} \ln((x^2 + (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2})) c d^2 + 3 / 32 b / a (a/b)^{1/4} 2^{1/2} \ln((x^2 + (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2})) c^2 d + 3 / 32 a^2 (a/b)^{1/4} 2^{1/2} \ln((x^2 + (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2}) / (x^2 - (a/b)^{1/4} 2^{1/2} x + (a/b)^{1/2})) c^3 + 9 / 16 b^3 a (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) d^3 - 15 / 16 b^2 (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) c d^2 + 3 / 16 b / a (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) c^2 d + 3 / 16 a^2 (a/b)^{1/4} 2^{1/2} \arctan(2^{1/2} / (a/b)^{1/4} x + 1) c^3$

**maxima [A]** time = 1.31, size = 405, normalized size = 1.28

$$\frac{(b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) x}{4 (a b^4 x^4 + a^2 b^3)} + \frac{b d^3 x^5 + 5 (3 b c d^2 - 2 a d^3) x}{5 b^3} + \frac{2 \sqrt{2} (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) \arctan\left(\frac{\sqrt{2} (2 \sqrt{b} x)}{2 \sqrt{a}}\right)}{3 \sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^3/(b\*x^4+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} (b^3 c^3 - 3 a b^2 c^2 d + 3 a^2 b c d^2 - a^3 d^3) x / (a b^4 x^4 + a^2 b^3) + \frac{1}{5} (b d^3 x^5 + 5 (3 b c d^2 - 2 a d^3) x) / b^3 + \frac{3}{32} (2 \sqrt{2}) (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) \arctan(1/2 \sqrt{2} (2 \sqrt{b} x + \sqrt{2} a^{1/4} b^{1/4}) / \sqrt{a} \sqrt{b}) / (\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}) + 2 \sqrt{2} (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) \arctan(1/2 \sqrt{2} (2 \sqrt{b} x - \sqrt{2} a^{1/4} b^{1/4}) / \sqrt{a} \sqrt{b}) / (\sqrt{a} \sqrt{\sqrt{a} \sqrt{b}}) + \sqrt{2} (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) \log(\sqrt{b} x^2 + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a}) / (a^{3/4} b^{1/4}) - \sqrt{2} (b^3 c^3 + a b^2 c^2 d - 5 a^2 b c d^2 + 3 a^3 d^3) \log(\sqrt{b} x^2 - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{a}) / (a^{3/4} b^{1/4}) / (a b^3)$

**mupad [B]** time = 1.53, size = 1616, normalized size = 5.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^3/(a + b\*x^4)^2,x)

[Out]  $(d^3x^5)/(5b^2) - x((2ad^3)/b^3 - (3cd^2)/b^2) - (x(a^3d^3 - b^3c^3 + 3ab^2c^2d - 3a^2b^2cd^2))/(4a(ab^3 + b^4x^4)) + (\operatorname{atan}(((ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) - (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)))/(16(-a)^{7/4}b^{13/4})) * 3i)/(16(-a)^{7/4}b^{13/4}) + ((ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) + (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)))/(16(-a)^{7/4}b^{13/4})) * 3i)/(16(-a)^{7/4}b^{13/4}) + ((3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) - (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)))/(16(-a)^{7/4}b^{13/4})) * 3i)/(16(-a)^{7/4}b^{13/4}) + ((ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) + (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)))/(16(-a)^{7/4}b^{13/4})) * 3i)/(8(-a)^{7/4}b^{13/4}) + (3 \operatorname{atan}(((3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) - ((ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)) * 3i)/(16(-a)^{7/4}b^{13/4}))) / (16(-a)^{7/4}b^{13/4}) + (3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) + (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)) * 3i)/(16(-a)^{7/4}b^{13/4}))) / (16(-a)^{7/4}b^{13/4}) + (3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) - ((ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)) * 3i)/(16(-a)^{7/4}b^{13/4}))) * (ad - b^2c)^2(3ad + bc) / (8(-a)^{7/4}b^{13/4}) + (3 \operatorname{atan}(((3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) - ((ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)) * 3i)/(16(-a)^{7/4}b^{13/4}))) / (16(-a)^{7/4}b^{13/4}) + (3(ad - b^2c)^2(3ad + bc)((9x(9a^6d^6 + b^6c^6 - 9a^2b^4c^4d^2 - 4a^3b^3c^3d^3 + 31a^4b^2c^2d^4 + 2ab^5c^5d - 30a^5b^2cd^5)))/(4a^2b^3) + (3(ad - b^2c)^2(3ad + bc)(36a^3d^3 + 12b^3c^3 + 12ab^2c^2d - 60a^2b^2cd^2)) * 3i)/(16(-a)^{7/4}b^{13/4}))) * (ad - b^2c)^2(3ad + bc) / (8(-a)^{7/4}b^{13/4})$

**sympy** [A] time = 6.99, size = 337, normalized size = 1.06

$$x \left( -\frac{2ad^3}{b^3} + \frac{3cd^2}{b^2} \right) + \frac{x(-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3)}{4a^2b^3 + 4ab^4x^4} + \operatorname{RootSum} \left( 65536t^4a^7b^{13} + 6561a^{12}d^{12} - 43740a^{11}t \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*3/(b\*x\*\*4+a)\*\*2,x)

[Out]  $x(-2ad^3/b^3 + 3cd^2/b^2) + x(-a^3d^3 + 3a^2bcd^2 - 3ab^2c^2d + b^3c^3)/(4a^2b^3 + 4ab^4x^4) + \operatorname{RootSum}(65536*_t^4a^7b^{13} + 6561a^{12}d^{12} - 43740a^{11}b^2cd^{11} + 118098a^{10}b^2c^2d^{10} - 156492a^9b^3c^3d^9 + 84159a^8b^4c^4d^8 + 26568a^7b^5c^5d^7 - 52164a^6b^6c^6d^6 + 11016a^5b^7c^7d^5 + 10287a^4b^8c^8d^4 - 3564a^3b^9c^9d^3 - 1134a^2b^{10}c^{10}d^2 + 324ab^{11}c^{11}d + 81b^{12}c^{12}, \operatorname{Lambda}(t, t \log(16*_ta^2b^3/(9a^3d^3 - 15a^2b^2cd^2 + 3ab^2c^2d + 3b^3c^3) + x))) + d^3x^5/(5b^2)$

$$3.169 \quad \int \frac{(c+dx^4)^2}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=291

$$\frac{(bc-ad)(5ad+3bc) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4} b^{9/4}} + \frac{(bc-ad)(5ad+3bc) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4} b^{9/4}}$$

[Out]  $d^2x/b^2+1/4*(-a*d+b*c)^2*x/a/b^2/(b*x^4+a)+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*\arctan(-1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(7/4)}/b^{(9/4)}*2^{(1/2)}+1/16*(-a*d+b*c)*(5*a*d+3*b*c)*\arctan(1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(7/4)}/b^{(9/4)}*2^{(1/2)}-1/32*(-a*d+b*c)*(5*a*d+3*b*c)*\ln(-a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(7/4)}/b^{(9/4)}*2^{(1/2)}+1/32*(-a*d+b*c)*(5*a*d+3*b*c)*\ln(a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(7/4)}/b^{(9/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.38, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {390, 385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(bc-ad)(5ad+3bc) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4} b^{9/4}} + \frac{(bc-ad)(5ad+3bc) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4} b^{9/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^2/(a + b\*x^4)^2, x]

[Out]  $(d^2*x)/b^2 + ((b*c - a*d)^2*x)/(4*a*b^2*(a + b*x^4)) - ((b*c - a*d)*(3*b*c + 5*a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*b^{(9/4)}) + ((b*c - a*d)*(3*b*c + 5*a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*b^{(9/4)}) - ((b*c - a*d)*(3*b*c + 5*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*b^{(9/4)}) + ((b*c - a*d)*(3*b*c + 5*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*b^{(9/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c + dx^4)^2}{(a + bx^4)^2} dx &= \int \left( \frac{d^2}{b^2} + \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^4}{b^2(a + bx^4)^2} \right) dx \\
&= \frac{d^2x}{b^2} + \frac{\int \frac{b^2c^2 - a^2d^2 + 2bd(bc - ad)x^4}{(a + bx^4)^2} dx}{b^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{4ab^2(a + bx^4)} + \frac{((bc - ad)(3bc + 5ad)) \int \frac{1}{a + bx^4} dx}{4ab^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{4ab^2(a + bx^4)} + \frac{((bc - ad)(3bc + 5ad)) \int \frac{\sqrt{a} - \sqrt{b}x^2}{a + bx^4} dx}{8a^{3/2}b^2} + \frac{((bc - ad)(3bc + 5ad)) \int \frac{\sqrt{a} + \sqrt{b}x^2}{a + bx^4} dx}{8a^{3/2}b^2} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{4ab^2(a + bx^4)} + \frac{((bc - ad)(3bc + 5ad)) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{16a^{3/2}b^{5/2}} + \frac{((bc - ad)(3bc + 5ad)) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt[4]{b}} + x^2} dx}{16a^{3/2}b^{5/2}} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{4ab^2(a + bx^4)} - \frac{(bc - ad)(3bc + 5ad) \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2} a^{7/4} b^{9/4}} + \frac{(bc - ad)(3bc + 5ad) \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2} a^{7/4} b^{9/4}} \\
&= \frac{d^2x}{b^2} + \frac{(bc - ad)^2x}{4ab^2(a + bx^4)} - \frac{(bc - ad)(3bc + 5ad) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} b^{9/4}} + \frac{(bc - ad)(3bc + 5ad) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2} a^{7/4} b^{9/4}}
\end{aligned}$$

**Mathematica [A]** time = 0.23, size = 297, normalized size = 1.02

$$\frac{\sqrt{2}(5a^2d^2 - 2abcd - 3b^2c^2) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} + \frac{\sqrt{2}(-5a^2d^2 + 2abcd + 3b^2c^2) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2\right)}{a^{7/4}} + \frac{2\sqrt{2}(5a^2d^2 - 2abcd - 3b^2c^2)}{32b^{9/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)^2/(a + b\*x^4)^2,x]

[Out] (32\*b^(1/4)\*d^2\*x + (8\*b^(1/4)\*(b\*c - a\*d)^2\*x)/(a\*(a + b\*x^4)) + (2\*Sqrt[2]\*(-3\*b^2\*c^2 - 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (2\*Sqrt[2]\*(3\*b^2\*c^2 + 2\*a\*b\*c\*d - 5\*a^2\*d^2)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]\*(-3\*b^2\*c^2 - 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4) + (Sqrt[2]\*(3\*b^2\*c^2 + 2\*a\*b\*c\*d - 5\*a^2\*d^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4))/(32\*b^(9/4))

**fricas [B]** time = 1.08, size = 1335, normalized size = 4.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a)^2,x, algorithm="fricas")

[Out] 1/16\*(16\*a\*b\*d^2\*x^5 - 4\*(a\*b^3\*x^4 + a^2\*b^2)\*(-81\*b^8\*c^8 + 216\*a\*b^7\*c^7\*d - 324\*a^2\*b^6\*c^6\*d^2 - 984\*a^3\*b^5\*c^5\*d^3 + 646\*a^4\*b^4\*c^4\*d^4 + 1640\*a^5\*b^3\*c^3\*d^5 - 900\*a^6\*b^2\*c^2\*d^6 - 1000\*a^7\*b\*c\*d^7 + 625\*a^8\*d^8)/(a^7\*b^9)^(1/4)\*arctan((a^5\*b^7\*x\*(-81\*b^8\*c^8 + 216\*a\*b^7\*c^7\*d - 324\*a^2\*b^6\*c^6\*d^2 - 984\*a^3\*b^5\*c^5\*d^3 + 646\*a^4\*b^4\*c^4\*d^4 + 1640\*a^5\*b^3\*c^3\*d^5 - 900\*a^6\*b^2\*c^2\*d^6 - 1000\*a^7\*b\*c\*d^7 + 625\*a^8\*d^8)/(a^7\*b^9)^(1/4)))/a^(7/4) + (32\*b^(1/4)\*d^2\*x + (8\*b^(1/4)\*(b\*c - a\*d)^2\*x)/(a\*(a + b\*x^4)) + (2\*Sqrt[2]\*(-3\*b^2\*c^2 - 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (2\*Sqrt[2]\*(3\*b^2\*c^2 + 2\*a\*b\*c\*d - 5\*a^2\*d^2)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)])/a^(7/4) + (Sqrt[2]\*(-3\*b^2\*c^2 - 2\*a\*b\*c\*d + 5\*a^2\*d^2)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4) + (Sqrt[2]\*(3\*b^2\*c^2 + 2\*a\*b\*c\*d - 5\*a^2\*d^2)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2])/a^(7/4))/(32\*b^(9/4))



$$\begin{aligned} & d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(3/4)} \\ & - a^5b^7\sqrt{((a^4b^4\sqrt{-(81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9)) + (9b^4c^4 + 12a^2b^3c^3d - 26a^2b^2c^2d^2 - 20a^3b^2cd^3 + 25a^4d^4) \\ & *x^2)/(9b^4c^4 + 12a^2b^3c^3d - 26a^2b^2c^2d^2 - 20a^3b^2cd^3 + 25a^4d^4))} \\ & *(-81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(3/4)}) \\ & /((27b^6c^6 + 54a^2b^5c^5d - 99a^2b^4c^4d^2 - 172a^3b^3c^3d^3 + 165a^4b^2c^2d^4 + 150a^5b^2cd^5 - 125a^6d^6)) - (a^2b^2x^4 + a^2b^2) \\ & *(-81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(1/4)} \\ & * \log(a^2b^2 * (-81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(1/4)} \\ & - (3b^2c^2 + 2a^2b^2cd - 5a^2d^2)x) + (a^2b^2x^4 + a^2b^2) * (-81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(1/4)} \\ & * \log(-a^2b^2 * (-81b^8c^8 + 216a^2b^7c^7d - 324a^2b^6c^6d^2 - 984a^3b^5c^5d^3 + 646a^4b^4c^4d^4 + 1640a^5b^3c^3d^5 - 900a^6b^2c^2d^6 - 1000a^7b^3cd^7 + 625a^8d^8)/(a^7b^9))^{(1/4)} \\ & - (3b^2c^2 + 2a^2b^2cd - 5a^2d^2)x) + 4*(b^2c^2 - 2a^2b^2cd + 5a^2d^2)x)/(a^2b^2x^4 + a^2b^2) \end{aligned}$$

**giac** [A] time = 0.17, size = 376, normalized size = 1.29

$$\frac{d^2x}{b^2} + \frac{\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} b^2 c^2 + 2 (ab^3)^{\frac{1}{4}} abcd - 5 (ab^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^3} + \frac{\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} b^2 c^2 + 2 (ab^3)^{\frac{1}{4}} abcd - 5 (ab^3)^{\frac{1}{4}} a^2 d^2 \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a)^2,x, algorithm="giac")

[Out]  $d^2x/b^2 + 1/16*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c^2 + 2*(a*b^3)^{(1/4)}*a*b*c*d - 5*(a*b^3)^{(1/4)}*a^2*d^2)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a^2*b^3) + 1/16*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c^2 + 2*(a*b^3)^{(1/4)}*a*b*c*d - 5*(a*b^3)^{(1/4)}*a^2*d^2)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(a^2*b^3) + 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c^2 + 2*(a*b^3)^{(1/4)}*a*b*c*d - 5*(a*b^3)^{(1/4)}*a^2*d^2)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^2*b^3) - 1/32*\sqrt{2}*(3*(a*b^3)^{(1/4)}*b^2*c^2 + 2*(a*b^3)^{(1/4)}*a*b*c*d - 5*(a*b^3)^{(1/4)}*a^2*d^2)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(a^2*b^3) + 1/4*(b^2*c^2*x - 2*a*b*c*d*x + a^2*d^2*x)/((b*x^4 + a)*a*b^2)$

**maple** [B] time = 0.06, size = 475, normalized size = 1.63

$$\frac{a d^2 x}{4 (b x^4 + a) b^2} + \frac{c^2 x}{4 (b x^4 + a) a} - \frac{c d x}{2 (b x^4 + a) b} + \frac{\left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} c d \arctan \left( \frac{\sqrt{2} x}{\left( \frac{a}{b} \right)^{\frac{1}{4}}} - 1 \right)}{8 a b} + \frac{\left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} c d \arctan \left( \frac{\sqrt{2} x}{\left( \frac{a}{b} \right)^{\frac{1}{4}}} + 1 \right)}{8 a b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^2/(b\*x^4+a)^2,x)

[Out]  $1/b^2*d^2*x + 1/4/b^2*a*x/(b*x^4+a)*d^2 - 1/2/b*x/(b*x^4+a)*c*d + 1/4/a*x/(b*x^4+a)*c^2 - 5/16/b^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)/(a/b)^{(1/4)}*x-1)*d^2 + 1/8$

$$\frac{1}{b/a} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \arctan\left( \frac{2^{1/2}}{\left( \frac{a}{b} \right)^{1/4} x - 1} \right) c^* d + \frac{3}{16} \frac{1}{a^2} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \arctan\left( \frac{2^{1/2}}{\left( \frac{a}{b} \right)^{1/4} x - 1} \right) c^{*2} - \frac{5}{32} \frac{1}{b^2} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \ln\left( \frac{(x^2 + \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})}{(x^2 - \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})} \right) d^2 + \frac{1}{16} \frac{1}{b/a} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \ln\left( \frac{(x^2 + \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})}{(x^2 - \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})} \right) c^* d + \frac{3}{32} \frac{1}{a^2} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \ln\left( \frac{(x^2 + \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})}{(x^2 - \left( \frac{a}{b} \right)^{1/4} 2^{1/2} x + \left( \frac{a}{b} \right)^{1/2})} \right) c^{*2} - \frac{5}{16} \frac{1}{b^2} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \arctan\left( \frac{2^{1/2}}{\left( \frac{a}{b} \right)^{1/4} x + 1} \right) d^2 + \frac{1}{8} \frac{1}{b/a} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \arctan\left( \frac{2^{1/2}}{\left( \frac{a}{b} \right)^{1/4} x + 1} \right) c^* d + \frac{3}{16} \frac{1}{a^2} \left( \frac{a}{b} \right)^{1/4} 2^{1/2} \arctan\left( \frac{2^{1/2}}{\left( \frac{a}{b} \right)^{1/4} x + 1} \right) c^{*2}$$

**maxima** [A] time = 1.26, size = 319, normalized size = 1.10

$$\frac{(b^2 c^2 - 2abcd + a^2 d^2)x}{4(ab^3 x^4 + a^2 b^2)} + \frac{d^2 x}{b^2} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a\frac{1}{4}b\frac{1}{4}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x + \sqrt{2}a\frac{1}{4}b\frac{1}{4}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^2/(b\*x^4+a)^2,x, algorithm="maxima")

[Out]  $\frac{1}{4} \left( \frac{b^2 c^2 - 2abcd + a^2 d^2}{ab^3 x^4 + a^2 b^2} \right) x + \frac{d^2 x}{b^2} + \frac{1}{32} \left( \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{1/2\sqrt{2}\sqrt{b}x + \sqrt{2}a^{1/4}b^{1/4}}{\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{1/2\sqrt{2}\sqrt{b}x - \sqrt{2}a^{1/4}b^{1/4}}{\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{1/2\sqrt{2}\sqrt{b}x + \sqrt{2}a^{1/4}b^{1/4}}{\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \arctan\left(\frac{1/2\sqrt{2}\sqrt{b}x - \sqrt{2}a^{1/4}b^{1/4}}{\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \log\left(\frac{\sqrt{b}x^2 + \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a}}{a^{3/4}b^{1/4}}\right) - \sqrt{2}(3b^2 c^2 + 2abcd - 5a^2 d^2) \log\left(\frac{\sqrt{b}x^2 - \sqrt{2}a^{1/4}b^{1/4}x + \sqrt{a}}{a^{3/4}b^{1/4}}\right)}{a^{3/4}b^{1/4}} \right)$

**mupad** [B] time = 0.30, size = 1254, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^2/(a + b\*x^4)^2,x)

[Out]  $\frac{(d^2 x)/b^2 + (x(a^2 d^2 + b^2 c^2 - 2ab^2 cd))/(4a(a^2 b^2 + b^3 x^4)) + \left( \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} - \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} + \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} \right) \frac{1}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} - \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} + \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} \right) \frac{1}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} - \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} + \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} \right) \frac{1}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} - \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} + \frac{\operatorname{atan}\left(\frac{(ad - bc)(5ad + 3bc)(x(25a^4 d^4 + 9b^4 c^4 - 26a^2 b^2 c^2 d^2 + 12ab^3 c^3 d - 20a^3 b^2 c^2 d^3))}{4a^2 b} + \frac{(ad - bc)(5ad + 3bc)(12b^3 c^2 - 20a^2 b^2 d^2 + 8ab^2 cd)}{16(-a)^{7/4} b^{9/4}} \right)}{16(-a)^{7/4} b^{9/4}} \right) \frac{1}{16(-a)^{7/4} b^{9/4}}$

$$\begin{aligned}
& - 20a^3b^2cd^3)/(4a^2b) + ((a^2d - b^2c)(5a^2d + 3b^2c)(12b^3c^2 - \\
& 20a^2b^2d^2 + 8a^2b^2cd^2) * i) / (16(-a)^{7/4} b^{9/4})) / (16(-a)^{7/4} b^{9/4}) \\
& + ((a^2d - b^2c)(5a^2d + 3b^2c) * ((x(25a^4d^4 + 9b^4c^4 - 26a^2b^2c^2d^2 + 12a^2b^3c^3d - 20a^3b^2cd^3) / (4a^2b) - \\
& (a^2d - b^2c)(5a^2d + 3b^2c)(12b^3c^2 - 20a^2b^2d^2 + 8a^2b^2cd^2) * i) / (16(-a)^{7/4} b^{9/4})) * i) / (16(-a)^{7/4} b^{9/4}) \\
& - ((a^2d - b^2c)(5a^2d + 3b^2c) * ((x(25a^4d^4 + 9b^4c^4 - 26a^2b^2c^2d^2 + 12a^2b^3c^3d - 20a^3b^2cd^3) / (4a^2b) + \\
& (a^2d - b^2c)(5a^2d + 3b^2c)(12b^3c^2 - 20a^2b^2d^2 + 8a^2b^2cd^2) * i) / (16(-a)^{7/4} b^{9/4})) * i) / (16(-a)^{7/4} b^{9/4})) * (a^2d - b^2c) \\
& * (5a^2d + 3b^2c) / (8(-a)^{7/4} b^{9/4})
\end{aligned}$$

**sympy [A]** time = 2.17, size = 219, normalized size = 0.75

$$\frac{x(a^2d^2 - 2abcd + b^2c^2)}{4a^2b^2 + 4ab^3x^4} + \text{RootSum}\left(65536t^4a^7b^9 + 625a^8d^8 - 1000a^7bcd^7 - 900a^6b^2c^2d^6 + 1640a^5b^3c^3d^5 + 6\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)\*\*2/(b\*x\*\*4+a)\*\*2,x)

[Out] x\*(a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d + b\*\*2\*c\*\*2)/(4\*a\*\*2\*b\*\*2 + 4\*a\*b\*\*3\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*9 + 625\*a\*\*8\*d\*\*8 - 1000\*a\*\*7\*b\*c\*d\*\*7 - 900\*a\*\*6\*b\*\*2\*c\*\*2\*d\*\*6 + 1640\*a\*\*5\*b\*\*3\*c\*\*3\*d\*\*5 + 646\*a\*\*4\*b\*\*4\*c\*\*4\*d\*\*4 - 984\*a\*\*3\*b\*\*5\*c\*\*5\*d\*\*3 - 324\*a\*\*2\*b\*\*6\*c\*\*6\*d\*\*2 + 216\*a\*b\*\*7\*c\*\*7\*d + 81\*b\*\*8\*c\*\*8, Lambda(\_t, \_t\*log(-16\*\_t\*a\*\*2\*b\*\*2/(5\*a\*\*2\*d\*\*2 - 2\*a\*b\*c\*d - 3\*b\*\*2\*c\*\*2) + x))) + d\*\*2\*x/b\*\*2

$$3.170 \quad \int \frac{c+dx^4}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=245

$$\frac{(ad+3bc)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

[Out]  $\frac{1}{4}*(-a*d+b*c)*x/a/b/(b*x^4+a)+1/16*(a*d+3*b*c)*\arctan(-1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(7/4)}/b^{(5/4)}*2^{(1/2)}+1/16*(a*d+3*b*c)*\arctan(1+b^{(1/4)}*x*2^{(1/2)}/a^{(1/4)})/a^{(7/4)}/b^{(5/4)}*2^{(1/2)}-1/32*(a*d+3*b*c)*\ln(-a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(7/4)}/b^{(5/4)}*2^{(1/2)}+1/32*(a*d+3*b*c)*\ln(a^{(1/4)}*b^{(1/4)}*x*2^{(1/2)}+a^{(1/2)}+x^2*b^{(1/2)})/a^{(7/4)}/b^{(5/4)}*2^{(1/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$ , Rules used = {385, 211, 1165, 628, 1162, 617, 204}

$$\frac{(ad+3bc)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(ad+3bc)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} - \frac{(ad+3bc)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x+\sqrt{a}+\sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)/(a + b\*x^4)^2, x]

[Out]  $((b*c - a*d)*x)/(4*a*b*(a + b*x^4)) - ((3*b*c + a*d)*\text{ArcTan}[1 - (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + ((3*b*c + a*d)*\text{ArcTan}[1 + (\text{Sqrt}[2]*b^{(1/4)}*x)/a^{(1/4)}])/(8*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) - ((3*b*c + a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)}) + ((3*b*c + a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)}*b^{(1/4)}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{(7/4)}*b^{(5/4)})$

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 211

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x]] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p+1))/(a\*b\*n\*(p+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rule 617

Int[((a\_) + (b\_.)\*(x\_) + (c\_.)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b

], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[d\*Log[RemoveContent[a + b\*x + c\*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 1162

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(2\*d)/e, 2]}, Dist[e/(2\*c), Int[1/Simp[d/e + q\*x + x^2, x], x], x] + Dist[e/(2\*c), Int[1/Simp[d/e - q\*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && PosQ[d\*e]

### Rule 1165

Int[((d\_) + (e\_)\*(x\_)^2)/((a\_) + (c\_)\*(x\_)^4), x\_Symbol] := With[{q = Rt[(-2\*d)/e, 2]}, Dist[e/(2\*c\*q), Int[(q - 2\*x)/Simp[d/e + q\*x - x^2, x], x], x] + Dist[e/(2\*c\*q), Int[(q + 2\*x)/Simp[d/e - q\*x - x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c\*d^2 - a\*e^2, 0] && NegQ[d\*e]

### Rubi steps

$$\begin{aligned} \int \frac{c + dx^4}{(a + bx^4)^2} dx &= \frac{(bc - ad)x}{4ab(a + bx^4)} + \frac{(3bc + ad) \int \frac{1}{a + bx^4} dx}{4ab} \\ &= \frac{(bc - ad)x}{4ab(a + bx^4)} + \frac{(3bc + ad) \int \frac{\sqrt{a} - \sqrt{b}x^2}{a + bx^4} dx}{8a^{3/2}b} + \frac{(3bc + ad) \int \frac{\sqrt{a} + \sqrt{b}x^2}{a + bx^4} dx}{8a^{3/2}b} \\ &= \frac{(bc - ad)x}{4ab(a + bx^4)} + \frac{(3bc + ad) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} - \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt{b}} + x^2} dx}{16a^{3/2}b^{3/2}} + \frac{(3bc + ad) \int \frac{1}{\frac{\sqrt{a}}{\sqrt{b}} + \frac{\sqrt{2} \sqrt[4]{a}x}{\sqrt{b}} + x^2} dx}{16a^{3/2}b^{3/2}} - \frac{(3bc + ad)}{16\sqrt{2}a^{7/4}b^{5/4}} \\ &= \frac{(bc - ad)x}{4ab(a + bx^4)} - \frac{(3bc + ad) \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc + ad) \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2}a^{7/4}b^{5/4}} \\ &= \frac{(bc - ad)x}{4ab(a + bx^4)} - \frac{(3bc + ad) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} + \frac{(3bc + ad) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}b^{5/4}} - \frac{(3bc + ad)}{16\sqrt{2}a^{7/4}b^{5/4}} \end{aligned}$$

**Mathematica [A]** time = 0.17, size = 212, normalized size = 0.87

$$\frac{-\frac{8a^{3/4}\sqrt[4]{b}x(ad-bc)}{a+bx^4} - \sqrt{2}(ad+3bc)\log(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2) + \sqrt{2}(ad+3bc)\log(\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2)}{32a^{7/4}b^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^4)/(a + b\*x^4)^2, x]

[Out] ((-8\*a^(3/4)\*b^(1/4)\*(-b\*c) + a\*d)\*x)/(a + b\*x^4) - 2\*Sqrt[2]\*(3\*b\*c + a\*d)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + 2\*Sqrt[2]\*(3\*b\*c + a\*d)\*ArcTan[

$1 + (\text{Sqrt}[2]*b^{(1/4)*x}/a^{(1/4)}) - \text{Sqrt}[2]*(3*b*c + a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{(1/4)*b^{(1/4)*x} + \text{Sqrt}[b]*x^2] + \text{Sqrt}[2]*(3*b*c + a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{(1/4)*b^{(1/4)*x} + \text{Sqrt}[b]*x^2}]/(32*a^{(7/4)*b^{(5/4)})}$

**fricas [B]** time = 0.88, size = 711, normalized size = 2.90

$$4(ab^2x^4 + a^2b) \left( -\frac{81b^4c^4 + 108ab^3c^3d + 54a^2b^2c^2d^2 + 12a^3bcd^3 + a^4d^4}{a^7b^5} \right)^{\frac{1}{4}} \arctan \left( \frac{a^5b^4x \left( -\frac{81b^4c^4 + 108ab^3c^3d + 54a^2b^2c^2d^2 + 12a^3bcd^3 + a^4d^4}{a^7b^5} \right)^{\frac{3}{4}}}{-a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a)^2,x, algorithm="fricas")

[Out]  $\frac{1}{16} * (4 * (a * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(1/4)} * \arctan(- (a^5 * b^4 * x * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(3/4)} - a^5 * b^4 * \text{sqrt}((a^4 * b^2 * \text{sqrt}(-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5)) + (9 * b^2 * c^2 + 6 * a * b * c * d + a^2 * d^2) * x^2) / (9 * b^2 * c^2 + 6 * a * b * c * d + a^2 * d^2)) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(3/4)} / (27 * b^3 * c^3 + 27 * a * b^2 * c^2 * d + 9 * a^2 * b * c * d^2 + a^3 * d^3)) + (a * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(1/4)} * \log(a^2 * b * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(1/4)} + (3 * b * c + a * d) * x - (a * b^2 * x^4 + a^2 * b) * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(1/4)} * \log(-a^2 * b * (-81 * b^4 * c^4 + 108 * a * b^3 * c^3 * d + 54 * a^2 * b^2 * c^2 * d^2 + 12 * a^3 * b * c * d^3 + a^4 * d^4) / (a^7 * b^5))^{(1/4)} + (3 * b * c + a * d) * x + 4 * (b * c - a * d) * x / (a * b^2 * x^4 + a^2 * b)$

**giac [A]** time = 0.17, size = 266, normalized size = 1.09

$$\frac{\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} bc + (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x + \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^2} + \frac{\sqrt{2} \left( 3 (ab^3)^{\frac{1}{4}} bc + (ab^3)^{\frac{1}{4}} ad \right) \arctan \left( \frac{\sqrt{2} \left( 2x - \sqrt{2} \left( \frac{a}{b} \right)^{\frac{1}{4}} \right)}{2 \left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16 a^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a)^2,x, algorithm="giac")

[Out]  $\frac{1}{16} * \text{sqrt}(2) * (3 * (a * b^3)^{(1/4)} * b * c + (a * b^3)^{(1/4)} * a * d) * \arctan(1/2 * \text{sqrt}(2) * (2 * x + \text{sqrt}(2) * (a/b)^{(1/4)}) / (a/b)^{(1/4)}) / (a^2 * b^2) + 1/16 * \text{sqrt}(2) * (3 * (a * b^3)^{(1/4)} * b * c + (a * b^3)^{(1/4)} * a * d) * \arctan(1/2 * \text{sqrt}(2) * (2 * x - \text{sqrt}(2) * (a/b)^{(1/4)}) / (a/b)^{(1/4)}) / (a^2 * b^2) + 1/32 * \text{sqrt}(2) * (3 * (a * b^3)^{(1/4)} * b * c + (a * b^3)^{(1/4)} * a * d) * \log(x^2 + \text{sqrt}(2) * x * (a/b)^{(1/4)} + \text{sqrt}(a/b)) / (a^2 * b^2) - 1/32 * \text{sqrt}(2) * (3 * (a * b^3)^{(1/4)} * b * c + (a * b^3)^{(1/4)} * a * d) * \log(x^2 - \text{sqrt}(2) * x * (a/b)^{(1/4)} + \text{sqrt}(a/b)) / (a^2 * b^2) + 1/4 * (b * c * x - a * d * x) / ((b * x^4 + a) * a * b)$

**maple [A]** time = 0.05, size = 295, normalized size = 1.20

$$\frac{\left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x}{\left( \frac{a}{b} \right)^{\frac{1}{4}}} - 1 \right)}{16ab} + \frac{\left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} d \arctan \left( \frac{\sqrt{2} x}{\left( \frac{a}{b} \right)^{\frac{1}{4}}} + 1 \right)}{16ab} + \frac{\left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} d \ln \left( \frac{x^2 + \left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}}{x^2 - \left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} x + \sqrt{\frac{a}{b}}} \right)}{32ab} + \frac{3 \left( \frac{a}{b} \right)^{\frac{1}{4}} \sqrt{2} c \arctan \left( \frac{\sqrt{2} x}{\left( \frac{a}{b} \right)^{\frac{1}{4}}} \right)}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)/(b\*x^4+a)^2,x)

[Out] -1/4\*(a\*d-b\*c)/a/b\*x/(b\*x^4+a)+1/16/a/b\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*d+3/16/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x-1)\*c+1/32/a/b\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*d+3/32/a^2\*(a/b)^(1/4)\*2^(1/2)\*ln((x^2+(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2))/(x^2-(a/b)^(1/4)\*2^(1/2)\*x+(a/b)^(1/2)))\*c+1/16/a/b\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*d+3/16/a^2\*(a/b)^(1/4)\*2^(1/2)\*arctan(2^(1/2)/(a/b)^(1/4)\*x+1)\*c

**maxima** [A] time = 1.35, size = 236, normalized size = 0.96

$$\frac{(bc - ad)x}{4(ab^2x^4 + a^2b)} + \frac{2\sqrt{2}(3bc + ad) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx} + \sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{2\sqrt{2}(3bc + ad) \arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx} - \sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}}\right)}{2\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3bc + ad) \log\left(\sqrt{bx^2 + \dots}\right)}{\frac{3}{a^{\frac{1}{4}}b^{\frac{1}{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)/(b\*x^4+a)^2,x, algorithm="maxima")

[Out] 1/4\*(b\*c - a\*d)\*x/(a\*b^2\*x^4 + a^2\*b) + 1/32\*(2\*sqrt(2))\*(3\*b\*c + a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x + sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + 2\*sqrt(2)\*(3\*b\*c + a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/sqrt(a)\*sqrt(sqrt(a)\*sqrt(b)) + sqrt(2)\*(3\*b\*c + a\*d)\*log(sqrt(b)\*x^2 + sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4)) - sqrt(2)\*(3\*b\*c + a\*d)\*log(sqrt(b)\*x^2 - sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4))/(a\*b)

**mupad** [B] time = 1.53, size = 740, normalized size = 3.02

$$\operatorname{atan} \left( \frac{\left( \frac{x(a^2bd^2+6ab^2cd+9b^3c^2)}{4a^2} - \frac{(ad+3bc)(12cb^3+4adab^2)li}{16(-a)^{7/4}b^{5/4}} \right)_{(ad+3bc)} + \left( \frac{x(a^2bd^2+6ab^2cd+9b^3c^2)}{4a^2} + \frac{(ad+3bc)(12cb^3+4adab^2)li}{16(-a)^{7/4}b^{5/4}} \right)_{(ad+3bc)}}{16(-a)^{7/4}b^{5/4}} \right) (a d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)/(a + b\*x^4)^2,x)

[Out] (atan((((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) - ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d))/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)) + (((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) + ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d))/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)))/((((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) - ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d))/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4)) - (((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) + ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d))/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4))))\*(a\*d + 3\*b\*c)\*1i)/(8\*(-a)^(7/4)\*b^(5/4)) + (atan((((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) - ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4)) + (((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) + ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4)))/((((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) - ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4)) - (((x\*(9\*b^3\*c^2 + a^2\*b\*d^2 + 6\*a\*b^2\*c\*d))/(4\*a^2) + ((a\*d + 3\*b\*c)\*(12\*b^3\*c + 4\*a\*b^2\*d)\*1i)/(16\*(-a)^(7/4)\*b^(5/4)))\*(a\*d + 3\*b\*c))/(16\*(-a)^(7/4)\*b^(5/4))))\*(a\*d + 3\*b\*c))

$$a^{7/4}b^{5/4}) \cdot (ad + 3bc) \cdot i / (16(-a)^{7/4}b^{5/4}) - (((x(9b^3c^2 + a^2bd^2 + 6ab^2cd)) / (4a^2) + ((ad + 3bc) \cdot (12b^3c + 4ab^2d) \cdot i) / (16(-a)^{7/4}b^{5/4})) \cdot (ad + 3bc) \cdot i) / (16(-a)^{7/4}b^{5/4})) \cdot (ad + 3bc) / (8(-a)^{7/4}b^{5/4}) - (x(ad - bc)) / (4ab(a + bx^4))$$

**sympy [A]** time = 0.97, size = 112, normalized size = 0.46

$$\frac{x(-ad + bc)}{4a^2b + 4ab^2x^4} + \text{RootSum}\left(65536t^4a^7b^5 + a^4d^4 + 12a^3bcd^3 + 54a^2b^2c^2d^2 + 108ab^3c^3d + 81b^4c^4, \left(t \mapsto t \log\left(\frac{16}{ad - \dots}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x\*\*4+c)/(b\*x\*\*4+a)\*\*2,x)

[Out] x\*(-a\*d + b\*c)/(4\*a\*\*2\*b + 4\*a\*b\*\*2\*x\*\*4) + RootSum(65536\*\_t\*\*4\*a\*\*7\*b\*\*5 + a\*\*4\*d\*\*4 + 12\*a\*\*3\*b\*c\*d\*\*3 + 54\*a\*\*2\*b\*\*2\*c\*\*2\*d\*\*2 + 108\*a\*b\*\*3\*c\*\*3\*d + 81\*b\*\*4\*c\*\*4, Lambda(\_t, \_t\*log(16\*\_t\*a\*\*2\*b/(a\*d + 3\*b\*c) + x)))



$$3.171 \quad \int \frac{1}{(a+bx^4)^2(c+dx^4)} dx$$

**Optimal.** Leaf size=513

$$\frac{b^{3/4}(3bc-7ad) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2} - \frac{b^{3/4}(3bc-7ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2}$$

[Out]  $\frac{1}{4} b x / a (-a d + b c) / (b x^4 + a) + 1/16 b^{3/4} (-7 a d + 3 b c) \arctan(-1 + b^{1/4} x^2)^{1/2} / a^{1/4} / a^{7/4} / (-a d + b c)^{2 \cdot 2^{1/2}} + 1/16 b^{3/4} (-7 a d + 3 b c) \arctan(1 + b^{1/4} x^2)^{1/2} / a^{1/4} / a^{7/4} / (-a d + b c)^{2 \cdot 2^{1/2}} + 1/4 d^{7/4} \arctan(-1 + d^{1/4} x^2)^{1/2} / c^{1/4} / c^{3/4} / (-a d + b c)^{2 \cdot 2^{1/2}} + 1/4 d^{7/4} \arctan(1 + d^{1/4} x^2)^{1/2} / c^{1/4} / c^{3/4} / (-a d + b c)^{2 \cdot 2^{1/2}} - 1/32 b^{3/4} (-7 a d + 3 b c) \ln(-a^{1/4} b^{1/4} x^2 + a^{1/2} + x^2 b^{1/2}) / a^{7/4} / (-a d + b c)^{2 \cdot 2^{1/2}} + 1/32 b^{3/4} (-7 a d + 3 b c) \ln(a^{1/4} b^{1/4} x^2 + a^{1/2} + x^2 b^{1/2}) / a^{7/4} / (-a d + b c)^{2 \cdot 2^{1/2}} - 1/8 d^{7/4} \ln(-c^{1/4} d^{1/4} x^2 + c^{1/2} + x^2 d^{1/2}) / c^{3/4} / (-a d + b c)^{2 \cdot 2^{1/2}} + 1/8 d^{7/4} \ln(c^{1/4} d^{1/4} x^2 + c^{1/2} + x^2 d^{1/2}) / c^{3/4} / (-a d + b c)^{2 \cdot 2^{1/2}}$

**Rubi [A]** time = 0.43, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 8, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$ , Rules used = {414, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{3/4}(3bc-7ad) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2} - \frac{b^{3/4}(3bc-7ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^2\*(c + d\*x^4)), x]

[Out]  $\frac{(b x) / (4 a (b c - a d) (a + b x^4)) - (b^{3/4} (3 b c - 7 a d) \operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2] b^{1/4} x) / a^{1/4}]) / (8 \operatorname{Sqrt}[2] a^{7/4} (b c - a d)^2) + (b^{3/4} (3 b c - 7 a d) \operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2] b^{1/4} x) / a^{1/4}]) / (8 \operatorname{Sqrt}[2] a^{7/4} (b c - a d)^2) - (d^{7/4} \operatorname{ArcTan}[1 - (\operatorname{Sqrt}[2] d^{1/4} x) / c^{1/4}]) / (2 \operatorname{Sqrt}[2] c^{3/4} (b c - a d)^2) + (d^{7/4} \operatorname{ArcTan}[1 + (\operatorname{Sqrt}[2] d^{1/4} x) / c^{1/4}]) / (2 \operatorname{Sqrt}[2] c^{3/4} (b c - a d)^2) - (b^{3/4} (3 b c - 7 a d) \operatorname{Log}[\operatorname{Sqrt}[a] - \operatorname{Sqrt}[2] a^{1/4} b^{1/4} x + \operatorname{Sqrt}[b] x^2]) / (16 \operatorname{Sqrt}[2] a^{7/4} (b c - a d)^2) + (b^{3/4} (3 b c - 7 a d) \operatorname{Log}[\operatorname{Sqrt}[a] + \operatorname{Sqrt}[2] a^{1/4} b^{1/4} x + \operatorname{Sqrt}[b] x^2]) / (16 \operatorname{Sqrt}[2] a^{7/4} (b c - a d)^2) - (d^{7/4} \operatorname{Log}[\operatorname{Sqrt}[c] - \operatorname{Sqrt}[2] c^{1/4} d^{1/4} x + \operatorname{Sqrt}[d] x^2]) / (4 \operatorname{Sqrt}[2] c^{3/4} (b c - a d)^2) + (d^{7/4} \operatorname{Log}[\operatorname{Sqrt}[c] + \operatorname{Sqrt}[2] c^{1/4} d^{1/4} x + \operatorname{Sqrt}[d] x^2]) / (4 \operatorname{Sqrt}[2] c^{3/4} (b c - a d)^2)}$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4), x], x] + Dist[1/(2\*r), Int[(r + s\*x^2)/(a + b\*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] && AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
- Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^2(c+dx^4)} dx &= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{\int \frac{-3bc+4ad-3bdx^4}{(a+bx^4)(c+dx^4)} dx}{4a(bc-ad)} \\
&= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2 \int \frac{1}{c+dx^4} dx}{(bc-ad)^2} + \frac{(b(3bc-7ad)) \int \frac{1}{a+bx^4} dx}{4a(bc-ad)^2} \\
&= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^2 \int \frac{\sqrt{c}-\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc-ad)^2} + \frac{d^2 \int \frac{\sqrt{c}+\sqrt{d}x^2}{c+dx^4} dx}{2\sqrt{c}(bc-ad)^2} + \frac{(b(3bc-7ad)) \int}{8a^{3/2}(bc-} \\
&= \frac{bx}{4a(bc-ad)(a+bx^4)} + \frac{d^{3/2} \int \frac{1}{\frac{\sqrt{c}}{\sqrt{a}} - \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + x^2} dx}{4\sqrt{c}(bc-ad)^2} + \frac{d^{3/2} \int \frac{1}{\frac{\sqrt{c}}{\sqrt{a}} + \frac{\sqrt{2} \sqrt[4]{c}x}{\sqrt[4]{a}} + x^2} dx}{4\sqrt{c}(bc-ad)^2} - \frac{d^{7/4}}{4} \\
&= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad) \log(\sqrt{a} - \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad) \log(\sqrt{a} + \sqrt{2} \sqrt[4]{a} \sqrt[4]{b}x + \sqrt{b}x^2)}{16\sqrt{2}a^{7/4}(bc-ad)^2} \\
&= \frac{bx}{4a(bc-ad)(a+bx^4)} - \frac{b^{3/4}(3bc-7ad) \tan^{-1}\left(1 - \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2} + \frac{b^{3/4}(3bc-7ad) \tan^{-1}\left(1 + \frac{\sqrt{2} \sqrt[4]{b}x}{\sqrt[4]{a}}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^2}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 499, normalized size = 0.97

$$8a^{3/4}bc^{3/4}x(bc-ad) - 8\sqrt{2}a^{7/4}d^{7/4}(a+bx^4)\tan^{-1}\left(1 - \frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}}\right) + 8\sqrt{2}a^{7/4}d^{7/4}(a+bx^4)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}x}{\sqrt[4]{c}} + 1\right) -$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)^2\*(c + d\*x^4)),x]

[Out] (8\*a^(3/4)\*b\*c^(3/4)\*(b\*c - a\*d)\*x - 2\*Sqrt[2]\*b^(3/4)\*c^(3/4)\*(3\*b\*c - 7\*a\*d)\*(a + b\*x^4)\*ArcTan[1 - (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] + 2\*Sqrt[2]\*b^(3/4)\*c^(3/4)\*(3\*b\*c - 7\*a\*d)\*(a + b\*x^4)\*ArcTan[1 + (Sqrt[2]\*b^(1/4)\*x)/a^(1/4)] - 8\*Sqrt[2]\*a^(7/4)\*d^(7/4)\*(a + b\*x^4)\*ArcTan[1 - (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] + 8\*Sqrt[2]\*a^(7/4)\*d^(7/4)\*(a + b\*x^4)\*ArcTan[1 + (Sqrt[2]\*d^(1/4)\*x)/c^(1/4)] - Sqrt[2]\*b^(3/4)\*c^(3/4)\*(3\*b\*c - 7\*a\*d)\*(a + b\*x^4)\*Log[Sqrt[a] - Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2] + Sqrt[2]\*b^(3/4)\*c^(3/4)\*(3\*b\*c - 7\*a\*d)\*(a + b\*x^4)\*Log[Sqrt[a] + Sqrt[2]\*a^(1/4)\*b^(1/4)\*x + Sqrt[b]\*x^2] - 4\*Sqrt[2]\*a^(7/4)\*d^(7/4)\*(a + b\*x^4)\*Log[Sqrt[c] - Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2] + 4\*Sqrt[2]\*a^(7/4)\*d^(7/4)\*(a + b\*x^4)\*Log[Sqrt[c] + Sqrt[2]\*c^(1/4)\*d^(1/4)\*x + Sqrt[d]\*x^2])/(32\*a^(7/4)\*c^(3/4)\*(b\*c - a\*d)^2\*(a + b\*x^4))

**fricas [B]** time = 36.68, size = 3299, normalized size = 6.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c),x, algorithm="fricas")

[Out] -1/16\*(4\*((a\*b^2\*c - a^2\*b\*d)\*x^4 + a^2\*b\*c - a^3\*d)\*(-(81\*b^7\*c^4 - 756\*a\*b^6\*c^3\*d + 2646\*a^2\*b^5\*c^2\*d^2 - 4116\*a^3\*b^4\*c\*d^3 + 2401\*a^4\*b^3\*d^4)/(a^7\*b^8\*c^8 - 8\*a^8\*b^7\*c^7\*d + 28\*a^9\*b^6\*c^6\*d^2 - 56\*a^10\*b^5\*c^5\*d^3 +

$$\begin{aligned}
& 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8)^{(1/4)} \arctan\left(\frac{(a^5b^6c^6 - 6a^6b^5c^5d + 15a^7b^4c^4d^2 - 20a^8b^3c^3d^3 + 15a^9b^2c^2d^4 - 6a^{10}b^1c^1d^5 + a^{11}d^6) \times (-81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4)}{(a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8)}\right)^{(3/4)} - (a^5b^6c^6 - 6a^6b^5c^5d + 15a^7b^4c^4d^2 - 20a^8b^3c^3d^3 + 15a^9b^2c^2d^4 - 6a^{10}b^1c^1d^5 + a^{11}d^6) \times (-81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4) / (a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8) \times \sqrt{\left(\frac{(9b^4c^2 - 42ab^3cd + 49a^2b^2d^2) \times x^2 + (a^4b^4c^4 - 4a^5b^3c^3d + 6a^6b^2c^2d^2 - 4a^7b^1c^1d^3 + a^8d^4) \times \sqrt{-(81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4)}}{(a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8)}\right)} / (27b^5c^3 - 189a^2b^4c^2d + 441a^3b^3c^1d^2 - 343a^4b^2c^0d^3) - 16(-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(1/4)} \times ((a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d) \times \arctan\left(\frac{(b^6c^8 - 6a^2b^5c^7d + 15a^3b^4c^6d^2 - 20a^4b^3c^5d^3 + 15a^5b^2c^4d^4 - 6a^6b^1c^3d^5 + a^6c^2d^6) \times (-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(3/4)} \times x - (b^6c^8 - 6a^2b^5c^7d + 15a^3b^4c^6d^2 - 20a^4b^3c^5d^3 + 15a^5b^2c^4d^4 - 6a^6b^1c^3d^5 + a^6c^2d^6) \times (-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(3/4)} \times \sqrt{\left(\frac{d^4 \times x^2 + (b^4c^6 - 4a^2b^3c^5d + 6a^3b^2c^4d^2 - 4a^4b^1c^3d^3 + a^4c^2d^4) \times \sqrt{-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8)}}{d^4}\right)} / d^5) - 4(-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(1/4)} \times ((a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d) \times \log(d^2 \times x + (-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(1/4)} \times (b^2c^3 - 2a^2b^1c^2d + a^2c^1d^2)) + 4(-d^7/(b^8c^{11} - 8a^2b^7c^{10}d + 28a^3b^6c^9d^2 - 56a^4b^5c^8d^3 + 70a^5b^4c^7d^4 - 56a^6b^3c^6d^5 + 28a^7b^2c^5d^6 - 8a^8b^1c^4d^7 + a^8c^3d^8))^{(1/4)} \times ((a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d) \times \log\left(\frac{(a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d}{(a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d} \times \frac{(-81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4)}{(a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8)}\right) \times x + (a^2b^2c^2 - 2a^3b^1c^1d + a^4d^2) \times (-81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4) / (a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8))^{(1/4)} - ((a^2b^2c - a^2b^2d) \times x^4 + a^2b^2c - a^3d) \times (-81b^7c^4 - 756a^2b^6c^3d + 2646a^3b^5c^2d^2 - 4116a^4b^4c^1d^3 + 2401a^5b^3c^0d^4) / (a^7b^8c^8 - 8a^8b^7c^7d + 28a^9b^6c^6d^2 - 56a^{10}b^5c^5d^3 + 70a^{11}b^4c^4d^4 - 56a^{12}b^3c^3d^5 + 28a^{13}b^2c^2d^6 - 8a^{14}b^1c^1d^7 + a^{15}d^8))^{(1/4)} \times
\end{aligned}$$

$\log(-(3*b^2*c - 7*a*b*d)*x - (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2))*(-(81*b^7*c^4 - 756*a*b^6*c^3*d + 2646*a^2*b^5*c^2*d^2 - 4116*a^3*b^4*c*d^3 + 2401*a^4*b^3*d^4)/(a^7*b^8*c^8 - 8*a^8*b^7*c^7*d + 28*a^9*b^6*c^6*d^2 - 56*a^10*b^5*c^5*d^3 + 70*a^11*b^4*c^4*d^4 - 56*a^12*b^3*c^3*d^5 + 28*a^13*b^2*c^2*d^6 - 8*a^14*b*c*d^7 + a^15*d^8))^{(1/4)} - 4*b*x)/((a*b^2*c - a^2*b*d)*x^4 + a^2*b*c - a^3*d)$

**giac [A]** time = 0.22, size = 667, normalized size = 1.30

$$\frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3 - 2\sqrt{2}abc^2d + \sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}} d \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{c}{d}\right)^{\frac{1}{4}}\right)}{2\left(\frac{c}{d}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}b^2c^3 - 2\sqrt{2}abc^2d + \sqrt{2}a^2cd^2\right)} + \frac{(cd^3)^{\frac{1}{4}} d \log\left(x^2 + \sqrt{2}x\left(\frac{c}{d}\right)^{\frac{1}{4}} + \left(\frac{c}{d}\right)^{\frac{1}{2}}\right)}{4\left(\sqrt{2}b^2c^3 - 2\sqrt{2}abc^2d + \sqrt{2}a^2cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c),x, algorithm="giac")

[Out]  $\frac{1}{2}*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + \frac{1}{2}*(c*d^3)^{(1/4)}*d*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(c/d)^{(1/4)})/(c/d)^{(1/4)})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + \frac{1}{4}*(c*d^3)^{(1/4)}*d*\log(x^2 + \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) - \frac{1}{4}*(c*d^3)^{(1/4)}*d*\log(x^2 - \sqrt{2}*x*(c/d)^{(1/4)} + \sqrt{c/d})/(\sqrt{2}*b^2*c^3 - 2*\sqrt{2}*a*b*c^2*d + \sqrt{2}*a^2*c*d^2) + \frac{1}{8}*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x + \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + \frac{1}{8}*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\arctan(1/2*\sqrt{2}*(2*x - \sqrt{2}*(a/b)^{(1/4)})/(a/b)^{(1/4)})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + \frac{1}{16}*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\log(x^2 + \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) - \frac{1}{16}*(3*(a*b^3)^{(1/4)}*b*c - 7*(a*b^3)^{(1/4)}*a*d)*\log(x^2 - \sqrt{2}*x*(a/b)^{(1/4)} + \sqrt{a/b})/(\sqrt{2}*a^2*b^2*c^2 - 2*\sqrt{2}*a^3*b*c*d + \sqrt{2}*a^4*d^2) + \frac{1}{4}*b*x/((b*x^4 + a)*(a*b*c - a^2*d))$

**maple [A]** time = 0.06, size = 550, normalized size = 1.07

$$\frac{b^2cx}{4(ad-bc)^2(bx^4+a)a} - \frac{bdx}{4(ad-bc)^2(bx^4+a)} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}bd\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}-1\right)}{16(ad-bc)^2a} - \frac{7\left(\frac{a}{b}\right)^{\frac{1}{4}}\sqrt{2}bd\arctan\left(\frac{\sqrt{2}x}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{16(ad-bc)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^2/(d\*x^4+c),x)

[Out]  $\frac{1}{8}*d^2/(a*d-b*c)^2*(c/d)^{(1/4)}/c*2^{(1/2)}*\ln((x^2+(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)})/(x^2-(c/d)^{(1/4)}*2^{(1/2)}*x+(c/d)^{(1/2)}))+\frac{1}{4}*d^2/(a*d-b*c)^2*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x+1)+\frac{1}{4}*d^2/(a*d-b*c)^2*(c/d)^{(1/4)}/c*2^{(1/2)}*\arctan(2^{(1/2)}/(c/d)^{(1/4)}*x-1)-\frac{1}{4}*b/(a*d-b*c)^2*x/(b*x^4+a)*d+\frac{1}{4}*b^2/(a*d-b*c)^2/a*x/(b*x^4+a)*c-\frac{7}{16}*b/(a*d-b*c)^2/a*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*d+\frac{3}{16}*b^2/(a*d-b*c)^2/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x+1)*c-\frac{7}{16}*b/(a*d-b*c)^2/a*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*d+\frac{3}{16}*b^2/(a*d-b*c)^2/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\arctan(2^{(1/2)}/(a/b)^{(1/4)}*x-1)*c-\frac{7}{32}*b/(a*d-b*c)^2/a*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))*d+\frac{3}{32}*b^2/(a*d-b*c)^2/a^2*(a/b)^{(1/4)}*2^{(1/2)}*\ln((x^2+(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)})/(x^2-(a/b)^{(1/4)}*2^{(1/2)}*x+(a/b)^{(1/2)}))*c$

**maxima** [A] time = 1.46, size = 470, normalized size = 0.92

$$\frac{\left( \frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx}+\sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{a}\sqrt{b}} \right) + \frac{2\sqrt{2}(3bc-7ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{bx}-\sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{a}\sqrt{b}} \right) + \frac{\sqrt{2}(3bc-7ad)\log\left(\sqrt{bx^2}+\sqrt{2a^{\frac{1}{4}}b^{\frac{1}{4}}x}+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32(ab^2c^2 - 2a^2bcd + a^3d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c),x, algorithm="maxima")

[Out] 1/32\*(2\*sqrt(2)\*(3\*b\*c - 7\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x + sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/(sqrt(a)\*sqrt(sqrt(a)\*sqrt(b))) + 2\*sqrt(2)\*(3\*b\*c - 7\*a\*d)\*arctan(1/2\*sqrt(2)\*(2\*sqrt(b)\*x - sqrt(2)\*a^(1/4)\*b^(1/4))/sqrt(sqrt(a)\*sqrt(b)))/(sqrt(a)\*sqrt(sqrt(a)\*sqrt(b))) + sqrt(2)\*(3\*b\*c - 7\*a\*d)\*log(sqrt(b)\*x^2 + sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4)) - sqrt(2)\*(3\*b\*c - 7\*a\*d)\*log(sqrt(b)\*x^2 - sqrt(2)\*a^(1/4)\*b^(1/4)\*x + sqrt(a))/(a^(3/4)\*b^(1/4))\*b/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2) + 1/4\*b\*x/((a\*b^2\*c - a^2\*b\*d)\*x^4 + a^2\*b\*c - a^3\*d) + 1/8\*(2\*sqrt(2)\*d^2\*arctan(1/2\*sqrt(2)\*(2\*sqrt(d)\*x + sqrt(2)\*c^(1/4)\*d^(1/4))/sqrt(sqrt(c)\*sqrt(d)))/(sqrt(c)\*sqrt(sqrt(c)\*sqrt(d))) + 2\*sqrt(2)\*d^2\*arctan(1/2\*sqrt(2)\*(2\*sqrt(d)\*x - sqrt(2)\*c^(1/4)\*d^(1/4))/sqrt(sqrt(c)\*sqrt(d)))/(sqrt(c)\*sqrt(sqrt(c)\*sqrt(d))) + sqrt(2)\*d^(7/4)\*log(sqrt(d)\*x^2 + sqrt(2)\*c^(1/4)\*d^(1/4)\*x + sqrt(c))/c^(3/4) - sqrt(2)\*d^(7/4)\*log(sqrt(d)\*x^2 - sqrt(2)\*c^(1/4)\*d^(1/4)\*x + sqrt(c))/c^(3/4))/(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)

**mupad** [B] time = 3.82, size = 21975, normalized size = 42.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^2\*(c + d\*x^4)),x)

[Out] 2\*atan((((-(81\*b^7\*c^4 + 2401\*a^4\*b^3\*d^4 - 4116\*a^3\*b^4\*c\*d^3 + 2646\*a^2\*b^5\*c^2\*d^2 - 756\*a\*b^6\*c^3\*d)/(65536\*a^15\*d^8 + 65536\*a^7\*b^8\*c^8 - 524288\*a^8\*b^7\*c^7\*d + 1835008\*a^9\*b^6\*c^6\*d^2 - 3670016\*a^10\*b^5\*c^5\*d^3 + 4587520\*a^11\*b^4\*c^4\*d^4 - 3670016\*a^12\*b^3\*c^3\*d^5 + 1835008\*a^13\*b^2\*c^2\*d^6 - 524288\*a^14\*b\*c\*d^7))^(1/4)\*(((28\*a^4\*b^6\*d^11 + (81\*b^10\*c^4\*d^7)/16 - (675\*a\*b^9\*c^3\*d^8)/16 - (2145\*a^3\*b^7\*c\*d^10)/16 + (1971\*a^2\*b^8\*c^2\*d^9)/16)\*1i)/(a^7\*d^3 - a^4\*b^3\*c^3 + 3\*a^5\*b^2\*c^2\*d - 3\*a^6\*b\*c\*d^2) + (-(81\*b^7\*c^4 + 2401\*a^4\*b^3\*d^4 - 4116\*a^3\*b^4\*c\*d^3 + 2646\*a^2\*b^5\*c^2\*d^2 - 756\*a\*b^6\*c^3\*d)/(65536\*a^15\*d^8 + 65536\*a^7\*b^8\*c^8 - 524288\*a^8\*b^7\*c^7\*d + 1835008\*a^9\*b^6\*c^6\*d^2 - 3670016\*a^10\*b^5\*c^5\*d^3 + 4587520\*a^11\*b^4\*c^4\*d^4 - 3670016\*a^12\*b^3\*c^3\*d^5 + 1835008\*a^13\*b^2\*c^2\*d^6 - 524288\*a^14\*b\*c\*d^7))^(1/4)\*((3072\*a^4\*b^14\*c^11\*d^4 - 4096\*a^14\*b^4\*c\*d^14 - 28672\*a^5\*b^13\*c^10\*d^5 + 114688\*a^6\*b^12\*c^9\*d^6 - 253952\*a^7\*b^11\*c^8\*d^7 + 329728\*a^8\*b^10\*c^7\*d^8 - 229376\*a^9\*b^9\*c^6\*d^9 + 28672\*a^10\*b^8\*c^5\*d^10 + 90112\*a^11\*b^7\*c^4\*d^11 - 78848\*a^12\*b^6\*c^3\*d^12 + 28672\*a^13\*b^5\*c^2\*d^13))/(a^7\*d^3 - a^4\*b^3\*c^3 + 3\*a^5\*b^2\*c^2\*d - 3\*a^6\*b\*c\*d^2) - (x\*(65536\*a^15\*b^4\*d^17 - 524288\*a^14\*b^5\*c\*d^16 + 36864\*a^2\*b^17\*c^13\*d^4 - 466944\*a^3\*b^16\*c^12\*d^5 + 2609152\*a^4\*b^15\*c^11\*d^6 - 8486912\*a^5\*b^14\*c^10\*d^7 + 17833984\*a^6\*b^13\*c^9\*d^8 - 25280512\*a^7\*b^12\*c^8\*d^9 + 24190976\*a^8

$$\begin{aligned}
& *b^{11}c^7d^{10} - 14516224a^9b^{10}c^6d^{11} + 3362816a^{10}b^9c^5d^{12} + 2 \\
& 809856a^{11}b^8c^4d^{13} - 3469312a^{12}b^7c^3d^{14} + 1835008a^{13}b^6c^2 \\
& *d^{15}) * i) / (64(a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5d + 15a^6b^4c^4d \\
& ^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^1c^1d^5)) * i) + (x(3 \\
& 185a^4b^7d^{13} + 81b^{11}c^4d^9 - 756a^10c^3d^{10} - 4788a^3b^8c^1d \\
& ^{12} + 2790a^2b^9c^2d^{11})) / (64(a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5d \\
& + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^1c^1 \\
& *d^5)) * (- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^1d^3 + 2646a^2b^5 \\
& *c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a \\
& ^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520 \\
& *a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 5 \\
& 24288a^{14}b^1c^1d^7))^{(1/4)} - (((- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b \\
& ^4c^1d^3 + 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 65536 \\
& a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10} \\
& *b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 183500 \\
& 8a^{13}b^2c^2d^6 - 524288a^{14}b^1c^1d^7))^{(1/4)} * (((28a^4b^6d^{11} + (81b \\
& ^{10}c^4d^7) / 16 - (675a^1b^9c^3d^8) / 16 - (2145a^3b^7c^1d^{10}) / 16 + (1971 \\
& *a^2b^8c^2d^9) / 16) * i) / (a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - 3a^6b \\
& *c^1d^2) + (- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^1d^3 + 2646a^2 \\
& *b^5c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 65536a^7b^8c^8 - 52428 \\
& 8a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587 \\
& 520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 \\
& - 524288a^{14}b^1c^1d^7))^{(3/4)} * (((- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3 \\
& *b^4c^1d^3 + 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 6553 \\
& 6a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10} \\
& *b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835 \\
& 008a^{13}b^2c^2d^6 - 524288a^{14}b^1c^1d^7))^{(1/4)} * (3072a^4b^{14}c^{11}d^4 \\
& - 4096a^{14}b^4c^1d^{14} - 28672a^5b^{13}c^{10}d^5 + 114688a^6b^{12}c^9d^6 \\
& - 253952a^7b^{11}c^8d^7 + 329728a^8b^{10}c^7d^8 - 229376a^9b^9c^6d^9 \\
& + 28672a^{10}b^8c^5d^{10} + 90112a^{11}b^7c^4d^{11} - 78848a^{12}b^6c^3d \\
& ^{12} + 28672a^{13}b^5c^2d^{13})) / (a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - \\
& 3a^6b^1c^1d^2) + (x(65536a^{15}b^4d^{17} - 524288a^{14}b^5c^1d^{16} + 36864 \\
& a^2b^{17}c^{13}d^4 - 466944a^3b^{16}c^{12}d^5 + 2609152a^4b^{15}c^{11}d^6 - \\
& 8486912a^5b^{14}c^{10}d^7 + 17833984a^6b^{13}c^9d^8 - 25280512a^7b^{12}c^ \\
& ^8d^9 + 24190976a^8b^{11}c^7d^{10} - 14516224a^9b^{10}c^6d^{11} + 3362816 \\
& a^{10}b^9c^5d^{12} + 2809856a^{11}b^8c^4d^{13} - 3469312a^{12}b^7c^3d^{14} + \\
& 1835008a^{13}b^6c^2d^{15}) * i) / (64(a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5 \\
& *d + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^1c^1 \\
& *d^5)) * i) - (x(3185a^4b^7d^{13} + 81b^{11}c^4d^9 - 756a^10c^3d^{10} - \\
& 4788a^3b^8c^1d^{12} + 2790a^2b^9c^2d^{11})) / (64(a^{10}d^6 + a^4b^6c^6 \\
& - 6a^5b^5c^5d + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - \\
& 6a^9b^1c^1d^5)) * (- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^1d^3 + \\
& 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 65536a^7b^8c^8 \\
& - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 \\
& + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2 \\
& *d^6 - 524288a^{14}b^1c^1d^7))^{(1/4)} / (((- (81b^7c^4 + 2401a^4b^3d^4 - \\
& 4116a^3b^4c^1d^3 + 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / (65 \\
& 536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6 \\
& *d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12} \\
& *b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^1c^1d^7))^{(1/4)} * (((28 \\
& *a^4b^6d^{11} + (81b^{10}c^4d^7) / 16 - (675a^1b^9c^3d^8) / 16 - (2145a^3b^7 \\
& *c^1d^{10}) / 16 + (1971a^2b^8c^2d^9) / 16) * i) / (a^7d^3 - a^4b^3c^3 + 3a^ \\
& ^5b^2c^2d - 3a^6b^1c^1d^2) + (- (81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4 \\
& *c^1d^3 + 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / (65536a^{15}d^8 + 6553 \\
& 6a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10} \\
& *b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835 \\
& 008a^{13}b^2c^2d^6 - 524288a^{14}b^1c^1d^7))^{(3/4)} * (((- (81b^7c^4 + 2401a^ \\
& ^4b^3d^4 - 4116a^3b^4c^1d^3 + 2646a^2b^5c^2d^2 - 756a^1b^6c^3d) / ( \\
& 65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6
\end{aligned}$$

$$\begin{aligned}
& *c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520*a^{11}*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 524288*a^{14}*b*c*d^7)^{(1/4)}*(30 \\
& 72*a^4*b^{14}*c^{11}*d^4 - 4096*a^{14}*b^4*c*d^{14} - 28672*a^5*b^{13}*c^{10}*d^5 + 114 \\
& 688*a^6*b^{12}*c^9*d^6 - 253952*a^7*b^{11}*c^8*d^7 + 329728*a^8*b^{10}*c^7*d^8 - \\
& 229376*a^9*b^9*c^6*d^9 + 28672*a^{10}*b^8*c^5*d^{10} + 90112*a^{11}*b^7*c^4*d^{11} \\
& - 78848*a^{12}*b^6*c^3*d^{12} + 28672*a^{13}*b^5*c^2*d^{13}))/ (a^7*d^3 - a^4*b^3*c^3 \\
& + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^{15}*b^4*d^{17} - 524288*a^{14}*b^5*c*d^{16} + 36864*a^2*b^{17}*c^{13}*d^4 - 466944*a^3*b^{16}*c^{12}*d^5 + 2609152 \\
& *a^4*b^{15}*c^{11}*d^6 - 8486912*a^5*b^{14}*c^{10}*d^7 + 17833984*a^6*b^{13}*c^9*d^8 \\
& - 25280512*a^7*b^{12}*c^8*d^9 + 24190976*a^8*b^{11}*c^7*d^{10} - 14516224*a^9*b^{10}*c^6*d^{11} + 3362816*a^{10}*b^9*c^5*d^{12} + 2809856*a^{11}*b^8*c^4*d^{13} - 346931 \\
& 2*a^{12}*b^7*c^3*d^{14} + 1835008*a^{13}*b^6*c^2*d^{15})*1i)/(64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i + (x*(3185*a^4*b^7*d^{13} + 81*b^{11}*c^4 \\
& *d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8*c*d^{12} + 2790*a^2*b^9*c^2*d^{11})*1 \\
& i)/(64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20* \\
& a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*(-(81*b^7*c^4 + 240 \\
& 1*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d \\
& )/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9* \\
& b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520*a^{11}*b^4*c^4*d^4 - 3670016 \\
& *a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 524288*a^{14}*b*c*d^7)^{(1/4)} \\
& + ((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2 \\
& *d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520*a^{11} \\
& *b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 524288 \\
& *a^{14}*b*c*d^7)^{(1/4)}*((28*a^4*b^6*d^{11} + (81*b^{10}*c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^{10})/16 + (1971*a^2*b^8*c^2*d^9)/16)*1i)/( \\
& a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (-(81*b^7*c^4 + \\
& 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d \\
& )/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520*a^{11}*b^4*c^4*d^4 - 3670 \\
& 016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 524288*a^{14}*b*c*d^7)^{(3/ \\
& 4)}*((-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5*c^2 \\
& *d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520*a^ \\
& 11*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 5242 \\
& 88*a^{14}*b*c*d^7)^{(1/4)}*(3072*a^4*b^{14}*c^{11}*d^4 - 4096*a^{14}*b^4*c*d^{14} - 28 \\
& 672*a^5*b^{13}*c^{10}*d^5 + 114688*a^6*b^{12}*c^9*d^6 - 253952*a^7*b^{11}*c^8*d^7 + \\
& 329728*a^8*b^{10}*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^{10}*b^8*c^5*d^{10} \\
& + 90112*a^{11}*b^7*c^4*d^{11} - 78848*a^{12}*b^6*c^3*d^{12} + 28672*a^{13}*b^5*c^2*d^{13}))/ (a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536 \\
& *a^{15}*b^4*d^{17} - 524288*a^{14}*b^5*c*d^{16} + 36864*a^2*b^{17}*c^{13}*d^4 - 466944* \\
& a^3*b^{16}*c^{12}*d^5 + 2609152*a^4*b^{15}*c^{11}*d^6 - 8486912*a^5*b^{14}*c^{10}*d^7 + \\
& 17833984*a^6*b^{13}*c^9*d^8 - 25280512*a^7*b^{12}*c^8*d^9 + 24190976*a^8*b^{11}* \\
& c^7*d^{10} - 14516224*a^9*b^{10}*c^6*d^{11} + 3362816*a^{10}*b^9*c^5*d^{12} + 2809856 \\
& *a^{11}*b^8*c^4*d^{13} - 3469312*a^{12}*b^7*c^3*d^{14} + 1835008*a^{13}*b^6*c^2*d^{15} \\
& *1i)/(64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 2 \\
& 0*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))*1i - (x*(3185 \\
& *a^4*b^7*d^{13} + 81*b^{11}*c^4*d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8*c*d^{12} \\
& + 2790*a^2*b^9*c^2*d^{11})*1i)/(64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d \\
& + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c \\
& *d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^2*b^5 \\
& *c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 4587520 \\
& *a^{11}*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 - 5 \\
& 24288*a^{14}*b*c*d^7)^{(1/4)}))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4 \\
& *c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}* \\
& b^5*c^5*d^3 + 4587520*a^{11}*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008
\end{aligned}$$



$$\begin{aligned} & *a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7)^{(1/4)} - \operatorname{atan}\left(\frac{(-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)}{(65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7)}\right)^{(1/4)} \\ & * \left( \frac{(28a^4b^6d^{11} + (81b^{10}c^4d^7)/16 - (675a^2b^9c^3d^8)/16 - (2145a^3b^7c^2d^{10})/16 + (1971a^2b^8c^2d^9)/16)}{(a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - 3a^6b^2c^2d^2) + (-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)} \right) / (65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7) \\ & ^{(3/4)} * \left( \frac{(-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)}{(65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7)} \right)^{(1/4)} * \\ & (3072a^4b^{14}c^{11}d^4 - 4096a^{14}b^4c^2d^{14} - 28672a^5b^{13}c^{10}d^5 + 114688a^6b^{12}c^9d^6 - 253952a^7b^{11}c^8d^7 + 329728a^8b^{10}c^7d^8 - 229376a^9b^9c^6d^9 + 28672a^{10}b^8c^5d^{10} + 90112a^{11}b^7c^4d^{11} - 78848a^{12}b^6c^3d^{12} + 28672a^{13}b^5c^2d^{13})) / (a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - 3a^6b^2c^2d^2) - \\ & (x * (65536a^{15}b^4d^{17} - 524288a^{14}b^5c^2d^{16} + 36864a^2b^{17}c^{13}d^4 - 466944a^3b^{16}c^{12}d^5 + 2609152a^4b^{15}c^{11}d^6 - 8486912a^5b^{14}c^{10}d^7 + 17833984a^6b^{13}c^9d^8 - 25280512a^7b^{12}c^8d^9 + 24190976a^8b^{11}c^7d^{10} - 14516224a^9b^{10}c^6d^{11} + 3362816a^{10}b^9c^5d^{12} + 2809856a^{11}b^8c^4d^{13} - 3469312a^{12}b^7c^3d^{14} + 1835008a^{13}b^6c^2d^{15})) / (64 * (a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5d + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^2c^2d^5))) * i - \\ & (x * (3185a^4b^7d^{13} + 81b^{11}c^4d^9 - 756a^2b^{10}c^3d^{10} - 4788a^3b^8c^2d^{12} + 2790a^2b^9c^2d^{11})) * i) / (64 * (a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5d + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^2c^2d^5)) * \\ & (-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d) / (65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7) \\ & ^{(1/4)} - \left( \frac{(-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)}{(65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7)} \right)^{(1/4)} * \\ & \left( \frac{(28a^4b^6d^{11} + (81b^{10}c^4d^7)/16 - (675a^2b^9c^3d^8)/16 - (2145a^3b^7c^2d^{10})/16 + (1971a^2b^8c^2d^9)/16)}{(a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - 3a^6b^2c^2d^2) + (-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)} \right) / (65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7) \\ & ^{(3/4)} * \left( \frac{(-81b^7c^4 + 2401a^4b^3d^4 - 4116a^3b^4c^2d^3 + 2646a^2b^5c^2d^2 - 756a^2b^6c^3d)}{(65536a^{15}d^8 + 65536a^7b^8c^8 - 524288a^8b^7c^7d + 1835008a^9b^6c^6d^2 - 3670016a^{10}b^5c^5d^3 + 4587520a^{11}b^4c^4d^4 - 3670016a^{12}b^3c^3d^5 + 1835008a^{13}b^2c^2d^6 - 524288a^{14}b^2c^2d^7)} \right)^{(1/4)} * \\ & (3072a^4b^{14}c^{11}d^4 - 4096a^{14}b^4c^2d^{14} - 28672a^5b^{13}c^{10}d^5 + 114688a^6b^{12}c^9d^6 - 253952a^7b^{11}c^8d^7 + 329728a^8b^{10}c^7d^8 - 229376a^9b^9c^6d^9 + 28672a^{10}b^8c^5d^{10} + 90112a^{11}b^7c^4d^{11} - 78848a^{12}b^6c^3d^{12} + 28672a^{13}b^5c^2d^{13})) / (a^7d^3 - a^4b^3c^3 + 3a^5b^2c^2d - 3a^6b^2c^2d^2) + \\ & (x * (65536a^{15}b^4d^{17} - 524288a^{14}b^5c^2d^{16} + 36864a^2b^{17}c^{13}d^4 - 466944a^3b^{16}c^{12}d^5 + 2609152a^4b^{15}c^{11}d^6 - 8486912a^5b^{14}c^{10}d^7 + 17833984a^6b^{13}c^9d^8 - 25280512a^7b^{12}c^8d^9 + 24190976a^8b^{11}c^7d^{10} - 14516224a^9b^{10}c^6d^{11} + 3362816a^{10}b^9c^5d^{12} + 2809856a^{11}b^8c^4d^{13} - 3469312a^{12}b^7c^3d^{14} + 1835008a^{13}b^6c^2d^{15})) / (64 * (a^{10}d^6 + a^4b^6c^6 - 6a^5b^5c^5d + 15a^6b^4c^4d^2 - 20a^7b^3c^3d^3 + 15a^8b^2c^2d^4 - 6a^9b^2c^2d^5))) * i \end{aligned}$$



$$\begin{aligned}
& 2*c^2*d^6 - 524288*a^{14}*b*c*d^7))^{(1/4)}*(3072*a^4*b^{14}*c^{11}*d^4 - 4096*a^{14} \\
& *b^4*c*d^{14} - 28672*a^5*b^{13}*c^{10}*d^5 + 114688*a^6*b^{12}*c^9*d^6 - 253952*a^ \\
& 7*b^{11}*c^8*d^7 + 329728*a^8*b^{10}*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^ \\
& ^{10}*b^8*c^5*d^{10} + 90112*a^{11}*b^7*c^4*d^{11} - 78848*a^{12}*b^6*c^3*d^{12} + 2867 \\
& 2*a^{13}*b^5*c^2*d^{13}))/((a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c* \\
& d^2) + (x*(65536*a^{15}*b^4*d^{17} - 524288*a^{14}*b^5*c*d^{16} + 36864*a^2*b^{17}*c^ \\
& ^{13}*d^4 - 466944*a^3*b^{16}*c^{12}*d^5 + 2609152*a^4*b^{15}*c^{11}*d^6 - 8486912*a^5 \\
& *b^{14}*c^{10}*d^7 + 17833984*a^6*b^{13}*c^9*d^8 - 25280512*a^7*b^{12}*c^8*d^9 + 24 \\
& 190976*a^8*b^{11}*c^7*d^{10} - 14516224*a^9*b^{10}*c^6*d^{11} + 3362816*a^{10}*b^9*c^ \\
& ^5*d^{12} + 2809856*a^{11}*b^8*c^4*d^{13} - 3469312*a^{12}*b^7*c^3*d^{14} + 1835008*a^ \\
& ^{13}*b^6*c^2*d^{15}))/((64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^ \\
& ^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5)))) + ( \\
& x*(3185*a^4*b^7*d^{13} + 81*b^{11}*c^4*d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8 \\
& *c*d^{12} + 2790*a^2*b^9*c^2*d^{11}))/((64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^ \\
& ^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9 \\
& *b*c*d^5)))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^3*b^4*c*d^3 + 2646*a^ \\
& ^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 65536*a^7*b^8*c^8 - 5242 \\
& 88*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^{10}*b^5*c^5*d^3 + 458 \\
& 7520*a^{11}*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 1835008*a^{13}*b^2*c^2*d^6 \\
& - 524288*a^{14}*b*c*d^7))^{(1/4)}))*(-(81*b^7*c^4 + 2401*a^4*b^3*d^4 - 4116*a^ \\
& ^3*b^4*c*d^3 + 2646*a^2*b^5*c^2*d^2 - 756*a*b^6*c^3*d)/(65536*a^{15}*d^8 + 655 \\
& 36*a^7*b^8*c^8 - 524288*a^8*b^7*c^7*d + 1835008*a^9*b^6*c^6*d^2 - 3670016*a^ \\
& ^{10}*b^5*c^5*d^3 + 4587520*a^{11}*b^4*c^4*d^4 - 3670016*a^{12}*b^3*c^3*d^5 + 183 \\
& 5008*a^{13}*b^2*c^2*d^6 - 524288*a^{14}*b*c*d^7))^{(1/4)}*2i - \operatorname{atan}((( -d^7/(256*b \\
& ^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 143 \\
& 36*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^ \\
& ^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d))^{(1/4)}*(( -d^7/(256*b^8*c^{11} + 256*a^8*c^ \\
& ^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 \\
& + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 20 \\
& 48*a*b^7*c^{10}*d))^{(1/4)}*(( -d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b \\
& *c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7 \\
& *d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d))^{( \\
& 3/4)}*(((-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^ \\
& ^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b \\
& ^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d))^{(1/4)}*(3072*a^4*b^1 \\
& 4*c^{11}*d^4 - 4096*a^{14}*b^4*c*d^{14} - 28672*a^5*b^{13}*c^{10}*d^5 + 114688*a^6*b^ \\
& ^{12}*c^9*d^6 - 253952*a^7*b^{11}*c^8*d^7 + 329728*a^8*b^{10}*c^7*d^8 - 229376*a^9 \\
& *b^9*c^6*d^9 + 28672*a^{10}*b^8*c^5*d^{10} + 90112*a^{11}*b^7*c^4*d^{11} - 78848*a^ \\
& ^{12}*b^6*c^3*d^{12} + 28672*a^{13}*b^5*c^2*d^{13}))/((a^7*d^3 - a^4*b^3*c^3 + 3*a^5* \\
& b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^{15}*b^4*d^{17} - 524288*a^{14}*b^5*c*d^ \\
& ^{16} + 36864*a^2*b^{17}*c^{13}*d^4 - 466944*a^3*b^{16}*c^{12}*d^5 + 2609152*a^4*b^{15}* \\
& c^{11}*d^6 - 8486912*a^5*b^{14}*c^{10}*d^7 + 17833984*a^6*b^{13}*c^9*d^8 - 25280512 \\
& *a^7*b^{12}*c^8*d^9 + 24190976*a^8*b^{11}*c^7*d^{10} - 14516224*a^9*b^{10}*c^6*d^{11} \\
& + 3362816*a^{10}*b^9*c^5*d^{12} + 2809856*a^{11}*b^8*c^4*d^{13} - 3469312*a^{12}*b^7 \\
& *c^3*d^{14} + 1835008*a^{13}*b^6*c^2*d^{15}))/((64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5 \\
& *b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - \\
& 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^{11} + (81*b^{10}*c^4*d^7)/16 - (675*a*b^9*c^ \\
& ^3*d^8)/16 - (2145*a^3*b^7*c*d^{10})/16 + (1971*a^2*b^8*c^2*d^9)/16)/(a^7*d^3 \\
& - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*1i - (x*(3185*a^4*b^7*d^{1 \\
& 3} + 81*b^{11}*c^4*d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8*c*d^{12} + 2790*a^2* \\
& b^9*c^2*d^{11})*1i)/(64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^ \\
& ^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) - ( - \\
& d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9 \\
& *d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^ \\
& ^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d))^{(1/4)}*(( -d^7/(256*b^8*c^{11} + \\
& 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^ \\
& ^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^ \\
& ^5*d^6 - 2048*a*b^7*c^{10}*d))^{(1/4)}*(( -d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - \\
& 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a
\end{aligned}$$

$$\begin{aligned}
& ^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7* \\
& c^{10}*d)^{(3/4)}*((-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 \\
& + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 1 \\
& 4336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)^{(1/4)}*(30 \\
& 72*a^4*b^{14}*c^{11}*d^4 - 4096*a^{14}*b^4*c*d^{14} - 28672*a^5*b^{13}*c^{10}*d^5 + 114 \\
& 688*a^6*b^{12}*c^9*d^6 - 253952*a^7*b^{11}*c^8*d^7 + 329728*a^8*b^{10}*c^7*d^8 - \\
& 229376*a^9*b^9*c^6*d^9 + 28672*a^{10}*b^8*c^5*d^{10} + 90112*a^{11}*b^7*c^4*d^{11} \\
& - 78848*a^{12}*b^6*c^3*d^{12} + 28672*a^{13}*b^5*c^2*d^{13}))/((a^7*d^3 - a^4*b^3*c^ \\
& 3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^{15}*b^4*d^{17} - 524288*a^1 \\
& 4*b^5*c*d^{16} + 36864*a^2*b^{17}*c^{13}*d^4 - 466944*a^3*b^{16}*c^{12}*d^5 + 2609152 \\
& *a^4*b^{15}*c^{11}*d^6 - 8486912*a^5*b^{14}*c^{10}*d^7 + 17833984*a^6*b^{13}*c^9*d^8 \\
& - 25280512*a^7*b^{12}*c^8*d^9 + 24190976*a^8*b^{11}*c^7*d^{10} - 14516224*a^9*b^{1 \\
& 0*c^6*d^{11} + 3362816*a^{10}*b^9*c^5*d^{12} + 2809856*a^{11}*b^8*c^4*d^{13} - 346931 \\
& 2*a^{12}*b^7*c^3*d^{14} + 1835008*a^{13}*b^6*c^2*d^{15}))/((64*(a^{10}*d^6 + a^4*b^6*c \\
& ^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2 \\
& *c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^{11} + (81*b^{10}*c^4*d^7)/16 - (67 \\
& 5*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^{10})/16 + (1971*a^2*b^8*c^2*d^9)/16) \\
& /((a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2))*i + (x*(3185*a \\
& ^4*b^7*d^{13} + 81*b^{11}*c^4*d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8*c*d^{12} + \\
& 2790*a^2*b^9*c^2*d^{11})*i)/((64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + \\
& 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d \\
& ^5))))/((-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a \\
& ^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5* \\
& b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)^{(1/4)}*((-d^7/(256* \\
& b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14 \\
& 336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168* \\
& a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)^{(1/4)}*((-d^7/(256*b^8*c^{11} + 256*a^8* \\
& c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 \\
& + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2 \\
& 048*a*b^7*c^{10}*d)^{(3/4)}*((-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7 \\
& *b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c \\
& ^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)) \\
& ^{(1/4)}*(3072*a^4*b^{14}*c^{11}*d^4 - 4096*a^{14}*b^4*c*d^{14} - 28672*a^5*b^{13}*c^{10} \\
& *d^5 + 114688*a^6*b^{12}*c^9*d^6 - 253952*a^7*b^{11}*c^8*d^7 + 329728*a^8*b^{10}* \\
& c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^{10}*b^8*c^5*d^{10} + 90112*a^{11}*b^7 \\
& *c^4*d^{11} - 78848*a^{12}*b^6*c^3*d^{12} + 28672*a^{13}*b^5*c^2*d^{13}))/((a^7*d^3 - \\
& a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^{15}*b^4*d^{17} - \\
& 524288*a^{14}*b^5*c*d^{16} + 36864*a^2*b^{17}*c^{13}*d^4 - 466944*a^3*b^{16}*c^{12}*d^5 \\
& + 2609152*a^4*b^{15}*c^{11}*d^6 - 8486912*a^5*b^{14}*c^{10}*d^7 + 17833984*a^6*b^{1 \\
& 3*c^9*d^8 - 25280512*a^7*b^{12}*c^8*d^9 + 24190976*a^8*b^{11}*c^7*d^{10} - 145162 \\
& 24*a^9*b^{10}*c^6*d^{11} + 3362816*a^{10}*b^9*c^5*d^{12} + 2809856*a^{11}*b^8*c^4*d^{1 \\
& 3} - 3469312*a^{12}*b^7*c^3*d^{14} + 1835008*a^{13}*b^6*c^2*d^{15}))/((64*(a^{10}*d^6 + \\
& a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + \\
& 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^{11} + (81*b^{10}*c^4*d^7) \\
& )/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^{10})/16 + (1971*a^2*b^8*c^ \\
& 2*d^9)/16)/((a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) - (x* \\
& (3185*a^4*b^7*d^{13} + 81*b^{11}*c^4*d^9 - 756*a*b^{10}*c^3*d^{10} - 4788*a^3*b^8*c \\
& *d^{12} + 2790*a^2*b^9*c^2*d^{11}))/((64*(a^{10}*d^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5 \\
& *d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b \\
& *c*d^5))) + (-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 71 \\
& 68*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336* \\
& a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)^{(1/4)}*((-d^7/( \\
& 256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 \\
& - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7 \\
& 168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^{10}*d)^{(1/4)}*((-d^7/(256*b^8*c^{11} + 256* \\
& a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8 \\
& *d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 \\
& - 2048*a*b^7*c^{10}*d)^{(3/4)}*((-d^7/(256*b^8*c^{11} + 256*a^8*c^3*d^8 - 2048 \\
& *a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b
\end{aligned}$$

$$\begin{aligned}
& ^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10 \\
& *d))^{(1/4)}*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13* \\
& c^10*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^ \\
& ^10*c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11 \\
& *b^7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^ \\
& 3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) + (x*(65536*a^15*b^4*d^1 \\
& 7 - 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12 \\
& *d^5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6 \\
& *b^13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14 \\
& 516224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4 \\
& *d^13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15))/(64*(a^10*d \\
& ^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^ \\
& 3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) + (28*a^4*b^6*d^11 + (81*b^10*c^4 \\
& *d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^2*b^ \\
& 8*c^2*d^9)/16)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2)) + \\
& (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 4788*a^3*b \\
& ^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6*a^5*b^5 \\
& *c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d^4 - 6*a \\
& ^9*b*c*d^5))))*(-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 \\
& + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14 \\
& 336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(1/4)}*2i + \\
& 2*atan((( -d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168* \\
& a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5 \\
& *b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(1/4)}*(( -d^7/(256 \\
& *b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 1 \\
& 4336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168 \\
& *a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(1/4)}*(( -d^7/(256*b^8*c^11 + 256*a^8 \\
& *c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^ \\
& 3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - \\
& 2048*a*b^7*c^10*d))^{(3/4)}*(( -d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^ \\
& 7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4* \\
& c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d) \\
& )^{(1/4)}*(3072*a^4*b^14*c^11*d^4 - 4096*a^14*b^4*c*d^14 - 28672*a^5*b^13*c^1 \\
& 0*d^5 + 114688*a^6*b^12*c^9*d^6 - 253952*a^7*b^11*c^8*d^7 + 329728*a^8*b^10 \\
& *c^7*d^8 - 229376*a^9*b^9*c^6*d^9 + 28672*a^10*b^8*c^5*d^10 + 90112*a^11*b^ \\
& 7*c^4*d^11 - 78848*a^12*b^6*c^3*d^12 + 28672*a^13*b^5*c^2*d^13))/(a^7*d^3 - \\
& a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c*d^2) - (x*(65536*a^15*b^4*d^17 - \\
& 524288*a^14*b^5*c*d^16 + 36864*a^2*b^17*c^13*d^4 - 466944*a^3*b^16*c^12*d^ \\
& 5 + 2609152*a^4*b^15*c^11*d^6 - 8486912*a^5*b^14*c^10*d^7 + 17833984*a^6*b^ \\
& 13*c^9*d^8 - 25280512*a^7*b^12*c^8*d^9 + 24190976*a^8*b^11*c^7*d^10 - 14516 \\
& 224*a^9*b^10*c^6*d^11 + 3362816*a^10*b^9*c^5*d^12 + 2809856*a^11*b^8*c^4*d^ \\
& 13 - 3469312*a^12*b^7*c^3*d^14 + 1835008*a^13*b^6*c^2*d^15)*1i)/(64*(a^10*d \\
& ^6 + a^4*b^6*c^6 - 6*a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^ \\
& 3 + 15*a^8*b^2*c^2*d^4 - 6*a^9*b*c*d^5))) *1i + ((28*a^4*b^6*d^11 + (81*b^10 \\
& *c^4*d^7)/16 - (675*a*b^9*c^3*d^8)/16 - (2145*a^3*b^7*c*d^10)/16 + (1971*a^ \\
& 2*b^8*c^2*d^9)/16)*1i)/(a^7*d^3 - a^4*b^3*c^3 + 3*a^5*b^2*c^2*d - 3*a^6*b*c \\
& *d^2)) + (x*(3185*a^4*b^7*d^13 + 81*b^11*c^4*d^9 - 756*a*b^10*c^3*d^10 - 47 \\
& 88*a^3*b^8*c*d^12 + 2790*a^2*b^9*c^2*d^11))/(64*(a^10*d^6 + a^4*b^6*c^6 - 6 \\
& *a^5*b^5*c^5*d + 15*a^6*b^4*c^4*d^2 - 20*a^7*b^3*c^3*d^3 + 15*a^8*b^2*c^2*d \\
& ^4 - 6*a^9*b*c*d^5))) - (-d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b* \\
& c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7* \\
& d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(1 \\
& /4)}*(( -d^7/(256*b^8*c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2* \\
& b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3 \\
& *c^6*d^5 + 7168*a^6*b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(1/4)}*(( -d^7/(256*b^8 \\
& *c^11 + 256*a^8*c^3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336 \\
& *a^3*b^5*c^8*d^3 + 17920*a^4*b^4*c^7*d^4 - 14336*a^5*b^3*c^6*d^5 + 7168*a^6 \\
& *b^2*c^5*d^6 - 2048*a*b^7*c^10*d))^{(3/4)}*(( -d^7/(256*b^8*c^11 + 256*a^8*c^ \\
& 3*d^8 - 2048*a^7*b*c^4*d^7 + 7168*a^2*b^6*c^9*d^2 - 14336*a^3*b^5*c^8*d^3 +
\end{aligned}$$



$$\begin{aligned}
& c^9 d^6 - 253952 a^7 b^{11} c^8 d^7 + 329728 a^8 b^{10} c^7 d^8 - 229376 a^9 b^9 c^6 d^9 + 28672 a^{10} b^8 c^5 d^{10} + 90112 a^{11} b^7 c^4 d^{11} - 78848 a^{12} b^6 c^3 d^{12} + 28672 a^{13} b^5 c^2 d^{13} \\
& \left. \right) / (a^7 d^3 - a^4 b^3 c^3 + 3 a^5 b^2 c^2 d - 3 a^6 b c d^2) + (x (65536 a^{15} b^4 d^{17} - 524288 a^{14} b^5 c d^{16} + 36864 a^2 b^{17} c^{13} d^4 - 466944 a^3 b^{16} c^{12} d^5 + 2609152 a^4 b^{15} c^{11} d^6 - 8486912 a^5 b^{14} c^{10} d^7 + 17833984 a^6 b^{13} c^9 d^8 - 25280512 a^7 b^{12} c^8 d^9 + 24190976 a^8 b^{11} c^7 d^{10} - 14516224 a^9 b^{10} c^6 d^{11} + 3362816 a^{10} b^9 c^5 d^{12} + 2809856 a^{11} b^8 c^4 d^{13} - 3469312 a^{12} b^7 c^3 d^{14} + 1835008 a^{13} b^6 c^2 d^{15}) * i) / (64 (a^{10} d^6 + a^4 b^6 c^6 - 6 a^5 b^5 c^5 d + 15 a^6 b^4 c^4 d^2 - 20 a^7 b^3 c^3 d^3 + 15 a^8 b^2 c^2 d^4 - 6 a^9 b c d^5)) * i + ((28 a^4 b^6 d^{11} + (81 b^{10} c^4 d^7) / 16 - (675 a b^9 c^3 d^8) / 16 - (2145 a^3 b^7 c d^{10}) / 16 + (1971 a^2 b^8 c^2 d^9) / 16) * i) / (a^7 d^3 - a^4 b^3 c^3 + 3 a^5 b^2 c^2 d - 3 a^6 b c d^2) * i - (x (3185 a^4 b^7 d^{13} + 81 b^{11} c^4 d^9 - 756 a b^{10} c^3 d^{10} - 4788 a^3 b^8 c d^{12} + 2790 a^2 b^9 c^2 d^{11}) * i) / (64 (a^{10} d^6 + a^4 b^6 c^6 - 6 a^5 b^5 c^5 d + 15 a^6 b^4 c^4 d^2 - 20 a^7 b^3 c^3 d^3 + 15 a^8 b^2 c^2 d^4 - 6 a^9 b c d^5)))) * (-d^7 / (256 b^8 c^{11} + 256 a^8 c^3 d^8 - 2048 a^7 b c^4 d^7 + 7168 a^2 b^6 c^9 d^2 - 14336 a^3 b^5 c^8 d^3 + 17920 a^4 b^4 c^7 d^4 - 14336 a^5 b^3 c^6 d^5 + 7168 a^6 b^2 c^5 d^6 - 2048 a b^7 c^{10} d))^{1/4} - (b x) / (4 a (a + b x^4) * (a d - b c))
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c),x)

[Out] Timed out

$$3.172 \quad \int \frac{1}{(a+bx^4)^2(c+dx^4)^2} dx$$

**Optimal.** Leaf size=596

$$\frac{b^{7/4}(3bc - 11ad) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc - ad)^3} + \frac{b^{7/4}(3bc - 11ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc - ad)^3} - \frac{b^{7/4}(3bc - 11ad)}{16\sqrt{2} a^{7/4}(bc - ad)^3}$$

[Out]  $\frac{1}{4} d (a d + b c) x / a / c / (-a d + b c)^2 / (d x^4 + c) + \frac{1}{4} b x / a / (-a d + b c) / (b x^4 + a) / (d x^4 + c) + \frac{1}{16} b^{7/4} (-11 a d + 3 b c) \arctan(-1 + b^{1/4} x^2^{1/2} / a^{1/4}) / a^{7/4} / (-a d + b c)^3 x^{2^{1/2}} + \frac{1}{16} b^{7/4} (-11 a d + 3 b c) \arctan(1 + b^{1/4} x^2^{1/2} / a^{1/4}) / a^{7/4} / (-a d + b c)^3 x^{2^{1/2}} + \frac{1}{16} d^{7/4} (-3 a d + 11 b c) \arctan(-1 + d^{1/4} x^2^{1/2} / c^{1/4}) / c^{7/4} / (-a d + b c)^3 x^{2^{1/2}} + \frac{1}{16} d^{7/4} (-3 a d + 11 b c) \arctan(1 + d^{1/4} x^2^{1/2} / c^{1/4}) / c^{7/4} / (-a d + b c)^3 x^{2^{1/2}} - \frac{1}{32} b^{7/4} (-11 a d + 3 b c) \ln(-a^{1/4} b^{1/4} x^2^{1/2} + a^{1/2} + x^2 b^{1/2}) / a^{7/4} / (-a d + b c)^3 x^{2^{1/2}} + \frac{1}{32} b^{7/4} (-11 a d + 3 b c) \ln(a^{1/4} b^{1/4} x^2^{1/2} + a^{1/2} + x^2 b^{1/2}) / a^{7/4} / (-a d + b c)^3 x^{2^{1/2}} - \frac{1}{32} d^{7/4} (-3 a d + 11 b c) \ln(-c^{1/4} d^{1/4} x^2^{1/2} + c^{1/2} + x^2 d^{1/2}) / c^{7/4} / (-a d + b c)^3 x^{2^{1/2}} + \frac{1}{32} d^{7/4} (-3 a d + 11 b c) \ln(c^{1/4} d^{1/4} x^2^{1/2} + c^{1/2} + x^2 d^{1/2}) / c^{7/4} / (-a d + b c)^3 x^{2^{1/2}}$

**Rubi [A]** time = 0.74, antiderivative size = 596, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 9, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$ , Rules used = {414, 527, 522, 211, 1165, 628, 1162, 617, 204}

$$\frac{b^{7/4}(3bc - 11ad) \log\left(-\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc - ad)^3} + \frac{b^{7/4}(3bc - 11ad) \log\left(\sqrt{2} \sqrt[4]{a} \sqrt[4]{b} x + \sqrt{a} + \sqrt{b} x^2\right)}{16\sqrt{2} a^{7/4}(bc - ad)^3} - \frac{b^{7/4}(3bc - 11ad)}{16\sqrt{2} a^{7/4}(bc - ad)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^2\*(c + d\*x^4)^2), x]

[Out]  $\frac{d(b c + a d) x}{4 a^2 c (b c - a d)^2 (c + d x^4)} + \frac{b x}{4 a (b c - a d) (a + b x^4) (c + d x^4)} - \frac{b^{7/4} (3 b c - 11 a d) \operatorname{ArcTan}\left[\frac{1 - (\sqrt{2} b^{1/4} x) / a^{1/4}}{8 \sqrt{2} a^{7/4} (b c - a d)^3}\right]}{8 \sqrt{2} a^{7/4} (b c - a d)^3} + \frac{b^{7/4} (3 b c - 11 a d) \operatorname{ArcTan}\left[\frac{1 + (\sqrt{2} b^{1/4} x) / a^{1/4}}{8 \sqrt{2} a^{7/4} (b c - a d)^3}\right]}{8 \sqrt{2} a^{7/4} (b c - a d)^3} - \frac{d^{7/4} (11 b c - 3 a d) \operatorname{ArcTan}\left[\frac{1 - (\sqrt{2} d^{1/4} x) / c^{1/4}}{8 \sqrt{2} c^{7/4} (b c - a d)^3}\right]}{8 \sqrt{2} c^{7/4} (b c - a d)^3} + \frac{d^{7/4} (11 b c - 3 a d) \operatorname{ArcTan}\left[\frac{1 + (\sqrt{2} d^{1/4} x) / c^{1/4}}{8 \sqrt{2} c^{7/4} (b c - a d)^3}\right]}{8 \sqrt{2} c^{7/4} (b c - a d)^3} - \frac{b^{7/4} (3 b c - 11 a d) \operatorname{Log}\left[\frac{\sqrt{a} - \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2}{16 \sqrt{2} a^{7/4} (b c - a d)^3}\right]}{16 \sqrt{2} a^{7/4} (b c - a d)^3} + \frac{b^{7/4} (3 b c - 11 a d) \operatorname{Log}\left[\frac{\sqrt{a} + \sqrt{2} a^{1/4} b^{1/4} x + \sqrt{b} x^2}{16 \sqrt{2} a^{7/4} (b c - a d)^3}\right]}{16 \sqrt{2} a^{7/4} (b c - a d)^3} - \frac{d^{7/4} (11 b c - 3 a d) \operatorname{Log}\left[\frac{\sqrt{c} - \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2}{16 \sqrt{2} c^{7/4} (b c - a d)^3}\right]}{16 \sqrt{2} c^{7/4} (b c - a d)^3} + \frac{d^{7/4} (11 b c - 3 a d) \operatorname{Log}\left[\frac{\sqrt{c} + \sqrt{2} c^{1/4} d^{1/4} x + \sqrt{d} x^2}{16 \sqrt{2} c^{7/4} (b c - a d)^3}\right]}{16 \sqrt{2} c^{7/4} (b c - a d)^3}$

**Rule 204**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

**Rule 211**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2\*r), Int[(r - s\*x^2)/(a + b\*x^4),



$x], x] + \text{Dist}[1/(2*r), \text{Int}[(r + s*x^2)/(a + b*x^4), x], x]] /; \text{FreeQ}[\{a, b\}, x] \&\& (\text{GtQ}[a/b, 0] \mid\mid (\text{PosQ}[a/b] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, a]] \&\& \text{AtomQ}[\text{SplitProduct}[\text{SumBaseQ}, b]]))$

#### Rule 414

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_)*((c_ + (d_)*(x_)^(n_))^(q_)), x\_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*\text{Simp}[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[p, -1] \&\& !( !\text{IntegerQ}[p] \&\& \text{IntegerQ}[q] \&\& \text{LtQ}[q, -1]) \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 522

$\text{Int}[(e_ + (f_)*(x_)^(n_))/((a_ + (b_)*(x_)^(n_))*((c_ + (d_)*(x_)^(n_))), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^n), x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[1/(c + d*x^n), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x]$

#### Rule 527

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_)*((c_ + (d_)*(x_)^(n_))^(q_))*((e_ + (f_)*(x_)^(n_))), x\_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p + 1)), \text{Int}[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

#### Rule 617

$\text{Int}[(a_ + (b_)*(x_) + (c_)*(x_)^2)^(-1), x\_Symbol] \rightarrow \text{With}[\{q = 1 - 4*S\text{implify}[(a*c)/b^2]\}, \text{Dist}[-2/b, \text{Subst}[\text{Int}[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; \text{RationalQ}[q] \&\& (\text{EqQ}[q^2, 1] \mid\mid !\text{RationalQ}[b^2 - 4*a*c])] /; \text{FreeQ}[\{a, b, c\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

#### Rule 628

$\text{Int}[(d_ + (e_)*(x_))/((a_ + (b_)*(x_) + (c_)*(x_)^2), x\_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

#### Rule 1162

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[(2*d)/e, 2]\}, \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e + q*x + x^2, x], x], x] + \text{Dist}[e/(2*c), \text{Int}[1/\text{Simp}[d/e - q*x + x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{PosQ}[d*e]$

#### Rule 1165

$\text{Int}[(d_ + (e_)*(x_)^2)/((a_ + (c_)*(x_)^4), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(2*d)/e, 2]\}, \text{Dist}[e/(2*c*q), \text{Int}[(q - 2*x)/\text{Simp}[d/e + q*x - x^2, x], x], x] + \text{Dist}[e/(2*c*q), \text{Int}[(q + 2*x)/\text{Simp}[d/e - q*x - x^2, x], x], x]] /; \text{FreeQ}[\{a, c, d, e\}, x] \&\& \text{EqQ}[c*d^2 - a*e^2, 0] \&\& \text{NegQ}[d*e]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^2(c+dx^4)^2} dx &= \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{\int \frac{-3bc+4ad-7bdx^4}{(a+bx^4)(c+dx^4)^2} dx}{4a(bc-ad)} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{\int \frac{-4(3b^2c^2-8abcd+3a^2d^2)}{(a+bx^4)(c+dx^4)} dx}{16ac(bc-ad)} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^2(3bc-11ad)) \int \frac{1}{a+bx^4} dx}{4a(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^2(3bc-11ad)) \int \frac{\sqrt{a}}{a+bx^4} dx}{8a^{3/2}(bc-ad)} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} + \frac{(b^{3/2}(3bc-11ad)) \int \frac{1}{a+bx^4} dx}{16a^{3/2}(bc-ad)} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{b^{7/4}(3bc-11ad) \log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} \\
&= \frac{d(bc+ad)x}{4ac(bc-ad)^2(c+dx^4)} + \frac{bx}{4a(bc-ad)(a+bx^4)(c+dx^4)} - \frac{b^{7/4}(3bc-11ad) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}
\end{aligned}$$

**Mathematica [A]** time = 6.19, size = 629, normalized size = 1.06

$$\frac{b^{7/4}(11ad-3bc) \log\left(\frac{-\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc) \log\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}\right)}{16\sqrt{2}a^{7/4}(bc-ad)^3} - \frac{b^{7/4}(11ad-3bc) \tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x + \sqrt{a} + \sqrt{b}x^2}\right)}{8\sqrt{2}a^{7/4}(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)^2\*(c + d\*x^4)^2), x]

[Out]  $(b^2x)/(4a*(-(b*c) + a*d)^2*(a + b*x^4)) + (d^2x)/(4c*(b*c - a*d)^2*(c + d*x^4)) - (b^{7/4}*(-3*b*c + 11*a*d)*\text{ArcTan}[(-(\text{Sqrt}[2]*a^{1/4}) + 2*b^{1/4}*x)/(\text{Sqrt}[2]*a^{1/4})])/(8*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (b^{7/4}*(-3*b*c + 11*a*d)*\text{ArcTan}[(\text{Sqrt}[2]*a^{1/4} + 2*b^{1/4}*x)/(\text{Sqrt}[2]*a^{1/4})])/(8*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[(-(\text{Sqrt}[2]*c^{1/4}) + 2*d^{1/4}*x)/(\text{Sqrt}[2]*c^{1/4})])/(8*\text{Sqrt}[2]*c^{7/4}*(-(b*c) + a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{ArcTan}[(\text{Sqrt}[2]*c^{1/4} + 2*d^{1/4}*x)/(\text{Sqrt}[2]*c^{1/4})])/(8*\text{Sqrt}[2]*c^{7/4}*(-(b*c) + a*d)^3) + (b^{7/4}*(-3*b*c + 11*a*d)*\text{Log}[\text{Sqrt}[a] - \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) - (b^{7/4}*(-3*b*c + 11*a*d)*\text{Log}[\text{Sqrt}[a] + \text{Sqrt}[2]*a^{1/4}*b^{1/4}*x + \text{Sqrt}[b]*x^2])/(16*\text{Sqrt}[2]*a^{7/4}*(b*c - a*d)^3) + (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] - \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(-(b*c) + a*d)^3) - (d^{7/4}*(11*b*c - 3*a*d)*\text{Log}[\text{Sqrt}[c] + \text{Sqrt}[2]*c^{1/4}*d^{1/4}*x + \text{Sqrt}[d]*x^2])/(16*\text{Sqrt}[2]*c^{7/4}*(-(b*c) + a*d)^3)$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

giac [B] time = 0.22, size = 967, normalized size = 1.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c)^2,x, algorithm="giac")

[Out] 
$$\frac{1}{8} \cdot (3 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot b^2 \cdot c - 11 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot a \cdot b \cdot d) \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2x + \sqrt{2} \cdot (a/b)^{\frac{1}{4}}) / (a/b)^{\frac{1}{4}}\right) / (\sqrt{2} \cdot a^2 \cdot b^3 \cdot c^3 - 3 \sqrt{2} \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \sqrt{2} \cdot a^4 \cdot b \cdot c \cdot d^2 - \sqrt{2} \cdot a^5 \cdot d^3) + \frac{1}{8} \cdot (3 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot b^2 \cdot c - 11 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot a \cdot b \cdot d) \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2x - \sqrt{2} \cdot (a/b)^{\frac{1}{4}}) / (a/b)^{\frac{1}{4}}\right) / (\sqrt{2} \cdot a^2 \cdot b^3 \cdot c^3 - 3 \sqrt{2} \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \sqrt{2} \cdot a^4 \cdot b \cdot c \cdot d^2 - \sqrt{2} \cdot a^5 \cdot d^3) + \frac{1}{8} \cdot (11 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot b \cdot c \cdot d - 3 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot a \cdot d^2) \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2x + \sqrt{2} \cdot (c/d)^{\frac{1}{4}}) / (c/d)^{\frac{1}{4}}\right) / (\sqrt{2} \cdot b^3 \cdot c^5 - 3 \sqrt{2} \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \sqrt{2} \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - \sqrt{2} \cdot a^3 \cdot c^2 \cdot d^3) + \frac{1}{8} \cdot (11 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot b \cdot c \cdot d - 3 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot a \cdot d^2) \cdot \arctan\left(\frac{1}{2} \sqrt{2} \cdot (2x - \sqrt{2} \cdot (c/d)^{\frac{1}{4}}) / (c/d)^{\frac{1}{4}}\right) / (\sqrt{2} \cdot b^3 \cdot c^5 - 3 \sqrt{2} \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \sqrt{2} \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - \sqrt{2} \cdot a^3 \cdot c^2 \cdot d^3) + \frac{1}{16} \cdot (3 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot b^2 \cdot c - 11 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot a \cdot b \cdot d) \cdot \log(x^2 + \sqrt{2} \cdot x \cdot (a/b)^{\frac{1}{4}} + \sqrt{a/b}) / (\sqrt{2} \cdot a^2 \cdot b^3 \cdot c^3 - 3 \sqrt{2} \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \sqrt{2} \cdot a^4 \cdot b \cdot c \cdot d^2 - \sqrt{2} \cdot a^5 \cdot d^3) - \frac{1}{16} \cdot (3 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot b^2 \cdot c - 11 \cdot (a \cdot b^3)^{\frac{1}{4}} \cdot a \cdot b \cdot d) \cdot \log(x^2 - \sqrt{2} \cdot x \cdot (a/b)^{\frac{1}{4}} + \sqrt{a/b}) / (\sqrt{2} \cdot a^2 \cdot b^3 \cdot c^3 - 3 \sqrt{2} \cdot a^3 \cdot b^2 \cdot c^2 \cdot d + 3 \sqrt{2} \cdot a^4 \cdot b \cdot c \cdot d^2 - \sqrt{2} \cdot a^5 \cdot d^3) + \frac{1}{16} \cdot (11 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot b \cdot c \cdot d - 3 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot a \cdot d^2) \cdot \log(x^2 + \sqrt{2} \cdot x \cdot (c/d)^{\frac{1}{4}} + \sqrt{c/d}) / (\sqrt{2} \cdot b^3 \cdot c^5 - 3 \sqrt{2} \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \sqrt{2} \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - \sqrt{2} \cdot a^3 \cdot c^2 \cdot d^3) - \frac{1}{16} \cdot (11 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot b \cdot c \cdot d - 3 \cdot (c \cdot d^3)^{\frac{1}{4}} \cdot a \cdot d^2) \cdot \log(x^2 - \sqrt{2} \cdot x \cdot (c/d)^{\frac{1}{4}} + \sqrt{c/d}) / (\sqrt{2} \cdot b^3 \cdot c^5 - 3 \sqrt{2} \cdot a \cdot b^2 \cdot c^4 \cdot d + 3 \sqrt{2} \cdot a^2 \cdot b \cdot c^3 \cdot d^2 - \sqrt{2} \cdot a^3 \cdot c^2 \cdot d^3) + \frac{1}{4} \cdot (b^2 \cdot c \cdot d \cdot x^5 + a \cdot b \cdot d^2 \cdot x^5 + b^2 \cdot c^2 \cdot x^4 + a^2 \cdot d^2 \cdot x^4) / ((b \cdot d \cdot x^8 + b \cdot c \cdot x^4 + a \cdot d \cdot x^4 + a \cdot c) \cdot (a \cdot b^2 \cdot c^3 - 2 \cdot a^2 \cdot b \cdot c^2 \cdot d + a^3 \cdot c \cdot d^2))$$

maple [A] time = 0.07, size = 784, normalized size = 1.32

$$\frac{a d^3 x}{4(ad-bc)^3(dx^4+c)c} - \frac{b^3 c x}{4(ad-bc)^3(bx^4+a)a} + \frac{b^2 d x}{4(ad-bc)^3(bx^4+a)} - \frac{b d^2 x}{4(ad-bc)^3(dx^4+c)} + \frac{3 \left(\frac{c}{d}\right)^{\frac{1}{4}} \sqrt{2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^2/(d\*x^4+c)^2,x)

[Out] 
$$\frac{1}{4} \cdot d^3 / (a \cdot d - b \cdot c)^3 / c \cdot x / (d \cdot x^4 + c) \cdot a^{-1/4} \cdot d^2 / (a \cdot d - b \cdot c)^3 \cdot x / (d \cdot x^4 + c) \cdot b + 3/16 \cdot d^3 / (a \cdot d - b \cdot c)^3 / c^2 \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan(2^{\frac{1}{2}} / (c/d)^{\frac{1}{4}} \cdot x - 1) \cdot a - 1/16 \cdot d^2 / (a \cdot d - b \cdot c)^3 / c \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan(2^{\frac{1}{2}} / (c/d)^{\frac{1}{4}} \cdot x - 1) \cdot b + 3/32 \cdot d^3 / (a \cdot d - b \cdot c)^3 / c^2 \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln((x^2 + (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (c/d)^{\frac{1}{2}}) / (x^2 - (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (c/d)^{\frac{1}{2}})) \cdot a - 11/32 \cdot d^2 / (a \cdot d - b \cdot c)^3 / c \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln((x^2 + (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (c/d)^{\frac{1}{2}}) / (x^2 - (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (c/d)^{\frac{1}{2}})) \cdot b + 3/16 \cdot d^3 / (a \cdot d - b \cdot c)^3 / c^2 \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan(2^{\frac{1}{2}} / (c/d)^{\frac{1}{4}} \cdot x + 1) \cdot a - 11/16 \cdot d^2 / (a \cdot d - b \cdot c)^3 / c \cdot (c/d)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \arctan(2^{\frac{1}{2}} / (c/d)^{\frac{1}{4}} \cdot x + 1) \cdot b + 1/4 \cdot b^2 / (a \cdot d - b \cdot c)^3 \cdot x / (b \cdot x^4 + a) \cdot d - 1/4 \cdot b^3 / (a \cdot d - b \cdot c)^3 / a \cdot x / (b \cdot x^4 + a) \cdot c + 11/32 \cdot b^2 / (a \cdot d - b \cdot c)^3 / a \cdot (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln((x^2 + (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}}) / (x^2 - (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}})) \cdot d - 3/32 \cdot b^3 / (a \cdot d - b \cdot c)^3 / a^2 \cdot (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln((x^2 + (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}}) / (x^2 - (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}})) \cdot d - 3/32 \cdot b^3 / (a \cdot d - b \cdot c)^3 / a^2 \cdot (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot \ln((x^2 + (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}}) / (x^2 - (a/b)^{\frac{1}{4}} \cdot 2^{\frac{1}{2}} \cdot x + (a/b)^{\frac{1}{2}})) \cdot d$$

$$\begin{aligned} & /b)^{(1/4)} * 2^{(1/2)} * x + (a/b)^{(1/2)}) / (x^2 - (a/b)^{(1/4)} * 2^{(1/2)} * x + (a/b)^{(1/2)}) * c \\ & + 11/16 * b^2 / (a * d - b * c)^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} * x - 1) \\ & * d - 3/16 * b^3 / (a * d - b * c)^3 * a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} \\ & * x - 1) * c + 11/16 * b^2 / (a * d - b * c)^3 * a * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} \\ & * x + 1) * d - 3/16 * b^3 / (a * d - b * c)^3 * a^2 * (a/b)^{(1/4)} * 2^{(1/2)} * \arctan(2^{(1/2)} / (a/b)^{(1/4)} \\ & * x + 1) * c \end{aligned}$$

**maxima [A]** time = 1.29, size = 670, normalized size = 1.12

$$\frac{\left( \frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}\right) + \frac{2\sqrt{2}(3bc-11ad)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{b}x-\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}\right)}{2\sqrt{a}\sqrt{b}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}} + \frac{\sqrt{2}(3bc-11ad)\log\left(\sqrt{b}x^2+\sqrt{2}a^{\frac{1}{4}}b^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}b^{\frac{1}{4}}}}{32\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^2/(d\*x^4+c)^2,x, algorithm="maxima")

[Out]  $\frac{1}{32} * (2 * \sqrt{2}) * (3 * b * c - 11 * a * d) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{b} * x + \sqrt{2}) * a^{1/4} * b^{1/4}) / \sqrt{a} * \sqrt{b} + 2 * \sqrt{2} * (3 * b * c - 11 * a * d) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{b} * x - \sqrt{2}) * a^{1/4} * b^{1/4}) / \sqrt{a} * \sqrt{b} + \sqrt{2} * (3 * b * c - 11 * a * d) * \log(\sqrt{b} * x^2 + \sqrt{2} * a^{1/4} * b^{1/4} * x + \sqrt{a}) / (a^{3/4} * b^{1/4}) - \sqrt{2} * (3 * b * c - 11 * a * d) * \log(\sqrt{b} * x^2 - \sqrt{2} * a^{1/4} * b^{1/4} * x + \sqrt{a}) / (a^{3/4} * b^{1/4}) * b^2 / (a * b^3 * c^3 - 3 * a^2 * b^2 * c^2 * d + 3 * a^3 * b * c * d^2 - a^4 * d^3) + 1/4 * ((b^2 * c * d + a * b * d^2) * x^5 + (b^2 * c^2 + a^2 * d^2) * x) / ((a * b^3 * c^3 * d - 2 * a^2 * b^2 * c^2 * d^2 + a^3 * b * c * d^3) * x^8 + a^2 * b^2 * c^4 - 2 * a^3 * b * c^3 * d + a^4 * c^2 * d^2 + (a * b^3 * c^4 - a^2 * b^2 * c^3 * d - a^3 * b * c^2 * d^2 + a^4 * c * d^3) * x^4) + 1/32 * (2 * \sqrt{2}) * (11 * b * c * d^2 - 3 * a * d^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{d} * x + \sqrt{2}) * c^{1/4} * d^{1/4}) / \sqrt{c} * \sqrt{d} + 2 * \sqrt{2} * (11 * b * c * d^2 - 3 * a * d^3) * \arctan(1/2 * \sqrt{2} * (2 * \sqrt{2} * \sqrt{d} * x - \sqrt{2}) * c^{1/4} * d^{1/4}) / \sqrt{c} * \sqrt{d} + \sqrt{2} * (11 * b * c * d^2 - 3 * a * d^3) * \log(\sqrt{d} * x^2 + \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{3/4} * d^{1/4}) - \sqrt{2} * (11 * b * c * d^2 - 3 * a * d^3) * \log(\sqrt{d} * x^2 - \sqrt{2} * c^{1/4} * d^{1/4} * x + \sqrt{c}) / (c^{3/4} * d^{1/4}) / (b^3 * c^4 - 3 * a * b^2 * c^3 * d + 3 * a^2 * b * c^2 * d^2 - a^3 * c * d^3)$

**mupad [B]** time = 5.62, size = 37266, normalized size = 62.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^2\*(c + d\*x^4)^2),x)

[Out]  $\frac{(x * (a^2 * d^2 + b^2 * c^2)) / (4 * a * c * (a^2 * d^2 + b^2 * c^2 - 2 * a * b * c * d)) + (b * d * x^5 * (a * d + b * c)) / (4 * a * c * (a^2 * d^2 + b^2 * c^2 - 2 * a * b * c * d)) / (a * c + x^4 * (a * d + b * c) + b * d * x^8) - \operatorname{atan}\left(\frac{(-81 * a^4 * d^{11} + 14641 * b^4 * c^4 * d^7 - 15972 * a * b^3 * c^3 * d^8 + 6534 * a^2 * b^2 * c^2 * d^9 - 1188 * a^3 * b * c * d^{10})}{(65536 * b^{12} * c^{19} + 65536 * a^{12} * c^7 * d^{12} - 786432 * a^{11} * b * c^8 * d^{11} + 4325376 * a^2 * b^{10} * c^{17} * d^2 - 14417920 * a^3 * b^9 * c^{16} * d^3 + 32440320 * a^4 * b^8 * c^{15} * d^4 - 51904512 * a^5 * b^7 * c^{14} * d^5 + 60555264 * a^6 * b^6 * c^{13} * d^6 - 51904512 * a^7 * b^5 * c^{12} * d^7 + 32440320 * a^8 * b^4 * c^{11} * d^8 - 14417920 * a^9 * b^3 * c^{10} * d^9 + 4325376 * a^{10} * b^2 * c^9 * d^{10} - 786432 * a * b^{11} * c^{18} * d)}{(-81 * a^4 * d^{11} + 14641 * b^4 * c^4 * d^7 - 15972 * a * b^3 * c^3 * d^8 + 6534 * a^2 * b^2 * c^2 * d^9 - 1188 * a^3 * b * c * d^{10})}{(65536 * b^{12} * c^{19} + 65536 * a^{12} * c^7 * d^{12} - 786432 * a^{11} * b * c^8 * d^{11} + 4325376 * a^2 * b^{10} * c^{17} * d^2 - 14417920 * a^3 * b^9 * c^{16} * d^3 + 32440320 * a^4 * b^8 * c^{15} * d^4 - 51904512 * a^5 * b^7 * c^{14} * d^5 +$

$$\begin{aligned}
& 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^11b^1c^{18}d^{11} \\
& \left. \left( \frac{1}{4} \left( \frac{891a^8b^7d^{15}}{64} + \frac{891b^{15}c^8d^7}{64} - \frac{3105a^11c^{18}d^8}{16} - \frac{3105a^7b^8c^4d^{14}}{16} + \frac{31509a^2b^{13}c^6d^9}{32} - \frac{33069a^3b^{12}c^5d^{10}}{16} \right. \right. \right. \\
& \left. \left. \left. + \frac{60307a^4b^{11}c^4d^{11}}{32} - \frac{33069a^5b^{10}c^3d^{12}}{16} + \frac{31509a^6b^9c^2d^{13}}{32} \right) \right) / (a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^1c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6) \\
& + \left( - (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^1c^1d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^1c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}b^1c^{18}d^{11}) \right) \left. \left( \frac{3}{4} \right) \right. \\
& \left. \left( (x(589824a^2b^{23}c^{21}d^4 - 11403264a^3b^{22}c^{20}d^5 + 98762752a^4b^{21}c^{19}d^6 - 510394368a^5b^{20}c^{18}d^7 + 1766916096a^6b^{19}c^{17}d^8 - 4344840192a^7b^{18}c^{16}d^9 + 7796490240a^8b^{17}c^{15}d^{10} - 10168369152a^9b^{16}c^{14}d^{11} + 9007726592a^{10}b^{15}c^{13}d^{12} - 3635478528a^{11}b^{14}c^{12}d^{13} - 3635478528a^{12}b^{13}c^{11}d^{14} + 9007726592a^{13}b^{12}c^{10}d^{15} - 10168369152a^{14}b^{11}c^9d^{16} + 7796490240a^{15}b^{10}c^8d^{17} - 4344840192a^{16}b^9c^7d^{18} + 1766916096a^{17}b^8c^6d^{19} - 510394368a^{18}b^7c^5d^{20} + 98762752a^{19}b^6c^4d^{21} - 11403264a^{20}b^5c^3d^{22} + 589824a^{21}b^4c^2d^{23})) / (1024(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^1c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10})) + \left( - (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^1c^1d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^1c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}b^1c^{18}d^{11}) \right) \left. \left( \frac{1}{4} \right) \right. \\
& \left. \left( 3072a^4b^{19}c^{19}d^4 - 45056a^5b^{18}c^{18}d^5 + 292864a^6b^{17}c^{17}d^6 - 1115136a^7b^{16}c^{16}d^7 + 2745344a^8b^{15}c^{15}d^8 - 4483072a^9b^{14}c^{14}d^9 + 4595712a^{10}b^{13}c^{13}d^{10} - 1993728a^{11}b^{12}c^{12}d^{11} - 1993728a^{12}b^{11}c^{11}d^{12} + 4595712a^{13}b^{10}c^{10}d^{13} - 4483072a^{14}b^9c^9d^{14} + 2745344a^{15}b^8c^8d^{15} - 1115136a^{16}b^7c^7d^{16} + 292864a^{17}b^6c^6d^{17} - 45056a^{18}b^5c^5d^{18} + 3072a^{19}b^4c^4d^{19} \right) / (a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^1c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6) \right) \left. \right) * i + (x(9801a^8b^9d^{17} + 9801b^{17}c^8d^9 - 149094a^16c^7d^{10} - 149094a^7b^{10}c^4d^{16} + 1001520a^2b^{15}c^6d^{11} - 3484602a^3b^{14}c^5d^{12} + 5769038a^4b^{13}c^4d^{13} - 3484602a^5b^{12}c^3d^{14} + 1001520a^6b^{11}c^2d^{15})) * i) / (1024(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^1c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10})) - \left( - (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^1c^1d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^1c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}b^1c^{18}d^{11}) \right) \left. \left( \frac{1}{4} \right) \right. \\
& \left. \left( - (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^1c^1d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^1c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}b^1c^{18}d^{11}) \right) \left. \right)
\end{aligned}$$

$$\begin{aligned}
& \left( 9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{1/4} * \left( \left( \frac{891*a^8*b^7*d^{15}}{64} + \frac{891*b^{15}*c^8*d^7}{64} - \frac{3105*a*b^{14}*c^7*d^8}{16} - \frac{3105*a^7*b^8*c*d^{14}}{16} + \frac{31509*a^2*b^{13}*c^6*d^9}{32} - \frac{33069*a^3*b^{12}*c^5*d^{10}}{16} + \frac{60307*a^4*b^{11}*c^4*d^{11}}{32} - \frac{33069*a^5*b^{10}*c^3*d^{12}}{16} + \frac{31509*a^6*b^9*c^2*d^{13}}{32} \right) / \left( a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6 \right) - \left( -81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10} \right) / \left( 65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{3/4} * \left( x*(589824*a^2*b^{23}*c^{21}*d^4 - 11403264*a^3*b^{22}*c^{20}*d^5 + 98762752*a^4*b^{21}*c^{19}*d^6 - 510394368*a^5*b^{20}*c^{18}*d^7 + 1766916096*a^6*b^{19}*c^{17}*d^8 - 4344840192*a^7*b^{18}*c^{16}*d^9 + 7796490240*a^8*b^{17}*c^{15}*d^{10} - 10168369152*a^9*b^{16}*c^{14}*d^{11} + 9007726592*a^{10}*b^{15}*c^{13}*d^{12} - 3635478528*a^{11}*b^{14}*c^{12}*d^{13} - 3635478528*a^{12}*b^{13}*c^{11}*d^{14} + 9007726592*a^{13}*b^{12}*c^{10}*d^{15} - 10168369152*a^{14}*b^{11}*c^9*d^{16} + 7796490240*a^{15}*b^{10}*c^8*d^{17} - 4344840192*a^{16}*b^9*c^7*d^{18} + 1766916096*a^{17}*b^8*c^6*d^{19} - 510394368*a^{18}*b^7*c^5*d^{20} + 98762752*a^{19}*b^6*c^4*d^{21} - 11403264*a^{20}*b^5*c^3*d^{22} + 589824*a^{21}*b^4*c^2*d^{23}) \right) / \left( 1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10}) \right) - \left( -81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10} \right) / \left( 65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{1/4} * \left( 3072*a^4*b^{19}*c^{19}*d^4 - 45056*a^5*b^{18}*c^{18}*d^5 + 292864*a^6*b^{17}*c^{17}*d^6 - 1115136*a^7*b^{16}*c^{16}*d^7 + 2745344*a^8*b^{15}*c^{15}*d^8 - 4483072*a^9*b^{14}*c^{14}*d^9 + 4595712*a^{10}*b^{13}*c^{13}*d^{10} - 1993728*a^{11}*b^{12}*c^{12}*d^{11} - 1993728*a^{12}*b^{11}*c^{11}*d^{12} + 4595712*a^{13}*b^{10}*c^{10}*d^{13} - 4483072*a^{14}*b^9*c^9*d^{14} + 2745344*a^{15}*b^8*c^8*d^{15} - 1115136*a^{16}*b^7*c^7*d^{16} + 292864*a^{17}*b^6*c^6*d^{17} - 45056*a^{18}*b^5*c^5*d^{18} + 3072*a^{19}*b^4*c^4*d^{19} \right) / \left( a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6 \right) * i - \left( x*(9801*a^8*b^9*d^{17} + 9801*b^{17}*c^8*d^9 - 149094*a*b^{16}*c^7*d^{10} - 149094*a^7*b^{10}*c^d^{16} + 1001520*a^2*b^{15}*c^6*d^{11} - 3484602*a^3*b^{14}*c^5*d^{12} + 5769038*a^4*b^{13}*c^4*d^{13} - 3484602*a^5*b^{12}*c^3*d^{14} + 1001520*a^6*b^{11}*c^2*d^{15}) * i \right) / \left( 1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10}) \right) / \left( -81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10} \right) / \left( 65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{1/4} * \left( -81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10} \right) / \left( 65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{1/4} * \left( \frac{891*a^8*b^7*d^{15}}{64} + \frac{891*b^{15}*c^8*d^7}{64} - \frac{3105*a*b^{14}*c^7*d^8}{16} - \frac{3105*a^7*b^8*c*d^{14}}{16} + \frac{31509*a^2*b^{13}*c^6*d^9}{32} - \frac{33069*a^3*b^{12}*c^5*d^{10}}{16} + \frac{60307*a^4*b^{11}*c^4*d^{11}}{32} - \frac{33069*a^5*b^{10}*c^3*d^{12}}{16} + \frac{31509*a^6*b^9*c^2*d^{13}}{32} \right) / \left( a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6 \right) - \left( -81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10} \right) / \left( 65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d \right)^{1/4} * \left( \frac{891*a^8*b^7*d^{15}}{64} + \frac{891*b^{15}*c^8*d^7}{64} - \frac{3105*a*b^{14}*c^7*d^8}{16} - \frac{3105*a^7*b^8*c*d^{14}}{16} + \frac{31509*a^2*b^{13}*c^6*d^9}{32} - \frac{33069*a^3*b^{12}*c^5*d^{10}}{16} + \frac{60307*a^4*b^{11}*c^4*d^{11}}{32} - \frac{33069*a^5*b^{10}*c^3*d^{12}}{16} + \frac{31509*a^6*b^9*c^2*d^{13}}{32} \right) / \left( a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6 \right)
\end{aligned}$$

$$\begin{aligned}
& c*d^{14}/16 + (31509*a^2*b^{13}*c^6*d^9)/32 - (33069*a^3*b^{12}*c^5*d^{10})/16 + (60307*a^4*b^{11}*c^4*d^{11})/32 - (33069*a^5*b^{10}*c^3*d^{12})/16 + (31509*a^6*b^9*c^2*d^{13})/32 \\
& / (a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 \\
& + 28*a^{10}*b^2*c^6*d^6) + (- (81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10}) / (65536*b^{12}*c^{19} \\
& + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 \\
& + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d))^{(3/4)} * ((x*(589824*a^2*b^{23}*c^{21}*d^4 - 11403264*a^3*b^{22}*c^{20}*d^5 \\
& + 98762752*a^4*b^{21}*c^{19}*d^6 - 510394368*a^5*b^{20}*c^{18}*d^7 + 1766916096*a^6*b^{19}*c^{17}*d^8 - 4344840192*a^7*b^{18}*c^{16}*d^9 + 7796490240*a^8*b^{17}*c^{15}*d^{10} - 10168369152*a^9*b^{16}*c^{14}*d^{11} \\
& + 9007726592*a^{10}*b^{15}*c^{13}*d^{12} - 3635478528*a^{11}*b^{14}*c^{12}*d^{13} - 3635478528*a^{12}*b^{13}*c^{11}*d^{14} + 9007726592*a^{13}*b^{12}*c^{10}*d^{15} - 10168369152*a^{14}*b^{11}*c^9*d^{16} \\
& + 7796490240*a^{15}*b^{10}*c^8*d^{17} - 4344840192*a^{16}*b^9*c^7*d^{18} + 1766916096*a^{17}*b^8*c^6*d^{19} - 510394368*a^{18}*b^7*c^5*d^{20} + 98762752*a^{19}*b^6*c^4*d^{21} - 11403264*a^{20}*b^5*c^3*d^{22} \\
& + 589824*a^{21}*b^4*c^2*d^{23})) / (1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 \\
& + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10})) + ((- (81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10}) / (65536*b^{12}*c^{19} \\
& + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 \\
& + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d))^{(1/4)} * (3072*a^4*b^{19}*c^{19}*d^4 - 45056*a^5*b^{18}*c^{18}*d^5 + 292864*a^6*b^{17}*c^{17}*d^6 - 1115136*a^7*b^{16}*c^{16}*d^7 \\
& + 2745344*a^8*b^{15}*c^{15}*d^8 - 4483072*a^9*b^{14}*c^{14}*d^9 + 4595712*a^{10}*b^{13}*c^{13}*d^{10} - 1993728*a^{11}*b^{12}*c^{12}*d^{11} - 1993728*a^{12}*b^{11}*c^{11}*d^{12} + 4595712*a^{13}*b^{10}*c^{10}*d^{13} - 4483072*a^{14}*b^9*c^9*d^{14} \\
& + 2745344*a^{15}*b^8*c^8*d^{15} - 1115136*a^{16}*b^7*c^7*d^{16} + 292864*a^{17}*b^6*c^6*d^{17} - 45056*a^{18}*b^5*c^5*d^{18} + 3072*a^{19}*b^4*c^4*d^{19})) / (a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 \\
& + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6)) + (x*(9801*a^8*b^9*d^{17} + 9801*b^{17}*c^8*d^9 - 149094*a*b^{16}*c^7*d^{10} - 149094*a^7*b^{10}*c*d^{16} + 1001520*a^2*b^{15}*c^6*d^{11} - 3484602*a^3*b^{14}*c^5*d^{12} \\
& + 5769038*a^4*b^{13}*c^4*d^{13} - 3484602*a^5*b^{12}*c^3*d^{14} + 1001520*a^6*b^{11}*c^2*d^{15})) / (1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12}*d^4 \\
& - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10})) + (- (81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10}) / (65536*b^{12}*c^{19} \\
& + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 \\
& + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d))^{(1/4)} * ((- (81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10}) / (65536*b^{12}*c^{19} \\
& + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 \\
& + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d))^{(1/4)} * ((891*a^8*b^7*d^{15})/64 + (891*b^{15}*c^8*d^7)/64 - (3105*a*b^{14}*c^7*d^8)/16 - (3105*a^7*b^8*c*d^{14})/16 \\
& + (31509*a^2*b^{13}*c^6*d^9)/32 - (33069*a^3*b^{12}*c^5*d^{10})/16 + (60307*a^4*b^{11}*c^4*d^{11})/32 - (33069*a^5*b^{10}*c^3*d^{12})/16 +
\end{aligned}$$

$$\begin{aligned}
& ((31509a^6b^9c^2d^{13})/32)/(a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^6c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6) - (- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10})/(65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^6c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}c^{18}d))^{(3/4)} * ((x*(589824a^2b^{23}c^{21}d^4 - 11403264a^3b^{22}c^{20}d^5 + 98762752a^4b^{21}c^{19}d^6 - 510394368a^5b^{20}c^{18}d^7 + 1766916096a^6b^{19}c^{17}d^8 - 4344840192a^7b^{18}c^{16}d^9 + 7796490240a^8b^{17}c^{15}d^{10} - 10168369152a^9b^{16}c^{14}d^{11} + 9007726592a^{10}b^{15}c^{13}d^{12} - 3635478528a^{11}b^{14}c^{12}d^{13} - 3635478528a^{12}b^{13}c^{11}d^{14} + 9007726592a^{13}b^{12}c^{10}d^{15} - 10168369152a^{14}b^{11}c^9d^{16} + 7796490240a^{15}b^{10}c^8d^{17} - 4344840192a^{16}b^9c^7d^{18} + 1766916096a^{17}b^8c^6d^{19} - 510394368a^{18}b^7c^5d^{20} + 98762752a^{19}b^6c^4d^{21} - 11403264a^{20}b^5c^3d^{22} + 589824a^{21}b^4c^2d^{23}))/((1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^6c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10}))) - ((- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10})/(65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^6c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}c^{18}d))^{(1/4)} * (3072a^4b^{19}c^{19}d^4 - 45056a^5b^{18}c^{18}d^5 + 292864a^6b^{17}c^{17}d^6 - 1115136a^7b^{16}c^{16}d^7 + 2745344a^8b^{15}c^{15}d^8 - 4483072a^9b^{14}c^{14}d^9 + 4595712a^{10}b^{13}c^{13}d^{10} - 1993728a^{11}b^{12}c^{12}d^{11} - 1993728a^{12}b^{11}c^{11}d^{12} + 4595712a^{13}b^{10}c^{10}d^{13} - 4483072a^{14}b^9c^9d^{14} + 2745344a^{15}b^8c^8d^{15} - 1115136a^{16}b^7c^7d^{16} + 292864a^{17}b^6c^6d^{17} - 45056a^{18}b^5c^5d^{18} + 3072a^{19}b^4c^4d^{19}))/((a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^6c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6))) - (x*(9801a^8b^9d^{17} + 9801b^{17}c^8d^9 - 149094a^8b^{16}c^7d^{10} - 149094a^7b^{10}c^6d^{16} + 1001520a^2b^{15}c^6d^{11} - 3484602a^3b^{14}c^5d^{12} + 5769038a^4b^{13}c^4d^{13} - 3484602a^5b^{12}c^3d^{14} + 1001520a^6b^{11}c^2d^{15}))/((1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^6c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10})))))) * (- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10})/(65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^6c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}c^{18}d))^{(1/4)} * 2i + 2*atan((((- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10})/(65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^6c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}c^{18}d))^{(1/4)} * (- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10})/(65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^6c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432a^{11}c^{18}d))^{(1/4)} * 2i + 2*atan))))))
\end{aligned}$$



$$\begin{aligned}
& 4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432 \\
& (ab^{11}c^{18}d)^{(1/4)} * (((891a^8b^7d^{15})/64 + (891b^{15}c^8d^7)/64 - ( \\
& 3105a^7b^8c^7d^8)/16 - (3105a^7b^8c^7d^8)/16 + (31509a^2b^{13}c^6d^9)/32 - (33069a^3b^{12}c^5d^{10})/16 + (60307a^4b^{11}c^4d^{11})/32 - (3306 \\
& 9a^5b^{10}c^3d^{12})/16 + (31509a^6b^9c^2d^{13})/32) * i) / (a^4b^8c^{12} + \\
& a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^6c^5d^7 + 28a^6b^6c^{10}d^2 - \\
& 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6) + (- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} \\
& - 786432a^{11}b^8c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 1 \\
& 4417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432ab^{11}c^{18}d \\
& ))^{(3/4)} * ((x*(589824a^2b^{23}c^{21}d^4 - 11403264a^3b^{22}c^{20}d^5 + 98762 \\
& 752a^4b^{21}c^{19}d^6 - 510394368a^5b^{20}c^{18}d^7 + 1766916096a^6b^{19}c^{17}d^8 - 4344840192a^7b^{18}c^{16}d^9 + 7796490240a^8b^{17}c^{15}d^{10} - 10 \\
& 168369152a^9b^{16}c^{14}d^{11} + 9007726592a^{10}b^{15}c^{13}d^{12} - 3635478528a^{11}b^{14}c^{12}d^{13} - 3635478528a^{12}b^{13}c^{11}d^{14} + 9007726592a^{13}b^{12} \\
& c^{10}d^{15} - 10168369152a^{14}b^{11}c^9d^{16} + 7796490240a^{15}b^{10}c^8d^{17} \\
& - 4344840192a^{16}b^9c^7d^{18} + 1766916096a^{17}b^8c^6d^{19} - 510394368a^{18}b^7c^5d^{20} + 98762752a^{19}b^6c^4d^{21} - 11403264a^{20}b^5c^3d^{22} \\
& + 589824a^{21}b^4c^2d^{23}) * i) / (1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^6c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 6 \\
& 6a^{14}b^2c^6d^{10})) + ((- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10}) / (65536b^{12}c^{19} + 65536 \\
& a^{12}c^7d^{12} - 786432a^{11}b^8c^8d^{11} + 4325376a^2b^{10}c^{17}d^2 - 14417 \\
& 920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432 \\
& ab^{11}c^{18}d)^{(1/4)} * (3072a^4b^{19}c^{19}d^4 - 45056a^5b^{18}c^{18}d^5 + \\
& 292864a^6b^{17}c^{17}d^6 - 1115136a^7b^{16}c^{16}d^7 + 2745344a^8b^{15}c^{15}d^8 - 4483072a^9b^{14}c^{14}d^9 + 4595712a^{10}b^{13}c^{13}d^{10} - 1993728a^{11}b^{12}c^{12}d^{11} - 1993728a^{12}b^{11}c^{11}d^{12} + 4595712a^{13}b^{10}c^{10}d^{13} - 4483072a^{14}b^9c^9d^{14} + 2745344a^{15}b^8c^8d^{15} - 1115136a^{16}b^7c^7d^{16} + 292864a^{17}b^6c^6d^{17} - 45056a^{18}b^5c^5d^{18} + 3072a^{19}b^4c^4d^{19})) / (a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^6c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6) * i) - (x*(9801a^8b^9d^{17} + 9 \\
& 801b^{17}c^8d^9 - 149094a^7b^{10}c^7d^{10} - 149094a^7b^{10}c^7d^{10} + 100152 \\
& 0a^2b^{15}c^6d^{11} - 3484602a^3b^{14}c^5d^{12} + 5769038a^4b^{13}c^4d^{13} \\
& - 3484602a^5b^{12}c^3d^{14} + 1001520a^6b^{11}c^2d^{15})) / (1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^6c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10})) - ((- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^8c^8d^{11} + 4325376 \\
& a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432ab^{11}c^{18}d)^{(1/4)} * ((- (81a^4d^{11} + 14641b^4c^4d^7 - 15972a^3b^3c^3d^8 + 6534a^2b^2c^2d^9 - 1188a^3b^3c^3d^{10}) / (65536b^{12}c^{19} + 65536a^{12}c^7d^{12} - 786432a^{11}b^8c^8d^{11} + 4325376 \\
& a^2b^{10}c^{17}d^2 - 14417920a^3b^9c^{16}d^3 + 32440320a^4b^8c^{15}d^4 - 51904512a^5b^7c^{14}d^5 + 60555264a^6b^6c^{13}d^6 - 51904512a^7b^5c^{12}d^7 + 32440320a^8b^4c^{11}d^8 - 14417920a^9b^3c^{10}d^9 + 4325376a^{10}b^2c^9d^{10} - 786432ab^{11}c^{18}d)^{(1/4)} * (((891a^8b^7d^{15})/64 +
\end{aligned}$$

$$\begin{aligned}
& (891*b^{15}*c^8*d^7)/64 - (3105*a*b^{14}*c^7*d^8)/16 - (3105*a^7*b^8*c*d^{14})/16 \\
& + (31509*a^2*b^{13}*c^6*d^9)/32 - (33069*a^3*b^{12}*c^5*d^{10})/16 + (60307*a^4* \\
& b^{11}*c^4*d^{11})/32 - (33069*a^5*b^{10}*c^3*d^{12})/16 + (31509*a^6*b^9*c^2*d^{13}) \\
& /32)*i)/(a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 \\
& + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b \\
& ^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6) - ((-81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15 \\
& 972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10})/(65536*b^{12}*c \\
& ^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}* \\
& d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5* \\
& b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 3244 \\
& 0320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d) \\
& )^{3/4}*((x*(589824*a^2*b^{23}*c^{21}*d^4 - 11403264* \\
& a^3*b^{22}*c^{20}*d^5 + 98762752*a^4*b^{21}*c^{19}*d^6 - 510394368*a^5*b^{20}*c^{18}*d^ \\
& 7 + 1766916096*a^6*b^{19}*c^{17}*d^8 - 4344840192*a^7*b^{18}*c^{16}*d^9 + 779649024 \\
& 0*a^8*b^{17}*c^{15}*d^{10} - 10168369152*a^9*b^{16}*c^{14}*d^{11} + 9007726592*a^{10}*b^{15}*c^{13}*d^{12} - 3635478528*a^{11}*b^{14}*c^{12}*d^{13} - 3635478528*a^{12}*b^{13}*c^{11}*d^{14} + 9007726592*a^{13}*b^{12}*c^{10}*d^{15} - 10168369152*a^{14}*b^{11}*c^9*d^{16} + 7796490240*a^{15}*b^{10}*c^8*d^{17} - 4344840192*a^{16}*b^9*c^7*d^{18} + 1766916096*a^{17}*b^8*c^6*d^{19} - 510394368*a^{18}*b^7*c^5*d^{20} + 98762752*a^{19}*b^6*c^4*d^{21} - 11403264*a^{20}*b^5*c^3*d^{22} + 589824*a^{21}*b^4*c^2*d^{23})*i)/(1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10})) - ((-81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2*d^9 - 1188*a^3*b*c*d^{10})/(65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432*a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 32440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13}*d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9*b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d) \\
& )^{1/4}*(3072*a^4*b^{19}*c^{19}*d^4 - 4 \\
& 5056*a^5*b^{18}*c^{18}*d^5 + 292864*a^6*b^{17}*c^{17}*d^6 - 1115136*a^7*b^{16}*c^{16}*d^ \\
& ^7 + 2745344*a^8*b^{15}*c^{15}*d^8 - 4483072*a^9*b^{14}*c^{14}*d^9 + 4595712*a^{10}*b \\
& ^{13}*c^{13}*d^{10} - 1993728*a^{11}*b^{12}*c^{12}*d^{11} - 1993728*a^{12}*b^{11}*c^{11}*d^{12} + \\
& 4595712*a^{13}*b^{10}*c^{10}*d^{13} - 4483072*a^{14}*b^9*c^9*d^{14} + 2745344*a^{15}*b^8 \\
& *c^8*d^{15} - 1115136*a^{16}*b^7*c^7*d^{16} + 292864*a^{17}*b^6*c^6*d^{17} - 45056*a^ \\
& ^{18}*b^5*c^5*d^{18} + 3072*a^{19}*b^4*c^4*d^{19}))/ (a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8 \\
& *a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^ \\
& ^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28*a^{10}*b^2*c^6*d^6))*i) + \\
& (x*(9801*a^8*b^9*d^{17} + 9801*b^{17}*c^8*d^9 - 149094*a*b^{16}*c^7*d^{10} - 149094 \\
& *a^7*b^{10}*c^d^{16} + 1001520*a^2*b^{15}*c^6*d^{11} - 3484602*a^3*b^{14}*c^5*d^{12} + \\
& 5769038*a^4*b^{13}*c^4*d^{13} - 3484602*a^5*b^{12}*c^3*d^{14} + 1001520*a^6*b^{11}*c^ \\
& ^2*d^{15}))/ (1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d - 12*a^{15} \\
& *b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a^8*b^8*c^{12} \\
& *d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b^5*c^9*d^ \\
& 7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6*d^{10}))))/ \\
& ((-81*a^4*d^{11} + 14641*b^4*c^4*d^7 - 15972*a*b^3*c^3*d^8 + 6534*a^2*b^2*c^2 \\
& *d^9 - 1188*a^3*b*c*d^{10})/(65536*b^{12}*c^{19} + 65536*a^{12}*c^7*d^{12} - 786432* \\
& a^{11}*b*c^8*d^{11} + 4325376*a^2*b^{10}*c^{17}*d^2 - 14417920*a^3*b^9*c^{16}*d^3 + 3 \\
& 2440320*a^4*b^8*c^{15}*d^4 - 51904512*a^5*b^7*c^{14}*d^5 + 60555264*a^6*b^6*c^{13} \\
& *d^6 - 51904512*a^7*b^5*c^{12}*d^7 + 32440320*a^8*b^4*c^{11}*d^8 - 14417920*a^9 \\
& *b^3*c^{10}*d^9 + 4325376*a^{10}*b^2*c^9*d^{10} - 786432*a*b^{11}*c^{18}*d) \\
& )^{1/4}*(( \\
& ((891*a^8*b^7*d^{15})/64 + (891*b^{15}*c^8*d^7)/64 - (3105*a*b^{14}*c^7*d^8)/16 - \\
& (3105*a^7*b^8*c*d^{14})/16 + (31509*a^2*b^{13}*c^6*d^9)/32 - (33069*a^3*b^{12}*c^5*d^{10})/16
\end{aligned}$$









$$\begin{aligned}
& - 14417920a^{10}b^9c^9d^3 + 32440320a^{11}b^8c^8d^4 - 51904512a^{12}b^7c^7d^5 + 60555264a^{13}b^6c^6d^6 - 51904512a^{14}b^5c^5d^7 + 32440320 \\
& a^{15}b^4c^4d^8 - 14417920a^{16}b^3c^3d^9 + 4325376a^{17}b^2c^2d^{10} - 786432a^{18}b^1c^1d^{11})^{(3/4)} * ((x*(589824a^2b^{23}c^{21}d^4 - 11403264a^3b^{22}c^{20}d^5 + 98762752a^4b^{21}c^{19}d^6 - 510394368a^5b^{20}c^{18}d^7 + \\
& 1766916096a^6b^{19}c^{17}d^8 - 4344840192a^7b^{18}c^{16}d^9 + 7796490240a^8b^{17}c^{15}d^{10} - 10168369152a^9b^{16}c^{14}d^{11} + 9007726592a^{10}b^{15}c^{13}d^{12} - 3635478528a^{11}b^{14}c^{12}d^{13} - 3635478528a^{12}b^{13}c^{11}d^{14} + \\
& 9007726592a^{13}b^{12}c^{10}d^{15} - 10168369152a^{14}b^{11}c^9d^{16} + 7796490240a^{15}b^{10}c^8d^{17} - 4344840192a^{16}b^9c^7d^{18} + 1766916096a^{17}b^8c^6d^{19} - 510394368a^{18}b^7c^5d^{20} + 98762752a^{19}b^6c^4d^{21} - 11403 \\
& 264a^{20}b^5c^3d^{22} + 589824a^{21}b^4c^2d^{23}))/ (1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^1c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 9 \\
& 24a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 220a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10})) + ((-(81b^{11}c^4 + 14641a^4b^7d^4 - 15972a^3b^8c^3d^3 + 6534a^2b^9c^2d^2 - 1188a^10c^3d)/ (65536 \\
& a^{19}d^{12} + 65536a^7b^{12}c^{12} - 786432a^8b^{11}c^{11}d + 4325376a^9b^{10}c^{10}d^2 - 14417920a^{10}b^9c^9d^3 + 32440320a^{11}b^8c^8d^4 - 51904512a^{12}b^7c^7d^5 + 60555264a^{13}b^6c^6d^6 - 51904512a^{14}b^5c^5d^7 \\
& + 32440320a^{15}b^4c^4d^8 - 14417920a^{16}b^3c^3d^9 + 4325376a^{17}b^2c^2d^{10} - 786432a^{18}b^1c^1d^{11}))^{(1/4)} * (3072a^4b^{19}c^{19}d^4 - 45056a^5b^{18}c^{18}d^5 + 292864a^6b^{17}c^{17}d^6 - 1115136a^7b^{16}c^{16}d^7 + 27 \\
& 45344a^8b^{15}c^{15}d^8 - 4483072a^9b^{14}c^{14}d^9 + 4595712a^{10}b^{13}c^{13}d^{10} - 1993728a^{11}b^{12}c^{12}d^{11} - 1993728a^{12}b^{11}c^{11}d^{12} + 4595712a^{13}b^{10}c^{10}d^{13} - 4483072a^{14}b^9c^9d^{14} + 2745344a^{15}b^8c^8d^{15} - 1115136a^{16}b^7c^7d^{16} + 292864a^{17}b^6c^6d^{17} - 45056a^{18}b^5c^5d^{18} + 3072a^{19}b^4c^4d^{19}))/ (a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^1c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70 \\
& a^8b^4c^8d^4 - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6)) + ((891a^8b^7d^{15})/64 + (891b^{15}c^8d^7)/64 - (3105a^14c^7d^8)/16 - (3105a^7b^8c^14)/16 + (31509a^2b^{13}c^6d^9)/32 - (33069a^3b^{12}c^5d^{10})/16 \\
& + (60307a^4b^{11}c^4d^{11})/32 - (33069a^5b^{10}c^3d^{12})/16 + (31509a^6b^9c^2d^{13})/32)/ (a^4b^8c^{12} + a^{12}c^4d^8 - 8a^5b^7c^{11}d - 8a^{11}b^1c^5d^7 + 28a^6b^6c^{10}d^2 - 56a^7b^5c^9d^3 + 70a^8b^4c^8d^4 \\
& - 56a^9b^3c^7d^5 + 28a^{10}b^2c^6d^6)) + (x*(9801a^8b^9d^{17} + 9801b^{17}c^8d^9 - 149094a^16c^7d^{10} - 149094a^7b^{10}c^16 + 1001520a^2b^{15}c^6d^{11} - 3484602a^3b^{14}c^5d^{12} + 5769038a^4b^{13}c^4d^{13} - \\
& 3484602a^5b^{12}c^3d^{14} + 1001520a^6b^{11}c^2d^{15}))/ (1024*(a^4b^{12}c^{16} + a^{16}c^4d^{12} - 12a^5b^{11}c^{15}d - 12a^{15}b^1c^5d^{11} + 66a^6b^{10}c^{14}d^2 - 220a^7b^9c^{13}d^3 + 495a^8b^8c^{12}d^4 - 792a^9b^7c^{11}d^5 + 924a^{10}b^6c^{10}d^6 - 792a^{11}b^5c^9d^7 + 495a^{12}b^4c^8d^8 - 2 \\
& 20a^{13}b^3c^7d^9 + 66a^{14}b^2c^6d^{10}))) - ((-(81b^{11}c^4 + 14641a^4b^7d^4 - 15972a^3b^8c^3d^3 + 6534a^2b^9c^2d^2 - 1188a^10c^3d)/ (65536a^{19}d^{12} + 65536a^7b^{12}c^{12} - 786432a^8b^{11}c^{11}d + 4325376a^9b^{10}c^{10}d^2 - 14417920a^{10}b^9c^9d^3 + 32440320a^{11}b^8c^8d^4 - 51 \\
& 904512a^{12}b^7c^7d^5 + 60555264a^{13}b^6c^6d^6 - 51904512a^{14}b^5c^5d^7 + 32440320a^{15}b^4c^4d^8 - 14417920a^{16}b^3c^3d^9 + 4325376a^{17}b^2c^2d^{10} - 786432a^{18}b^1c^1d^{11}))^{(1/4)} * ((-(81b^{11}c^4 + 14641a^4b^7d^4 - 15972a^3b^8c^3d^3 + 6534a^2b^9c^2d^2 - 1188a^10c^3d)/ (65 \\
& 536a^{19}d^{12} + 65536a^7b^{12}c^{12} - 786432a^8b^{11}c^{11}d + 4325376a^9b^{10}c^{10}d^2 - 14417920a^{10}b^9c^9d^3 + 32440320a^{11}b^8c^8d^4 - 51904512a^{12}b^7c^7d^5 + 60555264a^{13}b^6c^6d^6 - 51904512a^{14}b^5c^5d^7 + 32440320a^{15}b^4c^4d^8 - 14417920a^{16}b^3c^3d^9 + 4325376a^{17} \\
& b^2c^2d^{10} - 786432a^{18}b^1c^1d^{11}))^{(1/4)} * ((-(81b^{11}c^4 + 14641a^4b^7d^4 - 15972a^3b^8c^3d^3 + 6534a^2b^9c^2d^2 - 1188a^10c^3d)/ (65536a^{19}d^{12} + 65536a^7b^{12}c^{12} - 786432a^8b^{11}c^{11}d + 4325376a^9b^{10}c^{10}d^2 - 14417920a^{10}b^9c^9d^3 + 32440320a^{11}b^8c^8d^4 - 519 \\
& 04512a^{12}b^7c^7d^5 + 60555264a^{13}b^6c^6d^6 - 51904512a^{14}b^5c^5d^7
\end{aligned}$$





$$\begin{aligned}
& 1*b^{11}*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a^2*b^9*c^2*d^2 \\
& - 1188*a*b^{10}*c^3*d)/(65536*a^{19}*d^{12} + 65536*a^7*b^{12}*c^{12} - 786432*a^8*b \\
& ^{11}*c^{11}*d + 4325376*a^9*b^{10}*c^{10}*d^2 - 14417920*a^{10}*b^9*c^9*d^3 + 324403 \\
& 20*a^{11}*b^8*c^8*d^4 - 51904512*a^{12}*b^7*c^7*d^5 + 60555264*a^{13}*b^6*c^6*d^6 \\
& - 51904512*a^{14}*b^5*c^5*d^7 + 32440320*a^{15}*b^4*c^4*d^8 - 14417920*a^{16}*b^3 \\
& *c^3*d^9 + 4325376*a^{17}*b^2*c^2*d^{10} - 786432*a^{18}*b*c*d^{11}))^{(3/4)}*((x*(5 \\
& 89824*a^2*b^{23}*c^{21}*d^4 - 11403264*a^3*b^{22}*c^{20}*d^5 + 98762752*a^4*b^{21}*c^{19} \\
& *d^6 - 510394368*a^5*b^{20}*c^{18}*d^7 + 1766916096*a^6*b^{19}*c^{17}*d^8 - 43448 \\
& 40192*a^7*b^{18}*c^{16}*d^9 + 7796490240*a^8*b^{17}*c^{15}*d^{10} - 10168369152*a^9*b \\
& ^{16}*c^{14}*d^{11} + 9007726592*a^{10}*b^{15}*c^{13}*d^{12} - 3635478528*a^{11}*b^{14}*c^{12} \\
& *d^{13} - 3635478528*a^{12}*b^{13}*c^{11}*d^{14} + 9007726592*a^{13}*b^{12}*c^{10}*d^{15} - 10 \\
& 168369152*a^{14}*b^{11}*c^9*d^{16} + 7796490240*a^{15}*b^{10}*c^8*d^{17} - 4344840192*a \\
& ^{16}*b^9*c^7*d^{18} + 1766916096*a^{17}*b^8*c^6*d^{19} - 510394368*a^{18}*b^7*c^5*d^{20} \\
& + 98762752*a^{19}*b^6*c^4*d^{21} - 11403264*a^{20}*b^5*c^3*d^{22} + 589824*a^{21} \\
& *b^4*c^2*d^{23})*i)/(1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5*b^{11}*c^{15}*d \\
& - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13}*d^3 + 495*a \\
& ^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 - 792*a^{11}*b \\
& ^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66*a^{14}*b^2*c^6* \\
& d^{10})) + ((-(81*b^{11}*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d^3 + 6534*a \\
& ^2*b^9*c^2*d^2 - 1188*a*b^{10}*c^3*d)/(65536*a^{19}*d^{12} + 65536*a^7*b^{12}*c^{12} \\
& - 786432*a^8*b^{11}*c^{11}*d + 4325376*a^9*b^{10}*c^{10}*d^2 - 14417920*a^{10}*b^9*c^9 \\
& *d^3 + 32440320*a^{11}*b^8*c^8*d^4 - 51904512*a^{12}*b^7*c^7*d^5 + 60555264*a^{13} \\
& *b^6*c^6*d^6 - 51904512*a^{14}*b^5*c^5*d^7 + 32440320*a^{15}*b^4*c^4*d^8 - 14 \\
& 417920*a^{16}*b^3*c^3*d^9 + 4325376*a^{17}*b^2*c^2*d^{10} - 786432*a^{18}*b*c*d^{11}) \\
& )^{(1/4)}*(3072*a^4*b^{19}*c^{19}*d^4 - 45056*a^5*b^{18}*c^{18}*d^5 + 292864*a^6*b^{17} \\
& *c^{17}*d^6 - 1115136*a^7*b^{16}*c^{16}*d^7 + 2745344*a^8*b^{15}*c^{15}*d^8 - 4483072 \\
& *a^9*b^{14}*c^{14}*d^9 + 4595712*a^{10}*b^{13}*c^{13}*d^{10} - 1993728*a^{11}*b^{12}*c^{12} \\
& *d^{11} - 1993728*a^{12}*b^{11}*c^{11}*d^{12} + 4595712*a^{13}*b^{10}*c^{10}*d^{13} - 4483072*a \\
& ^{14}*b^9*c^9*d^{14} + 2745344*a^{15}*b^8*c^8*d^{15} - 1115136*a^{16}*b^7*c^7*d^{16} + \\
& 292864*a^{17}*b^6*c^6*d^{17} - 45056*a^{18}*b^5*c^5*d^{18} + 3072*a^{19}*b^4*c^4*d^{19} \\
& ))/(a^4*b^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28* \\
& a^6*b^6*c^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7 \\
& *d^5 + 28*a^{10}*b^2*c^6*d^6))*i + (((891*a^8*b^7*d^{15})/64 + (891*b^{15}*c^8*d \\
& ^7)/64 - (3105*a*b^{14}*c^7*d^8)/16 - (3105*a^7*b^8*c*d^{14})/16 + (31509*a^2*b \\
& ^{13}*c^6*d^9)/32 - (33069*a^3*b^{12}*c^5*d^{10})/16 + (60307*a^4*b^{11}*c^4*d^{11})/ \\
& 32 - (33069*a^5*b^{10}*c^3*d^{12})/16 + (31509*a^6*b^9*c^2*d^{13})/32)*i)/(a^4*b \\
& ^8*c^{12} + a^{12}*c^4*d^8 - 8*a^5*b^7*c^{11}*d - 8*a^{11}*b*c^5*d^7 + 28*a^6*b^6*c \\
& ^{10}*d^2 - 56*a^7*b^5*c^9*d^3 + 70*a^8*b^4*c^8*d^4 - 56*a^9*b^3*c^7*d^5 + 28 \\
& *a^{10}*b^2*c^6*d^6)) - (x*(9801*a^8*b^9*d^{17} + 9801*b^{17}*c^8*d^9 - 149094*a* \\
& b^{16}*c^7*d^{10} - 149094*a^7*b^{10}*c*d^{16} + 1001520*a^2*b^{15}*c^6*d^{11} - 348460 \\
& 2*a^3*b^{14}*c^5*d^{12} + 5769038*a^4*b^{13}*c^4*d^{13} - 3484602*a^5*b^{12}*c^3*d^{14} \\
& + 1001520*a^6*b^{11}*c^2*d^{15}))/((1024*(a^4*b^{12}*c^{16} + a^{16}*c^4*d^{12} - 12*a^5 \\
& *b^{11}*c^{15}*d - 12*a^{15}*b*c^5*d^{11} + 66*a^6*b^{10}*c^{14}*d^2 - 220*a^7*b^9*c^{13} \\
& *d^3 + 495*a^8*b^8*c^{12}*d^4 - 792*a^9*b^7*c^{11}*d^5 + 924*a^{10}*b^6*c^{10}*d^6 \\
& - 792*a^{11}*b^5*c^9*d^7 + 495*a^{12}*b^4*c^8*d^8 - 220*a^{13}*b^3*c^7*d^9 + 66* \\
& a^{14}*b^2*c^6*d^{10}))) + ((-(81*b^{11}*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c* \\
& *d^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^{10}*c^3*d)/(65536*a^{19}*d^{12} + 65536*a \\
& ^7*b^{12}*c^{12} - 786432*a^8*b^{11}*c^{11}*d + 4325376*a^9*b^{10}*c^{10}*d^2 - 14417920 \\
& *a^{10}*b^9*c^9*d^3 + 32440320*a^{11}*b^8*c^8*d^4 - 51904512*a^{12}*b^7*c^7*d^5 + \\
& 60555264*a^{13}*b^6*c^6*d^6 - 51904512*a^{14}*b^5*c^5*d^7 + 32440320*a^{15}*b^4 \\
& *c^4*d^8 - 14417920*a^{16}*b^3*c^3*d^9 + 4325376*a^{17}*b^2*c^2*d^{10} - 786432*a^{18} \\
& *b*c*d^{11}))^{(1/4)}*((-(81*b^{11}*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d \\
& ^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^{10}*c^3*d)/(65536*a^{19}*d^{12} + 65536*a^7 \\
& ^{12}*c^{12} - 786432*a^8*b^{11}*c^{11}*d + 4325376*a^9*b^{10}*c^{10}*d^2 - 14417920 \\
& *a^{10}*b^9*c^9*d^3 + 32440320*a^{11}*b^8*c^8*d^4 - 51904512*a^{12}*b^7*c^7*d^5 + \\
& 60555264*a^{13}*b^6*c^6*d^6 - 51904512*a^{14}*b^5*c^5*d^7 + 32440320*a^{15}*b^4 \\
& *c^4*d^8 - 14417920*a^{16}*b^3*c^3*d^9 + 4325376*a^{17}*b^2*c^2*d^{10} - 786432*a^{18} \\
& *b*c*d^{11}))^{(1/4)}*((-(81*b^{11}*c^4 + 14641*a^4*b^7*d^4 - 15972*a^3*b^8*c*d \\
& ^3 + 6534*a^2*b^9*c^2*d^2 - 1188*a*b^{10}*c^3*d)/(65536*a^{19}*d^{12} + 65536*a^7
\end{aligned}$$





$$\begin{aligned} & \left( a^{16} b^3 c^3 d^9 + 4325376 a^{17} b^2 c^2 d^{10} - 786432 a^{18} b c d^{11} \right)^{3/4} \cdot \\ & \left( (x \cdot (589824 a^2 b^{23} c^{21} d^4 - 11403264 a^3 b^{22} c^{20} d^5 + 98762752 a^4 b^{21} c^{19} d^6 - 510394368 a^5 b^{20} c^{18} d^7 + 1766916096 a^6 b^{19} c^{17} d^8 - \right. \\ & 4344840192 a^7 b^{18} c^{16} d^9 + 7796490240 a^8 b^{17} c^{15} d^{10} - 10168369152 a^9 b^{16} c^{14} d^{11} + 9007726592 a^{10} b^{15} c^{13} d^{12} - 3635478528 a^{11} b^{14} \\ & c^{12} d^{13} - 3635478528 a^{12} b^{13} c^{11} d^{14} + 9007726592 a^{13} b^{12} c^{10} d^{15} - 10168369152 a^{14} b^{11} c^9 d^{16} + 7796490240 a^{15} b^{10} c^8 d^{17} - 434484 \\ & 0192 a^{16} b^9 c^7 d^{18} + 1766916096 a^{17} b^8 c^6 d^{19} - 510394368 a^{18} b^7 c^5 d^{20} + 98762752 a^{19} b^6 c^4 d^{21} - 11403264 a^{20} b^5 c^3 d^{22} + 589824 \\ & a^{21} b^4 c^2 d^{23}) \cdot i) / (1024 (a^4 b^{12} c^{16} + a^{16} c^4 d^{12} - 12 a^5 b^{11} c^{15} d - 12 a^{15} b c^5 d^{11} + 66 a^6 b^{10} c^{14} d^2 - 220 a^7 b^9 c^{13} d^3 + \\ & 495 a^8 b^8 c^{12} d^4 - 792 a^9 b^7 c^{11} d^5 + 924 a^{10} b^6 c^{10} d^6 - 792 a^{11} b^5 c^9 d^7 + 495 a^{12} b^4 c^8 d^8 - 220 a^{13} b^3 c^7 d^9 + 66 a^{14} b^2 \\ & c^6 d^{10})) - ((- (81 b^{11} c^4 + 14641 a^4 b^7 d^4 - 15972 a^3 b^8 c d^3 + 6534 a^2 b^9 c^2 d^2 - 1188 a b^{10} c^3 d) / (65536 a^{19} d^{12} + 65536 a^7 b^{12} \\ & c^{12} - 786432 a^8 b^{11} c^{11} d + 4325376 a^9 b^{10} c^{10} d^2 - 14417920 a^{10} b^9 c^9 d^3 + 32440320 a^{11} b^8 c^8 d^4 - 51904512 a^{12} b^7 c^7 d^5 + 60555 \\ & 264 a^{13} b^6 c^6 d^6 - 51904512 a^{14} b^5 c^5 d^7 + 32440320 a^{15} b^4 c^4 d^8 - 14417920 a^{16} b^3 c^3 d^9 + 4325376 a^{17} b^2 c^2 d^{10} - 786432 a^{18} b c \\ & d^{11}))^{1/4} \cdot (3072 a^4 b^{19} c^{19} d^4 - 45056 a^5 b^{18} c^{18} d^5 + 292864 a^6 b^{17} c^{17} d^6 - 1115136 a^7 b^{16} c^{16} d^7 + 2745344 a^8 b^{15} c^{15} d^8 - 4 \\ & 483072 a^9 b^{14} c^{14} d^9 + 4595712 a^{10} b^{13} c^{13} d^{10} - 1993728 a^{11} b^{12} c^{12} d^{11} - 1993728 a^{12} b^{11} c^{11} d^{12} + 4595712 a^{13} b^{10} c^{10} d^{13} - 448 \\ & 3072 a^{14} b^9 c^9 d^{14} + 2745344 a^{15} b^8 c^8 d^{15} - 1115136 a^{16} b^7 c^7 d^{16} + 292864 a^{17} b^6 c^6 d^{17} - 45056 a^{18} b^5 c^5 d^{18} + 3072 a^{19} b^4 c^4 \\ & d^{19})) / (a^4 b^8 c^{12} + a^{12} c^4 d^8 - 8 a^5 b^7 c^{11} d - 8 a^{11} b c^5 d^7 + 28 a^6 b^6 c^{10} d^2 - 56 a^7 b^5 c^9 d^3 + 70 a^8 b^4 c^8 d^4 - 56 a^9 b^3 c^7 d^5 \\ & + 28 a^{10} b^2 c^6 d^6)) \cdot i - (((891 a^8 b^7 d^{15}) / 64 + (891 b^{15} c^8 d^7) / 64 - (3105 a b^{14} c^7 d^8) / 16 - (3105 a^7 b^8 c d^{14}) / 16 + (31509 \\ & a^2 b^{13} c^6 d^9) / 32 - (33069 a^3 b^{12} c^5 d^{10}) / 16 + (60307 a^4 b^{11} c^4 d^{11}) / 32 - (33069 a^5 b^{10} c^3 d^{12}) / 16 + (31509 a^6 b^9 c^2 d^{13}) / 32) \cdot i) / \\ & (a^4 b^8 c^{12} + a^{12} c^4 d^8 - 8 a^5 b^7 c^{11} d - 8 a^{11} b c^5 d^7 + 28 a^6 b^6 c^{10} d^2 - 56 a^7 b^5 c^9 d^3 + 70 a^8 b^4 c^8 d^4 - 56 a^9 b^3 c^7 d^5 \\ & + 28 a^{10} b^2 c^6 d^6)) \cdot i - (x \cdot (9801 a^8 b^9 d^{17} + 9801 b^{17} c^8 d^9 - 149094 a a b^{16} c^7 d^{10} - 149094 a^7 b^{10} c d^{16} + 1001520 a^2 b^{15} c^6 d^{11} \\ & - 3484602 a^3 b^{14} c^5 d^{12} + 5769038 a^4 b^{13} c^4 d^{13} - 3484602 a^5 b^{12} c^3 d^{14} + 1001520 a^6 b^{11} c^2 d^{15}) \cdot i) / (1024 (a^4 b^{12} c^{16} + a^{16} c^4 \\ & d^{12} - 12 a^5 b^{11} c^{15} d - 12 a^{15} b c^5 d^{11} + 66 a^6 b^{10} c^{14} d^2 - 220 a^7 b^9 c^{13} d^3 + 495 a^8 b^8 c^{12} d^4 - 792 a^9 b^7 c^{11} d^5 + 924 a^{10} \\ & b^6 c^{10} d^6 - 792 a^{11} b^5 c^9 d^7 + 495 a^{12} b^4 c^8 d^8 - 220 a^{13} b^3 c^7 d^9 + 66 a^{14} b^2 c^6 d^{10})) \cdot i) \cdot (- (81 b^{11} c^4 + 14641 a^4 b^7 d^4 - 159 \\ & 72 a^3 b^8 c d^3 + 6534 a^2 b^9 c^2 d^2 - 1188 a b^{10} c^3 d) / (65536 a^{19} d^{12} + 65536 a^7 b^{12} c^{12} - 786432 a^8 b^{11} c^{11} d + 4325376 a^9 b^{10} c^{10} d^2 \\ & - 14417920 a^{10} b^9 c^9 d^3 + 32440320 a^{11} b^8 c^8 d^4 - 51904512 a^{12} b^7 c^7 d^5 + 60555264 a^{13} b^6 c^6 d^6 - 51904512 a^{14} b^5 c^5 d^7 + 32440 \\ & 320 a^{15} b^4 c^4 d^8 - 14417920 a^{16} b^3 c^3 d^9 + 4325376 a^{17} b^2 c^2 d^{10} - 786432 a^{18} b c d^{11}))^{1/4} \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*2/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

$$3.173 \quad \int \frac{(a-bx^4)^{5/2}}{c-dx^4} dx$$

**Optimal.** Leaf size=321

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{21d^3\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^3 \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{2\sqrt[4]{b}cd^3\sqrt{a-bx^4}}$$

[Out]  $\frac{1}{7} b x (-b x^4 + a)^{3/2} / d - 1 / 21 b (-13 a d + 7 b c) x (-b x^4 + a)^{1/2} / d^2 + 1 / 21 a^{1/4} b^{3/4} (47 a^2 d^2 - 56 a b c d + 21 b^2 c^2) \text{EllipticF}(b^{1/4} x / a^{1/4}, I) (1 - b x^4 / a)^{1/2} / d^3 / (-b x^4 + a)^{1/2} - 1 / 2 a^{1/4} (-a d + b c)^3 \text{EllipticPi}(b^{1/4} x / a^{1/4}, -a^{1/2} d^{1/2} / b^{1/2} / c^{1/2}, I) (1 - b x^4 / a)^{1/2} / b^{1/4} / c / d^3 / (-b x^4 + a)^{1/2} - 1 / 2 a^{1/4} (-a d + b c)^3 \text{EllipticPi}(b^{1/4} x / a^{1/4}, a^{1/2} d^{1/2} / b^{1/2} / c^{1/2}, I) (1 - b x^4 / a)^{1/2} / b^{1/4} / c / d^3 / (-b x^4 + a)^{1/2}$

**Rubi [A]** time = 0.38, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {416, 528, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{21d^3\sqrt{a-bx^4}} - \frac{bx\sqrt{a-bx^4}(7bc-13ad)}{21d^2} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (b^2c^2 - 13ad^2)}{21d^2}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(5/2)/(c - d\*x^4), x]

[Out]  $-(b(7bc - 13ad)x\sqrt{a - bx^4}) / (21d^2) + (bx(a - bx^4)^{3/2}) / (7d) + (a^{1/4}b^{3/4}(21b^2c^2 - 56abcd + 47a^2d^2)\sqrt{1 - (bx^4)/a}\text{EllipticF}[\text{ArcSin}[(b^{1/4}x)/a^{1/4}], -1]) / (21d^3\sqrt{a - bx^4}) - (a^{1/4}(bc - ad)^3\sqrt{1 - (bx^4)/a}\text{EllipticPi}[-((\sqrt{a}\sqrt{d})/(\sqrt{b}\sqrt{c}))], \text{ArcSin}[(b^{1/4}x)/a^{1/4}], -1]) / (2b^{1/4}cd^3\sqrt{a - bx^4}) - (a^{1/4}(bc - ad)^3\sqrt{1 - (bx^4)/a}\text{EllipticPi}[(\sqrt{a}\sqrt{d})/(\sqrt{b}\sqrt{c}), \text{ArcSin}[(b^{1/4}x)/a^{1/4}], -1]) / (2b^{1/4}cd^3\sqrt{a - bx^4})$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (bx^4)/a]/Sqrt[a + bx^4], Int[1/Sqrt[1 + (bx^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + bx^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + bx^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)], x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

### Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^4)^{5/2}}{c - dx^4} dx &= \frac{bx(a - bx^4)^{3/2}}{7d} - \frac{\int \frac{\sqrt{a - bx^4}(a(bc - 7ad) - b(7bc - 13ad)x^4)}{c - dx^4} dx}{7d} \\
&= -\frac{b(7bc - 13ad)x\sqrt{a - bx^4}}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d} + \frac{\int \frac{a(7b^2c^2 - 16abcd + 21a^2d^2) - b(21b^2c^2 - 56abcd + 47a^2d^2)}{\sqrt{a - bx^4}(c - dx^4)} dx}{21d^2} \\
&= -\frac{b(7bc - 13ad)x\sqrt{a - bx^4}}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d} - \frac{(bc - ad)^3 \int \frac{1}{\sqrt{a - bx^4}(c - dx^4)} dx}{d^3} + \frac{b(21b^2c^2 - 56abcd + 47a^2d^2)\sqrt{a - bx^4}}{21d^3} \\
&= -\frac{b(7bc - 13ad)x\sqrt{a - bx^4}}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d} - \frac{(bc - ad)^3 \int \frac{1}{\left(1 - \frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{2cd^3} - \frac{b(21b^2c^2 - 56abcd + 47a^2d^2)\sqrt{a - bx^4}}{21d^3} \\
&= -\frac{b(7bc - 13ad)x\sqrt{a - bx^4}}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a} b^{3/4} (21b^2c^2 - 56abcd + 47a^2d^2) \sqrt{1 - \frac{\sqrt{a}x^2}{\sqrt{c}}}}{21d^3 \sqrt{a - bx^4}} \\
&= -\frac{b(7bc - 13ad)x\sqrt{a - bx^4}}{21d^2} + \frac{bx(a - bx^4)^{3/2}}{7d} + \frac{\sqrt[4]{a} b^{3/4} (21b^2c^2 - 56abcd + 47a^2d^2) \sqrt{1 - \frac{\sqrt{a}x^2}{\sqrt{c}}}}{21d^3 \sqrt{a - bx^4}}
\end{aligned}$$

**Mathematica [C]** time = 0.82, size = 290, normalized size = 0.90

$$x \frac{\left( \frac{bx^4 \sqrt{1 - \frac{bx^4}{a}} (47a^2d^2 - 56abcd + 21b^2c^2) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{c} + \frac{25a^2c(21a^2d^2 - 16abcd + 7b^2c^2) F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c - dx^4) \left( 2x^4 \left( 2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right) + 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right)}{105d^2 \sqrt{a - bx^4}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a - b\*x^4)^(5/2)/(c - d\*x^4), x]

[Out] (x\*(5\*b\*(-a + b\*x^4)\*(7\*b\*c - 16\*a\*d + 3\*b\*d\*x^4) - (b\*(21\*b^2\*c^2 - 56\*a\*b\*c\*d + 47\*a^2\*d^2)\*x^4\*sqrt[1 - (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])/c + (25\*a^2\*c\*(7\*b^2\*c^2 - 16\*a\*b\*c\*d + 21\*a^2\*d^2)\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/((c - d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(105\*d^2\*sqrt[a - b\*x^4])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(-bx^4 + a)^{5/2}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c),x, algorithm="giac")

[Out] integrate(-(-b\*x^4 + a)^(5/2)/(d\*x^4 - c), x)

**maple** [C] time = 0.31, size = 408, normalized size = 1.27

$$\frac{\sqrt{-bx^4+a} b^2 x^5}{7d} \frac{\left( \frac{-\frac{5ab^2}{7d} + \frac{(3ad-bc)b^2}{d^2}}{3b} - \frac{(3a^2d^2-3abcd+b^2c^2)b}{d^3} \right) \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}x, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4+a}} + \left(-\frac{5}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(5/2)/(-d\*x^4+c),x)

[Out]  $-1/7*b^2/d*x^5*(-b*x^4+a)^{(1/2)}+1/3*(b^2/d^2*(3*a*d-b*c)-5/7*b^2/d*a)/b*x*($   
 $-b*x^4+a)^{(1/2)}-(-b*(3*a^2*d^2-3*a*b*c*d+b^2*c^2)/d^3+1/3*(b^2/d^2*(3*a*d-b$   
 $*c)-5/7*b^2/d*a)/b*a)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(1-1/a^{(1/2)}*b^{(1/2)}*x^2)^{($   
 $1/2)}*(1+1/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticF}((1/a^{(1/2)}*$   
 $b^{(1/2)})^{(1/2)}*x, I)+1/8/d^4*\sum((-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c$   
 $^3)/\_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\_alpha^2*b*x^2+2*a)/(($   
 $a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*\_alpha^3*d/$   
 $c*(1-1/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}*(1+1/a^{(1/2)}*b^{(1/2)}*x^2)^{(1/2)}/(-b*x^4+a$   
 $)^{(1/2)}*\operatorname{EllipticPi}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, a^{(1/2)}/b^{(1/2)}*\_alpha^2/c*d$   
 $, (-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}), \_alpha=\operatorname{RootOf}(\_Z^4*$   
 $d-c))$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(-bx^4+a)^{5/2}}{dx^4-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c),x, algorithm="maxima")

[Out] -integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a-bx^4)^{5/2}}{c-dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(5/2)/(c - d\*x^4),x)

[Out] int((a - b\*x^4)^(5/2)/(c - d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a^2\sqrt{a-bx^4}}{-c+dx^4} dx - \int \frac{b^2x^8\sqrt{a-bx^4}}{-c+dx^4} dx - \int \left( -\frac{2abx^4\sqrt{a-bx^4}}{-c+dx^4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c),x)
```

```
[Out] -Integral(a**2*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(b**2*x**8*sqrt(a - b*x**4)/(-c + d*x**4), x) - Integral(-2*a*b*x**4*sqrt(a - b*x**4)/(-c + d*x**4), x)
```

$$3.174 \quad \int \frac{(a-bx^4)^{3/2}}{c-dx^4} dx$$

**Optimal.** Leaf size=277

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (3bc - 5ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b} cd^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}}}{\sqrt{a - bx^4}}$$

[Out]  $1/3*b*x*(-b*x^4+a)^{(1/2)}/d-1/3*a^{(1/4)}*b^{(3/4)}*(-5*a*d+3*b*c)*\text{EllipticF}(b^{(1/4)}*x/a^{(1/4)}, I)*(1-b*x^4/a)^{(1/2)}/d^2/(-b*x^4+a)^{(1/2)}+1/2*a^{(1/4)}*(-a*d+b*c)^2*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)}, -a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/d^2/(-b*x^4+a)^{(1/2)}+1/2*a^{(1/4)}*(-a*d+b*c)^2*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)}, a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/d^2/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.26, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {416, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (3bc - 5ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)^2 \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b} cd^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}}}{\sqrt{a - bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(3/2)/(c - d\*x^4), x]

[Out]  $(b*x*\text{Sqrt}[a - b*x^4])/(3*d) - (a^{(1/4)}*b^{(3/4)}*(3*b*c - 5*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(3*d^2*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(b*c - a*d)^2*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^2*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(b*c - a*d)^2*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^2*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp

```
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x_)^(n_)], x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

### Rule 1218

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := Dist[Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(a - bx^4)^{3/2}}{c - dx^4} dx &= \frac{bx\sqrt{a - bx^4}}{3d} - \frac{\int \frac{a(bc - 3ad) - b(3bc - 5ad)x^4}{\sqrt{a - bx^4}(c - dx^4)} dx}{3d} \\ &= \frac{bx\sqrt{a - bx^4}}{3d} - \frac{(b(3bc - 5ad)) \int \frac{1}{\sqrt{a - bx^4}} dx}{3d^2} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt{a - bx^4}(c - dx^4)} dx}{d^2} \\ &= \frac{bx\sqrt{a - bx^4}}{3d} + \frac{(bc - ad)^2 \int \frac{1}{\left(1 - \frac{\sqrt{dx^2}}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{2cd^2} + \frac{(bc - ad)^2 \int \frac{1}{\left(1 + \frac{\sqrt{dx^2}}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{2cd^2} - \frac{(b(3bc - 5ad)) \int \frac{1}{\sqrt{a - bx^4}} dx}{3d^2} \\ &= \frac{bx\sqrt{a - bx^4}}{3d} - \frac{\sqrt[4]{a} b^{3/4} (3bc - 5ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\left((bc - ad)^2 \sqrt{1 - \frac{bx^4}{a}}\right)}{2cd^2 \sqrt{a - bx^4}} \\ &= \frac{bx\sqrt{a - bx^4}}{3d} - \frac{\sqrt[4]{a} b^{3/4} (3bc - 5ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{3d^2 \sqrt{a - bx^4}} + \frac{\sqrt[4]{a} (bc - ad)^2 \sqrt{1 - \frac{bx^4}{a}}}{2\sqrt[4]{a} cd^2} \end{aligned}$$

**Mathematica [C]** time = 0.46, size = 341, normalized size = 1.23

$$\frac{x \left( 5 \left( 5ac(3a^2d - abdx^4 + b^2x^4(dx^4 - c)) F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + 2bx^4(a - bx^4)(c - dx^4) \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right) \right)}{(dx^4 - c) \left( 2x^4 \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right) + 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right)} + \frac{bx^4 \sqrt{a - bx^4}}{15d\sqrt{a - bx^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*x^4)^(3/2)/(c - d*x^4), x]
```

```
[Out] -1/15*(x*((b*(-3*b*c + 5*a*d))*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1,
  9/4, (b*x^4)/a, (d*x^4)/c])/c + (5*(5*a*c*(3*a^2*d - a*b*d*x^4 + b^2*x^4*(
-c + d*x^4))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(a
- b*x^4)*(c - d*x^4)*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c
] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((-c + d*x^4)*(
5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*Appel
lF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4
, (b*x^4)/a, (d*x^4)/c]))))/d*Sqrt[a - b*x^4])
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(-bx^4 + a)^{\frac{3}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

```
[Out] integrate(-(-b*x^4 + a)^(3/2)/(d*x^4 - c), x)
```

**maple** [C] time = 0.29, size = 311, normalized size = 1.12

$$\frac{\sqrt{-bx^4 + a} \, bx}{3d} - \frac{\left(\frac{ab}{3d} - \frac{(2ad-bc)b}{d^2}\right) \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} x, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4 + a}} + \frac{(-a^2d^2 + 2abcd - b^2c^2)}{\dots} \left( \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] 1/3*b*x*(-b*x^4+a)^(1/2)/d-(-b*(2*a*d-b*c)/d^2+1/3*b/d*a)/(1/a^(1/2)*b^(1/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x,I)+1/8/d^3*sum((-a^2*d^2+2*a*b*c*d-b^2*c^2)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2)),_alpha=RootOf(_Z^4*d-c))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{dx^4 - c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(3/2)/(-d\*x^4+c),x, algorithm="maxima")

[Out] -integrate((-b\*x^4 + a)^(3/2)/(d\*x^4 - c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - b x^4)^{3/2}}{c - d x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(3/2)/(c - d\*x^4), x)

[Out] int((a - b\*x^4)^(3/2)/(c - d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{a\sqrt{a - bx^4}}{-c + dx^4} dx - \int \left( -\frac{bx^4\sqrt{a - bx^4}}{-c + dx^4} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c), x)

[Out] -Integral(a\*sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4), x) - Integral(-b\*x\*\*4\*sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4), x)

$$3.175 \quad \int \frac{\sqrt{a-bx^4}}{c-dx^4} dx$$

**Optimal.** Leaf size=240

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc-a)}{2\sqrt[4]{a}}$$

[Out]  $a^{1/4} b^{3/4} \text{EllipticF}(b^{1/4} x/a^{1/4}, I) (1-bx^4/a)^{1/2} / d / (-bx^4+a)^{1/2} - 1/2 a^{1/4} (-a*d+bc) \text{EllipticPi}(b^{1/4} x/a^{1/4}, -a^{1/2} d^{1/2} / b^{1/2} / c^{1/2}, I) (1-bx^4/a)^{1/2} / b^{1/4} / c / d / (-bx^4+a)^{1/2} - 1/2 a^{1/4} (-a*d+bc) \text{EllipticPi}(b^{1/4} x/a^{1/4}, a^{1/2} d^{1/2} / b^{1/2} / c^{1/2}, I) (1-bx^4/a)^{1/2} / b^{1/4} / c / d / (-bx^4+a)^{1/2}$

**Rubi [A]** time = 0.16, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$ , Rules used = {406, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc-ad)\Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc-a)}{2\sqrt[4]{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(c - d\*x^4), x]

[Out]  $(a^{1/4} b^{3/4} \text{Sqrt}[1 - (b*x^4)/a] \text{EllipticF}[\text{ArcSin}[(b^{1/4} x)/a^{1/4}], -1]) / (d \text{Sqrt}[a - b*x^4]) - (a^{1/4} (b*c - a*d) \text{Sqrt}[1 - (b*x^4)/a] \text{EllipticPi}[-((\text{Sqrt}[a] \text{Sqrt}[d]) / (\text{Sqrt}[b] \text{Sqrt}[c])), \text{ArcSin}[(b^{1/4} x)/a^{1/4}], -1]) / (2*b^{1/4} * c * d \text{Sqrt}[a - b*x^4]) - (a^{1/4} (b*c - a*d) \text{Sqrt}[1 - (b*x^4)/a] \text{EllipticPi}[(\text{Sqrt}[a] \text{Sqrt}[d]) / (\text{Sqrt}[b] \text{Sqrt}[c]), \text{ArcSin}[(b^{1/4} x)/a^{1/4}], -1]) / (2*b^{1/4} * c * d \text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 406

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^4]/((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] := Dist[b/d, Int[1/Sqrt[a + b\*x^4], x], x] - Dist[(b\*c - a\*d)/d, Int[1/(Sqrt[a + b\*x^4]\*(c + d\*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 1218

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a-bx^4}}{c-dx^4} dx &= \frac{b \int \frac{1}{\sqrt{a-bx^4}} dx}{d} + \frac{(-bc+ad) \int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx}{d} \\ &= \frac{(-bc+ad) \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{2cd} + \frac{(-bc+ad) \int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{2cd} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right) \int \frac{1}{\sqrt{1-\frac{bx^4}{a}}} dx}{d\sqrt{a-bx^4}} \\ &= \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} + \frac{\left((-bc+ad)\sqrt{1-\frac{bx^4}{a}}\right) \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}} dx}{2cd\sqrt{a-bx^4}} + \dots \\ &= \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-ad)\sqrt{1-\frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}cd\sqrt{a-bx^4}} \end{aligned}$$

**Mathematica [C]** time = 0.21, size = 155, normalized size = 0.65

$$\frac{5acx\sqrt{a-bx^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{(c-dx^4)\left(2x^4\left(bcF_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 2adF_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) - 5acF_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a - b*x^4]/(c - d*x^4), x]
```

```
[Out] (-5*a*c*x*Sqrt[a - b*x^4]*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]
)/(c - d*x^4)*(-5*a*c*AppellF1[1/4, -1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] +
2*x^4*(-2*a*d*AppellF1[5/4, -1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*Appel
lF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x^4+a)^(1/2)/(-d*x^4+c), x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{-bx^4+a}}{dx^4-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(-d\*x^4+c),x, algorithm="giac")

[Out] integrate(-sqrt(-b\*x^4 + a)/(d\*x^4 - c), x)

maple [C] time = 0.31, size = 259, normalized size = 1.08

$$\frac{\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} b \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}x, i\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4+a} d} + \frac{(-ad+bc) \left( 2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{RootOf}(dZ^4-c)^3 d \operatorname{EllipticPi}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}c\right) \right)}{8d^2 \operatorname{RootOf}(dZ^4-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(-d\*x^4+c),x)

[Out] b/d/(1/a^(1/2)\*b^(1/2))^(1/2)\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF((1/a^(1/2)\*b^(1/2))^(1/2)\*x, I)+1/8/d^2\*sum((-a\*d+b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi((1/a^(1/2)\*b^(1/2))^(1/2)\*x, \_alpha^2\*a^(1/2)/b^(1/2)/c\*d, (-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2))),\_alpha=RootOf(\_Z^4\*d-c))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{-bx^4+a}}{dx^4-c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(-d\*x^4+c),x, algorithm="maxima")

[Out] -integrate(sqrt(-b\*x^4 + a)/(d\*x^4 - c), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a-bx^4}}{c-dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(1/2)/(c - d\*x^4),x)

[Out] int((a - b\*x^4)^(1/2)/(c - d\*x^4), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{a-bx^4}}{-c+dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c),x)

[Out] -Integral(sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4), x)



$$3.176 \quad \int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx$$

**Optimal.** Leaf size=162

$$\frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} \Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

[Out]  $1/2*a^{(1/4)}*EllipticPi(b^{(1/4)*x/a^{(1/4)}, -a^{(1/2)*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I) * (1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-b*x^4+a)^{(1/2)} + 1/2*a^{(1/4)}*EllipticPi(b^{(1/4)*x/a^{(1/4)}, a^{(1/2)*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I) * (1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {409, 1219, 1218}

$$\frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} \Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)), x]

[Out]  $(a^{(1/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]))], \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1)]/(2*b^{(1/4)}*c*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1)]/(2*b^{(1/4)}*c*\text{Sqrt}[a - b*x^4])$

**Rule 409**

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

**Rule 1218**

Int[1/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1\*EllipticPi[-(e/(d\*q^2)), ArcSin[q\*x], -1)]/(d\*Sqrt[a\*q]), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

**Rule 1219**

Int[1/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] := Dist[Sqrt[1 + (c\*x^4)/a]/Sqrt[a + c\*x^4], Int[1/((d + e\*x^2)\*Sqrt[1 + (c\*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

**Rubi steps**

$$\int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx = \frac{\int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{2c} + \frac{\int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{2c}$$

$$= \frac{\sqrt{1-\frac{bx^4}{a}} \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}} dx}{2c\sqrt{a-bx^4}} + \frac{\sqrt{1-\frac{bx^4}{a}} \int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{1-\frac{bx^4}{a}}} dx}{2c\sqrt{a-bx^4}}$$

$$= \frac{\sqrt[4]{a} \sqrt{1-\frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1-\frac{bx^4}{a}} \Pi\left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c\sqrt{a-bx^4}}$$

**Mathematica [C]** time = 0.10, size = 156, normalized size = 0.96

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{\sqrt{a-bx^4}(dx^4-c) \left(2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)),x]

[Out] (-5\*a\*c\*x\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/(Sqrt[a - b\*x^4] \* (-c + d\*x^4) \* (5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4+a)^(1/2)/(-d\*x^4+c)),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4+a}(dx^4-c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-b\*x^4+a)^(1/2)/(-d\*x^4+c)),x, algorithm="giac")

[Out] integrate(-1/(sqrt(-b\*x^4 + a)\*(d\*x^4 - c)), x)

**maple [C]** time = 0.31, size = 183, normalized size = 1.13

$$\frac{2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1}\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}\text{RootOf}(d\_Z^4-c)^3 d \text{EllipticPi}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, x, \frac{\text{RootOf}(d\_Z^4-c)^2 \sqrt{a} d}{\sqrt{b} c}, \sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}c} - \frac{\text{arctanh}\left(\frac{-2\text{RootOf}(d\_Z^4-c)^2 bx^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{-bx^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}$$

$$8d \text{RootOf}(d\_Z^4 - c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x)`

[Out] 
$$-1/8/d*\text{sum}(1/_\alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\text{arctanh}(1/2*(-2*_\alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*_\alpha^3*d/c*(-1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*\text{EllipticPi}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, \_\alpha^2*a^{(1/2)}/b^{(1/2)}/c*d, (-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)})), \_\alpha=\text{RootOf}(\_Z^4*d-c))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt{-bx^4 + a}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x^4+a)^(1/2)/(-d*x^4+c),x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(-b*x^4 + a)*(d*x^4 - c)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a - bx^4}(c - dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a - b*x^4)^(1/2)*(c - d*x^4)),x)`

[Out] `int(1/((a - b*x^4)^(1/2)*(c - d*x^4)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-c\sqrt{a - bx^4} + dx^4\sqrt{a - bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(1/2)/(-d*x**4+c),x)`

[Out] `-Integral(1/(-c*sqrt(a - b*x**4) + d*x**4*sqrt(a - b*x**4)), x)`

$$3.177 \quad \int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)} dx$$

**Optimal.** Leaf size=281

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4} \sqrt{a - bx^4} (bc - ad)} + \frac{bx}{2a \sqrt{a - bx^4} (bc - ad)} - \frac{\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2 \sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)} - \frac{\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}}}{2 \sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)}$$

[Out]  $1/2*b*x/a/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}+1/2*b^{(3/4)}*EllipticF(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/a^{(3/4)}/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}-1/2*a^{(1/4)}*d*EllipticPi(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}-1/2*a^{(1/4)}*d*EllipticPi(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.22, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4} \sqrt{a - bx^4} (bc - ad)} + \frac{bx}{2a \sqrt{a - bx^4} (bc - ad)} - \frac{\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2 \sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)} - \frac{\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}}}{2 \sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)),x]

[Out]  $(b*x)/(2*a*(b*c - a*d)*Sqrt[a - b*x^4]) + (b^{(3/4)}*Sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*a^{(3/4)}*(b*c - a*d)*Sqrt[a - b*x^4]) - (a^{(1/4)}*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[-((Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]))], ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)*Sqrt[a - b*x^4]) - (a^{(1/4)}*d*Sqrt[1 - (b*x^4)/a]*EllipticPi[(Sqrt[a]*Sqrt[d])/(Sqrt[b]*Sqrt[c]), ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)*Sqrt[a - b*x^4])$

**Rule 221**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

**Rule 224**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

**Rule 409**

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

**Rule 414**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c +

$d*x^n)^q*$ Simp[b\*c + n\*(p + 1)\*(b\*c - a\*d) + d\*b\*(n\*(p + q + 2) + 1)\*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rule 523

Int[((e\_) + (f\_)\*(x\_)^(n\_))/(((a\_) + (b\_)\*(x\_)^(n\_))\*Sqrt[(c\_) + (d\_)\*(x\_)^(n\_)]), x\_Symbol] := Dist[f/b, Int[1/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e - a\*f)/b, Int[1/((a + b\*x^n)\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

### Rule 1218

Int[1/(((d\_) + (e\_)\*(x\_)^2)\*Sqrt[(a\_) + (c\_)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1\*EllipticPi[-(e/(d\*q^2)), ArcSin[q\*x], -1])/(d\*Sqrt[a]\*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

### Rule 1219

Int[1/(((d\_) + (e\_)\*(x\_)^2)\*Sqrt[(a\_) + (c\_)\*(x\_)^4]), x\_Symbol] := Dist[Sqrt[1 + (c\*x^4)/a]/Sqrt[a + c\*x^4], Int[1/((d + e\*x^2)\*Sqrt[1 + (c\*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a - bx^4)^{3/2} (c - dx^4)} dx &= \frac{bx}{2a(bc - ad)\sqrt{a - bx^4}} + \frac{\int \frac{bc - 2ad - bdx^4}{\sqrt{a - bx^4}(c - dx^4)} dx}{2a(bc - ad)} \\ &= \frac{bx}{2a(bc - ad)\sqrt{a - bx^4}} + \frac{b \int \frac{1}{\sqrt{a - bx^4}} dx}{2a(bc - ad)} - \frac{d \int \frac{1}{\sqrt{a - bx^4}(c - dx^4)} dx}{bc - ad} \\ &= \frac{bx}{2a(bc - ad)\sqrt{a - bx^4}} - \frac{d \int \frac{1}{\left(1 - \frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{2c(bc - ad)} - \frac{d \int \frac{1}{\left(1 + \frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{2c(bc - ad)} + \frac{\left(b\sqrt{1 - \frac{bx^4}{a}}\right) \int \frac{1}{\left(1 - \frac{bx^4}{a}\right)\sqrt{a - bx^4}} dx}{2c(bc - ad)} \\ &= \frac{bx}{2a(bc - ad)\sqrt{a - bx^4}} + \frac{b^{3/4}\sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4}(bc - ad)\sqrt{a - bx^4}} - \frac{\left(d\sqrt{1 - \frac{bx^4}{a}}\right) \int \frac{1}{\left(1 - \frac{bx^4}{a}\right)\sqrt{a - bx^4}} dx}{2c(bc - ad)} \\ &= \frac{bx}{2a(bc - ad)\sqrt{a - bx^4}} + \frac{b^{3/4}\sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{2a^{3/4}(bc - ad)\sqrt{a - bx^4}} - \frac{\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} \Pi\left(\frac{1}{2}, \frac{bx^4}{a}, \sqrt[4]{a}\right)}{2\sqrt[4]{b}c(bc - ad)} \end{aligned}$$

**Mathematica [C]** time = 0.32, size = 381, normalized size = 1.36

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \left(bdx^4 \sqrt{1 - \frac{bx^4}{a}} (dx^4 - c) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 5c(2ad - 2bc + bdx^4)\right) + 2bx^5 (c - dx^4)}{10ac\sqrt{a - bx^4} (dx^4 - c) (ad - bc) \left(2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 5c\right) + 2bx^5 (c - dx^4)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)),x]

```
[Out] (5*a*c*x*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c]*(-5*c*(-2*b*c + 2
*a*d + b*d*x^4) + b*d*x^4*Sqrt[1 - (b*x^4)/a]*(-c + d*x^4)*AppellF1[5/4, 1/
2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]) + 2*b*x^5*(c - d*x^4)*(5*c - d*x^4*Sqrt[1
- (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])*(2*a*d*Appe
llF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/
4, (b*x^4)/a, (d*x^4)/c]))/(10*a*c*(-(b*c) + a*d)*Sqrt[a - b*x^4]*(-c + d*x
^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*
AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1
, 9/4, (b*x^4)/a, (d*x^4)/c])))
```

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="fricas")
```

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x, algorithm="giac")
```

[Out] integrate(-1/((-b\*x^4 + a)^(3/2)\*(d\*x^4 - c)), x)

**maple** [C] time = 0.36, size = 301, normalized size = 1.07

$$\frac{bx \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} b \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}x, i\right)}{2(ad-bc)\sqrt{-\left(x^4-\frac{a}{b}\right)}b a} - \frac{2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1}\sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1}\operatorname{RootOf}(d_...)}{2(ad-bc)\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-b*x^4+a)^(3/2)/(-d*x^4+c),x)
```

```
[Out] -1/2*b/a*x/(a*d-b*c)/(-x^4-a/b)*b)^(1/2)-1/2/a*b/(a*d-b*c)/(1/a^(1/2)*b^(1
/2))^(1/2)*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)
/(-b*x^4+a)^(1/2)*EllipticF((1/a^(1/2)*b^(1/2))^(1/2)*x, I)-1/8*sum(1/(a*d-b
*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((
a*d-b*c)/d)^(1/2)/(-b*x^4+a)^(1/2))-2/(1/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/
c*(-1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(1/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(-b*x^4+
a)^(1/2)*EllipticPi((1/a^(1/2)*b^(1/2))^(1/2)*x,_alpha^2*a^(1/2)/b^(1/2)/c*
d,(-1/a^(1/2)*b^(1/2))^(1/2)/(1/a^(1/2)*b^(1/2))^(1/2))),_alpha=RootOf(_Z^4
*d-c))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c),x, algorithm="maxima")

[Out] -integrate(1/((-b\*x^4 + a)^(3/2)\*(d\*x^4 - c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - bx^4)^{3/2} (c - dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)),x)

[Out] int(1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{-ac\sqrt{a-bx^4} + adx^4\sqrt{a-bx^4} + bcx^4\sqrt{a-bx^4} - bdx^8\sqrt{a-bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c),x)

[Out] -Integral(1/(-a\*c\*sqrt(a - b\*x\*\*4) + a\*d\*x\*\*4\*sqrt(a - b\*x\*\*4) + b\*c\*x\*\*4\*sqrt(a - b\*x\*\*4) - b\*d\*x\*\*8\*sqrt(a - b\*x\*\*4)), x)

$$3.178 \quad \int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)} dx$$

**Optimal.** Leaf size=334

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (5bc - 11ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{12a^{7/4} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(5bc - 11ad)}{12a^2 \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{a} d^2 \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{2\sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)^2}$$

[Out]  $1/6*b*x/a/(-a*d+b*c)/(-b*x^4+a)^{(3/2)}+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(-b*x^4+a)^{(1/2)}+1/12*b^{(3/4)}*(-11*a*d+5*b*c)*\text{EllipticF}(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/a^{(7/4)}/(-a*d+b*c)^2/(-b*x^4+a)^{(1/2)}+1/2*a^{(1/4)}*d^2*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^2/(-b*x^4+a)^{(1/2)}+1/2*a^{(1/4)}*d^2*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^2/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.40, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (5bc - 11ad) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{12a^{7/4} \sqrt{a - bx^4} (bc - ad)^2} + \frac{bx(5bc - 11ad)}{12a^2 \sqrt{a - bx^4} (bc - ad)^2} + \frac{\sqrt[4]{a} d^2 \sqrt{1 - \frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right)}{2\sqrt[4]{b} c \sqrt{a - bx^4} (bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)),x]

[Out]  $(b*x)/(6*a*(b*c - a*d)*(a - b*x^4)^{(3/2)}) + (b*(5*b*c - 11*a*d)*x)/(12*a^2*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) + (b^{(3/4)}*(5*b*c - 11*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(12*a^{(7/4)}*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*d^2*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*d^2*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 414



```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a - bx^4)^{5/2} (c - dx^4)} dx &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{\int \frac{5bc - 6ad - 5bdx^4}{(a - bx^4)^{3/2} (c - dx^4)} dx}{6a(bc - ad)} \\
 &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2 \sqrt{a - bx^4}} + \frac{\int \frac{5b^2c^2 - 11abcd + 12a^2d^2 - bd(5bc - 11ad)}{\sqrt{a - bx^4} (c - dx^4)} dx}{12a^2(bc - ad)^2} \\
 &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2 \sqrt{a - bx^4}} + \frac{d^2 \int \frac{1}{\sqrt{a - bx^4} (c - dx^4)} dx}{(bc - ad)^2} + \frac{b(5bc - 11ad)}{12a^2(bc - ad)^2} \\
 &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2 \sqrt{a - bx^4}} + \frac{d^2 \int \frac{1}{\left(1 - \frac{\sqrt{d}x^2}{\sqrt{c}}\right) \sqrt{a - bx^4}} dx}{2c(bc - ad)^2} + \frac{d^2}{2c} \\
 &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2 \sqrt{a - bx^4}} + \frac{b^{3/4}(5bc - 11ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{12a^{7/4}(bc - ad)^2 \sqrt{a - bx^4}} \\
 &= \frac{bx}{6a(bc - ad)(a - bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2 \sqrt{a - bx^4}} + \frac{b^{3/4}(5bc - 11ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{12a^{7/4}(bc - ad)^2 \sqrt{a - bx^4}}
 \end{aligned}$$

**Mathematica [C]** time = 0.85, size = 422, normalized size = 1.26

$$x \frac{\left( \frac{bdx^4 \sqrt{1 - \frac{bx^4}{a}} (11ad - 5bc) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{c} - \frac{5 \left( 2bx^4(dx^4 - c)(13a^2d - ab(7c + 11dx^4) + 5b^2cx^4) \left( 2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right)}{(a - bx^4)(dx^4 - c) \left( 2x^4 \left( 2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \right) \right)} \right)}{60a^2 \sqrt{a - bx^4} (bc - ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a - b*x^4)^(5/2)*(c - d*x^4)),x]
```

```
[Out] (x*((b*d*(-5*b*c + 11*a*d)*x^4*Sqrt[1 - (b*x^4)/a]*AppellF1[5/4, 1/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])/c - (5*(5*a*c*(12*a^3*d^2 + a^2*b*d*(-24*c + d*x^4) + 5*b^3*c*x^4*(-2*c + d*x^4) + a*b^2*(12*c^2 + 15*c*d*x^4 - 11*d^2*x^8))*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*b*x^4*(-c + d*x^4)*(13*a^2*d + 5*b^2*c*x^4 - a*b*(7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c])))/((a - b*x^4)*(-c + d*x^4)*(5*a*c*AppellF1[1/4, 1/2, 1, 5/4, (b*x^4)/a, (d*x^4)/c] + 2*x^4*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, (b*x^4)/a, (d*x^4)/c] + b*c*AppellF1[5/4, 3/2, 1, 9/4, (b*x^4)/a, (d*x^4)/c]))))/(60*a^2*(b*c - a*d)^2*Sqrt[a - b*x^4])
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-b*x^4+a)^(5/2)/(-d*x^4+c),x, algorithm="fricas")
```

```
[Out] Timed out
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c),x, algorithm="giac")

[Out] integrate(-1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)), x)

**maple** [C] time = 0.32, size = 361, normalized size = 1.08

$$d \left[ \frac{2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{RootOf}(d\_Z^4-c)^3 d \operatorname{EllipticPi}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}, x, \frac{\operatorname{RootOf}(d\_Z^4-c)^2 \sqrt{a} d}{\sqrt{b} c}, \sqrt{-\frac{\sqrt{b}}{\sqrt{a}}}\right)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4+a} c} - \frac{\operatorname{arctanh}\left(\frac{-2\operatorname{RootOf}(d\_Z^4-c)^2 bx^2+2a}{2\sqrt{\frac{ad-bc}{d}} \sqrt{-bx^4+a}}\right)}{\sqrt{\frac{ad-bc}{d}}}\right] \\ \hline 8(ad-bc)^2 \operatorname{RootOf}(d\_Z^4-c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c),x)

[Out] 
$$-1/6/a*x/b/(a*d-b*c)*(-b*x^4+a)^{(1/2)}/(x^4-a/b)^2-1/12*b/a^2*x*(11*a*d-5*b*c)/(a*d-b*c)^2/(-(x^4-a/b)*b)^{(1/2)}-1/12/a^2*b*(11*a*d-5*b*c)/(a*d-b*c)^2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(-1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticF}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x,I)-1/8*d*\sum(1/(a*d-b*c)^2/_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*_alpha^3*d/c*(-1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticPi}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x,_alpha^2*a^{(1/2)}/b^{(1/2)}/c*d,(-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)})),_alpha=\operatorname{RootOf}(_Z^4*d-c))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c),x, algorithm="maxima")

[Out] -integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - bx^4)^{\frac{5}{2}}(c - dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)),x)

[Out] int(1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*4+a)\*\*(5/2)/(-d\*x\*\*4+c),x)

[Out] Timed out

$$3.179 \quad \int \frac{(a+bx^4)^{3/2}}{c+dx^4} dx$$

**Optimal.** Leaf size=926

$$\frac{\sqrt[4]{b} (\sqrt{b} \sqrt{-c} - \sqrt{a} \sqrt{d}) (\sqrt{b} x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) (bc-ad)^2 \sqrt[4]{b} (\sqrt{b} \sqrt{-c} + \sqrt{a} \sqrt{d})}{4\sqrt[4]{a} \sqrt{-c} d^2 (bc+ad) \sqrt{bx^4+a}}$$

[Out]  $-1/4*(-a*d+b*c)^{(3/2)}*\arctan(x*(-a*d+b*c)^{(1/2)/(-c)^{(1/4)/d^{(1/4)/(b*x^4+a)^{(1/2))}}/(-c)^{(3/4)/d^{(7/4)}-1/4*(a*d-b*c)^{(3/2)}*\arctan(x*(a*d-b*c)^{(1/2)/(-c)^{(1/4)/d^{(1/4)/(b*x^4+a)^{(1/2))}}/(-c)^{(3/4)/d^{(7/4)}+1/3*b*x*(b*x^4+a)^{(1/2)/d-1/6*b^{(3/4)}*(-5*a*d+3*b*c)}*(\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})^2)^{(1/2)/\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})})*\text{EllipticF}(\sin(2*\arctan(b^{(1/4)*x/a^{(1/4))}), 1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})}*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)/a^{(1/4)/d^2/(b*x^4+a)^{(1/2)+1/4*b^{(1/4)}*(-a*d+b*c)^2*(\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})^2)^{(1/2)/\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})})*\text{EllipticF}(\sin(2*\arctan(b^{(1/4)*x/a^{(1/4))}), 1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})}*(b^{(1/2)*(-c)^{(1/2)-a^{(1/2)*d^{(1/2)}}*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)/a^{(1/4)/d^2/(a*d+b*c)/(-c)^{(1/2)/(b*x^4+a)^{(1/2)+1/8*(-a*d+b*c)^2*(\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})^2)^{(1/2)/\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})})*\text{EllipticPi}(\sin(2*\arctan(b^{(1/4)*x/a^{(1/4))}), 1/4*(b^{(1/2)*(-c)^{(1/2)+a^{(1/2)*d^{(1/2)}})^2/a^{(1/2)/b^{(1/2)/(-c)^{(1/2)/d^{(1/2)}, 1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})}*(b^{(1/2)*(-c)^{(1/2)-a^{(1/2)*d^{(1/2)}})^2*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)/a^{(1/4)/b^{(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^{(1/2)+1/4*b^{(1/4)}*(-a*d+b*c)^2*(\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})^2)^{(1/2)/\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})})*\text{EllipticF}(\sin(2*\arctan(b^{(1/4)*x/a^{(1/4))}), 1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})}*(b^{(1/2)*(-c)^{(1/2)+a^{(1/2)*d^{(1/2)}}*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)/a^{(1/4)/d^2/(a*d+b*c)/(-c)^{(1/2)/(b*x^4+a)^{(1/2)+1/8*(-a*d+b*c)^2*(\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})^2)^{(1/2)/\cos(2*\arctan(b^{(1/4)*x/a^{(1/4))})})*\text{EllipticPi}(\sin(2*\arctan(b^{(1/4)*x/a^{(1/4))}), -1/4*(b^{(1/2)*(-c)^{(1/2)-a^{(1/2)*d^{(1/2)}})^2/a^{(1/2)/b^{(1/2)/(-c)^{(1/2)/d^{(1/2)}, 1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})}*(b^{(1/2)*(-c)^{(1/2)+a^{(1/2)*d^{(1/2)}})^2*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)/a^{(1/4)/b^{(1/4)/c/d^2/(a*d+b*c)/(b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 1.66, antiderivative size = 926, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {416, 523, 220, 409, 1217, 1707}

$$\frac{\sqrt[4]{b} (\sqrt{b} \sqrt{-c} - \sqrt{a} \sqrt{d}) (\sqrt{b} x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right) (bc-ad)^2 \sqrt[4]{b} (\sqrt{b} \sqrt{-c} + \sqrt{a} \sqrt{d})}{4\sqrt[4]{a} \sqrt{-c} d^2 (bc+ad) \sqrt{bx^4+a}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(3/2)/(c + d\*x^4), x]

[Out]  $(b*x*\text{Sqrt}[a + b*x^4])/(3*d) - ((b*c - a*d)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[b*c - a*d]*x)/((-c)^{(1/4)*d^{(1/4)*\text{Sqrt}[a + b*x^4]})]/(4*(-c)^{(3/4)*d^{(7/4)}} - ((-b*c) + a*d)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[-(b*c) + a*d]*x)/((-c)^{(1/4)*d^{(1/4)*\text{Sqrt}[a + b*x^4]})]/(4*(-c)^{(3/4)*d^{(7/4)}} - (b^{(3/4)}*(3*b*c - 5*a*d)*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)*x}/a^{(1/4)}], 1/2])/(6*a^{(1/4)*d^2*\text{Sqrt}[a + b*x^4]} + (b^{(1/4)}*(\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d]))*(b*c - a*d)^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)*x}/a^{(1/4)}], 1/2])/(4*a^{(1/4)*\text{Sqrt}[-c]*d^2*(b*c + a*d)*\text{Sqrt}[a + b*x^4]} + (b^{(1/4)}*(\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d]))*(b*c - a*d)^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)*x}/a^{(1/4)}], 1/2])$

$$\frac{a + b*x^4}{(\sqrt{a} + \sqrt{b}*x^2)^2} * \text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2] / (4*a^{1/4}*\sqrt{-c}*d^2*(b*c + a*d)*\sqrt{a + b*x^4}) + ((\sqrt{b}*\sqrt{-c} + \sqrt{a}*\sqrt{d})^2*(b*c - a*d)^2*(\sqrt{a} + \sqrt{b}*x^2)*\sqrt{(a + b*x^4)/(\sqrt{a} + \sqrt{b}*x^2)^2} * \text{EllipticPi}[-(\sqrt{b}*\sqrt{-c} - \sqrt{a}*\sqrt{d})^2/(4*\sqrt{a}*\sqrt{b}*\sqrt{-c}*\sqrt{d}), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]) / (8*a^{1/4}*b^{1/4}*c*d^2*(b*c + a*d)*\sqrt{a + b*x^4}) + ((\sqrt{b}*\sqrt{-c} - \sqrt{a}*\sqrt{d})^2*(b*c - a*d)^2*(\sqrt{a} + \sqrt{b}*x^2)*\sqrt{(a + b*x^4)/(\sqrt{a} + \sqrt{b}*x^2)^2} * \text{EllipticPi}[(\sqrt{b}*\sqrt{-c} + \sqrt{a}*\sqrt{d})^2/(4*\sqrt{a}*\sqrt{b}*\sqrt{-c}*\sqrt{d}), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]) / (8*a^{1/4}*b^{1/4}*c*d^2*(b*c + a*d)*\sqrt{a + b*x^4})$$

### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

### Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)), x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]
```

### Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

### Rule 1707

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A*q*Sqrt[a + c*x^4]), x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a + bx^4)^{3/2}}{c + dx^4} dx &= \frac{bx\sqrt{a + bx^4}}{3d} + \frac{\int \frac{-a(bc-3ad)-b(3bc-5ad)x^4}{\sqrt{a+bx^4}(c+dx^4)} dx}{3d} \\
&= \frac{bx\sqrt{a + bx^4}}{3d} - \frac{(b(3bc - 5ad)) \int \frac{1}{\sqrt{a+bx^4}} dx}{3d^2} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx}{d^2} \\
&= \frac{bx\sqrt{a + bx^4}}{3d} - \frac{b^{3/4}(3bc - 5ad)(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a + bx^4}} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx}{d^2} \\
&= \frac{bx\sqrt{a + bx^4}}{3d} - \frac{b^{3/4}(3bc - 5ad)(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{6\sqrt[4]{a}d^2\sqrt{a + bx^4}} + \frac{(\sqrt{b} - \sqrt{a}) \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx}{d^2} \\
&= \frac{bx\sqrt{a + bx^4}}{3d} - \frac{(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}} - \frac{(-bc + ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}d^{7/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.52, size = 346, normalized size = 0.37

$$\frac{x \left( 5(2bx^4(a+bx^4)(c+dx^4)) \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 5ac(3a^2d+abdx^4+b^2x^4(c+dx^4))F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}{(c+dx^4) \left( 2x^4 \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)} 15d\sqrt{a + bx^4}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(3/2)/(c + d\*x^4), x]

[Out] (x\*((b\*(-3\*b\*c + 5\*a\*d))\*x^4\*Sqrt[1 + (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c + (5\*(-5\*a\*c\*(3\*a^2\*d + a\*b\*d\*x^4 + b^2\*x^4\*(c + d\*x^4))\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*b\*x^4\*(a + b\*x^4)\*(c + d\*x^4)\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))/((c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(15\*d\*Sqrt[a + b\*x^4])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/2)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/2)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x)

**maple** [C] time = 0.48, size = 322, normalized size = 0.35

$$\frac{\sqrt{bx^4+a} \, bx}{3d} + \frac{\left(-\frac{ab}{3d} + \frac{(2ad-bc)b}{d^2}\right) \sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{bx^4+a}} \left( -a^2d^2 + 2abcd - b^2c^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(3/2)/(d\*x^4+c),x)

[Out]  $\frac{1}{3}bx(bx^4+a)^{1/2}/d + ((2ad-bc)bd/d^2 - 1/3ab/d)/(I/a^{1/2}b^{1/2})^{1/2} * (1 - I/a^{1/2}b^{1/2}x^2)^{1/2} * (1 + I/a^{1/2}b^{1/2}x^2)^{1/2} / (bx^4+a)^{1/2} * \operatorname{EllipticF}(x(I/a^{1/2}b^{1/2})^{1/2}, I) - 1/8/d^3 * \sum((-a^2d^2 + 2ab^2cd - b^2c^2)/\alpha^3 * (-1/((ad-bc)/d)^{1/2} * \operatorname{arctanh}(1/2 * (2\alpha^2 * bx^2 + 2a)/((ad-bc)/d)^{1/2}) / (bx^4+a)^{1/2}) + 2/(I/a^{1/2}b^{1/2})^{1/2} * \alpha^3 d/c * (1 - I/a^{1/2}b^{1/2}x^2)^{1/2} * (1 + I/a^{1/2}b^{1/2}x^2)^{1/2} / (bx^4+a)^{1/2} * \operatorname{EllipticPi}(x(I/a^{1/2}b^{1/2})^{1/2}, I * a^{1/2}/b^{1/2}) * \alpha^2/c * d, (-I/a^{1/2}b^{1/2})^{1/2} / (I/a^{1/2}b^{1/2})^{1/2}), \alpha = \operatorname{RootOf}(\_Z^4d+c)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{2}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/2)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(3/2)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{3/2}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(3/2)/(c + d\*x^4),x)

[Out] int((a + b\*x^4)^(3/2)/(c + d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{3}{2}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(3/2)/(d\*x\*\*4+c),x)

[Out] Integral((a + b\*x\*\*4)\*\*(3/2)/(c + d\*x\*\*4), x)



$$3.180 \quad \int \frac{\sqrt{a+bx^4}}{c+dx^4} dx$$

**Optimal.** Leaf size=881

$$\frac{(bc - ad) (\sqrt{b} x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} \Pi \left( \frac{(\sqrt{b} \sqrt{-c} + \sqrt{a} \sqrt{d})^2}{4\sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}}; 2 \tan^{-1} \left( \frac{\sqrt[4]{b} x}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right) (\sqrt{b} \sqrt{-c} - \sqrt{a} \sqrt{d})^2 \sqrt[4]{b} (bc - ad)}{8\sqrt[4]{a} \sqrt[4]{b} cd(bc + ad) \sqrt{bx^4 + a}}$$

[Out]  $\frac{1}{4} \arctan(x \sqrt{-a+bx^4}) / (-c)^{1/4} / d^{1/4} / (bx^4+a)^{1/2} * (-a+bx^4)^{1/2} / (-c)^{3/4} / d^{3/4} - \frac{1}{4} \arctan(x \sqrt{a-bx^4}) / (-c)^{1/4} / d^{1/4} / (bx^4+a)^{1/2} * (a-bx^4)^{1/2} / (-c)^{3/4} / d^{3/4} + \frac{1}{2} b^{3/4} * (\cos(2 \arctan(b^{1/4} x / a^{1/4}))^2)^{1/2} / \cos(2 \arctan(b^{1/4} x / a^{1/4})) * \text{EllipticF}(\sin(2 \arctan(b^{1/4} x / a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2} + x^2 b^{1/2}) * ((bx^4+a) / (a^{1/2} + x^2 b^{1/2}))^2)^{1/2} / a^{1/4} / d / (bx^4+a)^{1/2} - \frac{1}{4} b^{1/4} * (-a+bx^4) * (\cos(2 \arctan(b^{1/4} x / a^{1/4}))^2)^{1/2} / \cos(2 \arctan(b^{1/4} x / a^{1/4})) * \text{EllipticF}(\sin(2 \arctan(b^{1/4} x / a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2} + x^2 b^{1/2}) * (b^{1/2} * (-c)^{1/2} - a^{1/2} * d^{1/2}) * ((bx^4+a) / (a^{1/2} + x^2 b^{1/2}))^2)^{1/2} / a^{1/4} / d / (a+bx^4) / (-c)^{1/2} / (bx^4+a)^{1/2} - \frac{1}{8} * (-a+bx^4) * (\cos(2 \arctan(b^{1/4} x / a^{1/4}))^2)^{1/2} / \cos(2 \arctan(b^{1/4} x / a^{1/4})) * \text{EllipticPi}(\sin(2 \arctan(b^{1/4} x / a^{1/4})), 1/4 * (b^{1/2} * (-c)^{1/2} + a^{1/2} * d^{1/2}))^2 / a^{1/2} / b^{1/2} / (-c)^{1/2} / d^{1/2}, 1/2, 2^{1/2}) * (a^{1/2} + x^2 b^{1/2}) * (b^{1/2} * (-c)^{1/2} - a^{1/2} * d^{1/2})^2 * ((bx^4+a) / (a^{1/2} + x^2 b^{1/2}))^2)^{1/2} / a^{1/4} / b^{1/4} / c / d / (a+bx^4) / (bx^4+a)^{1/2} - \frac{1}{8} * (-a+bx^4) * (\cos(2 \arctan(b^{1/4} x / a^{1/4}))^2)^{1/2} / \cos(2 \arctan(b^{1/4} x / a^{1/4})) * \text{EllipticPi}(\sin(2 \arctan(b^{1/4} x / a^{1/4})), -1/4 * (b^{1/2} * (-c)^{1/2} - a^{1/2} * d^{1/2}))^2 / a^{1/2} / b^{1/2} / (-c)^{1/2} / d^{1/2}, 1/2, 2^{1/2}) * (a^{1/2} + x^2 b^{1/2}) * (b^{1/2} * (-c)^{1/2} + a^{1/2} * d^{1/2})^2 * ((bx^4+a) / (a^{1/2} + x^2 b^{1/2}))^2)^{1/2} / a^{1/4} / b^{1/4} / c / d / (a+bx^4) / (bx^4+a)^{1/2} - \frac{1}{4} b^{1/4} * (-a+bx^4) * (\cos(2 \arctan(b^{1/4} x / a^{1/4}))^2)^{1/2} / \cos(2 \arctan(b^{1/4} x / a^{1/4})) * \text{EllipticF}(\sin(2 \arctan(b^{1/4} x / a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2} + x^2 b^{1/2}) * (b^{1/2} + a^{1/2} * d^{1/2} / (-c)^{1/2}) * ((bx^4+a) / (a^{1/2} + x^2 b^{1/2}))^2)^{1/2} / a^{1/4} / d / (a+bx^4) / (bx^4+a)^{1/2}$

**Rubi [A]** time = 0.88, antiderivative size = 881, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {406, 220, 409, 1217, 1707}

$$\frac{(bc - ad) (\sqrt{b} x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} \Pi \left( \frac{(\sqrt{b} \sqrt{-c} + \sqrt{a} \sqrt{d})^2}{4\sqrt{a} \sqrt{b} \sqrt{-c} \sqrt{d}}; 2 \tan^{-1} \left( \frac{\sqrt[4]{b} x}{\sqrt[4]{a}} \right) \middle| \frac{1}{2} \right) (\sqrt{b} \sqrt{-c} - \sqrt{a} \sqrt{d})^2 \sqrt[4]{b} (bc - ad)}{8\sqrt[4]{a} \sqrt[4]{b} cd(bc + ad) \sqrt{bx^4 + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^4]/(c + d\*x^4), x]

[Out]  $(\text{Sqrt}[b*c - a*d] * \text{ArcTan}[(\text{Sqrt}[b*c - a*d] * x) / ((-c)^{1/4} * d^{1/4} * \text{Sqrt}[a + b*x^4])]) / (4 * (-c)^{3/4} * d^{3/4}) - (\text{Sqrt}[-(b*c) + a*d] * \text{ArcTan}[(\text{Sqrt}[-(b*c) + a*d] * x) / ((-c)^{1/4} * d^{1/4} * \text{Sqrt}[a + b*x^4])]) / (4 * (-c)^{3/4} * d^{3/4}) + (b^{3/4} * (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2) * \text{Sqrt}[(a + b*x^4) / (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2)^2] * \text{EllipticF}[2 * \text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2]) / (2 * a^{1/4} * d * \text{Sqrt}[a + b*x^4]) - (b^{1/4} * (\text{Sqrt}[b] * \text{Sqrt}[-c] - \text{Sqrt}[a] * \text{Sqrt}[d]) * (b*c - a*d) * (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2) * \text{Sqrt}[(a + b*x^4) / (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2)^2] * \text{EllipticF}[2 * \text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2]) / (4 * a^{1/4} * \text{Sqrt}[-c] * d * (b*c + a*d) * \text{Sqrt}[a + b*x^4]) - (b^{1/4} * (\text{Sqrt}[b] + (\text{Sqrt}[a] * \text{Sqrt}[d]) / \text{Sqrt}[-c]) * (b*c - a*d) * (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2) * \text{Sqrt}[(a + b*x^4) / (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2)^2] * \text{EllipticF}[2 * \text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2]) / (4 * a^{1/4} * d * (b*c + a*d) * \text{Sqrt}[a + b*x^4]) - ((\text{Sqrt}[b] * \text{Sqrt}[-c] + \text{Sqrt}[a] * \text{Sqrt}[d])^2 * (b*c - a*d) * (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2) * \text{Sqrt}[(a + b*x^4) / (\text{Sqrt}[a] + \text{Sqrt}[b] * x^2)^2] * \text{EllipticF}[\text{ArcTan}[(b^{1/4} * x) / a^{1/4}], 1/2]) / (2 * a^{1/4} * d * \text{Sqrt}[a + b*x^4])$

```

qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[-(Sqrt[b]*Sqrt[-c] -
Sqrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*
x)/a^(1/4)], 1/2)]/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4]) - ((
Sqrt[b]*Sqrt[-c] - Sqrt[a]*Sqrt[d])^2*(b*c - a*d)*(Sqrt[a] + Sqrt[b]*x^2)*S
qrt[(a + b*x^4)/(Sqrt[a] + Sqrt[b]*x^2)^2]*EllipticPi[(Sqrt[b]*Sqrt[-c] + S
qrt[a]*Sqrt[d])^2/(4*Sqrt[a]*Sqrt[b]*Sqrt[-c]*Sqrt[d]), 2*ArcTan[(b^(1/4)*x
)/a^(1/4)], 1/2)]/(8*a^(1/4)*b^(1/4)*c*d*(b*c + a*d)*Sqrt[a + b*x^4])

```

#### Rule 220

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]

```

#### Rule 406

```

Int[Sqrt[(a_) + (b_.)*(x_)^4]/((c_) + (d_.)*(x_)^4), x_Symbol] := Dist[b/d,
Int[1/Sqrt[a + b*x^4], x], x] - Dist[(b*c - a*d)/d, Int[1/(Sqrt[a + b*x^4]
*(c + d*x^4)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

```

#### Rule 409

```

Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]

```

#### Rule 1217

```

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]

```

#### Rule 1707

```

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e
+ (a*e)/d, 2]*x)/Sqrt[a + c*x^4]])/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] +
Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*Ell
ipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2])/(4*d*e*A
*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e
^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]

```

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^4}}{c+dx^4} dx &= \frac{b \int \frac{1}{\sqrt{a+bx^4}} dx}{d} - \frac{(bc-ad) \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx}{d} \\
&= \frac{b^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a} d \sqrt{a+bx^4}} - \frac{(bc-ad) \int \frac{1}{\left(1 - \frac{\sqrt{a} x^2}{\sqrt{c}}\right) \sqrt{a+bx^4}} dx}{2cd} \\
&= \frac{b^{3/4} (\sqrt{a} + \sqrt{b} x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b} x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt{a}}\right) \middle| \frac{1}{2}\right)}{2\sqrt[4]{a} d \sqrt{a+bx^4}} - \frac{(\sqrt{b} (\sqrt{b} \sqrt{-c} - \sqrt{a} \sqrt{d}) (bc-ad))}{2\sqrt{-c} d (bc+ad)} \\
&= \frac{\sqrt{bc-ad} \tan^{-1}\left(\frac{\sqrt{bc-ad} x}{\sqrt[4]{-c} \sqrt[4]{d} \sqrt{a+bx^4}}\right)}{4(-c)^{3/4} d^{3/4}} - \frac{\sqrt{-bc+ad} \tan^{-1}\left(\frac{\sqrt{-bc+ad} x}{\sqrt[4]{-c} \sqrt[4]{d} \sqrt{a+bx^4}}\right)}{4(-c)^{3/4} d^{3/4}} + \frac{b^{3/4} (\sqrt{a} + \sqrt{b} x^2)}{2\sqrt{-c} d (bc+ad)}
\end{aligned}$$

**Mathematica [C]** time = 0.19, size = 161, normalized size = 0.18

$$\frac{5acx\sqrt{a+bx^4} F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4) \left(2x^4 \left(bc F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2ad F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5ac F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b\*x^4]/(c + d\*x^4), x]

[Out] (5\*a\*c\*x\*Sqrt[a + b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, -1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(-2\*a\*d\*AppellF1[5/4, -1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/2)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^4+a}}{dx^4+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/2)/(d\*x^4+c), x, algorithm="giac")

[Out] integrate(sqrt(b\*x^4 + a)/(d\*x^4 + c), x)

**maple** [C] time = 0.33, size = 273, normalized size = 0.31

$$\frac{\sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} b \operatorname{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x, i\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{bx^4+a} d} \left( -ad + bc \right) \frac{2\sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{RootOf}(d\_Z^4+c)^3 d \operatorname{EllipticPi}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a} c\right)}{8d^2 \operatorname{RootOf}(d\_Z^4+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^4+a)^(1/2)/(d*x^4+c), x)
```

```
[Out] b/d/(I/a^(1/2)*b^(1/2))^(1/2)*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticF((I/a^(1/2)*b^(1/2))^(1/2)*x, I)-1/8/d^2*sum((-a*d+b*c)/_alpha^3*(-1/((a*d-b*c)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^(1/2)/(b*x^4+a)^(1/2))+2/(I/a^(1/2)*b^(1/2))^(1/2)*_alpha^3*d/c*(-I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)*(I/a^(1/2)*b^(1/2)*x^2+1)^(1/2)/(b*x^4+a)^(1/2)*EllipticPi((I/a^(1/2)*b^(1/2))^(1/2)*x, I*_alpha^2*a^(1/2)/b^(1/2)/c*d, (-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)), _alpha=RootOf(_Z^4*d+c))
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^4+a}}{dx^4+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(1/2)/(d*x^4+c), x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*x^4+a)/(d*x^4+c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{bx^4+a}}{d x^4+c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

```
[Out] int((a + b*x^4)^(1/2)/(c + d*x^4), x)
```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^4}}{c+dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(1/2)/(d*x**4+c), x)
```

```
[Out] Integral(sqrt(a + b*x**4)/(c + d*x**4), x)
```

$$3.181 \quad \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx$$

**Optimal.** Leaf size=742

$$\frac{\sqrt[4]{d} \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d} \tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}} + \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}} + \sqrt{b}\right) F\left(2\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}$$

[Out]  $-1/4*d^{(1/4)}*\arctan(x*(-a*d+b*c)^{(1/2)/(-c)^{(1/4)}/d^{(1/4)/(b*x^4+a)^{(1/2))}/(-c)^{(3/4)/(-a*d+b*c)^{(1/2)}-1/4*d^{(1/4)}*\arctan(x*(a*d-b*c)^{(1/2)/(-c)^{(1/4)}/d^{(1/4)/(b*x^4+a)^{(1/2))}/(-c)^{(3/4)/(a*d-b*c)^{(1/2)}+1/8*(\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))))^2)^{(1/2)}/\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))})*EllipticPi(\sin(2*\arctan(b^{(1/4)}*x/a^{(1/4))}),1/4*(b^{(1/2)}*(-c)^{(1/2)+a^{(1/2)}*d^{(1/2)})^2/a^{(1/2)}/b^{(1/2)/(-c)^{(1/2)}/d^{(1/2)},1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})*(b^{(1/2)}*(-c)^{(1/2)-a^{(1/2)}*d^{(1/2)})^2*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/b^{(1/4)}/c/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/8*(\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))))^2)^{(1/2)}/\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))})*EllipticPi(\sin(2*\arctan(b^{(1/4)}*x/a^{(1/4))}),-1/4*(b^{(1/2)}*(-c)^{(1/2)-a^{(1/2)}*d^{(1/2)})^2/a^{(1/2)}/b^{(1/2)/(-c)^{(1/2)}/d^{(1/2)},1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})*(b^{(1/2)}*(-c)^{(1/2)+a^{(1/2)}*d^{(1/2)})^2*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/b^{(1/4)}/c/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/4*b^{(1/4)}*(\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))))^2)^{(1/2)}/\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))})*EllipticF(\sin(2*\arctan(b^{(1/4)}*x/a^{(1/4))}),1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})*(b^{(1/2)+a^{(1/2)}*d^{(1/2)}/(-c)^{(1/2)}*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/4*b^{(1/4)}*(\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))))^2)^{(1/2)}/\cos(2*\arctan(b^{(1/4)}*x/a^{(1/4))})*EllipticF(\sin(2*\arctan(b^{(1/4)}*x/a^{(1/4))}),1/2*2^{(1/2)}*(a^{(1/2)+x^2*b^{(1/2)})*(c*b^{(1/2)+a^{(1/2)}*(-c)^{(1/2)}*d^{(1/2)}*((b*x^4+a)/(a^{(1/2)+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/c/(a*d+b*c)/(b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.63, antiderivative size = 742, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {409, 1217, 220, 1707}

$$\frac{\sqrt[4]{d} \tan^{-1}\left(\frac{x\sqrt{bc-ad}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc-ad}} - \frac{\sqrt[4]{d} \tan^{-1}\left(\frac{x\sqrt{ad-bc}}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{ad-bc}} + \frac{\sqrt[4]{b}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} \left(\frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}} + \sqrt{b}\right) F\left(2\right)}{4\sqrt[4]{a}\sqrt{a+bx^4}(ad+bc)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b\*x^4]\*(c + d\*x^4)), x]

[Out]  $-(d^{(1/4)}*\text{ArcTan}[(\text{Sqrt}[b*c - a*d]*x)/((-c)^{(1/4)}*d^{(1/4)}*\text{Sqrt}[a + b*x^4])]/(4*(-c)^{(3/4)}*\text{Sqrt}[b*c - a*d]) - (d^{(1/4)}*\text{ArcTan}[(\text{Sqrt}[-(b*c) + a*d]*x)/((-c)^{(1/4)}*d^{(1/4)}*\text{Sqrt}[a + b*x^4])]/(4*(-c)^{(3/4)}*\text{Sqrt}[-(b*c) + a*d]) + (b^{(1/4)}*(\text{Sqrt}[b] + (\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[-c]))*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/ (4*a^{(1/4)}*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + (b^{(1/4)}*(\text{Sqrt}[b]*c + \text{Sqrt}[a]*\text{Sqrt}[-c]*\text{Sqrt}[d])*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/ (4*a^{(1/4)}*c*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + ((\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d])^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticPi}[-(\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}], 1/2])/ (8*a^{(1/4)}*b^{(1/4)}*c*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + ((\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d])^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticPi}[(\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}$

$[(b^{(1/4)}x)/a^{(1/4)}, 1/2]]/(8*a^{(1/4)}*b^{(1/4)}*c*(b*c + a*d)*Sqrt[a + b*x^4])$

Rule 220

`Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x], 1/2]]/(2*q*Sqrt[a + b*x^4]), x]] /; FreeQ[{a, b}, x] && PosQ[b/a]`

Rule 409

`Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]`

Rule 1217

`Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]`

Rule 1707

`Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*ArcTan[(Rt[(c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + c*x^4]]]/(2*d*e*Rt[(c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*d*e*A*B))], 2*ArcTan[q*x], 1/2]]/(4*d*e*A*q*Sqrt[a + c*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ[c*A^2 - a*B^2, 0]`

Rubi steps

$$\int \frac{1}{\sqrt{a + bx^4} (c + dx^4)} dx = \frac{\int \frac{1}{\left(1 - \frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}} dx}{2c} + \frac{\int \frac{1}{\left(1 + \frac{\sqrt{d}x^2}{\sqrt{-c}}\right)\sqrt{a+bx^4}} dx}{2c}$$

$$= \frac{\left(\sqrt{b} \left(\sqrt{b} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right)\right) \int \frac{1}{\sqrt{a+bx^4}} dx}{2(bc + ad)} + \frac{\left(\sqrt{b} \left(\sqrt{b}c + \sqrt{a}\sqrt{-c}\sqrt{d}\right)\right) \int \frac{1}{\sqrt{a+bx^4}} dx}{2c(bc + ad)} - \frac{\left(\sqrt{a}\sqrt{d}\right) \int \frac{1}{\sqrt{a+bx^4}} dx}{4(-c)^{3/4}\sqrt{bc - ad}}$$

$$= -\frac{\sqrt[4]{d} \tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{bc - ad}} - \frac{\sqrt[4]{d} \tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{a+bx^4}}\right)}{4(-c)^{3/4}\sqrt{-bc + ad}} + \frac{\sqrt[4]{b} \left(\sqrt{b} + \frac{\sqrt{a}\sqrt{d}}{\sqrt{-c}}\right) \left(\sqrt{a}\sqrt{d}\right) \int \frac{1}{\sqrt{a+bx^4}} dx}{4(-c)^{3/4}\sqrt{-bc + ad}}$$

**Mathematica [C]** time = 0.03, size = 161, normalized size = 0.22

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{\sqrt{a + bx^4} (c + dx^4) \left(2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a + b\*x^4]\*(c + d\*x^4)),x]

[Out] (-5\*a\*c\*x\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/(Sqrt[a + b\*x^4]\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/2)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/2)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/(sqrt(b\*x^4 + a)\*(d\*x^4 + c)), x)

**maple** [C] time = 0.33, size = 191, normalized size = 0.26

$$\frac{2\sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1}\sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1}\text{RootOf}(d\_Z^4+c)^3 d \text{EllipticPi}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}, x, \frac{i\text{RootOf}(d\_Z^4+c)^2\sqrt{a}d}{\sqrt{b}c}, \sqrt{\frac{-i\sqrt{b}}{\sqrt{a}}}\right) + \text{arctanh}\left(\frac{2\text{RootOf}(d\_Z^4+c)^2bx^2+2a}{2\sqrt{\frac{ad-bc}{d}}\sqrt{bx^4+a}}\right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}c - \sqrt{\frac{ad-bc}{d}}}$$

$$8d \text{RootOf}(d\_Z^4 + c)^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(1/2)/(d\*x^4+c),x)

[Out] 1/8/d\*sum(1/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(b\*x^4+a)^(1/2))+2/(I/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha a^3\*d/c\*(-I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticPi((I/a^(1/2)\*b^(1/2))^(1/2)\*x, I\*\_alpha^2\*a^(1/2)/b^(1/2)/c\*d, (-I/a^(1/2)\*b^(1/2))^(1/2)/(I/a^(1/2)\*b^(1/2))^(1/2)),\_alpha=RootOf(\_Z^4\*d+c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/2)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b\*x^4 + a)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{bx^4 + a}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^4)^(1/2)*(c + d*x^4)),x)`

[Out] `int(1/((a + b*x^4)^(1/2)*(c + d*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + bx^4} (c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(1/2)/(d*x**4+c),x)`

[Out] `Integral(1/(sqrt(a + b*x**4)*(c + d*x**4)), x)`



$$3.182 \quad \int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)} dx$$

**Optimal.** Leaf size=913

$$\frac{d(\sqrt{b}x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} \Pi\left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right) (\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})^2 d^{5/4} \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-c}\sqrt[4]{d}}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}} + \frac{d^{5/4} \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-c}\sqrt[4]{d}}\right)}{4(-c)^{3/4}(bc-}$$

[Out]  $\frac{1}{4}d^{5/4} \arctan(x(-a+d+bc)^{1/2}/(-c)^{1/4}/d^{1/4}/(bx^4+a)^{1/2})/(-c)^{3/4}/(-a+d+bc)^{3/2} - \frac{1}{4}d^{5/4} \arctan(x(a-d-bc)^{1/2}/(-c)^{1/4}/d^{1/4}/(bx^4+a)^{1/2})/(-c)^{3/4}/(a-d-bc)^{3/2} + \frac{1}{2}bx/a/(-a+d+bc)/(bx^4+a)^{1/2} + \frac{1}{4}b^{3/4}(\cos(2\arctan(b^{1/4}x/a^{1/4}))^2)^{1/2}/\cos(2\arctan(b^{1/4}x/a^{1/4})) * \text{EllipticF}(\sin(2\arctan(b^{1/4}x/a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2}+x^2b^{1/2}) * ((bx^4+a)/(a^{1/2}+x^2b^{1/2}))^{1/2} / a^{5/4} / (-a+d+bc) / (bx^4+a)^{1/2} - \frac{1}{8}d * (\cos(2\arctan(b^{1/4}x/a^{1/4}))^2)^{1/2} / \cos(2\arctan(b^{1/4}x/a^{1/4})) * \text{EllipticPi}(\sin(2\arctan(b^{1/4}x/a^{1/4})), 1/4, (b^{1/2}*(-c)^{1/2}+a^{1/2})d^{1/2})^2/a^{1/2}/b^{1/2}/(-c)^{1/2}/d^{1/2}, 1/2, 2^{1/2}) * (a^{1/2}+x^2b^{1/2}) * (b^{1/2}*(-c)^{1/2}-a^{1/2})d^{1/2})^2 * ((bx^4+a)/(a^{1/2}+x^2b^{1/2}))^{1/2} / a^{1/4}/b^{1/4}/c/(-a^2d^2+b^2c^2)/(bx^4+a)^{1/2} - \frac{1}{8}d * (\cos(2\arctan(b^{1/4}x/a^{1/4}))^2)^{1/2} / \cos(2\arctan(b^{1/4}x/a^{1/4})) * \text{EllipticPi}(\sin(2\arctan(b^{1/4}x/a^{1/4})), -1/4, (b^{1/2}*(-c)^{1/2}-a^{1/2})d^{1/2})^2/a^{1/2}/b^{1/2}/(-c)^{1/2}/d^{1/2}, 1/2, 2^{1/2}) * (a^{1/2}+x^2b^{1/2}) * (b^{1/2}*(-c)^{1/2}+a^{1/2})d^{1/2})^2 * ((bx^4+a)/(a^{1/2}+x^2b^{1/2}))^{1/2} / a^{1/4}/b^{1/4}/c/(-a^2d^2+b^2c^2)/(bx^4+a)^{1/2} - \frac{1}{4}b^{1/4}d * (\cos(2\arctan(b^{1/4}x/a^{1/4}))^2)^{1/2} / \cos(2\arctan(b^{1/4}x/a^{1/4})) * \text{EllipticF}(\sin(2\arctan(b^{1/4}x/a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2}+x^2b^{1/2}) * (b^{1/2}+a^{1/2})d^{1/2}/(-c)^{1/2}) * ((bx^4+a)/(a^{1/2}+x^2b^{1/2}))^{1/2} / a^{1/4}/(-a+d+bc)/(a+d+bc)/(bx^4+a)^{1/2} - \frac{1}{4}b^{1/4}d * (\cos(2\arctan(b^{1/4}x/a^{1/4}))^2)^{1/2} / \cos(2\arctan(b^{1/4}x/a^{1/4})) * \text{EllipticF}(\sin(2\arctan(b^{1/4}x/a^{1/4})), 1/2, 2^{1/2}) * (a^{1/2}+x^2b^{1/2}) * (c*b^{1/2}+a^{1/2}) * (-c)^{1/2}d^{1/2}) * ((bx^4+a)/(a^{1/2}+x^2b^{1/2}))^{1/2} / a^{1/4}/c/(-a^2d^2+b^2c^2)/(bx^4+a)^{1/2}$

**Rubi [A]** time = 1.13, antiderivative size = 913, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {414, 523, 220, 409, 1217, 1707}

$$\frac{d(\sqrt{b}x^2 + \sqrt{a}) \sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})^2}} \Pi\left(\frac{(\sqrt{b}\sqrt{-c}+\sqrt{a}\sqrt{d})^2}{4\sqrt{a}\sqrt{b}\sqrt{-c}\sqrt{d}}; 2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\middle|\frac{1}{2}\right) (\sqrt{b}\sqrt{-c} - \sqrt{a}\sqrt{d})^2 d^{5/4} \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-c}\sqrt[4]{d}}\right)}{8\sqrt[4]{a}\sqrt[4]{b}c(bc-ad)(bc+ad)\sqrt{bx^4+a}} + \frac{d^{5/4} \tan^{-1}\left(\frac{\sqrt{b}}{\sqrt[4]{-c}\sqrt[4]{d}}\right)}{4(-c)^{3/4}(bc-}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(3/2)\*(c + d\*x^4)), x]

[Out]  $\frac{(bx)/(2a*(bc-a*d)*\text{Sqrt}[a+b*x^4]) + (d^{5/4}*\text{ArcTan}[(\text{Sqrt}[b*c-a*d]*x)/((-c)^{1/4}*d^{1/4}*\text{Sqrt}[a+b*x^4])])/(4*(-c)^{3/4}*(bc-a*d)^{3/2}) - (d^{5/4}*\text{ArcTan}[(\text{Sqrt}[-(b*c)+a*d]*x)/((-c)^{1/4}*d^{1/4}*\text{Sqrt}[a+b*x^4])])/(4*(-c)^{3/4}*(-(b*c)+a*d)^{3/2}) + (b^{3/4}*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4})*x/a^{1/4}], 1/2])/(4*a^{5/4}*(bc-a*d)*\text{Sqrt}[a+b*x^4]) - (b^{1/4}*(\text{Sqrt}[b]+(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[-c]))*d*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4})*x/a^{1/4}], 1/2])/(4*a^{1/4}*(bc-a*d)*(bc+a*d)*\text{Sqrt}[a+b*x^4]) - (b^{1/4}*(\text{Sqrt}[b]*c+\text{Sqrt}[a]*\text{Sqrt}[-c]*\text{Sqrt}[d])*d*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[$

$a] + \text{Sqrt}[b]*x^2)^2 * \text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]/(4*a^{1/4}*c*(b^2*c^2 - a^2*d^2)*\text{Sqrt}[a + b*x^4]) - ((\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d])^2*d*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)]^2 * \text{EllipticPi}[-(\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2)]/(8*a^{1/4}*b^{1/4}*c*(b*c - a*d)*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) - ((\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d])^2*d*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)]^2 * \text{EllipticPi}[(\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2)]/(8*a^{1/4}*b^{1/4}*c*(b*c - a*d)*(b*c + a*d)*\text{Sqrt}[a + b*x^4])$

### Rule 220

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^4], x\_Symbol] \text{ :> With}[\{q = \text{Rt}[b/a, 4]\}, \text{Simp}[(1 + q^2*x^2)*\text{Sqrt}[(a + b*x^4)/(a*(1 + q^2*x^2)^2]]*\text{EllipticF}[2*\text{ArcTan}[q*x], 1/2]]/(2*q*\text{Sqrt}[a + b*x^4]), x]] \text{ /; FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[b/a]$

### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)^4]*((c_) + (d_)*(x_)^4)), x\_Symbol] \text{ :> Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] \text{ /; FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

### Rule 414

$\text{Int}(((a_) + (b_)*(x_)^{(n_)})^{(p_)*((c_) + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \text{ :> -Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x], x] \text{ /; FreeQ}\{a, b, c, d, n, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !( \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1]) \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

### Rule 523

$\text{Int}(((e_) + (f_)*(x_)^{(n_)})/(((a_) + (b_)*(x_)^{(n_)})*\text{Sqrt}[(c_) + (d_)*(x_)^{(n_)}]), x\_Symbol] \text{ :> Dist}[f/b, \text{Int}[1/\text{Sqrt}[c + d*x^n], x], x] + \text{Dist}[(b*e - a*f)/b, \text{Int}[1/((a + b*x^n)*\text{Sqrt}[c + d*x^n]), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f, n, x\}$

### Rule 1217

$\text{Int}[1/(((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (c_)*(x_)^4]), x\_Symbol] \text{ :> With}[\{q = \text{Rt}[c/a, 2]\}, \text{Dist}[(c*d + a*e*q)/(c*d^2 - a*e^2), \text{Int}[1/\text{Sqrt}[a + c*x^4], x], x] - \text{Dist}[(a*e*(e + d*q))/(c*d^2 - a*e^2), \text{Int}[(1 + q*x^2)/((d + e*x^2)*\text{Sqrt}[a + c*x^4]), x], x]] \text{ /; FreeQ}\{a, c, d, e, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[c/a]$

### Rule 1707

$\text{Int}(((A_) + (B_)*(x_)^2)/(((d_) + (e_)*(x_)^2)*\text{Sqrt}[(a_) + (c_)*(x_)^4]), x\_Symbol] \text{ :> With}[\{q = \text{Rt}[B/A, 2]\}, -\text{Simp}[(B*d - A*e)*\text{ArcTan}[(\text{Rt}[(c*d)/e + (a*e)/d, 2]*x)/\text{Sqrt}[a + c*x^4]]/(2*d*e*\text{Rt}[(c*d)/e + (a*e)/d, 2]), x] + \text{Simp}[(B*d + A*e)*(A + B*x^2)*\text{Sqrt}[(A^2*(a + c*x^4))/(a*(A + B*x^2)^2)]*\text{EllipticPi}[\text{Cancel}[-(B*d - A*e)^2/(4*d*e*A*B)], 2*\text{ArcTan}[q*x], 1/2)]/(4*d*e*A*q*\text{Sqrt}[a + c*x^4]), x]] \text{ /; FreeQ}\{a, c, d, e, A, B, x\} \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ \text{NeQ}[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[c/a] \ \&\& \ \text{EqQ}[c*A^2 - a*B^2, 0]$

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/2}(c+dx^4)} dx &= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} - \frac{\int \frac{-bc+2ad-bdx^4}{\sqrt{a+bx^4}(c+dx^4)} dx}{2a(bc-ad)} \\
&= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b \int \frac{1}{\sqrt{a+bx^4}} dx}{2a(bc-ad)} - \frac{d \int \frac{1}{\sqrt{a+bx^4}(c+dx^4)} dx}{bc-ad} \\
&= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} \\
&= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{b^{3/4}(\sqrt{a} + \sqrt{b}x^2) \sqrt{\frac{a+bx^4}{(\sqrt{a} + \sqrt{b}x^2)^2}} F\left(2 \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| \frac{1}{2}\right)}{4a^{5/4}(bc-ad)\sqrt{a+bx^4}} \\
&= \frac{bx}{2a(bc-ad)\sqrt{a+bx^4}} + \frac{d^{5/4} \tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c} \sqrt[4]{d} \sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(bc-ad)^{3/2}} - \frac{d^{5/4} \tan^{-1}\left(\frac{\sqrt{-bc+ad}x}{\sqrt[4]{-c} \sqrt[4]{d} \sqrt{a+bx^4}}\right)}{4(-c)^{3/4}(-bc+ad)^{3/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.28, size = 331, normalized size = 0.36

$$\frac{x \left( \frac{5(2bx^4(c+dx^4) \left( 2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5ac(2ad-b(2c+dx^4))F_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}{(c+dx^4) \left( 5acF_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2x^4 \left( 2adF_1\left(\frac{5}{4}, \frac{1}{2}, 2; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}, \frac{3}{2}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) \right)} - \frac{bdx^4 \sqrt{\frac{bx^4}{a} + 1} F_1\left(\frac{1}{4}, \frac{1}{2}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{10a\sqrt{a+bx^4}(ad-bc)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(3/2)\*(c + d\*x^4)), x]

[Out] (x\*(-((b\*d\*x^4\*Sqrt[1 + (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c) + (5\*(5\*a\*c\*(2\*a\*d - b\*(2\*c + d\*x^4))\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*b\*x^4\*(c + d\*x^4)\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] - 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((10\*a\*(-(b\*c) + a\*d)\*Sqrt[a + b\*x^4]))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/2)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/2)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(3/2)\*(d\*x^4 + c)), x)

**maple** [C] time = 0.29, size = 313, normalized size = 0.34

$$\frac{bx \sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} b \operatorname{EllipticF}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x, i\right)}{2(ad-bc)\sqrt{\left(x^4+\frac{a}{b}\right)ba}} + \frac{2\sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{RootOf}(d\_Z^4+cx^4+a)}{2(ad-bc)\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{bx^4+a}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(3/2)/(d\*x^4+c),x)

[Out] 
$$-1/2*b/a*x/(a*d-b*c)/((x^4+a/b)*b)^{(1/2)}-1/2/a*b/(a*d-b*c)/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(-I/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(I/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(b*x^4+a)^{(1/2)}*\operatorname{EllipticF}((I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, I)+1/8*\sum(1/(a*d-b*c))/\_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(2*\_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(b*x^4+a)^{(1/2)})+2/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*\_alpha^3*d/c*(-I/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(I/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(b*x^4+a)^{(1/2)}*\operatorname{EllipticPi}((I/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, I*\_alpha^2*a^{(1/2)}/b^{(1/2)}/c*d, (-I/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(I/a^{(1/2)}*b^{(1/2)})^{(1/2)}), \_alpha=\operatorname{RootOf}(d\_Z^4+cx^4+c))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/2)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(3/2)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{3/2}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(3/2)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(3/2)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{3}{2}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(3/2)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(3/2)\*(c + d\*x\*\*4)), x)

$$3.183 \quad \int \frac{1}{(a+bx^4)^{5/2}(c+dx^4)} dx$$

**Optimal.** Leaf size=976

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}} + \frac{\sqrt[4]{b}(\sqrt{bc}-\sqrt{a}\sqrt{-c}\sqrt{d})(\sqrt{b}x^2+\sqrt{a})\sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})}}}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{bx^4}}$$

[Out]  $1/6*b*x/a/(-a*d+b*c)/(b*x^4+a)^{(3/2)}-1/4*d^{(9/4)}*\arctan(x*(-a*d+b*c)^{(1/2)}/(-c)^{(1/4)}/d^{(1/4)}/(b*x^4+a)^{(1/2))}/(-c)^{(3/4)}/(-a*d+b*c)^{(5/2)}-1/4*d^{(9/4)}*\arctan(x*(a*d-b*c)^{(1/2)}/(-c)^{(1/4)}/d^{(1/4)}/(b*x^4+a)^{(1/2))}/(-c)^{(3/4)}/(a*d-b*c)^{(5/2)}+1/12*b*(-11*a*d+5*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^{(1/2)}+1/24*b^{(3/4)}*(-11*a*d+5*b*c)*(cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))^2)^{(1/2)}/cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))*EllipticF(sin(2*\arctan(b^{(1/4)}*x/a^{(1/4)})),1/2*2^{(1/2)}*(a^{(1/2)}+x^2*b^{(1/2)})*(b*x^4+a)/(a^{(1/2)}+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(9/4)}/(-a*d+b*c)^2/(b*x^4+a)^{(1/2)}+1/8*d^2*(cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))^2)^{(1/2)}/cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))*EllipticPi(sin(2*\arctan(b^{(1/4)}*x/a^{(1/4)})),1/4*(b^{(1/2)}*(-c)^{(1/2)}+a^{(1/2)}*d^{(1/2)})^2/a^{(1/2)}/b^{(1/2)}/(-c)^{(1/2)}/d^{(1/2)},1/2*2^{(1/2)}*(a^{(1/2)}+x^2*b^{(1/2)})*(b^{(1/2)}*(-c)^{(1/2)}-a^{(1/2)}*d^{(1/2)})^2*(b*x^4+a)/(a^{(1/2)}+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/b^{(1/4)}/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/8*d^2*(cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))^2)^{(1/2)}/cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))*EllipticPi(sin(2*\arctan(b^{(1/4)}*x/a^{(1/4)})),-1/4*(b^{(1/2)}*(-c)^{(1/2)}-a^{(1/2)}*d^{(1/2)})^2/a^{(1/2)}/b^{(1/2)}/(-c)^{(1/2)}/d^{(1/2)},1/2*2^{(1/2)}*(a^{(1/2)}+x^2*b^{(1/2)})*(b^{(1/2)}*(-c)^{(1/2)}+a^{(1/2)}*d^{(1/2)})^2*(b*x^4+a)/(a^{(1/2)}+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/b^{(1/4)}/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/4*b^{(1/4)}*d^2*(cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))^2)^{(1/2)}/cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))*EllipticF(sin(2*\arctan(b^{(1/4)}*x/a^{(1/4)})),1/2*2^{(1/2)}*(a^{(1/2)}+x^2*b^{(1/2)})*(c*b^{(1/2)}-a^{(1/2)}*(-c)^{(1/2)}*d^{(1/2)})*(b*x^4+a)/(a^{(1/2)}+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^{(1/2)}+1/4*b^{(1/4)}*d^2*(cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))^2)^{(1/2)}/cos(2*\arctan(b^{(1/4)}*x/a^{(1/4)}))*EllipticF(sin(2*\arctan(b^{(1/4)}*x/a^{(1/4)})),1/2*2^{(1/2)}*(a^{(1/2)}+x^2*b^{(1/2)})*(c*b^{(1/2)}+a^{(1/2)}*(-c)^{(1/2)}*d^{(1/2)})*(b*x^4+a)/(a^{(1/2)}+x^2*b^{(1/2)})^2)^{(1/2)}/a^{(1/4)}/c/(-a*d+b*c)^2/(a*d+b*c)/(b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 1.68, antiderivative size = 976, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {414, 527, 523, 220, 409, 1217, 1707}

$$\frac{\tan^{-1}\left(\frac{\sqrt{bc-ad}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(bc-ad)^{5/2}} - \frac{\tan^{-1}\left(\frac{\sqrt{ad-bc}x}{\sqrt[4]{-c}\sqrt[4]{d}\sqrt{bx^4+a}}\right)d^{9/4}}{4(-c)^{3/4}(ad-bc)^{5/2}} + \frac{\sqrt[4]{b}(\sqrt{bc}-\sqrt{a}\sqrt{-c}\sqrt{d})(\sqrt{b}x^2+\sqrt{a})\sqrt{\frac{bx^4+a}{(\sqrt{b}x^2+\sqrt{a})}}}{4\sqrt[4]{a}c(bc-ad)^2(bc+ad)\sqrt{bx^4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)),x]

[Out]  $(b*x)/(6*a*(b*c-a*d)*(a+b*x^4)^{(3/2)})+(b*(5*b*c-11*a*d)*x)/(12*a^2*(b*c-a*d)^2*\text{Sqrt}[a+b*x^4])-(d^{(9/4)}*\text{ArcTan}[(\text{Sqrt}[b*c-a*d]*x)/((-c)^{(1/4)}*d^{(1/4)}*\text{Sqrt}[a+b*x^4])]/(4*(-c)^{(3/4)}*(b*c-a*d)^{(5/2)})-(d^{(9/4)}*\text{ArcTan}[(\text{Sqrt}[-(b*c)+a*d]*x)/((-c)^{(1/4)}*d^{(1/4)}*\text{Sqrt}[a+b*x^4])]/(4*(-c)^{(3/4)}*(-(b*c)+a*d)^{(5/2)})+(b^{(3/4)}*(5*b*c-11*a*d)*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}],1/2]/(24*a^{(9/4)}*(b*c-a*d)^2*\text{Sqrt}[a+b*x^4])+(b^{(1/4)}*(\text{Sqrt}[b]*c-\text{Sqrt}[a]*\text{Sqrt}[-c]*\text{Sqrt}[d])*d^2*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}],1/2]/(24*a^{(9/4)}*(b*c-a*d)^2*\text{Sqrt}[a+b*x^4])+(b^{(1/4)}*(\text{Sqrt}[b]*c-\text{Sqrt}[a]*\text{Sqrt}[-c]*\text{Sqrt}[d])*d^2*(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)*\text{Sqrt}[(a+b*x^4)/(\text{Sqrt}[a]+\text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{(1/4)}*x)/a^{(1/4)}],1/2]/(24*a^{(9/4)}*(b*c-a*d)^2*\text{Sqrt}[a+b*x^4])$

$$\begin{aligned} & /4]], 1/2]]/(4*a^{1/4}*c*(b*c - a*d)^2*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + (b^{1/4}*(\text{Sqrt}[b]*c + \text{Sqrt}[a]*\text{Sqrt}[-c]*\text{Sqrt}[d])*d^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt} \\ & [(a + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticF}[2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}], 1/2]]/(4*a^{1/4}*c*(b*c - a*d)^2*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + ((\text{Sqr} \\ & \text{t}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]*\text{Sqrt}[d])^2*d^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a + b* \\ & x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticPi}[-(\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqr} \\ & \text{t}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4}] \\ & , 1/2]]/(8*a^{1/4}*b^{1/4}*c*(b*c - a*d)^2*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) + ( \\ & (\text{Sqrt}[b]*\text{Sqrt}[-c] - \text{Sqrt}[a]*\text{Sqrt}[d])^2*d^2*(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)*\text{Sqrt}[(a \\ & + b*x^4)/(\text{Sqrt}[a] + \text{Sqrt}[b]*x^2)^2]*\text{EllipticPi}[(\text{Sqrt}[b]*\text{Sqrt}[-c] + \text{Sqrt}[a]* \\ & \text{Sqrt}[d])^2/(4*\text{Sqrt}[a]*\text{Sqrt}[b]*\text{Sqrt}[-c]*\text{Sqrt}[d]), 2*\text{ArcTan}[(b^{1/4}*x)/a^{1/4} \\ & ], 1/2]]/(8*a^{1/4}*b^{1/4}*c*(b*c - a*d)^2*(b*c + a*d)*\text{Sqrt}[a + b*x^4]) \end{aligned}$$

#### Rule 220

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[b/a, 4]}, Simp[(
(1 + q^2*x^2)*Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]*EllipticF[2*ArcTan[q*x]
, 1/2]]/(2*q*Sqrt[a + b*x^4]), x] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

#### Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

#### Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

#### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x_)
^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

#### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

#### Rule 1217

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/Sqrt[a + c*x^4]
, x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^
2)*Sqrt[a + c*x^4]), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2
, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

## Rule 1707

Int[((A\_) + (B\_.)\*(x\_)^2)/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B\*d - A\*e)\*ArcTan[(Rt[(c\*d)/e + (a\*e)/d, 2]\*x)/Sqrt[a + c\*x^4]])/(2\*d\*e\*Rt[(c\*d)/e + (a\*e)/d, 2]), x] + Simp[((B\*d + A\*e)\*(A + B\*x^2)\*Sqrt[(A^2\*(a + c\*x^4))/(a\*(A + B\*x^2)^2)]\*EllipticPi[Cancel[-((B\*d - A\*e)^2/(4\*d\*e\*A\*B))], 2\*ArcTan[q\*x], 1/2])/(4\*d\*e\*A\*q\*Sqrt[a + c\*x^4]), x]] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c\*d^2 + a\*e^2, 0] && NeQ[c\*d^2 - a\*e^2, 0] && PosQ[c/a] && EqQ[c\*A^2 - a\*B^2, 0]

## Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^4)^{5/2} (c + dx^4)} dx &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} - \frac{\int \frac{-5bc + 6ad - 5bdx^4}{(a + bx^4)^{3/2}(c + dx^4)} dx}{6a(bc - ad)} \\ &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2\sqrt{a + bx^4}} + \frac{\int \frac{5b^2c^2 - 11abcd + 12a^2d^2 + bd(5b^2c - 11ad)}{\sqrt{a + bx^4}(c + dx^4)} dx}{12a^2(bc - ad)^2} \\ &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2\sqrt{a + bx^4}} + \frac{d^2 \int \frac{1}{\sqrt{a + bx^4}(c + dx^4)} dx}{(bc - ad)^2} + \dots \\ &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2\sqrt{a + bx^4}} + \frac{b^{3/4}(5bc - 11ad)(\sqrt{a} + \sqrt{c + dx^4})}{24a^9} \\ &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2\sqrt{a + bx^4}} + \frac{b^{3/4}(5bc - 11ad)(\sqrt{a} + \sqrt{c + dx^4})}{24a^9} \\ &= \frac{bx}{6a(bc - ad)(a + bx^4)^{3/2}} + \frac{b(5bc - 11ad)x}{12a^2(bc - ad)^2\sqrt{a + bx^4}} - \frac{d^{9/4} \tan^{-1}\left(\frac{\sqrt{bc - ad}x}{\sqrt[4]{-c} \sqrt[4]{d} \sqrt{a + bx^4}}\right)}{4(-c)^{3/4}(bc - ad)^{5/2}} \end{aligned}$$

**Mathematica [C]** time = 0.99, size = 429, normalized size = 0.44

$$x \left( \frac{5 \left( 2bx^4(c + dx^4)(13a^2d + ab(11dx^4 - 7c) - 5b^2cx^4) \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5ac(12a^3d^2 - a^2bd(24c + dx^4)) + ab^2(13a^2d^2 - a^2bd(24c + dx^4))}{(a + bx^4)(c + dx^4) \left( 2x^4 \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 5acF_1\left(\frac{1}{4}; \frac{1}{2}, 1, \frac{5}{4}, -\frac{(b*x^4)}{a}, -\frac{(d*x^4)}{c}\right) + 2*b*x^4*(c + d*x^4)*(13*a^2*d - 5*b^2*c*x^4 + a*b*(-7*c + 11*d*x^4))*(2*a*d*AppellF1[5/4, 1/2, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + b*c*AppellF1[5/4, 3/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])} \right) \right) / ((a + b*x^4)*(c + d*x^4)*(-5*a*c*AppellF1[5/4, 1/2, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)), x]

[Out] -1/60\*(x\*((b\*d\*(-5\*b\*c + 11\*a\*d)\*x^4\*Sqrt[1 + (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c + (5\*(5\*a\*c\*(12\*a^3\*d^2 + 5\*b^3\*c\*x^4\*(2\*c + d\*x^4) - a^2\*b\*d\*(24\*c + d\*x^4) + a\*b^2\*(12\*c^2 - 15\*c\*d\*x^4 - 11\*d^2\*x^8))\*AppellF1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*b\*x^4\*(c + d\*x^4)\*(13\*a^2\*d - 5\*b^2\*c\*x^4 + a\*b\*(-7\*c + 11\*d\*x^4))\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((a + b\*x^4)\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

11F1[1/4, 1/2, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((a^2\*(b\*c - a\*d)^2\*Sqrt[a + b\*x^4])

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/2)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/2)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(5/2)\*(d\*x^4 + c)), x)

**maple** [C] time = 0.34, size = 371, normalized size = 0.38

$$d \frac{\left( 2\sqrt{-\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{i\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{RootOf}(d\_Z^4+c)^3 d \operatorname{EllipticPi}\left(\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}x, \frac{i\operatorname{RootOf}(d\_Z^4+c)^2 \sqrt{a} d}{\sqrt{b}c}, \sqrt{\frac{-i\sqrt{b}}{\sqrt{a}}}\right) - \operatorname{arctanh}\left(\frac{2\operatorname{RootOf}(d\_Z^4+c)^2 b x^2+2a}{2\sqrt{\frac{ad-bc}{d}} \sqrt{bx^4+a}}\right) \right)}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}} \sqrt{bx^4+a} c \sqrt{\frac{ad-bc}{d}}} \frac{1}{8(ad-bc)^2 \operatorname{RootOf}(d\_Z^4+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(5/2)/(d\*x^4+c),x)

[Out] -1/6/a\*x/b/(a\*d-b\*c)\*(b\*x^4+a)^(1/2)/(x^4+a/b)^2-1/12\*b/a^2\*x\*(11\*a\*d-5\*b\*c)/(a\*d-b\*c)^2/((x^4+a/b)\*b)^(1/2)-1/12/a^2\*b\*(11\*a\*d-5\*b\*c)/(a\*d-b\*c)^2/(I/a^(1/2)\*b^(1/2))^(1/2)\*(-I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticF((I/a^(1/2)\*b^(1/2))^(1/2)\*x,I)+1/8\*d\*sum(1/(a\*d-b\*c)^2/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(b\*x^4+a)^(1/2))+2/(I/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(I/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(b\*x^4+a)^(1/2)\*EllipticPi((I/a^(1/2)\*b^(1/2))^(1/2)\*x,I\*\_alpha^2\*a^(1/2)/b^(1/2)/c\*d,(-I/a^(1/2)\*b^(1/2))^(1/2)/(I/a^(1/2)\*b^(1/2))^(1/2))),\_alpha=RootOf(\_Z^4\*d+c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{2}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/2)/(d\*x^4+c),x, algorithm="maxima")



[Out] integrate(1/((b\*x^4 + a)^(5/2)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{5/2} (dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)), x)

[Out] int(1/((a + b\*x^4)^(5/2)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{5/2} (c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(5/2)/(d\*x\*\*4+c), x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(5/2)\*(c + d\*x\*\*4)), x)

$$3.184 \quad \int \frac{(a-bx^4)^{7/2}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=426

$$\frac{bx\sqrt{a-bx^4} (21a^2d^2 - 122abcd + 77b^2c^2)}{84cd^3} + \frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (21a^3d^3 + 349a^2bcd^2 - 553ab^2c^2d + 231b^3c^3) F\left(\sin^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)\right)}{84cd^4\sqrt{a-bx^4}}$$

[Out] 1/28\*b\*(-7\*a\*d+11\*b\*c)\*x\*(-b\*x^4+a)^(3/2)/c/d^2-1/4\*(-a\*d+b\*c)\*x\*(-b\*x^4+a)^(5/2)/c/d/(-d\*x^4+c)-1/84\*b\*(21\*a^2\*d^2-122\*a\*b\*c\*d+77\*b^2\*c^2)\*x\*(-b\*x^4+a)^(1/2)/c/d^3+1/84\*a^(1/4)\*b^(3/4)\*(21\*a^3\*d^3+349\*a^2\*b\*c\*d^2-553\*a\*b^2\*c^2\*d+231\*b^3\*c^3)\*EllipticF(b^(1/4)\*x/a^(1/4),I)\*(1-b\*x^4/a)^(1/2)/c/d^4/(-b\*x^4+a)^(1/2)-1/8\*a^(1/4)\*(-a\*d+b\*c)^3\*(3\*a\*d+11\*b\*c)\*EllipticPi(b^(1/4)\*x/a^(1/4),-a^(1/2)\*d^(1/2)/b^(1/2)/c^(1/2),I)\*(1-b\*x^4/a)^(1/2)/b^(1/4)/c^2/d^4/(-b\*x^4+a)^(1/2)-1/8\*a^(1/4)\*(-a\*d+b\*c)^3\*(3\*a\*d+11\*b\*c)\*EllipticPi(b^(1/4)\*x/a^(1/4),a^(1/2)\*d^(1/2)/b^(1/2)/c^(1/2),I)\*(1-b\*x^4/a)^(1/2)/b^(1/4)/c^2/d^4/(-b\*x^4+a)^(1/2)

**Rubi [A]** time = 0.54, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$\frac{bx\sqrt{a-bx^4} (21a^2d^2 - 122abcd + 77b^2c^2)}{84cd^3} + \frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (349a^2bcd^2 + 21a^3d^3 - 553ab^2c^2d + 231b^3c^3) F\left(\sin^{-1}\left(\frac{\sqrt{a-bx^4}}{\sqrt{a}}\right)\right)}{84cd^4\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(7/2)/(c - d\*x^4)^2, x]

[Out] -(b\*(77\*b^2\*c^2 - 122\*a\*b\*c\*d + 21\*a^2\*d^2)\*x\*Sqrt[a - b\*x^4])/(84\*c\*d^3) + (b\*(11\*b\*c - 7\*a\*d)\*x\*(a - b\*x^4)^(3/2))/(28\*c\*d^2) - ((b\*c - a\*d)\*x\*(a - b\*x^4)^(5/2))/(4\*c\*d\*(c - d\*x^4)) + (a^(1/4)\*b^(3/4)\*(231\*b^3\*c^3 - 553\*a\*b^2\*c^2\*d + 349\*a^2\*b\*c\*d^2 + 21\*a^3\*d^3)\*Sqrt[1 - (b\*x^4)/a]\*EllipticF[ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(84\*c\*d^4\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)^3\*(11\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[-((Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^4\*Sqrt[a - b\*x^4]) - (a^(1/4)\*(b\*c - a\*d)^3\*(11\*b\*c + 3\*a\*d)\*Sqrt[1 - (b\*x^4)/a]\*EllipticPi[(Sqrt[a]\*Sqrt[d])/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/a^(1/4)], -1])/(8\*b^(1/4)\*c^2\*d^4\*Sqrt[a - b\*x^4])

**Rule 221**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

**Rule 224**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

**Rule 409**

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a,

b, c, d}, x] && NeQ[b\*c - a\*d, 0]

### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rule 523

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/(((a\_) + (b\_.)\*(x\_)^(n\_))\*Sqrt[(c\_) + (d\_.)\*(x\_)^(n\_)]), x\_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e - a\*f)/b, Int[1/((a + b\*x^n)\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

### Rule 528

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(f\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(b\*(n\*(p + q + 1) + 1)), x] + Dist[1/(b\*(n\*(p + q + 1) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*(b\*e - a\*f + b\*e\*n\*(p + q + 1)) + (d\*(b\*e - a\*f) + f\*n\*q\*(b\*c - a\*d) + b\*d\*e\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n\*(p + q + 1) + 1, 0]

### Rule 1218

Int[1/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] :> With[{q = Rt[-(c/a), 4]}, Simp[(1\*EllipticPi[-(e/(d\*q^2)), ArcSin[q\*x], -1])/(d\*Sqrt[a]\*q), x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

### Rule 1219

Int[1/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] :> Dist[Sqrt[1 + (c\*x^4)/a]/Sqrt[a + c\*x^4], Int[1/((d + e\*x^2)\*Sqrt[1 + (c\*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{(a - bx^4)^{7/2}}{(c - dx^4)^2} dx &= -\frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} - \int \frac{(a - bx^4)^{3/2}(-a(bc + 3ad) + b(11bc - 7ad)x^4)}{c - dx^4} dx \\
&= \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} + \int \frac{\sqrt{a - bx^4}(-a(11b^2c^2 - 14abcd - 21a^2d^2) + b(77b^2c^2 - 122abcd + 21a^2d^2)x)}{c - dx^4} dx \\
&= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)x\sqrt{a - bx^4}}{84cd^3} + \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} \\
&= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)x\sqrt{a - bx^4}}{84cd^3} + \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} \\
&= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)x\sqrt{a - bx^4}}{84cd^3} + \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} \\
&= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)x\sqrt{a - bx^4}}{84cd^3} + \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)} \\
&= -\frac{b(77b^2c^2 - 122abcd + 21a^2d^2)x\sqrt{a - bx^4}}{84cd^3} + \frac{b(11bc - 7ad)x(a - bx^4)^{3/2}}{28cd^2} - \frac{(bc - ad)x(a - bx^4)^{5/2}}{4cd(c - dx^4)}
\end{aligned}$$

**Mathematica** [C] time = 0.92, size = 477, normalized size = 1.12

$$bx^5 \sqrt{1 - \frac{bx^4}{a}} \left( 21a^3d^3 + 349a^2bcd^2 - 553ab^2c^2d + 231b^3c^3 \right) F_1 \left( \frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c} \right) + \frac{5c \left( 2x^5(bx^4 - a)(21a^3d^3 - 63a^2bcd^2 + a^3d^3) \right)}{c^2d^3 \sqrt{a - bx^4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a - b\*x^4)^(7/2)/(c - d\*x^4)^2, x]

[Out] 
$$-\frac{1}{420} (b(231b^3c^3 - 553a^2b^2c^2d + 349a^2b^2c^2d + 21a^3d^3)) x^5 \sqrt{1 - \frac{bx^4}{a}} \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + (5c^2d^3 \sqrt{a - bx^4} (21a^3d^3 - 63a^2bcd^2 + a^3d^3) + (5c^2d^3 \sqrt{a - bx^4} (-84a^4d^3 + 29a^2b^2c^2d^2x^4 + 21a^3b^2d^3x^4 + a^2b^3c^2d^4(111c - 104dx^4) + b^4c^2x^4(-77c^2 + 44cdx^4 + 12d^2x^8)) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^5(-a + bx^4)(-63a^2b^2c^2d^2 + 21a^3d^3 + a^2b^2c^2d(155c - 92dx^4) + b^3c^2(-77c^2 + 44cdx^4 + 12d^2x^8)) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c^2 \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right])) / (c^2d^3 \sqrt{a - bx^4})$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(7/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{\frac{7}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(7/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((-b\*x^4 + a)^(7/2)/(d\*x^4 - c)^2, x)

**maple** [C] time = 0.29, size = 540, normalized size = 1.27

$$-\frac{\sqrt{-bx^4 + a} b^3 x^5}{7d^2} + \frac{\left( \frac{\left( \frac{5ab^3}{7d^2} - \frac{2(2ad-bc)b^3}{d^3} \right) a}{3b} + \frac{(6a^2d^2 - 8abcd + 3b^2c^2)b^2}{d^4} + \frac{(a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3)b}{4cd^4} \right) \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(7/2)/(-d\*x^4+c)^2,x)

[Out] 
$$-1/4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c/d^3*x*(-b*x^4+a)^{(1/2)}/(d*x^4-c)-1/7*b^3/d^2*x^5*(-b*x^4+a)^{(1/2)}-1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*x*(-b*x^4+a)^{(1/2)}+(b^2*(6*a^2*d^2-8*a*b*c*d+3*b^2*c^2)/d^4+1/4*b/d^4*(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/c+1/3*(-2*b^3/d^3*(2*a*d-b*c)+5/7*b^3/d^2*a)/b*a)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(-1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticF((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x,I)-1/32/d^5/c*sum((3*a^4*d^4+2*a^3*b*c*d^3-24*a^2*b^2*c^2*d^2+30*a*b^3*c^3*d-11*b^4*c^4)/_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*_alpha^3*d/c*(-1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticPi((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x,_alpha^2*a^{(1/2)}/b^{(1/2)}/c*d,(-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}),_alpha=RootOf(_Z^4*d-c))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{\frac{7}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(7/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((-b\*x^4 + a)^(7/2)/(d\*x^4 - c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - bx^4)^{7/2}}{(c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - b*x^4)^(7/2)/(c - d*x^4)^2,x)
```

```
[Out] int((a - b*x^4)^(7/2)/(c - d*x^4)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(7/2)/(-d*x**4+c)**2,x)
```

```
[Out] Timed out
```

$$3.185 \quad \int \frac{(a-bx^4)^{5/2}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=365

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (-3a^2d^2 - 26abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (3ad + 7bc)(bc - ad)^2 \Pi\left(-\frac{\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}{bc - ad}\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

[Out]  $-1/4*(-a*d+b*c)*x*(-b*x^4+a)^{(3/2)}/c/d/(-d*x^4+c)+1/12*b*(-3*a*d+7*b*c)*x*(-b*x^4+a)^{(1/2)}/c/d^2-1/12*a^{(1/4)}*b^{(3/4)}*(-3*a^2*d^2-26*a*b*c*d+21*b^2*c^2)*\text{EllipticF}(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/c/d^3/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*(-a*d+b*c)^2*(3*a*d+7*b*c)*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d^3/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*(-a*d+b*c)^2*(3*a*d+7*b*c)*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d^3/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.40, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {413, 528, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (-3a^2d^2 - 26abcd + 21b^2c^2) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{12cd^3\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (3ad + 7bc)(bc - ad)^2 \Pi\left(-\frac{\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}{bc - ad}\right)}{8\sqrt[4]{b}c^2d^3\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(5/2)/(c - d\*x^4)^2, x]

[Out]  $(b*(7*b*c - 3*a*d)*x*\text{Sqrt}[a - b*x^4]/(12*c*d^2) - ((b*c - a*d)*x*(a - b*x^4)^{(3/2)})/(4*c*d*(c - d*x^4)) - (a^{(1/4)}*b^{(3/4)}*(21*b^2*c^2 - 26*a*b*c*d - 3*a^2*d^2)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(12*c*d^3*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(b*c - a*d)^2*(7*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^3*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(b*c - a*d)^2*(7*b*c + 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^3*\text{Sqrt}[a - b*x^4])$

**Rule 221**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

**Rule 224**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

**Rule 409**

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

Rubi steps



$$\begin{aligned}
\int \frac{(a - bx^4)^{5/2}}{(c - dx^4)^2} dx &= -\frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} - \frac{\int \frac{\sqrt{a - bx^4}(-a(bc + 3ad) + b(7bc - 3ad)x^4)}{c - dx^4} dx}{4cd} \\
&= \frac{b(7bc - 3ad)x\sqrt{a - bx^4}}{12cd^2} - \frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} + \frac{\int \frac{-a(7b^2c^2 - 6abcd - 9a^2d^2) + b(21b^2c^2 - 26abcd - 3a^2d^2)}{\sqrt{a - bx^4}(c - dx^4)} dx}{12cd^2} \\
&= \frac{b(7bc - 3ad)x\sqrt{a - bx^4}}{12cd^2} - \frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} + \frac{((bc - ad)^2(7bc + 3ad)) \int \frac{1}{\sqrt{a - bx^4}(c - dx^4)} dx}{4cd^3} \\
&= \frac{b(7bc - 3ad)x\sqrt{a - bx^4}}{12cd^2} - \frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} + \frac{((bc - ad)^2(7bc + 3ad)) \int \frac{1}{\left(1 - \frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a - bx^4}} dx}{8c^2d^3} \\
&= \frac{b(7bc - 3ad)x\sqrt{a - bx^4}}{12cd^2} - \frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2 - 26abcd - 3a^2d^2)\sqrt{a - bx^4}}{12cd^3\sqrt{a - bx^4}} \\
&= \frac{b(7bc - 3ad)x\sqrt{a - bx^4}}{12cd^2} - \frac{(bc - ad)x(a - bx^4)^{3/2}}{4cd(c - dx^4)} - \frac{\sqrt[4]{a}b^{3/4}(21b^2c^2 - 26abcd - 3a^2d^2)\sqrt{a - bx^4}}{12cd^3\sqrt{a - bx^4}}
\end{aligned}$$

**Mathematica [C]** time = 0.67, size = 396, normalized size = 1.08

$$bx^5 \sqrt{1 - \frac{bx^4}{a}} (3a^2d^2 + 26abcd - 21b^2c^2) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + \frac{5c(2x^5(a - bx^4)(3a^2d^2 - 6abcd + b^2c(7c - 4dx^4)))(2adF_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + 2a^2c^2)}{(dx^4 - c)(2x^4(2a^2c^2 - 6abcd + b^2c(7c - 4dx^4)) + 2a^2c^2)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a - b\*x^4)^(5/2)/(c - d\*x^4)^2,x]

[Out] 
$$-\frac{1}{60} \frac{(b(-21b^2c^2 + 26abc^2d + 3a^2d^2)x^5 \sqrt{1 - \frac{bx^4}{a}} \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + (5c(5a^2c^2x^5(12a^3d^2 + 2a^2b^2cdx^4 - 3a^2b^2d^2x^4 + b^3c^2x^4(-7c + 4dx^4)) \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^5(a - bx^4)(-6abc^2d + 3a^2d^2 + b^2c(7c - 4dx^4)) \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]))}{(-c + dx^4)(5a^2c^2 \text{AppellF1}\left[\frac{1}{4}, \frac{1}{2}, 1, \frac{5}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + 2x^4(2a^2d^2 \text{AppellF1}\left[\frac{5}{4}, \frac{1}{2}, 2, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right] + b^2c \text{AppellF1}\left[\frac{5}{4}, \frac{3}{2}, 1, \frac{9}{4}, \frac{bx^4}{a}, \frac{dx^4}{c}\right]))}{(c^2d^2 \sqrt{a - bx^4})}$$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{5/2}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c)^2, x)

maple [C] time = 0.34, size = 412, normalized size = 1.13

$$\frac{\sqrt{-bx^4+a} b^2 x}{3d^2} + \frac{\left(-\frac{ab^2}{3d^2} + \frac{(3ad-2bc)b^2}{d^3} + \frac{(a^2d^2-2abcd+b^2c^2)b}{4cd^3}\right) \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}}+1} \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}x, i\right) (a^2d^2 - \dots)}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x)

[Out]  $-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c/d^2*x*(-b*x^4+a)^{(1/2)}/(d*x^4-c)+1/3*b^2/d^2*x*(-b*x^4+a)^{(1/2)}+(b^2*(3*a*d-2*b*c)/d^3+1/4*b/d^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/c-1/3*b^2/d^2*a)/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*(-1/a^{(1/2)}*b^{(1/2)}*x^{2+1})^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^{2+1})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticF}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, I)-1/32/c/d^4*\sum((3*a^3*d^3+a^2*b*c*d^2-11*a*b^2*c^2*d+7*b^3*c^3)/\_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*\operatorname{arctanh}(1/2*(-2*\_alpha^2*b*x^{2+2}*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)}))-2/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*\_alpha^3*d/c*(-1/a^{(1/2)}*b^{(1/2)}*x^{2+1})^{(1/2)}*(1/a^{(1/2)}*b^{(1/2)}*x^{2+1})^{(1/2)}/(-b*x^4+a)^{(1/2)}*\operatorname{EllipticPi}((1/a^{(1/2)}*b^{(1/2)})^{(1/2)}*x, \_alpha^2*a^{(1/2)}/b^{(1/2)}/c*d, (-1/a^{(1/2)}*b^{(1/2)})^{(1/2)}/(1/a^{(1/2)}*b^{(1/2)})^{(1/2)}), \_alpha=\operatorname{RootOf}(\_Z^4*d-c))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{\frac{5}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((-b\*x^4 + a)^(5/2)/(d\*x^4 - c)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - bx^4)^{\frac{5}{2}}}{(c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(5/2)/(c - d\*x^4)^2,x)

[Out] int((a - b\*x^4)^(5/2)/(c - d\*x^4)^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a - bx^4)^{\frac{5}{2}}}{(-c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b*x**4+a)**(5/2)/(-d*x**4+c)**2,x)
```

```
[Out] Integral((a - b*x**4)**(5/2)/(-c + d*x**4)**2, x)
```

$$3.186 \quad \int \frac{(a-bx^4)^{3/2}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=309

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 3bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2 \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 d^2 \sqrt{a - bx^4}}$$

[Out]  $-1/4*(-a*d+b*c)*x*(-b*x^4+a)^{(1/2)}/c/d/(-d*x^4+c)+1/4*a^{(1/4)}*b^{(3/4)}*(a*d+3*b*c)*\text{EllipticF}(b^{(1/4)}*x/a^{(1/4)}, I)*(1-b*x^4/a)^{(1/2)}/c/d^2/(-b*x^4+a)^{(1/2)}-3/8*a^{(1/4)}*(-a*d+b*c)*(a*d+b*c)*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)}, -a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d^2/(-b*x^4+a)^{(1/2)}-3/8*a^{(1/4)}*(-a*d+b*c)*(a*d+b*c)*\text{EllipticPi}(b^{(1/4)}*x/a^{(1/4)}, a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d^2/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.28, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {413, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 3bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2 \sqrt{a - bx^4}} - \frac{3\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - ad)(ad + bc) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 d^2 \sqrt{a - bx^4}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^4)^(3/2)/(c - d\*x^4)^2, x]

[Out]  $-((b*c - a*d)*x*\text{Sqrt}[a - b*x^4])/(4*c*d*(c - d*x^4)) + (a^{(1/4)}*b^{(3/4)}*(3*b*c + a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*c*d^2*\text{Sqrt}[a - b*x^4]) - (3*a^{(1/4)}*(b*c - a*d)*(b*c + a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b*x^4]) - (3*a^{(1/4)}*(b*c - a*d)*(b*c + a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d^2*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 413

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
+ q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_)*(x_)^(n_))/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_) + (d_)*(x
_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

### Rule 1218

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_)*(x_)^2)*Sqrt[(a_) + (c_)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a
]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2} dx = -\frac{(bc - ad)x\sqrt{a - bx^4}}{4cd(c - dx^4)} - \frac{\int \frac{-a(bc+3ad)+b(3bc+ad)x^4}{\sqrt{a-bx^4}(c-dx^4)} dx}{4cd}$$

$$= -\frac{(bc - ad)x\sqrt{a - bx^4}}{4cd(c - dx^4)} + \frac{\left(3\left(a^2 - \frac{b^2c^2}{d^2}\right)\right) \int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx}{4c} + \frac{(b(3bc + ad)) \int \frac{1}{\sqrt{a-bx^4}} dx}{4cd^2}$$

$$= -\frac{(bc - ad)x\sqrt{a - bx^4}}{4cd(c - dx^4)} + \frac{\left(3\left(a^2 - \frac{b^2c^2}{d^2}\right)\right) \int \frac{1}{\left(1 - \frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2} + \frac{\left(3\left(a^2 - \frac{b^2c^2}{d^2}\right)\right) \int \frac{1}{\left(1 + \frac{\sqrt{d}x^2}{\sqrt{c}}\right)}}{8c^2}$$

$$= -\frac{(bc - ad)x\sqrt{a - bx^4}}{4cd(c - dx^4)} + \frac{\sqrt[4]{a} b^{3/4} (3bc + ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2\sqrt{a - bx^4}} + \frac{\left(3\left(a^2 - \frac{b^2c^2}{d^2}\right)\right)}{4cd^2}$$

$$= -\frac{(bc - ad)x\sqrt{a - bx^4}}{4cd(c - dx^4)} + \frac{\sqrt[4]{a} b^{3/4} (3bc + ad) \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd^2\sqrt{a - bx^4}} + \frac{3\sqrt[4]{a} \left(a^2 - \frac{b^2c^2}{d^2}\right)}{4cd^2}$$

**Mathematica [C]** time = 0.44, size = 342, normalized size = 1.11

$$x \frac{\left(5c \left(-5ac(4a^2d - abdx^4 + b^2cx^4) F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 2x^4(a - bx^4)(ad - bc) \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right)\right)}{2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)} - bx^4 \sqrt{1 - \frac{bx^4}{a}}}{20c^2d\sqrt{a - bx^4} (dx^4 - c)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a - b\*x^4)^(3/2)/(c - d\*x^4)^2,x]

[Out] (x\*(-(b\*(3\*b\*c + a\*d)\*x^4\*Sqrt[1 - (b\*x^4)/a]\*(-c + d\*x^4)\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]) + (5\*c\*(-5\*a\*c\*(4\*a^2\*d + b^2\*c\*x^4 - a\*b\*d\*x^4)\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] - 2\*(-(b\*c) + a\*d)\*x^4\*(a - b\*x^4)\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(20\*c^2\*d\*Sqrt[a - b\*x^4]\*(-c + d\*x^4))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((-b\*x^4 + a)^(3/2)/(d\*x^4 - c)^2, x)

**maple** [C] time = 0.34, size = 329, normalized size = 1.06

$$\frac{\left(\frac{b^2}{d^2} + \frac{(ad-bc)b}{4cd^2}\right) \sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}x, i\right) \frac{(ad-bc)\sqrt{-bx^4+a}x}{4(dx^4-c)cd} - \frac{3(a^2d^2 - b^2c^2)}{2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}} + 1}}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}}\sqrt{-bx^4+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x)

[Out] -1/4\*(a\*d-b\*c)/d/c\*x\*(-b\*x^4+a)^(1/2)/(d\*x^4-c)+(b^2/d^2+1/4\*b/d^2\*(a\*d-b\*c)/c)/(1/a^(1/2)\*b^(1/2))^(1/2)\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF((1/a^(1/2)\*b^(1/2))^(1/2)\*x, I)-3/32/c/d^3\*sum((a^2\*d^2-b^2\*c^2)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi((1/a^(1/2)\*b^(1/2))^(1/2)\*x, \_alpha^2\*a^(1/2)/b^(1/2)/c\*d, (-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)), \_alpha=RootOf(\_Z^4\*d-c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(-bx^4 + a)^{\frac{3}{2}}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((-b\*x^4 + a)^(3/2)/(d\*x^4 - c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a - bx^4)^{3/2}}{(c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(3/2)/(c - d\*x^4)^2,x)

[Out] int((a - b\*x^4)^(3/2)/(c - d\*x^4)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a - bx^4)^{\frac{3}{2}}}{(-c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c)\*\*2,x)

[Out] Integral((a - b\*x\*\*4)\*\*(3/2)/(-c + d\*x\*\*4)\*\*2, x)

$$3.187 \quad \int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx$$

**Optimal.** Leaf size=276

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - 3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - 3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

[Out]  $1/4*x*(-b*x^4+a)^{(1/2)}/c/(-d*x^4+c)+1/4*a^{(1/4)}*b^{(3/4)}*EllipticF(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/c/d/(-b*x^4+a)^{(1/2)}-1/8*a^{(1/4)}*(-3*a*d+b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d/(-b*x^4+a)^{(1/2)}-1/8*a^{(1/4)}*(-3*a*d+b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/d/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.21, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {412, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - 3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} - \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (bc - 3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(c - d\*x^4)^2,x]

[Out]  $(x*\text{Sqrt}[a - b*x^4])/(4*c*(c - d*x^4)) + (a^{(1/4)}*b^{(3/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*c*d*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d*\text{Sqrt}[a - b*x^4]) - (a^{(1/4)}*(b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 412

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] + Dist[1/



$a*n*(p + 1)$ , Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1)\*Simp[c\*(n\*(p + 1) + 1) + d\*(n\*(p + q + 1) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rule 523

Int[((e\_) + (f\_)\*(x\_)^(n\_))/(((a\_) + (b\_)\*(x\_)^(n\_))\*Sqrt[(c\_) + (d\_)\*(x\_)^(n\_)], x\_Symbol] := Dist[f/b, Int[1/Sqrt[c + d\*x^n], x], x] + Dist[(b\*e - a\*f)/b, Int[1/((a + b\*x^n)\*Sqrt[c + d\*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

### Rule 1218

Int[1/(((d\_) + (e\_)\*(x\_)^2)\*Sqrt[(a\_) + (c\_)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1\*EllipticPi[-(e/(d\*q^2)), ArcSin[q\*x], -1])/(d\*Sqrt[a]\*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

### Rule 1219

Int[1/(((d\_) + (e\_)\*(x\_)^2)\*Sqrt[(a\_) + (c\_)\*(x\_)^4]), x\_Symbol] := Dist[Sqrt[1 + (c\*x^4)/a]/Sqrt[a + c\*x^4], Int[1/((d + e\*x^2)\*Sqrt[1 + (c\*x^4)/a]), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a-bx^4}}{(c-dx^4)^2} dx &= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} - \frac{\int \frac{-3a+bx^4}{\sqrt{a-bx^4}(c-dx^4)} dx}{4c} \\ &= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{b \int \frac{1}{\sqrt{a-bx^4}} dx}{4cd} + \frac{(-bc+3ad) \int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx}{4cd} \\ &= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} - \frac{(bc-3ad) \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2d} - \frac{(bc-3ad) \int \frac{1}{\left(1+\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2d} + \frac{\left(b\sqrt{1-\frac{bx^4}{a}}\right) \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{4c} \\ &= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right) - 1}{4cd\sqrt{a-bx^4}} - \frac{\left((bc-3ad)\sqrt{1-\frac{bx^4}{a}}\right) \int \frac{1}{\left(1-\frac{\sqrt{d}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2d\sqrt{a-bx^4}} \\ &= \frac{x\sqrt{a-bx^4}}{4c(c-dx^4)} + \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right)\right) - 1}{4cd\sqrt{a-bx^4}} - \frac{\sqrt[4]{a}(bc-3ad)\sqrt{1-\frac{bx^4}{a}} \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}\right)}{8\sqrt[4]{b}c^2d\sqrt{a-bx^4}} \end{aligned}$$

**Mathematica [C]** time = 0.13, size = 233, normalized size = 0.84

$$x \left( \frac{75a^2 F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bc F_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)\right) + 5ac F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{c^2} + \frac{bx^4 \sqrt{1-\frac{bx^4}{a}} (c-dx^4) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{c^2} - \frac{5(a-bx^4)}{c} \right) / (20\sqrt{a-bx^4}(dx^4-c))$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - b\*x^4]/(c - d\*x^4)^2,x]

[Out] (x\*((-5\*(a - b\*x^4))/c + (b\*x^4\*Sqrt[1 - (b\*x^4)/a]\*(c - d\*x^4)\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])/c^2 - (75\*a^2\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(20\*Sqrt[a - b\*x^4]\*(-c + d\*x^4))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4 + a}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(sqrt(-b\*x^4 + a)/(d\*x^4 - c)^2, x)

**maple** [C] time = 0.31, size = 294, normalized size = 1.07

$$\frac{\sqrt{-\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} b \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} x, i\right) \sqrt{-bx^4 + a} x}{4 \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4 + a} cd} - \frac{(3ad - bc)}{4(dx^4 - c)c} \left( \frac{2 \sqrt{-\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \operatorname{RootOf}(d\_Z^4 - c)}{\sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x)

[Out] -1/4/c\*x\*(-b\*x^4+a)^(1/2)/(d\*x^4-c)+1/4\*b/c/d/(1/a^(1/2)\*b^(1/2))^(1/2)\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF((1/a^(1/2)\*b^(1/2))^(1/2)\*x,I)-1/32/c/d^2\*sum((3\*a\*d-b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2))\*EllipticPi((1/a^(1/2)\*b^(1/2))^(1/2)\*x,\_alpha^2\*a^(1/2)/b^(1/2)/c\*d,(-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)),\_alpha=RootOf(\_Z^4\*d-c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4 + a}}{(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(sqrt(-b\*x^4 + a)/(d\*x^4 - c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a - bx^4}}{(c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - b\*x^4)^(1/2)/(c - d\*x^4)^2,x)

[Out] int((a - b\*x^4)^(1/2)/(c - d\*x^4)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a - bx^4}}{(-c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c)\*\*2,x)

[Out] Integral(sqrt(a - b\*x\*\*4)/(-c + d\*x\*\*4)\*\*2, x)

$$3.188 \quad \int \frac{1}{\sqrt{a-bx^4} (c-dx^4)^2} dx$$

**Optimal.** Leaf size=310

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a-bx^4}(bc-ad)}$$

[Out]  $-1/4*d*x*(-b*x^4+a)^{(1/2)}/c/(-a*d+b*c)/(-d*x^4+c)-1/4*a^{(1/4)}*b^{(3/4)}*EllipticF(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/c/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*(-3*a*d+5*b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*(-3*a*d+5*b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.25, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {414, 523, 224, 221, 409, 1219, 1218}

$$\frac{\sqrt[4]{a} b^{3/4} \sqrt{1 - \frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c\sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a-bx^4}(bc-ad)} + \frac{\sqrt[4]{a} \sqrt{1 - \frac{bx^4}{a}} (5bc-3ad) \Pi\left(-\frac{\sqrt{a}\sqrt{d}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a-bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)^2),x]

[Out]  $-(d*x*\text{Sqrt}[a - b*x^4])/(4*c*(b*c - a*d)*(c - d*x^4)) - (a^{(1/4)}*b^{(3/4)}*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*c*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/((8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4]) + (a^{(1/4)}*(5*b*c - 3*a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c -

```
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] := Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a-bx^4} (c-dx^4)^2} dx &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)(c-dx^4)} - \frac{\int \frac{-4bc+3ad-bdx^4}{\sqrt{a-bx^4}(c-dx^4)} dx}{4c(bc-ad)} \\ &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)(c-dx^4)} - \frac{b \int \frac{1}{\sqrt{a-bx^4}} dx}{4c(bc-ad)} + \frac{(5bc-3ad) \int \frac{1}{\sqrt{a-bx^4}(c-dx^4)} dx}{4c(bc-ad)} \\ &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)(c-dx^4)} + \frac{(5bc-3ad) \int \frac{1}{\left(1-\frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2(bc-ad)} + \frac{(5bc-3ad) \int \frac{1}{\left(1+\frac{\sqrt{a}x^2}{\sqrt{c}}\right)\sqrt{a-bx^4}} dx}{8c^2(bc-ad)} \\ &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)(c-dx^4)} - \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{(5bc-3ad) \sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8c^2(bc-ad)\sqrt{a-bx^4}} \\ &= -\frac{dx\sqrt{a-bx^4}}{4c(bc-ad)(c-dx^4)} - \frac{\sqrt[4]{a} b^{3/4} \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{4c(bc-ad)\sqrt{a-bx^4}} + \frac{\sqrt[4]{a} (5bc-3ad) \sqrt{1-\frac{bx^4}{a}} F\left(\sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a}}\right) \middle| -1\right)}{8c^2(bc-ad)\sqrt{a-bx^4}} \end{aligned}$$

**Mathematica [C]** time = 0.28, size = 386, normalized size = 1.25

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) \left(bdx^4 \sqrt{1-\frac{bx^4}{a}} (dx^4-c) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 5c(-4ad+4bc+bdx^4)\right) + 2dx^5}{20c^2\sqrt{a-bx^4} (dx^4-c) (bc-ad) \left(2x^4 \left(2ad F_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) - 5c(-4ad+4bc+bdx^4)\right) + 2dx^5\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[a - b\*x^4]\*(c - d\*x^4)^2),x]

[Out] (5\*a\*c\*x\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c]\*(-5\*c\*(4\*b\*c - 4\*a\*d + b\*d\*x^4) + b\*d\*x^4\*Sqrt[1 - (b\*x^4)/a]\*(-c + d\*x^4)\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]) + 2\*d\*x^5\*(5\*c\*(a - b\*x^4) + b\*x^4\*Sqrt[1 - (b\*x^4)/a]\*(-c + d\*x^4)\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]))/(20\*c^2\*(b\*c - a\*d)\*Sqrt[a - b\*x^4]\*(-c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4 + a} (dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/(sqrt(-b\*x^4 + a)\*(d\*x^4 - c)^2), x)

**maple** [C] time = 0.34, size = 322, normalized size = 1.04

$$\frac{\sqrt{-\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} b \operatorname{EllipticF}\left(\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} x, i\right) \sqrt{-b x^4 + a} dx}{4(ad - bc) \sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-b x^4 + a} c} - \frac{(3ad - 5bc)}{4(ad - bc)(dx^4 - c)^2} \left( 2\sqrt{-\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \operatorname{Ro} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x)

[Out] -1/4\*d/(a\*d-b\*c)/c\*x\*(-b\*x^4+a)^(1/2)/(d\*x^4-c)+1/4\*b/(a\*d-b\*c)/c/(1/a^(1/2)\*b^(1/2))^(1/2)\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF((1/a^(1/2)\*b^(1/2))^(1/2)\*x,I)-1/32/c/d\*sum((3\*a\*d-5\*b\*c)/(a\*d-b\*c)/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi((1/a^(1/2)\*b^(1/2))^(1/2)\*x,\_alpha^2\*a^(1/2)/b^(1/2)/c\*d,(-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2))),\_alpha=RootOf(\_Z^4\*d-c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-bx^4 + a} (dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(1/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/(sqrt(-b\*x^4 + a)\*(d\*x^4 - c)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{a - bx^4} (c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - b\*x^4)^(1/2)\*(c - d\*x^4)^2), x)

[Out] int(1/((a - b\*x^4)^(1/2)\*(c - d\*x^4)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - bx^4} (-c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*4+a)\*\*(1/2)/(-d\*x\*\*4+c)\*\*2,x)

[Out] Integral(1/(sqrt(a - b\*x\*\*4)\*(-c + d\*x\*\*4)\*\*2), x)

$$3.189 \quad \int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2} dx$$

**Optimal.** Leaf size=362

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 2bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4a^{3/4} c \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a - bx^4} (bc - ad)^2}$$

[Out]  $\frac{1}{4} b^* (a*d + 2*b*c) * x/a/c / (-a*d + b*c)^{2/(-b*x^4 + a)^{(1/2)} - 1/4} * d*x/c / (-a*d + b*c) / (-d*x^4 + c) / (-b*x^4 + a)^{(1/2)} + 1/4 * b^{(3/4)} * (a*d + 2*b*c) * \text{EllipticF}(b^{(1/4)} * x/a^{(1/4)}, I) * (1 - b*x^4/a)^{(1/2)} / a^{(3/4)} / c / (-a*d + b*c)^{2/(-b*x^4 + a)^{(1/2)} - 3/8} * a^{(1/4)} * d * (-a*d + 3*b*c) * \text{EllipticPi}(b^{(1/4)} * x/a^{(1/4)}, -a^{(1/2)} * d^{(1/2)} / b^{(1/2)} / c^{(1/2)}, I) * (1 - b*x^4/a)^{(1/2)} / b^{(1/4)} / c^2 / (-a*d + b*c)^{2/(-b*x^4 + a)^{(1/2)} - 3/8} * a^{(1/4)} * d * (-a*d + 3*b*c) * \text{EllipticPi}(b^{(1/4)} * x/a^{(1/4)}, a^{(1/2)} * d^{(1/2)} / b^{(1/2)} / c^{(1/2)}, I) * (1 - b*x^4/a)^{(1/2)} / b^{(1/4)} / c^2 / (-a*d + b*c)^{2/(-b*x^4 + a)^{(1/2)}$

**Rubi [A]** time = 0.40, antiderivative size = 362, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{b^{3/4} \sqrt{1 - \frac{bx^4}{a}} (ad + 2bc) F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{4a^{3/4} c \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a - bx^4} (bc - ad)^2} - \frac{3\sqrt[4]{a} d \sqrt{1 - \frac{bx^4}{a}} (3bc - ad) \Pi\left(-\frac{\sqrt{a} \sqrt{d}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 \sqrt{a - bx^4} (bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)^2), x]

[Out]  $(b*(2*b*c + a*d)*x)/(4*a*c*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) - (d*x)/(4*c*(b*c - a*d)*\text{Sqrt}[a - b*x^4]*(c - d*x^4)) + (b^{(3/4)}*(2*b*c + a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticF}[\text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(4*a^{(3/4)}*c*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) - (3*a^{(1/4)}*d*(3*b*c - a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[-((\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4]) - (3*a^{(1/4)}*d*(3*b*c - a*d)*\text{Sqrt}[1 - (b*x^4)/a]*\text{EllipticPi}[(\text{Sqrt}[a]*\text{Sqrt}[d])/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)^2*\text{Sqrt}[a - b*x^4])$

#### Rule 221

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

#### Rule 224

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 414



```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a-bx^4)^{3/2}(c-dx^4)^2} dx &= -\frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} - \frac{\int \frac{-4bc+3ad-5bdx^4}{(a-bx^4)^{3/2}(c-dx^4)} dx}{4c(bc-ad)} \\
&= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} - \frac{\int \frac{-2(2b^2c^2-8abcd+3a^2d^2)+}{\sqrt{a-bx^4}(c-dx^4)} dx}{8ac(bc-ad)} \\
&= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} - \frac{(3d(3bc-ad)) \int \frac{dx}{\sqrt{a-bx^4}}}{4c(bc-ad)} \\
&= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} - \frac{(3d(3bc-ad)) \int \frac{dx}{(1-\frac{bx^4}{a})}}{8c^2(bc-ad)} \\
&= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}}{4a^{3/4}c(bc-ad)} \\
&= \frac{b(2bc+ad)x}{4ac(bc-ad)^2\sqrt{a-bx^4}} - \frac{dx}{4c(bc-ad)\sqrt{a-bx^4}(c-dx^4)} + \frac{b^{3/4}(2bc+ad)\sqrt{1-\frac{bx^4}{a}}}{4a^{3/4}c(bc-ad)}
\end{aligned}$$

**Mathematica** [C] time = 0.52, size = 374, normalized size = 1.03

$$x \frac{c \left( 25ac(4a^2d^2 - abd(8c + dx^4)) + 2b^2c(2c - dx^4) \right) F_1 \left( \frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c} \right) - 10x^4(-a^2d^2 + abd^2x^4 - 2b^2c(c - dx^4)) \left( 2adF_1 \left( \frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c} \right) + bcF_1 \left( \frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a} \right) \right) + (c - dx^4) \left( 2x^4 \left( 2adF_1 \left( \frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c} \right) + bcF_1 \left( \frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}; \frac{bx^4}{a} \right) \right) + 5acF_1 \left( \frac{1}{4}; \frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, \frac{dx^4}{c} \right) \right)}{20ac^2\sqrt{a-bx^4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)^2), x]

[Out] (x\*(-(b\*d\*(2\*b\*c + a\*d)\*x^4\*Sqrt[1 - (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c]) + (c\*(25\*a\*c\*(4\*a^2\*d^2 + 2\*b^2\*c\*(2\*c - d\*x^4) - a\*b\*d\*(8\*c + d\*x^4))\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] - 10\*x^4\*(-(a^2\*d^2) + a\*b\*d^2\*x^4 - 2\*b^2\*c\*(c - d\*x^4))\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(c - d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))/(20\*a\*c^2\*(b\*c - a\*d)^2\*Sqrt[a - b\*x^4])

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((-b\*x^4 + a)^(3/2)\*(d\*x^4 - c)^2), x)

**maple** [C] time = 0.35, size = 375, normalized size = 1.04

$$\frac{\frac{b^2 x}{2(ad-bc)^2 \sqrt{-\left(x^4 - \frac{a}{b}\right) b a}} - \frac{\sqrt{-bx^4 + a} d^2 x}{4(ad-bc)^2 (dx^4 - c)^2} + \frac{\left(\frac{b^2}{2(ad-bc)^2 a} + \frac{bd}{4(ad-bc)^2 c}\right) \sqrt{-\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \sqrt{\frac{\sqrt{b} x^2}{\sqrt{a}} + 1} \text{Ellip}}{\sqrt{\frac{\sqrt{b}}{\sqrt{a}}} \sqrt{-bx^4 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x)

[Out] 1/2\*b^2/a\*x/(a\*d-b\*c)^2/(-(x^4-a/b)\*b)^(1/2)-1/4\*d^2/(a\*d-b\*c)^2/c\*x\*(-b\*x^4+a)^(1/2)/(d\*x^4-c)+(1/2/a\*b^2/(a\*d-b\*c)^2+1/4\*d\*b/(a\*d-b\*c)^2/c)/(1/a^(1/2)\*b^(1/2))^(1/2)\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticF((1/a^(1/2)\*b^(1/2))^(1/2)\*x,I)-3/32/c\*sum((a\*d-3\*b\*c)/(a\*d-b\*c)^2/\_alpha^3\*(-1/((a\*d-b\*c)/d)^(1/2)\*arctanh(1/2\*(-2\*\_alpha^2\*b\*x^2+2\*a)/((a\*d-b\*c)/d)^(1/2)/(-b\*x^4+a)^(1/2))-2/(1/a^(1/2)\*b^(1/2))^(1/2)\*\_alpha^3\*d/c\*(-1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)\*(1/a^(1/2)\*b^(1/2)\*x^2+1)^(1/2)/(-b\*x^4+a)^(1/2)\*EllipticPi((1/a^(1/2)\*b^(1/2))^(1/2)\*x,\_alpha^2\*a^(1/2)/b^(1/2)/c\*d,(-1/a^(1/2)\*b^(1/2))^(1/2)/(1/a^(1/2)\*b^(1/2))^(1/2)),\_alpha=RootOf(\_Z^4\*d-c))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^4 + a)^{\frac{3}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(3/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((-b\*x^4 + a)^(3/2)\*(d\*x^4 - c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - bx^4)^{3/2} (c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)^2),x)

[Out] int(1/((a - b\*x^4)^(3/2)\*(c - d\*x^4)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x\*\*4+a)\*\*(3/2)/(-d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

$$3.190 \quad \int \frac{1}{(a-bx^4)^{5/2}(c-dx^4)^2} dx$$

**Optimal.** Leaf size=439

$$\frac{bx(-3a^2d^2 - 17abcd + 5b^2c^2)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2 - 17abcd + 5b^2c^2)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right) - 1}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}$$

[Out]  $1/12*b*(3*a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/(-b*x^4+a)^{(3/2)}-1/4*d*x/c/(-a*d+b*c)/(-b*x^4+a)^{(3/2)}/(-d*x^4+c)+1/12*b*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*x/a^2/c/(-a*d+b*c)^3/(-b*x^4+a)^{(1/2)}+1/12*b^{(3/4)}*(-3*a^2*d^2-17*a*b*c*d+5*b^2*c^2)*EllipticF(b^{(1/4)}*x/a^{(1/4)},I)*(1-b*x^4/a)^{(1/2)}/a^{(7/4)}/c/(-a*d+b*c)^3/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*d^2*(-3*a*d+13*b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},-a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)^3/(-b*x^4+a)^{(1/2)}+1/8*a^{(1/4)}*d^2*(-3*a*d+13*b*c)*EllipticPi(b^{(1/4)}*x/a^{(1/4)},a^{(1/2)}*d^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(1-b*x^4/a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)^3/(-b*x^4+a)^{(1/2)}$

**Rubi [A]** time = 0.54, antiderivative size = 439, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {414, 527, 523, 224, 221, 409, 1219, 1218}

$$\frac{bx(-3a^2d^2 - 17abcd + 5b^2c^2)}{12a^2c\sqrt{a-bx^4}(bc-ad)^3} + \frac{b^{3/4}\sqrt{1-\frac{bx^4}{a}}(-3a^2d^2 - 17abcd + 5b^2c^2)F\left(\sin^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a}}\right)\right) - 1}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3} + \frac{\sqrt[4]{a}d^2\sqrt{1-\frac{bx^4}{a}}}{12a^{7/4}c\sqrt{a-bx^4}(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)^2), x]

[Out]  $(b*(2*b*c + 3*a*d)*x)/(12*a*c*(b*c - a*d)^2*(a - b*x^4)^{(3/2)}) + (b*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*x)/(12*a^2*c*(b*c - a*d)^3*sqrt[a - b*x^4]) - (d*x)/(4*c*(b*c - a*d)*(a - b*x^4)^{(3/2)}*(c - d*x^4)) + (b^{(3/4)}*(5*b^2*c^2 - 17*a*b*c*d - 3*a^2*d^2)*sqrt[1 - (b*x^4)/a]*EllipticF[ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(12*a^{(7/4)}*c*(b*c - a*d)^3*sqrt[a - b*x^4]) + (a^{(1/4)}*d^2*(13*b*c - 3*a*d)*sqrt[1 - (b*x^4)/a]*EllipticPi[-((sqrt[a]*sqrt[d])/(sqrt[b]*sqrt[c])), ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)^3*sqrt[a - b*x^4]) + (a^{(1/4)}*d^2*(13*b*c - 3*a*d)*sqrt[1 - (b*x^4)/a]*EllipticPi[(sqrt[a]*sqrt[d])/(sqrt[b]*sqrt[c]), ArcSin[(b^{(1/4)}*x)/a^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*(b*c - a*d)^3*sqrt[a - b*x^4])$

**Rule 221**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Simp[EllipticF[ArcSin[(Rt[-b, 4]\*x)/Rt[a, 4]], -1]/(Rt[a, 4]\*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[b/a] && GtQ[a, 0]

**Rule 224**

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^4], x\_Symbol] := Dist[Sqrt[1 + (b\*x^4)/a]/Sqrt[a + b\*x^4], Int[1/Sqrt[1 + (b\*x^4)/a], x], x] /; FreeQ[{a, b}, x] && NegQ[b/a] && !GtQ[a, 0]

**Rule 409**

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a,

b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

#### Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)], x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
- a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

#### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

#### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a*q]), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

#### Rule 1219

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :> Dist[
Sqrt[1 + (c*x^4)/a]/Sqrt[a + c*x^4], Int[1/((d + e*x^2)*Sqrt[1 + (c*x^4)/a]
), x], x] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && !GtQ[a, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a - bx^4)^{5/2} (c - dx^4)^2} dx &= -\frac{dx}{4c(bc - ad)(a - bx^4)^{3/2} (c - dx^4)} - \frac{\int \frac{-4bc+3ad-9bdx^4}{(a-bx^4)^{5/2} (c-dx^4)} dx}{4c(bc - ad)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} - \frac{dx}{4c(bc - ad)(a - bx^4)^{3/2} (c - dx^4)} - \int \frac{-2(10b^2c^2 - 2)}{\dots} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)} \\
&= \frac{b(2bc + 3ad)x}{12ac(bc - ad)^2 (a - bx^4)^{3/2}} + \frac{b(5b^2c^2 - 17abcd - 3a^2d^2)x}{12a^2c(bc - ad)^3 \sqrt{a - bx^4}} - \frac{d}{4c(bc - ad)(a - \dots)}
\end{aligned}$$

**Mathematica [C]** time = 0.88, size = 382, normalized size = 0.87

$$x \left( \frac{bdx^4 \sqrt{1 - \frac{bx^4}{a}} (3a^2d^2 + 17abcd - 5b^2c^2) F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right)}{a^2c^2} + 5 \left( \frac{2b^3c}{a^2 - abx^4} + \frac{5b^3c}{a^2} + \frac{5(-9a^3d^3 + 36a^2bcd^2 - 17ab^2c^2d + 5b^3c^3)}{a(c - dx^4) \left( 2x^4 \left( 2adF_1\left(\frac{5}{4}; \frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, \frac{dx^4}{c}\right) + bcF_1\left(\frac{5}{4}; \frac{3}{2}, 1; \frac{9}{4}\right) \right) \right)}{60\sqrt{a - bx^4} (bc - ad)^3} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a - b\*x^4)^(5/2)\*(c - d\*x^4)^2), x]

[Out] (x\*((b\*d\*(-5\*b^2\*c^2 + 17\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^4\*Sqrt[1 - (b\*x^4)/a]\*AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])/(a^2\*c^2) + 5\*((5\*b^3\*c)/a^2 - (17\*b^2\*d)/a - (2\*b^2\*d)/(a - b\*x^4) + (2\*b^3\*c)/(a^2 - a\*b\*x^4) - (3\*a\*d^3)/(c^2 - c\*d\*x^4) + (3\*b\*d^3\*x^4)/(c^2 - c\*d\*x^4) + (5\*(5\*b^3\*c^3 - 17\*a\*b^2\*c^2\*d + 36\*a^2\*b\*c\*d^2 - 9\*a^3\*d^3)\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c])/(a\*(c - d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 1/2, 1, 5/4, (b\*x^4)/a, (d\*x^4)/c] + 2\*x^4\*(2\*a\*d\*AppellF1[5/4, 1/2, 2, 9/4, (b\*x^4)/a, (d\*x^4)/c] + b\*c\*AppellF1[5/4, 3/2, 1, 9/4, (b\*x^4)/a, (d\*x^4)/c])))))/(60\*(b\*c - a\*d)^3\*Sqrt[a - b\*x^4])

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x)

**maple** [C] time = 0.35, size = 484, normalized size = 1.10

$$\frac{\sqrt{-bx^4 + a} b d^3 x}{4(a^2 d^2 - 2abcd + b^2 c^2)(ad - bc)(bdx^4 - bc)c} + \frac{(17ad - 5bc)b^2 x}{12(ad - bc)^3 \sqrt{-\left(x^4 - \frac{a}{b}\right) b a^2}} \left[ \frac{d(3ad - 13bc)}{2\sqrt{-\frac{\sqrt{b}x^2}{\sqrt{a}} + 1} \sqrt{\dots}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x)

[Out] 
$$-1/4*b*d^3/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c*x*(-b*x^4+a)^{(1/2)}/(b*d*x^4-b*c)+1/6/(a*d-b*c)^2/a*x*(-b*x^4+a)^{(1/2)}/(x^4-a/b)^2+1/12*b^2/a^2*x*(17*a*d-5*b*c)/(a*d-b*c)^3/(-(x^4-a/b)*b)^{(1/2)}+(1/4*b*d^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/c+1/12/a^2*b^2*(17*a*d-5*b*c)/(a*d-b*c)^3)/(1/a^{(1/2)*b^{(1/2)}})^{(1/2)}*(-1/a^{(1/2)*b^{(1/2)}}*x^2+1)^{(1/2)}*(1/a^{(1/2)*b^{(1/2)}}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticF((1/a^{(1/2)*b^{(1/2)}})^{(1/2)}*x,I)-1/32/c*d*sum((3*a*d-13*b*c)/(a*d-b*c)^3/_alpha^3*(-1/((a*d-b*c)/d)^{(1/2)}*arctanh(1/2*(-2*_alpha^2*b*x^2+2*a)/((a*d-b*c)/d)^{(1/2)}/(-b*x^4+a)^{(1/2)})-2/(1/a^{(1/2)*b^{(1/2)}})^{(1/2)}*_alpha^3*d/c*(-1/a^{(1/2)*b^{(1/2)}}*x^2+1)^{(1/2)}*(1/a^{(1/2)*b^{(1/2)}}*x^2+1)^{(1/2)}/(-b*x^4+a)^{(1/2)}*EllipticPi((1/a^{(1/2)*b^{(1/2)}})^{(1/2)}*x,_alpha^2*a^{(1/2)}/b^{(1/2)}/c*d,(-1/a^{(1/2)*b^{(1/2)}})^{(1/2)}/(1/a^{(1/2)*b^{(1/2)}})^{(1/2)})),_alpha=RootOf(_Z^4*d-c))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(-bx^4 + a)^{\frac{5}{2}}(dx^4 - c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-b\*x^4+a)^(5/2)/(-d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((-b\*x^4 + a)^(5/2)\*(d\*x^4 - c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a - bx^4)^{5/2} (c - dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)`

[Out] `int(1/((a - b*x^4)^(5/2)*(c - d*x^4)^2), x)`

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a - bx^4)^{\frac{5}{2}} (-c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-b*x**4+a)**(5/2)/(-d*x**4+c)**2, x)`

[Out] `Integral(1/((a - b*x**4)**(5/2)*(-c + d*x**4)**2), x)`

$$3.191 \quad \int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx$$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}}$$

[Out]  $1/4*\arctan(a^{(1/4)*b^{(1/4)}*x*2^{(1/2)}/(b*x^4+a)^{(1/2)})/a^{(1/4)}/b^{(1/4)}/c*2^{(1/2)}+1/4*\arctanh(a^{(1/4)*b^{(1/4)}*x*2^{(1/2)}/(b*x^4+a)^{(1/2)})/a^{(1/4)}/b^{(1/4)}/c*2^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$ , Rules used = {404, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{bx}}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{bc}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x^4]/(a\*c - b\*c\*x^4), x]

[Out] ArcTan[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(Sqrt[2]\*a^(1/4)\*b^(1/4)\*x)/Sqrt[a + b\*x^4]]/(2\*Sqrt[2]\*a^(1/4)\*b^(1/4)\*c)

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 404

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^4]/((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] := Dist[a/c, Subst[Int[1/(1 - 4\*a\*b\*x^4), x], x, x/Sqrt[a + b\*x^4]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && PosQ[a\*b]

#### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a+bx^4}}{ac-bcx^4} dx &= \frac{\text{Subst}\left(\int \frac{1}{1-4abx^4} dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{c} \\ &= \frac{\text{Subst}\left(\int \frac{1}{1-2\sqrt{a}\sqrt{b}x^2} dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} + \frac{\text{Subst}\left(\int \frac{1}{1+2\sqrt{a}\sqrt{b}x^2} dx, x, \frac{x}{\sqrt{a+bx^4}}\right)}{2c} \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}x}{\sqrt{a+bx^4}}\right)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{b}c} \end{aligned}$$

**Mathematica [C]** time = 0.17, size = 155, normalized size = 1.50

$$\frac{5ax\sqrt{a+bx^4}F_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)}{c(a-bx^4)\left(2bx^4\left(2F_1\left(\frac{5}{4};-\frac{1}{2},2;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)+F_1\left(\frac{5}{4};\frac{1}{2},1;\frac{9}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)+5aF_1\left(\frac{1}{4};-\frac{1}{2},1;\frac{5}{4};-\frac{bx^4}{a},\frac{bx^4}{a}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a + b\*x^4]/(a\*c - b\*c\*x^4),x]

[Out] (5\*a\*x\*Sqrt[a + b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, -((b\*x^4)/a), (b\*x^4)/a])/(c\*(a - b\*x^4)\*(5\*a\*AppellF1[1/4, -1/2, 1, 5/4, -((b\*x^4)/a), (b\*x^4)/a] + 2\*b\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, -((b\*x^4)/a), (b\*x^4)/a] + AppellF1[5/4, 1/2, 1, 9/4, -((b\*x^4)/a), (b\*x^4)/a])))

**fricas [B]** time = 2.75, size = 315, normalized size = 3.06

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{\left(\frac{1}{4}\right)^{\frac{1}{4}}\sqrt{bx^4+a}c\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}-\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}bcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}}{\sqrt{b}}\right)+\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}\log\left(\frac{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}abcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}{4\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\left(\frac{1}{abc^4}\right)^{\frac{3}{4}}+2\left(\frac{1}{4}\right)^{\frac{1}{4}}abcx^2\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}+\sqrt{bx^4+a}\left(\frac{1}{abc^4}\right)^{\frac{1}{4}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/2)/(-b\*c\*x^4+a\*c),x, algorithm="fricas")

[Out] -(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*arctan(((1/4)^(1/4)\*sqrt(b\*x^4 + a)\*c\*(1/(a\*b\*c^4))^(1/4) - (2\*(1/4)^(3/4)\*a\*b\*c^3\*(1/(a\*b\*c^4))^(3/4) + (1/4)^(1/4)\*b\*c\*x^2\*(1/(a\*b\*c^4))^(1/4))/sqrt(b))/x) + 1/4\*(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*log((4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(1/(a\*b\*c^4))^(3/4) + 2\*(1/4)^(1/4)\*a\*c\*x\*(1/(a\*b\*c^4))^(1/4) + sqrt(b\*x^4 + a)\*(a\*c^2\*sqrt(1/(a\*b\*c^4)) + x^2))/(b\*x^4 - a)) - 1/4\*(1/4)^(1/4)\*(1/(a\*b\*c^4))^(1/4)\*log(-(4\*(1/4)^(3/4)\*a\*b\*c^3\*x^3\*(1/(a\*b\*c^4))^(3/4) + 2\*(1/4)^(1/4)\*a\*c\*x\*(1/(a\*b\*c^4))^(1/4) - sqrt(b\*x^4 + a)\*(a\*c^2\*sqrt(1/(a\*b\*c^4)) + x^2))/(b\*x^4 - a))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{\sqrt{bx^4+a}}{bcx^4-ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/2)/(-b\*c\*x^4+a\*c),x, algorithm="giac")

[Out] integrate(-sqrt(b\*x^4 + a)/(b\*c\*x^4 - a\*c), x)

**maple** [A] time = 0.21, size = 103, normalized size = 1.00

$$-\frac{\sqrt{2} \arctan\left(\frac{\sqrt{bx^4+a} \sqrt{2}}{2(ab)^{\frac{1}{4}}x}\right)}{4(ab)^{\frac{1}{4}}c} + \frac{\sqrt{2} \ln\left(\frac{\frac{\sqrt{bx^4+a} \sqrt{2}}{2x} + (ab)^{\frac{1}{4}}}{\frac{\sqrt{bx^4+a} \sqrt{2}}{2x} - (ab)^{\frac{1}{4}}}\right)}{8(ab)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(1/2)/(-b\*c\*x^4+a\*c), x)

[Out] -1/4/c\*2^(1/2)/(a\*b)^(1/4)\*arctan(1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x/(a\*b)^(1/4))+1/8/c\*2^(1/2)/(a\*b)^(1/4)\*ln(((1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x+(a\*b)^(1/4))/(1/2\*(b\*x^4+a)^(1/2)\*2^(1/2)/x-(a\*b)^(1/4)))

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{bx^4+a}}{bcx^4-ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/2)/(-b\*c\*x^4+a\*c), x, algorithm="maxima")

[Out] -integrate(sqrt(b\*x^4+a)/(b\*c\*x^4-a\*c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{bx^4+a}}{ac-bcx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b\*x^4)^(1/2)/(a\*c-b\*c\*x^4), x)

[Out] int((a+b\*x^4)^(1/2)/(a\*c-b\*c\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sqrt{a+bx^4}}{-a+bx^4} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(1/2)/(-b\*c\*x\*\*4+a\*c), x)

[Out] -Integral(sqrt(a+b\*x\*\*4)/(-a+b\*x\*\*4), x)/c

$$3.192 \quad \int \frac{\sqrt{a-bx^4}}{ac+bcx^4} dx$$

Optimal. Leaf size=116

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}+\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}-\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

[Out] 1/2\*arctan(b^(1/4)\*x\*(a^(1/2)+x^2\*b^(1/2))/a^(1/4)/(-b\*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c+1/2\*arctanh(b^(1/4)\*x\*(a^(1/2)-x^2\*b^(1/2))/a^(1/4)/(-b\*x^4+a)^(1/2))/a^(1/4)/b^(1/4)/c

Rubi [A] time = 0.02, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {405}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}+\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}-\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^4]/(a\*c + b\*c\*x^4), x]

[Out] ArcTan[(b^(1/4)\*x\*(Sqrt[a] + Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c) + ArcTanh[(b^(1/4)\*x\*(Sqrt[a] - Sqrt[b]\*x^2))/(a^(1/4)\*Sqrt[a - b\*x^4])]/(2\*a^(1/4)\*b^(1/4)\*c)

Rule 405

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^4]/((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] := With[{q = Rt[-(a\*b), 4]}, Simp[(a\*ArcTan[(q\*x\*(a + q^2\*x^2))/(a\*Sqrt[a + b\*x^4]])]/(2\*c\*q), x] + Simp[(a\*ArcTanh[(q\*x\*(a - q^2\*x^2))/(a\*Sqrt[a + b\*x^4]])]/(2\*c\*q), x]] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && NegQ[a\*b]

Rubi steps

$$\int \frac{\sqrt{a-bx^4}}{ac+bcx^4} dx = \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}+\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}x(\sqrt{a}-\sqrt{bx^2})}{\sqrt[4]{a}\sqrt{a-bx^4}}\right)}{2\sqrt[4]{a}\sqrt[4]{b}c}$$

Mathematica [C] time = 0.17, size = 155, normalized size = 1.34

$$\frac{5ax\sqrt{a-bx^4}F_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, -\frac{bx^4}{a}\right)}{c(a+bx^4)\left(5aF_1\left(\frac{1}{4}; -\frac{1}{2}, 1; \frac{5}{4}; \frac{bx^4}{a}, -\frac{bx^4}{a}\right) - 2bx^4\left(2F_1\left(\frac{5}{4}; -\frac{1}{2}, 2; \frac{9}{4}; \frac{bx^4}{a}, -\frac{bx^4}{a}\right) + F_1\left(\frac{5}{4}; \frac{1}{2}, 1; \frac{9}{4}; \frac{bx^4}{a}, -\frac{bx^4}{a}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[a - b\*x^4]/(a\*c + b\*c\*x^4), x]

[Out] (5\*a\*x\*Sqrt[a - b\*x^4]\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, -((b\*x^4)/a)])/(c\*(a + b\*x^4)\*(5\*a\*AppellF1[1/4, -1/2, 1, 5/4, (b\*x^4)/a, -((b\*x^4)/a)] - 2\*b\*x^4\*(2\*AppellF1[5/4, -1/2, 2, 9/4, (b\*x^4)/a, -((b\*x^4)/a)] + AppellF1[5/4, 1/2, 1, 9/4, (b\*x^4)/a, -((b\*x^4)/a)]))

**fricas** [B] time = 1.92, size = 339, normalized size = 2.92

$$-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\sqrt{-\frac{1}{b}}\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bcx^2\sqrt{-\frac{1}{b}}+\sqrt{-bx^4+a}c\right)\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(b\*c\*x^4+a\*c),x, algorithm="fricas")

[Out]  $-\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}\arctan\left(\frac{2\left(\frac{1}{4}\right)^{\frac{3}{4}}abc^3\sqrt{-\frac{1}{b}}\left(-\frac{1}{abc^4}\right)^{\frac{3}{4}}+\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(bcx^2\sqrt{-\frac{1}{b}}+\sqrt{-bx^4+a}c\right)\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}}{x}\right)-\frac{1}{4}\left(\frac{1}{4}\right)^{\frac{1}{4}}\left(-\frac{1}{abc^4}\right)^{\frac{1}{4}}$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(b\*c\*x^4+a\*c),x, algorithm="giac")

[Out] integrate(sqrt(-b\*x^4+a)/(b\*c\*x^4+a\*c), x)

**maple** [A] time = 0.22, size = 158, normalized size = 1.36

$$\frac{\arctan\left(-\frac{\sqrt{-bx^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4(ab)^{\frac{1}{4}}c}-\frac{\arctan\left(\frac{\sqrt{-bx^4+a}}{(ab)^{\frac{1}{4}}x}+1\right)}{4(ab)^{\frac{1}{4}}c}-\frac{\ln\left(\frac{\frac{(ab)^{\frac{1}{4}}\sqrt{-bx^4+a}}{x}+\frac{-bx^4+a}{2x^2}+\sqrt{ab}}{\frac{(ab)^{\frac{1}{4}}\sqrt{-bx^4+a}}{x}+\frac{-bx^4+a}{2x^2}+\sqrt{ab}}}\right)}{8(ab)^{\frac{1}{4}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^4+a)^(1/2)/(b\*c\*x^4+a\*c),x)

[Out]  $-\frac{1}{8}c/(ab)^{\frac{1}{4}}*\ln\left(\frac{1}{2}*(-bx^4+a)/x^2-(ab)^{\frac{1}{4}}*(-bx^4+a)^{\frac{1}{2}}/x+(ab)^{\frac{1}{2}}\right)/(1/2*(-bx^4+a)/x^2+(ab)^{\frac{1}{4}}*(-bx^4+a)^{\frac{1}{2}}/x+(ab)^{\frac{1}{2}})$   
 $-\frac{1}{4}c/(ab)^{\frac{1}{4}}*\arctan(1/(ab)^{\frac{1}{4}}*(-bx^4+a)^{\frac{1}{2}}/x+1)+\frac{1}{4}c/(ab)^{\frac{1}{4}}*\arctan(-1/(ab)^{\frac{1}{4}}*(-bx^4+a)^{\frac{1}{2}}/x+1)$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^4+a}}{bcx^4+ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b\*x^4+a)^(1/2)/(b\*c\*x^4+a\*c),x, algorithm="maxima")

[Out] integrate(sqrt(-b\*x^4+a)/(b\*c\*x^4+a\*c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a-bx^4}}{bcx^4+ac} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)`

[Out] `int((a - b*x^4)^(1/2)/(a*c + b*c*x^4), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{a-bx^4}}{a+bx^4} dx}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-b*x**4+a)**(1/2)/(b*c*x**4+a*c), x)`

[Out] `Integral(sqrt(a - b*x**4)/(a + b*x**4), x)/c`

$$3.193 \quad \int \frac{(a+bx^4)^{7/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=211

$$\frac{b^{3/4}(4bc-7ad) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{b^{3/4}(4bc-7ad) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{(bc-ad)^{7/4}}{2c^{3/4}d^2}$$

[Out]  $1/4*b*x*(b*x^4+a)^{(3/4)}/d-1/8*b^{(3/4)}*(-7*a*d+4*b*c)*\arctan(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^2+1/2*(-a*d+b*c)^{(7/4)}*\arctan((-a*d+b*c)^{(1/4)}*x/c^{(1/4)})/(b*x^4+a)^{(1/4)}/c^{(3/4)}/d^2-1/8*b^{(3/4)}*(-7*a*d+4*b*c)*\operatorname{arctanh}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^2+1/2*(-a*d+b*c)^{(7/4)}*\operatorname{arctanh}((-a*d+b*c)^{(1/4)}*x/c^{(1/4)})/(b*x^4+a)^{(1/4)}/c^{(3/4)}/d^2$

**Rubi [A]** time = 0.22, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {416, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{3/4}(4bc-7ad) \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{b^{3/4}(4bc-7ad) \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc-ad)^{7/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} + \frac{(bc-ad)^{7/4}}{2c^{3/4}d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(7/4)/(c + d\*x^4), x]

[Out]  $(b*x*(a + b*x^4)^{(3/4)})/(4*d) - (b^{(3/4)}*(4*b*c - 7*a*d)*\operatorname{ArcTan}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)})]/(8*d^2) + ((b*c - a*d)^{(7/4)}*\operatorname{ArcTan}[(b*c - a*d)^{(1/4)}*x]/(c^{(1/4)}*(a + b*x^4)^{(1/4)}))/((2*c^{(3/4)}*d^2) - (b^{(3/4)}*(4*b*c - 7*a*d)*\operatorname{ArcTanh}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)})]/(8*d^2) + ((b*c - a*d)^{(7/4)}*\operatorname{ArcTanh}[(b*c - a*d)^{(1/4)}*x]/(c^{(1/4)}*(a + b*x^4)^{(1/4)}))/((2*c^{(3/4)}*d^2)$

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ



[a/b, 0]

### Rule 240

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[a^(p + 1/n), Subst[Int[1/(1 - b\*x^n)^(p + 1/n + 1), x], x, x/(a + b\*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1)/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rule 530

Int((((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)))/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[f/d, Int[(a + b\*x^n)^p, x], x] + Dist[(d\*e - c\*f)/d, Int[(a + b\*x^n)^p/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p, n}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^4)^{7/4}}{c + dx^4} dx &= \frac{bx(a + bx^4)^{3/4}}{4d} + \frac{\int \frac{-a(bc-4ad)-b(4bc-7ad)x^4}{\sqrt[4]{a+bx^4}(c+dx^4)} dx}{4d} \\ &= \frac{bx(a + bx^4)^{3/4}}{4d} - \frac{(b(4bc - 7ad)) \int \frac{1}{\sqrt[4]{a+bx^4}} dx}{4d^2} + \frac{(bc - ad)^2 \int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx}{d^2} \\ &= \frac{bx(a + bx^4)^{3/4}}{4d} - \frac{(b(4bc - 7ad)) \operatorname{Subst}\left(\int \frac{1}{1-bx^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4d^2} + \frac{(bc - ad)^2 \operatorname{Subst}\left(\int \frac{1}{c-(bx^4+d)} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d^2} \\ &= \frac{bx(a + bx^4)^{3/4}}{4d} - \frac{(b(4bc - 7ad)) \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} - \frac{(b(4bc - 7ad)) \operatorname{Subst}\left(\int \frac{1}{c-(bx^4+d)} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} \\ &= \frac{bx(a + bx^4)^{3/4}}{4d} - \frac{b^{3/4}(4bc - 7ad) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^2} + \frac{(bc - ad)^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d^2} - \frac{b^{3/4}}{8d^2} \end{aligned}$$

**Mathematica [C]** time = 0.63, size = 364, normalized size = 1.73

$$5\sqrt[4]{c} \left( 4a^2 d \sqrt[4]{a + bx^4} \log\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 4b^2 c^{3/4} x^5 \sqrt[4]{bc-ad} + 4abc^{3/4} x \sqrt[4]{bc-ad} + a \sqrt[4]{a + bx^4} (bc - 4ad) \log\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) \right)$$



$$\frac{c^5 d^2 - 35 a^3 b^4 c^4 d^3 + 35 a^4 b^3 c^3 d^4 - 21 a^5 b^2 c^2 d^5 + 7 a^6 b c d^6 - a^7 d^7}{(c^3 d^8)^{3/4}} - \frac{(b^5 c^5 - 5 a b^4 c^4 d + 10 a^2 b^3 c^3 d^2 - 10 a^3 b^2 c^2 d^3 + 5 a^4 b c d^4 - a^5 d^5) (b x^4 + a)^{1/4}}{x} - d \frac{(256 b^7 c^4 - 1792 a b^6 c^3 d + 4704 a^2 b^5 c^2 d^2 - 5488 a^3 b^4 c d^3 + 2401 a^4 b^3 d^4)/d^8)^{1/4} \log(-d^6 x ((256 b^7 c^4 - 1792 a b^6 c^3 d + 4704 a^2 b^5 c^2 d^2 - 5488 a^3 b^4 c d^3 + 2401 a^4 b^3 d^4)/d^8)^{3/4} + (64 b^5 c^3 - 336 a b^4 c^2 d + 588 a^2 b^3 c d^2 - 343 a^3 b^2 d^3) (b x^4 + a)^{1/4})}{x} + d \frac{(256 b^7 c^4 - 1792 a b^6 c^3 d + 4704 a^2 b^5 c^2 d^2 - 5488 a^3 b^4 c d^3 + 2401 a^4 b^3 d^4)/d^8)^{1/4} \log((d^6 x ((256 b^7 c^4 - 1792 a b^6 c^3 d + 4704 a^2 b^5 c^2 d^2 - 5488 a^3 b^4 c d^3 + 2401 a^4 b^3 d^4)/d^8)^{3/4} - (64 b^5 c^3 - 336 a b^4 c^2 d + 588 a^2 b^3 c d^2 - 343 a^3 b^2 d^3) (b x^4 + a)^{1/4})}{x}}{d}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b x^4 + a)^{7/4}}{d x^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(7/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(b x^4 + a)^{7/4}}{d x^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(7/4)/(d\*x^4+c),x)

[Out] int((b\*x^4+a)^(7/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b x^4 + a)^{7/4}}{d x^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(7/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(b x^4 + a)^{7/4}}{d x^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(7/4)/(c + d\*x^4),x)

[Out] int((a + b\*x^4)^(7/4)/(c + d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b x^4)^{7/4}}{c + d x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(7/4)/(d*x**4+c),x)
```

```
[Out] Integral((a + b*x**4)**(7/4)/(c + d*x**4), x)
```

$$3.194 \quad \int \frac{(a+bx^4)^{3/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=173

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} - \frac{(bc-ad)^{3/4} \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

[Out] 1/2\*b^(3/4)\*arctan(b^(1/4)\*x/(b\*x^4+a)^(1/4))/d-1/2\*(-a\*d+b\*c)^(3/4)\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/d+1/2\*b^(3/4)\*arctanh(b^(1/4)\*x/(b\*x^4+a)^(1/4))/d-1/2\*(-a\*d+b\*c)^(3/4)\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/d

**Rubi [A]** time = 0.10, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {408, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{bx}}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc-ad)^{3/4} \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} - \frac{(bc-ad)^{3/4} \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(3/4)/(c + d\*x^4), x]

[Out] (b^(3/4)\*ArcTan[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*d) - ((b\*c - a\*d)^(3/4)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*d) + (b^(3/4)\*ArcTanh[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*d) - ((b\*c - a\*d)^(3/4)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*d)

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 240

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[a^(p + 1/n), Subst[Int
[1/(1 - b*x^n)^(p + 1/n + 1), x], x, x/(a + b*x^n)^(1/n)], x] /; FreeQ[{a,
b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/
n]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] := Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

Rubi steps

$$\int \frac{(a + bx^4)^{3/4}}{c + dx^4} dx = \frac{b \int \frac{1}{\sqrt[4]{a+bx^4}} dx}{d} - \frac{(bc - ad) \int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx}{d}$$

$$= \frac{b \operatorname{Subst}\left(\int \frac{1}{1-bx^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d} - \frac{(bc - ad) \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$= \frac{b \operatorname{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} + \frac{b \operatorname{Subst}\left(\int \frac{1}{1+\sqrt{b}x^2} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc - ad) \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{d}$$

$$= \frac{b^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc - ad)^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}d} + \frac{b^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d} - \frac{(bc - ad)^{3/4}}{2d}$$

**Mathematica [C]** time = 0.18, size = 161, normalized size = 0.93

$$\frac{5acx(a + bx^4)^{3/4} F_1\left(\frac{1}{4}; -\frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c + dx^4) \left( x^4 \left( 3bcF_1\left(\frac{5}{4}; \frac{1}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 4adF_1\left(\frac{5}{4}; -\frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5acF_1\left(\frac{1}{4}; -\frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(3/4)/(c + d*x^4), x]
```

```
[Out] (5*a*c*x*(a + b*x^4)^(3/4)*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d*x
^4)/c)]/((c + d*x^4)*(5*a*c*AppellF1[1/4, -3/4, 1, 5/4, -((b*x^4)/a), -((d
*x^4)/c)] + x^4*(-4*a*d*AppellF1[5/4, -3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)
/c)] + 3*b*c*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)]))
```

**fricas [B]** time = 1.06, size = 844, normalized size = 4.88

$$\left(\frac{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3}{c^3d^4}\right)^{\frac{1}{4}} \arctan\left(\frac{cdx\sqrt{\frac{(b^3c^4d^2 - 3ab^2c^3d^3 + 3a^2bc^2d^4 - a^3cd^5)x^2\sqrt{\frac{b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3}{c^3d^4}} + (b^4c^4 - \dots)}}{x^2}}{\dots}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="fricas")

[Out] ((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4)\*arctan(-(c\*d\*x\*sqrt((b^3\*c^4\*d^2 - 3\*a\*b^2\*c^3\*d^3 + 3\*a^2\*b\*c^2\*d^4 - a^3\*c\*d^5)\*x^2\*sqrt((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4)) + (b^4\*c^4 - 4\*a\*b^3\*c^3\*d + 6\*a^2\*b^2\*c^2\*d^2 - 4\*a^3\*b\*c\*d^3 + a^4\*d^4)\*sqrt(b\*x^4 + a))/x^2)\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4) - (b^2\*c^3\*d - 2\*a\*b\*c^2\*d^2 + a^2\*c\*d^3)\*(b\*x^4 + a)^(1/4)\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4))/((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)\*x)) + (b^3/d^4)^(1/4)\*arctan(-(b\*x^4 + a)^(1/4)\*b^2\*d\*(b^3/d^4)^(1/4) - d\*x\*(b^3/d^4)^(1/4)\*sqrt((b^3\*d^2\*x^2\*sqrt(b^3/d^4) + sqrt(b\*x^4 + a)\*b^4)/x^2))/(b^3\*x)) - 1/4\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4)\*log((c^2\*d^3\*x\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(3/4) + (b\*x^4 + a)^(1/4)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2))/x) + 1/4\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(1/4)\*log(-(c^2\*d^3\*x\*((b^3\*c^3 - 3\*a\*b^2\*c^2\*d + 3\*a^2\*b\*c\*d^2 - a^3\*d^3)/(c^3\*d^4))^(3/4) - (b\*x^4 + a)^(1/4)\*(b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2))/x) + 1/4\*(b^3/d^4)^(1/4)\*log((d^3\*x\*(b^3/d^4)^(3/4) + (b\*x^4 + a)^(1/4)\*b^2)/x) - 1/4\*(b^3/d^4)^(1/4)\*log(-(d^3\*x\*(b^3/d^4)^(3/4) - (b\*x^4 + a)^(1/4)\*b^2)/x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(3/4)/(d\*x^4+c),x)

[Out] int((b\*x^4+a)^(3/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^4)^(3/4)/(c + d*x^4), x)`

[Out] `int((a + b*x^4)^(3/4)/(c + d*x^4), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{3}{4}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**(3/4)/(d*x**4+c), x)`

[Out] `Integral((a + b*x**4)**(3/4)/(c + d*x**4), x)`



$$3.195 \quad \int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx$$

Optimal. Leaf size=105

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

[Out] 1/2\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(1/4)+1/2\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(1/4)

**Rubi [A]** time = 0.06, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$ , Rules used = {377, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)), x]

[Out] ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4)) + ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))]/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4))

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx &= \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}-\sqrt{bc-ad}x^2} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{c}+\sqrt{bc-ad}x^2} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}} \\
&= \frac{\tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.05, size = 84, normalized size = 0.80

$$\frac{\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right) + \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}\sqrt[4]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)), x]

[Out] (ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))] + ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(1/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c), x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)), x)

**maple [F]** time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c), x)

[Out] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^4 + a)^{1/4} (dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^4} (c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(1/4)\*(c + d\*x\*\*4)), x)

$$3.196 \quad \int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} dx$$

Optimal. Leaf size=134

$$-\frac{d \tan^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d \tanh^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} + \frac{bx}{a \sqrt[4]{a+bx^4}(bc-ad)}$$

[Out] b\*x/a/(-a\*d+b\*c)/(b\*x^4+a)^(1/4)-1/2\*d\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(5/4)-1/2\*d\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(5/4)

**Rubi [A]** time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {382, 377, 212, 208, 205}

$$-\frac{d \tan^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d \tanh^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} + \frac{bx}{a \sqrt[4]{a+bx^4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)),x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*(a + b\*x^4)^(1/4)) - (d\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4)) - (d\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(5/4))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)], Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+2)+1, 0] && (LtQ[p, -1] || !LtQ

[q, -1]) && NeQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^4)^{5/4}(c+dx^4)} dx &= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d \int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx}{bc-ad} \\ &= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc-ad} \\ &= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d \operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\sqrt{bc-ad}x^2}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)} - \frac{d \operatorname{Subst}\left(\int \frac{1}{\sqrt{c-\sqrt{bc-ad}x^2}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2\sqrt{c}(bc-ad)} \\ &= \frac{bx}{a(bc-ad)\sqrt[4]{a+bx^4}} - \frac{d \tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} - \frac{d \tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{5/4}} \end{aligned}$$

**Mathematica [C]** time = 0.58, size = 256, normalized size = 1.91

$$\frac{45c^3(a+bx^4)^2 {}_2F_1\left(\frac{1}{4}, 1; \frac{5}{4}; \frac{(bc-ad)x^4}{c(bx^4+a)}\right) - 45c^3(a+bx^4)^2 + 36c^2dx^4(a+bx^4)^2 {}_2F_1\left(\frac{1}{4}, 1; \frac{5}{4}; \frac{(bc-ad)x^4}{c(bx^4+a)}\right) - 36c^2dx^4}{9c^3x^3(a+bx^4)^{9/4}(ad)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)), x]

[Out]  $-1/9*(-45*c^3*(a + b*x^4)^2 - 36*c^2*d*x^4*(a + b*x^4)^2 + 45*c^3*(a + b*x^4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]) + 36*c^2*d*x^4*(a + b*x^4)^2*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]) + 4*c*(b*c - a*d)^2*x^8*Hypergeometric2F1[2, 9/4, 13/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]) + 4*d*(b*c - a*d)^2*x^12*Hypergeometric2F1[2, 9/4, 13/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/(c^3*(-(b*c) + a*d)*x^3*(a + b*x^4)^(9/4))$

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)^{5/4}(dx^4+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c), x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{5}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(5/4)\*(c + d\*x\*\*4)), x)

$$3.197 \quad \int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=180

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2} + \frac{d^2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}$$

[Out]  $1/5*b*x/a/(-a*d+b*c)/(b*x^4+a)^{(5/4)}+1/5*b*(-9*a*d+4*b*c)*x/a^2/(-a*d+b*c)^{2/(b*x^4+a)^{(1/4)}+1/2*d^2*arctan((-a*d+b*c)^{(1/4)*x/c^{(1/4)/(b*x^4+a)^{(1/4)})/c^{(3/4)/(-a*d+b*c)^{(9/4)}+1/2*d^2*arctanh((-a*d+b*c)^{(1/4)*x/c^{(1/4)/(b*x^4+a)^{(1/4)})/c^{(3/4)/(-a*d+b*c)^{(9/4)}}$

**Rubi [A]** time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(4bc-9ad)}{5a^2\sqrt[4]{a+bx^4}(bc-ad)^2} + \frac{d^2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{d^2 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{9/4}} + \frac{bx}{5a(a+bx^4)^{5/4}(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)), x]

[Out]  $(b*x)/(5*a*(b*c - a*d)*(a + b*x^4)^{(5/4)}) + (b*(4*b*c - 9*a*d)*x)/(5*a^2*(b*c - a*d)^2*(a + b*x^4)^{(1/4)}) + (d^2*ArcTan[((b*c - a*d)^{(1/4)*x)/(c^{(1/4)*(a + b*x^4)^{(1/4)})}]))/(2*c^{(3/4)*(b*c - a*d)^{(9/4)})} + (d^2*ArcTanh[((b*c - a*d)^{(1/4)*x)/(c^{(1/4)*(a + b*x^4)^{(1/4)})}]))/(2*c^{(3/4)*(b*c - a*d)^{(9/4)})}$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a + bx^4)^{9/4} (c + dx^4)} dx = \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} - \frac{\int \frac{-4bc + 5ad - 4bdx^4}{(a + bx^4)^{5/4} (c + dx^4)} dx}{5a(bc - ad)}$$

$$= \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} + \frac{b(4bc - 9ad)x}{5a^2(bc - ad)^2 \sqrt[4]{a + bx^4}} + \frac{\int \frac{5a^2 d^2}{\sqrt[4]{a + bx^4} (c + dx^4)} dx}{5a^2(bc - ad)^2}$$

$$= \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} + \frac{b(4bc - 9ad)x}{5a^2(bc - ad)^2 \sqrt[4]{a + bx^4}} + \frac{d^2 \int \frac{1}{\sqrt[4]{a + bx^4} (c + dx^4)} dx}{(bc - ad)^2}$$

$$= \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} + \frac{b(4bc - 9ad)x}{5a^2(bc - ad)^2 \sqrt[4]{a + bx^4}} + \frac{d^2 \operatorname{Subst}\left(\int \frac{1}{c - (bc - ad)x^4} dx, x\right)}{(bc - ad)^2}$$

$$= \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} + \frac{b(4bc - 9ad)x}{5a^2(bc - ad)^2 \sqrt[4]{a + bx^4}} + \frac{d^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{c} - \sqrt{bc - ad}x^2} dx\right)}{2\sqrt{c}(bc - ad)^2}$$

$$= \frac{bx}{5a(bc - ad)(a + bx^4)^{5/4}} + \frac{b(4bc - 9ad)x}{5a^2(bc - ad)^2 \sqrt[4]{a + bx^4}} + \frac{d^2 \tan^{-1}\left(\frac{\sqrt[4]{bc - ad}x}{\sqrt[4]{c} \sqrt[4]{a + bx^4}}\right)}{2c^{3/4}(bc - ad)^{9/4}} + \frac{d^2 \operatorname{ta}}{2}$$

**Mathematica [C]** time = 2.37, size = 621, normalized size = 3.45

$$80c^2 x^{12} (bc - ad)^3 {}_3F_2\left(2, 2, \frac{13}{4}; 1, \frac{17}{4}; \frac{(bc - ad)x^4}{c(bx^4 + a)}\right) + 80d^2 x^{20} (bc - ad)^3 {}_3F_2\left(2, 2, \frac{13}{4}; 1, \frac{17}{4}; \frac{(bc - ad)x^4}{c(bx^4 + a)}\right) + 160cdx^{16} (bc - ad)^3$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(9/4)*(c + d*x^4)),x]
```

```
[Out] (-585*c^4*(b*c - a*d)*x^4*(a + b*x^4)^2 - 936*c^3*d*(b*c - a*d)*x^8*(a + b*
x^4)^2 - 416*c^2*d^2*(b*c - a*d)*x^12*(a + b*x^4)^2 - 2925*c^5*(a + b*x^4)^
```



$3 - 4680*c^4*d*x^4*(a + b*x^4)^3 - 2080*c^3*d^2*x^8*(a + b*x^4)^3 + 2925*c^5*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 4680*c^4*d*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 2080*c^3*d^2*x^8*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 280*c^2*(b*c - a*d)^3*x^12*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 520*c*d*(b*c - a*d)^3*x^16*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 240*d^2*(b*c - a*d)^3*x^20*Hypergeometric2F1[2, 13/4, 17/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*c^2*(b*c - a*d)^3*x^12*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 160*c*d*(b*c - a*d)^3*x^16*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 80*d^2*(b*c - a*d)^3*x^20*HypergeometricPFQ[{2, 2, 13/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]/(325*c^4*(b*c - a*d)^2*x^7*(a + b*x^4)^(13/4))$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^4)^(9/4)*(c + d*x^4)),x)`

[Out] `int(1/((a + b*x^4)^(9/4)*(c + d*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{9}{4}} (c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(9/4)/(d*x**4+c),x)`

[Out] `Integral(1/((a + b*x**4)**(9/4)*(c + d*x**4)), x)`

$$3.198 \quad \int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=233

$$\frac{bx(8bc-17ad)}{45a^2(a+bx^4)^{5/4}(bc-ad)^2} + \frac{bx(113a^2d^2-100abcd+32b^2c^2)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3} - \frac{d^3 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}$$

[Out] 1/9\*b\*x/a/(-a\*d+b\*c)/(b\*x^4+a)^(9/4)+1/45\*b\*(-17\*a\*d+8\*b\*c)\*x/a^2/(-a\*d+b\*c)^2/(b\*x^4+a)^(5/4)+1/45\*b\*(113\*a^2\*d^2-100\*a\*b\*c\*d+32\*b^2\*c^2)\*x/a^3/(-a\*d+b\*c)^3/(b\*x^4+a)^(1/4)-1/2\*d^3\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(13/4)-1/2\*d^3\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(3/4)/(-a\*d+b\*c)^(13/4)

**Rubi [A]** time = 0.29, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, number of rules / integrand size = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(113a^2d^2-100abcd+32b^2c^2)}{45a^3\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{bx(8bc-17ad)}{45a^2(a+bx^4)^{5/4}(bc-ad)^2} - \frac{d^3 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}} - \frac{d^3 \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{2c^{3/4}(bc-ad)^{13/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)), x]

[Out] (b\*x)/(9\*a\*(b\*c - a\*d)\*(a + b\*x^4)^(9/4)) + (b\*(8\*b\*c - 17\*a\*d)\*x)/(45\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(32\*b^2\*c^2 - 100\*a\*b\*c\*d + 113\*a^2\*d^2)\*x)/(45\*a^3\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d^3\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4)) - (d^3\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(2\*c^(3/4)\*(b\*c - a\*d)^(13/4))

**Rule 12**

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

**Rule 377**

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

#### Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Simp[(b*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(p+1)*(b*c - a*d)), x] + Dist[1/(a*n*(p+1)*(b*c - a*d)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]
```

#### Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p+1)*(c + d*x^n)^(q+1))/(a*n*(b*c - a*d)*(p+1)), x] + Dist[1/(a*n*(b*c - a*d)*(p+1)), Int[(a + b*x^n)^(p+1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^4)^{13/4}(c+dx^4)} dx &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} - \frac{\int \frac{-8bc+9ad-8bdx^4}{(a+bx^4)^{9/4}(c+dx^4)} dx}{9a(bc-ad)} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{\int \frac{32b^2c^2-68abcd+45a^2d^2+4ba^3}{(a+bx^4)^{5/4}(c+dx^4)} dx}{45a^2(bc-ad)^2} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+10a^3d^2)}{45a^3(bc-ad)^3\sqrt[4]{a}} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+10a^3d^2)}{45a^3(bc-ad)^3\sqrt[4]{a}} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+10a^3d^2)}{45a^3(bc-ad)^3\sqrt[4]{a}} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+10a^3d^2)}{45a^3(bc-ad)^3\sqrt[4]{a}} \\ &= \frac{bx}{9a(bc-ad)(a+bx^4)^{9/4}} + \frac{b(8bc-17ad)x}{45a^2(bc-ad)^2(a+bx^4)^{5/4}} + \frac{b(32b^2c^2-100abcd+10a^3d^2)}{45a^3(bc-ad)^3\sqrt[4]{a}} \end{aligned}$$

**Mathematica [A]** time = 5.44, size = 231, normalized size = 0.99

$$\frac{bx \left( (a + bx^4)^2 (113a^2d^2 - 100abcd + 32b^2c^2) + 5a^2(bc - ad)^2 + a(a + bx^4)(ad - bc)(17ad - 8bc) \right) d^3 \left( -\log \right)}{45a^3 (a + bx^4)^{9/4} (bc - ad)^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)),x]

[Out] (b\*x\*(5\*a^2\*(b\*c - a\*d)^2 + a\*(-(b\*c) + a\*d)\*(-8\*b\*c + 17\*a\*d)\*(a + b\*x^4) + (32\*b^2\*c^2 - 100\*a\*b\*c\*d + 113\*a^2\*d^2)\*(a + b\*x^4)^2)/(45\*a^3\*(b\*c - a\*d)^3\*(a + b\*x^4)^(9/4)) - (d^3\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]))/(4\*c^(3/4)\*(b\*c - a\*d)^(13/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(13/4)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(13/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(13/4)\*(d\*x^4 + c)), x)

**maple [F]** time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(13/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(13/4)/(d\*x^4+c),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{13}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(13/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(13/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{13/4} (dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(13/4)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{13}{4}} (c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(13/4)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(13/4)\*(c + d\*x\*\*4)), x)

$$3.199 \quad \int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=316

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (6bc - 11ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{12d^2 (a + bx^4)^{3/4}} - \frac{bx^4 \sqrt{a + bx^4} (6bc - 11ad)}{12d^2} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (bc - ad)}{12d^2}$$

[Out]  $-1/12*b*(-11*a*d+6*b*c)*x*(b*x^4+a)^{(1/4)}/d^2+1/6*b*x*(b*x^4+a)^{(5/4)}/d+1/12*b^{(3/2)}*(-11*a*d+6*b*c)*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})*a^{(1/2)}/d^2/(b*x^4+a)^{(3/4)}+1/2*(-a*d+b*c)^2*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/d^2+1/2*(-a*d+b*c)^2*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/d^2$

**Rubi [A]** time = 0.34, antiderivative size = 316, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {416, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (6bc - 11ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{12d^2 (a + bx^4)^{3/4}} - \frac{bx^4 \sqrt{a + bx^4} (6bc - 11ad)}{12d^2} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (bc - ad)}{12d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(9/4)/(c + d\*x^4), x]

[Out]  $-(b*(6*b*c - 11*a*d)*x*(a + b*x^4)^{(1/4)})/(12*d^2) + (b*x*(a + b*x^4)^{(5/4)})/(6*d) + (\operatorname{Sqrt}[a]*b^{(3/2)}*(6*b*c - 11*a*d)*(1 + a/(b*x^4))^{(3/4)}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/(12*d^2*(a + b*x^4)^{(3/4)}) + ((b*c - a*d)^2*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d])/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])], \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^2) + ((b*c - a*d)^2*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1])/(2*b^{(1/4)}*c*d^2)$

#### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

#### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 335

$\text{Int}[(x_)^{(m_.)}*((a_) + (b_.)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{m+2}, x], x, 1/x] /; \text{FreeQ}\{a, b, p, x\} \&\& \text{ILtQ}[n, 0] \&\& \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_) + (b_.)*(x_)^4)^{(1/4)} / ((c_) + (d_.)*(x_)^4), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4]*\text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4]*(c - (b*c - a*d)*x^4)), x], x, x/(a + b*x^4)^{(1/4)}], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{NeQ}[b*c - a*d, 0]$

#### Rule 416

$\text{Int}[(a_) + (b_.)*(x_)^{(n_)})^{(p_)}*((c_) + (d_.)*(x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow \text{Simp}[(d*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)})/(b*(n*(p+q) + 1)), x] + \text{Dist}[1/(b*(n*(p+q) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^{(q-2)}*\text{Simp}[c*(b*c*(n*(p+q) + 1) - a*d) + d*(b*c*(n*(p+2*q-1) + 1) - a*d*(n*(q-1) + 1))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, p, x\} \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[q, 1] \&\& \text{NeQ}[n*(p+q) + 1, 0] \&\& !\text{IGtQ}[p, 1] \&\& \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 528

$\text{Int}[(a_) + (b_.)*(x_)^{(n_)})^{(p_)}*((c_) + (d_.)*(x_)^{(n_)})^{(q_)}*((e_) + (f_.)*(x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow \text{Simp}[(f*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q)/(b*(n*(p+q+1) + 1)), x] + \text{Dist}[1/(b*(n*(p+q+1) + 1)), \text{Int}[(a + b*x^n)^p*(c + d*x^n)^{(q-1)}*\text{Simp}[c*(b*e - a*f + b*e*n*(p+q+1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p+q+1))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p, x\} \&\& \text{GtQ}[q, 0] \&\& \text{NeQ}[n*(p+q+1) + 1, 0]$

#### Rule 529

$\text{Int}[(e_) + (f_.)*(x_)^4) / (((a_) + (b_.)*(x_)^4)^{(3/4)}*((c_) + (d_.)*(x_)^4)), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)}/(c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, x\}$

#### Rule 1218

$\text{Int}[1/(((d_) + (e_.)*(x_)^2)*\text{Sqrt}[(a_) + (c_.)*(x_)^4]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1*\text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1])/(d*\text{Sqrt}[a]*q), x] /; \text{FreeQ}\{a, c, d, e, x\} \&\& \text{NegQ}[c/a] \&\& \text{GtQ}[a, 0]$

#### Rubi steps



$$\begin{aligned}
\int \frac{(a+bx^4)^{9/4}}{c+dx^4} dx &= \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\int \frac{\sqrt[4]{a+bx^4}(-a(bc-6ad)-b(6bc-11ad)x^4)}{c+dx^4} dx}{6d} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\int \frac{a(6b^2c^2-13abcd+12a^2d^2)+b(12b^2c^2-30abcd+23a^2d^2)}{(a+bx^4)^{3/4}(c+dx^4)} dx}{12d^2} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} - \frac{(ab(6bc-11ad)) \int \frac{1}{(a+bx^4)^{3/4}} dx}{12d^2} + \frac{(bc-ad)}{6d} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} - \frac{(ab(6bc-11ad)(1+\frac{a}{bx^4})^{3/4}x^3) \int \frac{1}{(1+\frac{a}{bx^4})^3} dx}{12d^2(a+bx^4)^{3/4}} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{(ab(6bc-11ad)(1+\frac{a}{bx^4})^{3/4}x^3) \text{Subst} \left( \int \frac{1}{(1+\frac{a}{bx^4})^3} dx \right)}{12d^2(a+bx^4)^{3/4}} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{(bc-ad)^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \frac{a}{a+bx^4}\right)}{2\sqrt[4]{b}cd^2} \\
&= -\frac{b(6bc-11ad)x\sqrt[4]{a+bx^4}}{12d^2} + \frac{bx(a+bx^4)^{5/4}}{6d} + \frac{\sqrt{a}b^{3/2}(6bc-11ad)(1+\frac{a}{bx^4})^{3/4}x^3 F\left(\frac{1}{2}, \frac{a}{a+bx^4}\right)}{12d^2(a+bx^4)^{3/4}}
\end{aligned}$$

**Mathematica [C]** time = 0.70, size = 294, normalized size = 0.93

$$\frac{x \left( \frac{bx^4 \left( \frac{bx^4}{a} + 1 \right)^{3/4} (23a^2d^2 - 30abcd + 12b^2c^2) F_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{c} - \frac{25a^2c(12a^2d^2 - 13abcd + 6b^2c^2) F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4) \left( x^4 \left( 4ad F_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 5a^2d^2 \right)}{60d^2(a+bx^4)^{3/4}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(9/4)/(c + d\*x^4), x]

[Out] (x\*(5\*b\*(a + b\*x^4)\*(-6\*b\*c + 13\*a\*d + 2\*b\*d\*x^4) + (b\*(12\*b^2\*c^2 - 30\*a\*b\*c\*d + 23\*a^2\*d^2)\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]/c - (25\*a^2\*c\*(6\*b^2\*c^2 - 13\*a\*b\*c\*d + 12\*a^2\*d^2)\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))))/(60\*d^2\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c), x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(9/4)/(d\*x^4+c),x)

[Out] int((b\*x^4+a)^(9/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(9/4)/(c + d\*x^4),x)

[Out] int((a + b\*x^4)^(9/4)/(c + d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{9}{4}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c),x)

[Out] Integral((a + b\*x\*\*4)\*\*(9/4)/(c + d\*x\*\*4), x)

$$3.200 \quad \int \frac{(a+bx^4)^{5/4}}{c+dx^4} dx$$

**Optimal.** Leaf size=274

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (bc-ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2d(a+bx^4)^{3/4} 2\sqrt[4]{b} cd}$$

[Out]  $1/2*b*x*(b*x^4+a)^{(1/4)}/d-1/2*b^{(3/2)}*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})*a^{(1/2)}/d/(b*x^4+a)^{(3/4)}-1/2*(-a*d+b*c)*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/d-1/2*(-a*d+b*c)*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/d$

**Rubi [A]** time = 0.16, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {408, 195, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (bc-ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2d(a+bx^4)^{3/4} 2\sqrt[4]{b} cd}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(5/4)/(c + d\*x^4), x]

[Out]  $(b*x*(a + b*x^4)^{(1/4)})/(2*d) - (\operatorname{Sqrt}[a]*b^{(3/2)}*(1 + a/(b*x^4))^{(3/4)}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/(2*d*(a + b*x^4)^{(3/4)}) - ((b*c - a*d)*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d])/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])], \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1))/(2*b^{(1/4)}*c*d) - ((b*c - a*d)*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1))/(2*b^{(1/4)}*c*d)$

#### Rule 195

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^p)/(n\*p + 1), x] + Dist[(a\*n\*p)/(n\*p + 1), Int[(a + b\*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2\*p] || (EqQ[n, 2] && IntegerQ[4\*p]) || (EqQ[n, 2] && IntegerQ[3\*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

#### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] :> Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

#### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] :> Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

#### Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

### Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

### Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] := Dist[Sq
rt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c -
a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && N
eQ[b*c - a*d, 0]
```

### Rule 408

```
Int[((a_) + (b_.)*(x_)^4)^(p_)/((c_) + (d_.)*(x_)^4), x_Symbol] := Dist[b/d
, Int[(a + b*x^4)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^4)^(p
- 1)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
(EqQ[p, 3/4] || EqQ[p, 5/4])
```

### Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[
1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(
2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[b*c - a*d, 0]
```

### Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[
{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*
Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^4)^{5/4}}{c+dx^4} dx &= \frac{b \int \sqrt[4]{a+bx^4} dx}{d} - \frac{(bc-ad) \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{d} \\
&= \frac{bx \sqrt[4]{a+bx^4}}{2d} + \frac{(ab) \int \frac{1}{(a+bx^4)^{3/4}} dx}{2d} - \frac{\left( (bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \right) \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-a)x^4)} dx \right)}{d} \\
&= \frac{bx \sqrt[4]{a+bx^4}}{2d} + \frac{\left( ab \left( 1 + \frac{a}{bx^4} \right)^{3/4} x^3 \right) \int \frac{1}{\left( 1 + \frac{a}{bx^4} \right)^{3/4} x^3} dx}{2d (a+bx^4)^{3/4}} - \frac{\left( (bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \right) \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-a)x^4)} dx \right)}{d} \\
&= \frac{bx \sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi \left( -\frac{\sqrt{bc-ad}}{\sqrt{b} \sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{b} x}{\sqrt[4]{a+bx^4}} \right) \middle| -1 \right)}{2 \sqrt[4]{b} cd} - \frac{(bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-a)x^4)} dx \right)}{d} \\
&= \frac{bx \sqrt[4]{a+bx^4}}{2d} - \frac{(bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi \left( -\frac{\sqrt{bc-ad}}{\sqrt{b} \sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{b} x}{\sqrt[4]{a+bx^4}} \right) \middle| -1 \right)}{2 \sqrt[4]{b} cd} - \frac{(bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-a)x^4)} dx \right)}{d} \\
&= \frac{bx \sqrt[4]{a+bx^4}}{2d} - \frac{\sqrt{a} b^{3/2} \left( 1 + \frac{a}{bx^4} \right)^{3/4} x^3 F \left( \frac{1}{2} \cot^{-1} \left( \frac{\sqrt{b} x^2}{\sqrt{a}} \right) \middle| 2 \right)}{2d (a+bx^4)^{3/4}} - \frac{(bc-ad) \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-a)x^4)} dx \right)}{d}
\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 346, normalized size = 1.26

$$x \frac{\left( 5(bx^4(a+bx^4)(c+dx^4) \left( 4adF_1 \left( \frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bcF_1 \left( \frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 5ac(2a^2d+abdx^4+b^2x^4(c+dx^4)) F_1 \left( \frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)}{(c+dx^4) \left( x^4 \left( 4adF_1 \left( \frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) + 3bcF_1 \left( \frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right) - 5acF_1 \left( \frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c} \right) \right)}{10d(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(5/4)/(c + d\*x^4), x]

[Out] (x\*((b\*(-2\*b\*c + 3\*a\*d))\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -(b\*x^4)/a, -(d\*x^4)/c])/c + (5\*(-5\*a\*c\*(2\*a^2\*d + a\*b\*d\*x^4 + b^2\*x^4\*(c + d\*x^4))\*AppellF1[1/4, 3/4, 1, 5/4, -(b\*x^4)/a, -(d\*x^4)/c] + b\*x^4\*(a + b\*x^4)\*(c + d\*x^4)\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -(b\*x^4)/a, -(d\*x^4)/c] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -(b\*x^4)/a, -(d\*x^4)/c]))/(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -(b\*x^4)/a, -(d\*x^4)/c] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -(b\*x^4)/a, -(d\*x^4)/c] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -(b\*x^4)/a, -(d\*x^4)/c])))/(10\*d\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(5/4)/(d\*x^4+c),x)

[Out] int((b\*x^4+a)^(5/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(5/4)/(c + d\*x^4),x)

[Out] int((a + b\*x^4)^(5/4)/(c + d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{5}{4}}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c),x)

[Out] Integral((a + b\*x\*\*4)\*\*(5/4)/(c + d\*x\*\*4), x)

$$3.201 \quad \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx$$

**Optimal.** Leaf size=166

$$\frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c}$$

[Out] 1/2\*EllipticPi(b^(1/4)\*x/(b\*x^4+a)^(1/4), -(-a\*d+b\*c)^(1/2)/b^(1/2)/c^(1/2), I)\*(a/(b\*x^4+a))^(1/2)\*(b\*x^4+a)^(1/2)/b^(1/4)/c+1/2\*EllipticPi(b^(1/4)\*x/(b\*x^4+a)^(1/4), (-a\*d+b\*c)^(1/2)/b^(1/2)/c^(1/2), I)\*(a/(b\*x^4+a))^(1/2)\*(b\*x^4+a)^(1/2)/b^(1/4)/c

**Rubi [A]** time = 0.09, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {407, 409, 1218}

$$\frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(1/4)/(c + d\*x^4), x]

[Out] (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c) + (Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(2\*b^(1/4)\*c)

#### Rule 407

Int[((a\_) + (b\_.)\*(x\_)^4)^((c\_) + (d\_.)\*(x\_)^4), x\_Symbol] := Dist[Sqrt[a + b\*x^4]\*Sqrt[a/(a + b\*x^4)], Subst[Int[1/(Sqrt[1 - b\*x^4]\*(c - (b\*c - a\*d)\*x^4)), x], x, x/(a + b\*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 409

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^4]\*((c\_) + (d\_.)\*(x\_)^4)), x\_Symbol] := Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 - Rt[-(d/c), 2]\*x^2)), x], x] + Dist[1/(2\*c), Int[1/(Sqrt[a + b\*x^4]\*(1 + Rt[-(d/c), 2]\*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0]

#### Rule 1218

Int[1/(((d\_) + (e\_.)\*(x\_)^2)\*Sqrt[(a\_) + (c\_.)\*(x\_)^4]), x\_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1\*EllipticPi[-(e/(d\*q^2)), ArcSin[q\*x], -1])/(d\*Sqrt[a]\*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]

#### Rubi steps

$$\int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx = \left( \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \right) \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4} (c-(bc-ad)x^4)} dx, x, \frac{x}{\sqrt[4]{a+bx^4}} \right)$$

$$= \frac{\left( \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \right) \text{Subst} \left( \int \frac{1}{\left(1-\frac{\sqrt{bc-ad}x^2}{\sqrt{c}}\right) \sqrt{1-bx^4}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}} \right)}{2c} + \frac{\left( \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \right) \text{Subst} \left( \int \frac{1}{\sqrt{1-bx^4}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}} \right)}{2c}$$

$$= \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi \left( -\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}} \right) \middle| -1 \right)}{2\sqrt[4]{b}c} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi \left( \frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1} \left( \frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}} \right) \right)}{2\sqrt[4]{b}c}$$

**Mathematica** [C] time = 0.17, size = 160, normalized size = 0.96

$$\frac{5acx\sqrt[4]{a+bx^4} F_1\left(\frac{1}{4}; -\frac{1}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(c+dx^4) \left( x^4 \left( bcF_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 4adF_1\left(\frac{5}{4}; -\frac{1}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5acF_1\left(\frac{1}{4}; -\frac{1}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(1/4)/(c + d\*x^4), x]

[Out] (5\*a\*c\*x\*(a + b\*x^4)^(1/4)\*AppellF1[1/4, -1/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, -1/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(-4\*a\*d\*AppellF1[5/4, -1/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*c\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c), x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(1/4)/(d\*x^4+c), x)

[Out] int((b\*x^4+a)^(1/4)/(d\*x^4+c), x)



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^4 + a)^{1/4}}{dx^4 + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(1/4)/(c + d\*x^4),x)

[Out] int((a + b\*x^4)^(1/4)/(c + d\*x^4), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx^4}}{c + dx^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c),x)

[Out] Integral((a + b\*x\*\*4)\*\*(1/4)/(c + d\*x\*\*4), x)

$$3.202 \quad \int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=259

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right) d\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right) d\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}}{\sqrt{a} (a+bx^4)^{3/4} (bc-ad) \quad 2\sqrt[4]{b}c(bc-ad)}$$

[Out]  $-b^{3/2}*(1+a/b/x^4)^{3/4}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{1/2}/a^{1/2}))^2)^{1/2}/\cos(1/2*\operatorname{arccot}(x^2*b^{1/2}/a^{1/2}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{1/2}/a^{1/2})), 2^{1/2})/(-a*d+b*c)/(b*x^4+a)^{3/4}/a^{1/2}-1/2*d*\operatorname{EllipticPi}(b^{1/4}*x/(b*x^4+a)^{1/4}, -(-a*d+b*c)^{1/2}/b^{1/2}/c^{1/2}, I)*(a/(b*x^4+a))^{1/2}*(b*x^4+a)^{1/2}/b^{1/4}/c/(-a*d+b*c)-1/2*d*\operatorname{EllipticPi}(b^{1/4}*x/(b*x^4+a)^{1/4}, (-a*d+b*c)^{1/2}/b^{1/2}/c^{1/2}, I)*(a/(b*x^4+a))^{1/2}*(b*x^4+a)^{1/2}/b^{1/4}/c/(-a*d+b*c)$

**Rubi [A]** time = 0.15, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {410, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right) d\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right) d\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4}}{\sqrt{a} (a+bx^4)^{3/4} (bc-ad) \quad 2\sqrt[4]{b}c(bc-ad)}$$

Antiderivative was successfully verified.

[In] `Int[1/((a + b*x^4)^(3/4)*(c + d*x^4)), x]`

[Out]  $-(b^{3/2}*(1 + a/(b*x^4))^{3/4}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/(\operatorname{Sqrt}[a]*(b*c - a*d)*(a + b*x^4)^{3/4}) - (d*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d])/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])], \operatorname{ArcSin}[(b^{1/4}*x)/(a + b*x^4)^{1/4}], -1])/ (2*b^{1/4}*c*(b*c - a*d) - (d*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{1/4}*x)/(a + b*x^4)^{1/4}], -1])/ (2*b^{1/4}*c*(b*c - a*d))$

#### Rule 231

`Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] := Simp[(2*EllipticF[(1*ArcTan[Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]`

#### Rule 237

`Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] := Dist[(x^3*(1 + a/(b*x^4))^(3/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]`

#### Rule 275

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

#### Rule 335

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int`

egerQ[m]

Rule 407

```
Int[((a_) + (b_.)*(x_)^4)^(1/4)/((c_) + (d_.)*(x_)^4), x_Symbol] := Dist[Sqrt[a + b*x^4]*Sqrt[a/(a + b*x^4)], Subst[Int[1/(Sqrt[1 - b*x^4]*(c - (b*c - a*d)*x^4)), x], x, x/(a + b*x^4)^(1/4)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 409

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^4]*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 - Rt[-(d/c), 2]*x^2)), x], x] + Dist[1/(2*c), Int[1/(Sqrt[a + b*x^4]*(1 + Rt[-(d/c), 2]*x^2)), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 410

```
Int[1/(((a_) + (b_.)*(x_)^4)^(3/4)*((c_) + (d_.)*(x_)^4)), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^4)^(3/4), x], x] - Dist[d/(b*c - a*d), Int[(a + b*x^4)^(1/4)/(c + d*x^4), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 1218

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := With[{q = Rt[-(c/a), 4]}, Simp[(1*EllipticPi[-(e/(d*q^2)), ArcSin[q*x], -1])/(d*Sqrt[a]*q), x]] /; FreeQ[{a, c, d, e}, x] && NegQ[c/a] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + bx^4)^{3/4} (c + dx^4)} dx &= \frac{b \int \frac{1}{(a+bx^4)^{3/4}} dx}{bc - ad} - \frac{d \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{bc - ad} \\
&= \frac{\left(b \left(1 + \frac{a}{bx^4}\right)^{3/4} x^3\right) \int \frac{1}{\left(1 + \frac{a}{bx^4}\right)^{3/4} x^3} dx}{(bc - ad) (a + bx^4)^{3/4}} - \frac{\left(d \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4} (c-bx^4)} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{bc - ad} \\
&= -\frac{\left(b \left(1 + \frac{a}{bx^4}\right)^{3/4} x^3\right) \text{Subst}\left(\int \frac{x}{\left(1 + \frac{ax^4}{b}\right)^{3/4}} dx, x, \frac{1}{x}\right)}{(bc - ad) (a + bx^4)^{3/4}} - \frac{\left(d \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-bx^4} (c-bx^4)} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{2c} \\
&= -\frac{d \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a+bx^4}}\right)\right) - 1}{2 \sqrt[4]{b} c (bc - ad)} - \frac{d \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a+bx^4}}\right)\right)}{2 \sqrt[4]{b} c (bc - ad)} \\
&= -\frac{b^{3/2} \left(1 + \frac{a}{bx^4}\right)^{3/4} x^3 F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right)\right)}{\sqrt{a} (bc - ad) (a + bx^4)^{3/4}} - \frac{d \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b} \sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{a+bx^4}}\right)\right)}{2 \sqrt[4]{b} c (bc - ad)}
\end{aligned}$$

**Mathematica** [C] time = 0.04, size = 161, normalized size = 0.62

$$\frac{5acx F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(a + bx^4)^{3/4} (c + dx^4) \left( x^4 \left( 4ad F_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) - 5ac F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)),x]

[Out] (-5\*a\*c\*x\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((a + b\*x^4)^(3/4)\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)), x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(3/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(3/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{3/4} (dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)`

[Out] `int(1/((a + b*x^4)^(3/4)*(c + d*x^4)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{3}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(3/4)/(d*x**4+c), x)`

[Out] `Integral(1/((a + b*x**4)**(3/4)*(c + d*x**4)), x)`

$$3.203 \quad \int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=304

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (2bc - 5ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2} (a + bx^4)^{3/4} (bc - ad)^2} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c(bc - ad)^2} + \dots$$

[Out]  $1/3*b*x/a/(-a*d+b*c)/(b*x^4+a)^{(3/4)}-1/3*b^{(3/2)}*(-5*a*d+2*b*c)*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})/a^{(3/2)}/(-a*d+b*c)^2/(b*x^4+a)^{(3/4)}+1/2*d^2*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^2+1/2*d^2*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^2$

**Rubi [A]** time = 0.24, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (2bc - 5ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right)}{3a^{3/2} (a + bx^4)^{3/4} (bc - ad)^2} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{2\sqrt[4]{b}c(bc - ad)^2} + \dots$$

Antiderivative was successfully verified.

[In] `Int[1/((a + b*x^4)^(7/4)*(c + d*x^4)), x]`

[Out]  $(b*x)/(3*a*(b*c - a*d)*(a + b*x^4)^{(3/4)}) - (b^{(3/2)}*(2*b*c - 5*a*d)*(1 + a/(b*x^4))^{(3/4)}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/(3*a^{(3/2)}*(b*c - a*d)^2*(a + b*x^4)^{(3/4)}) + (d^2*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)^2) + (d^2*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1])/(2*b^{(1/4)}*c*(b*c - a*d)^2)$

#### Rule 231

`Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] := Simp[(2*EllipticF[(1*ArcTan[Rt[b/a, 2]*x])/2, 2])/(a^(3/4)*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]`

#### Rule 237

`Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] := Dist[(x^3*(1 + a/(b*x^4))^(3/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]`

#### Rule 275

`Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]`

#### Rule 335

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \text{ :> } -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] \text{ /; } \text{FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_ + (b_)*(x_)^4)^{(1/4)} / ((c_ + (d_)*(x_)^4)), x\_Symbol] \text{ :> } \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4]) * (c - (b*c - a*d)*x^4)], x], x, x/(a + b*x^4)^{(1/4)}, x] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_ + (b_)*(x_)^4) * ((c_ + (d_)*(x_)^4))], x\_Symbol] \text{ :> } \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 414

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)} * ((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \text{ :> } -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q+1)}) / (a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^q * \text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, n, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !( \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1]) \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 529

$\text{Int}[(e_ + (f_)*(x_)^4) / (((a_ + (b_)*(x_)^4)^{(3/4)} * ((c_ + (d_)*(x_)^4))), x\_Symbol] \text{ :> } \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] \text{ /; } \text{FreeQ}\{a, b, c, d, e, f\}, x\}$

#### Rule 1218

$\text{Int}[1/(((d_ + (e_)*(x_)^2) * \text{Sqrt}[(a_ + (c_)*(x_)^4])), x\_Symbol] \text{ :> } \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a]*q), x] \text{ /; } \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{7/4}(c+dx^4)} dx &= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{\int \frac{-2bc+3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)} dx}{3a(bc-ad)} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{d^2 \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{(bc-ad)^2} + \frac{(b(2bc-5ad)) \int \frac{1}{(a+bx^4)^{3/4}} dx}{3a(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3\right) \int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4} x^3} dx}{3a(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\left(d^2 \sqrt{\frac{a}{a+bx^4}}\right)}{3a(bc-ad)^2(a+bx^4)^{3/4}} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{\left(b(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3\right) \text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}} dx, x, \frac{bx^4}{a}\right)}{3a(bc-ad)^2(a+bx^4)^{3/4}} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right) - 1}{2\sqrt[4]{b}c(bc-ad)^2} \\
&= \frac{bx}{3a(bc-ad)(a+bx^4)^{3/4}} - \frac{b^{3/2}(2bc-5ad)\left(1+\frac{a}{bx^4}\right)^{3/4} x^3 F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right) \Big|_2}{3a^{3/2}(bc-ad)^2(a+bx^4)^{3/4}} + \dots
\end{aligned}$$

**Mathematica [C]** time = 0.27, size = 332, normalized size = 1.09

$$x \frac{\left(5\left(bx^4(c+dx^4)\left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5ac(3ad-b(3c+dx^4))F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)}{(c+dx^4)\left(5acF_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - x^4\left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)\right)} - \frac{2bdx^4\left(\frac{bx^4}{a}+1\right)^{3/4} F_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{15a(a+bx^4)^{3/4}(ad-bc)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)), x]

[Out] (x\*((-2\*b\*d\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]/c + (5\*(5\*a\*c\*(3\*a\*d - b\*(3\*c + d\*x^4))\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*x^4\*(c + d\*x^4)\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(c + d\*x^4)\*(5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] - x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(15\*a\*(-(b\*c) + a\*d)\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c), x, algorithm="fricas")

[Out] Timed out



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(7/4)\*(d\*x^4 + c)), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(7/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(7/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(7/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{7}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(7/4)\*(c + d\*x\*\*4)), x)

$$3.204 \quad \int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)} dx$$

**Optimal.** Leaf size=357

$$\frac{bx(6bc - 13ad)}{21a^2 (a + bx^4)^{3/4} (bc - ad)^2} - \frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (47a^2d^2 - 38abcd + 12b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle| 2\right)}{21a^{5/2} (a + bx^4)^{3/4} (bc - ad)^3} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a}}{21a^{5/2} (a + bx^4)^{3/4} (bc - ad)^3}$$

[Out]  $\frac{1}{7}bx/a/(-a*d+b*c)/(b*x^4+a)^{(7/4)}+1/21*b*(-13*a*d+6*b*c)*x/a^2/(-a*d+b*c)^2/(b*x^4+a)^{(3/4)}-1/21*b^{(3/2)}*(47*a^2*d^2-38*a*b*c*d+12*b^2*c^2)*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)})),2^{(1/2)})/a^{(5/2)}/(-a*d+b*c)^3/(b*x^4+a)^{(3/4)}-1/2*d^3*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)},-(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^3-1/2*d^3*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)},(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)},I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c/(-a*d+b*c)^3$

**Rubi [A]** time = 0.40, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (47a^2d^2 - 38abcd + 12b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\middle| 2\right)}{21a^{5/2} (a + bx^4)^{3/4} (bc - ad)^3} + \frac{bx(6bc - 13ad)}{21a^2 (a + bx^4)^{3/4} (bc - ad)^2} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a}}{21a^{5/2} (a + bx^4)^{3/4} (bc - ad)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(11/4)\*(c + d\*x^4)),x]

[Out]  $(b*x)/(7*a*(b*c - a*d)*(a + b*x^4)^{(7/4)}) + (b*(6*b*c - 13*a*d)*x)/(21*a^2*(b*c - a*d)^2*(a + b*x^4)^{(3/4)}) - (b^{(3/2)}*(12*b^2*c^2 - 38*a*b*c*d + 47*a^2*d^2)*(1 + a/(b*x^4))^{(3/4)}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/(21*a^{(5/2)}*(b*c - a*d)^3*(a + b*x^4)^{(3/4)}) - (d^3*\operatorname{Sqrt}[a/(a + b*x^4)])*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*(b*c - a*d)^3) - (d^3*\operatorname{Sqrt}[a/(a + b*x^4)])*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(2*b^{(1/4)}*c*(b*c - a*d)^3)$

### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 335

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p / x^{(m+2)}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x\} \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_ + (b_.) * (x_)^4)^{(1/4)} / ((c_) + (d_.) * (x_)^4), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4] * (c - (b*c - a*d)*x^4)), x], x, x/(a + b*x^4)^{(1/4)}], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_.) * (x_)^4] * ((c_) + (d_.) * (x_)^4)), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 414

$\text{Int}[(a_ + (b_.) * (x_)^{(n_)})^{(p_)} * ((c_) + (d_.) * (x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q+1)}) / (a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^q * \text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !( \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1]) \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 527

$\text{Int}[(a_ + (b_.) * (x_)^{(n_)})^{(p_)} * ((c_) + (d_.) * (x_)^{(n_)})^{(q_)} * ((e_) + (f_.) * (x_)^{(n_)}), x\_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q+1)}) / (a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^q * \text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, q\}, x\} \ \&\& \ \text{LtQ}[p, -1]$

#### Rule 529

$\text{Int}[(e_ + (f_.) * (x_)^4) / (((a_) + (b_.) * (x_)^4)^{(3/4)} * ((c_) + (d_.) * (x_)^4)), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x\}$

#### Rule 1218

$\text{Int}[1/(((d_) + (e_.) * (x_)^2) * \text{Sqrt}[(a_) + (c_.) * (x_)^4]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a]*q), x] /; \text{FreeQ}\{a, c, d, e\}, x\} \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{11/4}(c+dx^4)} dx &= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} - \frac{\int \frac{-6bc+7ad-6bdx^4}{(a+bx^4)^{7/4}(c+dx^4)} dx}{7a(bc-ad)} \\
&= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{\int \frac{12b^2c^2-26abcd+21a^2d^2+2bd^3}{(a+bx^4)^{3/4}(c+dx^4)} dx}{21a^2(bc-ad)^2} \\
&= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3 \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{(bc-ad)^3} + \frac{b(12b^2c^2-38abcd+4a^2d^2)}{21a^2(bc-ad)^2} \\
&= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} + \frac{b(12b^2c^2-38abcd+4a^2d^2)}{21a^2(bc-ad)^2} \\
&= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{d^3 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} \Pi\left(-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{a}{a+bx^4}\right)}{2\sqrt[4]{b}c} \\
&= \frac{bx}{7a(bc-ad)(a+bx^4)^{7/4}} + \frac{b(6bc-13ad)x}{21a^2(bc-ad)^2(a+bx^4)^{3/4}} - \frac{b^{3/2}(12b^2c^2-38abcd+4a^2d^2)}{21a^5}
\end{aligned}$$

**Mathematica [C]** time = 0.77, size = 430, normalized size = 1.20

$$x \frac{5(bx^4(c+dx^4)(16a^2d+ab(13dx^4-9c)-6b^2cx^4) \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5ac(21a^3d^2+a^2bd(5dx^4-42c)+ab^2(21c^2-d^3))}{(a+bx^4)(c+dx^4) \left(x^4 \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5acF_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{(b*x^4)}{a}, -\frac{(d*x^4)}{c}\right) + 3*b*c*AppellF1\left[\frac{5}{4}, \frac{7}{4}, 1, \frac{9}{4}, -\frac{(b*x^4)}{a}, -\frac{(d*x^4)}{c}\right]\right)}{105a^2(a+bx^4)^{3/4}(bc-ad)^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(11/4)\*(c + d\*x^4)), x]

[Out] (x\*((-2\*b\*d\*(-6\*b\*c + 13\*a\*d)\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c - (5\*(5\*a\*c\*(21\*a^3\*d^2 + 6\*b^3\*c\*x^4\*(3\*c + d\*x^4) + a^2\*b\*d\*(-42\*c + 5\*d\*x^4) + a\*b^2\*(21\*c^2 - 30\*c\*d\*x^4 - 13\*d^2\*x^8))\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + b\*x^4\*(c + d\*x^4)\*(16\*a^2\*d - 6\*b^2\*c\*x^4 + a\*b\*(-9\*c + 13\*d\*x^4))\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((a + b\*x^4)\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(105\*a^2\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(11/4)/(d\*x^4+c),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(11/4)/(d\*x^4+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(11/4)\*(d\*x^4 + c)), x)

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(11/4)/(d\*x^4+c),x)

[Out] int(1/(b\*x^4+a)^(11/4)/(d\*x^4+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(11/4)/(d\*x^4+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(11/4)\*(d\*x^4 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{11}{4}}(dx^4 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(11/4)\*(c + d\*x^4)),x)

[Out] int(1/((a + b\*x^4)^(11/4)\*(c + d\*x^4)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{11}{4}}(c + dx^4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(11/4)/(d\*x\*\*4+c),x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(11/4)\*(c + d\*x\*\*4)), x)

$$3.205 \quad \int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=280

$$\frac{b^{7/4}(8bc - 11ad) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} - \frac{b^{7/4}(8bc - 11ad) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

[Out]  $1/4*b*(-a*d+2*b*c)*x*(b*x^4+a)^{(3/4)}/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^{(7/4)}/c/d/(d*x^4+c)-1/8*b^{(7/4)}*(-11*a*d+8*b*c)*\arctan(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^3+1/8*(-a*d+b*c)^{(7/4)}*(3*a*d+8*b*c)*\arctan((-a*d+b*c)^{(1/4)}*x/c^{(1/4)/(b*x^4+a)^{(1/4)})}/c^{(7/4)}/d^3-1/8*b^{(7/4)}*(-11*a*d+8*b*c)*\operatorname{arctanh}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^3+1/8*(-a*d+b*c)^{(7/4)}*(3*a*d+8*b*c)*\operatorname{arctanh}((-a*d+b*c)^{(1/4)}*x/c^{(1/4)/(b*x^4+a)^{(1/4)})}/c^{(7/4)}/d^3$

**Rubi [A]** time = 0.36, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {413, 528, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{7/4}(8bc - 11ad) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} - \frac{b^{7/4}(8bc - 11ad) \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc - ad)^{7/4}(3ad + 8bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(11/4)/(c + d\*x^4)^2, x]

[Out]  $(b*(2*b*c - a*d)*x*(a + b*x^4)^{(3/4)})/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4)^{(7/4)})/(4*c*d*(c + d*x^4)) - (b^{(7/4)}*(8*b*c - 11*a*d)*\operatorname{ArcTan}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}])/(8*d^3) + ((b*c - a*d)^{(7/4)}*(8*b*c + 3*a*d)*\operatorname{ArcTan}[(b*c - a*d)^{(1/4)}*x/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*d^3) - (b^{(7/4)}*(8*b*c - 11*a*d)*\operatorname{ArcTanh}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}])/(8*d^3) + ((b*c - a*d)^{(7/4)}*(8*b*c + 3*a*d)*\operatorname{ArcTanh}[(b*c - a*d)^{(1/4)}*x/(c^{(1/4)}*(a + b*x^4)^{(1/4)})])/(8*c^{(7/4)}*d^3)$

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 240

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[a^(p + 1/n), Subst[Int[1/(1 - b\*x^n)^(p + 1/n + 1), x], x, x/(a + b\*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 528

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(f\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(b\*(n\*(p + q + 1) + 1)), x] + Dist[1/(b\*(n\*(p + q + 1) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*(b\*e - a\*f + b\*e\*n\*(p + q + 1)) + (d\*(b\*e - a\*f) + f\*n\*q\*(b\*c - a\*d) + b\*d\*e\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n\*(p + q + 1) + 1, 0]

Rule 530

Int((((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)))/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[f/d, Int[(a + b\*x^n)^p, x], x] + Dist[(d\*e - c\*f)/d, Int[(a + b\*x^n)^p/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p, n}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^4)^{11/4}}{(c+dx^4)^2} dx &= -\frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} + \frac{\int \frac{(a+bx^4)^{3/4}(a(bc+3ad)+4b(2bc-ad)x^4)}{c+dx^4} dx}{4cd} \\
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} + \frac{\int \frac{-4a(2b^2c^2-2abcd-3a^2d^2)-4b^2c(8bc-11ad)x^4}{\sqrt[4]{a+bx^4}(c+dx^4)}}{16cd^2} \\
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad)) \int \frac{1}{\sqrt[4]{a+bx^4}} dx}{4d^3} + \frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd^2} \\
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad)) \text{Subst}\left(\int \frac{1}{1-bx^4} dx, x, \sqrt[4]{a+bx^4}\right)}{4d^3} \\
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{(b^2(8bc-11ad)) \text{Subst}\left(\int \frac{1}{1-\sqrt{b}x^2} dx, x, \sqrt[4]{a+bx^4}\right)}{8d^3} \\
&= \frac{b(2bc-ad)x(a+bx^4)^{3/4}}{4cd^2} - \frac{(bc-ad)x(a+bx^4)^{7/4}}{4cd(c+dx^4)} - \frac{b^{7/4}(8bc-11ad) \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{8d^3} + \frac{(bc-ad)x(a+bx^4)^{3/4}}{4cd^2}
\end{aligned}$$

**Mathematica [C]** time = 0.96, size = 560, normalized size = 2.00

$$\frac{1}{80} \left( \frac{15a^3 \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) + \log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right) \right)}{c^{7/4}\sqrt[4]{bc-ad}} + \frac{10a^2b \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) \right)}{c^{7/4}\sqrt[4]{bc-ad}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(11/4)/(c + d\*x^4)^2,x]

[Out] ((20\*x\*(a + b\*x^4)^(3/4)\*(b^2 + (b\*c - a\*d)^2/(c\*(c + d\*x^4))))/d^2 - (32\*b^3\*x^5\*(1 + (b\*x^4)/a)^(1/4)\*AppellF1[5/4, 1/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/d^2\*(a + b\*x^4)^(1/4) + (44\*a\*b^2\*x^5\*(1 + (b\*x^4)/a)^(1/4)\*AppellF1[5/4, 1/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c\*d\*(a + b\*x^4)^(1/4) + (15\*a^3\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]))/c^(7/4)\*(b\*c - a\*d)^(1/4) - (10\*a\*b^2\*c^(1/4)\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]))/d^2\*(b\*c - a\*d)^(1/4) + (10\*a^2\*b\*(2\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(b + a\*x^4)^(1/4))] - Log[c^(1/4) - ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)] + Log[c^(1/4) + ((b\*c - a\*d)^(1/4)\*x)/(b + a\*x^4)^(1/4)]))/c^(3/4)\*d\*(b\*c - a\*d)^(1/4))/80

**fricas [B]** time = 45.48, size = 3308, normalized size = 11.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(11/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] 1/16\*(4\*(c\*d^3\*x^4 + c^2\*d^2)\*((4096\*b^11\*c^11 - 22528\*a\*b^10\*c^10\*d + 46464\*a^2\*b^9\*c^9\*d^2 - 37664\*a^3\*b^8\*c^8\*d^3 - 5071\*a^4\*b^7\*c^7\*d^4 + 25641\*a^5\*b^6\*c^6\*d^5 - 11200\*a^6\*b^5\*c^5\*d^6 + 3584\*a^7\*b^4\*c^4\*d^7 - 768\*a^8\*b^3\*c^3\*d^8 + 128\*a^9\*b^2\*c^2\*d^9 - 16\*a^10\*b\*c\*d^10 + a^11\*d^11)/(c^11\*d^11) + 22528\*a\*b^10\*c^10\*d + 46464\*a^2\*b^9\*c^9\*d^2 - 37664\*a^3\*b^8\*c^8\*d^3 - 5071\*a^4\*b^7\*c^7\*d^4 + 25641\*a^5\*b^6\*c^6\*d^5 - 11200\*a^6\*b^5\*c^5\*d^6 + 3584\*a^7\*b^4\*c^4\*d^7 - 768\*a^8\*b^3\*c^3\*d^8 + 128\*a^9\*b^2\*c^2\*d^9 - 16\*a^10\*b\*c\*d^10 + a^11\*d^11)/(c^11\*d^11))





$$\begin{aligned}
 & *b^9*c^9*d^2 - 37664*a^3*b^8*c^8*d^3 - 5071*a^4*b^7*c^7*d^4 + 25641*a^5*b^6 \\
 & *c^6*d^5 - 7931*a^6*b^5*c^5*d^6 - 6259*a^7*b^4*c^4*d^7 + 2739*a^8*b^3*c^3*d \\
 & ^8 + 891*a^9*b^2*c^2*d^9 - 297*a^{10}*b*c*d^{10} - 81*a^{11}*d^{11})/(c^7*d^{12})^{(3 \\
 & /4) - (512*b^8*c^8 - 1984*a*b^7*c^7*d + 2456*a^2*b^6*c^6*d^2 - 413*a^3*b^5* \\
 & c^5*d^3 - 1175*a^4*b^4*c^4*d^4 + 478*a^5*b^3*c^3*d^5 + 234*a^6*b^2*c^2*d^6 \\
 & - 81*a^7*b*c*d^7 - 27*a^8*d^8)*(b*x^4 + a)^{(1/4))/x) - (c*d^3*x^4 + c^2*d^2 \\
 & )*((4096*b^{11}*c^4 - 22528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3* \\
 & b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^{12})^{(1/4)*\log(-(d^9*x*((4096*b^{11}*c^4 - 22 \\
 & 528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4* \\
 & b^7*d^4)/d^{12})^{(3/4) + (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 \\
 & - 1331*a^3*b^5*d^3)*(b*x^4 + a)^{(1/4))/x) + (c*d^3*x^4 + c^2*d^2)*((4096*b \\
 & ^{11}*c^4 - 22528*a*b^{10}*c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 \\
 & + 14641*a^4*b^7*d^4)/d^{12})^{(1/4)*\log((d^9*x*((4096*b^{11}*c^4 - 22528*a*b^{10} \\
 & c^3*d + 46464*a^2*b^9*c^2*d^2 - 42592*a^3*b^8*c*d^3 + 14641*a^4*b^7*d^4)/d^ \\
 & 12)^{(3/4) - (512*b^8*c^3 - 2112*a*b^7*c^2*d + 2904*a^2*b^6*c*d^2 - 1331*a^3 \\
 & *b^5*d^3)*(b*x^4 + a)^{(1/4))/x) + 4*(b^2*c*d*x^5 + (2*b^2*c^2 - 2*a*b*c*d + \\
 & a^2*d^2)*x)*(b*x^4 + a)^{(3/4)))/(c*d^3*x^4 + c^2*d^2)
 \end{aligned}$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(11/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2, x)

**maple** [F] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(11/4)/(d\*x^4+c)^2,x)

[Out] int((b\*x^4+a)^(11/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(11/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(11/4)/(d\*x^4 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{\frac{11}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^4)^(11/4)/(c + d*x^4)^2,x)
```

```
[Out] int((a + b*x^4)^(11/4)/(c + d*x^4)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**4+a)**(11/4)/(d*x**4+c)**2,x)
```

```
[Out] Timed out
```

$$3.206 \quad \int \frac{(a+bx^4)^{7/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=230

$$\frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} + \frac{b^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

[Out]  $-1/4*(-a*d+b*c)*x*(b*x^4+a)^{(3/4)}/c/d/(d*x^4+c)+1/2*b^{(7/4)}*\arctan(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^2-1/8*(-a*d+b*c)^{(3/4)}*(3*a*d+4*b*c)*\arctan((-a*d+b*c)^{(1/4)}*x/c^{(1/4)}/(b*x^4+a)^{(1/4)})/c^{(7/4)}/d^2+1/2*b^{(7/4)}*\operatorname{arctanh}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)})/d^2-1/8*(-a*d+b*c)^{(3/4)}*(3*a*d+4*b*c)*\operatorname{arctanh}((-a*d+b*c)^{(1/4)}*x/c^{(1/4)}/(b*x^4+a)^{(1/4)})/c^{(7/4)}/d^2$

**Rubi [A]** time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {413, 530, 240, 212, 206, 203, 377, 208, 205}

$$\frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} + \frac{b^{7/4} \tanh^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)}{2d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2} - \frac{(bc-ad)^{3/4}(3ad+4bc) \tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(7/4)/(c + d\*x^4)^2, x]

[Out]  $-((b*c - a*d)*x*(a + b*x^4)^{(3/4)})/(4*c*d*(c + d*x^4)) + (b^{(7/4)}*\operatorname{ArcTan}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}])/(2*d^2) - ((b*c - a*d)^{(3/4)}*(4*b*c + 3*a*d)*\operatorname{ArcTan}[(b^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)}])/(8*c^{(7/4)}*d^2) + (b^{(7/4)}*\operatorname{ArcTanh}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}])/(2*d^2) - ((b*c - a*d)^{(3/4)}*(4*b*c + 3*a*d)*\operatorname{ArcTanh}[(b^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)}])/(8*c^{(7/4)}*d^2)$

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 240

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[a^(p + 1/n), Subst[Int[1/(1 - b\*x^n)^(p + 1/n + 1), x], x, x/(a + b\*x^n)^(1/n)], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[-1, p, 0] && NeQ[p, -2^(-1)] && IntegerQ[p + 1/n]

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

### Rule 530

Int[(((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)))/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[f/d, Int[(a + b\*x^n)^p, x], x] + Dist[(d\*e - c\*f)/d, Int[(a + b\*x^n)^p/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, p, n}, x]

### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^4)^{7/4}}{(c + dx^4)^2} dx &= -\frac{(bc - ad)x(a + bx^4)^{3/4}}{4cd(c + dx^4)} + \frac{\int \frac{a(bc + 3ad) + 4b^2cx^4}{\sqrt[4]{a + bx^4}(c + dx^4)} dx}{4cd} \\ &= -\frac{(bc - ad)x(a + bx^4)^{3/4}}{4cd(c + dx^4)} + \frac{b^2 \int \frac{1}{\sqrt[4]{a + bx^4}} dx}{d^2} - \frac{((bc - ad)(4bc + 3ad)) \int \frac{1}{\sqrt[4]{a + bx^4}(c + dx^4)} dx}{4cd^2} \\ &= -\frac{(bc - ad)x(a + bx^4)^{3/4}}{4cd(c + dx^4)} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1 - bx^4} dx, x, \frac{x}{\sqrt[4]{a + bx^4}}\right)}{d^2} - \frac{((bc - ad)(4bc + 3ad)) \text{Subst}\left(\int \frac{1}{1 - bx^4} dx, x, \frac{x}{\sqrt[4]{a + bx^4}}\right)}{4cd^2} \\ &= -\frac{(bc - ad)x(a + bx^4)^{3/4}}{4cd(c + dx^4)} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1 - \sqrt{b}x^2} dx, x, \frac{x}{\sqrt[4]{a + bx^4}}\right)}{2d^2} + \frac{b^2 \text{Subst}\left(\int \frac{1}{1 + \sqrt{b}x^2} dx, x, \frac{x}{\sqrt[4]{a + bx^4}}\right)}{2d^2} \\ &= -\frac{(bc - ad)x(a + bx^4)^{3/4}}{4cd(c + dx^4)} + \frac{b^{7/4} \tan^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a + bx^4}}\right)}{2d^2} - \frac{(bc - ad)^{3/4}(4bc + 3ad) \tan^{-1}\left(\frac{\sqrt[4]{bc - ad}x}{\sqrt[4]{c} \sqrt[4]{a + bx^4}}\right)}{8c^{7/4}d^2} \end{aligned}$$

**Mathematica** [C] time = 0.64, size = 358, normalized size = 1.56

$$\frac{15a^2 \left( -\log\left(\sqrt[4]{c} - \frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}}\right) + \log\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{ax^4+b}} + \sqrt[4]{c}\right) + 2 \tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{ax^4+b}}\right) \right)}{\sqrt[4]{bc-ad}} + \frac{16b^2c^{3/4}x^5\sqrt[4]{\frac{bx^4}{a}} + {}_1F_1\left(\frac{5}{4}; \frac{1}{4}; 1; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{d\sqrt[4]{a+bx^4}} - \frac{20c^{3/4}x(a+bx^4)^{3/4}(bc-ad)}{d(c+dx^4)}$$


---

80c<sup>7/4</sup>

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*x^4)^(7/4)/(c + d*x^4)^2,x]
```

```
[Out] ((-20*c^(3/4)*(b*c - a*d)*x*(a + b*x^4)^(3/4))/(d*(c + d*x^4)) + (16*b^2*c^(3/4)*x^5*(1 + (b*x^4)/a)^(1/4)*AppellF1[5/4, 1/4, 1, 9/4, -((b*x^4)/a), -(d*x^4)/c])/(d*(a + b*x^4)^(1/4)) + (15*a^2*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))]) - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)])/(b*c - a*d)^(1/4) + (5*a*b*c*(2*ArcTan[((b*c - a*d)^(1/4)*x)/(c^(1/4)*(b + a*x^4)^(1/4))]) - Log[c^(1/4) - ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)] + Log[c^(1/4) + ((b*c - a*d)^(1/4)*x)/(b + a*x^4)^(1/4)])/(d*(b*c - a*d)^(1/4)))/(80*c^(7/4))
```

**fricas** [B] time = 3.26, size = 1667, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^4+a)^(7/4)/(d*x^4+c)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*(b*x^4 + a)^(3/4)*(b*c - a*d)*x - 4*(c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*arctan(-(c^2*d^2*x*sqrt(((256*b^7*c^10*d^4 - 672*a^2*b^5*c^8*d^6 - 112*a^3*b^4*c^7*d^7 + 609*a^4*b^3*c^6*d^8 + 189*a^5*b^2*c^5*d^9 - 189*a^6*b*c^4*d^10 - 81*a^7*c^3*d^11)*x^2*sqrt((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8)) + (4096*b^10*c^10 + 2048*a*b^9*c^9*d - 14592*a^2*b^8*c^8*d^2 - 9472*a^3*b^7*c^7*d^3 + 18928*a^4*b^6*c^6*d^4 + 15624*a^5*b^5*c^5*d^5 - 9639*a^6*b^4*c^4*d^6 - 11124*a^7*b^3*c^3*d^7 + 486*a^8*b^2*c^2*d^8 + 2916*a^9*b*c*d^9 + 729*a^10*d^10)*sqrt(b*x^4 + a))/x^2)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4) - (64*b^5*c^7*d^2 + 16*a*b^4*c^6*d^3 - 116*a^2*b^3*c^5*d^4 - 45*a^3*b^2*c^4*d^5 + 54*a^4*b*c^3*d^6 + 27*a^5*c^2*d^7)*(b*x^4 + a)^(1/4)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4))/((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)) - 16*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^(1/4)*arctan(-(b*x^4 + a)^(1/4)*b^5*d^2*(b^7/d^8)^(1/4) - d^2*x*(b^7/d^8)^(1/4)*sqrt((b^7*d^4*x^2*sqrt(b^7/d^8) + sqrt(b*x^4 + a)*b^10)/x^2))/(b^7*x)) + (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log(((c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) + (64*b^5*c^5 + 16*a*b^4*c^4*d - 116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^(1/4))/x) - (c*d^2*x^4 + c^2*d)*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(1/4)*log(-(c^5*d^6*x*((256*b^7*c^7 - 672*a^2*b^5*c^5*d^2 - 112*a^3*b^4*c^4*d^3 + 609*a^4*b^3*c^3*d^4 + 189*a^5*b^2*c^2*d^5 - 189*a^6*b*c*d^6 - 81*a^7*d^7)/(c^7*d^8))^(3/4) - (64*b^5*c^5 + 16*a*b^4*c^4*d -
```

$$116*a^2*b^3*c^3*d^2 - 45*a^3*b^2*c^2*d^3 + 54*a^4*b*c*d^4 + 27*a^5*d^5)*(b*x^4 + a)^{(1/4)}/x - 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^{(1/4)}*\log((d^6*x*(b^7/d^8)^{(3/4)} + (b*x^4 + a)^{(1/4)}*b^5)/x) + 4*(c*d^2*x^4 + c^2*d)*(b^7/d^8)^{(1/4)}*\log(-(d^6*x*(b^7/d^8)^{(3/4)} - (b*x^4 + a)^{(1/4)}*b^5)/x))/(c*d^2*x^4 + c^2*d)$$

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c)^2, x)

**maple** [F] time = 0.65, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x)

[Out] int((b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{7}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(7/4)/(d\*x^4 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{7/4}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(7/4)/(c + d\*x^4)^2,x)

[Out] int((a + b\*x^4)^(7/4)/(c + d\*x^4)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

$$3.207 \quad \int \frac{(a+bx^4)^{3/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=135

$$\frac{3a \tan^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{8c^{7/4} \sqrt[4]{bc-ad}} + \frac{3a \tanh^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{8c^{7/4} \sqrt[4]{bc-ad}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

[Out]  $1/4*x*(b*x^4+a)^{(3/4)}/c/(d*x^4+c)+3/8*a*\arctan((-a*d+b*c)^{(1/4)}*x/c^{(1/4)})/(b*x^4+a)^{(1/4)}/c^{(7/4)}/(-a*d+b*c)^{(1/4)}+3/8*a*\operatorname{arctanh}((-a*d+b*c)^{(1/4)}*x/c^{(1/4)})/(b*x^4+a)^{(1/4)}/c^{(7/4)}/(-a*d+b*c)^{(1/4)}$

**Rubi [A]** time = 0.08, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {378, 377, 212, 208, 205}

$$\frac{3a \tan^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{8c^{7/4} \sqrt[4]{bc-ad}} + \frac{3a \tanh^{-1}\left(\frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}}\right)}{8c^{7/4} \sqrt[4]{bc-ad}} + \frac{x(a+bx^4)^{3/4}}{4c(c+dx^4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(3/4)/(c + d\*x^4)^2, x]

[Out]  $(x*(a + b*x^4)^{(3/4)})/(4*c*(c + d*x^4)) + (3*a*\operatorname{ArcTan}(((b*c - a*d)^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)})))/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)}) + (3*a*\operatorname{ArcTanh}(((b*c - a*d)^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)})))/(8*c^{(7/4)}*(b*c - a*d)^{(1/4)})$

**Rule 205**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

**Rule 208**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

**Rule 212**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

**Rule 377**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(a\*n\*(p+1)), x] - Dist[(c\*q)/(a\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p+q+1) + 1, 0]



&& GtQ[q, 0] && NeQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{(a + bx^4)^{3/4}}{(c + dx^4)^2} dx &= \frac{x(a + bx^4)^{3/4}}{4c(c + dx^4)} + \frac{(3a) \int \frac{1}{\sqrt[4]{a+bx^4}(c+dx^4)} dx}{4c} \\
 &= \frac{x(a + bx^4)^{3/4}}{4c(c + dx^4)} + \frac{(3a) \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c} \\
 &= \frac{x(a + bx^4)^{3/4}}{4c(c + dx^4)} + \frac{(3a) \text{Subst}\left(\int \frac{1}{\sqrt{c-\sqrt{bc-ad}x^2}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}} + \frac{(3a) \text{Subst}\left(\int \frac{1}{\sqrt{c+\sqrt{bc-ad}x^2}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}} \\
 &= \frac{x(a + bx^4)^{3/4}}{4c(c + dx^4)} + \frac{3a \tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}} + \frac{3a \tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}\sqrt[4]{bc-ad}}
 \end{aligned}$$

**Mathematica [C]** time = 0.06, size = 78, normalized size = 0.58

$$\frac{x(a + bx^4)^{3/4} {}_2F_1\left(-\frac{3}{4}, \frac{1}{4}, \frac{5}{4}, \frac{(ad-bc)x^4}{a(dx^4+c)}\right)}{c^2 \left(\frac{bx^4}{a} + 1\right)^{3/4} \sqrt[4]{\frac{dx^4}{c} + 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(3/4)/(c + d\*x^4)^2,x]

[Out] (x\*(a + b\*x^4)^(3/4)\*Hypergeometric2F1[-3/4, 1/4, 5/4, ((-b\*c) + a\*d)\*x^4/(a\*(c + d\*x^4))]/(c^2\*(1 + (b\*x^4)/a)^(3/4)\*(1 + (d\*x^4)/c)^(1/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{3/4}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(3/4)/(d\*x^4 + c)^2, x)

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{3/4}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)`

[Out] `int((b*x^4+a)^(3/4)/(d*x^4+c)^2,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{3}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+a)^(3/4)/(d*x^4+c)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^(3/4)/(d*x^4 + c)^2, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(bx^4 + a)^{3/4}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^4)^(3/4)/(c + d*x^4)^2,x)`

[Out] `int((a + b*x^4)^(3/4)/(c + d*x^4)^2, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{3}{4}}}{(c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**(3/4)/(d*x**4+c)**2,x)`

[Out] `Integral((a + b*x**4)**(3/4)/(c + d*x**4)**2, x)`

$$3.208 \quad \int \frac{1}{\sqrt[4]{a+bx^4} (c+dx^4)^2} dx$$

**Optimal.** Leaf size=162

$$\frac{(4bc - 3ad) \tan^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{5/4}} + \frac{(4bc - 3ad) \tanh^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{5/4}} - \frac{dx (a + bx^4)^{3/4}}{4c (c + dx^4) (bc - ad)}$$

[Out]  $-1/4*d*x*(b*x^4+a)^{(3/4)}/c/(-a*d+b*c)/(d*x^4+c)+1/8*(-3*a*d+4*b*c)*\arctan(($   
 $-a*d+b*c)^{(1/4)}*x/c^{(1/4)}/(b*x^4+a)^{(1/4)})/c^{(7/4)}/(-a*d+b*c)^{(5/4)}+1/8*(-3$   
 $*a*d+4*b*c)*\operatorname{arctanh}((-a*d+b*c)^{(1/4)}*x/c^{(1/4)}/(b*x^4+a)^{(1/4)})/c^{(7/4)}/(-$   
 $*d+b*c)^{(5/4)}$

**Rubi [A]** time = 0.11, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {382, 377, 212, 208, 205}

$$\frac{(4bc - 3ad) \tan^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{5/4}} + \frac{(4bc - 3ad) \tanh^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{5/4}} - \frac{dx (a + bx^4)^{3/4}}{4c (c + dx^4) (bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x*(a + b*x^4)^{(3/4)})/(4*c*(b*c - a*d)*(c + d*x^4)) + ((4*b*c - 3*a*d)*\operatorname{ArcTan}(((b*c - a*d)^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)})))/(8*c^{(7/4)}*(b*c - a*d)^{(5/4)}) + ((4*b*c - 3*a*d)*\operatorname{ArcTanh}(((b*c - a*d)^{(1/4)}*x)/(c^{(1/4)}*(a + b*x^4)^{(1/4)})))/(8*c^{(7/4)}*(b*c - a*d)^{(5/4)})$

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), I

nt[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{a+bx^4} (c+dx^4)^2} dx &= -\frac{dx (a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)} + \frac{(4bc-3ad) \int \frac{1}{\sqrt[4]{a+bx^4} (c+dx^4)} dx}{4c(bc-ad)} \\ &= -\frac{dx (a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)} + \frac{(4bc-3ad) \text{Subst}\left(\int \frac{1}{c-(bc-ad)x^4} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{4c(bc-ad)} \\ &= -\frac{dx (a+bx^4)^{3/4}}{4c(bc-ad)(c+dx^4)} + \frac{(4bc-3ad) \text{Subst}\left(\int \frac{1}{\sqrt{c-\sqrt{bc-ad}x^2}} dx, x, \frac{x}{\sqrt[4]{a+bx^4}}\right)}{8c^{3/2}(bc-ad)} + \frac{(4bc-3ad) \tan^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} + \frac{(4bc-3ad) \tanh^{-1}\left(\frac{\sqrt[4]{bc-ad}x}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{5/4}} \end{aligned}$$

**Mathematica [C]** time = 0.14, size = 99, normalized size = 0.61

$$\frac{x \left( (c+dx^4) (4bc-3ad) {}_2F_1\left(\frac{1}{4}, 1; \frac{5}{4}; \frac{(bc-ad)x^4}{c(bx^4+a)}\right) - cd(a+bx^4) \right)}{4c^2 \sqrt[4]{a+bx^4} (c+dx^4) (bc-ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2), x]

[Out] (x\*(-(c\*d\*(a + b\*x^4)) + (4\*b\*c - 3\*a\*d)\*(c + d\*x^4)\*Hypergeometric2F1[1/4, 1, 5/4, ((b\*c - a\*d)\*x^4)/(c\*(a + b\*x^4))]))/(4\*c^2\*(b\*c - a\*d)\*(a + b\*x^4)^(1/4)\*(c + d\*x^4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4+a)^{\frac{1}{4}}(dx^4+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)^2), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x)

[Out] int(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(1/4)\*(d\*x^4 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(bx^4 + a)^{\frac{1}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2),x)

[Out] int(1/((a + b\*x^4)^(1/4)\*(c + d\*x^4)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{a + bx^4} (c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(1/4)\*(c + d\*x\*\*4)\*\*2), x)

$$3.209 \quad \int \frac{1}{(a+bx^4)^{5/4} (c+dx^4)^2} dx$$

**Optimal.** Leaf size=205

$$\frac{d(8bc - 3ad) \tan^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{9/4}} - \frac{d(8bc - 3ad) \tanh^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{9/4}} + \frac{bx(ad + 4bc)}{4ac \sqrt[4]{a + bx^4} (bc - ad)^2} - \frac{dx}{4c \sqrt[4]{a + bx^4} (c + dx^4)}$$

[Out] 1/4\*b\*(a\*d+4\*b\*c)\*x/a/c/(-a\*d+b\*c)^2/(b\*x^4+a)^(1/4)-1/4\*d\*x/c/(-a\*d+b\*c)/(b\*x^4+a)^(1/4)/(d\*x^4+c)-1/8\*d\*(-3\*a\*d+8\*b\*c)\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(7/4)/(-a\*d+b\*c)^(9/4)-1/8\*d\*(-3\*a\*d+8\*b\*c)\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(7/4)/(-a\*d+b\*c)^(9/4)

**Rubi [A]** time = 0.18, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, number of rules / integrand size = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{d(8bc - 3ad) \tan^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{9/4}} - \frac{d(8bc - 3ad) \tanh^{-1} \left( \frac{x \sqrt[4]{bc-ad}}{\sqrt[4]{c} \sqrt[4]{a+bx^4}} \right)}{8c^{7/4}(bc - ad)^{9/4}} + \frac{bx(ad + 4bc)}{4ac \sqrt[4]{a + bx^4} (bc - ad)^2} - \frac{dx}{4c \sqrt[4]{a + bx^4} (c + dx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)^2), x]

[Out] (b\*(4\*b\*c + a\*d)\*x)/(4\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(1/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(1/4)\*(c + d\*x^4)) - (d\*(8\*b\*c - 3\*a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(9/4)) - (d\*(8\*b\*c - 3\*a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(9/4))

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(a + bx^4)^{5/4} (c + dx^4)^2} dx = -\frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} + \frac{\int \frac{4bc - 3ad - 4bdx^4}{(a + bx^4)^{5/4} (c + dx^4)} dx}{4c(bc - ad)}$$

$$= \frac{b(4bc + ad)x}{4ac(bc - ad)^2\sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} - \frac{\int \frac{ad(8bc - 3ad)}{\sqrt[4]{a + bx^4} (c + dx^4)} dx}{4ac(bc - ad)^2}$$

$$= \frac{b(4bc + ad)x}{4ac(bc - ad)^2\sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} - \frac{(d(8bc - 3ad)) \int \frac{dx}{c + dx^4}}{4c(bc - ad)}$$

$$= \frac{b(4bc + ad)x}{4ac(bc - ad)^2\sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} - \frac{(d(8bc - 3ad)) \operatorname{Subst}\left(\int \frac{dx}{c + dx^4}\right)}{4c(bc - ad)}$$

$$= \frac{b(4bc + ad)x}{4ac(bc - ad)^2\sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} - \frac{(d(8bc - 3ad)) \operatorname{Subst}\left(\int \frac{dx}{c + dx^4}\right)}{4c(bc - ad)}$$

$$= \frac{b(4bc + ad)x}{4ac(bc - ad)^2\sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)\sqrt[4]{a + bx^4} (c + dx^4)} - \frac{d(8bc - 3ad) \tan^{-1}\left(\frac{dx}{\sqrt[4]{c + dx^4}}\right)}{8c^{7/4}(bc - ad)}$$

**Mathematica [C]** time = 2.27, size = 625, normalized size = 3.05

$$c(a + bx^4)^{3/4} \left( \frac{320d^2x^{20}(bc - ad)^3 {}_3F_2\left(2, 2, \frac{9}{4}; 1, \frac{17}{4}; \frac{(bc - ad)x^4}{c(bx^4 + a)}\right)}{c^5(a + bx^4)^3} + \frac{640dx^{16}(bc - ad)^3 {}_3F_2\left(2, 2, \frac{9}{4}; 1, \frac{17}{4}; \frac{(bc - ad)x^4}{c(bx^4 + a)}\right)}{c^4(a + bx^4)^3} + \frac{320x^{12}(bc - ad)^3 {}_3F_2\left(2, 2, \frac{9}{4}; 1, \frac{17}{4}; \frac{(bc - ad)x^4}{c(bx^4 + a)}\right)}{c^3(a + bx^4)^3} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(5/4)\*(c + d\*x^4)^2), x]

[Out] (c\*(a + b\*x^4)^(3/4)\*(-47385 - (94770\*d\*x^4)/c - (44460\*d^2\*x^8)/c^2 + (5148\*(b\*c - a\*d)\*x^4)/(c\*(a + b\*x^4)) + (14976\*d\*(b\*c - a\*d)\*x^8)/(c^2\*(a + b\*x^4)))/c^5

$x^4)) + (7488*d^2*(b*c - a*d)*x^{12})/(c^3*(a + b*x^4)) + 47385*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + (94770*d*x^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c + (44460*d^2*x^8*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^2 - (14625*(b*c - a*d)*x^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c*(a + b*x^4) + (33930*d*(-(b*c) + a*d)*x^8*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^2*(a + b*x^4) + (16380*d^2*(-(b*c) + a*d)*x^{12}*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^3*(a + b*x^4) + (320*(b*c - a*d)^3*x^{12}*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^3*(a + b*x^4)^3 + (640*d*(b*c - a*d)^3*x^{16}*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^4*(a + b*x^4)^3 + (320*d^2*(b*c - a*d)^3*x^{20}*HypergeometricPFQ[{2, 2, 9/4}, {1, 17/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))])/c^5*(a + b*x^4)^3))/(2340*(b*c - a*d)^2*x^7*(c + d*x^4))$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)^2), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x)

[Out] int(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(5/4)\*(d\*x^4 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{5}{4}}(dx^4 + c)^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)`

[Out] `int(1/((a + b*x^4)^(5/4)*(c + d*x^4)^2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{5}{4}} (c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)**(5/4)/(d*x**4+c)**2, x)`

[Out] `Integral(1/((a + b*x**4)**(5/4)*(c + d*x**4)**2), x)`

$$3.210 \quad \int \frac{1}{(a+bx^4)^{9/4}(c+dx^4)^2} dx$$

**Optimal.** Leaf size=266

$$\frac{bx(-5a^2d^2 - 56abcd + 16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} - \frac{1}{4c(a+bx^4)}$$

[Out] 1/20\*b\*(5\*a\*d+4\*b\*c)\*x/a/c/(-a\*d+b\*c)^2/(b\*x^4+a)^(5/4)+1/20\*b\*(-5\*a^2\*d^2-56\*a\*b\*c\*d+16\*b^2\*c^2)\*x/a^2/c/(-a\*d+b\*c)^3/(b\*x^4+a)^(1/4)-1/4\*d\*x/c/(-a\*d+b\*c)/(b\*x^4+a)^(5/4)/(d\*x^4+c)+3/8\*d^2\*(-a\*d+4\*b\*c)\*arctan((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(7/4)/(-a\*d+b\*c)^(13/4)+3/8\*d^2\*(-a\*d+4\*b\*c)\*arctanh((-a\*d+b\*c)^(1/4)\*x/c^(1/4)/(b\*x^4+a)^(1/4))/c^(7/4)/(-a\*d+b\*c)^(13/4)

**Rubi [A]** time = 0.29, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, number of rules / integrand size = 0.333, Rules used = {414, 527, 12, 377, 212, 208, 205}

$$\frac{bx(-5a^2d^2 - 56abcd + 16b^2c^2)}{20a^2c\sqrt[4]{a+bx^4}(bc-ad)^3} + \frac{3d^2(4bc-ad)\tan^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} + \frac{3d^2(4bc-ad)\tanh^{-1}\left(\frac{x\sqrt[4]{bc-ad}}{\sqrt[4]{c}\sqrt[4]{a+bx^4}}\right)}{8c^{7/4}(bc-ad)^{13/4}} - \frac{1}{4c(a+bx^4)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2), x]

[Out] (b\*(4\*b\*c + 5\*a\*d)\*x)/(20\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(5/4)) + (b\*(16\*b^2\*c^2 - 56\*a\*b\*c\*d - 5\*a^2\*d^2)\*x)/(20\*a^2\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(1/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(5/4)\*(c + d\*x^4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTan[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4)) + (3\*d^2\*(4\*b\*c - a\*d)\*ArcTanh[((b\*c - a\*d)^(1/4)\*x)/(c^(1/4)\*(a + b\*x^4)^(1/4))])/(8\*c^(7/4)\*(b\*c - a\*d)^(13/4))

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(b\*c - a\*d)\*(p+1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p+1) + d\*(b\*e - a\*f)\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + bx^4)^{9/4} (c + dx^4)^2} dx &= -\frac{dx}{4c(bc - ad)(a + bx^4)^{5/4} (c + dx^4)} + \frac{\int \frac{4bc - 3ad - 8bdx^4}{(a + bx^4)^{9/4} (c + dx^4)} dx}{4c(bc - ad)} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4} (c + dx^4)} - \frac{\int \frac{-16b^2c^2 + 40}{(a + bx^4)^{9/4} (c + dx^4)} dx}{4c(bc - ad)} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} + \frac{b(16b^2c^2 - 56abcd - 5a^2d^2)x}{20a^2c(bc - ad)^3 \sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4}} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} + \frac{b(16b^2c^2 - 56abcd - 5a^2d^2)x}{20a^2c(bc - ad)^3 \sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4}} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} + \frac{b(16b^2c^2 - 56abcd - 5a^2d^2)x}{20a^2c(bc - ad)^3 \sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4}} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} + \frac{b(16b^2c^2 - 56abcd - 5a^2d^2)x}{20a^2c(bc - ad)^3 \sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4}} \\
 &= \frac{b(4bc + 5ad)x}{20ac(bc - ad)^2 (a + bx^4)^{5/4}} + \frac{b(16b^2c^2 - 56abcd - 5a^2d^2)x}{20a^2c(bc - ad)^3 \sqrt[4]{a + bx^4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{5/4}}
 \end{aligned}$$

**Mathematica** [C] time = 5.01, size = 1216, normalized size = 4.57

$$69120d^3(bc - ad)^4 {}_3F_2\left(2, 2, \frac{13}{4}; 1, \frac{21}{4}; \frac{(bc-ad)x^4}{c(bx^4+a)}\right)x^{28} + 11520d^3(bc - ad)^4 {}_4F_3\left(2, 2, 2, \frac{13}{4}; 1, 1, \frac{21}{4}; \frac{(bc-ad)x^4}{c(bx^4+a)}\right)x^{28} + 21$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2), x]

[Out] 
$$\begin{aligned} & -1/198900*(285532*c^5*(b*c - a*d)^2*x^8*(a + b*x^4)^2 + 933504*c^4*d*(b*c - a*d)^2*x^{12}*(a + b*x^4)^2 + 891072*c^3*d^2*(b*c - a*d)^2*x^{16}*(a + b*x^4)^2 + 282880*c^2*d^3*(b*c - a*d)^2*x^{20}*(a + b*x^4)^2 + 9793836*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3 + 27973296*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3 + 25968384*c^4*d^2*(b*c - a*d)*x^{12}*(a + b*x^4)^3 + 8146944*c^3*d^3*(b*c - a*d)*x^{16}*(a + b*x^4)^3 - 23529870*c^7*(a + b*x^4)^4 - 65547495*c^6*d*x^4*(a + b*x^4)^4 - 60505380*c^5*d^2*x^8*(a + b*x^4)^4 - 18935280*c^4*d^3*x^{12}*(a + b*x^4)^4 - 14499810*c^6*(b*c - a*d)*x^4*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 41082795*c^5*d*(b*c - a*d)*x^8*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 38069460*c^4*d^2*(b*c - a*d)*x^{12}*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] - 11934000*c^3*d^3*(b*c - a*d)*x^{16}*(a + b*x^4)^3*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 23529870*c^7*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 65547495*c^6*d*x^4*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 60505380*c^5*d^2*x^8*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 18935280*c^4*d^3*x^{12}*(a + b*x^4)^4*Hypergeometric2F1[1/4, 1, 5/4, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 77760*c^3*(b*c - a*d)^4*x^{16}*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 224640*c^2*d*(b*c - a*d)^4*x^{20}*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 216000*c*d^2*(b*c - a*d)^4*x^{24}*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 69120*d^3*(b*c - a*d)^4*x^{28}*HypergeometricPFQ[{2, 2, 13/4}, {1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 11520*c^3*(b*c - a*d)^4*x^{16}*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c^2*d*(b*c - a*d)^4*x^{20}*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 34560*c*d^2*(b*c - a*d)^4*x^{24}*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))] + 11520*d^3*(b*c - a*d)^4*x^{28}*HypergeometricPFQ[{2, 2, 2, 13/4}, {1, 1, 21/4}, ((b*c - a*d)*x^4)/(c*(a + b*x^4))]/(c^8*(-b + (a*d)/c)^3*x^{11}*(a + b*x^4)^(13/4)*(c + d*x^4)) \end{aligned}$$

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)^2), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x)

[Out] int(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(9/4)\*(d\*x^4 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{9}{4}} (dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2),x)

[Out] int(1/((a + b\*x^4)^(9/4)\*(c + d\*x^4)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{9}{4}} (c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(9/4)\*(c + d\*x\*\*4)\*\*2), x)

$$3.211 \quad \int \frac{(a+bx^4)^{9/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=353

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4cd^2 (a + bx^4)^{3/4}} - \frac{3\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (bc - ad)(ad + 2bc) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\right)}{8\sqrt[4]{b} c^2 d^2}$$

[Out]  $1/4*b*(-a*d+3*b*c)*x*(b*x^4+a)^{(1/4)}/c/d^2-1/4*(-a*d+b*c)*x*(b*x^4+a)^{(5/4)}/c/d/(d*x^4+c)-1/4*b^{(3/2)}*(-a*d+3*b*c)*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)}))*\text{EllipticF}(\sin(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})*a^{(1/2)}/c/d^2/(b*x^4+a)^{(3/4)}-3/8*(-a*d+b*c)*(a*d+2*b*c)*\text{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/d^2-3/8*(-a*d+b*c)*(a*d+2*b*c)*\text{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/d^2$

**Rubi [A]** time = 0.35, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {413, 528, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4cd^2 (a + bx^4)^{3/4}} - \frac{3\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (bc - ad)(ad + 2bc) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\right)}{8\sqrt[4]{b} c^2 d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(9/4)/(c + d\*x^4)^2, x]

[Out]  $(b*(3*b*c - a*d)*x*(a + b*x^4)^{(1/4)})/(4*c*d^2) - ((b*c - a*d)*x*(a + b*x^4)^{(5/4)})/(4*c*d*(c + d*x^4)) - (\text{Sqrt}[a]*b^{(3/2)}*(3*b*c - a*d)*(1 + a/(b*x^4)))^{(3/4)}*x^3*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]]/2, 2]/(4*c*d^2*(a + b*x^4)^{(3/4)}) - (3*(b*c - a*d)*(2*b*c + a*d)*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[-(\text{Sqrt}[b*c - a*d])/(\text{Sqrt}[b]*\text{Sqrt}[c])], \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(8*b^{(1/4)}*c^2*d^2) - (3*(b*c - a*d)*(2*b*c + a*d)*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1)]/(8*b^{(1/4)}*c^2*d^2)$

**Rule 231**

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

**Rule 237**

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

**Rule 275**

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_.))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 335

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 407

$\text{Int}(((a_) + (b_)*(x_)^4)^{(1/4)} / ((c_) + (d_)*(x_)^4), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4] * (c - (b*c - a*d)*x^4)), x], x, x/(a + b*x^4)^{(1/4)}], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_)*(x_)^4] * ((c_) + (d_)*(x_)^4)), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 413

$\text{Int}(((a_) + (b_)*(x_)^{(n_)})^{(p_)} * ((c_) + (d_)*(x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow \text{Simp}(((a*d - c*b)*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q-1)}) / (a*b*n*(p+1)), x] - \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q-2)} * \text{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 528

$\text{Int}(((a_) + (b_)*(x_)^{(n_)})^{(p_)} * ((c_) + (d_)*(x_)^{(n_)})^{(q_)} * ((e_) + (f_)*(x_)^{(n_)}), x\_Symbol] \rightarrow \text{Simp}[(f*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^q] / (b*(n*(p+q+1) + 1)), x] + \text{Dist}[1/(b*(n*(p+q+1) + 1)), \text{Int}[(a + b*x^n)^p * (c + d*x^n)^{(q-1)} * \text{Simp}[c*(b*e - a*f + b*e*n*(p+q+1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p+q+1))*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x \ \&\& \ \text{GtQ}[q, 0] \ \&\& \ \text{NeQ}[n*(p+q+1) + 1, 0]$

Rule 529

$\text{Int}(((e_) + (f_)*(x_)^4) / (((a_) + (b_)*(x_)^4)^{(3/4)} * ((c_) + (d_)*(x_)^4)), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x]$

Rule 1218

$\text{Int}[1/(((d_) + (e_)*(x_)^2) * \text{Sqrt}[(a_) + (c_)*(x_)^4]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a*q]), x] /; \text{FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\int \frac{(a + bx^4)^{9/4}}{(c + dx^4)^2} dx = -\frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} + \frac{\int \frac{\sqrt[4]{a+bx^4}(a(bc+3ad)+2b(3bc-ad)x^4)}{c+dx^4} dx}{4cd}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} + \frac{\int \frac{-2a(3b^2c^2-2abcd-3a^2d^2)-4b(3b^2c^2-3abcd-a^2d^2)}{(a+bx^4)^{3/4}(c+dx^4)} dx}{8cd^2}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} + \frac{(ab(3bc - ad)) \int \frac{1}{(a+bx^4)^{3/4}} dx}{4cd^2} - \frac{(3(bc - a}}{4cd^2}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} + \frac{(ab(3bc - ad)(1 + \frac{a}{bx^4})^{3/4} x^3) \int \frac{1}{(1 + \frac{a}{bx^4})^{3/4}} dx}{4cd^2(a + bx^4)^{3/4}}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} - \frac{(ab(3bc - ad)(1 + \frac{a}{bx^4})^{3/4} x^3) \text{Subst} \left( \int \frac{1}{(1 + \frac{a}{bx^4})^{3/4}} dx \right)}{4cd^2(a + bx^4)^{3/4}}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} - \frac{3(bc - ad)(2bc + ad)\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} \Pi}{8\sqrt[4]{b} c^2 d^2}$$

$$= \frac{b(3bc - ad)x\sqrt[4]{a + bx^4}}{4cd^2} - \frac{(bc - ad)x(a + bx^4)^{5/4}}{4cd(c + dx^4)} - \frac{\sqrt{a} b^{3/2} (3bc - ad) (1 + \frac{a}{bx^4})^{3/4} x^3 F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{a+bx^4}}{\sqrt{a}}\right)\right)}{4cd^2(a + bx^4)^{3/4}}$$

**Mathematica [C]** time = 0.59, size = 392, normalized size = 1.11

$$\frac{2bx^5 \left(\frac{bx^4}{a} + 1\right)^{3/4} (a^2d^2 + 3abcd - 3b^2c^2) F_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + \frac{5c \left(x^5(a+bx^4)(a^2d^2-2abcd+b^2c(3c+2dx^4))\right) \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + (c+dx^4) \left(x^4 \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)\right)}{20c^2d^2(a + bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(9/4)/(c + d\*x^4)^2,x]

[Out] (2\*b\*(-3\*b^2\*c^2 + 3\*a\*b\*c\*d + a^2\*d^2)\*x^5\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + (5\*c\*(-5\*a\*c\*x\*(4\*a^3\*d^2 + a^2\*b\*d^2\*x^4 + b^3\*c\*x^4\*(3\*c + 2\*d\*x^4))\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^5\*(a + b\*x^4)\*(-2\*a\*b\*c\*d + a^2\*d^2 + b^2\*c\*(3\*c + 2\*d\*x^4))\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(20\*c^2\*d^2\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c)^2, x)

**maple** [F] time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x)

[Out] int((b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(9/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(9/4)/(d\*x^4 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{\frac{9}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(9/4)/(c + d\*x^4)^2,x)

[Out] int((a + b\*x^4)^(9/4)/(c + d\*x^4)^2, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(9/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

$$3.212 \quad \int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=298

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4cd (a+bx^4)^{3/4}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 d} + \dots$$

[Out]  $-1/4*(-a*d+b*c)*x*(b*x^4+a)^{(1/4)}/c/d/(d*x^4+c)+1/4*b^{(3/2)}*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)}))*\text{EllipticF}(\sin(1/2*\arccot(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})*a^{(1/2)}/c/d/(b*x^4+a)^{(3/4)}+1/8*(3*a*d+2*b*c)*\text{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/d+1/8*(3*a*d+2*b*c)*\text{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/d$

**Rubi [A]** time = 0.24, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {413, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4cd (a+bx^4)^{3/4}} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (3ad+2bc) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 d} + \dots$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(5/4)/(c + d\*x^4)^2, x]

[Out]  $-((b*c - a*d)*x*(a + b*x^4)^{(1/4)})/(4*c*d*(c + d*x^4)) + (\text{Sqrt}[a]*b^{(3/2)}*(1 + a/(b*x^4))^{(3/4)}*x^3*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[b]*x^2)/\text{Sqrt}[a]]/2, 2])/(4*c*d*(a + b*x^4)^{(3/4)}) + ((2*b*c + 3*a*d)*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[-(\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c])), \text{ArcSin}[b^{(1/4)}*x/(a + b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d) + ((2*b*c + 3*a*d)*\text{Sqrt}[a/(a + b*x^4)]*\text{Sqrt}[a + b*x^4]*\text{EllipticPi}[\text{Sqrt}[b*c - a*d]/(\text{Sqrt}[b]*\text{Sqrt}[c]), \text{ArcSin}[b^{(1/4)}*x/(a + b*x^4)^{(1/4)}], -1])/(8*b^{(1/4)}*c^2*d)$

### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 335

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{m+2}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_) + (b_.) * (x_)^4)^{(1/4)} / ((c_) + (d_.) * (x_)^4), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4]) * (c - (b*c - a*d)*x^4)], x], x, x/(a + b*x^4)^{(1/4)}], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_) + (b_.) * (x_)^4] * ((c_) + (d_.) * (x_)^4)), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2] * x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2] * x^2)), x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 413

$\text{Int}[(a_) + (b_.) * (x_)^{(n_)})^{(p_)} * ((c_) + (d_.) * (x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow \text{Simp}[(a*d - c*b) * x * (a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q-1)} / (a*b*n*(p+1)), x] - \text{Dist}[1/(a*b*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q-2)} * \text{Simp}[c*(a*d - c*b*(n*(p+1) + 1)) + d*(a*d*(n*(q-1) + 1) - b*c*(n*(p+q) + 1)) * x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{GtQ}[q, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 529

$\text{Int}[(e_) + (f_.) * (x_)^4) / (((a_) + (b_.) * (x_)^4)^{(3/4)} * ((c_) + (d_.) * (x_)^4)), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f) / (b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f) / (b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x]$

#### Rule 1218

$\text{Int}[1/(((d_) + (e_.) * (x_)^2) * \text{Sqrt}[(a_) + (c_.) * (x_)^4]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a]*q), x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{(a+bx^4)^{5/4}}{(c+dx^4)^2} dx &= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\int \frac{a(bc+3ad)+2b(bc+ad)x^4}{(a+bx^4)^{3/4}(c+dx^4)} dx}{4cd} \\
&= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} - \frac{(ab) \int \frac{1}{(a+bx^4)^{3/4}} dx}{4cd} - \frac{(-2bc-3ad) \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{4cd} \\
&= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3} dx}{4cd(a+bx^4)^{3/4}} - \frac{(-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}}{4cd} \\
&= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}} dx, x, \frac{1}{x}\right)}{4cd(a+bx^4)^{3/4}} - \frac{(-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}}{4cd} \\
&= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{bx^4}}{\sqrt[4]{a+bx^4}}\right)\right) - 1}{8\sqrt[4]{b}c^2d} + \frac{(-2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}}{4cd} \\
&= -\frac{(bc-ad)x^4\sqrt[4]{a+bx^4}}{4cd(c+dx^4)} + \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{bx^2}}{\sqrt{a}}\right)\right) - 2}{4cd(a+bx^4)^{3/4}} + \frac{(2bc+3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}}{4cd}
\end{aligned}$$

**Mathematica [C]** time = 0.49, size = 341, normalized size = 1.14

$$x \frac{\left(5c\left(x^4(a+bx^4)(ad-bc)\left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5ac(4a^2d+abdx^4-b^2cx^4)F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)}{(c+dx^4)\left(x^4\left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5acF_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)} + 2bx^4 \frac{\left(\frac{bx^4}{a}\right)}{20c^2d(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(5/4)/(c + d\*x^4)^2,x]

[Out] (x\*(2\*b\*(b\*c + a\*d)\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)] + (5\*c\*(-5\*a\*c\*(4\*a^2\*d - b^2\*c\*x^4 + a\*b\*d\*x^4)\*AppellF1[1/4, 3/4, 1, 5/4, -(b\*x^4)/a, -((d\*x^4)/c)] + (-b\*c) + a\*d)\*x^4\*(a + b\*x^4)\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)]))/((c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -(b\*x^4)/a, -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -(b\*x^4)/a, -((d\*x^4)/c)])))/((20\*c^2\*d\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2, x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x)

[Out] int((b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(5/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(5/4)/(d\*x^4 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{\frac{5}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(5/4)/(c + d\*x^4)^2,x)

[Out] int((a + b\*x^4)^(5/4)/(c + d\*x^4)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^4)^{\frac{5}{4}}}{(c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(5/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Integral((a + b\*x\*\*4)\*\*(5/4)/(c + d\*x\*\*4)\*\*2, x)

$$3.213 \quad \int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2} dx$$

**Optimal.** Leaf size=308

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4c(a+bx^4)^{3/4}(bc-ad)} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (2bc-3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 (bc-ad)}$$

[Out]  $\frac{1}{4} x (b x^4 + a)^{1/4} / (d x^4 + c) - 1/4 b^{3/2} (1 + a/b x^4)^{3/4} x^3 (\cos(1/2 \operatorname{arccot}(x^2 b^{1/2} / a^{1/2}))^2)^{1/2} / \cos(1/2 \operatorname{arccot}(x^2 b^{1/2} / a^{1/2})) * \operatorname{EllipticF}(\sin(1/2 \operatorname{arccot}(x^2 b^{1/2} / a^{1/2})), 2^{1/2}) * a^{1/2} / c / (-a d + b c) / (b x^4 + a)^{3/4} + 1/8 (-3 a d + 2 b c) * \operatorname{EllipticPi}(b^{1/4} x / (b x^4 + a)^{1/4}, -(-a d + b c)^{1/2} / b^{1/2} / c^{1/2}, I) * (a / (b x^4 + a))^{1/2} * (b x^4 + a)^{1/2} / b^{1/4} / c^2 / (-a d + b c) + 1/8 (-3 a d + 2 b c) * \operatorname{EllipticPi}(b^{1/4} x / (b x^4 + a)^{1/4}, (-a d + b c)^{1/2} / b^{1/2} / c^{1/2}, I) * (a / (b x^4 + a))^{1/2} * (b x^4 + a)^{1/2} / b^{1/4} / c^2 / (-a d + b c)$

**Rubi [A]** time = 0.20, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {412, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{\sqrt{a} b^{3/2} x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b} x^2}{\sqrt{a}}\right) \middle| 2\right)}{4c(a+bx^4)^{3/4}(bc-ad)} + \frac{\sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (2bc-3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b} x}{\sqrt[4]{bx^4+a}}\right) \middle| -1\right)}{8\sqrt[4]{b} c^2 (bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^(1/4)/(c + d\*x^4)^2, x]

[Out]  $(x * (a + b x^4)^{1/4}) / (4 c * (c + d x^4)) - (\operatorname{Sqrt}[a] * b^{3/2} * (1 + a / (b x^4))^{3/4} * x^3 * \operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b] * x^2) / \operatorname{Sqrt}[a]] / 2, 2]) / (4 c * (b c - a d) * (a + b x^4)^{3/4}) + ((2 * b c - 3 * a d) * \operatorname{Sqrt}[a / (a + b x^4)] * \operatorname{Sqrt}[a + b x^4] * \operatorname{EllipticPi}[-(\operatorname{Sqrt}[b c - a d]) / (\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c])], \operatorname{ArcSin}[(b^{1/4} * x) / (a + b x^4)^{1/4}], -1]) / (8 * b^{1/4} * c^2 * (b c - a d)) + ((2 * b c - 3 * a d) * \operatorname{Sqrt}[a / (a + b x^4)] * \operatorname{Sqrt}[a + b x^4] * \operatorname{EllipticPi}[\operatorname{Sqrt}[b c - a d] / (\operatorname{Sqrt}[b] * \operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{1/4} * x) / (a + b x^4)^{1/4}], -1]) / (8 * b^{1/4} * c^2 * (b c - a d))$

### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

### Rule 335

$\text{Int}[(x_)^{(m_.)} * ((a_) + (b_.) * (x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p / x^{(m+2)}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_) + (b_.) * (x_)^4)^{(1/4)} / ((c_) + (d_.) * (x_)^4), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/\text{Sqrt}[1 - b*x^4] * (c - (b*c - a*d) * x^4)], x], x, x/(a + b*x^4)^{(1/4)}, x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.) * (x_)^4] * ((c_) + (d_.) * (x_)^4), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2] * x^2)], x], x] + \text{Dist}[1/(2*c), \text{Int}[1/\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2] * x^2)], x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 412

$\text{Int}[(a_) + (b_.) * (x_)^{(n_)})^{(p_)} * ((c_) + (d_.) * (x_)^{(n_)})^{(q_)}, x\_Symbol] \rightarrow -\text{Simp}[(x * (a + b*x^n)^{(p+1)} * (c + d*x^n)^q) / (a*n*(p+1)), x] + \text{Dist}[1/(a*n*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q-1)} * \text{Simp}[c*(n*(p+1) + 1) + d*(n*(p+q+1) + 1) * x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ \text{LtQ}[0, q, 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 529

$\text{Int}[(e_) + (f_.) * (x_)^4) / (((a_) + (b_.) * (x_)^4)^{(3/4)} * ((c_) + (d_.) * (x_)^4)), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f) / (b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f) / (b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x]$

#### Rule 1218

$\text{Int}[1/(((d_) + (e_.) * (x_)^2) * \text{Sqrt}[(a_) + (c_.) * (x_)^4]), x\_Symbol] \rightarrow \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a]*q), x] /; \text{FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt[4]{a+bx^4}}{(c+dx^4)^2} dx &= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\int \frac{-3a-2bx^4}{(a+bx^4)^{3/4}(c+dx^4)} dx}{4c} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} + \frac{(ab) \int \frac{1}{(a+bx^4)^{3/4}} dx}{4c(bc-ad)} + \frac{(2bc-3ad) \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{4c(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} + \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3} dx}{4c(bc-ad)(a+bx^4)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right) \text{Subst}\left(\int \frac{1}{c+dx^4} dx, x, \sqrt[4]{a+bx^4}\right)}{4c(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\left(ab\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}} dx, x, \frac{1}{x}\right)}{4c(bc-ad)(a+bx^4)^{3/4}} + \frac{\left((2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\right) \text{Subst}\left(\int \frac{1}{c+dx^4} dx, x, \sqrt[4]{a+bx^4}\right)}{4c(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right) - 1}{8\sqrt[4]{b}c^2(bc-ad)} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right) + 1}{8\sqrt[4]{b}c^2(bc-ad)} \\
&= \frac{x\sqrt[4]{a+bx^4}}{4c(c+dx^4)} - \frac{\sqrt{a}b^{3/2}\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right) - 2}{4c(bc-ad)(a+bx^4)^{3/4}} + \frac{(2bc-3ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4} \Pi\left(\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right) + 1}{8\sqrt[4]{b}c^2(bc-ad)}
\end{aligned}$$

**Mathematica [C]** time = 0.31, size = 233, normalized size = 0.76

$$x \left( \frac{5 \left( \frac{a+bx^4}{c} \frac{15a^2 F_1\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{x^4 \left( 4ad F_1\left(\frac{5}{4}, \frac{3}{4}, 2; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bc F_1\left(\frac{5}{4}, \frac{7}{4}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 5ac F_1\left(\frac{1}{4}, \frac{3}{4}, 1; \frac{5}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right)}{c+dx^4} + \frac{2bx^4 \left(\frac{bx^4}{a} + 1\right)^{3/4} F_1\left(\frac{5}{4}, \frac{3}{4}, 1; \frac{9}{4}, -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{c^2} \right)}{20(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^4)^(1/4)/(c + d\*x^4)^2,x]

[Out] (x\*((2\*b\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/c^2 + (5\*((a + b\*x^4)/c - (15\*a^2\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)])/(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/(20\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2, x)

**maple** [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x)

[Out] int((b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^4 + a)^{\frac{1}{4}}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^(1/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^(1/4)/(d\*x^4 + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(bx^4 + a)^{1/4}}{(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^(1/4)/(c + d\*x^4)^2,x)

[Out] int((a + b\*x^4)^(1/4)/(c + d\*x^4)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[4]{a + bx^4}}{(c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*(1/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Integral((a + b\*x\*\*4)\*\*(1/4)/(c + d\*x\*\*4)\*\*2, x)

$$3.214 \quad \int \frac{1}{(a+bx^4)^{3/4} (c+dx^4)^2} dx$$

**Optimal.** Leaf size=330

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (4bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right)}{4\sqrt{a}c (a + bx^4)^{3/4} (bc - ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (2bc - ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\right)}{8\sqrt[4]{b}c^2(bc - ad)^2}$$

[Out]  $-1/4*d*x*(b*x^4+a)^{(1/4)}/c/(-a*d+b*c)/(d*x^4+c)-1/4*b^{(3/2)}*(-a*d+4*b*c)*(1+a/b/x^4)^{(3/4)}*x^3*(\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))^2)^{(1/2)}/\cos(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)}))*\operatorname{EllipticF}(\sin(1/2*\operatorname{arccot}(x^2*b^{(1/2)}/a^{(1/2)})), 2^{(1/2)})/c/(-a*d+b*c)^2/(b*x^4+a)^{(3/4)}/a^{(1/2)}-3/8*d*(-a*d+2*b*c)*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, -(-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)^2-3/8*d*(-a*d+2*b*c)*\operatorname{EllipticPi}(b^{(1/4)}*x/(b*x^4+a)^{(1/4)}, (-a*d+b*c)^{(1/2)}/b^{(1/2)}/c^{(1/2)}, I)*(a/(b*x^4+a))^{(1/2)}*(b*x^4+a)^{(1/2)}/b^{(1/4)}/c^2/(-a*d+b*c)^2$

**Rubi [A]** time = 0.25, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$ , Rules used = {414, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (4bc - ad) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right)}{4\sqrt{a}c (a + bx^4)^{3/4} (bc - ad)^2} - \frac{3d\sqrt{\frac{a}{a+bx^4}} \sqrt{a + bx^4} (2bc - ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{bx^4+a}}\right)\right)}{8\sqrt[4]{b}c^2(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)^2), x]

[Out]  $-(d*x*(a + b*x^4)^{(1/4)})/(4*c*(b*c - a*d)*(c + d*x^4)) - (b^{(3/2)}*(4*b*c - a*d)*(1 + a/(b*x^4))^{(3/4)}*x^3*\operatorname{EllipticF}[\operatorname{ArcCot}[(\operatorname{Sqrt}[b]*x^2)/\operatorname{Sqrt}[a]]/2, 2])/ (4*\operatorname{Sqrt}[a]*c*(b*c - a*d)^2*(a + b*x^4)^{(3/4)}) - (3*d*(2*b*c - a*d)*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[-(\operatorname{Sqrt}[b*c - a*d])/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c])], \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1))/(8*b^{(1/4)}*c^2*(b*c - a*d)^2) - (3*d*(2*b*c - a*d)*\operatorname{Sqrt}[a/(a + b*x^4)]*\operatorname{Sqrt}[a + b*x^4]*\operatorname{EllipticPi}[\operatorname{Sqrt}[b*c - a*d]/(\operatorname{Sqrt}[b]*\operatorname{Sqrt}[c]), \operatorname{ArcSin}[(b^{(1/4)}*x)/(a + b*x^4)^{(1/4)}], -1))/(8*b^{(1/4)}*c^2*(b*c - a*d)^2)$

#### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

#### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

#### Rule 335

$\text{Int}[(x_)^{(m_)} * ((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \text{ :> } -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] \text{ /; FreeQ}\{a, b, p\}, x] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

#### Rule 407

$\text{Int}[(a_ + (b_)*(x_)^4)^{(1/4)} / ((c_ + (d_)*(x_)^4)), x\_Symbol] \text{ :> } \text{Dist}[\text{Sqrt}[a + b*x^4] * \text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4]) * (c - (b*c - a*d)*x^4)], x], x, x/(a + b*x^4)^{(1/4)}, x] \text{ /; FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_ + (b_)*(x_)^4] * ((c_ + (d_)*(x_)^4))), x\_Symbol] \text{ :> } \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4] * (1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] \text{ /; FreeQ}\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

#### Rule 414

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)} * ((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \text{ :> } -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)} * (c + d*x^n)^{(q+1)}) / (a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)} * (c + d*x^n)^q * \text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] \text{ /; FreeQ}\{a, b, c, d, n, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !( \ \&\& \ \text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1]) \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

#### Rule 529

$\text{Int}[(e_ + (f_)*(x_)^4) / (((a_ + (b_)*(x_)^4)^{(3/4)} * ((c_ + (d_)*(x_)^4))), x\_Symbol] \text{ :> } \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)} / (c + d*x^4), x], x] \text{ /; FreeQ}\{a, b, c, d, e, f\}, x]$

#### Rule 1218

$\text{Int}[1/(((d_ + (e_)*(x_)^2) * \text{Sqrt}[(a_ + (c_)*(x_)^4])), x\_Symbol] \text{ :> } \text{With}\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1 * \text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1]) / (d * \text{Sqrt}[a]*q), x] \text{ /; FreeQ}\{a, c, d, e\}, x] \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+bx^4)^{3/4}(c+dx^4)^2} dx &= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\int \frac{4bc-3ad-2bdx^4}{(a+bx^4)^{3/4}(c+dx^4)} dx}{4c(bc-ad)} \\
&= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{(3d(2bc-ad)) \int \frac{\sqrt[4]{a+bx^4}}{c+dx^4} dx}{4c(bc-ad)^2} + \frac{(b(4bc-ad)) \int \frac{1}{(a+bx^4)^{3/4}} dx}{4c(bc-ad)^2} \\
&= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} + \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \int \frac{1}{\left(1+\frac{a}{bx^4}\right)^{3/4}x^3} dx}{4c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{(3d(2bc-ad)) \int \frac{1}{(a+bx^4)^{3/4}} dx}{4c(bc-ad)^2} \\
&= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{\left(b(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3\right) \text{Subst}\left(\int \frac{x}{\left(1+\frac{ax^4}{b}\right)^{3/4}} dx, x, \frac{1}{x}\right)}{4c(bc-ad)^2(a+bx^4)^{3/4}} \\
&= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{3d(2bc-ad)\sqrt{\frac{a}{a+bx^4}}\sqrt{a+bx^4}\Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}; \sin^{-1}\left(\frac{\sqrt[4]{b}x}{\sqrt[4]{a+bx^4}}\right)\right)}{8\sqrt[4]{b}c^2(bc-ad)^2} \\
&= -\frac{dx\sqrt[4]{a+bx^4}}{4c(bc-ad)(c+dx^4)} - \frac{b^{3/2}(4bc-ad)\left(1+\frac{a}{bx^4}\right)^{3/4}x^3F\left(\frac{1}{2}\cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right)\right)\Big|_2}{4\sqrt{a}c(bc-ad)^2(a+bx^4)^{3/4}} - \frac{3d}{4c(bc-ad)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.34, size = 337, normalized size = 1.02

$$x \left( \frac{2bdx^4 \left(\frac{bx^4}{a} + 1\right)^{3/4} F_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{ad-bc} + \frac{c \left(25ac(4ad-4bc+bdx^4) F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 5dx^4(a+bx^4) \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right)}{(c+dx^4)(bc-ad) \left(x^4 \left(4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{3}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) - 5acF_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)} \right)}{20c^2(a+bx^4)^{3/4}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)^2), x]

[Out] (x\*((2\*b\*d\*x^4\*(1 + (b\*x^4)/a)^(3/4)\*AppellF1[5/4, 3/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])/(-b\*c) + a\*d) + (c\*(25\*a\*c\*(-4\*b\*c + 4\*a\*d + b\*d\*x^4)\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] - 5\*d\*x^4\*(a + b\*x^4)\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((b\*c - a\*d)\*(c + d\*x^4)\*(-5\*a\*c\*AppellF1[1/4, 3/4, 1, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + x^4\*(4\*a\*d\*AppellF1[5/4, 3/4, 2, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 3\*b\*c\*AppellF1[5/4, 7/4, 1, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)])))/((20\*c^2\*(a + b\*x^4)^(3/4))

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)^2), x)

**maple** [F] time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x)

[Out] int(1/(b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(3/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(3/4)\*(d\*x^4 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{3}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)^2),x)

[Out] int(1/((a + b\*x^4)^(3/4)\*(c + d\*x^4)^2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4)^{\frac{3}{4}}(c + dx^4)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(3/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Integral(1/((a + b\*x\*\*4)\*\*(3/4)\*(c + d\*x\*\*4)\*\*2), x)

$$3.215 \quad \int \frac{1}{(a+bx^4)^{7/4} (c+dx^4)^2} dx$$

**Optimal.** Leaf size=390

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3a^2d^2 - 32abcd + 8b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right) d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc - 3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right)}{12a^{3/2}c (a+bx^4)^{3/4} (bc-ad)^3} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc - 3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

[Out] 1/12\*b\*(3\*a\*d+4\*b\*c)\*x/a/c/(-a\*d+b\*c)^(2/(b\*x^4+a)^(3/4)-1/4\*d\*x/c/(-a\*d+b\*c)/(b\*x^4+a)^(3/4)/(d\*x^4+c)-1/12\*b^(3/2)\*(3\*a^2\*d^2-32\*a\*b\*c\*d+8\*b^2\*c^2)\*(1+a/b/x^4)^(3/4)\*x^3\*(cos(1/2\*arccot(x^2\*b^(1/2)/a^(1/2)))^2)^(1/2)/cos(1/2\*arccot(x^2\*b^(1/2)/a^(1/2)))\*EllipticF(sin(1/2\*arccot(x^2\*b^(1/2)/a^(1/2))),2^(1/2))/a^(3/2)/c/(-a\*d+b\*c)^3/(b\*x^4+a)^(3/4)+1/8\*d^2\*(-3\*a\*d+10\*b\*c)\*EllipticPi(b^(1/4)\*x/(b\*x^4+a)^(1/4),-(-a\*d+b\*c)^(1/2)/b^(1/2)/c^(1/2),I)\*(a/(b\*x^4+a))^(1/2)\*(b\*x^4+a)^(1/2)/b^(1/4)/c^2/(-a\*d+b\*c)^3+1/8\*d^2\*(-3\*a\*d+10\*b\*c)\*EllipticPi(b^(1/4)\*x/(b\*x^4+a)^(1/4),(-a\*d+b\*c)^(1/2)/b^(1/2)/c^(1/2),I)\*(a/(b\*x^4+a))^(1/2)\*(b\*x^4+a)^(1/2)/b^(1/4)/c^2/(-a\*d+b\*c)^3

**Rubi [A]** time = 0.41, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$ , Rules used = {414, 527, 529, 237, 335, 275, 231, 407, 409, 1218}

$$\frac{b^{3/2}x^3 \left(\frac{a}{bx^4} + 1\right)^{3/4} (3a^2d^2 - 32abcd + 8b^2c^2) F\left(\frac{1}{2} \cot^{-1}\left(\frac{\sqrt{b}x^2}{\sqrt{a}}\right) \middle| 2\right) d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc - 3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right)}{12a^{3/2}c (a+bx^4)^{3/4} (bc-ad)^3} + \frac{d^2 \sqrt{\frac{a}{a+bx^4}} \sqrt{a+bx^4} (10bc - 3ad) \Pi\left(-\frac{\sqrt{bc-ad}}{\sqrt{b}\sqrt{c}}\right)}{8\sqrt[4]{b}c^2(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)^2), x]

[Out] (b\*(4\*b\*c + 3\*a\*d)\*x)/(12\*a\*c\*(b\*c - a\*d)^2\*(a + b\*x^4)^(3/4)) - (d\*x)/(4\*c\*(b\*c - a\*d)\*(a + b\*x^4)^(3/4)\*(c + d\*x^4)) - (b^(3/2)\*(8\*b^2\*c^2 - 32\*a\*b\*c\*d + 3\*a^2\*d^2)\*(1 + a/(b\*x^4))^(3/4)\*x^3\*EllipticF[ArcCot[(Sqrt[b]\*x^2)/Sqrt[a]]/2, 2])/(12\*a^(3/2)\*c\*(b\*c - a\*d)^3\*(a + b\*x^4)^(3/4)) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[-(Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c])), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3) + (d^2\*(10\*b\*c - 3\*a\*d)\*Sqrt[a/(a + b\*x^4)]\*Sqrt[a + b\*x^4]\*EllipticPi[Sqrt[b\*c - a\*d]/(Sqrt[b]\*Sqrt[c]), ArcSin[(b^(1/4)\*x)/(a + b\*x^4)^(1/4)], -1])/(8\*b^(1/4)\*c^2\*(b\*c - a\*d)^3)

#### Rule 231

Int[((a\_) + (b\_.)\*(x\_)^2)^(-3/4), x\_Symbol] := Simp[(2\*EllipticF[(1\*ArcTan[Rt[b/a, 2]\*x])/2, 2])/(a^(3/4)\*Rt[b/a, 2]), x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b/a]

#### Rule 237

Int[((a\_) + (b\_.)\*(x\_)^4)^(-3/4), x\_Symbol] := Dist[(x^3\*(1 + a/(b\*x^4))^(3/4))/(a + b\*x^4)^(3/4), Int[1/(x^3\*(1 + a/(b\*x^4))^(3/4)), x], x] /; FreeQ[{a, b}, x]

#### Rule 275

Int[(x\_)^(m\_.)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)\*(a + b\*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 335

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p/x^{(m+2)}, x], x, 1/x] /; \text{FreeQ}\{a, b, p\}, x \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

Rule 407

$\text{Int}[(a_ + (b_)*(x_)^4)^{(1/4)} / ((c_ + (d_)*(x_)^4)), x\_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[a + b*x^4]*\text{Sqrt}[a/(a + b*x^4)], \text{Subst}[\text{Int}[1/(\text{Sqrt}[1 - b*x^4]*(c - (b*c - a*d)*x^4)), x], x, x/(a + b*x^4)^{(1/4)}], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 409

$\text{Int}[1/(\text{Sqrt}[(a_ + (b_)*(x_)^4]*((c_ + (d_)*(x_)^4))), x\_Symbol] \rightarrow \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 - \text{Rt}[-(d/c), 2]*x^2)), x], x] + \text{Dist}[1/(2*c), \text{Int}[1/(\text{Sqrt}[a + b*x^4]*(1 + \text{Rt}[-(d/c), 2]*x^2)), x], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0]$

Rule 414

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] \rightarrow -\text{Simp}[(b*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(p+1)*(b*c - a*d)), x] + \text{Dist}[1/(a*n*(p+1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[b*c + n*(p+1)*(b*c - a*d) + d*b*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{LtQ}[p, -1] \ \&\& \ !( \ !\text{IntegerQ}[p] \ \&\& \ \text{IntegerQ}[q] \ \&\& \ \text{LtQ}[q, -1]) \ \&\& \ \text{IntBinomialQ}[a, b, c, d, n, p, q, x]$

Rule 527

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}*((e_ + (f_)*(x_)^{(n_)})), x\_Symbol] \rightarrow -\text{Simp}[(b*e - a*f)*x*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)})/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, q\}, x \ \&\& \ \text{LtQ}[p, -1]$

Rule 529

$\text{Int}[(e_ + (f_)*(x_)^4) / (((a_ + (b_)*(x_)^4)^{(3/4)}*((c_ + (d_)*(x_)^4))), x\_Symbol] \rightarrow \text{Dist}[(b*e - a*f)/(b*c - a*d), \text{Int}[1/(a + b*x^4)^{(3/4)}, x], x] - \text{Dist}[(d*e - c*f)/(b*c - a*d), \text{Int}[(a + b*x^4)^{(1/4)}/(c + d*x^4), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x]$

Rule 1218

$\text{Int}[1/(((d_ + (e_)*(x_)^2)*\text{Sqrt}[(a_ + (c_)*(x_)^4])), x\_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[-(c/a), 4]\}, \text{Simp}[(1*\text{EllipticPi}[-(e/(d*q^2)), \text{ArcSin}[q*x], -1)]/(d*\text{Sqrt}[a*q]), x]] /; \text{FreeQ}\{a, c, d, e\}, x \ \&\& \ \text{NegQ}[c/a] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + bx^4)^{7/4} (c + dx^4)^2} dx &= -\frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} + \frac{\int \frac{4bc - 3ad - 6bdx^4}{(a + bx^4)^{7/4} (c + dx^4)} dx}{4c(bc - ad)} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} - \frac{\int \frac{-8b^2c^2 + 24abcd}{(a + bx^4)^{7/4} (c + dx^4)} dx}{12ac} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} + \frac{(d^2(10bc - 3ad))}{4c(bc - ad)} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} + \frac{(b(8b^2c^2 - 32abcd))}{4c(bc - ad)} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} - \frac{(b(8b^2c^2 - 32abcd))}{4c(bc - ad)} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} - \frac{(d^2(10bc - 3ad))}{4c(bc - ad)} \\
 &= \frac{b(4bc + 3ad)x}{12ac(bc - ad)^2 (a + bx^4)^{3/4}} - \frac{dx}{4c(bc - ad)(a + bx^4)^{3/4} (c + dx^4)} + \frac{b^{3/2}(8b^2c^2 - 32abcd)}{4c(bc - ad)}
 \end{aligned}$$

**Mathematica [C]** time = 0.58, size = 387, normalized size = 0.99

$$x \frac{c \left( 25ac(12a^2d^2 + 3abd(dx^4 - 8c) + 4b^2c(3c + dx^4)) F_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 5x^4(3a^2d^2 + 3abd^2x^4 + 4b^2c(c + dx^4)) \left( 4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) \right)}{(c + dx^4) \left( 5acF_1\left(\frac{1}{4}; \frac{3}{4}, 1; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - x^4 \left( 4adF_1\left(\frac{5}{4}; \frac{3}{4}, 2; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + 3bcF_1\left(\frac{5}{4}; \frac{7}{4}, 1; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) \right)}{60ac^2 (a + bx^4)^{3/4} (bc - ad)^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*x^4)^(7/4)*(c + d*x^4)^2), x]
```

```
[Out] (x*(2*b*d*(4*b*c + 3*a*d)*x^4*(1 + (b*x^4)/a)^(3/4)*AppellF1[5/4, 3/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + (c*(25*a*c*(12*a^2*d^2 + 3*a*b*d*(-8*c + d*x^4) + 4*b^2*c*(3*c + d*x^4))*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - 5*x^4*(3*a^2*d^2 + 3*a*b*d^2*x^4 + 4*b^2*c*(c + d*x^4))*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/(c + d*x^4)*(5*a*c*AppellF1[1/4, 3/4, 1, 5/4, -((b*x^4)/a), -((d*x^4)/c)] - x^4*(4*a*d*AppellF1[5/4, 3/4, 2, 9/4, -((b*x^4)/a), -((d*x^4)/c)] + 3*b*c*AppellF1[5/4, 7/4, 1, 9/4, -((b*x^4)/a), -((d*x^4)/c)])))/(60*a*c^2*(b*c - a*d)^2*(a + b*x^4)^(3/4))
```

**fricas [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^4 + a)^(7/4)\*(d\*x^4 + c)^2), x)

**maple** [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x)

[Out] int(1/(b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^4+a)^(7/4)/(d\*x^4+c)^2,x, algorithm="maxima")

[Out] integrate(1/((b\*x^4 + a)^(7/4)\*(d\*x^4 + c)^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^4 + a)^{\frac{7}{4}}(dx^4 + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)^2),x)

[Out] int(1/((a + b\*x^4)^(7/4)\*(c + d\*x^4)^2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x\*\*4+a)\*\*(7/4)/(d\*x\*\*4+c)\*\*2,x)

[Out] Timed out

$$3.216 \quad \int \frac{1}{\sqrt[4]{1+x^4}(2+x^4)} dx$$

**Optimal.** Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{3/4}}$$

[Out] 1/4\*arctan(1/2\*x\*2^(3/4)/(x^4+1)^(1/4))\*2^(1/4)+1/4\*arctanh(1/2\*x\*2^(3/4)/(x^4+1)^(1/4))\*2^(1/4)

**Rubi [A]** time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{3/4}} + \frac{\tanh^{-1}\left(\frac{x}{\sqrt[4]{2}\sqrt[4]{x^4+1}}\right)}{2^{3/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x^4)^(1/4)\*(2 + x^4)),x]

[Out] ArcTan[x/(2^(1/4)\*(1 + x^4)^(1/4))]/(2\*2^(3/4)) + ArcTanh[x/(2^(1/4)\*(1 + x^4)^(1/4))]/(2\*2^(3/4))

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTan[Rt[b, 2]\*x]/Rt[a, 2])/Rt[a, 2]\*Rt[b, 2], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/Rt[a, 2]\*Rt[-b, 2], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{1+x^4} (2+x^4)} dx &= \text{Subst} \left( \int \frac{1}{2-x^4} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right) \\ &= \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2-x^2}} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt{2}} + \frac{\text{Subst} \left( \int \frac{1}{\sqrt{2+x^2}} dx, x, \frac{x}{\sqrt[4]{1+x^4}} \right)}{2\sqrt{2}} \\ &= \frac{\tan^{-1} \left( \frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{2 \cdot 2^{3/4}} + \frac{\tanh^{-1} \left( \frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{2 \cdot 2^{3/4}} \end{aligned}$$

**Mathematica [A]** time = 0.02, size = 44, normalized size = 0.83

$$\frac{\tan^{-1} \left( \frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right) + \tanh^{-1} \left( \frac{x}{\sqrt[4]{2} \sqrt[4]{1+x^4}} \right)}{2 \cdot 2^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x^4)^(1/4)\*(2 + x^4)),x]

[Out] (ArcTan[x/(2^(1/4)\*(1 + x^4)^(1/4))] + ArcTanh[x/(2^(1/4)\*(1 + x^4)^(1/4))])/(2\*2^(3/4))

**fricas [B]** time = 16.99, size = 208, normalized size = 3.92

$$-\frac{1}{16} \cdot 8^{\frac{3}{4}} \arctan \left( -\frac{8^{\frac{3}{4}}(x^4+1)^{\frac{1}{4}}x^3 + 4 \cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{3}{4}}x - 2^{\frac{1}{4}}(8^{\frac{3}{4}}\sqrt{x^4+1}x^2 + 8^{\frac{1}{4}}(3x^4+2))}{2(x^4+2)} \right) + \frac{1}{64} \cdot 8^{\frac{3}{4}} \log \left( \frac{8\sqrt{2}(x^4+2)}{8^{\frac{3}{4}}(x^4+1)^{\frac{1}{4}}x^3 + 4 \cdot 8^{\frac{1}{4}}(x^4+1)^{\frac{3}{4}}x - 2^{\frac{1}{4}}(8^{\frac{3}{4}}\sqrt{x^4+1}x^2 + 8^{\frac{1}{4}}(3x^4+2))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="fricas")

[Out] -1/16\*8^(3/4)\*arctan(-1/2\*(8^(3/4)\*(x^4 + 1)^(1/4)\*x^3 + 4\*8^(1/4)\*(x^4 + 1)^(3/4)\*x - 2^(1/4)\*(8^(3/4)\*sqrt(x^4 + 1)\*x^2 + 8^(1/4)\*(3\*x^4 + 2)))/(x^4 + 2)) + 1/64\*8^(3/4)\*log((8\*sqrt(2)\*(x^4 + 1)^(1/4)\*x^3 + 8\*8^(1/4)\*sqrt(x^4 + 1)\*x^2 + 8^(3/4)\*(3\*x^4 + 2) + 16\*(x^4 + 1)^(3/4)\*x)/(x^4 + 2)) - 1/64\*8^(3/4)\*log((8\*sqrt(2)\*(x^4 + 1)^(1/4)\*x^3 - 8\*8^(1/4)\*sqrt(x^4 + 1)\*x^2 - 8^(3/4)\*(3\*x^4 + 2) + 16\*(x^4 + 1)^(3/4)\*x)/(x^4 + 2))

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4+2)(x^4+1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 2)\*(x^4 + 1)^(1/4)), x)

**maple [C]** time = 3.24, size = 211, normalized size = 3.98

$$\text{RootOf} \left( -Z^2 + \text{RootOf} \left( -Z^4 - 2 \right)^2 \right) \ln \left( -\frac{-3x^4 \text{RootOf} \left( -Z^2 + \text{RootOf} \left( -Z^4 - 2 \right)^2 \right) - 2(x^4+1)^{\frac{1}{4}}x^3 \text{RootOf} \left( -Z^4 - 2 \right)^2 + 2\sqrt{x^4+1} x^2 \text{RootOf} \left( -Z^2 + \text{RootOf} \left( -Z^4 - 2 \right)^2 \right)}{2(x^4+2)(x^4+1)^{\frac{1}{4}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^4+1)^(1/4)/(x^4+2),x)`

[Out] 
$$-1/8*\text{RootOf}(\_Z^2+\text{RootOf}(\_Z^4-2)^2)*\ln(-(2*(x^4+1)^{(1/2)}*\text{RootOf}(\_Z^4-2)^2*\text{RootOf}(\_Z^2+\text{RootOf}(\_Z^4-2)^2)*x^2-2*(x^4+1)^{(1/4)}*\text{RootOf}(\_Z^4-2)^2*x^3-3*\text{RootOf}(\_Z^2+\text{RootOf}(\_Z^4-2)^2)*x^4+4*(x^4+1)^{(3/4)}*x-2*\text{RootOf}(\_Z^2+\text{RootOf}(\_Z^4-2)^2)))/(x^4+2))+1/8*\text{RootOf}(\_Z^4-2)*\ln((2*(x^4+1)^{(1/2)}*\text{RootOf}(\_Z^4-2)^3*x^2+2*(x^4+1)^{(1/4)}*\text{RootOf}(\_Z^4-2)^2*x^3+3*\text{RootOf}(\_Z^4-2)*x^4+4*(x^4+1)^{(3/4)}*x+2*\text{RootOf}(\_Z^4-2)))/(x^4+2))$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 2)(x^4 + 1)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x^4+1)^(1/4)/(x^4+2),x, algorithm="maxima")`

[Out] `integrate(1/((x^4 + 2)*(x^4 + 1)^(1/4)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(x^4 + 1)^{\frac{1}{4}}(x^4 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^4 + 1)^(1/4)*(x^4 + 2)),x)`

[Out] `int(1/((x^4 + 1)^(1/4)*(x^4 + 2)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x^4 + 1}(x^4 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**4+1)**(1/4)/(x**4+2),x)`

[Out] `Integral(1/((x**4 + 1)**(1/4)*(x**4 + 2)), x)`

$$3.217 \quad \int \frac{1}{(a-(a-b)x^4)\sqrt[4]{a+bx^4}} dx$$

Optimal. Leaf size=57

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

[Out] 1/2\*arctan(a^(1/4)\*x/(b\*x^4+a)^(1/4))/a^(5/4)+1/2\*arctanh(a^(1/4)\*x/(b\*x^4+a)^(1/4))/a^(5/4)

**Rubi [A]** time = 0.02, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$ , Rules used = {377, 212, 206, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{a}x}{\sqrt[4]{a+bx^4}}\right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

[In] Int[1/((a - (a - b)\*x^4)\*(a + b\*x^4)^(1/4)),x]

[Out] ArcTan[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4)) + ArcTanh[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)]/(2\*a^(5/4))

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 212

Int[((a\_) + (b\_.)\*(x\_)^4)^(-1), x\_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2\*a), Int[1/(r - s\*x^2), x], x] + Dist[r/(2\*a), Int[1/(r + s\*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

#### Rule 377

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)/((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Subst[Int[1/(c - (b\*c - a\*d)\*x^n), x], x, x/(a + b\*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*p + 1, 0] && IntegerQ[n]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a - (a - b)x^4) \sqrt[4]{a + bx^4}} dx &= \text{Subst} \left( \int \frac{1}{a - (ab - a(-a + b))x^4} dx, x, \frac{x}{\sqrt[4]{a + bx^4}} \right) \\ &= \frac{\text{Subst} \left( \int \frac{1}{1 - \sqrt{a}x^2} dx, x, \frac{x}{\sqrt[4]{a + bx^4}} \right)}{2a} + \frac{\text{Subst} \left( \int \frac{1}{1 + \sqrt{a}x^2} dx, x, \frac{x}{\sqrt[4]{a + bx^4}} \right)}{2a} \\ &= \frac{\tan^{-1} \left( \frac{\sqrt[4]{a}x}{\sqrt[4]{a + bx^4}} \right)}{2a^{5/4}} + \frac{\tanh^{-1} \left( \frac{\sqrt[4]{a}x}{\sqrt[4]{a + bx^4}} \right)}{2a^{5/4}} \end{aligned}$$

**Mathematica** [A] time = 0.03, size = 48, normalized size = 0.84

$$\frac{\tan^{-1} \left( \frac{\sqrt[4]{a}x}{\sqrt[4]{a + bx^4}} \right) + \tanh^{-1} \left( \frac{\sqrt[4]{a}x}{\sqrt[4]{a + bx^4}} \right)}{2a^{5/4}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a - (a - b)\*x^4)\*(a + b\*x^4)^(1/4)),x]

[Out] (ArcTan[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)] + ArcTanh[(a^(1/4)\*x)/(a + b\*x^4)^(1/4)])/(2\*a^(5/4))

**fricas** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(a-b)\*x^4)/(b\*x^4+a)^(1/4),x, algorithm="fricas")

[Out] Timed out

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{((a - b)x^4 - a)(bx^4 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(a-b)\*x^4)/(b\*x^4+a)^(1/4),x, algorithm="giac")

[Out] integrate(-1/(((a - b)\*x^4 - a)\*(b\*x^4 + a)^(1/4)), x)

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{1}{(-(a - b)x^4 + a)(bx^4 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a-(a-b)\*x^4)/(b\*x^4+a)^(1/4),x)

[Out] int(1/(a-(a-b)\*x^4)/(b\*x^4+a)^(1/4),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{((a - b)x^4 - a)(bx^4 + a)^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(a-b)\*x^4)/(b\*x^4+a)^(1/4),x, algorithm="maxima")

[Out] -integrate(1/(((a - b)\*x^4 - a)\*(b\*x^4 + a)^(1/4)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^4 + a)^{1/4} (a - x^4 (a - b))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^4)^(1/4)\*(a - x^4\*(a - b))),x)

[Out] int(1/((a + b\*x^4)^(1/4)\*(a - x^4\*(a - b))), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{ax^4 \sqrt[4]{a + bx^4} - a \sqrt[4]{a + bx^4} - bx^4 \sqrt[4]{a + bx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a-(a-b)\*x\*\*4)/(b\*x\*\*4+a)\*\*(1/4),x)

[Out] -Integral(1/(a\*x\*\*4\*(a + b\*x\*\*4)\*\*(1/4) - a\*(a + b\*x\*\*4)\*\*(1/4) - b\*x\*\*4\*(a + b\*x\*\*4)\*\*(1/4)), x)

### 3.218 $\int (a + bx^4)^p (c + dx^4)^q dx$

**Optimal.** Leaf size=79

$$x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

[Out]  $x*(b*x^4+a)^p*(d*x^4+c)^q*AppellF1(1/4, -p, -q, 5/4, -b*x^4/a, -d*x^4/c)/((1+b*x^4/a)^p)/((1+d*x^4/c)^q)$

**Rubi [A]** time = 0.05, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$x (a + bx^4)^p \left(\frac{bx^4}{a} + 1\right)^{-p} (c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^p\*(c + d\*x^4)^q,x]

[Out]  $(x*(a + b*x^4)^p*(c + d*x^4)^q*AppellF1[1/4, -p, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/((1 + (b*x^4)/a)^p*(1 + (d*x^4)/c)^q)$

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (a + bx^4)^p (c + dx^4)^q dx &= \left( (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} \right) \int \left(1 + \frac{bx^4}{a}\right)^p (c + dx^4)^q dx \\ &= \left( (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} (c + dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} \right) \int \left(1 + \frac{bx^4}{a}\right)^p \left(1 + \frac{dx^4}{c}\right)^q dx \\ &= x (a + bx^4)^p \left(1 + \frac{bx^4}{a}\right)^{-p} (c + dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \end{aligned}$$

**Mathematica [B]** time = 0.26, size = 172, normalized size = 2.18

$$\frac{5acx (a + bx^4)^p (c + dx^4)^q F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{4x^4 \left( bcp F_1\left(\frac{5}{4}; 1 - p, -q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) + adq F_1\left(\frac{5}{4}; -p, 1 - q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) \right) + 5ac F_1\left(\frac{1}{4}; -p, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.



[In] Integrate[(a + b\*x^4)^p\*(c + d\*x^4)^q,x]

[Out] (5\*a\*c\*x\*(a + b\*x^4)^p\*(c + d\*x^4)^q\*AppellF1[1/4, -p, -q, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/(5\*a\*c\*AppellF1[1/4, -p, -q, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 4\*x^4\*(b\*c\*p\*AppellF1[5/4, 1 - p, -q, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] + a\*d\*q\*AppellF1[5/4, -p, 1 - q, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas** [F] time = 1.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4 + a\right)^p\left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^p\*(d\*x^4+c)^q,x, algorithm="fricas")

[Out] integral((b\*x^4 + a)^p\*(d\*x^4 + c)^q, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^p\*(d\*x^4+c)^q,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^p\*(d\*x^4 + c)^q, x)

**maple** [F] time = 0.55, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^p\*(d\*x^4+c)^q,x)

[Out] int((b\*x^4+a)^p\*(d\*x^4+c)^q,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^p\*(d\*x^4+c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)^p\*(d\*x^4 + c)^q, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^4 + a)^p (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^4)^p\*(c + d\*x^4)^q,x)

[Out] int((a + b\*x^4)^p\*(c + d\*x^4)^q, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*4+a)\*\*p\*(d\*x\*\*4+c)\*\*q,x)

[Out] Timed out

### 3.219 $\int (a + bx^4)^2 (c + dx^4)^q dx$

**Optimal.** Leaf size=176

$$\frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (a^2 d^2 (16q^2 + 56q + 45) - 2abcd(4q + 9) + 5b^2 c^2) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) bx(c + dx^4)^{q+1}}{d^2(4q + 5)(4q + 9) d^2(4q + 5)(4q + 9)}$$

[Out]  $-b*(5*b*c - a*d*(13+4*q))*x*(d*x^4+c)^(1+q)/d^2/(16*q^2+56*q+45)+b*x*(b*x^4+a)$   
 $*(d*x^4+c)^(1+q)/d/(9+4*q)+(5*b^2*c^2-2*a*b*c*d*(9+4*q)+a^2*d^2*(16*q^2+56$   
 $*q+45))*x*(d*x^4+c)^q*hypergeom([1/4, -q], [5/4], -d*x^4/c)/d^2/(16*q^2+56*q+$   
 $45)/((1+d*x^4/c)^q)$

**Rubi [A]** time = 0.13, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 388, 246, 245}

$$\frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (a^2 d^2 (16q^2 + 56q + 45) - 2abcd(4q + 9) + 5b^2 c^2) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) bx(c + dx^4)^{q+1}}{d^2(4q + 5)(4q + 9) d^2(4q + 5)(4q + 9)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)^2\*(c + d\*x^4)^q,x]

[Out]  $-((b*(5*b*c - a*d*(13 + 4*q))*x*(c + d*x^4)^(1 + q))/(d^2*(5 + 4*q)*(9 + 4*q)) + (b*x*(a + b*x^4)*(c + d*x^4)^(1 + q))/(d*(9 + 4*q)) + ((5*b^2*c^2 - 2*a*b*c*d*(9 + 4*q) + a^2*d^2*(45 + 56*q + 16*q^2))*x*(c + d*x^4)^q*Hypergeometric2F1[1/4, -q, 5/4, -((d*x^4)/c)]/(d^2*(5 + 4*q)*(9 + 4*q)*(1 + (d*x^4)/c)^q)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d) + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^4)^2 (c + dx^4)^q dx &= \frac{bx(a + bx^4)(c + dx^4)^{1+q}}{d(9 + 4q)} + \frac{\int (c + dx^4)^q (-a(bc - ad(9 + 4q)) - b(5bc - ad(13 - \\ &= -\frac{b(5bc - ad(13 + 4q))x(c + dx^4)^{1+q}}{d^2(5 + 4q)(9 + 4q)} + \frac{bx(a + bx^4)(c + dx^4)^{1+q}}{d(9 + 4q)} + \frac{(5b^2c^2 - 2a \\ &= -\frac{b(5bc - ad(13 + 4q))x(c + dx^4)^{1+q}}{d^2(5 + 4q)(9 + 4q)} + \frac{bx(a + bx^4)(c + dx^4)^{1+q}}{d(9 + 4q)} + \frac{\left((5b^2c^2 - 2a \\ &= -\frac{b(5bc - ad(13 + 4q))x(c + dx^4)^{1+q}}{d^2(5 + 4q)(9 + 4q)} + \frac{bx(a + bx^4)(c + dx^4)^{1+q}}{d(9 + 4q)} + \frac{(5b^2c^2 - 2a \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 106, normalized size = 0.60

$$\frac{1}{45}x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left(45a^2 {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + bx^4 \left(18a {}_2F_1\left(\frac{5}{4}, -q; \frac{9}{4}; -\frac{dx^4}{c}\right) + 5bx^4 {}_2F_1\left(\frac{9}{4}, -q; \frac{13}{4}; -\frac{dx^4}{c}\right)\right)\right) / (45(1 + (dx^4)/c)^q)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)^2\*(c + d\*x^4)^q,x]

[Out] (x\*(c + d\*x^4)^q\*(45\*a^2\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)] + b\*x^4\*(18\*a\*Hypergeometric2F1[5/4, -q, 9/4, -((d\*x^4)/c)] + 5\*b\*x^4\*Hypergeometric2F1[9/4, -q, 13/4, -((d\*x^4)/c)]))/(45\*(1 + (d\*x^4)/c)^q)

**fricas [F]** time = 1.77, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2x^8 + 2abx^4 + a^2\right)(dx^4 + c)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^q,x, algorithm="fricas")

[Out] integral((b^2\*x^8 + 2\*a\*b\*x^4 + a^2)\*(d\*x^4 + c)^q, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)^2\*(d\*x^4+c)^q,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)^2\*(d\*x^4 + c)^q, x)

**maple [F]** time = 0.53, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)^2\*(d\*x^4+c)^q,x)

[Out] `int((b*x^4+a)^2*(d*x^4+c)^q,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^4+a)^2*(d*x^4+c)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^4 + a)^2*(d*x^4 + c)^q, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^4 + a)^2 (dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^4)^2*(c + d*x^4)^q,x)`

[Out] `int((a + b*x^4)^2*(c + d*x^4)^q, x)`

**sympy** [C] time = 166.03, size = 119, normalized size = 0.68

$$\frac{a^2 c^q x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -q \left| \frac{dx^4 e^{i\pi}}{c} \right.\right)}{4 \Gamma\left(\frac{5}{4}\right)} + \frac{abc^q x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, -q \left| \frac{dx^4 e^{i\pi}}{c} \right.\right)}{2 \Gamma\left(\frac{9}{4}\right)} + \frac{b^2 c^q x^9 \Gamma\left(\frac{9}{4}\right) {}_2F_1\left(\frac{9}{4}, -q \left| \frac{dx^4 e^{i\pi}}{c} \right.\right)}{4 \Gamma\left(\frac{13}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)**2*(d*x**4+c)**q,x)`

[Out] `a**2*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4, ), d*x**4*exp_polar(I*pi)/c)/(4*gamma(5/4)) + a*b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4, ), d*x**4*exp_polar(I*pi)/c)/(2*gamma(9/4)) + b**2*c**q*x**9*gamma(9/4)*hyper((9/4, -q), (13/4, ), d*x**4*exp_polar(I*pi)/c)/(4*gamma(13/4))`

### 3.220 $\int (a + bx^4)(c + dx^4)^q dx$

Optimal. Leaf size=93

$$\frac{bx(c + dx^4)^{q+1}}{d(4q + 5)} - \frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} (bc - ad(4q + 5)) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right)}{d(4q + 5)}$$

[Out] b\*x\*(d\*x^4+c)^(1+q)/d/(5+4\*q)-(b\*c-a\*d\*(5+4\*q))\*x\*(d\*x^4+c)^q\*hypergeom([1/4, -q], [5/4], -d\*x^4/c)/d/(5+4\*q)/((1+d\*x^4/c)^q)

**Rubi [A]** time = 0.04, antiderivative size = 85, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {388, 246, 245}

$$x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left(a - \frac{bc}{4dq + 5d}\right) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + \frac{bx(c + dx^4)^{q+1}}{d(4q + 5)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^4)\*(c + d\*x^4)^q, x]

[Out] (b\*x\*(c + d\*x^4)^(1 + q))/(d\*(5 + 4\*q)) + ((a - (b\*c)/(5\*d + 4\*d\*q))\*x\*(c + d\*x^4)^q\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)])/(1 + (d\*x^4)/c)^q

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (a + bx^4)(c + dx^4)^q dx &= \frac{bx(c + dx^4)^{1+q}}{d(5 + 4q)} - \left(-a + \frac{bc}{5d + 4dq}\right) \int (c + dx^4)^q dx \\ &= \frac{bx(c + dx^4)^{1+q}}{d(5 + 4q)} - \left(\left(-a + \frac{bc}{5d + 4dq}\right)(c + dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q}\right) \int \left(1 + \frac{dx^4}{c}\right)^q dx \\ &= \frac{bx(c + dx^4)^{1+q}}{d(5 + 4q)} + \left(a - \frac{bc}{5d + 4dq}\right) x(c + dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 90, normalized size = 0.97

$$\frac{x(c + dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} \left( (ad(4q + 5) - bc) {}_2F_1\left(\frac{1}{4}, -q; \frac{5}{4}; -\frac{dx^4}{c}\right) + b(c + dx^4) \left(\frac{dx^4}{c} + 1\right)^q \right)}{d(4q + 5)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^4)\*(c + d\*x^4)^q,x]

[Out] (x\*(c + d\*x^4)^q\*(b\*(c + d\*x^4)\*(1 + (d\*x^4)/c)^q + (-b\*c) + a\*d\*(5 + 4\*q))\*Hypergeometric2F1[1/4, -q, 5/4, -((d\*x^4)/c)]/(d\*(5 + 4\*q)\*(1 + (d\*x^4)/c)^q)

**fricas [F]** time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(bx^4 + a\right)\left(dx^4 + c\right)^q, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^q,x, algorithm="fricas")

[Out] integral((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)(dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^q,x, algorithm="giac")

[Out] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

**maple [F]** time = 0.38, size = 0, normalized size = 0.00

$$\int (bx^4 + a)(dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^4+a)\*(d\*x^4+c)^q,x)

[Out] int((b\*x^4+a)\*(d\*x^4+c)^q,x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^4 + a)(dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^4+a)\*(d\*x^4+c)^q,x, algorithm="maxima")

[Out] integrate((b\*x^4 + a)\*(d\*x^4 + c)^q, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (bx^4 + a)(dx^4 + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^4)*(c + d*x^4)^q, x)`

[Out] `int((a + b*x^4)*(c + d*x^4)^q, x)`

**sympy [C]** time = 61.01, size = 75, normalized size = 0.81

$$\frac{ac^q x \Gamma\left(\frac{1}{4}\right) {}_2F_1\left(\frac{1}{4}, -q \middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{5}{4}\right)} + \frac{bc^q x^5 \Gamma\left(\frac{5}{4}\right) {}_2F_1\left(\frac{5}{4}, -q \middle| \frac{dx^4 e^{i\pi}}{c}\right)}{4\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**4+a)*(d*x**4+c)**q, x)`

[Out] `a*c**q*x*gamma(1/4)*hyper((1/4, -q), (5/4, ), d*x**4*exp_polar(I*pi)/c)/(4*gamma(5/4)) + b*c**q*x**5*gamma(5/4)*hyper((5/4, -q), (9/4, ), d*x**4*exp_polar(I*pi)/c)/(4*gamma(9/4))`

$$3.221 \quad \int \frac{(c+dx^4)^q}{a+bx^4} dx$$

**Optimal.** Leaf size=57

$$\frac{x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 1, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a}$$

[Out] x\*(d\*x^4+c)^q\*AppellF1(1/4,1,-q,5/4,-b\*x^4/a,-d\*x^4/c)/a/((1+d\*x^4/c)^q)

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 1, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^q/(a + b\*x^4), x]

[Out] (x\*(c + d\*x^4)^q\*AppellF1[1/4, 1, -q, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)])/(a\*(1 + (d\*x^4)/c)^q)

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^4)^q}{a+bx^4} dx &= \left( (c+dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} \right) \int \frac{\left(1 + \frac{dx^4}{c}\right)^q}{a+bx^4} dx \\ &= \frac{x(c+dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4}; 1, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a} \end{aligned}$$

**Mathematica [B]** time = 0.22, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4}; -q, 1; \frac{5}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)}{(a+bx^4) \left(4x^4 \left(adqF_1\left(\frac{5}{4}; 1-q, 1; \frac{9}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right) - bcF_1\left(\frac{5}{4}; -q, 2; \frac{9}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)\right) + 5acF_1\left(\frac{1}{4}; -q, 1; \frac{5}{4}; -\frac{dx^4}{c}, -\frac{bx^4}{a}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^4)^q/(a + b\*x^4), x]



```
[Out] (5*a*c*x*(c + d*x^4)^q*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
]/((a + b*x^4)*(5*a*c*AppellF1[1/4, -q, 1, 5/4, -((d*x^4)/c), -((b*x^4)/a)
] + 4*x^4*(a*d*q*AppellF1[5/4, 1 - q, 1, 9/4, -((d*x^4)/c), -((b*x^4)/a)] -
b*c*AppellF1[5/4, -q, 2, 9/4, -((d*x^4)/c), -((b*x^4)/a)]))
```

**fricas** [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(dx^4 + c)^q}{bx^4 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="fricas")
```

```
[Out] integral((d*x^4 + c)^q/(b*x^4 + a), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="giac")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^4+c)^q/(b*x^4+a),x)
```

```
[Out] int((d*x^4+c)^q/(b*x^4+a),x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^4+c)^q/(b*x^4+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x^4 + c)^q/(b*x^4 + a), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx^4 + c)^q}{bx^4 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^4)^q/(a + b*x^4),x)
```

```
[Out] int((c + d*x^4)^q/(a + b*x^4), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a),x)
```

```
[Out] Timed out
```

$$3.222 \quad \int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx$$

**Optimal.** Leaf size=57

$$\frac{x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2}$$

[Out]  $x*(d*x^4+c)^q*AppellF1(1/4,2,-q,5/4,-b*x^4/a,-d*x^4/c)/a^2/((1+d*x^4/c)^q)$

**Rubi [A]** time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(c+dx^4)^q \left(\frac{dx^4}{c} + 1\right)^{-q} F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^4)^q/(a + b\*x^4)^2,x]

[Out]  $(x*(c + d*x^4)^q*AppellF1[1/4, 2, -q, 5/4, -((b*x^4)/a), -((d*x^4)/c)])/(a^2*(1 + (d*x^4)/c)^q)$

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^4)^q}{(a+bx^4)^2} dx &= \left( (c+dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} \right) \int \frac{\left(1 + \frac{dx^4}{c}\right)^q}{(a+bx^4)^2} dx \\ &= \frac{x(c+dx^4)^q \left(1 + \frac{dx^4}{c}\right)^{-q} F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{a^2} \end{aligned}$$

**Mathematica [B]** time = 0.24, size = 162, normalized size = 2.84

$$\frac{5acx(c+dx^4)^q F_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}{(a+bx^4)^2 \left(4x^4 \left(adqF_1\left(\frac{5}{4}; 2, 1-q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right) - 2bcF_1\left(\frac{5}{4}; 3, -q; \frac{9}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)\right) + 5acF_1\left(\frac{1}{4}; 2, -q; \frac{5}{4}; -\frac{bx^4}{a}, -\frac{dx^4}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^4)^q/(a + b\*x^4)^2,x]

[Out] (5\*a\*c\*x\*(c + d\*x^4)^q\*AppellF1[1/4, 2, -q, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)]/((a + b\*x^4)^2\*(5\*a\*c\*AppellF1[1/4, 2, -q, 5/4, -((b\*x^4)/a), -((d\*x^4)/c)] + 4\*x^4\*(a\*d\*q\*AppellF1[5/4, 2, 1 - q, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)] - 2\*b\*c\*AppellF1[5/4, 3, -q, 9/4, -((b\*x^4)/a), -((d\*x^4)/c)]))

**fricas** [F] time = 1.19, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^4 + c)^q}{b^2x^8 + 2abx^4 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^q/(b\*x^4+a)^2,x, algorithm="fricas")

[Out] integral((d\*x^4 + c)^q/(b^2\*x^8 + 2\*a\*b\*x^4 + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^q/(b\*x^4+a)^2,x, algorithm="giac")

[Out] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x)

**maple** [F] time = 0.56, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^4+c)^q/(b\*x^4+a)^2,x)

[Out] int((d\*x^4+c)^q/(b\*x^4+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^4+c)^q/(b\*x^4+a)^2,x, algorithm="maxima")

[Out] integrate((d\*x^4 + c)^q/(b\*x^4 + a)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(dx^4 + c)^q}{(bx^4 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^4)^q/(a + b\*x^4)^2,x)

```
[Out] int((c + d*x^4)^q/(a + b*x^4)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**4+c)**q/(b*x**4+a)**2,x)
```

```
[Out] Timed out
```

$$3.223 \quad \int \frac{1}{\sqrt[5]{a+bx^5}(c+dx^5)} dx$$

**Optimal.** Leaf size=545

$$\frac{\log\left(\sqrt[5]{c} - \frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} - \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{1}{5}(5-2\sqrt{5})} - \frac{2\sqrt{\frac{2}{5+\sqrt{5}}x\sqrt[5]{bc-ad}}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1}\left(\frac{\sqrt{\frac{2}{5}(5+\sqrt{5})}}{\sqrt[5]{c}\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}}$$

[Out]  $-1/5*\ln(c^{(1/5)}-(-a*d+b*c)^{(1/5)}*x/(b*x^5+a)^{(1/5)})/c^{(4/5)}/(-a*d+b*c)^{(1/5)}$   
 $+1/20*\ln((2*(-a*d+b*c)^{(2/5)}*x^2+c^{(1/5)}*(-a*d+b*c)^{(1/5)}*x*(b*x^5+a)^{(1/5)}$   
 $+2*c^{(2/5)}*(b*x^5+a)^{(2/5)}-c^{(1/5)}*(-a*d+b*c)^{(1/5)}*x*(b*x^5+a)^{(1/5)}*5^{(1/2)})/(b*x^5+a)^{(2/5)}*(-5^{(1/2)}+1)/c^{(4/5)}/(-a*d+b*c)^{(1/5)}$   
 $+1/20*\ln((2*(-a*d+b*c)^{(2/5)}*x^2+c^{(1/5)}*(-a*d+b*c)^{(1/5)}*x*(b*x^5+a)^{(1/5)}+2*c^{(2/5)}*(b*x^5+a)^{(2/5)}+c^{(1/5)}*(-a*d+b*c)^{(1/5)}*x*(b*x^5+a)^{(1/5)}*5^{(1/2)})/(b*x^5+a)^{(2/5)}$   
 $*(5^{(1/2)}+1)/c^{(4/5)}/(-a*d+b*c)^{(1/5)}$   
 $+1/10*\arctan(1/5*(-a*d+b*c)^{(1/5)}*x*(50+10*5^{(1/2)})^{(1/2)}/c^{(1/5)}/(b*x^5+a)^{(1/5)}+1/5*(25+10*5^{(1/2)})^{(1/2)})*(10-2*5^{(1/2)})^{(1/2)}/c^{(4/5)}/(-a*d+b*c)^{(1/5)}$   
 $+1/10*\arctan(-1/5*(25-10*5^{(1/2)})^{(1/2)}+2*(-a*d+b*c)^{(1/5)}*x*2^{(1/2)}/(5+5^{(1/2)})^{(1/2)}/c^{(1/5)}/(b*x^5+a)^{(1/5)})*(10+2*5^{(1/2)})^{(1/2)}/c^{(4/5)}/(-a*d+b*c)^{(1/5)}$

**Rubi [A]** time = 1.09, antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {377, 202, 634, 618, 204, 628, 31}

$$\frac{\log\left(\sqrt[5]{c} - \frac{x\sqrt[5]{bc-ad}}{\sqrt[5]{a+bx^5}}\right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{(1-\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}+2x^2(bc-ad)^{2/5}-\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}} + \frac{(1+\sqrt{5}) \log\left(\frac{2c^{2/5}(a+bx^5)^{2/5}+2x^2(bc-ad)^{2/5}+\sqrt{5}\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}+\sqrt[5]{c}x\sqrt[5]{a+bx^5}\sqrt[5]{bc-ad}}{(a+bx^5)^{2/5}}\right)}{20c^{4/5}\sqrt[5]{bc-ad}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^5)^(1/5)\*(c + d\*x^5)),x]

[Out]  $-(\text{Sqrt}[(5 + \text{Sqrt}[5])/2]*\text{ArcTan}[\text{Sqrt}[(5 - 2*\text{Sqrt}[5])/5] - (2*\text{Sqrt}[2/(5 + \text{Sqrt}[5]))*(b*c - a*d)^{(1/5)}*x)/(c^{(1/5)}*(a + b*x^5)^{(1/5)})]/(5*c^{(4/5)}*(b*c - a*d)^{(1/5)}) + (\text{Sqrt}[(5 - \text{Sqrt}[5])/2]*\text{ArcTan}[\text{Sqrt}[(5 + 2*\text{Sqrt}[5])/5] + (\text{Sqrt}[(2*(5 + \text{Sqrt}[5]))/5]*(b*c - a*d)^{(1/5)}*x)/(c^{(1/5)}*(a + b*x^5)^{(1/5)})]/(5*c^{(4/5)}*(b*c - a*d)^{(1/5)}) - \text{Log}[c^{(1/5)} - ((b*c - a*d)^{(1/5)}*x)/(a + b*x^5)^{(1/5)}]/(5*c^{(4/5)}*(b*c - a*d)^{(1/5)}) + ((1 - \text{Sqrt}[5])*\text{Log}[(2*(b*c - a*d)^{(2/5)}*x^2 + c^{(1/5)}*(b*c - a*d)^{(1/5)}*x*(a + b*x^5)^{(1/5)} - \text{Sqrt}[5]*c^{(1/5)}*(b*c - a*d)^{(1/5)}*x*(a + b*x^5)^{(1/5)} + 2*c^{(2/5)}*(a + b*x^5)^{(2/5)})/(a + b*x^5)^{(2/5)}]/(20*c^{(4/5)}*(b*c - a*d)^{(1/5)}) + ((1 + \text{Sqrt}[5])*\text{Log}[(2*(b*c - a*d)^{(2/5)}*x^2 + c^{(1/5)}*(b*c - a*d)^{(1/5)}*x*(a + b*x^5)^{(1/5)} + \text{Sqrt}[5]*c^{(1/5)}*(b*c - a*d)^{(1/5)}*x*(a + b*x^5)^{(1/5)} + 2*c^{(2/5)}*(a + b*x^5)^{(2/5)})/(a + b*x^5)^{(2/5)}]/(20*c^{(4/5)}*(b*c - a*d)^{(1/5)})$

**Rule 31**

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

**Rule 202**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(m\_), x\_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s\*Cos[(2\*k - 1)\*Pi/n]\*x)/(r^2 + 2\*r\*s\*Cos[(2\*k - 1)\*Pi/n]\*x + s^2\*x^2), x]; (r \* Int[1/(r - s\*x), x])/(a\*n) + Dist[(2\*r)/(a\*n), Sum[u, {k, 1, (n - 1)/2}],

$x], x]] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{IGtQ}\{(n - 3)/2, 0\} \&\& \text{NegQ}\{a/b\}$

#### Rule 204

$\text{Int}[(a_ + (b_ \cdot)(x_ )^2)^{-1}, x\_ \text{Symbol}] \rightarrow -\text{Simp}[\text{ArcTan}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[-a, 2]] / (\text{Rt}[-a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x\} \&\& \text{PosQ}\{a/b\} \&\& (\text{LtQ}\{a, 0\} \parallel \text{LtQ}\{b, 0\})$

#### Rule 377

$\text{Int}[(a_ + (b_ \cdot)(x_ )^{(n_ )})^{(p_ )} / ((c_ ) + (d_ \cdot)(x_ )^{(n_ )}), x\_ \text{Symbol}] \rightarrow \text{Subst}[\text{Int}[1/(c - (b \cdot c - a \cdot d) \cdot x^n), x], x, x/(a + b \cdot x^n)^{1/n}] /; \text{FreeQ}\{a, b, c, d\}, x\} \&\& \text{NeQ}\{b \cdot c - a \cdot d, 0\} \&\& \text{EqQ}\{n \cdot p + 1, 0\} \&\& \text{IntegerQ}\{n\}$

#### Rule 618

$\text{Int}[(a_ \cdot) + (b_ \cdot)(x_ ) + (c_ \cdot)(x_ )^2]^{-1}, x\_ \text{Symbol}] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4 \cdot a \cdot c - x^2, x], x], x, b + 2 \cdot c \cdot x], x] /; \text{FreeQ}\{a, b, c\}, x\} \&\& \text{NeQ}\{b^2 - 4 \cdot a \cdot c, 0\}$

#### Rule 628

$\text{Int}[(d_ ) + (e_ \cdot)(x_ )] / ((a_ \cdot) + (b_ \cdot)(x_ ) + (c_ \cdot)(x_ )^2), x\_ \text{Symbol}] \rightarrow \text{Simp}[(d \cdot \text{Log}[\text{RemoveContent}[a + b \cdot x + c \cdot x^2, x]]) / b, x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{EqQ}\{2 \cdot c \cdot d - b \cdot e, 0\}$

#### Rule 634

$\text{Int}[(d_ \cdot) + (e_ \cdot)(x_ )] / ((a_ ) + (b_ \cdot)(x_ ) + (c_ \cdot)(x_ )^2), x\_ \text{Symbol}] \rightarrow \text{Dist}[(2 \cdot c \cdot d - b \cdot e) / (2 \cdot c), \text{Int}[1/(a + b \cdot x + c \cdot x^2), x], x] + \text{Dist}[e / (2 \cdot c), \text{Int}[(b + 2 \cdot c \cdot x) / (a + b \cdot x + c \cdot x^2), x], x] /; \text{FreeQ}\{a, b, c, d, e\}, x\} \&\& \text{NeQ}\{2 \cdot c \cdot d - b \cdot e, 0\} \&\& \text{NeQ}\{b^2 - 4 \cdot a \cdot c, 0\} \&\& \text{!NiceSqrtQ}\{b^2 - 4 \cdot a \cdot c\}$

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[5]{a+bx^5} (c+dx^5)} dx &= \text{Subst} \left( \int \frac{1}{c - (bc-ad)x^5} dx, x, \frac{x}{\sqrt[5]{a+bx^5}} \right) \\ &= \frac{2 \text{Subst} \left( \int \frac{\sqrt[5]{c+\frac{1}{4}(1-\sqrt{5})\sqrt[5]{bc-ad}x}}{c^{2/5+\frac{1}{2}(1-\sqrt{5})}\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2} dx, x, \frac{x}{\sqrt[5]{a+bx^5}} \right)}{5c^{4/5}} + \frac{2 \text{Subst} \left( \int \frac{\sqrt[5]{c+\frac{1}{4}(1+\sqrt{5})\sqrt[5]{bc-ad}x}}{c^{2/5+\frac{1}{2}(1+\sqrt{5})}\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2} dx, x, \frac{x}{\sqrt[5]{a+bx^5}} \right)}{5c^{4/5}} \\ &= -\frac{\log \left( \sqrt[5]{c} - \frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}} \right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{(5-\sqrt{5}) \text{Subst} \left( \int \frac{1}{c^{2/5+\frac{1}{2}(1+\sqrt{5})}\sqrt[5]{c}\sqrt[5]{bc-ad}x+(bc-ad)^{2/5}x^2} dx, x, \frac{x}{\sqrt[5]{a+bx^5}} \right)}{20c^{3/5}} \\ &= -\frac{\log \left( \sqrt[5]{c} - \frac{\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}} \right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{(1-\sqrt{5}) \log \left( 2c^{2/5} + \frac{2(bc-ad)^{2/5}x^2}{(a+bx^5)^{2/5}} + \frac{\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}} - \frac{\sqrt{5}\sqrt[5]{c}\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}} \right)}{20c^{4/5}\sqrt[5]{bc-ad}} \\ &= \frac{\sqrt{\frac{1}{2}(5+\sqrt{5})} \tan^{-1} \left( \frac{(1-\sqrt{5})\sqrt[5]{c} + \frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}}}{\sqrt{2(5+\sqrt{5})}\sqrt[5]{c}} \right)}{5c^{4/5}\sqrt[5]{bc-ad}} + \frac{\sqrt{\frac{1}{2}(5-\sqrt{5})} \tan^{-1} \left( \frac{\sqrt{5+\sqrt{5}} \left( (1+\sqrt{5})\sqrt[5]{c} - \frac{4\sqrt[5]{bc-ad}x}{\sqrt[5]{a+bx^5}} \right)}{2\sqrt{10}\sqrt[5]{c}} \right)}{5c^{4/5}\sqrt[5]{bc-ad}} \end{aligned}$$

**Mathematica** [C] time = 0.02, size = 48, normalized size = 0.09

$$\frac{x {}_2F_1\left(\frac{1}{5}, 1; \frac{6}{5}; \frac{(bc-ad)x^5}{c(bx^5+a)}\right)}{c\sqrt[5]{a+bx^5}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^5)^(1/5)\*(c + d\*x^5)),x]

[Out] (x\*Hypergeometric2F1[1/5, 1, 6/5, ((b\*c - a\*d)\*x^5)/(c\*(a + b\*x^5))]/(c\*(a + b\*x^5)^(1/5))

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^5+a)^(1/5)/(d\*x^5+c),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (trace 0)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^5 + a)^{\frac{1}{5}}(dx^5 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^5+a)^(1/5)/(d\*x^5+c),x, algorithm="giac")

[Out] integrate(1/((b\*x^5 + a)^(1/5)\*(d\*x^5 + c)), x)

**maple** [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^5 + a)^{\frac{1}{5}}(dx^5 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^5+a)^(1/5)/(d\*x^5+c),x)

[Out] int(1/(b\*x^5+a)^(1/5)/(d\*x^5+c),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^5 + a)^{\frac{1}{5}}(dx^5 + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*x^5+a)^(1/5)/(d\*x^5+c),x, algorithm="maxima")

[Out] integrate(1/((b\*x^5 + a)^(1/5)\*(d\*x^5 + c)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(bx^5 + a)^{1/5}(dx^5 + c)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)`

[Out] `int(1/((a + b*x^5)^(1/5)*(c + d*x^5)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[5]{a + bx^5} (c + dx^5)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**5+a)**(1/5)/(d*x**5+c), x)`

[Out] `Integral(1/((a + b*x**5)**(1/5)*(c + d*x**5)), x)`

$$3.224 \quad \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=143

$$\frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 15abcd + 57b^2c^2) + \frac{bd(2ad+33bc)}{x}\right)}{15b^2} + \frac{c^2(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + x\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 - \frac{7}{5}d$$

[Out]  $c^2*(6*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(1/2)}-7/5*d*(c+d/x)^2*(a+b/x)^{(1/2)}-1/15*d*(-4*a^2*d^2+30*a*b*c*d+114*b^2*c^2+b*d*(2*a*d+33*b*c)/x)*(a+b/x)^{(1/2)}/b^2+(c+d/x)^3*x*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.12, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 97, 153, 147, 63, 208}

$$\frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 15abcd + 57b^2c^2) + \frac{bd(2ad+33bc)}{x}\right)}{15b^2} + \frac{c^2(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + x\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 - \frac{7}{5}d$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*(c + d/x)^3,x]

[Out]  $(-7*d*\operatorname{Sqrt}[a + b/x]*(c + d/x)^2)/5 - (d*\operatorname{Sqrt}[a + b/x]*(2*(57*b^2*c^2 + 15*a*b*c*d - 2*a^2*d^2) + (b*d*(33*b*c + 2*a*d))/x))/(15*b^2) + \operatorname{Sqrt}[a + b/x]*(c + d/x)^3*x + (c^2*(b*c + 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/\operatorname{Sqrt}[a]$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 97

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p)/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p - 1)\*Simp[d\*e\*n + c\*f\*p + d\*f\*(n + p)\*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 147

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))\*(g\_.) + (h\_.)\*(x\_)), x\_Symbol] := -Simp[((a\*d\*f\*h\*(n + 2) + b\*c\*f\*h\*(m + 2) - b\*d\*(f\*g + e\*h)\*(m + n + 3) - b\*d\*f\*h\*(m + n + 2)\*x)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/(b^2\*d^2\*(m + n + 2)\*(m + n + 3)), x] + Dist[(a^2\*d^2\*f\*h\*(n + 1)\*(n + 2) + a\*b\*d\*(n + 1)\*(2\*c\*f\*h\*(m + 1) - d\*(f\*g + e\*h)\*(m + n + 3)) + b^2\*(c^2\*f\*h\*(m + 1)\*(m + 2) - c\*d\*(f\*g + e\*h)\*(m + 1)\*(m + n + 3) + d^2\*e\*g\*(m + n + 2)\*(m + n + 3)))/(b^2\*d^2\*(m + n + 2)\*(m + n + 3)), Int[(a + b\*x)^m\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]

### Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegerQ[m]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx} (c + dx)^3}{x^2} dx, x, \frac{1}{x} \right) \\ &= \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 x - \text{Subst} \left( \int \frac{(c + dx)^2 \left(\frac{1}{2}(bc + 6ad) + \frac{7bdx}{2}\right)}{x\sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\ &= -\frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 + \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3 x - \frac{2 \text{Subst} \left( \int \frac{(c + dx) \left(\frac{5}{4}bc(bc + 6ad) + \frac{1}{4}bd(33bc + 2ad)\right)}{x\sqrt{a + bx}} dx, x, \frac{1}{x} \right)}{5b} \\ &= -\frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 - \frac{d\sqrt{a + \frac{b}{x}} \left(2(57b^2c^2 + 15abcd - 2a^2d^2) + \frac{bd(33bc + 2ad)}{x}\right)}{15b^2} + \dots \\ &= -\frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 - \frac{d\sqrt{a + \frac{b}{x}} \left(2(57b^2c^2 + 15abcd - 2a^2d^2) + \frac{bd(33bc + 2ad)}{x}\right)}{15b^2} + \dots \\ &= -\frac{7}{5}d\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 - \frac{d\sqrt{a + \frac{b}{x}} \left(2(57b^2c^2 + 15abcd - 2a^2d^2) + \frac{bd(33bc + 2ad)}{x}\right)}{15b^2} + \dots \end{aligned}$$

**Mathematica [A]** time = 0.19, size = 118, normalized size = 0.83

$$\frac{\sqrt{a + \frac{b}{x}} \left(4a^2d^3x^2 - 2abd^2x(15cx + d) - 3b^2(-5c^3x^3 + 30c^2dx^2 + 10cd^2x + 2d^3)\right)}{15b^2x^2} + \frac{c^2(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^3, x]
```

```
[Out] (Sqrt[a + b/x]*(4*a^2*d^3*x^2 - 2*a*b*d^2*x*(d + 15*c*x) - 3*b^2*(2*d^3 + 10*c*d^2*x + 30*c^2*d*x^2 - 5*c^3*x^3)))/(15*b^2*x^2) + (c^2*(b*c + 6*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.19, size = 306, normalized size = 2.14

$$\frac{15(b^3c^3 + 6ab^2c^2d)\sqrt{a}x^2 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(15ab^2c^3x^3 - 6ab^2d^3 - 2(45ab^2c^2d + 15a^2bcd^2 - 15a^2b^2c^2d^2 - 2a^3d^3)x^2 - 2(15a^2b^2c^2d^2 + a^2b^2d^3)x)\sqrt{((ax+b)/x)}}{30ab^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3\*(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/30\*(15\*(b^3\*c^3 + 6\*a\*b^2\*c^2\*d)\*sqrt(a)\*x^2\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(15\*a\*b^2\*c^3\*x^3 - 6\*a\*b^2\*d^3 - 2\*(45\*a\*b^2\*c^2\*d + 15\*a^2\*b\*c\*d^2 - 2\*a^3\*d^3)\*x^2 - 2\*(15\*a\*b^2\*c^2\*d^2 + a^2\*b\*d^3)\*x)\*sqrt((a\*x + b)/x))/(a\*b^2\*x^2), -1/15\*(15\*(b^3\*c^3 + 6\*a\*b^2\*c^2\*d)\*sqrt(-a)\*x^2\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (15\*a\*b^2\*c^3\*x^3 - 6\*a\*b^2\*d^3 - 2\*(45\*a\*b^2\*c^2\*d + 15\*a^2\*b\*c\*d^2 - 2\*a^3\*d^3)\*x^2 - 2\*(15\*a\*b^2\*c^2\*d^2 + a^2\*b\*d^3)\*x)\*sqrt((a\*x + b)/x))/(a\*b^2\*x^2)]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3\*(a+b/x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [7,-27,26]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [-89,63,-49]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [61.7937478349,-30,70]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [73.519035968,-9,-13]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parameters values [18,15.451549686,-33]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple** [A] time = 0.06, size = 248, normalized size = 1.73

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 90ab^2c^2dx^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}}{2\sqrt{a}}\sqrt{a}\right) + 15b^3c^3x^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}}{2\sqrt{a}}\sqrt{a}\right) + 180\sqrt{ax^2+bx} a^{\frac{3}{2}}b^3c^2dx^4 + 30 \right)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3\*(a+b/x)^(1/2),x)

[Out] 1/30\*((a\*x+b)/x)^(1/2)\*(180\*(a\*x^2+b\*x)^(1/2)\*a^(3/2)\*x^4\*b\*c^2\*d+30\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)\*x^4\*b^2\*c^3+90\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1

$(/2)/a^{(1/2)}) * x^4 * a * b^2 * c^2 * d + 15 * \ln(1/2 * (2 * a * x + b + 2 * (a * x^2 + b * x)^{(1/2)} * a^{(1/2)})) / a^{(1/2)}) * x^4 * b^3 * c^3 + 8 * (a * x^2 + b * x)^{(3/2)} * a^{(3/2)} * x * d^3 - 180 * (a * x^2 + b * x)^{(3/2)} * a^{(1/2)} * x^2 * b * c^2 * d - 60 * d^2 * c * (a * x^2 + b * x)^{(3/2)} * x * b * a^{(1/2)} - 12 * (a * x^2 + b * x)^{(3/2)} * a^{(1/2)} * b * d^3 / x^3 / ((a * x + b) * x)^{(1/2)} / b^2 / a^{(1/2)}$

**maxima** [A] time = 1.45, size = 164, normalized size = 1.15

$$\frac{1}{2} \left( 2 \sqrt{a + \frac{b}{x}} x - \frac{b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{\sqrt{a}} \right) c^3 - 3 \left( \sqrt{a} \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) + 2 \sqrt{a + \frac{b}{x}} \right) c^2 d - \frac{2}{15} d^3 \left( \frac{3 \left( a + \frac{b}{x} \right)^{\frac{5}{2}}}{b^2} - \frac{5 \left( a + \frac{b}{x} \right)^{\frac{3}{2}} a}{b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3\*(a+b/x)^(1/2),x, algorithm="maxima")

[Out]  $1/2 * (2 * \sqrt{a + b/x} * x - b * \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a}))) / \sqrt{a} * c^3 - 3 * (\sqrt{a} * \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a}))) + 2 * \sqrt{a + b/x} * c^2 * d - 2/15 * d^3 * (3 * (a + b/x)^{(5/2)} / b^2 - 5 * (a + b/x)^{(3/2)} * a / b^2) - 2 * (a + b/x)^{(3/2)} * c * d^2 / b$

**mupad** [B] time = 2.59, size = 173, normalized size = 1.21

$$\left( a + \frac{b}{x} \right)^{3/2} \left( \frac{6 a d^3 - 6 b c d^2}{3 b^2} - \frac{4 a d^3}{3 b^2} \right) + \sqrt{a + \frac{b}{x}} \left( 2 a \left( \frac{6 a d^3 - 6 b c d^2}{b^2} - \frac{4 a d^3}{b^2} \right) - \frac{6 d (a d - b c)^2}{b^2} + \frac{2 a^2 d^3}{b^2} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)\*(c + d/x)^3,x)

[Out]  $(a + b/x)^{(3/2)} * ((6 * a * d^3 - 6 * b * c * d^2) / (3 * b^2) - (4 * a * d^3) / (3 * b^2)) + (a + b/x)^{(1/2)} * (2 * a * ((6 * a * d^3 - 6 * b * c * d^2) / b^2 - (4 * a * d^3) / b^2) - (6 * d * (a * d - b * c)^2) / b^2 + (2 * a^2 * d^3) / b^2) + c^3 * x * (a + b/x)^{(1/2)} - (2 * d^3 * (a + b/x)^{(5/2)}) / (5 * b^2) - (c^2 * \operatorname{atan}(((a + b/x)^{(1/2)} * i) / a^{(1/2)})) * (6 * a * d + b * c) * i / a^{(1/2)}$

**sympy** [A] time = 57.22, size = 454, normalized size = 3.17

$$\frac{4 a^{\frac{11}{2}} b^{\frac{3}{2}} d^3 x^3 \sqrt{\frac{a x}{b} + 1}}{15 a^{\frac{7}{2}} b^3 x^2 + 15 a^{\frac{5}{2}} b^4 x^2} + \frac{2 a^{\frac{9}{2}} b^{\frac{5}{2}} d^3 x^2 \sqrt{\frac{a x}{b} + 1}}{15 a^{\frac{7}{2}} b^3 x^2 + 15 a^{\frac{5}{2}} b^4 x^2} - \frac{8 a^{\frac{7}{2}} b^{\frac{7}{2}} d^3 x \sqrt{\frac{a x}{b} + 1}}{15 a^{\frac{7}{2}} b^3 x^2 + 15 a^{\frac{5}{2}} b^4 x^2} - \frac{6 a^{\frac{5}{2}} b^{\frac{9}{2}} d^3 \sqrt{\frac{a x}{b} + 1}}{15 a^{\frac{7}{2}} b^3 x^2 + 15 a^{\frac{5}{2}} b^4 x^2} - \frac{4 a^6 b d^3}{15 a^{\frac{7}{2}} b^3 x^2 + 15 a^{\frac{5}{2}} b^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3\*(a+b/x)\*\*(1/2),x)

[Out]  $4 * a^{(11/2)} * b^{(3/2)} * d^{(3)} * x^{(3)} * \sqrt{a * x / b + 1} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) + 2 * a^{(9/2)} * b^{(5/2)} * d^{(3)} * x^{(2)} * \sqrt{a * x / b + 1} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) - 8 * a^{(7/2)} * b^{(7/2)} * d^{(3)} * x * \sqrt{a * x / b + 1} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) - 6 * a^{(5/2)} * b^{(9/2)} * d^{(3)} * \sqrt{a * x / b + 1} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) - 4 * a^{(6)} * b * d^{(3)} * x^{(7/2)} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) - 4 * a^{(5)} * b^{(2)} * d^{(3)} * x^{(5/2)} / (15 * a^{(7/2)} * b^{(3)} * x^{(7/2)} + 15 * a^{(5/2)} * b^{(4)} * x^{(5/2)}) - 6 * a * c^{(2)} * d * \operatorname{atan}(\sqrt{a + b/x} / \sqrt{-a}) / \sqrt{-a} + \sqrt{b} * c^{(3)} * \sqrt{x} * \sqrt{a * x / b + 1} - 6 * c^{(2)} * d * \sqrt{a + b/x} + 3 * c * d^{(2)} * \operatorname{Piecewise}((- \sqrt{a} / x, \operatorname{Eq}(b, 0)), (-2 * (a + b/x)^{(3/2)} / (3 * b), \operatorname{True})) + b * c^{(3)} * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / \sqrt{a}$

$$3.225 \quad \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=99

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{c \sqrt{a + \frac{b}{x}} (4ad + bc)}{a} + \frac{c(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b}$$

[Out]  $-2/3*d^2*(a+b/x)^{(3/2)}/b+c^2*(a+b/x)^{(3/2)}*x/a+c*(4*a*d+b*c)*\arctanh((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(1/2)}-c*(4*a*d+b*c)*(a+b/x)^{(1/2)}/a$

**Rubi [A]** time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{c \sqrt{a + \frac{b}{x}} (4ad + bc)}{a} + \frac{c(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*(c + d/x)^2,x]

[Out]  $-((c*(b*c + 4*a*d)*\text{Sqrt}[a + b/x])/a) - (2*d^2*(a + b/x)^{(3/2)})/(3*b) + (c^2*(a + b/x)^{(3/2)}*x)/a + (c*(b*c + 4*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/\text{Sqrt}[a]$

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 80

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

#### Rule 89

Int[((a\_.) + (b\_.)\*(x\_))^2\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[((b\*c - a\*d)^2\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d^2\*(d\*e - c\*f)\*(n + 1)), x] - Dist[1/(d^2\*(d\*e - c\*f)\*(n + 1)), Int[(c + d\*x)^(n + 1)\*(e + f\*x)^p\*Simp[a^2\*d^2\*f\*(n + p + 2) + b^2\*c\*(d\*e\*(n + 1)

```
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx} (c + dx)^2}{x^2} dx, x, \frac{1}{x} \right) \\ &= \frac{c^2 \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{\text{Subst} \left( \int \frac{\sqrt{a + bx} \left(\frac{1}{2}c(bc + 4ad) + ad^2x\right)}{x} dx, x, \frac{1}{x} \right)}{a} \\ &= -\frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{(c(bc + 4ad)) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right)}{2a} \\ &= -\frac{c(bc + 4ad)\sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{1}{2}(c(bc + 4ad)) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right) \\ &= -\frac{c(bc + 4ad)\sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{3/2}}{a} - \frac{(c(bc + 4ad)) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right)}{b} \\ &= -\frac{c(bc + 4ad)\sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{3/2}}{3b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{3/2}}{a} + \frac{c(bc + 4ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 84, normalized size = 0.85

$$\frac{\sqrt{a + \frac{b}{x}} \left(b(3c^2x^2 - 12cdx - 2d^2) - 2ad^2x\right)}{3bx} + \frac{c(4ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x)^2, x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2*x + b*(-2*d^2 - 12*c*d*x + 3*c^2*x^2)))/(3*b*x) +
(c*(b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

**fricas** [A] time = 1.01, size = 208, normalized size = 2.10

$$\frac{3(b^2c^2 + 4abcd)\sqrt{a}x \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(3abc^2x^2 - 2abd^2 - 2(6abcd + a^2d^2)x)\sqrt{\frac{ax+b}{x}}}{6abx}, \quad 3(b^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(a)*x*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x), -1/3*(3*(b^2*c^2 + 4*a*b*c*d)*sqrt(-a)*x*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*a*b*c^2*x^2 - 2*a*b*d^2 - 2*(6*a*b*c*d + a^2*d^2)*x)*sqrt((a*x + b)/x))/(a*b*x)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2*(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2
,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2
,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%
}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%
}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1
,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[
2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at param
eters values [-89,63,-49]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+
%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [61.7937478349
,-30,70]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%
},0,%%{1,[0,2,2]%%}] at parameters values [73.519035968,-9,-13]Warning, c
hoosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%
}] at parameters values [18,15.451549686,-33]Sign error (%%{-b,0%%}+%%
{2*sqrt(a)*sqrt(b),1/2%%}+%%{-2*a,1%%}+%%{a*sqrt(a)*sqrt(b)/b,3/2%%}+%%
{-a^2*sqrt(a)*sqrt(b)/(4*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order
reached or unable to make series expansion Error: Bad Argument Value
```

**maple** [B] time = 0.06, size = 191, normalized size = 1.93

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -12abcdx^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 3b^2c^2x^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 24\sqrt{ax^2+bx}a^{\frac{3}{2}}cdx^3 - 6\sqrt{ax^2+bx} \right)}{6\sqrt{(ax+b)x}\sqrt{a}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)^2*(a+b/x)^(1/2),x)
```

```
[Out] -1/6*((a*x+b)/x)^(1/2)/x^2*(-24*(a*x^2+b*x)^(1/2)*a^(3/2)*x^3*c*d-6*(a*x^2+b*x)^(1/2)*a^(1/2)*x^3*b*c^2-12*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2)))/a^(1/2))*x^3*a*b*c*d-3*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2)))/a^(1/2)
```



2)) \* x^3 \* b^2 \* c^2 + 24 \* (a \* x^2 + b \* x)^(3/2) \* a^(1/2) \* x \* c \* d + 4 \* d^2 \* (a \* x^2 + b \* x)^(3/2) \* a^(1/2) / ((a \* x + b) \* x)^(1/2) / b / a^(1/2)

**maxima** [A] time = 1.19, size = 126, normalized size = 1.27

$$\frac{1}{2} \left( 2 \sqrt{a + \frac{b}{x}} x - \frac{b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{\sqrt{a}} \right) c^2 - 2 \left( \sqrt{a} \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) + 2 \sqrt{a + \frac{b}{x}} \right) c d - \frac{2 \left( a + \frac{b}{x} \right)^{\frac{3}{2}} d^2}{3 b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2\*(a+b/x)^(1/2),x, algorithm="maxima")

[Out] 1/2\*(2\*sqrt(a + b/x)\*x - b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a))\*c^2 - 2\*(sqrt(a)\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2\*sqrt(a + b/x))\*c\*d - 2/3\*(a + b/x)^(3/2)\*d^2/b

**mupad** [B] time = 1.90, size = 99, normalized size = 1.00

$$\left( \frac{4 a d^2 - 4 b c d}{b} - \frac{4 a d^2}{b} \right) \sqrt{a + \frac{b}{x}} + c^2 x \sqrt{a + \frac{b}{x}} - \frac{2 d^2 \left( a + \frac{b}{x} \right)^{\frac{3}{2}}}{3 b} - \frac{c \operatorname{atan} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) (4 a d + b c)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)\*(c + d/x)^2,x)

[Out] ((4\*a\*d^2 - 4\*b\*c\*d)/b - (4\*a\*d^2)/b)\*(a + b/x)^(1/2) + c^2\*x\*(a + b/x)^(1/2) - (2\*d^2\*(a + b/x)^(3/2))/(3\*b) - (c\*atan(((a + b/x)^(1/2)\*1i)/a^(1/2))\*(4\*a\*d + b\*c)\*1i)/a^(1/2)

**sympy** [A] time = 36.58, size = 121, normalized size = 1.22

$$-\frac{4acd \operatorname{atan} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{-a}} \right)}{\sqrt{-a}} + \sqrt{b} c^2 \sqrt{x} \sqrt{\frac{ax}{b} + 1} - 4cd \sqrt{a + \frac{b}{x}} + d^2 \left( \begin{cases} -\frac{\sqrt{a}}{x} & \text{for } b = 0 \\ -\frac{2 \left( a + \frac{b}{x} \right)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{cases} \right) + \frac{bc^2 \operatorname{asinh} \left( \frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2\*(a+b/x)\*\*(1/2),x)

[Out] -4\*a\*c\*d\*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + sqrt(b)\*c\*\*2\*sqrt(x)\*sqrt(a\*x/b + 1) - 4\*c\*d\*sqrt(a + b/x) + d\*\*2\*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2\*(a + b/x)\*\*(3/2)/(3\*b), True)) + b\*c\*\*2\*asinh(sqrt(a)\*sqrt(x)/sqrt(b))/sqrt(a)

$$3.226 \quad \int \sqrt{a + \frac{b}{x}} \left( c + \frac{d}{x} \right) dx$$

**Optimal.** Leaf size=74

$$-\frac{\sqrt{a + \frac{b}{x}}(2ad + bc)}{a} + \frac{(2ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{cx\left(a + \frac{b}{x}\right)^{3/2}}{a}$$

[Out]  $c*(a+b/x)^{(3/2)*x/a+(2*a*d+b*c)*\text{arctanh}((a+b/x)^{(1/2)/a^{(1/2)})/a^{(1/2)}-(2*a*d+b*c)*(a+b/x)^{(1/2)/a}}$

**Rubi [A]** time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {375, 78, 50, 63, 208}

$$-\frac{\sqrt{a + \frac{b}{x}}(2ad + bc)}{a} + \frac{(2ad + bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}} + \frac{cx\left(a + \frac{b}{x}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*(c + d/x), x]

[Out]  $-(((b*c + 2*a*d)*\text{Sqrt}[a + b/x])/a) + (c*(a + b/x)^{(3/2)*x}/a) + ((b*c + 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/\text{Sqrt}[a]$

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[  
((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/  
(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b,  
c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ  
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n  
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[  
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +  
(d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ  
[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den  
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p  
\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(  
f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f  
\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x],  
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int  
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/  
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a + \frac{b}{x}} \left( c + \frac{d}{x} \right) dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx} (c + dx)}{x^2} dx, x, \frac{1}{x} \right) \\ &= \frac{c \left( a + \frac{b}{x} \right)^{3/2} x}{a} - \frac{\left( \frac{bc}{2} + ad \right) \text{Subst} \left( \int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x} \right)}{a} \\ &= -\frac{(bc + 2ad)\sqrt{a + \frac{b}{x}}}{a} + \frac{c \left( a + \frac{b}{x} \right)^{3/2} x}{a} - \frac{1}{2}(bc + 2ad) \text{Subst} \left( \int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\ &= -\frac{(bc + 2ad)\sqrt{a + \frac{b}{x}}}{a} + \frac{c \left( a + \frac{b}{x} \right)^{3/2} x}{a} - \frac{(bc + 2ad) \text{Subst} \left( \int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{b} \\ &= -\frac{(bc + 2ad)\sqrt{a + \frac{b}{x}}}{a} + \frac{c \left( a + \frac{b}{x} \right)^{3/2} x}{a} + \frac{(bc + 2ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 52, normalized size = 0.70

$$\sqrt{a + \frac{b}{x}} (cx - 2d) + \frac{(2ad + bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x]*(c + d/x), x]
```

```
[Out] Sqrt[a + b/x]*(-2*d + c*x) + ((b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]
```

**fricas [A]** time = 1.50, size = 128, normalized size = 1.73

$$\left[ \frac{(bc + 2ad)\sqrt{a} \log \left( 2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b \right) + 2(acx - 2ad)\sqrt{\frac{ax+b}{x}}}{2a}, -\frac{(bc + 2ad)\sqrt{-a} \arctan \left( \frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a} \right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)*(a+b/x)^(1/2), x, algorithm="fricas")
```

```
[Out] [1/2*((b*c + 2*a*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a, -((b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (a*c*x - 2*a*d)*sqrt((a*x + b)/x))/a]
```

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*(a+b/x)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes  
 constant sign by intervals (correct if the argument is real):Check [abs(x)]  
 Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2  
 ,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2  
 ,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86  
 ,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%  
 }+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%  
 }+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters v  
 alues [7,-27,26]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1  
 ,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[  
 2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at param  
 eters values [-89,63,-49]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+  
 %%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [61.7937478349  
 ,-30,70]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%  
 },0,%%{1,[0,2,2]%%}] at parameters values [73.519035968,-9,-13]Warning, c  
 hoosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%  
 }+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [18,15.451549686,-33]Sign error (%%{-b,0%%}+%%  
 {2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+  
 %%{-a^2\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order  
 reached or unable to make series expansion Error: Bad Argument Value

**maple [B]** time = 0.06, size = 163, normalized size = 2.20

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 2abd x^2 \ln \left( \frac{2ax+b+2\sqrt{ax^2+bx} \sqrt{a}}{2\sqrt{a}} \right) + b^2 c x^2 \ln \left( \frac{2ax+b+2\sqrt{ax^2+bx} \sqrt{a}}{2\sqrt{a}} \right) + 4\sqrt{ax^2+bx} a^{\frac{3}{2}} d x^2 + 2\sqrt{ax^2+bx} \sqrt{a} \right)}{2\sqrt{(ax+b)x} \sqrt{a} bx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)\*(a+b/x)^(1/2),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*(4\*a^(3/2)\*(a\*x^2+b\*x)^(1/2)\*x^2\*d+2\*a^(1/2)\*(a\*x^2+b\*x)^(1/2)\*x^2\*b\*c+2\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a\*b\*d+ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*b^2\*c-4\*a^(1/2)\*(a\*x^2+b\*x)^(3/2)\*d)/x/((a\*x+b)\*x)^(1/2)/b/a^(1/2)

**maxima [A]** time = 1.37, size = 106, normalized size = 1.43

$$\frac{1}{2} \left( 2 \sqrt{a + \frac{b}{x}} x - \frac{b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{\sqrt{a}} \right) c - \left( \sqrt{a} \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) + 2 \sqrt{a + \frac{b}{x}} \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*(a+b/x)^(1/2),x, algorithm="maxima")

[Out] 1/2\*(2\*sqrt(a + b/x)\*x - b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/sqrt(a))\*c - (sqrt(a)\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2\*sqrt(a + b/x))\*d

**mupad [B]** time = 1.96, size = 92, normalized size = 1.24

$$2\sqrt{a} d \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - 2d \sqrt{a + \frac{b}{x}} + c x \sqrt{ax^2 + bx} \sqrt{\frac{1}{x^2}} + \frac{bcx \ln \left( \frac{\frac{b}{2} + ax + \sqrt{a} \sqrt{ax^2 + bx}}{\sqrt{a}} \right) \sqrt{\frac{1}{x^2}}}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(1/2)*(c + d/x), x)`

[Out]  $2*a^{(1/2)}*d*atanh((a + b/x)^{(1/2)}/a^{(1/2)}) - 2*d*(a + b/x)^{(1/2)} + c*x*(b*x + a*x^2)^{(1/2)}*(1/x^2)^{(1/2)} + (b*c*x*\log((b/2 + a*x + a^{(1/2)}*(b*x + a*x^2)^{(1/2)})/a^{(1/2)})*(1/x^2)^{(1/2)})/(2*a^{(1/2)})$

**sympy [A]** time = 41.31, size = 87, normalized size = 1.18

$$-\frac{2ad \operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b}+1} - 2d\sqrt{a+\frac{b}{x}} + \frac{bc \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d/x)*(a+b/x)**(1/2), x)`

[Out]  $-2*a*d*atan(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} + \sqrt{b}*c*\sqrt{x}*\sqrt{a*x/b + 1} - 2*d*\sqrt{a + b/x} + b*c*asinh(\sqrt{a}*\sqrt{x}/\sqrt{b})/\sqrt{a}$

$$3.227 \quad \int \sqrt{a + \frac{b}{x}} dx$$

Optimal. Leaf size=39

$$x\sqrt{a + \frac{b}{x}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

[Out] b\*arctanh((a+b/x)^(1/2)/a^(1/2))/a^(1/2)+x\*(a+b/x)^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {242, 47, 63, 208}

$$x\sqrt{a + \frac{b}{x}} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x], x]

[Out] Sqrt[a + b/x]\*x + (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

#### Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

#### Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{b}{x}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx}}{x^2} dx, x, \frac{1}{x} \right) \\
&= \sqrt{a + \frac{b}{x}} x - \frac{1}{2} b \text{Subst} \left( \int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\
&= \sqrt{a + \frac{b}{x}} x - \text{Subst} \left( \int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right) \\
&= \sqrt{a + \frac{b}{x}} x + \frac{b \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 39, normalized size = 1.00

$$x\sqrt{a + \frac{b}{x}} + \frac{b \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x], x]

[Out] Sqrt[a + b/x]\*x + (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]

**fricas [A]** time = 1.08, size = 99, normalized size = 2.54

$$\left[ \frac{2ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}b \log(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b)}{2a}, \frac{ax\sqrt{\frac{ax+b}{x}} - \sqrt{-a}b \arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2), x, algorithm="fricas")

[Out] [1/2\*(2\*a\*x\*sqrt((a\*x + b)/x) + sqrt(a)\*b\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b))/a, (a\*x\*sqrt((a\*x + b)/x) - sqrt(-a)\*b\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a))/a]

**giac [B]** time = 0.20, size = 64, normalized size = 1.64

$$-\frac{b \log \left( \left| -2 \left( \sqrt{a} x - \sqrt{ax^2 + bx} \right) \sqrt{a} - b \right| \right) \text{sgn}(x)}{2\sqrt{a}} + \frac{b \log(|b|) \text{sgn}(x)}{2\sqrt{a}} + \sqrt{ax^2 + bx} \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2), x, algorithm="giac")

[Out] -1/2\*b\*log(abs(-2\*(sqrt(a)\*x - sqrt(a\*x^2 + b\*x))\*sqrt(a) - b))\*sgn(x)/sqrt(a) + 1/2\*b\*log(abs(b))\*sgn(x)/sqrt(a) + sqrt(a\*x^2 + b\*x)\*sgn(x)

**maple [B]** time = 0.05, size = 74, normalized size = 1.90

$$\frac{\sqrt{\frac{ax+b}{x}} \left( b \ln \left( \frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}} \right) + 2\sqrt{ax^2+bx}\sqrt{a} \right) x}{2\sqrt{(ax+b)x}\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x)^(1/2),x)`

[Out]  $\frac{1}{2} * \left( \frac{a*x+b}{x} \right)^{1/2} * x * \left( 2 * (a*x^2+b*x)^{1/2} * a^{1/2} + b * \ln \left( \frac{1}{2} * (2*a*x+b+2*(a*x^2+b*x)^{1/2} * a^{1/2}) / a^{1/2} \right) \right) / \left( (a*x+b)*x \right)^{1/2} / a^{1/2}$

**maxima** [A] time = 1.20, size = 50, normalized size = 1.28

$$\sqrt{a + \frac{b}{x}} x - \frac{b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)^(1/2),x, algorithm="maxima")`

[Out]  $\sqrt{a + b/x} * x - 1/2 * b * \log \left( \frac{\sqrt{a + b/x} - \sqrt{a}}{\sqrt{a + b/x} + \sqrt{a}} \right) / \sqrt{a}$

**mupad** [B] time = 0.08, size = 58, normalized size = 1.49

$$x \sqrt{a x^2 + b x} \sqrt{\frac{1}{x^2}} + \frac{b x \ln \left( \frac{\frac{b}{2} + a x + \sqrt{a} \sqrt{a x^2 + b x}}{\sqrt{a}} \right) \sqrt{\frac{1}{x^2}}}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(1/2),x)`

[Out]  $x * (b*x + a*x^2)^{1/2} * (1/x^2)^{1/2} + (b*x * \log \left( \frac{b/2 + a*x + a^{1/2} * (b*x + a*x^2)^{1/2}}{a^{1/2}} \right) * (1/x^2)^{1/2}) / (2 * a^{1/2})$

**sympy** [A] time = 2.19, size = 42, normalized size = 1.08

$$\sqrt{b} \sqrt{x} \sqrt{\frac{ax}{b} + 1} + \frac{b \operatorname{asinh} \left( \frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(1/2),x)`

[Out]  $\sqrt{b} * \sqrt{x} * \sqrt{a*x/b + 1} + b * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / \sqrt{a}$



$$3.228 \quad \int \frac{\sqrt{a + \frac{b}{x}}}{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=104

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2} + \frac{x\sqrt{a+\frac{b}{x}}}{c}$$

[Out]  $(-2*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}/a^{(1/2)}}{c^2/a^{(1/2)}+2*\operatorname{arctan}(d^{(1/2)}*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)})}d^{(1/2)}*(-a*d+b*c)^{(1/2)}/c^2+x*(a+b/x)^{(1/2)}/c}\right)$

**Rubi [A]** time = 0.11, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 99, 156, 63, 208, 205}

$$\frac{2\sqrt{d}\sqrt{bc-ad}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2} + \frac{(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^2} + \frac{x\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x), x]

[Out]  $(\operatorname{Sqrt}[a + b/x]*x)/c + (2*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[b*c - a*d]*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/(\operatorname{Sqrt}[b*c - a*d])])/c^2 + ((b*c - 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*c^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 99

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*e - a\*f)), x] - Dist[1/((m + 1)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[d\*e\*n + c\*f\*(m + p + 2) + d\*f\*(m + n + p + 2)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 156

Int[(((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] :> Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_ )^2)^{-1}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 375

$\text{Int}[(a_ + (b_ \cdot)(x_ )^{(n_ )})^{(p_ )} \cdot ((c_ ) + (d_ \cdot)(x_ )^{(n_ )})^{(q_ )}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x^n)^p \cdot (c + d/x^n)^q / x^2, x], x, 1/x] /; \text{FreeQ}[\{a, b, c, d, p, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{ILtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + \frac{b}{x}}}{c + \frac{d}{x}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx}}{x^2(c + dx)} dx, x, \frac{1}{x} \right) \\ &= \frac{\sqrt{a + \frac{b}{x}}}{c} - \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(bc - 2ad) - \frac{bdx}{2}}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x} \right)}{c} \\ &= \frac{\sqrt{a + \frac{b}{x}}}{c} - \frac{(bc - 2ad) \text{Subst} \left( \int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x} \right)}{2c^2} + \frac{(d(bc - ad)) \text{Subst} \left( \int \frac{1}{\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x} \right)}{c^2} \\ &= \frac{\sqrt{a + \frac{b}{x}}}{c} - \frac{(bc - 2ad) \text{Subst} \left( \int \frac{1}{\frac{a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{bc^2} + \frac{(2d(bc - ad)) \text{Subst} \left( \int \frac{1}{c - \frac{ad}{b} + \frac{dx^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{bc^2} \\ &= \frac{\sqrt{a + \frac{b}{x}}}{c} + \frac{2\sqrt{d} \sqrt{bc - ad} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{c^2} + \frac{(bc - 2ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a} c^2} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 100, normalized size = 0.96

$$\frac{2\sqrt{d} \sqrt{bc - ad} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right) + \frac{(bc - 2ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} + cx \sqrt{a + \frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x), x]

[Out] (c\*Sqrt[a + b/x]\*x + 2\*Sqrt[d]\*Sqrt[b\*c - a\*d]\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]] + ((b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^2

**fricas [A]** time = 0.97, size = 482, normalized size = 4.63

$$\left[ \frac{2acx\sqrt{\frac{ax+b}{x}} - (bc - 2ad)\sqrt{a} \log \left( 2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b \right) + 2\sqrt{-bcd + ad^2} a \log \left( \frac{bd - (bc - 2ad)x + 2\sqrt{-bcd + ad^2}x\sqrt{\frac{ax+b}{x}}}{cx+d} \right)}{2ac^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x), x, algorithm="fricas")

```
[Out] [1/2*(2*a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d))/(a*c^2), 1/2*(2*a*c*x*sqrt((a*x + b)/x) - 4*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + sqrt(-b*c*d + a*d^2)*a*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2)*x*sqrt((a*x + b)/x))/(c*x + d))/(a*c^2), (a*c*x*sqrt((a*x + b)/x) - 2*sqrt(b*c*d - a*d^2)*a*arctan(sqrt(b*c*d - a*d^2)*x*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b*c - 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a*c^2)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

**maple** [B] time = 0.11, size = 287, normalized size = 2.76

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 2a^3 d^2 \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)x} c}{cx+d} \right) - 2\sqrt{a} bcd \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)x} c}{cx+d} \right) + 2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)x} \right)}{2\sqrt{(ax+b)x} \sqrt{\frac{ad-bc}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*a*c*d-ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*(d*(a*d-b*c)/c^2)^(1/2)*b*c^2+2*ln((2*(d*(a*d-b*c)/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(3/2)*d^2-2*ln((2*(d*(a*d-b*c)/c^2)^(1/2)*((a*x+b)*x)^(1/2)*c-2*a*d*x+b*c*x-b*d)/(c*x+d))*a^(1/2)*b*c*d-2*((a*x+b)*x)^(1/2)*c^2*a^(1/2)*(d*(a*d-b*c)/c^2)^(1/2))/((a*x+b)*x)^(1/2)/c^3/a^(1/2)/(d*(a*d-b*c)/c^2)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)/(c + d/x), x)
```

**mupad** [B] time = 1.63, size = 149, normalized size = 1.43

$$\frac{x \sqrt{a + \frac{b}{x}}}{c} + \frac{\ln \left( \sqrt{a + \frac{b}{x}} - \sqrt{a} \right) \left( ad - \frac{bc}{2} \right)}{\sqrt{a} c^2} - \frac{\ln \left( \sqrt{a + \frac{b}{x}} + \sqrt{a} \right) (2ad - bc)}{2\sqrt{a} c^2} - \frac{\operatorname{atan} \left( \frac{b^4 d^3 \sqrt{a + \frac{b}{x}} \sqrt{ad^2 - bcd} 4i}{4ab^4 d^4 - 4b^5 c d^3} \right) \sqrt{a}}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(1/2)/(c + d/x),x)`

[Out]  $(x*(a + b/x)^{(1/2)})/c - (\operatorname{atan}((b^4*d^3*(a + b/x)^{(1/2)}*(a*d^2 - b*c*d)^{(1/2)})*4i)/(4*a*b^4*d^4 - 4*b^5*c*d^3)*(a*d^2 - b*c*d)^{(1/2)}*2i)/c^2 + (\log((a + b/x)^{(1/2)} - a^{(1/2)})*(a*d - (b*c)/2))/(a^{(1/2)}*c^2) - (\log((a + b/x)^{(1/2)} + a^{(1/2)})*(2*a*d - b*c))/(2*a^{(1/2)}*c^2)$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{a + \frac{b}{x}}}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(1/2)/(c+d/x),x)`

[Out] `Integral(x*sqrt(a + b/x)/(c*x + d), x)`

$$3.229 \quad \int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=147

$$\frac{\sqrt{d}(3bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{bc - ad}} + \frac{(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3} + \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{x\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

[Out]  $(-4*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}/a^{(1/2)}}{c^3/a^{(1/2)}+(-4*a*d+3*b*c)*\operatorname{arctan}(d^{(1/2)}*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)}*d^{(1/2)}/c^3/(-a*d+b*c)^{(1/2)}+2*d*(a+b/x)^{(1/2)}/c^2/(c+d/x)+x*(a+b/x)^{(1/2)}/c/(c+d/x)}\right)$

**Rubi [A]** time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{d}(3bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{bc - ad}} + \frac{(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3} + \frac{x\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x)^2, x]

[Out]  $(2*d*\operatorname{Sqrt}[a + b/x])/(c^2*(c + d/x)) + (\operatorname{Sqrt}[a + b/x]*x)/(c*(c + d/x)) + (\operatorname{Sqrt}[d]*(3*b*c - 4*a*d)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(c^3*\operatorname{Sqrt}[b*c - a*d]) + ((b*c - 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*c^3)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 99

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*e - a\*f)), x] - Dist[1/((m + 1)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[d\*e\*n + c\*f\*(m + p + 2) + d\*f\*(m + n + p + 2)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Simp[(b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x]

, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 156

Int[(((e\_.) + (f\_.)\*(x\_))^(p\_.)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^2} dx = -\text{Subst}\left(\int \frac{\sqrt{a + bx}}{x^2(c + dx)^2} dx, x, \frac{1}{x}\right)$$

$$= \frac{\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(bc - 4ad) - \frac{3bdx}{2}}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x}\right)}{c}$$

$$= \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}(bc - 4ad)(bc - ad) + bd(bc - ad)x}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{c^2(bc - ad)}$$

$$= \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{(bc - 4ad)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^3} + \frac{(d(3bc - 4ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^3}$$

$$= \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{(bc - 4ad)\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^3} + \frac{(d(3bc - 4ad))\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{bc^3}$$

$$= \frac{2d\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} + \frac{\sqrt{d}(3bc - 4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{bc - ad}} + \frac{(bc - 4ad)\tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^3}$$

**Mathematica [A]** time = 0.39, size = 122, normalized size = 0.83

$$\frac{cx\sqrt{a+\frac{b}{x}}(cx+2d)}{cx+d} + \frac{\sqrt{d}(3bc-4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{\sqrt{bc-ad}} + \frac{(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x)^2,x]

[Out] ((c\*Sqrt[a + b/x]\*x\*(2\*d + c\*x))/(d + c\*x) + (Sqrt[d]\*(3\*b\*c - 4\*a\*d)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/Sqrt[b\*c - a\*d] + ((b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a])/c^3

**fricas [A]** time = 1.36, size = 801, normalized size = 5.45

$$\frac{(bcd - 4ad^2 + (bc^2 - 4acd)x)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + (3abcd - 4a^2d^2 + (3abc^2 - 4a^2cd)x)\sqrt{a}}{2(ac^4x + ac^3d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="fricas")

[Out] [-1/2\*((b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + (3\*a\*b\*c\*d - 4\*a^2\*d^2 + (3\*a\*b\*c^2 - 4\*a^2\*c\*d)\*x)\*sqrt(-d/(b\*c - a\*d))\*log(-(2\*(b\*c - a\*d)\*x\*sqrt(-d/(b\*c - a\*d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) - 2\*(a\*c^2\*x^2 + 2\*a\*c\*d\*x)\*sqrt((a\*x + b)/x))/(a\*c^4\*x + a\*c^3\*d), -1/2\*(2\*(b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (3\*a\*b\*c\*d - 4\*a^2\*d^2 + (3\*a\*b\*c^2 - 4\*a^2\*c\*d)\*x)\*sqrt(-d/(b\*c - a\*d))\*log(-(2\*(b\*c - a\*d)\*x\*sqrt(-d/(b\*c - a\*d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) - 2\*(a\*c^2\*x^2 + 2\*a\*c\*d\*x)\*sqrt((a\*x + b)/x))/(a\*c^4\*x + a\*c^3\*d), 1/2\*(2\*(3\*a\*b\*c\*d - 4\*a^2\*d^2 + (3\*a\*b\*c^2 - 4\*a^2\*c\*d)\*x)\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*x\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)/(a\*d\*x + b\*d)) - (b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(a\*c^2\*x^2 + 2\*a\*c\*d\*x)\*sqrt((a\*x + b)/x))/(a\*c^4\*x + a\*c^3\*d), ((3\*a\*b\*c\*d - 4\*a^2\*d^2 + (3\*a\*b\*c^2 - 4\*a^2\*c\*d)\*x)\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*x\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)/(a\*d\*x + b\*d)) - (b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (a\*c^2\*x^2 + 2\*a\*c\*d\*x)\*sqrt((a\*x + b)/x))/(a\*c^4\*x + a\*c^3\*d)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Unable to divide, perhaps due to rounding error%%{[-2,0]:[1,0,%%{-1,[1]%%}]%%}, [4,6,4,0]%%}+%%{%%{8,[1]%%}, [3,5,4,1]%%}+%%{%%{[-4,0]:[1,0,%%{-1,[1]%%}]%%}, [2,5,5,1]%%}+%%{%%{[-8,[1]%%},0]:[1,0,%%{-1,[1]%%}]%%}, [2,4,4,2]%%}+%%{%%{8,[1]%%}, [1,4,5,2]%%}+%%{%%{[-2,0]:[1,0,%%

```
{-1, [1]]}, [0, 4, 6, 2] / {1, [1]}, [4, 2, 0, 0]} + {poly1[-4, [1], 0]: [1, 0, -1, [1]], [3, 1, 0, 1]} + {2, [1], [2, 1, 1, 1]} + {4, [2], [2, 0, 0, 2]} + {poly1[-4, [1], 0]: [1, 0, -1, [1]], [1, 0, 1, 2]} + {1, [1], [0, 0, 2, 2]}
Error: Bad Argument Value
```

**maple [B]** time = 0.07, size = 943, normalized size = 6.41

$$\sqrt{\frac{ax+b}{x}} \left( 4a^{\frac{7}{2}} c d^3 x \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}} \sqrt{(ax+b)xc}}{cx+d} \right) - 7a^{\frac{5}{2}} b c^2 d^2 x \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}} \sqrt{(ax+b)xc}}{cx+d} \right) + 3a^{\frac{3}{2}} b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(1/2)/(c+d/x)^2,x)

[Out]  $-1/2*((a*x+b)/x)^{(1/2)}*x*(4*a^{(7/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*x*c*d^3+2*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*c^4+4*a^{(7/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*d^4-2*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*c^3*d-7*a^{(5/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*x*b*c^2*d^2-4*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*c^2*d^2-7*a^{(5/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*b*c*d^3-2*c^4*((a*x+b)*x)^{(3/2)}*a^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}+4*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b*c^4+3*a^{(3/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*x*b^2*c^3*d+4*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b*c^3*d+3*a^{(3/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*b^2*c^2*d^2+4*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*c^2*d^2-5*a^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b*c^3*d+a*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^2*c^4+4*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*c*d^3-5*a^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b*c^2*d^2+a*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^2*c^3*d)/c^4/((a*x+b)*x)^{(1/2)}/(a*d-b*c)/(c*x+d)/a^{(3/2)}/((a*d-b*c)/c^2*d)^{(1/2)}$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^2,x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x)/(c + d/x)^2, x)

**mupad [B]** time = 2.26, size = 1195, normalized size = 8.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)/(c + d/x)^2,x)



```
[Out] - ((2*b*d*(a + b/x)^(3/2))/c^2 - (b*(a + b/x)^(1/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d - b*c) - d*(a + b/x)^2 - a^2*d + a*b*c) - (atanh((8*b^5*d^3*(a + b/x)^(1/2))/(a^(1/2)*(8*b^5*d^3 - (2*b^6*c*d^2)/a)) + (2*b^6*d^2*(a + b/x)^(1/2))/(a^(3/2)*((2*b^6*d^2)/a - (8*b^5*d^3)/c)))*(4*a*d - b*c))/(a^(1/2)*c^3) - (atan((((d*(a*d - b*c))^(1/2))*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4)))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2))*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c)*1i)/(2*(b*c^4 - a*c^3*d)))/((4*(16*a^2*b^3*d^5 + 3*b^5*c^2*d^3 - 16*a*b^4*c*d^4))/c^6 - ((d*(a*d - b*c))^(1/2))*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 - (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 - (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)) + ((d*(a*d - b*c))^(1/2))*((4*(a + b/x)^(1/2)*(16*a^2*b^2*d^5 + 5*b^4*c^2*d^3 - 16*a*b^3*c*d^4))/c^4 + (((2*(2*b^4*c^7*d^2 - 4*a*b^3*c^6*d^3))/c^6 + (2*(2*b^3*c^7*d^2 - 4*a*b^2*c^6*d^3)*(a + b/x)^(1/2)*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(c^4*(b*c^4 - a*c^3*d)))*(d*(a*d - b*c))^(1/2)*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*(4*a*d - 3*b*c))/(2*(b*c^4 - a*c^3*d)))*1i)/(b*c^4 - a*c^3*d)
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{a + \frac{b}{x}}}{(cx + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**2,x)
```

```
[Out] Integral(x**2*sqrt(a + b/x)/(c*x + d)**2, x)
```

$$3.230 \quad \int \frac{\sqrt{a+\frac{b}{x}}}{\left(c+\frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=213

$$\frac{\sqrt{d} (24a^2d^2 - 40abcd + 15b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{(bc-6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4} + \frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)}$$

[Out]  $(-6*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{1/2}/a^{1/2}}{c^4/a^{1/2}+1/4*(24*a^2*d^2-40*a*b*c*d+15*b^2*c^2)*\operatorname{arctan}(d^{1/2}*(a+b/x)^{1/2}/(-a*d+b*c)^{1/2})}*d^{1/2}/c^4/(-a*d+b*c)^{3/2}+3/2*d*(a+b/x)^{1/2}/c^2/(c+d/x)^2+1/4*d*(-12*a*d+11*b*c)*(a+b/x)^{1/2}/c^3/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^{1/2}/c/(c+d/x)^2\right)$

**Rubi [A]** time = 0.34, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 99, 151, 156, 63, 208, 205}

$$\frac{\sqrt{d} (24a^2d^2 - 40abcd + 15b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc-ad)^{3/2}} + \frac{d\sqrt{a+\frac{b}{x}}(11bc-12ad)}{4c^3\left(c+\frac{d}{x}\right)(bc-ad)} + \frac{3d\sqrt{a+\frac{b}{x}}}{2c^2\left(c+\frac{d}{x}\right)^2} + \frac{(bc-6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}c^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x)^3,x]

[Out]  $(3*d*\operatorname{Sqrt}[a + b/x])/(2*c^2*(c + d/x)^2) + (d*(11*b*c - 12*a*d)*\operatorname{Sqrt}[a + b/x])/(4*c^3*(b*c - a*d)*(c + d/x)) + (\operatorname{Sqrt}[a + b/x]*x)/(c*(c + d/x)^2) + (\operatorname{Sqrt}[d]*(15*b^2*c^2 - 40*a*b*c*d + 24*a^2*d^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{3/2}) + ((b*c - 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(\operatorname{Sqrt}[a]*c^4)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 99

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*e - a\*f)), x] - Dist[1/((m + 1)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[d\*e\*n + c\*f\*(m + p + 2) + d\*f\*(m + n + p + 2)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

### Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:= -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx}}{x^2(c + dx)^3} dx, x, \frac{1}{x} \right) \\
&= \frac{\sqrt{a + \frac{b}{x}}}{c \left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(bc - 6ad) - \frac{5bdx}{2}}{x\sqrt{a+bx}(c+dx)^3} dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{3d\sqrt{a + \frac{b}{x}}}{2c^2 \left(c + \frac{d}{x}\right)^2} + \frac{\sqrt{a + \frac{b}{x}}}{c \left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst} \left( \int \frac{-(bc - 6ad)(bc - ad) + \frac{9}{2}bd(bc - ad)x}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x} \right)}{2c^2(bc - ad)} \\
&= \frac{3d\sqrt{a + \frac{b}{x}}}{2c^2 \left(c + \frac{d}{x}\right)^2} + \frac{d(11bc - 12ad)\sqrt{a + \frac{b}{x}}}{4c^3(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c \left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst} \left( \int \frac{(bc - 6ad)(bc - ad)^2 - \frac{1}{4}bd(11bc - 12ad)(bc - ad)}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x} \right)}{2c^3(bc - ad)^2} \\
&= \frac{3d\sqrt{a + \frac{b}{x}}}{2c^2 \left(c + \frac{d}{x}\right)^2} + \frac{d(11bc - 12ad)\sqrt{a + \frac{b}{x}}}{4c^3(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c \left(c + \frac{d}{x}\right)^2} - \frac{(bc - 6ad) \text{Subst} \left( \int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x} \right)}{2c^4} + \frac{(bc - 6ad) \text{Subst} \left( \int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{bc^4} \\
&= \frac{3d\sqrt{a + \frac{b}{x}}}{2c^2 \left(c + \frac{d}{x}\right)^2} + \frac{d(11bc - 12ad)\sqrt{a + \frac{b}{x}}}{4c^3(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{c \left(c + \frac{d}{x}\right)^2} + \frac{\sqrt{d} (15b^2c^2 - 40abcd + 24a^2d^2) \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{a + \frac{b}{x}}} \right)}{4c^4(bc - ad)^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.82, size = 330, normalized size = 1.55

$$(cx + d) \left( \frac{1}{2} cd^{5/2} \sqrt{a + \frac{b}{x}} (ax + b) (12a^2d^2 - 17abcd + 4b^2c^2) + (cx + d) \left( -\frac{1}{2} ad^2 (24a^2d^2 - 40abcd + 15b^2c^2) \left( \sqrt{d} \sqrt{a + \frac{b}{x}} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x)^3,x]

[Out] (2\*c^3\*d^(3/2)\*(b\*c - a\*d)^2\*(a + b/x)^(3/2)\*x^3 + c^2\*d^(5/2)\*(2\*b\*c - 3\*a\*d)\*(b\*c - a\*d)\*Sqrt[a + b/x]\*x\*(b + a\*x) + (d + c\*x)\*((c\*d^(5/2)\*(4\*b^2\*c^2 - 17\*a\*b\*c\*d + 12\*a^2\*d^2)\*Sqrt[a + b/x]\*(b + a\*x))/2 + (d + c\*x)\*(-1/2\*(a\*d^2\*(15\*b^2\*c^2 - 40\*a\*b\*c\*d + 24\*a^2\*d^2)\*(Sqrt[d]\*Sqrt[a + b/x] - Sqrt[b\*c - a\*d])\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])) - d^(3/2)\*(b\*c - 6\*a\*d)\*(b\*c - a\*d)^2\*(2\*Sqrt[a + b/x] - 2\*Sqrt[a]\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])))/(2\*a\*c^4\*d^(3/2)\*(b\*c - a\*d)^2\*(d + c\*x)^2)

**fricas [B]** time = 2.25, size = 1749, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="fricas")

```
[Out] [-1/8*(4*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d +
6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)
*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (15*a*b^2*c^2*d^2 - 40*a^
2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x
^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*c -
a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d +
(b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3
*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b
)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d
- a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4 +
(15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d -
40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)
*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^2*c^2*d^2 -
7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(
b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x
*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18
*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(
a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*
c^5*d^2)*x), -1/8*(8*(b^2*c^2*d^2 - 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*
a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3
)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (15*a*b^2*c^2*d^2 - 40
*a^2*b*c*d^3 + 24*a^3*d^4 + (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)
*x^2 + 2*(15*a*b^2*c^3*d - 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(-d/(b*
c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*
d + (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*
c^3*d - 18*a^2*c^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x
+ b)/x))/(a*b*c^5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^
6*d - a^2*c^5*d^2)*x), 1/4*((15*a*b^2*c^2*d^2 - 40*a^2*b*c*d^3 + 24*a^3*d^4
+ (15*a*b^2*c^4 - 40*a^2*b*c^3*d + 24*a^3*c^2*d^2)*x^2 + 2*(15*a*b^2*c^3*d
- 40*a^2*b*c^2*d^2 + 24*a^3*c*d^3)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a
*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^2*c^2*d^2
- 7*a*b*c*d^3 + 6*a^2*d^4 + (b^2*c^4 - 7*a*b*c^3*d + 6*a^2*c^2*d^2)*x^2 +
2*(b^2*c^3*d - 7*a*b*c^2*d^2 + 6*a^2*c*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sq
rt((a*x + b)/x)/a) + (4*(a*b*c^4 - a^2*c^3*d)*x^3 + (17*a*b*c^3*d - 18*a^2*c
^2*d^2)*x^2 + (11*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*sqrt((a*x + b)/x))/(a*b*c^
5*d^2 - a^2*c^4*d^3 + (a*b*c^7 - a^2*c^6*d)*x^2 + 2*(a*b*c^6*d - a^2*c^5*d^
2)*x)]
```

**giac [B]** time = 0.41, size = 820, normalized size = 3.85

$$\frac{\left(15 \sqrt{a} b^2 c^2 d \arctan\left(\frac{\sqrt{a} d}{\sqrt{bcd-ad^2}}\right) - 40 a^{\frac{3}{2}} b c d^2 \arctan\left(\frac{\sqrt{a} d}{\sqrt{bcd-ad^2}}\right) + 24 a^{\frac{5}{2}} d^3 \arctan\left(\frac{\sqrt{a} d}{\sqrt{bcd-ad^2}}\right) - 2 \sqrt{bcd-ad^2} b\right)}{4 \left(\sqrt{bcd-ad^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] -1/4*(15*sqrt(a)*b^2*c^2*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 40*a^(3/
2)*b*c*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^3*arctan(sq
rt(a)*d/sqrt(b*c*d - a*d^2)) - 2*sqrt(b*c*d - a*d^2)*b^2*c^2*log(abs(b)) +
14*sqrt(b*c*d - a*d^2)*a*b*c*d*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d^2
*log(abs(b)) + 9*sqrt(b*c*d - a*d^2)*a*b*c*d - 10*sqrt(b*c*d - a*d^2)*a^2*d
^2)*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*b*c^5 - sqrt(b*c*d - a*d^2)*a^(3/2)
*c^4*d) - 1/4*(15*b^2*c^2*d*sgn(x) - 40*a*b*c*d^2*sgn(x) + 24*a^2*d^3*sgn(x
))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d
^2))/((b*c^5 - a*c^4*d)*sqrt(b*c*d - a*d^2)) + sqrt(a*x^2 + b*x)*sgn(x)/c^3
- 1/4*(9*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*d*sgn(x) - 32*(
```

$$\begin{aligned} & \sqrt{a}x - \sqrt{ax^2 + bx})^3 a^{3/2} b^2 c^2 d^2 \operatorname{sgn}(x) + 24(\sqrt{a}x - \sqrt{ax^2 + bx})^3 a^{5/2} c^2 d^3 \operatorname{sgn}(x) + 3(\sqrt{a}x - \sqrt{ax^2 + bx})^2 a^2 b^2 c^2 d^2 \operatorname{sgn}(x) - 40(\sqrt{a}x - \sqrt{ax^2 + bx})^2 a^2 b^2 c^2 d^2 \operatorname{sgn}(x) + 40(\sqrt{a}x - \sqrt{ax^2 + bx})^2 a^3 d^4 \operatorname{sgn}(x) + 7(\sqrt{a}x - \sqrt{ax^2 + bx}) \sqrt{a} b^3 c^2 d^2 \operatorname{sgn}(x) - 44(\sqrt{a}x - \sqrt{ax^2 + bx}) a^{3/2} b^2 c^2 d^3 \operatorname{sgn}(x) + 40(\sqrt{a}x - \sqrt{ax^2 + bx}) a^{5/2} b^2 d^4 \operatorname{sgn}(x) - 9 a^2 b^3 c^2 d^3 \operatorname{sgn}(x) + 10 a^2 b^2 d^4 \operatorname{sgn}(x) / ((\sqrt{a} b^2 c^5 - a^{3/2} c^4 d) ((\sqrt{a}x - \sqrt{ax^2 + bx})^2 c + 2(\sqrt{a}x - \sqrt{ax^2 + bx}) \sqrt{a} d + b d)^2) - 1/2 (b^2 c \operatorname{sgn}(x) - 6 a^2 d \operatorname{sgn}(x)) \log(\operatorname{abs}(2(\sqrt{a}x - \sqrt{ax^2 + bx}) \sqrt{a} + b)) / (\sqrt{a} c^4) \end{aligned}$$

**maple [B]** time = 0.07, size = 1972, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/x)^(1/2)/(c+d/x)^3,x)`

[Out] 
$$\begin{aligned} & -1/8((a*x+b)/x)^{1/2} * x * (55*a^{5/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x^2 * b^2 * c^4 * d^2 + 24*a^4 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * c^3 * d^3 - 36*a^{7/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x * c^3 * d^3 - 128*a^{7/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x * b^2 * c^4 + 110*a^{5/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x * b^2 * c^3 * d^3 + 48*a^4 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x * c^2 * d^4 + 46*a^{5/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * b^2 * c^3 * d^3 - 52*a^3 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * b^2 * c^2 * d^4 + 32*a^2 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * b^2 * c^3 * d^3 - 22*a^{3/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * b^2 * c^4 * d^2 - 4*a * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * b^3 * c^4 * d^2 + 14*a^{3/2} * ((a*x+b)*x)^{3/2} * ((a*d-b*c)/c^2*d)^{1/2} * x * b^2 * c^6 - 22*a^{3/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * b^2 * c^6 - 15*a^{3/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x^2 * b^3 * c^5 * d - 4*a * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * b^3 * c^6 + 10*a^{3/2} * ((a*x+b)*x)^{3/2} * ((a*d-b*c)/c^2*d)^{1/2} * b^2 * c^5 * d + 12*a^{7/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x^3 * c^5 * d - 14*a^{5/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x^3 * b^2 * c^6 - 12*a^{5/2} * ((a*x+b)*x)^{3/2} * ((a*d-b*c)/c^2*d)^{1/2} * x * c^5 * d - 64*a^{7/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x^2 * b^2 * c^3 * d^3 - 30*a^{3/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x * b^3 * c^4 * d^2 + 24*a^{9/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * d^6 + 64*a^2 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x * b^2 * c^4 * d^2 + 78*a^{5/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x * b^2 * c^4 * d^2 - 104*a^3 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x * b^2 * c^3 * d^3 + 18*a^{5/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * b^2 * c^5 * d - 52*a^3 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * b^2 * c^4 * d^2 + 32*a^2 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x^2 * b^2 * c^5 * d - 44*a^{3/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * x * b^2 * c^5 * d - 8*a * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * x * b^3 * c^5 * d + 48*a^{9/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x * c * d^5 - 24*a^{7/2} * ((a*x+b)*x)^{1/2} * ((a*d-b*c)/c^2*d)^{1/2} * c^2 * d^4 - 64*a^{7/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * b^2 * c^5 + 55*a^{5/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * b^2 * c^2 * d^4 + 24*a^4 * \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2} * a^{1/2}) / a^{1/2}) * ((a*d-b*c)/c^2*d)^{1/2} * c * d^5 + 24*a^{9/2} * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}) * ((a*x+b)*x)^{1/2} * c) / (c*x+d) * x^2 * c^2 * d^4 - 8*a^{5/2} * ((a*x+b)*x)^{3/2} * ((a$$

$$\frac{(d-bc)/c^2d)^{1/2}c^4d^2-15a^{3/2}\ln((-2ad*x+bc*x-bd+2((ad-bc)/c^2d)^{1/2}((a*x+b)*x)^{1/2}c)/(c*x+d))b^3c^3d^3/c^5/((a*x+b)*x)^{1/2}/(ad-bc)^2/(c*x+d)^2/a^{3/2}/((ad-bc)/c^2d)^{1/2}}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x)/(c + d/x)^3, x)

**mupad** [B] time = 3.73, size = 1895, normalized size = 8.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)/(c + d/x)^3,x)

[Out]  $(\log((a + b/x)^{1/2}*(d*(ad - bc)^3)^{1/2} - a^2d^2 - b^2c^2 + 2a*bc*d)*(d*(ad - bc)^3)^{1/2}*(3a^2d^2 + (15b^2c^2)/8 - 5a*bc*d))/(b^3c^7 - a^3c^4d^3 + 3a^2b*c^5d^2 - 3a*b^2c^6d) - ((b*(a + b/x)^{1/2}*(12a^2d^2 + 4b^2c^2 - 17a*bc*d))/(4c^3) + (b*(a + b/x)^{5/2}*(12ad^3 - 11b*c*d^2))/(4c^3*(ad - bc)) - (d*(a + b/x)^{3/2}*(17b^3c^2 + 24a^2b*d^2 - 40a*b^2c*d))/(4c^3*(ad - bc)))/((a + b/x)^2*(3a^2d^2 - 2b*c*d) - (a + b/x)*(3a^2d^2 + b^2c^2 - 4a*bc*d) - d^2*(a + b/x)^3 + a^3*d^2 + a*b^2c^2 - 2a^2b*c*d) - (\log((a + b/x)^{1/2}*(d*(ad - bc)^3)^{1/2} + a^2d^2 + b^2c^2 - 2a*bc*d)*(d*(ad - bc)^3)^{1/2}*(24a^2d^2 + 15b^2c^2 - 40a*bc*d))/(8*(b^3c^7 - a^3c^4d^3 + 3a^2b*c^5d^2 - 3a*b^2c^6d) - (\operatorname{atan}(\frac{((a + b/x)^{1/2}*(1152a^4b^2d^7 + 241b^6c^4d^3 - 1424a*b^5c^3d^4 - 3264a^3b^3c*d^6 + 3296a^2b^4c^2d^5))}{8*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d)} - ((6ad - bc)*((4b^6c^11d^2 - 21a*b^5c^10d^3 + 29a^2b^4c^9d^4 - 12a^3b^3c^8d^5))/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) - ((a + b/x)^{1/2}*(6ad - bc)*(64b^5c^11d^2 - 256a*b^4c^10d^3 + 320a^2b^3c^9d^4 - 128a^3b^2c^8d^5))/(16a^{1/2}*c^4*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d))))/(2a^{1/2}*c^4))*(6ad - bc)*1i)/(2a^{1/2}*c^4) + (((a + b/x)^{1/2}*(1152a^4b^2d^7 + 241b^6c^4d^3 - 1424a*b^5c^3d^4 - 3264a^3b^3c*d^6 + 3296a^2b^4c^2d^5))/(8*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d)) + ((6ad - bc)*((4b^6c^11d^2 - 21a*b^5c^10d^3 + 29a^2b^4c^9d^4 - 12a^3b^3c^8d^5))/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) + ((a + b/x)^{1/2}*(6ad - bc)*(64b^5c^11d^2 - 256a*b^4c^10d^3 + 320a^2b^3c^9d^4 - 128a^3b^2c^8d^5))/(16a^{1/2}*c^4*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d))))/(2a^{1/2}*c^4))*(6ad - bc))/((216a^4b^3d^7 + (165b^7c^4d^3)/8 - (805a*b^6c^3d^4)/4 - 594a^3b^4c*d^6 + 558a^2b^5c^2d^5)/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) - (((a + b/x)^{1/2}*(1152a^4b^2d^7 + 241b^6c^4d^3 - 1424a*b^5c^3d^4 - 3264a^3b^3c*d^6 + 3296a^2b^4c^2d^5))/(8*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d)) - ((6ad - bc)*((4b^6c^11d^2 - 21a*b^5c^10d^3 + 29a^2b^4c^9d^4 - 12a^3b^3c^8d^5))/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) - ((a + b/x)^{1/2}*(6ad - bc)*(64b^5c^11d^2 - 256a*b^4c^10d^3 + 320a^2b^3c^9d^4 - 128a^3b^2c^8d^5))/(16a^{1/2}*c^4*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d))))/(2a^{1/2}*c^4))*(6ad - bc))/((216a^4b^3d^7 + (165b^7c^4d^3)/8 - (805a*b^6c^3d^4)/4 - 594a^3b^4c*d^6 + 558a^2b^5c^2d^5)/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) - (((a + b/x)^{1/2}*(1152a^4b^2d^7 + 241b^6c^4d^3 - 1424a*b^5c^3d^4 - 3264a^3b^3c*d^6 + 3296a^2b^4c^2d^5))/(8*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d)) + ((6ad - bc)*((4b^6c^11d^2 - 21a*b^5c^10d^3 + 29a^2b^4c^9d^4 - 12a^3b^3c^8d^5))/(b^2c^11 + a^2c^9d^2 - 2a*bc^10*d) - ((a + b/x)^{1/2}*(6ad - bc)*(64b^5c^11d^2 - 256a*b^4c^10d^3 + 320a^2b^3c^9d^4 - 128a^3b^2c^8d^5))/(16a^{1/2}*c^4*(b^2c^8 + a^2c^6d^2 - 2a*bc^7*d))))/(2a^{1/2}*c^4))*(6ad - bc))$

$$\begin{aligned} & ^2*c^9*d^2 - 2*a*b*c^{10}*d) + ((a + b/x)^{(1/2)}*(6*a*d - b*c)*(64*b^5*c^{11}*d^2 \\ & - 256*a*b^4*c^{10}*d^3 + 320*a^2*b^3*c^9*d^4 - 128*a^3*b^2*c^8*d^5))/(16*a^{(1/2)}*c^4*(b^2*c^8 + a^2*c^6*d^2 - 2*a*b*c^7*d)))/ \\ & (2*a^{(1/2)}*c^4))*(6*a*d - b*c))/(2*a^{(1/2)}*c^4))*i)/(a^{(1/2)}*c^4) \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*3,x)

[Out] Timed out



$$3.231 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=164

$$\frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(\frac{3bd(2ad+19bc)}{x} + 2(13bc - ad)(2ad + 5bc)\right)}{35b^2} - 3c^2 \sqrt{a + \frac{b}{x}} (2ad+bc) + 3\sqrt{a} c^2 (2ad+bc) \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

[Out]  $-9/7*d*(a+b/x)^{(3/2)}*(c+d/x)^2 - 1/35*d*(a+b/x)^{(3/2)}*(2*(-a*d+13*b*c))*(2*a*d+5*b*c)+3*b*d*(2*a*d+19*b*c)/x/b^2+(a+b/x)^{(3/2)}*(c+d/x)^3*x+3*c^2*(2*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-3*c^2*(2*a*d+b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 97, 153, 147, 50, 63, 208}

$$\frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(\frac{3bd(2ad+19bc)}{x} + 2(13bc - ad)(2ad + 5bc)\right)}{35b^2} - 3c^2 \sqrt{a + \frac{b}{x}} (2ad+bc) + 3\sqrt{a} c^2 (2ad+bc) \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b/x)^{(3/2)}*(c + d/x)^3, x]$

[Out]  $-3*c^2*(b*c + 2*a*d)*\operatorname{Sqrt}[a + b/x] - (9*d*(a + b/x)^{(3/2)}*(c + d/x)^2)/7 - (d*(a + b/x)^{(3/2)}*(2*(13*b*c - a*d)*(5*b*c + 2*a*d) + (3*b*d*(19*b*c + 2*a*d))/x)/(35*b^2) + (a + b/x)^{(3/2)}*(c + d/x)^3*x + 3*\operatorname{Sqrt}[a]*c^2*(b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

#### Rule 50

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{m+1}*(c + d*x)^n/(b*(m + n + 1)), x] + \operatorname{Dist}[(n*(b*c - a*d))/(b*(m + n + 1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{n-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n, x\_Symbol] \rightarrow \operatorname{With}[p = \operatorname{Denominator}[m], \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p*(m+1)-1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 97

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{m+1}*(c + d*x)^n*(e + f*x)^p/(b*(m + 1)), x] - \operatorname{Dist}[1/(b*(m + 1)), \operatorname{Int}[(a + b*x)^{m+1}*(c + d*x)^{n-1}*(e + f*x)^{p-1}*\operatorname{Simp}[d*e*n + c*f*p + d*f*(n + p)*x, x], x]] /;$  FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

#### Rule 147

$\operatorname{Int}[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x\_Symbol] \rightarrow -\operatorname{Simp}[(a*d*f*h*(n + 2) + b*c*f*h*(m$

```
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rule 153

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 dx = -\text{Subst}\left(\int \frac{(a + bx)^{3/2}(c + dx)^3}{x^2} dx, x, \frac{1}{x}\right)$$

$$= \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x - \text{Subst}\left(\int \frac{\sqrt{a + bx}(c + dx)^2 \left(\frac{3}{2}(bc + 2ad) + \frac{9bdx}{2}\right)}{x} dx, x, \frac{1}{x}\right)$$

$$= -\frac{9}{7}d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 + \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x - \frac{2 \text{Subst}\left(\int \frac{\sqrt{a + bx}(c + dx) \left(\frac{21}{4}bc(bc + 2ad) + \frac{3bd(19bc + 2ad)}{x}\right)}{x} dx, x, \frac{1}{x}\right)}{7b}$$

$$= -\frac{9}{7}d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(2(13bc - ad)(5bc + 2ad) + \frac{3bd(19bc + 2ad)}{x}\right)}{35b^2} + \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x$$

$$= -3c^2(bc + 2ad)\sqrt{a + \frac{b}{x}} - \frac{9}{7}d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(2(13bc - ad)(5bc + 2ad) + \frac{3bd(19bc + 2ad)}{x}\right)}{35b^2} + \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x$$

$$= -3c^2(bc + 2ad)\sqrt{a + \frac{b}{x}} - \frac{9}{7}d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(2(13bc - ad)(5bc + 2ad) + \frac{3bd(19bc + 2ad)}{x}\right)}{35b^2} + \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x$$

$$= -3c^2(bc + 2ad)\sqrt{a + \frac{b}{x}} - \frac{9}{7}d \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{3/2} \left(2(13bc - ad)(5bc + 2ad) + \frac{3bd(19bc + 2ad)}{x}\right)}{35b^2} + \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3 x$$

**Mathematica [A]** time = 0.21, size = 159, normalized size = 0.97

$$\frac{\sqrt{a + \frac{b}{x}} \left( 4a^3 d^3 x^3 - 2a^2 b d^2 x^2 (21cx + d) + ab^2 x (35c^3 x^3 - 280c^2 dx^2 - 84cd^2 x - 16d^3) - 2b^3 (35c^3 x^3 + 35c^2 dx^2 + 35c^2 d^2 x + 35c^3 x^3) \right)}{35b^2 x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)\*(c + d/x)^3,x]

[Out] (Sqrt[a + b/x]\*(4\*a^3\*d^3\*x^3 - 2\*a^2\*b\*d^2\*x^2\*(d + 21\*c\*x) + a\*b^2\*x\*(-16\*d^3 - 84\*c\*d^2\*x - 280\*c^2\*d\*x^2 + 35\*c^3\*x^3) - 2\*b^3\*(5\*d^3 + 21\*c\*d^2\*x + 35\*c^2\*d\*x^2 + 35\*c^3\*x^3)))/(35\*b^2\*x^3) + 3\*Sqrt[a]\*c^2\*(b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**fricas [A]** time = 1.24, size = 380, normalized size = 2.32

$$\frac{105(b^3 c^3 + 2ab^2 c^2 d)\sqrt{a} x^3 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(35ab^2 c^3 x^4 - 10b^3 d^3 - 2(35b^3 c^3 + 140ab^2 c^2 d + 21a^2 b^2 c^2 d + 21a^2 b^2 c^2 d - 2a^3 d^3)x^3 - 2(35b^3 c^2 d + 42a^2 b^2 c^2 d + a^2 b^2 d^3)x^2 - 2(21b^3 c^2 d + 8a^2 b^2 d^3)x)\sqrt{(ax + b)/x}}{70b^2 x^3} + 3\sqrt{a}c^2(b c + 2a d)\operatorname{Arctanh}\left(\frac{\sqrt{ax + b}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^3,x, algorithm="fricas")

[Out] [1/70\*(105\*(b^3\*c^3 + 2\*a\*b^2\*c^2\*d)\*sqrt(a)\*x^3\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(35\*a\*b^2\*c^3\*x^4 - 10\*b^3\*d^3 - 2\*(35\*b^3\*c^3 + 140\*a\*b^2\*c^2\*d + 21\*a^2\*b^2\*c^2\*d - 2\*a^3\*d^3)\*x^3 - 2\*(35\*b^3\*c^2\*d + 42\*a\*b^2\*c^2\*d + a^2\*b^2\*d^3)\*x^2 - 2\*(21\*b^3\*c^2\*d + 8\*a\*b^2\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^3), -1/35\*(105\*(b^3\*c^3 + 2\*a\*b^2\*c^2\*d)\*sqrt(-a)\*x^3\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (35\*a\*b^2\*c^3\*x^4 - 10\*b^3\*d^3 - 2\*(35\*b^3\*c^3 + 140\*a\*b^2\*c^2\*d + 21\*a^2\*b^2\*c^2\*d - 2\*a^3\*d^3)\*x^3 - 2\*(35\*b^3\*c^2\*d + 42\*a\*b^2\*c^2\*d + a^2\*b^2\*d^3)\*x^2 - 2\*(21\*b^3\*c^2\*d + 8\*a\*b^2\*d^3)\*x)\*sqrt((a\*x + b)/x))/(b^2\*x^3)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2

\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}%+%%%{undef,7/2%%}%}Evaluation time: 0.8Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple [B]** time = 0.06, size = 353, normalized size = 2.15

$$\sqrt{\frac{ax+b}{x}} \left( 210a^2b^2c^2dx^5 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 105ab^3c^3x^5 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 420\sqrt{ax^2+bx} a^{\frac{5}{2}}b^2c^2dx^5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)\*(c+d/x)^3,x)

[Out] 1/70\*((a\*x+b)/x)^(1/2)\*(420\*(a\*x^2+b\*x)^(1/2)\*a^(5/2)\*x^5\*b\*c^2\*d+210\*(a\*x^2+b\*x)^(1/2)\*a^(3/2)\*x^5\*b^2\*c^3+8\*(a\*x^2+b\*x)^(3/2)\*a^(5/2)\*x^2\*d^3-420\*(a\*x^2+b\*x)^(3/2)\*a^(3/2)\*x^3\*b\*c^2\*d+210\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*x^5\*a^2\*b^2\*c^2\*d+105\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*x^5\*a\*b^3\*c^3-84\*(a\*x^2+b\*x)^(3/2)\*a^(3/2)\*x^2\*b\*c\*d^2-140\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*x^3\*b^2\*c^3-12\*(a\*x^2+b\*x)^(3/2)\*a^(3/2)\*x\*b\*d^3-140\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*x^2\*b^2\*c^2\*d-84\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*x\*b^2\*c\*d^2-20\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*b^2\*d^3)/x^4/b^2/((a\*x+b)\*x)^(1/2)/a^(1/2)

**maxima [A]** time = 1.49, size = 190, normalized size = 1.16

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{5}{2}}cd^2}{5b} + \frac{1}{2}\left(2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b \log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b\right)c^3 - \left(3a^{\frac{3}{2}} \log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) + 2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^3,x, algorithm="maxima")

[Out] -6/5\*(a + b/x)^(5/2)\*c\*d^2/b + 1/2\*(2\*sqrt(a + b/x)\*a\*x - 3\*sqrt(a)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))) - 4\*sqrt(a + b/x)\*b)\*c^3 - (3\*a^(3/2)\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2\*(a + b/x)^(3/2) + 6\*sqrt(a + b/x)\*a)\*c^2\*d - 2/35\*(5\*(a + b/x)^(7/2)/b^2 - 7\*(a + b/x)^(5/2)\*a/b^2)\*d^3

**mupad [B]** time = 3.88, size = 327, normalized size = 1.99

$$\left(a + \frac{b}{x}\right)^{5/2} \left(\frac{6ad^3 - 6bcd^2}{5b^2} - \frac{4ad^3}{5b^2}\right) + \sqrt{a + \frac{b}{x}} \left(\frac{2(ad - bc)^3}{b^2} + 2a \left(2a \left(\frac{6ad^3 - 6bcd^2}{b^2} - \frac{4ad^3}{b^2}\right) - \frac{6d(ad - bc)}{b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2)\*(c + d/x)^3,x)

[Out] (a + b/x)^(5/2)\*((6\*a\*d^3 - 6\*b\*c\*d^2)/(5\*b^2) - (4\*a\*d^3)/(5\*b^2)) + (a + b/x)^(1/2)\*((2\*(a\*d - b\*c)^3)/b^2 + 2\*a\*(2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2) - (6\*d\*(a\*d - b\*c)^2)/b^2 + (2\*a^2\*d^3)/b^2) - a^2\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2) + (a + b/x)^(3/2)\*((2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2))/3 - (2\*d\*(a\*d - b\*c)^2)/b^2 + (2\*a^2\*d^3)/(3\*b^2)) - (2\*d^3\*(a + b/x)^(7/2))/(7\*b^2) + a\*c^3\*x\*(a + b/x)^(1/2) - 2\*c^2\*atan((2\*c^2\*(a + b/x)^(1/2)\*(2\*a\*d + b\*c)\*(-9\*a)/4)^(1/2))/(6\*a^2\*c^2\*d + 3\*a\*b\*c^3)\*(2\*a\*d + b\*c)\*(-9\*a)/4)^(1/2)

sympy [A] time = 123.48, size = 1817, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)\*(c+d/x)\*\*3,x)

[Out] 
$$\begin{aligned} & -16*a^{19/2}*b^{11/2}*d^3*x^6*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & - 40*a^{17/2}*b^{13/2}*d^3*x^5*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & - 30*a^{15/2}*b^{15/2}*d^3*x^4*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & - 40*a^{13/2}*b^{17/2}*d^3*x^3*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 4*a^{13/2}*b^{3/2}*d^3*x^3*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) - 100*a^{11/2}*b^{19/2}*d^3*x^2*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 12*a^{11/2}*b^{5/2}*c*d^2*x^3*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) + 2*a^{11/2}*b^{5/2}*d^3*x^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & - 96*a^{9/2}*b^{21/2}*d^3*x*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 6*a^{9/2}*b^{7/2}*c*d^2*x^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) - 8*a^{9/2}*b^{7/2}*d^3*x*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & - 30*a^{7/2}*b^{23/2}*d^3*\sqrt{a*x/b + 1}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & - 24*a^{7/2}*b^{9/2}*c*d^2*x*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) - 6*a^{7/2}*b^{9/2}*d^3*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & - 18*a^{5/2}*b^{11/2}*c*d^2*\sqrt{a*x/b + 1}/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & + \sqrt{a}*b*c^3*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) + 16*a^{10}*b^5*d^3*x^{13/2}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 48*a^9*b^6*d^3*x^{11/2}/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 48*a^8*b^7*d^3*x^9/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & + 16*a^7*b^8*d^3*x^7/(105*a^{13/2}*b^7*x^{13/2} + 315*a^{11/2}*b^8*x^{11/2} + 315*a^{9/2}*b^9*x^9 + 105*a^{7/2}*b^{10}*x^7) \\ & - 4*a^7*b^8*d^3*x^7/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) - 12*a^6*b^2*c*d^2*x^7/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & - 4*a^6*b^2*d^3*x^5/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) - 12*a^5*b^3*c*d^2*x^5/(15*a^{7/2}*b^3*x^7 + 15*a^{5/2}*b^4*x^5) \\ & - 6*a^2*c^2*d*\operatorname{atan}(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} + a*\sqrt{b}*c^3*\sqrt{x}*\sqrt{a*x/b + 1} - 2*a*b*c^3*\operatorname{atan}(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} \\ & - 6*a*c^2*d*\sqrt{a + b/x} + 3*a*c*d^2*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True})) - 2*b*c^3*\sqrt{a + b/x} + 3*b*c^2*d*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True})) \end{aligned}$$

$$3.232 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=126

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{c \left(a + \frac{b}{x}\right)^{3/2} (4ad + 3bc)}{3a} - c \sqrt{a + \frac{b}{x}} (4ad + 3bc) + \sqrt{a} c (4ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b}$$

[Out]  $-1/3*c*(4*a*d+3*b*c)*(a+b/x)^{(3/2)}/a-2/5*d^2*(a+b/x)^{(5/2)}/b+c^2*(a+b/x)^{(5/2)*x}/a+c*(4*a*d+3*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-c*(4*a*d+3*b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 89, 80, 50, 63, 208}

$$\frac{c^2 x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{c \left(a + \frac{b}{x}\right)^{3/2} (4ad + 3bc)}{3a} - c \sqrt{a + \frac{b}{x}} (4ad + 3bc) + \sqrt{a} c (4ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b/x)^{(3/2)}*(c + d/x)^2, x]$

[Out]  $-(c*(3*b*c + 4*a*d)*\operatorname{Sqrt}[a + b/x]) - (c*(3*b*c + 4*a*d)*(a + b/x)^{(3/2)})/(3*a) - (2*d^2*(a + b/x)^{(5/2)})/(5*b) + (c^2*(a + b/x)^{(5/2)*x})/a + \operatorname{Sqrt}[a]*c*(3*b*c + 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

#### Rule 50

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^n/(b*(m + n + 1)), x] + \operatorname{Dist}[(n*(b*c - a*d))/(b*(m + n + 1)), \operatorname{Int}[(a + b*x)^m*(c + d*x)^{(n - 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{GtQ}[n, 0] \&\& \operatorname{NeQ}[m + n + 1, 0] \&\& !( \operatorname{IGtQ}[m, 0] \&\& ( !\operatorname{IntegerQ}[n] || (\operatorname{GtQ}[m, 0] \&\& \operatorname{LtQ}[m - n, 0]) ) ) \&\& !\operatorname{ILtQ}[m + n + 2, 0] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

#### Rule 80

$\operatorname{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/(d*f*(n + p + 2)), x] + \operatorname{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^p, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \operatorname{NeQ}[n + p + 2, 0]$

#### Rule 89

$\operatorname{Int}[(a_. + (b_.)*(x_.))^2*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(b*c - a*d)^2*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)})/(d^2*(d*e - c*f)*(n + 1)), x] - \operatorname{Dist}[1/(d^2*(d*e - c*f)*(n + 1)), \operatorname{Int}[(c$

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned} \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}(c+dx)^2}{x^2} dx, x, \frac{1}{x}\right) \\ &= \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2} \left(\frac{1}{2}c(3bc+4ad)+ad^2x\right)}{x} dx, x, \frac{1}{x}\right)}{a} \\ &= -\frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{(c(3bc+4ad)) \text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x}\right)}{2a} \\ &= -\frac{c(3bc+4ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{1}{2}(c(3bc+4ad)) \text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x}\right) \\ &= -c(3bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{c(3bc+4ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} \\ &= -c(3bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{c(3bc+4ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} \\ &= -c(3bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{c(3bc+4ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{5/2}}{a} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 106, normalized size = 0.84

$$\frac{c(4ad+3bc) \left( \sqrt{a + \frac{b}{x}} (4ax+b) - 3a^{3/2}x \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) \right)}{3ax} + \frac{c^2x \left(a + \frac{b}{x}\right)^{5/2}}{a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{5/2}}{5b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(3/2)*(c + d/x)^2,x]
```

```
[Out] (-2*d^2*(a + b/x)^(5/2))/(5*b) + (c^2*(a + b/x)^(5/2)*x)/a - (c*(3*b*c + 4*
a*d)*(Sqrt[a + b/x]*(b + 4*a*x) - 3*a^(3/2)*x*ArcTanh[Sqrt[a + b/x]/Sqrt[a]
]))/(3*a*x)
```

**fricas** [A] time = 1.09, size = 268, normalized size = 2.13

$$\frac{15(3b^2c^2 + 4abcd)\sqrt{a}x^2 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(15abc^2x^3 - 6b^2d^2 - 2(15b^2c^2 + 40abcd + 3a^2d^2))}{30bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^2,x, algorithm="fricas")

[Out] [1/30\*(15\*(3\*b^2\*c^2 + 4\*a\*b\*c\*d)\*sqrt(a)\*x^2\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(15\*a\*b\*c^2\*x^3 - 6\*b^2\*d^2 - 2\*(15\*b^2\*c^2 + 40\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(5\*b^2\*c\*d + 3\*a\*b\*d^2)\*x)\*sqrt((a\*x + b)/x))/(b\*x^2), -1/15\*(15\*(3\*b^2\*c^2 + 4\*a\*b\*c\*d)\*sqrt(-a)\*x^2\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (15\*a\*b\*c^2\*x^3 - 6\*b^2\*d^2 - 2\*(15\*b^2\*c^2 + 40\*a\*b\*c\*d + 3\*a^2\*d^2)\*x^2 - 4\*(5\*b^2\*c\*d + 3\*a\*b\*d^2)\*x)\*sqrt((a\*x + b)/x))/(b\*x^2)]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]  
Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Evaluation time: 0.43Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple** [B] time = 0.06, size = 260, normalized size = 2.06

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -60a^2bcdx^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 45ab^2c^2x^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 120\sqrt{ax^2+bx}a^{\frac{5}{2}}cdx^4 - 90\right)}{30bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)\*(c+d/x)^2,x)

[Out] -1/30\*((a\*x+b)/x)^(1/2)/x^3/b\*(-120\*(a\*x^2+b\*x)^(1/2)\*a^(5/2)\*x^4\*c\*d-90\*(a\*x^2+b\*x)^(1/2)\*a^(3/2)\*x^4\*b\*c^2-60\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^4\*a^2\*b\*c\*d-45\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^4\*a\*b^2\*c^2+120\*(a\*x^2+b\*x)^(3/2)\*a^(3/2)\*x^2\*c\*d+60\*(a\*x^2+b



$$*x)^{(3/2)} * a^{(1/2)} * x^2 * b * c^2 + 12 * (a * x^2 + b * x)^{(3/2)} * a^{(3/2)} * x * d^2 + 40 * (a * x^2 + b * x)^{(3/2)} * a^{(1/2)} * x * b * c * d + 12 * (a * x^2 + b * x)^{(3/2)} * a^{(1/2)} * b * d^2 / ((a * x + b) * x)^{(1/2)} / a^{(1/2)}$$

**maxima [A]** time = 1.16, size = 152, normalized size = 1.21

$$-\frac{2\left(a + \frac{b}{x}\right)^{\frac{5}{2}} d^2}{5b} + \frac{1}{2} \left( 2 \sqrt{a + \frac{b}{x}} a x - 3 \sqrt{a} b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) - 4 \sqrt{a + \frac{b}{x}} b \right) c^2 - \frac{2}{3} \left( 3 a^{\frac{3}{2}} \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x)^2,x, algorithm="maxima")

[Out] -2/5\*(a + b/x)^(5/2)\*d^2/b + 1/2\*(2\*sqrt(a + b/x)\*a\*x - 3\*sqrt(a)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4\*sqrt(a + b/x)\*b)\*c^2 - 2/3\*(3\*a^(3/2)\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2\*(a + b/x)^(3/2) + 6\*sqrt(a + b/x)\*a)\*c\*d

**mupad [B]** time = 2.58, size = 197, normalized size = 1.56

$$\sqrt{a + \frac{b}{x}} \left( 2a \left( \frac{4ad^2 - 4bcd}{b} - \frac{4ad^2}{b} \right) - \frac{2(ad - bc)^2}{b} + \frac{2a^2 d^2}{b} \right) + \left( \frac{4ad^2 - 4bcd}{3b} - \frac{4ad^2}{3b} \right) \left( a + \frac{b}{x} \right)^{3/2} - \frac{2d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2)\*(c + d/x)^2,x)

[Out] (a + b/x)^(1/2)\*(2\*a\*((4\*a\*d^2 - 4\*b\*c\*d)/b - (4\*a\*d^2)/b) - (2\*(a\*d - b\*c)^2)/b + (2\*a^2\*d^2)/b) + ((4\*a\*d^2 - 4\*b\*c\*d)/(3\*b) - (4\*a\*d^2)/(3\*b))\*(a + b/x)^(3/2) - (2\*d^2\*(a + b/x)^(5/2))/(5\*b) + a\*c^2\*x\*(a + b/x)^(1/2) - 2\*c\*atan((2\*c\*(a + b/x)^(1/2)\*(4\*a\*d + 3\*b\*c)\*(-a/4)^(1/2))/(3\*a\*b\*c^2 + 4\*a^2\*c\*d))\*(4\*a\*d + 3\*b\*c)\*(-a/4)^(1/2)

**sympy [A]** time = 94.07, size = 534, normalized size = 4.24

$$\frac{4a^{\frac{11}{2}} b^{\frac{5}{2}} d^2 x^3 \sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}} b^3 x^2 + 15a^{\frac{5}{2}} b^4 x^2} + \frac{2a^{\frac{9}{2}} b^{\frac{7}{2}} d^2 x^2 \sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}} b^3 x^2 + 15a^{\frac{5}{2}} b^4 x^2} - \frac{8a^{\frac{7}{2}} b^{\frac{9}{2}} d^2 x \sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}} b^3 x^2 + 15a^{\frac{5}{2}} b^4 x^2} - \frac{6a^{\frac{5}{2}} b^{\frac{11}{2}} d^2 \sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}} b^3 x^2 + 15a^{\frac{5}{2}} b^4 x^2} + \sqrt{a} b c^2 \operatorname{asinh}\left(\sqrt{\frac{ax}{b} + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)\*(c+d/x)\*\*2,x)

[Out] 4\*a\*\*(11/2)\*b\*\*(5/2)\*d\*\*2\*x\*\*3\*sqrt(a\*x/b + 1)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) + 2\*a\*\*(9/2)\*b\*\*(7/2)\*d\*\*2\*x\*\*2\*sqrt(a\*x/b + 1)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) - 8\*a\*\*(7/2)\*b\*\*(9/2)\*d\*\*2\*x\*sqrt(a\*x/b + 1)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) - 6\*a\*\*(5/2)\*b\*\*(11/2)\*d\*\*2\*sqrt(a\*x/b + 1)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) + sqrt(a)\*b\*c\*\*2\*asinh(sqrt(a)\*sqrt(x)/sqrt(b)) - 4\*a\*\*6\*b\*\*2\*d\*\*2\*x\*\*(7/2)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) - 4\*a\*\*5\*b\*\*3\*d\*\*2\*x\*\*(5/2)/(15\*a\*\*(7/2)\*b\*\*3\*x\*\*(7/2) + 15\*a\*\*(5/2)\*b\*\*4\*x\*\*(5/2)) - 4\*a\*\*2\*c\*d\*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) + a\*sqrt(b)\*c\*\*2\*sqrt(x)\*sqrt(a\*x/b + 1) - 2\*a\*b\*c\*\*2\*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 4\*a\*c\*d\*sqrt(a + b/x) + a\*d\*\*2\*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2\*(a + b/x)\*\*(3/2)/(3\*b), True)) - 2\*b\*c\*\*2\*sqrt(a + b/x) + 2\*b\*c\*d\*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2\*(a + b/x)\*\*(3/2)/(3\*b), True))

$$3.233 \quad \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right) dx$$

**Optimal.** Leaf size=100

$$-\frac{\left(a + \frac{b}{x}\right)^{3/2} (2ad + 3bc)}{3a} - \sqrt{a + \frac{b}{x}} (2ad + 3bc) + \sqrt{a} (2ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) + \frac{cx \left(a + \frac{b}{x}\right)^{5/2}}{a}$$

[Out]  $-1/3*(2*a*d+3*b*c)*(a+b/x)^{(3/2)}/a+c*(a+b/x)^{(5/2)*x}/a+(2*a*d+3*b*c)*\arctan h((a+b/x)^{(1/2)}/a^{(1/2)})*a^{(1/2)}-(2*a*d+3*b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {375, 78, 50, 63, 208}

$$-\frac{\left(a + \frac{b}{x}\right)^{3/2} (2ad + 3bc)}{3a} - \sqrt{a + \frac{b}{x}} (2ad + 3bc) + \sqrt{a} (2ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) + \frac{cx \left(a + \frac{b}{x}\right)^{5/2}}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(3/2)\*(c + d/x), x]

[Out]  $-((3*b*c + 2*a*d)*\text{Sqrt}[a + b/x]) - ((3*b*c + 2*a*d)*(a + b/x)^{(3/2)})/(3*a) + (c*(a + b/x)^{(5/2)*x})/a + \text{Sqrt}[a]*(3*b*c + 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]]$

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

## Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

## Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right) dx &= -\text{Subst} \left( \int \frac{(a + bx)^{3/2}(c + dx)}{x^2} dx, x, \frac{1}{x} \right) \\
 &= \frac{c \left(a + \frac{b}{x}\right)^{5/2} x}{a} - \frac{\left(\frac{3bc}{2} + ad\right) \text{Subst} \left( \int \frac{(a+bx)^{3/2}}{x} dx, x, \frac{1}{x} \right)}{a} \\
 &= -\frac{(3bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} + \frac{c \left(a + \frac{b}{x}\right)^{5/2} x}{a} - \frac{1}{2}(3bc + 2ad) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right) \\
 &= -(3bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{(3bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} + \frac{c \left(a + \frac{b}{x}\right)^{5/2} x}{a} - \frac{1}{2}(a(3bc + 2ad)) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right) \\
 &= -(3bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{(3bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} + \frac{c \left(a + \frac{b}{x}\right)^{5/2} x}{a} - \frac{(a(3bc + 2ad)) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right)}{2} \\
 &= -(3bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{(3bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2}}{3a} + \frac{c \left(a + \frac{b}{x}\right)^{5/2} x}{a} + \sqrt{a} (3bc + 2ad) \text{Subst} \left( \int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 73, normalized size = 0.73

$$\frac{\sqrt{a + \frac{b}{x}} (ax(3cx - 8d) - 2b(3cx + d))}{3x} + \sqrt{a} (2ad + 3bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)\*(c + d/x), x]

[Out] (Sqrt[a + b/x]\*(a\*x\*(-8\*d + 3\*c\*x) - 2\*b\*(d + 3\*c\*x)))/(3\*x) + Sqrt[a]\*(3\*b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**fricas [A]** time = 1.03, size = 164, normalized size = 1.64

$$\left[ \frac{3(3bc + 2ad)\sqrt{a}x \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(3acx^2 - 2bd - 2(3bc + 4ad)x)\sqrt{\frac{ax+b}{x}}}{6x}, -\frac{3(3bc + 2ad)\sqrt{a}}{6x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x), x, algorithm="fricas")

[Out] [1/6\*(3\*(3\*b\*c + 2\*a\*d)\*sqrt(a)\*x\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(3\*a\*c\*x^2 - 2\*b\*d - 2\*(3\*b\*c + 4\*a\*d)\*x)\*sqrt((a\*x + b)/x))/x, -1/3\*(3\*(3\*b\*c + 2\*a\*d)\*sqrt(-a)\*x\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (3\*a\*c\*x^2 - 2\*b\*d - 2\*(3\*b\*c + 4\*a\*d)\*x)\*sqrt((a\*x + b)/x))/x]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes  
 constant sign by intervals (correct if the argument is real):Check [abs(x)]  
 Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2  
 ,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2  
 ,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86  
 ,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%  
 }+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%  
 }+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters v  
 alues [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0  
 ,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]  
 Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1  
 ,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing  
 root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at  
 parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2\*sqrt  
 (a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2  
 \*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached  
 or unable to make series expansion Error: Bad Argument Value

**maple** [B] time = 0.06, size = 205, normalized size = 2.05

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 6a^2bdx^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 9ab^2cx^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 12\sqrt{ax^2+bx} a^{\frac{5}{2}}dx^3 + 18\sqrt{ax^2+bx} \right)}{6\sqrt{(ax+b)x}\sqrt{a}bx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)\*(c+d/x),x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)\*(12\*a^(5/2)\*(a\*x^2+b\*x)^(1/2)\*x^3\*d+18\*a^(3/2)\*(a\*x^2  
 +b\*x)^(1/2)\*x^3\*b\*c-12\*a^(3/2)\*(a\*x^2+b\*x)^(3/2)\*x\*d+6\*ln(1/2\*(2\*a\*x+b+2\*(a  
 \*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^3\*a^2\*b\*d+9\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*  
 x)^(1/2)\*a^(1/2))/a^(1/2))\*x^3\*a\*b^2\*c-12\*a^(1/2)\*(a\*x^2+b\*x)^(3/2)\*x\*b\*c-4  
 \*d\*(a\*x^2+b\*x)^(3/2)\*b\*a^(1/2))/x^2/((a\*x+b)\*x)^(1/2)/b/a^(1/2)

**maxima** [A] time = 1.33, size = 132, normalized size = 1.32

$$\frac{1}{2} \left( 2\sqrt{a+\frac{b}{x}}ax - 3\sqrt{a}b \log\left(\frac{\sqrt{a+\frac{b}{x}} - \sqrt{a}}{\sqrt{a+\frac{b}{x}} + \sqrt{a}}\right) - 4\sqrt{a+\frac{b}{x}}b \right) c - \frac{1}{3} \left( 3a^{\frac{3}{2}} \log\left(\frac{\sqrt{a+\frac{b}{x}} - \sqrt{a}}{\sqrt{a+\frac{b}{x}} + \sqrt{a}}\right) + 2\left(a+\frac{b}{x}\right)^{\frac{3}{2}} + 6\sqrt{a+\frac{b}{x}} \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)\*(c+d/x),x, algorithm="maxima")

[Out] 1/2\*(2\*sqrt(a + b/x)\*a\*x - 3\*sqrt(a)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a  
 + b/x) + sqrt(a))) - 4\*sqrt(a + b/x)\*b)\*c - 1/3\*(3\*a^(3/2)\*log((sqrt(a +  
 b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 2\*(a + b/x)^(3/2) + 6\*sqrt(a +  
 b/x)\*a)\*d

**mupad** [B] time = 2.51, size = 81, normalized size = 0.81

$$2a^{3/2}d \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{2d\left(a+\frac{b}{x}\right)^{3/2}}{3} - 2ad\sqrt{a+\frac{b}{x}} - \frac{2cx\left(a+\frac{b}{x}\right)^{3/2}}{\left(\frac{ax}{b}+1\right)^{3/2}} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{ax}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(3/2)*(c + d/x), x)`

[Out]  $2a^{3/2}d \operatorname{atanh}\left(\frac{a + b/x}{a}\right) - \frac{2d(a + b/x)^{3/2}}{3} - 2a^2d \frac{(a + b/x)^{1/2}}{a} - \frac{2c^2x^2(a + b/x)^{3/2} \operatorname{hypergeom}\left(\left[-\frac{3}{2}, -\frac{1}{2}\right], \frac{1}{2}, -\frac{(a + b/x)}{b}\right)}{(ax/b + 1)^{3/2}}$

**sympy** [A] time = 56.15, size = 163, normalized size = 1.63

$$\sqrt{a}bc \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - \frac{2a^2d \operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a\sqrt{b}c\sqrt{x}\sqrt{\frac{ax}{b} + 1} - \frac{2abc \operatorname{atan}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2ad\sqrt{a + \frac{b}{x}} - 2bc\sqrt{a + \frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(3/2)*(c+d/x), x)`

[Out]  $\sqrt{a}b^2c \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) - 2a^2d \operatorname{atan}\left(\frac{\sqrt{a + b/x}}{\sqrt{-a}}\right) + a^2c \sqrt{b} \sqrt{ax/b + 1} - 2a^2b^2c \operatorname{atan}\left(\frac{\sqrt{a + b/x}}{\sqrt{-a}}\right) - 2a^2d \sqrt{a + b/x} - 2b^2c \sqrt{a + b/x} + b^2d \operatorname{Piecewise}\left(\left(-\sqrt{a}/x, \operatorname{Eq}(b, 0)\right), \left(-2(a + b/x)^{3/2}/(3b), \operatorname{True}\right)\right)$

$$3.234 \quad \int \left(a + \frac{b}{x}\right)^{3/2} dx$$

**Optimal.** Leaf size=54

$$x \left(a + \frac{b}{x}\right)^{3/2} - 3b\sqrt{a + \frac{b}{x}} + 3\sqrt{a}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

[Out] (a+b/x)^(3/2)\*x+3\*b\*arctanh((a+b/x)^(1/2)/a^(1/2))\*a^(1/2)-3\*b\*(a+b/x)^(1/2)

**Rubi [A]** time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {242, 47, 50, 63, 208}

$$x \left(a + \frac{b}{x}\right)^{3/2} - 3b\sqrt{a + \frac{b}{x}} + 3\sqrt{a}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(3/2), x]

[Out] -3\*b\*Sqrt[a + b/x] + (a + b/x)^(3/2)\*x + 3\*Sqrt[a]\*b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

#### Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

#### Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

#### Rule 242

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x}\right)^{3/2} dx &= -\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x^2} dx, x, \frac{1}{x}\right) \\
 &= \left(a + \frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3b) \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{x}\right) \\
 &= -3b\sqrt{a + \frac{b}{x}} + \left(a + \frac{b}{x}\right)^{3/2} x - \frac{1}{2}(3ab) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right) \\
 &= -3b\sqrt{a + \frac{b}{x}} + \left(a + \frac{b}{x}\right)^{3/2} x - (3a) \text{Subst}\left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right) \\
 &= -3b\sqrt{a + \frac{b}{x}} + \left(a + \frac{b}{x}\right)^{3/2} x + 3\sqrt{a} b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 46, normalized size = 0.85

$$\sqrt{a + \frac{b}{x}}(ax - 2b) + 3\sqrt{a} b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2), x]

[Out] Sqrt[a + b/x]\*(-2\*b + a\*x) + 3\*Sqrt[a]\*b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**fricas [A]** time = 0.88, size = 100, normalized size = 1.85

$$\left[\frac{3}{2}\sqrt{a} b \log\left(2ax + 2\sqrt{a} x \sqrt{\frac{ax+b}{x}} + b\right) + (ax - 2b)\sqrt{\frac{ax+b}{x}}, -3\sqrt{-a} b \arctan\left(\frac{\sqrt{-a} \sqrt{\frac{ax+b}{x}}}{a}\right) + (ax - 2b)\sqrt{\frac{ax+b}{x}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2), x, algorithm="fricas")

[Out] [3/2\*sqrt(a)\*b\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + (a\*x - 2\*b)\*sqrt((a\*x + b)/x), -3\*sqrt(-a)\*b\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (a\*x - 2\*b)\*sqrt((a\*x + b)/x)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]

Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [82.1195442914,26,-89]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parameters values [42,43.9628838282,-9]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple [B]** time = 0.05, size = 100, normalized size = 1.85

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 3abx^2 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 6\sqrt{ax^2+bx} a^{\frac{3}{2}}x^2 - 4(ax^2+bx)^{\frac{3}{2}}\sqrt{a} \right)}{2\sqrt{(ax+b)x}\sqrt{a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*(6\*a^(3/2)\*(a\*x^2+b\*x)^(1/2)\*x^2+3\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a\*b-4\*(a\*x^2+b\*x)^(3/2)\*a^(1/2))/x/((a\*x+b)\*x)^(1/2)/a^(1/2)

**maxima [A]** time = 1.20, size = 63, normalized size = 1.17

$$\sqrt{a+\frac{b}{x}}ax - \frac{3}{2}\sqrt{a}b \log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right) - 2\sqrt{a+\frac{b}{x}}b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2),x, algorithm="maxima")

[Out] sqrt(a + b/x)\*a\*x - 3/2\*sqrt(a)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 2\*sqrt(a + b/x)\*b

**mupad [B]** time = 1.50, size = 34, normalized size = 0.63

$$\frac{2x\left(a+\frac{b}{x}\right)^{3/2} {}_2F_1\left(-\frac{3}{2},-\frac{1}{2};\frac{1}{2};-\frac{ax}{b}\right)}{\left(\frac{ax}{b}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2),x)

[Out] -(2\*x\*(a + b/x)^(3/2)\*hypergeom([-3/2, -1/2], 1/2, -(a\*x)/b))/((a\*x)/b + 1)^(3/2)

**sympy [B]** time = 2.69, size = 92, normalized size = 1.70

$$3\sqrt{a}b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) + \frac{a^2x^{\frac{3}{2}}}{\sqrt{b}\sqrt{\frac{ax}{b}+1}} - \frac{a\sqrt{b}\sqrt{x}}{\sqrt{\frac{ax}{b}+1}} - \frac{2b^{\frac{3}{2}}}{\sqrt{x}\sqrt{\frac{ax}{b}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((a+b/x)**(3/2),x)
```

```
[Out] 3*sqrt(a)*b*asinh(sqrt(a)*sqrt(x)/sqrt(b)) + a**2*x**(3/2)/(sqrt(b)*sqrt(a*  
x/b + 1)) - a*sqrt(b)*sqrt(x)/sqrt(a*x/b + 1) - 2*b**(3/2)/(sqrt(x)*sqrt(a*  
x/b + 1))
```

$$3.235 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=106

$$-\frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

[Out]  $(-2*a*d+3*b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}/a^{(1/2)}}{d^{(1/2)}\sqrt{a+b/x}}\right)*a^{(1/2)}/c^2-2*(-a*d+b*c)^{(3/2)}*\operatorname{arctan}\left(\frac{d^{(1/2)}*(a+b/x)^{(1/2)}/(-a*d+b*c)^{(1/2)}}{c^2/d^{(1/2)}+a*x*(a+b/x)^{(1/2)}\sqrt{a+b/x}}\right)/c$

**Rubi [A]** time = 0.13, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 98, 156, 63, 208, 205}

$$-\frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{ax\sqrt{a+\frac{b}{x}}}{c}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(a + \frac{b}{x}\right)^{3/2}/\left(c + \frac{d}{x}\right), x\right]$

[Out]  $(a*\operatorname{Sqrt}[a + b/x]*x)/c - (2*(b*c - a*d)^{(3/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(c^2*\operatorname{Sqrt}[d]) + (\operatorname{Sqrt}[a]*(3*b*c - 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/c^2$

### Rule 63

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)^{(m_.)}\right)\left((c_.) + (d_.)*(x_.)^{(n_.)}\right), x\_Symbol\right] :> \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x\right], x, (a + b*x)^{(1/p)}\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 98

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)^{(m_.)}\right)\left((c_.) + (d_.)*(x_.)^{(n_.)}\right)\left((e_.) + (f_.)*(x_.)^{(p_.)}\right), x\_Symbol\right] :> \operatorname{Simp}\left[\left((b*c - a*d)*(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}*(e + f*x)^{(p+1)}\right)/(b*(b*e - a*f)*(m+1)), x\right] + \operatorname{Dist}\left[1/(b*(b*e - a*f)*(m+1)), \operatorname{Int}\left[(a + b*x)^{(m+1)}*(c + d*x)^{(n-2)}*(e + f*x)^p*\operatorname{Simp}\left[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))\right]*x, x\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, p\}, x \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 1] \&\& (\operatorname{IntegersQ}[2*m, 2*n, 2*p] \|\| \operatorname{IntegersQ}[m, n+p] \|\| \operatorname{IntegersQ}[p, m+n])$

### Rule 156

$\operatorname{Int}\left[\left(\left((e_.) + (f_.)*(x_.)^{(p_.)}\right)\left((g_.) + (h_.)*(x_.)\right)\right)/\left(\left((a_.) + (b_.)*(x_.)^{(c_.)}\right)\left((d_.) + (e_.)*(x_.)\right)\right), x\_Symbol\right] :> \operatorname{Dist}\left[(b*g - a*h)/(b*c - a*d), \operatorname{Int}\left[(e + f*x)^p/(a + b*x), x\right], x\right] - \operatorname{Dist}\left[(d*g - c*h)/(b*c - a*d), \operatorname{Int}\left[(e + f*x)^p/(c + d*x), x\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h\}, x$

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{c + \frac{d}{x}} dx &= -\text{Subst}\left(\int \frac{(a + bx)^{3/2}}{x^2(c + dx)} dx, x, \frac{1}{x}\right) \\ &= \frac{a\sqrt{a + \frac{b}{x}}}{c} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc - 2ad) - \frac{1}{2}b(2bc - ad)x}{x\sqrt{a + bx}(c + dx)} dx, x, \frac{1}{x}\right)}{c} \\ &= \frac{a\sqrt{a + \frac{b}{x}}}{c} - \frac{(a(3bc - 2ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{2c^2} - \frac{(bc - ad)^2 \text{Subst}\left(\int \frac{1}{\sqrt{a + bx}(c + dx)} dx, x, \frac{1}{x}\right)}{c^2} \\ &= \frac{a\sqrt{a + \frac{b}{x}}}{c} - \frac{(a(3bc - 2ad)) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^2} - \frac{(2(bc - ad)^2) \text{Subst}\left(\int \frac{1}{c - \frac{ad}{b} + \frac{d}{b}x} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^2} \\ &= \frac{a\sqrt{a + \frac{b}{x}}}{c} - \frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2\sqrt{d}} + \frac{\sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 102, normalized size = 0.96

$$\frac{2(bc - ad)^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right) + \sqrt{a}(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + acx\sqrt{a + \frac{b}{x}}}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x), x]

[Out] (a\*c\*Sqrt[a + b/x]\*x - (2\*(b\*c - a\*d)^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/Sqrt[d] + Sqrt[a]\*(3\*b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]/c^2

**fricas [A]** time = 1.08, size = 519, normalized size = 4.90

$$\frac{2acx\sqrt{\frac{ax+b}{x}} - (3bc - 2ad)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(bc - ad)\sqrt{-\frac{bc-ad}{d}} \log\left(\frac{2dx\sqrt{-\frac{bc-ad}{d}}\sqrt{\frac{ax+b}{x}} + bd}{cx+d}\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")

[Out] [1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x) - (3\*b\*c - 2\*a\*d)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(b\*c - a\*d)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)))/c^2, (a\*c\*x\*sqrt((a\*x + b)/x) - (3\*b\*c - 2\*a\*d)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (b\*c - a\*d)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)))/c^2, 1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x) + 4\*(b\*c - a\*d)\*sqrt((b\*c - a\*d)/d)\*arctan(-d\*sqrt((b\*c - a\*d)/d)\*sqrt((a\*x + b)/x)/(b\*c - a\*d)) - (3\*b\*c - 2\*a\*d)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b))/c^2, (a\*c\*x\*sqrt((a\*x + b)/x) + 2\*(b\*c - a\*d)\*sqrt((b\*c - a\*d)/d)\*arctan(-d\*sqrt((b\*c - a\*d)/d)\*sqrt((a\*x + b)/x)/(b\*c - a\*d)) - (3\*b\*c - 2\*a\*d)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a))/c^2]

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Error: Bad Argument Type

**maple** [B] time = 0.06, size = 528, normalized size = 4.98

$$\frac{\sqrt{\frac{ax+b}{x}}}{x} \left( 2a^{\frac{5}{2}}d^3 \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)xc}}{cx+d} \right) - 4a^{\frac{3}{2}}bcd^2 \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)xc}}{cx+d} \right) + 2\sqrt{a} b^2c^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)/(c+d/x),x)

[Out] -1/2\*((a\*x+b)/x)^(1/2)\*x\*(2\*a^(5/2)\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*d^3-2\*a^(3/2)\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*c^2\*d-4\*a^(3/2)\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*b\*c\*d^2+2\*a^(1/2)\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*b\*c^3-2\*a^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*(a\*x^2+b\*x)^(1/2)\*b\*c^3+2\*a^(1/2)\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*b^2\*c^2\*d+2\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2))\*a^(1/2))/a^(1/2))\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^2\*c\*d^2-3\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2))\*a^(1/2))/a^(1/2))\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a\*b\*c^2\*d+ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2))\*a^(1/2))/a^(1/2))\*((a\*d-b\*c)/c^2\*d)^(1/2)\*b^2\*c^3-((a\*d-b\*c)/c^2\*d)^(1/2)\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2))\*a^(1/2))/a^(1/2))\*b^2\*c^3)/((a\*x+b)\*x)^(1/2)/d/c^3/a^(1/2)/((a\*d-b\*c)/c^2\*d)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x),x, algorithm="maxima")

[Out] integrate((a + b/x)^(3/2)/(c + d/x), x)

**mupad [B]** time = 1.68, size = 556, normalized size = 5.25

$$\frac{ax\sqrt{a+\frac{b}{x}}}{c} - \frac{\sqrt{a} \operatorname{atanh}\left(\frac{58a^{3/2}b^6d^2\sqrt{a+\frac{b}{x}}}{58a^2b^6d^2-24ab^7cd-\frac{46a^3b^5d^3}{c}+\frac{12a^4b^4d^4}{c^2}} + \frac{46a^{5/2}b^5d^3\sqrt{a+\frac{b}{x}}}{46a^3b^5d^3-58a^2b^6cd^2-\frac{12a^4b^4d^4}{c}+24ab^7c^2d}\right)}{c^2} + \frac{12a^4b^4d^4}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2)/(c + d/x),x)

[Out] (a\*x\*(a + b/x)^(1/2))/c - (a^(1/2)\*atanh((58\*a^(3/2)\*b^6\*d^2\*(a + b/x)^(1/2))/(58\*a^2\*b^6\*d^2 - 24\*a\*b^7\*c\*d - (46\*a^3\*b^5\*d^3)/c + (12\*a^4\*b^4\*d^4)/c^2) + (46\*a^(5/2)\*b^5\*d^3\*(a + b/x)^(1/2))/(46\*a^3\*b^5\*d^3 - 58\*a^2\*b^6\*c\*d^2 - (12\*a^4\*b^4\*d^4)/c + 24\*a\*b^7\*c^2\*d) + (12\*a^(7/2)\*b^4\*d^4\*(a + b/x)^(1/2))/(12\*a^4\*b^4\*d^4 - 46\*a^3\*b^5\*c\*d^3 + 58\*a^2\*b^6\*c^2\*d^2 - 24\*a\*b^7\*c^3\*d) - (24\*a^(1/2)\*b^7\*c\*d\*(a + b/x)^(1/2))/(58\*a^2\*b^6\*d^2 - 24\*a\*b^7\*c\*d - (46\*a^3\*b^5\*d^3)/c + (12\*a^4\*b^4\*d^4)/c^2))\*(2\*a\*d - 3\*b\*c))/c^2 + (2\*atanh((12\*a^2\*b^4\*d^2\*(a + b/x)^(1/2)\*(a^3\*d^4 - b^3\*c^3\*d + 3\*a\*b^2\*c^2\*d^2 - 3\*a^2\*b\*c\*d^3)^(1/2))/(12\*a^4\*b^4\*d^4 - 40\*a^3\*b^5\*c\*d^3 + 44\*a^2\*b^6\*c^2\*d^2 - 16\*a\*b^7\*c^3\*d) + (16\*a\*b^5\*d\*(a + b/x)^(1/2)\*(a^3\*d^4 - b^3\*c^3\*d + 3\*a\*b^2\*c^2\*d^2 - 3\*a^2\*b\*c\*d^3)^(1/2))/(40\*a^3\*b^5\*d^3 - 44\*a^2\*b^6\*c\*d^2 - (12\*a^4\*b^4\*d^4)/c + 16\*a\*b^7\*c^2\*d))\*(d\*(a\*d - b\*c)^3)^(1/2))/(c^2\*d)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{cx + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(3/2)/(c+d/x),x)

[Out] Integral(x\*(a + b/x)\*\*(3/2)/(c\*x + d), x)

$$3.236 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=156

$$\frac{(bc - 4ad)\sqrt{bc - ad} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right) + \sqrt{a}(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) - \sqrt{a + \frac{b}{x}}(bc - 2ad) + ax\sqrt{a + \frac{b}{x}}}{c^3\sqrt{d} + c^3 - c^2\left(c + \frac{d}{x}\right) + c\left(c + \frac{d}{x}\right)}$$

[Out]  $(-4*a*d+3*b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{1/2}/a^{1/2}}{\sqrt{bc-ad}}\right)*a^{1/2}/c^3 - (-4*a*d+b*c)*\operatorname{arctan}\left(\frac{d^{1/2}*(a+b/x)^{1/2}/(-a*d+b*c)^{1/2}}{\sqrt{a}}\right)*(-a*d+b*c)^{1/2}/c^3/d^{1/2} - (-2*a*d+b*c)*(a+b/x)^{1/2}/c^2/(c+d/x) + a*x*(a+b/x)^{1/2}/c/(c+d/x)$

**Rubi [A]** time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 98, 151, 156, 63, 208, 205}

$$\frac{\sqrt{a + \frac{b}{x}}(bc - 2ad) - (bc - 4ad)\sqrt{bc - ad} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right) + \sqrt{a}(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + ax\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right) - c^3\sqrt{d} + c^3 + c\left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(a + \frac{b}{x}\right)^{3/2}/\left(c + \frac{d}{x}\right)^2, x\right]$

[Out]  $-\left(\frac{(b*c - 2*a*d)*\operatorname{Sqrt}[a + b/x]}{c^2*(c + d/x)}\right) + \frac{(a*\operatorname{Sqrt}[a + b/x]*x)}{c*(c + d/x)} - \frac{(b*c - 4*a*d)*\operatorname{Sqrt}[b*c - a*d]*\operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x]}{\operatorname{Sqrt}[b*c - a*d]}\right]}{c^3*\operatorname{Sqrt}[d]} + \frac{(\operatorname{Sqrt}[a]*(3*b*c - 4*a*d)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a + b/x]}{\operatorname{Sqrt}[a]}\right])}{c^3}$

### Rule 63

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)^{m_.}\right)\left((c_.) + (d_.)*(x_.)^{n_.}\right), x\_Symbol\right] \rightarrow \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[\frac{p}{b}, \operatorname{Subst}\left[\operatorname{Int}\left[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x\right], x, (a + b*x)^{1/p}\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, x\} \ \&\& \operatorname{NeQ}[b*c - a*d, 0] \ \&\& \operatorname{LtQ}[-1, m, 0] \ \&\& \operatorname{LeQ}[-1, n, 0] \ \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]\right]$

### Rule 98

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)^{m_.}\right)\left((c_.) + (d_.)*(x_.)^{n_.}\right)\left((e_.) + (f_.)*(x_.)^{p_.}\right), x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\frac{(b*c - a*d)*(a + b*x)^{m+1}*(c + d*x)^{n-1}*(e + f*x)^{p+1}}{b*(b*e - a*f)*(m+1)}, x\right] + \operatorname{Dist}\left[\frac{1}{b*(b*e - a*f)*(m+1)}, \operatorname{Int}\left[(a + b*x)^{m+1}*(c + d*x)^{n-2}*(e + f*x)^p*\operatorname{Simp}\left[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))\right]*x, x\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, p\}, x \ \&\& \operatorname{LtQ}[m, -1] \ \&\& \operatorname{GtQ}[n, 1] \ \&\& (\operatorname{IntegersQ}[2*m, 2*n, 2*p] \ \|\ \operatorname{IntegersQ}[m, n+p] \ \|\ \operatorname{IntegersQ}[p, m+n])$

### Rule 151

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)^{m_.}\right)\left((c_.) + (d_.)*(x_.)^{n_.}\right)\left((e_.) + (f_.)*(x_.)^{p_.}\right)\left((g_.) + (h_.)*(x_.)^{q_.}\right), x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\frac{(b*g - a*h)*(a + b*x)^{m+1}*(c + d*x)^{n+1}*(e + f*x)^{p+1}}{(m+1)*(b*c - a*d)*(b*e - a*f)}, x\right] + \operatorname{Dist}\left[\frac{1}{(m+1)*(b*c - a*d)*(b*e - a*f)}, \operatorname{Int}\left[(a + b*x)^{m+1}*(c + d*x)^n*(e + f*x)^p*\operatorname{Simp}\left[(a*d*f*g - b*(d*e + c*f))*g + b*c*e*h\right]*(m+1) - (b*g\right.$

$- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]$   
 $, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 156

$\text{Int}[(((e_.) + (f_.)*(x_.))^(p_.)*((g_.) + (h_.)*(x_.)))/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))), x\_Symbol] := \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

$\text{Int}[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x\_Symbol] := -\text{Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] /;$  FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^2} dx = -\text{Subst}\left(\int \frac{(a + bx)^{3/2}}{x^2(c + dx)^2} dx, x, \frac{1}{x}\right)$$

$$= \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}a(3bc - 4ad) - \frac{1}{2}b(2bc - 3ad)x}{x\sqrt{a + bx}(c + dx)^2} dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{(bc - 2ad)\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a(3bc - 4ad)(bc - ad) + \frac{1}{2}b(bc - 2ad)(bc - ad)x}{x\sqrt{a + bx}(c + dx)} dx, x, \frac{1}{x}\right)}{c^2(bc - ad)}$$

$$= -\frac{(bc - 2ad)\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{(a(3bc - 4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{2c^3} - \frac{((bc - 4ad))}{c^2}$$

$$= -\frac{(bc - 2ad)\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{(a(3bc - 4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^3} - \frac{((bc - 4ad))}{c^2}$$

$$= -\frac{(bc - 2ad)\sqrt{a + \frac{b}{x}}}{c^2\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)} - \frac{(bc - 4ad)\sqrt{bc - ad} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^3\sqrt{d}} + \frac{\sqrt{a}(3bc - 4ad)}{c^2}$$

**Mathematica [A]** time = 0.39, size = 143, normalized size = 0.92

$$\frac{(4a^2d^2 - 5abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}(acx+2ad-bc)}{cx+d} + \sqrt{a}(3bc-4ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3 \sqrt{d}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^2,x]

[Out] ((c\*Sqrt[a + b/x]\*x\*(-(b\*c) + 2\*a\*d + a\*c\*x))/(d + c\*x) - ((b^2\*c^2 - 5\*a\*b\*c\*d + 4\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(Sqrt[d]\*Sqrt[b\*c - a\*d]) + Sqrt[a]\*(3\*b\*c - 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/c^3

**fricas [A]** time = 0.95, size = 769, normalized size = 4.93

$$\left[ \frac{(3bcd - 4ad^2 + (3bc^2 - 4acd)x)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + (bcd - 4ad^2 + (bc^2 - 4acd)x)\sqrt{-\frac{bc-ad}{d}}}{2(c^4x + c^3d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")

[Out] [-1/2\*((3\*b\*c\*d - 4\*a\*d^2 + (3\*b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + (b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)) - 2\*(a\*c^2\*x^2 - (b\*c^2 - 2\*a\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(c^4\*x + c^3\*d), 1/2\*(2\*(b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt((b\*c - a\*d)/d)\*arctan(-d\*sqrt((b\*c - a\*d)/d)\*sqrt((a\*x + b)/x)/(b\*c - a\*d)) - (3\*b\*c\*d - 4\*a\*d^2 + (3\*b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(a\*c^2\*x^2 - (b\*c^2 - 2\*a\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(c^4\*x + c^3\*d), -1/2\*(2\*(3\*b\*c\*d - 4\*a\*d^2 + (3\*b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-(b\*c - a\*d)/d)\*log((2\*d\*x\*sqrt(-(b\*c - a\*d)/d)\*sqrt((a\*x + b)/x) + b\*d - (b\*c - 2\*a\*d)\*x)/(c\*x + d)) - 2\*(a\*c^2\*x^2 - (b\*c^2 - 2\*a\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(c^4\*x + c^3\*d), ((b\*c\*d - 4\*a\*d^2 + (b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt((b\*c - a\*d)/d)\*arctan(-d\*sqrt((b\*c - a\*d)/d)\*sqrt((a\*x + b)/x)/(b\*c - a\*d)) - (3\*b\*c\*d - 4\*a\*d^2 + (3\*b\*c^2 - 4\*a\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (a\*c^2\*x^2 - (b\*c^2 - 2\*a\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(c^4\*x + c^3\*d)]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Unable to divide, perhaps due to rounding error%[[[-2,0]:[1,0,%{-1,[1]%%}]]], [4,6,4,0]%%}+%[[{8,[1]%%}, [3,5,4,1]%%}+%[[{-4,0]:[1,0,%{-1,[1]%%}]]], [2,5,5,1]%%}+%[[{[-8,[1]%%}, 0]:[1,0,%{-1,[1]%%}]]], [2,4,4,2]%%}+%[[{8,[1]%%}, [1,4,5,2]%%}+%[[{-2,0]:[1,0,%{-1,[1]%%}]]]]



```
%{-1, [1]%%}%%}, [0, 4, 6, 2]%%} / %%{%%{1, [1]%%}, [4, 2, 0, 0]%%}+%%{%%{poly1[%%{-4, [1]%%}, 0]: [1, 0, %%{-1, [1]%%}%%}, [3, 1, 0, 1]%%}+%%{%%{2, [1]%%}, [2, 1, 1, 1]%%}+%%{%%{4, [2]%%}, [2, 0, 0, 2]%%}+%%{%%{poly1[%%{-4, [1]%%}, 0]: [1, 0, %%{-1, [1]%%}%%}, [1, 0, 1, 2]%%}+%%{%%{1, [1]%%}, [0, 0, 2, 2]%%} E
rroor: Bad Argument Value
```

**maple [B]** time = 0.06, size = 834, normalized size = 5.35

$$\left( 4a^{\frac{7}{2}}c d^3 x \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)x} c}{cx+d} \right) - 5a^{\frac{5}{2}}b c^2 d^2 x \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{ad-bc}{c^2}} \sqrt{(ax+b)x} c}{cx+d} \right) + a^{\frac{3}{2}}b^2 c^3 dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(3/2)/(c+d/x)^2, x)

[Out] -1/2\*(4\*a^(7/2)\*c\*d^3\*x\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))+2\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(5/2)\*c^4\*x^2+4\*a^(7/2)\*d^4\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))-2\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(5/2)\*c^3\*d\*x-5\*a^(5/2)\*b\*c^2\*d^2\*x\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))-4\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(5/2)\*c^2\*d^2-5\*a^(5/2)\*b\*c\*d^3\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))-2\*((a\*x+b)\*x)^(3/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(3/2)\*c^4+2\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(3/2)\*b\*c^4\*x+a^(3/2)\*b^2\*c^3\*d\*x\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))+2\*((a\*x+b)\*x)^(1/2)\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(3/2)\*b\*c^3\*d+a^(3/2)\*b^2\*c^2\*d^2\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2))\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))+4\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^3\*c^2\*d^2\*x\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))-3\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^2\*b\*c^3\*d\*x\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))+4\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^3\*c\*d^3\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))-3\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^2\*b\*c^2\*d^2\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2)))\*x\*((a\*x+b)/x)^(1/2)/c^4/((a\*d-b\*c)/c^2\*d)^(1/2)/a^(3/2)/(c\*x+d)/d/((a\*x+b)\*x)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{\left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x)^2, x, algorithm="maxima")

[Out] integrate((a + b/x)^(3/2)/(c + d/x)^2, x)

**mupad [B]** time = 2.16, size = 448, normalized size = 2.87

$$\frac{\operatorname{atanh} \left( \frac{8 a^2 b^5 d^2 \sqrt{a+\frac{b}{x}} \sqrt{a d^2-b c d}}{8 a^3 b^5 d^3-10 a^2 b^6 c d^2+2 a b^7 c^2 d} - \frac{2 a b^6 d \sqrt{a+\frac{b}{x}} \sqrt{a d^2-b c d}}{2 a b^7 c d-10 a^2 b^6 d^2+\frac{8 a^3 b^5 d^3}{c}} \right) \sqrt{d(a d-b c)}(4 a d-b c) \sqrt{a} \operatorname{atanh} \left( \frac{6 \sqrt{a}}{6 a b^7 d-\frac{14}{c}} \right)}{c^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2)/(c + d/x)^2, x)

```
[Out] (atanh((8*a^2*b^5*d^2*(a + b/x)^(1/2)*(a*d^2 - b*c*d)^(1/2))/(8*a^3*b^5*d^3
- 10*a^2*b^6*c*d^2 + 2*a*b^7*c^2*d) - (2*a*b^6*d*(a + b/x)^(1/2)*(a*d^2 -
b*c*d)^(1/2))/(2*a*b^7*c*d - 10*a^2*b^6*d^2 + (8*a^3*b^5*d^3)/c))*(d*(a*d -
b*c))^(1/2)*(4*a*d - b*c))/(c^3*d) - (a^(1/2)*atanh((6*a^(1/2)*b^7*d*(a +
b/x)^(1/2))/(6*a*b^7*d - (14*a^2*b^6*d^2)/c + (8*a^3*b^5*d^3)/c^2) - (14*a^
(3/2)*b^6*d^2*(a + b/x)^(1/2))/(6*a*b^7*c*d - 14*a^2*b^6*d^2 + (8*a^3*b^5*d
^3)/c) + (8*a^(5/2)*b^5*d^3*(a + b/x)^(1/2))/(8*a^3*b^5*d^3 - 14*a^2*b^6*c*
d^2 + 6*a*b^7*c^2*d))*(4*a*d - 3*b*c))/c^3 - ((2*(a*b^2*c - a^2*b*d)*(a + b
/x)^(1/2))/c^2 + (b*(a + b/x)^(3/2)*(2*a*d - b*c))/c^2)/((a + b/x)*(2*a*d -
b*c) - d*(a + b/x)^2 - a^2*d + a*b*c)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**2,x)
```

```
[Out] Timed out
```

$$3.237 \quad \int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=209

$$\frac{3(8a^2d^2 - 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{3\sqrt{a}(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4} - \frac{3\sqrt{a+\frac{b}{x}}(bc - 4ad)}{4c^3\left(c + \frac{d}{x}\right)} - \frac{\sqrt{a+\frac{b}{x}}(b^2c^2 - 8abcd + 8a^2d^2)}{2c^2\left(c + \frac{d}{x}\right)^2}}{4c^4\sqrt{d}\sqrt{bc-ad}}$$

[Out]  $3*(-2*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}/a^{(1/2)}}{c^4-3/4*(8*a^2*d^2-8*a*b*c*d+b^2*c^2)}\right)*a^{(1/2)}/c^4-3/4*(8*a^2*d^2-8*a*b*c*d+b^2*c^2)*\operatorname{arctan}\left(\frac{d^{(1/2)}*(a+b/x)^{(1/2)}/(-a*d+b*c)^{(1/2)}}{c^4/d^{(1/2)}/(-a*d+b*c)^{(1/2)}-1/2*(-3*a*d+b*c)*(a+b/x)^{(1/2)}/c^2/(c+d/x)^2-3/4*(-4*a*d+b*c)*(a+b/x)^{(1/2)}/c^3/(c+d/x)+a*x*(a+b/x)^{(1/2)}/c/(c+d/x)^2}\right)$

**Rubi [A]** time = 0.34, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 98, 151, 156, 63, 208, 205}

$$\frac{3(8a^2d^2 - 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{3\sqrt{a+\frac{b}{x}}(bc - 4ad)}{4c^3\left(c + \frac{d}{x}\right)} - \frac{\sqrt{a+\frac{b}{x}}(bc - 3ad)}{2c^2\left(c + \frac{d}{x}\right)^2} + \frac{3\sqrt{a}(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^4}}{4c^4\sqrt{d}\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(a + \frac{b}{x}\right)^{3/2}/\left(c + \frac{d}{x}\right)^3, x\right]$

[Out]  $-\left(\frac{(b*c - 3*a*d)*\operatorname{Sqrt}[a + b/x]}{2*c^2*(c + d/x)^2} - \frac{(3*(b*c - 4*a*d)*\operatorname{Sqrt}[a + b/x])}{4*c^3*(c + d/x)} + \frac{(a*\operatorname{Sqrt}[a + b/x]*x)}{c*(c + d/x)^2} - \frac{(3*(b^2*c^2 - 8*a*b*c*d + 8*a^2*d^2)*\operatorname{ArcTan}\left[\frac{\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x]}{\operatorname{Sqrt}[b*c - a*d]}\right])}{4*c^4*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[b*c - a*d]} + \frac{(3*\operatorname{Sqrt}[a]*(b*c - 2*a*d)*\operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[a + b/x]}{\operatorname{Sqrt}[a]}\right])}{c^4}\right)$

### Rule 63

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}, x\_Symbol\right] \rightarrow \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[\frac{p}{b}, \operatorname{Subst}\left[\operatorname{Int}\left[x^{\left(p*(m+1)-1\right)}*\left(c - \frac{a*d}{b} + \frac{d*x^p}{b}\right)^n, x\right], x, \left(a + b*x\right)^{\left(1/p\right)}, x\right] \right] /; \operatorname{FreeQ}\left[\{a, b, c, d\}, x\right] \&\& \operatorname{NeQ}\left[b*c - a*d, 0\right] \&\& \operatorname{LtQ}\left[-1, m, 0\right] \&\& \operatorname{LeQ}\left[-1, n, 0\right] \&\& \operatorname{LeQ}\left[\operatorname{Denominator}[n], \operatorname{Denominator}[m]\right] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 98

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}*\left(\left(e_{.}\right) + \left(f_{.}\right)*\left(x_{.}\right)\right)^{\left(p_{.}\right)}, x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\frac{(b*c - a*d)*(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}*(e + f*x)^{(p+1)}}{b*(b*e - a*f)*(m+1)}, x\right] + \operatorname{Dist}\left[\frac{1}{b*(b*e - a*f)*(m+1)}, \operatorname{Int}\left[\left(a + b*x\right)^{(m+1)}*\left(c + d*x\right)^{(n-2)}*(e + f*x)^p*\operatorname{Simp}\left[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))\right]*x, x\right] \right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, p\}, x\right] \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 1] \&\& (\operatorname{IntegersQ}[2*m, 2*n, 2*p] \parallel \operatorname{IntegersQ}[m, n+p] \parallel \operatorname{IntegersQ}[p, m+n])$

### Rule 151

$\operatorname{Int}\left[\left(\left(a_{.}\right) + \left(b_{.}\right)*\left(x_{.}\right)\right)^{\left(m_{.}\right)}*\left(\left(c_{.}\right) + \left(d_{.}\right)*\left(x_{.}\right)\right)^{\left(n_{.}\right)}*\left(\left(e_{.}\right) + \left(f_{.}\right)*\left(x_{.}\right)\right)^{\left(p_{.}\right)}*\left(\left(g_{.}\right) + \left(h_{.}\right)*\left(x_{.}\right)\right), x\_Symbol\right] \rightarrow \operatorname{Simp}\left[\left((b*g - a*h)*(a + b*x)^{(m+1)}\right)\right]$

```

1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^(n)*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

```

### Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

### Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

### Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

### Rule 375

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x}\right)^{3/2}}{\left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst}\left(\int \frac{(a + bx)^{3/2}}{x^2(c + dx)^3} dx, x, \frac{1}{x}\right) \\
&= \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-\frac{3}{2}a(bc-2ad) - \frac{1}{2}b(2bc-5ad)x}{x\sqrt{a+bx}(c+dx)^3} dx, x, \frac{1}{x}\right)}{c} \\
&= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{2c^2\left(c + \frac{d}{x}\right)^2} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{3a(bc-2ad)(bc-ad) + \frac{3}{2}b(bc-3ad)(bc-ad)x}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x}\right)}{2c^2(bc - ad)} \\
&= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{2c^2\left(c + \frac{d}{x}\right)^2} - \frac{3(bc - 4ad)\sqrt{a + \frac{b}{x}}}{4c^3\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{-3a(bc-2ad)(bc-ad)^2 - \frac{3}{4}b(bc-3ad)(bc-ad)^2x}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{2c^3(bc - ad)} \\
&= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{2c^2\left(c + \frac{d}{x}\right)^2} - \frac{3(bc - 4ad)\sqrt{a + \frac{b}{x}}}{4c^3\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} - \frac{(3a(bc - 2ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^4} \\
&= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{2c^2\left(c + \frac{d}{x}\right)^2} - \frac{3(bc - 4ad)\sqrt{a + \frac{b}{x}}}{4c^3\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} - \frac{(3a(bc - 2ad)) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \frac{1}{x}\right)}{bc^4} \\
&= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{2c^2\left(c + \frac{d}{x}\right)^2} - \frac{3(bc - 4ad)\sqrt{a + \frac{b}{x}}}{4c^3\left(c + \frac{d}{x}\right)} + \frac{a\sqrt{a + \frac{b}{x}}}{c\left(c + \frac{d}{x}\right)^2} - \frac{3(b^2c^2 - 8abcd + 8a^2d^2) \tan^{-1}\left(\frac{\sqrt{a}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4\sqrt{d}\sqrt{bc-ad}}
\end{aligned}$$

**Mathematica [A]** time = 0.60, size = 168, normalized size = 0.80

$$\frac{3(8a^2d^2 - 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}(2a(2c^2x^2 + 9cdx + 6d^2) - bc(5cx + 3d))}{(cx+d)^2} + 12\sqrt{a}(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(3/2)/(c + d/x)^3, x]

[Out] ((c\*Sqrt[a + b/x]\*x\*(-(b\*c\*(3\*d + 5\*c\*x)) + 2\*a\*(6\*d^2 + 9\*c\*d\*x + 2\*c^2\*x^2)))/(d + c\*x)^2 - (3\*(b^2\*c^2 - 8\*a\*b\*c\*d + 8\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(Sqrt[d]\*Sqrt[b\*c - a\*d]) + 12\*Sqrt[a]\*(b\*c - 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]/(4\*c^4)

**fricas [B]** time = 1.23, size = 1765, normalized size = 8.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x)^3, x, algorithm="fricas")

```
[Out] [-1/8*(12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2))*sqrt((a*x + b)/x))/(c*x + d) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x), -1/8*(24*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(-b*c*d + a*d^2)*log((b*d - (b*c - 2*a*d)*x + 2*sqrt(-b*c*d + a*d^2))*sqrt((a*x + b)/x))/(c*x + d) - 2*(4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 6*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x), 1/4*(3*(b^2*c^2*d^2 - 8*a*b*c*d^3 + 8*a^2*d^4 + (b^2*c^4 - 8*a*b*c^3*d + 8*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d - 8*a*b*c^2*d^2 + 8*a^2*c*d^3)*x)*sqrt(b*c*d - a*d^2)*arctan(sqrt(b*c*d - a*d^2)*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 12*(b^2*c^2*d^3 - 3*a*b*c*d^4 + 2*a^2*d^5 + (b^2*c^4*d - 3*a*b*c^3*d^2 + 2*a^2*c^2*d^3)*x^2 + 2*(b^2*c^3*d^2 - 3*a*b*c^2*d^3 + 2*a^2*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (4*(a*b*c^4*d - a^2*c^3*d^2)*x^3 - (5*b^2*c^4*d - 23*a*b*c^3*d^2 + 18*a^2*c^2*d^3)*x^2 - 3*(b^2*c^3*d^2 - 5*a*b*c^2*d^3 + 4*a^2*c*d^4)*x)*sqrt((a*x + b)/x))/(b*c^5*d^3 - a*c^4*d^4 + (b*c^7*d - a*c^6*d^2)*x^2 + 2*(b*c^6*d^2 - a*c^5*d^3)*x)]
```

**giac [B]** time = 0.61, size = 727, normalized size = 3.48

$$\frac{\sqrt{ax^2 + bx} \operatorname{asgn}(x)}{c^3} + \frac{3(b^2c^2\operatorname{sgn}(x) - 8abcd\operatorname{sgn}(x) + 8a^2d^2\operatorname{sgn}(x)) \arctan\left(\frac{(\sqrt{ax - \sqrt{ax^2 + bx}})c + \sqrt{ad}}{\sqrt{bcd - ad^2}}\right)}{4\sqrt{bcd - ad^2}c^4} - \frac{3(abcs\operatorname{sgn}(x))}{c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")
```

```
[Out] sqrt(a*x^2 + b*x)*a*sgn(x)/c^3 + 3/4*(b^2*c^2*sgn(x) - 8*a*b*c*d*sgn(x) + 8*a^2*d^2*sgn(x))*arctan(-((sqrt(a)*x - sqrt(a*x^2 + b*x))*c + sqrt(a)*d)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*c^4) - 3/2*(a*b*c*sgn(x) - 2*a^2*d*sgn(x))*log(abs(2*(sqrt(a)*x - sqrt(a*x^2 + b*x))*sqrt(a) + b))/(sqrt(a)*c^4) + 1/4*(3*sqrt(a)*b^2*c^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) - 24*a^(3/2)*b*c*d*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 24*a^(5/2)*d^2*arctan(sqrt(a)*d/sqrt(b*c*d - a*d^2)) + 6*sqrt(b*c*d - a*d^2)*a*b*c*log(abs(b)) - 12*sqrt(b*c*d - a*d^2)*a^2*d*log(abs(b)) + 5*sqrt(b*c*d - a*d^2)*a*b*c - 10*sqrt(b*c*d - a*d^2)*a^2*d*sgn(x)/(sqrt(b*c*d - a*d^2)*sqrt(a)*c^4) + 1/4*(5*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*sqrt(a)*b^2*c^3*sgn(x) - 24*(sqrt(a)*x - sqrt(a*x^2 + b*x))^3*a^(3/2)*b*c^2*d*sgn(x) + 24*(sqrt(a)*x - sqrt(a*x^2 +
```

$$b*x))^3*a^{(5/2)*c*d^2*\text{sgn}(x) - (\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))^{2*a*b^2*c^2*d*\text{sgn}(x) - 24*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))^{2*a^2*b*c*d^2*\text{sgn}(x) + 40*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))^{2*a^3*d^3*\text{sgn}(x) + 3*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))*\text{sqrt}(a)*b^3*c^2*d*\text{sgn}(x) - 28*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))*a^{(3/2)*b^2*c*d^2*\text{sgn}(x) + 40*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))*a^{(5/2)*b*d^3*\text{sgn}(x) - 5*a*b^3*c*d^2*\text{sgn}(x) + 10*a^2*b^2*d^3*\text{sgn}(x))/((\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))^{2*c} + 2*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))*\text{sqrt}(a)*d + b*d)^2*\text{sqrt}(a)*c^4}$$

**maple [B]** time = 0.07, size = 1817, normalized size = 8.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a+b/x)^{(3/2)}/(c+d/x)^3, x)$

[Out] 
$$-1/8*(27*a^{(5/2)*b^2*c^4*d^2*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+24*((a*d-b*c)/c^2*d)^{(1/2)*a^4*c^3*d^3*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})-36*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(7/2)*c^3*d^3*x-96*a^{(7/2)*b*c^2*d^4*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+54*a^{(5/2)*b^2*c^3*d^3*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+48*((a*d-b*c)/c^2*d)^{(1/2)*a^4*c^2*d^4*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+30*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*b*c^3*d^3-36*((a*d-b*c)/c^2*d)^{(1/2)*a^3*b*c^2*d^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+12*((a*d-b*c)/c^2*d)^{(1/2)*a^2*b^2*c^3*d^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})-6*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(3/2)*b^2*c^4*d^2+6*((a*x+b)*x)^{(3/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(3/2)*b*c^6*x-6*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(3/2)*b^2*c^6*x^2-3*a^{(3/2)*b^3*c^5*d*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+2*((a*x+b)*x)^{(3/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(3/2)*b*c^5*d+12*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(7/2)*c^5*d*x^3-6*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*b*c^6*x^3-12*((a*x+b)*x)^{(3/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*c^5*d*x-48*a^{(7/2)*b*c^3*d^3*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))-6*a^{(3/2)*b^3*c^4*d^2*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+24*a^{(9/2)*d^6*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+24*((a*d-b*c)/c^2*d)^{(1/2)*a^2*b^2*c^4*d^2*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+54*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*b*c^4*d^2*x-72*((a*d-b*c)/c^2*d)^{(1/2)*a^3*b*c^3*d^3*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+18*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*b*c^5*d*x^2-36*((a*d-b*c)/c^2*d)^{(1/2)*a^3*b*c^4*d^2*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+12*((a*d-b*c)/c^2*d)^{(1/2)*a^2*b^2*c^5*d*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})-12*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(3/2)*b^2*c^5*d*x+48*a^{(9/2)*c*d^5*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))-24*((a*x+b)*x)^{(1/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(7/2)*c^2*d^4-48*a^{(7/2)*b*c*d^5*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+27*a^{(5/2)*b^2*c^2*d^4*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))+24*((a*d-b*c)/c^2*d)^{(1/2)*a^4*c*d^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)*a^{(1/2)})/a^{(1/2)})+24*a^{(9/2)*c^2*d^4*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d))-8*((a*x+b)*x)^{(3/2)*((a*d-b*c)/c^2*d)^{(1/2)*a^{(5/2)*c^4*d^2-3*a^{(3/2)*b^3*c^3*d^3*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2))*((a*x+b)*x)^{(1/2)*c}/(c*x+d)))*x*((a*x+b)/x)^{(1/2)/c^5/d/((a*d-b*c)/c^2*d)^{(1/2)/a^{(3/2)/(c*x+d)^2/(a*d-b*c)/((a*x+b)*x)^{(1/2)}$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{\left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate((a + b/x)^(3/2)/(c + d/x)^3, x)

**mupad** [B] time = 3.47, size = 1664, normalized size = 7.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(3/2)/(c + d/x)^3,x)

[Out] - ((3\*(a + b/x)^(1/2)\*(3\*a\*b^3\*c^2 + 4\*a^3\*b\*d^2 - 7\*a^2\*b^2\*c\*d))/(4\*c^3) - ((a + b/x)^(3/2)\*(5\*b^3\*c^2 + 24\*a^2\*b\*d^2 - 24\*a\*b^2\*c\*d))/(4\*c^3) + (3\*b\*(a + b/x)^(5/2)\*(4\*a\*d^2 - b\*c\*d))/(4\*c^3))/((a + b/x)^2\*(3\*a\*d^2 - 2\*b\*c\*d) - (a + b/x)\*(3\*a^2\*d^2 + b^2\*c^2 - 4\*a\*b\*c\*d) - d^2\*(a + b/x)^3 + a^3\*d^2 + a\*b^2\*c^2 - 2\*a^2\*b\*c\*d) - (3\*a^(1/2)\*atanh((27\*a^(1/2)\*b^7\*d\*(a + b/x)^(1/2))/(8\*((27\*a\*b^7\*d)/8 - (27\*a^2\*b^6\*d^2)/(4\*c)))) + (27\*a^(3/2)\*b^6\*d^2\*(a + b/x)^(1/2))/(4\*((27\*a^2\*b^6\*d^2)/4 - (27\*a\*b^7\*c\*d)/8)))\*(2\*a\*d - b\*c))/c^4 - (atan((((((a + b/x)^(1/2)\*(9\*b^6\*c^4\*d + 1152\*a^4\*b^2\*d^5 - 144\*a\*b^5\*c^3\*d^2 - 1728\*a^3\*b^3\*c\*d^4 + 864\*a^2\*b^4\*c^2\*d^3))/(8\*c^6) - (3\*(d\*(a\*d - b\*c))^(1/2)\*((9\*a\*b^4\*c^9\*d^2 - 12\*a^2\*b^3\*c^8\*d^3)/c^9 - (3\*(64\*b^3\*c^9\*d^2 - 128\*a\*b^2\*c^8\*d^3)\*(a + b/x)^(1/2)\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(64\*c^6\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(8\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d)\*3i)/(8\*(a\*c^4\*d^2 - b\*c^5\*d)) + (((a + b/x)^(1/2)\*(9\*b^6\*c^4\*d + 1152\*a^4\*b^2\*d^5 - 144\*a\*b^5\*c^3\*d^2 - 1728\*a^3\*b^3\*c\*d^4 + 864\*a^2\*b^4\*c^2\*d^3))/(8\*c^6) + (3\*(d\*(a\*d - b\*c))^(1/2)\*((9\*a\*b^4\*c^9\*d^2 - 12\*a^2\*b^3\*c^8\*d^3)/c^9 + (3\*(64\*b^3\*c^9\*d^2 - 128\*a\*b^2\*c^8\*d^3)\*(a + b/x)^(1/2)\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(64\*c^6\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(8\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d)\*3i)/(8\*(a\*c^4\*d^2 - b\*c^5\*d)))/((216\*a^5\*b^3\*d^5 - 378\*a^4\*b^4\*c\*d^4 - (189\*a^2\*b^6\*c^3\*d^2)/4 + 216\*a^3\*b^5\*c^2\*d^3 + (27\*a\*b^7\*c^4\*d)/8)/c^9 - (3\*((a + b/x)^(1/2)\*(9\*b^6\*c^4\*d + 1152\*a^4\*b^2\*d^5 - 144\*a\*b^5\*c^3\*d^2 - 1728\*a^3\*b^3\*c\*d^4 + 864\*a^2\*b^4\*c^2\*d^3))/(8\*c^6) - (3\*(d\*(a\*d - b\*c))^(1/2)\*((9\*a\*b^4\*c^9\*d^2 - 12\*a^2\*b^3\*c^8\*d^3)/c^9 - (3\*(64\*b^3\*c^9\*d^2 - 128\*a\*b^2\*c^8\*d^3)\*(a + b/x)^(1/2)\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(64\*c^6\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(8\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/((216\*a^5\*b^3\*d^5 - 378\*a^4\*b^4\*c\*d^4 - (189\*a^2\*b^6\*c^3\*d^2)/4 + 216\*a^3\*b^5\*c^2\*d^3 + (27\*a\*b^7\*c^4\*d)/8)/c^9 - (3\*((a + b/x)^(1/2)\*(9\*b^6\*c^4\*d + 1152\*a^4\*b^2\*d^5 - 144\*a\*b^5\*c^3\*d^2 - 1728\*a^3\*b^3\*c\*d^4 + 864\*a^2\*b^4\*c^2\*d^3))/(8\*c^6) + (3\*(d\*(a\*d - b\*c))^(1/2)\*((9\*a\*b^4\*c^9\*d^2 - 12\*a^2\*b^3\*c^8\*d^3)/c^9 - (3\*(64\*b^3\*c^9\*d^2 - 128\*a\*b^2\*c^8\*d^3)\*(a + b/x)^(1/2)\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(64\*c^6\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d))/(8\*(a\*c^4\*d^2 - b\*c^5\*d)))\*(d\*(a\*d - b\*c))^(1/2)\*(8\*a^2\*d^2 + b^2\*c^2 - 8\*a\*b\*c\*d)\*3i)/(4\*(a\*c^4\*d^2 - b\*c^5\*d))

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(3/2)/(c+d/x)**3,x)
```

```
[Out] Timed out
```

$$3.238 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 dx$$

**Optimal.** Leaf size=198

$$a^{3/2}c^2(6ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(-10a^2d^2+135abcd+469b^2c^2)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2} - \frac{1}{3}c^2\left(a+\frac{b}{x}\right)$$

[Out]  $-1/3*c^2*(6*a*d+5*b*c)*(a+b/x)^{(3/2)}-11/9*d*(a+b/x)^{(5/2)*(c+d/x)^2-1/315*d*(a+b/x)^{(5/2)*(-20*a^2*d^2+270*a*b*c*d+938*b^2*c^2+5*b*d*(10*a*d+89*b*c)/x)/b^2+(a+b/x)^{(5/2)*(c+d/x)^3*x+a^{(3/2)*c^2*(6*a*d+5*b*c)*\arctanh((a+b/x)^{(1/2)/a^{(1/2))}-a*c^2*(6*a*d+5*b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.16, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 97, 153, 147, 50, 63, 208}

$$-\frac{d\left(a+\frac{b}{x}\right)^{5/2}\left(2(-10a^2d^2+135abcd+469b^2c^2)+\frac{5bd(10ad+89bc)}{x}\right)}{315b^2} + a^{3/2}c^2(6ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{1}{3}c^2\left(a+\frac{b}{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x)^3,x]

[Out]  $-(a*c^2*(5*b*c+6*a*d)*\text{Sqrt}[a+b/x]) - (c^2*(5*b*c+6*a*d)*(a+b/x)^{(3/2)})/3 - (11*d*(a+b/x)^{(5/2)*(c+d/x)^2)/9 - (d*(a+b/x)^{(5/2)*(2*(469*b^2*c^2+135*a*b*c*d-10*a^2*d^2)+(5*b*d*(89*b*c+10*a*d))/x)})/(315*b^2) + (a+b/x)^{(5/2)*(c+d/x)^3*x+a^{(3/2)*c^2*(5*b*c+6*a*d)*\text{ArcTanh}[\text{Sqrt}[a+b/x]/\text{Sqrt}[a]]$

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 97

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p)/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p - 1)\*Simp[d\*e\*n + c\*f\*p + d\*f\*(n + p)\*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

#### Rule 147

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
*((g_.) + (h_.)*(x_)), x_Symbol] := -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]

```

### Rule 153

```

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))
^(p_.)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[(h*(a + b*x)^m*(c + d*x)^(n
+ 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p
+ 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n +
p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p
+ 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /
; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p +
2, 0] && IntegerQ[m]

```

### Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

### Rule 375

```

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]

```

### Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 dx &= -\text{Subst} \left( \int \frac{(a+bx)^{5/2}(c+dx)^3}{x^2} dx, x, \frac{1}{x} \right) \\
&= \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 x - \text{Subst} \left( \int \frac{(a+bx)^{3/2}(c+dx)^2 \left(\frac{1}{2}(5bc+6ad) + \frac{11bdx}{2}\right)}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 + \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3 x - \frac{2 \text{Subst} \left( \int \frac{(a+bx)^{3/2}(c+dx) \left(\frac{9}{4}bc(5bc+6ad) + \frac{11bdx}{2}\right)}{x} dx, x, \frac{1}{x} \right)}{9b} \\
&= -\frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{5/2} \left(2(469b^2c^2 + 135abcd - 10a^2d^2) + \frac{5bd(89bc+6ad)}{x}\right)}{315b^2} \\
&= -\frac{1}{3}c^2(5bc+6ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{5/2} \left(2(469b^2c^2 + 135abcd - 10a^2d^2) + \frac{5bd(89bc+6ad)}{x}\right)}{315b^2} \\
&= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{5/2} \left(2(469b^2c^2 + 135abcd - 10a^2d^2) + \frac{5bd(89bc+6ad)}{x}\right)}{315b^2} \\
&= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{5/2} \left(2(469b^2c^2 + 135abcd - 10a^2d^2) + \frac{5bd(89bc+6ad)}{x}\right)}{315b^2} \\
&= -ac^2(5bc+6ad)\sqrt{a+\frac{b}{x}} - \frac{1}{3}c^2(5bc+6ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{11}{9}d \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 - \frac{d \left(a + \frac{b}{x}\right)^{5/2} \left(2(469b^2c^2 + 135abcd - 10a^2d^2) + \frac{5bd(89bc+6ad)}{x}\right)}{315b^2}
\end{aligned}$$

**Mathematica** [A] time = 0.25, size = 201, normalized size = 1.02

$$a^{3/2}c^2(6ad+5bc) \tanh^{-1} \left( \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}} \right) + \frac{\sqrt{a+\frac{b}{x}} (20a^4d^3x^4 - 10a^3bd^2x^3(27cx+d) - 3a^2b^2x^2(-105c^3x^3 + 966c^2dx^2 - 105c^3x^3) - 2b^4(35d^3 + 135cd^2x + 189c^2d^2x^2 + 105c^3x^3) - 2ab^3x(95d^3 + 405cd^2x + 693c^2d^2x^2 + 735c^3x^3))}{(315b^2x^4) + a^{(3/2)}c^2(5bc+6ad)} \text{ArcTanh} \left[ \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}} \right]$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)\*(c + d/x)^3,x]

[Out] (Sqrt[a + b/x]\*(20\*a^4\*d^3\*x^4 - 10\*a^3\*b\*d^2\*x^3\*(d + 27\*c\*x) - 3\*a^2\*b^2\*x^2\*(50\*d^3 + 270\*c\*d^2\*x + 966\*c^2\*d\*x^2 - 105\*c^3\*x^3) - 2\*b^4\*(35\*d^3 + 135\*c\*d^2\*x + 189\*c^2\*d\*x^2 + 105\*c^3\*x^3) - 2\*a\*b^3\*x\*(95\*d^3 + 405\*c\*d^2\*x + 693\*c^2\*d\*x^2 + 735\*c^3\*x^3))/(315\*b^2\*x^4) + a^(3/2)\*c^2\*(5\*b\*c + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**fricas** [A] time = 0.85, size = 494, normalized size = 2.49

$$\left[ \frac{315(5ab^3c^3 + 6a^2b^2c^2d)\sqrt{a}x^4 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(315a^2b^2c^3x^5 - 70b^4d^3 - 2(735ab^3c^3 + 1449a^2b^2c^2d))\sqrt{a}}{315b^2x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)\*(c+d/x)^3,x, algorithm="fricas")

```
[Out] [1/630*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(a)*x^4*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4), -1/315*(315*(5*a*b^3*c^3 + 6*a^2*b^2*c^2*d)*sqrt(-a)*x^4*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (315*a^2*b^2*c^3*x^5 - 70*b^4*d^3 - 2*(735*a*b^3*c^3 + 1449*a^2*b^2*c^2*d + 135*a^3*b*c*d^2 - 10*a^4*d^3)*x^4 - 2*(105*b^4*c^3 + 693*a*b^3*c^2*d + 405*a^2*b^2*c*d^2 + 5*a^3*b*d^3)*x^3 - 6*(63*b^4*c^2*d + 135*a*b^3*c*d^2 + 25*a^2*b^2*d^3)*x^2 - 10*(27*b^4*c*d^2 + 19*a*b^3*d^3)*x)*sqrt((a*x + b)/x))/(b^2*x^4)]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2
,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2
,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%
}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%
}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0
,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{
1,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2*sqrt
(a)*sqrt(b),1/2%%}+%%{-2*a,1%%}+%%{a*sqrt(a)*sqrt(b)/b,3/2%%}+%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%}+%%{undef,7/2%%})Evaluation time: 1.79Lim
it: Max order reached or unable to make series expansion Error: Bad Argumen
t Value
```

**maple** [B] time = 0.07, size = 457, normalized size = 2.31

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 1890a^3b^2c^2d x^6 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 1575a^2b^3c^3x^6 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) + 3780\sqrt{ax^2+bx} a^{\frac{7}{2}} \right)}{x^5/b^2/((a*x+b)*x)^{(1/2)}/a^{(1/2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x)^3,x)
```

```
[Out] 1/630*((a*x+b)/x)^(1/2)*(3780*(a*x^2+b*x)^(1/2)*a^(7/2)*x^6*b*c^2*d+3150*(a
*x^2+b*x)^(1/2)*a^(5/2)*x^6*b^2*c^3+40*(a*x^2+b*x)^(3/2)*a^(7/2)*x^3*d^3-37
80*(a*x^2+b*x)^(3/2)*a^(5/2)*x^4*b*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*
b*c*d^2-2520*(a*x^2+b*x)^(3/2)*a^(3/2)*x^4*b^2*c^3+1890*ln(1/2*(2*a*x+b+2*(
a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^3*b^2*c^2*d+1575*ln(1/2*(2*a*x+b+2
*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(1/2))*x^6*a^2*b^3*c^3-60*(a*x^2+b*x)^(3/2)*a
^(5/2)*x^2*b*d^3-2016*(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b^2*c^2*d-1080*(a*x^2+b
*x)^(3/2)*a^(3/2)*x^2*b^2*c*d^2-420*(a*x^2+b*x)^(3/2)*a^(1/2)*x^3*b^3*c^3-2
40*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b^2*d^3-756*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^
3*c^2*d-540*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b^3*c*d^2-140*(a*x^2+b*x)^(3/2)*a^(
1/2)*b^3*d^3)/x^5/b^2/((a*x+b)*x)^{(1/2)}/a^{(1/2)}
```

**maxima [A]** time = 1.21, size = 219, normalized size = 1.11

$$-\frac{6\left(a+\frac{b}{x}\right)^{\frac{7}{2}}cd^2}{7b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^3-\frac{1}{5}\left(15a^{\frac{5}{2}}\log\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)\*(c+d/x)^3,x, algorithm="maxima")

[Out] -6/7\*(a + b/x)^(7/2)\*c\*d^2/b + 1/6\*(6\*sqrt(a + b/x)\*a^2\*x - 15\*a^(3/2)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) - 4\*(a + b/x)^(3/2)\*b - 24\*sqrt(a + b/x)\*a\*b)\*c^3 - 1/5\*(15\*a^(5/2)\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a))) + 6\*(a + b/x)^(5/2) + 10\*(a + b/x)^(3/2)\*a + 30\*sqrt(a + b/x)\*a^2)\*c^2\*d - 2/63\*(7\*(a + b/x)^(9/2)/b^2 - 9\*(a + b/x)^(7/2)\*a/b^2)\*d^3

**mupad [B]** time = 6.05, size = 487, normalized size = 2.46

$$\left(a+\frac{b}{x}\right)^{7/2}\left(\frac{6ad^3-6bcd^2}{7b^2}-\frac{4ad^3}{7b^2}\right)-\sqrt{a+\frac{b}{x}}\left(a^2\left(2a\left(\frac{6ad^3-6bcd^2}{b^2}-\frac{4ad^3}{b^2}\right)-\frac{6d(ad-bc)^2}{b^2}+\frac{2a^2d^3}{b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(5/2)\*(c + d/x)^3,x)

[Out] (a + b/x)^(7/2)\*((6\*a\*d^3 - 6\*b\*c\*d^2)/(7\*b^2) - (4\*a\*d^3)/(7\*b^2)) - (a + b/x)^(1/2)\*(a^2\*(2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2) - (6\*d\*(a\*d - b\*c)^2)/b^2 + (2\*a^2\*d^3)/b^2) - 2\*a\*((2\*(a\*d - b\*c)^3)/b^2 + 2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2) - (6\*d\*(a\*d - b\*c)^2)/b^2 + (2\*a^2\*d^3)/b^2) - a^2\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2)) + (a + b/x)^(3/2)\*((2\*(a\*d - b\*c)^3)/(3\*b^2) + (2\*a\*(2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2) - (6\*d\*(a\*d - b\*c)^2)/b^2 + (2\*a^2\*d^3)/b^2))/3 - (a^2\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2))/3) + (a + b/x)^(5/2)\*((2\*a\*((6\*a\*d^3 - 6\*b\*c\*d^2)/b^2 - (4\*a\*d^3)/b^2))/5 - (6\*d\*(a\*d - b\*c)^2)/(5\*b^2) + (2\*a^2\*d^3)/(5\*b^2)) - (2\*d^3\*(a + b/x)^(9/2))/(9\*b^2) + a^2\*c^3\*x\*(a + b/x)^(1/2) - a^(3/2)\*c^2\*atan(((a + b/x)^(1/2)\*i)/a^(1/2))\*(6\*a\*d + 5\*b\*c)\*i

**sympy [A]** time = 158.70, size = 5513, normalized size = 27.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)\*(c+d/x)\*\*3,x)

[Out] 32\*a\*\*(29/2)\*b\*\*(27/2)\*d\*\*3\*x\*\*10\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 176\*a\*\*(27/2)\*b\*\*(29/2)\*d\*\*3\*x\*\*9\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 396\*a\*\*(25/2)\*b\*\*(31/2)\*d\*\*3\*x\*\*8\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)\*b\*\*15\*x\*\*(21/2) + 1890\*a\*\*(19/2)\*b\*\*16\*x\*\*(19/2) + 4725\*a\*\*(17/2)\*b\*\*17\*x\*\*(17/2) + 6300\*a\*\*(15/2)\*b\*\*18\*x\*\*(15/2) + 4725\*a\*\*(13/2)\*b\*\*19\*x\*\*(13/2) + 1890\*a\*\*(11/2)\*b\*\*20\*x\*\*(11/2) + 315\*a\*\*(9/2)\*b\*\*21\*x\*\*(9/2)) + 462\*a\*\*(23/2)\*b\*\*(33/2)\*d\*\*3\*x\*\*7\*sqrt(a\*x/b + 1)/(315\*a\*\*(21/2)

$$\begin{aligned}
& ) * b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} \\
& + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)} + 210 a^{(21/2)} b^{(35/2)} d^3 x^6 \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1 \\
& 890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)} \\
& ) - 32 a^{(21/2)} b^{(11/2)} d^3 x^6 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 378 a^{(19/2)} b^{(37/2)} d^3 x^5 \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)}) - 48 a^{(19/2)} b^{(13/2)} c d^2 x^6 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 80 a^{(19/2)} b^{(13/2)} d^3 x^5 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 1134 a^{(17/2)} b^{(39/2)} d^3 x^4 \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)}) - 120 a^{(17/2)} b^{(15/2)} c d^2 x^5 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 60 a^{(17/2)} b^{(15/2)} d^3 x^4 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 1494 a^{(15/2)} b^{(41/2)} d^3 x^3 \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)}) - 90 a^{(15/2)} b^{(17/2)} c d^2 x^4 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 80 a^{(15/2)} b^{(17/2)} d^3 x^3 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) + 4 a^{(15/2)} b^{(3/2)} d^3 x^3 \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)}) - 1098 a^{(13/2)} b^{(43/2)} d^3 x^2 \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)}) - 120 a^{(13/2)} b^{(19/2)} c d^2 x^3 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 200 a^{(13/2)} b^{(19/2)} d^3 x^2 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) + 24 a^{(13/2)} b^{(5/2)} c d^2 x^3 \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)}) + 2 a^{(13/2)} b^{(5/2)} d^3 x^2 \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)}) + 15 a^{(5/2)} b^{(41/2)} d^3 x \sqrt{a x/b + 1} / (315 a^{(21/2)} b^{15} x^{(21/2)} + 1890 a^{(19/2)} b^{16} x^{(19/2)} + 4725 a^{(17/2)} b^{17} x^{(17/2)} + 6300 a^{(15/2)} b^{18} x^{(15/2)} + 4725 a^{(13/2)} b^{19} x^{(13/2)} + 1890 a^{(11/2)} b^{20} x^{(11/2)} + 315 a^{(9/2)} b^{21} x^{(9/2)}) - 300 a^{(11/2)} b^{(21/2)} c d^2 x^2 \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) - 192 a^{(11/2)} b^{(21/2)} d^3 x \sqrt{a x/b + 1} / (105 a^{(13/2)} b^7 x^{(13/2)} + 315 a^{(11/2)} b^8 x^{(11/2)} + 315 a^{(9/2)} b^9 x^{(9/2)} + 105 a^{(7/2)} b^{10} x^{(7/2)}) + 12 a^{(11/2)} b^{(7/2)} c^2 d x^3 \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)}) + 12 a^{(11/2)} b^{(7/2)} c d^2 x^2 \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)}) - 8 a^{(11/2)} b^{(7/2)} d^3 x \sqrt{a x/b + 1} / (15 a^{(7/2)} b^3 x^{(7/2)} + 15 a^{(5/2)} b^4 x^{(5/2)})
\end{aligned}$$

$$\begin{aligned}
& ) - 70*a^{(9/2)}*b^{(47/2)}*d^{*3}*sqrt(a*x/b + 1)/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) - 288*a^{(9/2)}*b^{(23/2)}*c*d^{*2}*x*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 60*a^{(9/2)}*b^{(23/2)}*d^{*3}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 6*a^{(9/2)}*b^{(9/2)}*c^{*2}*d*x^{*2}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 48*a^{(9/2)}*b^{(9/2)}*c*d^{*2}*x*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 6*a^{(9/2)}*b^{(9/2)}*d^{*3}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 90*a^{(7/2)}*b^{(25/2)}*c*d^{*2}*sqrt(a*x/b + 1)/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 24*a^{(7/2)}*b^{(11/2)}*c^{*2}*d*x*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 36*a^{(7/2)}*b^{(11/2)}*c*d^{*2}*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) - 18*a^{(5/2)}*b^{(13/2)}*c^{*2}*d*sqrt(a*x/b + 1)/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) + a^{(3/2)}*b*c^{*3}*asinh(sqrt(a)*sqrt(x)/sqrt(b)) - 32*a^{*15}*b^{*13}*d^{*3}*x^{(21/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) - 192*a^{*14}*b^{*14}*d^{*3}*x^{(19/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) - 480*a^{*13}*b^{*15}*d^{*3}*x^{(17/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) - 480*a^{*11}*b^{*17}*d^{*3}*x^{(13/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) + 32*a^{*11}*b^{*5}*d^{*3}*x^{(13/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 192*a^{*10}*b^{*18}*d^{*3}*x^{(11/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) + 48*a^{*10}*b^{*6}*c*d^{*2}*x^{(13/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 96*a^{*10}*b^{*6}*d^{*3}*x^{(11/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 32*a^{*9}*b^{*19}*d^{*3}*x^{(9/2)}/(315*a^{(21/2)}*b^{*15}*x^{(21/2)} + 1890*a^{(19/2)}*b^{*16}*x^{(19/2)} + 4725*a^{(17/2)}*b^{*17}*x^{(17/2)} + 6300*a^{(15/2)}*b^{*18}*x^{(15/2)} + 4725*a^{(13/2)}*b^{*19}*x^{(13/2)} + 1890*a^{(11/2)}*b^{*20}*x^{(11/2)} + 315*a^{(9/2)}*b^{*21}*x^{(9/2)}) + 144*a^{*9}*b^{*7}*c*d^{*2}*x^{(11/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 96*a^{*9}*b^{*7}*d^{*3}*x^{(9/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 144*a^{*8}*b^{*8}*c*d^{*2}*x^{(9/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) + 32*a^{*8}*b^{*8}*d^{*3}*x^{(7/2)}/(105*a^{(13/2)}*b^{*7}*x^{(13/2)} + 315*a^{(11/2)}*b^{*8}*x^{(11/2)} + 315*a^{(9/2)}*b^{*9}*x^{(9/2)} + 105*a^{(7/2)}*b^{*10}*x^{(7/2)}) - 4*a^{*8}*b^{*d^{*3}*x^{(7/2)}/(15*a^{(7/2)}*b^{*3}*x^{(7/2)} + 15*a^{(5/2)}*b^{*4}*x^{(5/2)}) + 48*a^{*7}*b^{*9}
\end{aligned}$$



```

*c*d**2*x**(7/2)/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/
2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 24*a**7*b*
*2*c*d**2*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2))
- 4*a**7*b**2*d**3*x**(5/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x
**(5/2)) - 12*a**6*b**3*c**2*d*x**(7/2)/(15*a**(7/2)*b**3*x**(7/2) + 15*a**
(5/2)*b**4*x**(5/2)) - 24*a**6*b**3*c*d**2*x**(5/2)/(15*a**(7/2)*b**3*x**(7
/2) + 15*a**(5/2)*b**4*x**(5/2)) - 12*a**5*b**4*c**2*d*x**(5/2)/(15*a**(7/2
)*b**3*x**(7/2) + 15*a**(5/2)*b**4*x**(5/2)) - 6*a**3*c**2*d*atan(sqrt(a +
b/x)/sqrt(-a))/sqrt(-a) + a**2*sqrt(b)*c**3*sqrt(x)*sqrt(a*x/b + 1) - 4*a**
2*b*c**3*atan(sqrt(a + b/x)/sqrt(-a))/sqrt(-a) - 6*a**2*c**2*d*sqrt(a + b/x
) + 3*a**2*c*d**2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3
*b), True)) - 4*a*b*c**3*sqrt(a + b/x) + 6*a*b*c**2*d*Piecewise((-sqrt(a)/x
, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b**2*c**3*Piecewise((-sqr
t(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))

```

$$3.239 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 dx$$

**Optimal.** Leaf size=152

$$a^{3/2}c(4ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a} - \frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a} - \frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc) - ac\sqrt{a+\frac{b}{x}}$$

[Out]  $-1/3*c*(4*a*d+5*b*c)*(a+b/x)^{(3/2)} - 1/5*c*(4*a*d+5*b*c)*(a+b/x)^{(5/2)}/a - 2/7*d^2*(a+b/x)^{(7/2)}/b + c^2*(a+b/x)^{(7/2)*x}/a + a^{(3/2)*c*(4*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)}) - a*c*(4*a*d+5*b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 89, 80, 50, 63, 208}

$$a^{3/2}c(4ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{c^2x\left(a+\frac{b}{x}\right)^{7/2}}{a} - \frac{c\left(a+\frac{b}{x}\right)^{5/2}(4ad+5bc)}{5a} - \frac{1}{3}c\left(a+\frac{b}{x}\right)^{3/2}(4ad+5bc) - ac\sqrt{a+\frac{b}{x}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x)^2,x]

[Out]  $-(a*c*(5*b*c + 4*a*d)*\operatorname{Sqrt}[a + b/x]) - (c*(5*b*c + 4*a*d)*(a + b/x)^{(3/2)})/3 - (c*(5*b*c + 4*a*d)*(a + b/x)^{(5/2)})/(5*a) - (2*d^2*(a + b/x)^{(7/2)})/(7*b) + (c^2*(a + b/x)^{(7/2)*x})/a + a^{(3/2)*c*(5*b*c + 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(a + b\*x)^(m + 1)\*(c + d\*x)^n/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 80

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

### Rule 89

Int[((a\_.) + (b\_.)\*(x\_))^(2)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*c - a\*d)^2\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1)

```
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2 dx &= -\text{Subst}\left(\int \frac{(a+bx)^{5/2}(c+dx)^2}{x^2} dx, x, \frac{1}{x}\right) \\
&= \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2} \left(\frac{1}{2}c(5bc+4ad)+ad^2x\right)}{x} dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{(c(5bc+4ad)) \text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x} dx, x, \frac{1}{x}\right)}{2a} \\
&= -\frac{c(5bc+4ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{1}{2}(c(5bc+4ad)) \text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x} dx, x, \frac{1}{x}\right) \\
&= -\frac{1}{3}c(5bc+4ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} \\
&= -ac(5bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{1}{3}c(5bc+4ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} \\
&= -ac(5bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{1}{3}c(5bc+4ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a} \\
&= -ac(5bc+4ad)\sqrt{a + \frac{b}{x}} - \frac{1}{3}c(5bc+4ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{c(5bc+4ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b} + \frac{c^2 \left(a + \frac{b}{x}\right)^{7/2}}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.16, size = 121, normalized size = 0.80

$$\frac{c(4ad+5bc) \left( \sqrt{a + \frac{b}{x}} (23a^2x^2 + 11abx + 3b^2) - 15a^{5/2}x^2 \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) \right)}{15ax^2} + \frac{c^2x \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{2d^2 \left(a + \frac{b}{x}\right)^{7/2}}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)\*(c + d/x)^2,x]

[Out]  $(-2*d^2*(a + b/x)^{(7/2)})/(7*b) + (c^2*(a + b/x)^{(7/2)*x})/a - (c*(5*b*c + 4*a*d)*(Sqrt[a + b/x]*(3*b^2 + 11*a*b*x + 23*a^2*x^2) - 15*a^{(5/2)*x^2}*ArcTan[h[Sqrt[a + b/x]/Sqrt[a]]]))/(15*a*x^2)$

**fricas** [A] time = 1.02, size = 350, normalized size = 2.30

$$\frac{105(5ab^2c^2 + 4a^2bcd)\sqrt{a}x^3 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(105a^2bc^2x^4 - 30b^3d^2 - 2(245ab^2c^2 + 322a^2b^2cd + 15a^3d^2)x^3 - 2(35b^3c^2 + 154a*b^2*c*d + 45a^2*b*d^2)x^2 - 6(14b^3*c*d + 15a*b^2*d^2)*x)\sqrt{(a*x + b)/x}}{210bx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)\*(c+d/x)^2,x, algorithm="fricas")

[Out]  $[1/210*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(a)*x^3*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*(105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b^2*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3), -1/105*(105*(5*a*b^2*c^2 + 4*a^2*b*c*d)*sqrt(-a)*x^3*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (105*a^2*b*c^2*x^4 - 30*b^3*d^2 - 2*(245*a*b^2*c^2 + 322*a^2*b^2*c*d + 15*a^3*d^2)*x^3 - 2*(35*b^3*c^2 + 154*a*b^2*c*d + 45*a^2*b*d^2)*x^2 - 6*(14*b^3*c*d + 15*a*b^2*d^2)*x)*sqrt((a*x + b)/x))/(b*x^3)]$

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)\*(c+d/x)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)]  
Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sqrt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2\*sqrt(a)\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Evaluation time: 0.81Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

**maple** [B] time = 0.06, size = 336, normalized size = 2.21

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -420a^3bcdx^5 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 525a^2b^2c^2x^5 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 840\sqrt{ax^2+bx}a^{\frac{7}{2}}cdx^5 \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)\*(c+d/x)^2,x)

[Out] 
$$-1/210*((a*x+b)/x)^(1/2)/x^4/b*(-840*(a*x^2+b*x)^(1/2)*a^(7/2)*x^5*c*d-1050*(a*x^2+b*x)^(1/2)*a^(5/2)*x^5*b*c^2-420*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2))*a^(1/2))/a^(1/2))*x^5*a^3*b*c*d-525*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2))*a^(1/2))/a^(1/2))*x^5*a^2*b^2*c^2+840*(a*x^2+b*x)^(3/2)*a^(5/2)*x^3*c*d+840*(a*x^2+b*x)^(3/2)*a^(3/2)*x^3*b*c^2+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d^2+448*(a*x^2+b*x)^(3/2)*a^(3/2)*x^2*b*c*d+140*(a*x^2+b*x)^(3/2)*a^(1/2)*x^2*b^2*c^2+120*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d^2+168*(a*x^2+b*x)^(3/2)*a^(1/2)*x*b^2*c*d+60*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d^2)/((a*x+b)*x)^(1/2)/a^(1/2)$$

**maxima** [A] time = 1.25, size = 181, normalized size = 1.19

$$-\frac{2\left(a+\frac{b}{x}\right)^{\frac{7}{2}}d^2}{7b}+\frac{1}{6}\left(6\sqrt{a+\frac{b}{x}}a^2x-15a^{\frac{3}{2}}b\log\left(\frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}}\right)-4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b-24\sqrt{a+\frac{b}{x}}ab\right)c^2-\frac{2}{15}\left(15a^{\frac{5}{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)\*(c+d/x)^2,x, algorithm="maxima")

[Out] 
$$-2/7*(a + b/x)^(7/2)*d^2/b + 1/6*(6*\sqrt{a + b/x}*a^2*x - 15*a^(3/2)*b*\log(\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a})) - 4*(a + b/x)^(3/2)*b - 24*\sqrt{a + b/x}*a*b)*c^2 - 2/15*(15*a^(5/2)*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a})) + 6*(a + b/x)^(5/2) + 10*(a + b/x)^(3/2)*a + 30*\sqrt{a + b/x}*a^2)*c*d$$

**mupad** [B] time = 3.79, size = 271, normalized size = 1.78

$$\left(a+\frac{b}{x}\right)^{3/2}\left(\frac{2a\left(\frac{4ad^2-4bcd}{b}-\frac{4ad^2}{b}\right)}{3}-\frac{2(ad-bc)^2}{3b}+\frac{2a^2d^2}{3b}\right)+\left(\frac{4ad^2-4bcd}{5b}-\frac{4ad^2}{5b}\right)\left(a+\frac{b}{x}\right)^{5/2}-\sqrt{a+\frac{b}{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(5/2)\*(c + d/x)^2,x)

[Out] 
$$(a + b/x)^(3/2)*((2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b))/3 - (2*(a*d - b*c)^2)/(3*b) + (2*a^2*d^2)/(3*b)) + ((4*a*d^2 - 4*b*c*d)/(5*b) - (4*a*d^2)/(5*b))*(a + b/x)^(5/2) - (a + b/x)^(1/2)*(a^2*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - 2*a*(2*a*((4*a*d^2 - 4*b*c*d)/b - (4*a*d^2)/b) - (2*(a*d - b*c)^2)/b + (2*a^2*d^2)/b) - (2*d^2*(a + b/x)^(7/2))/(7*b) + a^2*c^2*x*(a + b/x)^(1/2) - a^(3/2)*c*atan((a + b/x)^(1/2)*i)/a^(1/2)*(4*a*d + 5*b*c)*i$$

**sympy** [A] time = 112.03, size = 1841, normalized size = 12.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)\*(c+d/x)\*\*2,x)

[Out] 
$$-16*a**(19/2)*b**(13/2)*d**2*x**6*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(17/2)*b**(15/2)*d**2*x**5*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 30*a**(15/2)*b**(17/2)*d**2*x**4*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(13/2)*b**(19/2)*d**2*x**3*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(11/2)*b**(17/2)*d**2*x**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(9/2)*b**(15/2)*d**2*x*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(7/2)*b**(13/2)*d**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(5/2)*b**(11/2)*d**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(3/2)*b**(9/2)*d**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*a**(1/2)*b**(7/2)*d**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2)) - 40*d**2*\sqrt{a*x/b + 1}/(105*a**(13/2)*b**7*x**(13/2) + 315*a**(11/2)*b**8*x**(11/2) + 315*a**(9/2)*b**9*x**(9/2) + 105*a**(7/2)*b**10*x**(7/2))$$

$$\begin{aligned}
& 0*x^{7/2}) + 8*a^{13/2}*b^{5/2}*d^{2*x^3*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 100*a^{11/2}*b^{21/2}*d^{2*x^2*\sqrt{a*x/b + 1}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) + 8*a^{11/2}*b^{7/2}*c*d*x^3*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) + 4*a^{11/2}*b^{7/2}*d^{2*x^2*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 96*a^{9/2}*b^{23/2}*d^{2*x*\sqrt{a*x/b + 1}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) + 4*a^{9/2}*b^{9/2}*c*d*x^2*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 16*a^{9/2}*b^{9/2}*d^{2*x*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 30*a^{7/2}*b^{25/2}*d^{2*\sqrt{a*x/b + 1}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) - 16*a^{7/2}*b^{11/2}*c*d*x*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 12*a^{7/2}*b^{11/2}*d^{2*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 12*a^{5/2}*b^{13/2}*c*d*\sqrt{a*x/b + 1}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) + a^{3/2}*b*c^{2*asinh(\sqrt{a}*\sqrt{x}/\sqrt{b})} + 16*a^{10}*b^{6*d^{2*x^{13/2}}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) + 48*a^{9}*b^{7*d^{2*x^{11/2}}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) + 48*a^{8}*b^{8*d^{2*x^{9/2}}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) + 16*a^{7}*b^{9*d^{2*x^{7/2}}}/(105*a^{13/2}*b^{7*x^{13/2}} + 315*a^{11/2}*b^{8*x^{11/2}} + 315*a^{9/2}*b^{9*x^{9/2}} + 105*a^{7/2}*b^{10*x^{7/2}}) - 8*a^{7}*b^{2*d^{2*x^{7/2}}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 8*a^{6}*b^{3*c*d*x^{7/2}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 8*a^{6}*b^{3*d^{2*x^{5/2}}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 8*a^{5}*b^{4*c*d*x^{5/2}}/(15*a^{7/2}*b^{3*x^{7/2}} + 15*a^{5/2}*b^{4*x^{5/2}}) - 4*a^{3}*c*d*atan(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} + a^{2*\sqrt{b}*c^{2*\sqrt{x}}*\sqrt{a*x/b + 1}} - 4*a^{2}*b*c^{2*atan(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a}} - 4*a^{2}*c*d*\sqrt{a + b/x} + a^{2*d^{2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))} - 4*a*b*c^{2*\sqrt{a + b/x}} + 4*a*b*c*d*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True)) + b^{2*c^{2*Piecewise((-sqrt(a)/x, Eq(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), True))}
\end{aligned}$$

$$3.240 \quad \int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right) dx$$

**Optimal.** Leaf size=125

$$a^{3/2}(2ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{\left(a+\frac{b}{x}\right)^{5/2} (2ad+5bc)}{5a} - \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} (2ad+5bc) - a\sqrt{a+\frac{b}{x}} (2ad+5bc) + \frac{cx}{a}$$

[Out]  $-1/3*(2*a*d+5*b*c)*(a+b/x)^{(3/2)}-1/5*(2*a*d+5*b*c)*(a+b/x)^{(5/2)}/a+c*(a+b/x)^{(7/2)*x/a+a^{(3/2)}*(2*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})-a*(2*a*d+5*b*c)*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {375, 78, 50, 63, 208}

$$a^{3/2}(2ad+5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) - \frac{\left(a+\frac{b}{x}\right)^{5/2} (2ad+5bc)}{5a} - \frac{1}{3}\left(a+\frac{b}{x}\right)^{3/2} (2ad+5bc) - a\sqrt{a+\frac{b}{x}} (2ad+5bc) + \frac{cx}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)\*(c + d/x), x]

[Out]  $-(a*(5*b*c + 2*a*d)*\operatorname{Sqrt}[a + b/x]) - ((5*b*c + 2*a*d)*(a + b/x)^{(3/2)})/3 - ((5*b*c + 2*a*d)*(a + b/x)^{(5/2)})/(5*a) + (c*(a + b/x)^{(7/2)*x}/a + a^{(3/2)}*(5*b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[  
((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/  
(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b,  
c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ  
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n  
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[  
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b +  
(d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ  
[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den  
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p  
\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(  
f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f  
\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x],  
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || Int  
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/  
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

## Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:= -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right) dx &= -\text{Subst} \left( \int \frac{(a + bx)^{5/2} (c + dx)}{x^2} dx, x, \frac{1}{x} \right) \\
&= \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{\left(\frac{5bc}{2} + ad\right) \text{Subst} \left( \int \frac{(a+bx)^{5/2}}{x} dx, x, \frac{1}{x} \right)}{a} \\
&= -\frac{(5bc + 2ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} + \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{1}{2} (5bc + 2ad) \text{Subst} \left( \int \frac{(a + bx)^{3/2}}{x} dx, x, \frac{1}{x} \right) \\
&= -\frac{1}{3} (5bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{(5bc + 2ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} + \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a} - \frac{1}{2} (a(5bc + 2ad) \sqrt{a + \frac{b}{x}}) \\
&= -a(5bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{1}{3} (5bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{(5bc + 2ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} + \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a} \\
&= -a(5bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{1}{3} (5bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{(5bc + 2ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} + \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a} \\
&= -a(5bc + 2ad) \sqrt{a + \frac{b}{x}} - \frac{1}{3} (5bc + 2ad) \left(a + \frac{b}{x}\right)^{3/2} - \frac{(5bc + 2ad) \left(a + \frac{b}{x}\right)^{5/2}}{5a} + \frac{c \left(a + \frac{b}{x}\right)^{7/2}}{a}
\end{aligned}$$

**Mathematica [A]** time = 0.11, size = 94, normalized size = 0.75

$$a^{3/2}(2ad+5bc) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) + \frac{\sqrt{a + \frac{b}{x}} (a^2 x^2 (15cx - 46d) - 2abx(35cx + 11d) - 2b^2(5cx + 3d))}{15x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/x)^(5/2)*(c + d/x), x]
```

```
[Out] (Sqrt[a + b/x]*(-2*b^2*(3*d + 5*c*x) + a^2*x^2*(-46*d + 15*c*x) - 2*a*b*x*(11*d + 35*c*x)))/(15*x^2) + a^(3/2)*(5*b*c + 2*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]
```

**fricas [A]** time = 0.96, size = 222, normalized size = 1.78

$$\left[ \frac{15(5abc + 2a^2d)\sqrt{a}x^2 \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(15a^2cx^3 - 6b^2d - 2(35abc + 23a^2d)x^2 - 2(5b^2c + 3a^2d)x)}{30x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="fricas")
```



```
[Out] [1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x
+ b)/x) + b) + 2*(15*a^2*c*x^3 - 6*b^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2
*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x + b)/x))/x^2, -1/15*(15*(5*a*b*c + 2*a^2
*d)*sqrt(-a)*x^2*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (15*a^2*c*x^3 - 6*b
^2*d - 2*(35*a*b*c + 23*a^2*d)*x^2 - 2*(5*b^2*c + 11*a*b*d)*x)*sqrt((a*x +
b)/x))/x^2]
```

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2
,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2
,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86
,-97,-82]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%
}+%%{-2,[0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%
}+%%{2,[1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters v
alues [7,-27,26]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0
,1,1]%%},0,%%{1,[0,2,2]%%}] at parameters values [18.6420984049,-49,-86]
Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{
1,[0,2,2]%%}] at parameters values [78.6493344628,22,42]Warning, choosing
root of [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at
parameters values [-13,74.7709350525,24]Sign error (%%{-b,0%%}+%%{2*sqrt
(a)*sqrt(b),1/2%%}+%%{-2*a,1%%}+%%{a*sqrt(a)*sqrt(b)/b,3/2%%}+%%{-a^2
*sqrt(a)*sqrt(b)/(4*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached
or unable to make series expansion Error: Bad Argument Value
```

**maple** [B] time = 0.06, size = 253, normalized size = 2.02

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -30a^3bdx^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 75a^2b^2cx^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 60\sqrt{ax^2+bx} a^{\frac{7}{2}}dx^4 - 15 \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(5/2)*(c+d/x),x)
```

```
[Out] -1/30*((a*x+b)/x)^(1/2)/x^3/b*(-60*(a*x^2+b*x)^(1/2)*a^(7/2)*x^4*d-150*(a*x
^2+b*x)^(1/2)*a^(5/2)*x^4*b*c-30*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2
))/a^(1/2))*x^4*a^3*b*d-75*ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^(1/2)*a^(1/2))/a^(
1/2))*x^4*a^2*b^2*c+60*(a*x^2+b*x)^(3/2)*a^(5/2)*x^2*d+120*(a*x^2+b*x)^(3/2
)*a^(3/2)*x^2*b*c+32*(a*x^2+b*x)^(3/2)*a^(3/2)*x*b*d+20*(a*x^2+b*x)^(3/2)*a
^(1/2)*x*b^2*c+12*(a*x^2+b*x)^(3/2)*a^(1/2)*b^2*d)/((a*x+b)*x)^(1/2)/a^(1/2
)
```

**maxima** [A] time = 1.51, size = 161, normalized size = 1.29

$$\frac{1}{6} \left( 6\sqrt{a+\frac{b}{x}} a^2x - 15a^{\frac{3}{2}}b \log\left(\frac{\sqrt{a+\frac{b}{x}} - \sqrt{a}}{\sqrt{a+\frac{b}{x}} + \sqrt{a}}\right) - 4\left(a+\frac{b}{x}\right)^{\frac{3}{2}}b - 24\sqrt{a+\frac{b}{x}}ab \right) c - \frac{1}{15} \left( 15a^{\frac{5}{2}} \log\left(\frac{\sqrt{a+\frac{b}{x}} - \sqrt{a}}{\sqrt{a+\frac{b}{x}} + \sqrt{a}}\right) - 15a^{\frac{3}{2}}b \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(5/2)*(c+d/x),x, algorithm="maxima")
```

[Out]  $\frac{1}{6}*(6*\sqrt{a + b/x}*a^2*x - 15*a^{(3/2)}*b*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))) - 4*(a + b/x)^{(3/2)}*b - 24*\sqrt{a + b/x}*a*b)*c - 1/15*(15*a^{(5/2)}*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))) + 6*(a + b/x)^{(5/2)} + 10*(a + b/x)^{(3/2)}*a + 30*\sqrt{a + b/x}*a^2)*d$

**mupad [B]** time = 3.48, size = 99, normalized size = 0.79

$$-\frac{2d\left(a + \frac{b}{x}\right)^{5/2}}{5} - 2a^2d\sqrt{a + \frac{b}{x}} - \frac{2ad\left(a + \frac{b}{x}\right)^{3/2}}{3} - \frac{2cx\left(a + \frac{b}{x}\right)^{5/2}}{3\left(\frac{ax}{b} + 1\right)^{5/2}} - a^{5/2}d\operatorname{atan}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(5/2)*(c + d/x), x)`

[Out]  $-(2*d*(a + b/x)^{(5/2)})/5 - 2*a^2*d*(a + b/x)^{(1/2)} - a^{(5/2)}*d*\operatorname{atan}\left(\frac{(a + b/x)^{(1/2)}*1i}{a^{(1/2)}}\right)*2i - (2*a*d*(a + b/x)^{(3/2)})/3 - (2*c*x*(a + b/x)^{(5/2)}*\operatorname{hypergeom}([-5/2, -3/2], -1/2, -(a*x)/b))/(3*((a*x)/b + 1)^{(5/2)})$

**sympy [A]** time = 82.77, size = 520, normalized size = 4.16

$$\frac{4a^{\frac{11}{2}}b^{\frac{7}{2}}dx^3\sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + \frac{2a^{\frac{9}{2}}b^{\frac{9}{2}}dx^2\sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{8a^{\frac{7}{2}}b^{\frac{11}{2}}dx\sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} - \frac{6a^{\frac{5}{2}}b^{\frac{13}{2}}d\sqrt{\frac{ax}{b} + 1}}{15a^{\frac{7}{2}}b^3x^{\frac{7}{2}} + 15a^{\frac{5}{2}}b^4x^{\frac{5}{2}}} + a^{\frac{3}{2}}bc\operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(5/2)*(c+d/x), x)`

[Out]  $4*a^{(11/2)}*b^{(7/2)}*d*x**3*\sqrt{a*x/b + 1}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) + 2*a^{(9/2)}*b^{(9/2)}*d*x**2*\sqrt{a*x/b + 1}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) - 8*a^{(7/2)}*b^{(11/2)}*d*x*\sqrt{a*x/b + 1}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) - 6*a^{(5/2)}*b^{(13/2)}*d*\sqrt{a*x/b + 1}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) + a^{(3/2)}*b*c*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b}) - 4*a^{(5/2)}*b^{(3/2)}*d*x^{(7/2)}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) - 4*a^{(5/2)}*b^{(4/2)}*d*x^{(5/2)}/(15*a^{(7/2)}*b^{(3/2)}*x^{(7/2)} + 15*a^{(5/2)}*b^{(4/2)}*x^{(5/2)}) - 2*a^{(3/2)}*d*\operatorname{atan}(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} + a^{(2/2)}*\sqrt{b}*c*\sqrt{x}*\sqrt{a*x/b + 1} - 4*a^{(2/2)}*b*c*\operatorname{atan}(\sqrt{a + b/x}/\sqrt{-a})/\sqrt{-a} - 2*a^{(2/2)}*d*\sqrt{a + b/x} - 4*a*b*c*\sqrt{a + b/x} + 2*a*b*d*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True})) + b^{(2/2)}*c*\operatorname{Piecewise}((-sqrt(a)/x, \operatorname{Eq}(b, 0)), (-2*(a + b/x)**(3/2)/(3*b), \operatorname{True}))$

$$3.241 \quad \int \left(a + \frac{b}{x}\right)^{5/2} dx$$

**Optimal.** Leaf size=71

$$5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x\left(a + \frac{b}{x}\right)^{5/2} - \frac{5}{3}b\left(a + \frac{b}{x}\right)^{3/2} - 5ab\sqrt{a + \frac{b}{x}}$$

[Out]  $-5/3*b*(a+b/x)^{(3/2)}+(a+b/x)^{(5/2)}*x+5*a^{(3/2)}*b*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})-5*a*b*(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$ , Rules used = {242, 47, 50, 63, 208}

$$5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + x\left(a + \frac{b}{x}\right)^{5/2} - \frac{5}{3}b\left(a + \frac{b}{x}\right)^{3/2} - 5ab\sqrt{a + \frac{b}{x}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2), x]

[Out]  $-5*a*b*\operatorname{Sqrt}[a + b/x] - (5*b*(a + b/x)^{(3/2)})/3 + (a + b/x)^{(5/2)}*x + 5*a^{(3/2)}*b*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]]$

#### Rule 47

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + 1)), x] - Dist[(d\*n)/(b\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2\*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 50

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^n)/(b\*(m + n + 1)), x] + Dist[(n\*(b\*c - a\*d))/(b\*(m + n + 1)), Int[(a + b\*x)^m\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 242

`Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]`

### Rubi steps

$$\begin{aligned}
 \int \left(a + \frac{b}{x}\right)^{5/2} dx &= -\text{Subst}\left(\int \frac{(a + bx)^{5/2}}{x^2} dx, x, \frac{1}{x}\right) \\
 &= \left(a + \frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5b) \text{Subst}\left(\int \frac{(a + bx)^{3/2}}{x} dx, x, \frac{1}{x}\right) \\
 &= -\frac{5}{3}b \left(a + \frac{b}{x}\right)^{3/2} + \left(a + \frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5ab) \text{Subst}\left(\int \frac{\sqrt{a + bx}}{x} dx, x, \frac{1}{x}\right) \\
 &= -5ab\sqrt{a + \frac{b}{x}} - \frac{5}{3}b \left(a + \frac{b}{x}\right)^{3/2} + \left(a + \frac{b}{x}\right)^{5/2} x - \frac{1}{2}(5a^2b) \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x}\right) \\
 &= -5ab\sqrt{a + \frac{b}{x}} - \frac{5}{3}b \left(a + \frac{b}{x}\right)^{3/2} + \left(a + \frac{b}{x}\right)^{5/2} x - (5a^2) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right) \\
 &= -5ab\sqrt{a + \frac{b}{x}} - \frac{5}{3}b \left(a + \frac{b}{x}\right)^{3/2} + \left(a + \frac{b}{x}\right)^{5/2} x + 5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 64, normalized size = 0.90

$$5a^{3/2}b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) + \frac{\sqrt{a + \frac{b}{x}}(3a^2x^2 - 14abx - 2b^2)}{3x}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2), x]

[Out] (Sqrt[a + b/x]\*(-2\*b^2 - 14\*a\*b\*x + 3\*a^2\*x^2))/(3\*x) + 5\*a^(3/2)\*b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]

**fricas [A]** time = 0.94, size = 139, normalized size = 1.96

$$\left[ \frac{15a^{\frac{3}{2}}bx \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(3a^2x^2 - 14abx - 2b^2)\sqrt{\frac{ax+b}{x}}}{6x}, -\frac{15\sqrt{-a}abx \arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{3x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2), x, algorithm="fricas")

[Out] [1/6\*(15\*a^(3/2)\*b\*x\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(3\*a^2\*x^2 - 14\*a\*b\*x - 2\*b^2)\*sqrt((a\*x + b)/x))/x, -1/3\*(15\*sqrt(-a)\*a\*b\*x\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (3\*a^2\*x^2 - 14\*a\*b\*x - 2\*b^2)\*sqrt((a\*x + b)/x))/x]

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes  
 constant sign by intervals (correct if the argument is real):Check [abs(x)]  
 Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2  
 , [0,1,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{2  
 , [1,3,1]%%}+%%{-2,[1,1,1]%%}+%%{1,[0,2,2]%%}] at parameters values [86  
 ,-97,-82]Warning, choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%  
 %},0,%%{1,[0,2,2]%%}] at parameters values [82.1195442914,26,-89]Warning,  
 choosing root of [1,0,%%{-4,[1,0,0]%%}+%%{-2,[0,1,1]%%},0,%%{1,[0,2,2  
 ]%%}] at parameters values [85.3561567818,-64,-30]Warning, choosing root o  
 f [1,0,%%{-2,[1,0,1]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,2]%%}] at parame  
 ters values [42,43.9628838282,-9]Sign error (%%{-b,0%%}+%%{2\*sqrt(a)\*sq  
 rt(b),1/2%%}+%%{-2\*a,1%%}+%%{a\*sqrt(a)\*sqrt(b)/b,3/2%%}+%%{-a^2\*sqrt(a  
 )\*sqrt(b)/(4\*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached or una  
 ble to make series expansion Error: Bad Argument Value

**maple** [B] time = 0.06, size = 120, normalized size = 1.69

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 15a^2b x^3 \ln \left( \frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}} \right) + 30\sqrt{ax^2+bx} a^{\frac{5}{2}} x^3 - 24(ax^2+bx)^{\frac{3}{2}} a^{\frac{3}{2}} x - 4(ax^2+bx)^{\frac{3}{2}} \sqrt{a} b \right)}{6\sqrt{(ax+b)x}\sqrt{a}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2),x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)\*(30\*a^(5/2)\*(a\*x^2+b\*x)^(1/2)\*x^3-24\*a^(3/2)\*(a\*x^2+b  
 \*x)^(3/2)\*x+15\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^3\*a^  
 2\*b-4\*b\*(a\*x^2+b\*x)^(3/2)\*a^(1/2))/x^2/((a\*x+b)\*x)^(1/2)/a^(1/2)

**maxima** [A] time = 1.25, size = 78, normalized size = 1.10

$$\sqrt{a + \frac{b}{x}} a^2 x - \frac{5}{2} a^{\frac{3}{2}} b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right) - \frac{2}{3} \left( a + \frac{b}{x} \right)^{\frac{3}{2}} b - 4 \sqrt{a + \frac{b}{x}} ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2),x, algorithm="maxima")

[Out] sqrt(a + b/x)\*a^2\*x - 5/2\*a^(3/2)\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a +  
 b/x) + sqrt(a))) - 2/3\*(a + b/x)^(3/2)\*b - 4\*sqrt(a + b/x)\*a\*b

**mupad** [B] time = 1.63, size = 34, normalized size = 0.48

$$\frac{2x \left( a + \frac{b}{x} \right)^{5/2} {}_2F_1 \left( -\frac{5}{2}, -\frac{3}{2}; -\frac{1}{2}; -\frac{ax}{b} \right)}{3 \left( \frac{ax}{b} + 1 \right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(5/2),x)

[Out] -(2\*x\*(a + b/x)^(5/2)\*hypergeom([-5/2, -3/2], -1/2, -(a\*x)/b))/(3\*((a\*x)/b  
 + 1)^(5/2))

**sympy** [A] time = 4.36, size = 99, normalized size = 1.39

$$a^{\frac{5}{2}} x \sqrt{1 + \frac{b}{ax}} - \frac{14a^{\frac{3}{2}} b \sqrt{1 + \frac{b}{ax}}}{3} - \frac{5a^{\frac{3}{2}} b \log \left( \frac{b}{ax} \right)}{2} + 5a^{\frac{3}{2}} b \log \left( \sqrt{1 + \frac{b}{ax}} + 1 \right) - \frac{2\sqrt{a} b^2 \sqrt{1 + \frac{b}{ax}}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(5/2),x)
```

```
[Out] a**(5/2)*x*sqrt(1 + b/(a*x)) - 14*a**(3/2)*b*sqrt(1 + b/(a*x))/3 - 5*a**(3/2)*b*log(b/(a*x))/2 + 5*a**(3/2)*b*log(sqrt(1 + b/(a*x)) + 1) - 2*sqrt(a)*b**2*sqrt(1 + b/(a*x))/(3*x)
```

$$3.242 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{c + \frac{d}{x}} dx$$

**Optimal.** Leaf size=134

$$\frac{a^{3/2}(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{2(bc - ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2 d^{3/2}} - \frac{b\sqrt{a + \frac{b}{x}}(ad + 2bc)}{cd} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c}$$

[Out]  $a*(a+b/x)^{(3/2)*x}/c+2*(-a*d+b*c)^{(5/2)*\arctan(d^{(1/2)}*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)})}/c^2/d^{(3/2)+a^{(3/2)*(-2*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)/a^{(1/2)})}/c^2-b*(a*d+2*b*c)*(a+b/x)^{(1/2)/c/d}$

**Rubi [A]** time = 0.22, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 98, 154, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{c^2} + \frac{2(bc - ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}}\right)}{c^2 d^{3/2}} - \frac{b\sqrt{a + \frac{b}{x}}(ad + 2bc)}{cd} + \frac{ax\left(a + \frac{b}{x}\right)^{3/2}}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)/(c + d/x), x]

[Out]  $-((b*(2*b*c + a*d)*\operatorname{Sqrt}[a + b/x])/(c*d)) + (a*(a + b/x)^{(3/2)*x}/c + (2*(b*c - a*d)^{(5/2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(c^2*d^{(3/2)}) + (a^{(3/2)*(5*b*c - 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/c^2$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 98

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] + Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 2)\*(e + f\*x)^p\*Simp[a\*d\*(d\*e\*(n - 1) + c\*f\*(p + 1)) + b\*c\*(d\*e\*(m - n + 2) - c\*f\*(m + p + 2)) + d\*(a\*d\*f\*(n + p) + b\*(d\*e\*(m + 1) - c\*f\*(m + n + p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 154

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(h\*(a + b\*x)^m\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 2)), x] + Dist[1/(d\*f\*(m + n + p + 2)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*g\*(m + n + p + 2) - h\*(b\*c\*e\*m + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + (b\*d\*f\*g\*(m + n + p + 2) + h\*(a\*d\*f\*m - b\*(d\*e\*(m + n + 1) + c\*f\*(m + p + 1)))]\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2\*m, 2\*n, 2\*p]

Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{c + \frac{d}{x}} dx = -\text{Subst}\left(\int \frac{(a + bx)^{5/2}}{x^2(c + dx)} dx, x, \frac{1}{x}\right)$$

$$= \frac{a\left(a + \frac{b}{x}\right)^{3/2}}{c} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-2ad)-\frac{1}{2}b(2bc+ad)x\right)}{x(c+dx)} dx, x, \frac{1}{x}\right)}{c}$$

$$= -\frac{b(2bc + ad)\sqrt{a + \frac{b}{x}}}{cd} + \frac{a\left(a + \frac{b}{x}\right)^{3/2}}{c} + \frac{2\text{Subst}\left(\int \frac{-\frac{1}{4}a^2d(5bc-2ad)+\frac{1}{4}b(2b^2c^2-6abcd+a^2d^2)x}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{cd}$$

$$= -\frac{b(2bc + ad)\sqrt{a + \frac{b}{x}}}{cd} + \frac{a\left(a + \frac{b}{x}\right)^{3/2}}{c} - \frac{(a^2(5bc - 2ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^2} + \frac{(bc - ad)}{c^2}$$

$$= -\frac{b(2bc + ad)\sqrt{a + \frac{b}{x}}}{cd} + \frac{a\left(a + \frac{b}{x}\right)^{3/2}}{c} - \frac{(a^2(5bc - 2ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^2} + \frac{(bc - ad)}{c^2}$$

$$= -\frac{b(2bc + ad)\sqrt{a + \frac{b}{x}}}{cd} + \frac{a\left(a + \frac{b}{x}\right)^{3/2}}{c} + \frac{2(bc - ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2d^{3/2}} + \frac{a^{3/2}(5bc - 2ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2}$$

Mathematica [A] time = 0.27, size = 116, normalized size = 0.87

$$\frac{a^{3/2}(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{c\sqrt{a+\frac{b}{x}}(a^2dx-2b^2c)}{d} + \frac{2(bc-ad)^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{d^{3/2}}}{c^2}$$

Antiderivative was successfully verified.





2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*((a\*d-b\*c)/c^2\*d)^(1/2)\*x^2\*a\*b^2\*c^3\*d-ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*((a\*d-b\*c)/c^2\*d)^(1/2)\*x^2\*b^3\*c^4-8\*((a\*d-b\*c)/c^2\*d)^(1/2)\*(a\*x^2+b\*x)^(1/2)\*a^(3/2)\*x^2\*b\*c^3\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*(a\*x^2+b\*x)^(1/2)\*a^(1/2)\*x^2\*b^2\*c^4-4\*((a\*d-b\*c)/c^2\*d)^(1/2)\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a\*b^2\*c^3\*d+((a\*d-b\*c)/c^2\*d)^(1/2)\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*b^3\*c^4-2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(5/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*c^2\*d^2+4\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(3/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b\*c^3\*d-2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*a^(1/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b^2\*c^4+2\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*a^(7/2)\*x^2\*d^4-6\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*a^(5/2)\*x^2\*b\*c\*d^3+6\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*a^(3/2)\*x^2\*b^2\*c^2\*d^2-2\*ln((-2\*a\*d\*x+b\*c\*x-b\*d+2\*((a\*d-b\*c)/c^2\*d)^(1/2)\*((a\*x+b)\*x)^(1/2)\*c)/(c\*x+d))\*a^(1/2)\*x^2\*b^3\*c^3\*d+4\*((a\*d-b\*c)/c^2\*d)^(1/2)\*(a\*x^2+b\*x)^(3/2)\*a^(1/2)\*b\*c^3\*d)/((a\*x+b)\*x)^(1/2)/d^2/c^3/a^(1/2)/((a\*d-b\*c)/c^2\*d)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)/(c+d/x),x, algorithm="maxima")

[Out] integrate((a + b/x)^(5/2)/(c + d/x), x)

**mupad** [B] time = 2.16, size = 1427, normalized size = 10.65

$$\frac{a^2 b d \sqrt{a + \frac{b}{x}}}{c \left(d \left(a + \frac{b}{x}\right) - a d\right)} - \frac{2 b^2 \sqrt{a + \frac{b}{x}}}{d} + \frac{\operatorname{atan}\left(\frac{a^3 b^5 \sqrt{a + \frac{b}{x}} \sqrt{a^5 d^8 - 5 a^4 b c d^7 + 10 a^3 b^2 c^2 d^6 - 10 a^2 b^3 c^3 d^5 + 5 a b^4 c^4 d^4 - b^5 c^5 d^3} - 160i}{448 a^3 b^8 c^3 d - 340 a^6 b^5 d^4 - 128 a^2 b^9 c^4 + 740 a^5 b^6 c d^3 + \frac{16 a b^{10} c^5}{d} - 796 a^4 b^7 c^2 d^2 + \frac{60 a^7 b^4 d^5}{c}}\right)}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(5/2)/(c + d/x),x)

[Out] (atan((a^3\*b^5\*(a + b/x)^(1/2)\*(a^5\*d^8 - b^5\*c^5\*d^3 + 5\*a\*b^4\*c^4\*d^4 - 10\*a^2\*b^3\*c^3\*d^5 + 10\*a^3\*b^2\*c^2\*d^6 - 5\*a^4\*b\*c\*d^7)^(1/2)\*160i)/(448\*a^3\*b^8\*c^3\*d - 340\*a^6\*b^5\*d^4 - 128\*a^2\*b^9\*c^4 + 740\*a^5\*b^6\*c\*d^3 + (16\*a\*b^10\*c^5)/d - 796\*a^4\*b^7\*c^2\*d^2 + (60\*a^7\*b^4\*d^5)/c) - (a^2\*b^6\*(a + b/x)^(1/2)\*(a^5\*d^8 - b^5\*c^5\*d^3 + 5\*a\*b^4\*c^4\*d^4 - 10\*a^2\*b^3\*c^3\*d^5 + 10\*a^3\*b^2\*c^2\*d^6 - 5\*a^4\*b\*c\*d^7)^(1/2)\*80i)/(16\*a\*b^10\*c^4 + 740\*a^5\*b^6\*d^4 - 128\*a^2\*b^9\*c^3\*d - 796\*a^4\*b^7\*c\*d^3 + 448\*a^3\*b^8\*c^2\*d^2 - (340\*a^6\*b^5\*d^5)/c + (60\*a^7\*b^4\*d^6)/c^2) - (a^4\*b^4\*(a + b/x)^(1/2)\*(a^5\*d^8 - b^5\*c^5\*d^3 + 5\*a\*b^4\*c^4\*d^4 - 10\*a^2\*b^3\*c^3\*d^5 + 10\*a^3\*b^2\*c^2\*d^6 - 5\*a^4\*b\*c\*d^7)^(1/2)\*60i)/(448\*a^3\*b^8\*c^4 + 60\*a^7\*b^4\*d^4 - 796\*a^4\*b^7\*c^3\*d - 340\*a^6\*b^5\*c\*d^3 + (16\*a\*b^10\*c^6)/d^2 + 740\*a^5\*b^6\*c^2\*d^2 - (128\*a^2\*b^9\*c^5)/d) + (a\*b^7\*c\*(a + b/x)^(1/2)\*(a^5\*d^8 - b^5\*c^5\*d^3 + 5\*a\*b^4\*c^4\*d^4 - 10\*a^2\*b^3\*c^3\*d^5 + 10\*a^3\*b^2\*c^2\*d^6 - 5\*a^4\*b\*c\*d^7)^(1/2)\*16i)/(740\*a^5\*b^6\*d^5 - 796\*a^4\*b^7\*c\*d^4 - 128\*a^2\*b^9\*c^3\*d^2 + 448\*a^3\*b^8\*c^2\*d^3 - (340\*a^6\*b^5\*d^6)/c + (60\*a^7\*b^4\*d^7)/c^2 + 16\*a\*b^10\*c^4\*d)\*(d^3\*(a\*d - b\*c)^5)^(1/2)\*2i)/(c^2\*d^3) - (2\*b^2\*(a + b/x)^(1/2))/d + (atan((b^9\*c^3\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*40i)/(40\*a^2\*b^9\*c^3 - 790\*a^5\*b^6\*d^3 - 256\*a^3\*b^8\*c^2\*d + 696\*a^4\*b^7\*c\*d^2 + (370\*a^6\*b^5\*d^4)/c - (60\*a^7\*b^4\*d^5)/c^2) + (a\*b^8\*c^2\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*256i)/(256\*a^3\*b^8\*c^2

$$\begin{aligned}
& + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4 \\
& *d^4)/c^2 - 696*a^4*b^7*c*d) + (a^3*b^6*d^2*(a + b/x)^{(1/2)}*(a^3)^{(1/2)}*790 \\
& i)/(256*a^3*b^8*c^2 + 790*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d \\
& ^3)/c + (60*a^7*b^4*d^4)/c^2 - 696*a^4*b^7*c*d) - (a^4*b^5*d^3*(a + b/x)^{(1 \\
& /2)}*(a^3)^{(1/2)}*370i)/(256*a^3*b^8*c^3 - 370*a^6*b^5*d^3 - 696*a^4*b^7*c^2* \\
& d + 790*a^5*b^6*c*d^2 - (40*a^2*b^9*c^4)/d + (60*a^7*b^4*d^4)/c) + (a^5*b^4 \\
& *d^4*(a + b/x)^{(1/2)}*(a^3)^{(1/2)}*60i)/(256*a^3*b^8*c^4 + 60*a^7*b^4*d^4 - 6 \\
& 96*a^4*b^7*c^3*d - 370*a^6*b^5*c*d^3 + 790*a^5*b^6*c^2*d^2 - (40*a^2*b^9*c^ \\
& 5)/d) - (a^2*b^7*c*d*(a + b/x)^{(1/2)}*(a^3)^{(1/2)}*696i)/(256*a^3*b^8*c^2 + 7 \\
& 90*a^5*b^6*d^2 - (40*a^2*b^9*c^3)/d - (370*a^6*b^5*d^3)/c + (60*a^7*b^4*d^4 \\
& )/c^2 - 696*a^4*b^7*c*d))*(2*a*d - 5*b*c)*(a^3)^{(1/2)}*1i)/c^2 + (a^2*b*d*(a \\
& + b/x)^{(1/2)))/(c*(d*(a + b/x) - a*d))
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)/(c+d/x), x)

[Out] Timed out

$$3.243 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=166

$$\frac{a^{3/2}(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{(bc - ad)^{3/2}(4ad + bc) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3 d^{3/2}} + \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)(bc - ad)}{c^2 d \left(c + \frac{d}{x}\right)} + \frac{ax \left(a + \frac{b}{x}\right)}{c \left(c + \frac{d}{x}\right)}$$

[Out]  $a*(a+b/x)^{(3/2)*x/c/(c+d/x)-(-a*d+b*c)^{(3/2)*(4*a*d+b*c)*\arctan(d^{(1/2)*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)})/c^3/d^{(3/2)+a^{(3/2)*(-4*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)/a^{(1/2)})/c^3+(-2*a*d+b*c)*(-a*d+b*c)*(a+b/x)^{(1/2)/c^2/d/(c+d/x)}$

**Rubi [A]** time = 0.23, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 98, 149, 156, 63, 208, 205}

$$\frac{a^{3/2}(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{c^3} - \frac{(bc - ad)^{3/2}(4ad + bc) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3 d^{3/2}} + \frac{\sqrt{a + \frac{b}{x}}(bc - 2ad)(bc - ad)}{c^2 d \left(c + \frac{d}{x}\right)} + \frac{ax \left(a + \frac{b}{x}\right)}{c \left(c + \frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)/(c + d/x)^2,x]

[Out]  $((b*c - 2*a*d)*(b*c - a*d)*\operatorname{Sqrt}[a + b/x])/(c^2*d*(c + d/x)) + (a*(a + b/x)^{(3/2)*x)/(c*(c + d/x)) - ((b*c - a*d)^{(3/2)*(b*c + 4*a*d)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(c^3*d^{(3/2)}) + (a^{(3/2)*(5*b*c - 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/c^3$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 98

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((b\*c - a\*d)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] + Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 2)\*(e + f\*x)^p\*Simp[a\*d\*(d\*e\*(n - 1) + c\*f\*(p + 1)) + b\*c\*(d\*e\*(m - n + 2) - c\*f\*(m + p + 2)) + d\*(a\*d\*f\*(n + p) + b\*(d\*e\*(m + 1) - c\*f\*(m + n + p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 149

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] - Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[b\*c\*(f\*g - e\*h)\*(m + 1) + (b\*g - a\*h)\*(d\*e\*n + c\*f\*(p + 1)) + d\*(b\*(f\*g

$- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h, p\}, x] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{IntegerQ}[m]$

### Rule 156

$\text{Int}[\frac{((e_.) + (f_.)*(x_))^{(p_)}*((g_.) + (h_.)*(x_))}{((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))}, x\_Symbol] \rightarrow \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, h\}, x]$

### Rule 205

$\text{Int}[\frac{((a_.) + (b_.)*(x_)^2)^{-1}}{x\_Symbol}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{PosQ}[a/b]$

### Rule 208

$\text{Int}[\frac{((a_.) + (b_.)*(x_)^2)^{-1}}{x\_Symbol}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

### Rule 375

$\text{Int}[\frac{((a_.) + (b_.)*(x_)^{(n_}))^{(p_)}*((c_.) + (d_.)*(x_)^{(n_}))^{(q_.)}}{x\_Symbol}, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[\frac{(a + b/x^n)^p*(c + d/x^n)^q}{x^2}, x], x, 1/x] /; \text{FreeQ}[\{a, b, c, d, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{ILtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^2} dx &= -\text{Subst}\left(\int \frac{(a + bx)^{5/2}}{x^2(c + dx)^2} dx, x, \frac{1}{x}\right) \\ &= \frac{a\left(a + \frac{b}{x}\right)^{3/2} x}{c\left(c + \frac{d}{x}\right)} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-4ad)-\frac{1}{2}b(2bc-ad)x\right)}{x(c+dx)^2} dx, x, \frac{1}{x}\right)}{c} \\ &= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}a^2d(5bc-4ad)+\frac{1}{2}b(b^2c^2+2abcd-2a^2d^2)x}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{c^2d} \\ &= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2c^3} \\ &= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)} - \frac{(a^2(5bc-4ad))\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)}{bc^3} \\ &= \frac{(bc-2ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{c^2d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)} - \frac{(bc-ad)^{3/2}(bc+4ad)\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3d^{3/2}} + \dots \end{aligned}$$



Unable to divide, perhaps due to rounding error  
 $[-2, 0] : [1, 0, \{-1, [1]\}]$ ,  $[4, 6, 4, 0]$ ,  $\{8, [1]\}$ ,  $[3, 5, 4, 1]$ ,  $[-4, 0] : [1, 0, \{-1, [1]\}]$ ,  $[2, 5, 5, 1]$ ,  $[-8, [1]]$ ,  $[0] : [1, 0, \{-1, [1]\}]$ ,  $[2, 4, 4, 2]$ ,  $\{8, [1]\}$ ,  $[1, 4, 5, 2]$ ,  $[-2, 0] : [1, 0, \{-1, [1]\}]$ ,  $[0, 4, 6, 2]$  /  $\{1, [1]\}$ ,  $[4, 2, 0, 0]$ ,  $\{poly1[-4, [1]]\}$ ,  $[0] : [1, 0, \{-1, [1]\}]$ ,  $[3, 1, 0, 1]$ ,  $\{2, [1]\}$ ,  $[2, 1, 1, 1]$ ,  $\{4, [2]\}$ ,  $[2, 0, 0, 2]$ ,  $\{poly1[-4, [1]]\}$ ,  $[0] : [1, 0, \{-1, [1]\}]$ ,  $[1, 0, 1, 2]$ ,  $\{1, [1]\}$ ,  $[0, 0, 2, 2]$  Error: Bad Argument Value

**maple [B]** time = 0.06, size = 1323, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int (a+b/x)^{5/2} / (c+d/x)^2 dx$

[Out] 
$$-1/2*(2*a^{5/2}*((a*x+b)*x)^{1/2}*((a*d-b*c)/c^2*d)^{1/2}*x*b*c^4*d-5*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*((a*d-b*c)/c^2*d)^{1/2}*x*b*c^3*d^2-5*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*((a*d-b*c)/c^2*d)^{1/2}*b^3*c^4*d-a*((a*d-b*c)/c^2*d)^{1/2}*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*b^3*c^4*d-2*a^{3/2}*((a*d-b*c)/c^2*d)^{1/2}*(a*x^2+b*x)^{1/2}*x*b^2*c^5+a^{3/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*x*b^3*c^4*d+a*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*((a*d-b*c)/c^2*d)^{1/2}*x*b^3*c^5-a*((a*d-b*c)/c^2*d)^{1/2}*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*x*b^3*c^5-2*a^{3/2}*((a*d-b*c)/c^2*d)^{1/2}*(a*x^2+b*x)^{1/2}*b^2*c^4*d-2*a^{5/2}*((a*x+b)*x)^{1/2}*((a*d-b*c)/c^2*d)^{1/2}*x^2*b*c^5-2*a^{7/2}*((a*x+b)*x)^{1/2}*((a*d-b*c)/c^2*d)^{1/2}*x*c^3*d^2-7*a^{7/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*x*b*c^2*d^3+2*a^{5/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*x*b^2*c^3*d^2+4*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*((a*d-b*c)/c^2*d)^{1/2}*x*c^2*d^3+4*a^{5/2}*((a*x+b)*x)^{1/2}*((a*d-b*c)/c^2*d)^{1/2}*b*c^3*d^2+2*a^{7/2}*((a*x+b)*x)^{1/2}*((a*d-b*c)/c^2*d)^{1/2}*x^2*c^4*d+4*a^{9/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*d^5-2*a^{5/2}*((a*x+b)*x)^{3/2}*((a*d-b*c)/c^2*d)^{1/2}*c^4*d+4*a^{9/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*b*c*d^4+2*a^{5/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*b^2*c^2*d^3+4*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{1/2}*a^{1/2})/a^{1/2})*((a*d-b*c)/c^2*d)^{1/2}*c*d^4+2*a^{3/2}*((a*x+b)*x)^{3/2}*((a*d-b*c)/c^2*d)^{1/2}*b*c^5+a^{3/2}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{1/2}*((a*x+b)*x)^{1/2}*c)/(c*x+d))*b^3*c^3*d^2*x*((a*x+b)/x)^{1/2}/c^4/((a*d-b*c)/c^2*d)^{1/2}/a^{3/2}/(c*x+d)/d^2/((a*x+b)*x)^{1/2}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{\left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a+b/x)^{5/2} / (c+d/x)^2, x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((a + b/x)^{5/2} / (c + d/x)^2, x)$

**mupad [B]** time = 2.31, size = 1153, normalized size = 6.95

$$\frac{\sqrt{a+\frac{b}{x}}(2a^3bd^2-3a^2b^2cd+ab^3c^2)}{c^2d} - \frac{b\left(a+\frac{b}{x}\right)^{3/2}(2a^2d^2-2abcd+b^2c^2)}{c^2d} \operatorname{atanh}\left(\frac{10b^9\sqrt{a+\frac{b}{x}}\sqrt{a^3}}{10a^2b^9+\frac{32a^3b^8d}{c}-\frac{132a^4b^7d^2}{c^2}+\frac{130a^5b^6d^3}{c^3}-\frac{40a^6b^5d^4}{c^4}}\right) + \frac{32}{32}$$

$$\frac{\left(a+\frac{b}{x}\right)(2ad-bc)-d\left(a+\frac{b}{x}\right)^2-a^2d+abc}{c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^(5/2)/(c + d/x)^2,x)`

[Out] 
$$\left(\frac{(a + b/x)^{1/2}(a^3b^3c^2 + 2a^3b^2d^2 - 3a^2b^2c^2d)}{c^2d} - (b(a + b/x)^{3/2}(2a^2d^2 + b^2c^2 - 2ab^2cd)) / (c^2d)\right) / \left((a + b/x)(2ad - bc) - d(a + b/x)^2 - a^2d + abc\right) - \operatorname{atanh}\left(\frac{10b^9(a + b/x)^{1/2}(a^3)^{1/2}}{10a^2b^9 + (32a^3b^8d)/c - (132a^4b^7d^2)/c^2 + (130a^5b^6d^3)/c^3 - (40a^6b^5d^4)/c^4} + \frac{32}{32}\right) / \left(\frac{32a^3b^8 + (10a^2b^9c)/d - (132a^4b^7d)/c + (130a^5b^6d^2)/c^2 - (40a^6b^5d^3)/c^3 - (132a^2b^7d^2(a + b/x)^{1/2}(a^3)^{1/2}) / (32a^3b^8c - 132a^4b^7d + (10a^2b^9c^2)/d + (130a^5b^6d^2)/c - (40a^6b^5d^3)/c^2} + (130a^3b^6d^2(a + b/x)^{1/2}(a^3)^{1/2}) / (32a^3b^8c^2 + 130a^5b^6d^2 + (10a^2b^9c^3)/d - (40a^6b^5d^3)/c - 132a^4b^7c^2d - (40a^4b^5d^3(a + b/x)^{1/2}(a^3)^{1/2}) / (32a^3b^8c^3 - 40a^6b^5d^3 - 132a^4b^7c^2d + 130a^5b^6c^2d^2 + (10a^2b^9c^4)/d)\right) * (4ad - 5bc)(a^3)^{1/2} / c^3 + \operatorname{atanh}\left(\frac{30a^3b^6(a + b/x)^{1/2}(a^3d^6 - b^3c^3d^3 + 3ab^2c^2d^4 - 3a^2b^2c^2d^5)^{1/2}}{(14a^2b^9c^3 + 110a^5b^6d^3 - 4a^3b^8c^2d - 82a^4b^7c^2d^2 + (2ab^10c^4)/d - (40a^6b^5d^4)/c) + (18a^2b^7(a + b/x)^{1/2}(a^3d^6 - b^3c^3d^3 + 3ab^2c^2d^4 - 3a^2b^2c^2d^5)^{1/2}) / (2ab^10c^3 - 82a^4b^7d^3 + 14a^2b^9c^2d - 4a^3b^8c^2d^2 + (110a^5b^6d^4)/c - (40a^6b^5d^5)/c^2} + (40a^4b^5(a + b/x)^{1/2}(a^3d^6 - b^3c^3d^3 + 3ab^2c^2d^4 - 3a^2b^2c^2d^5)^{1/2}) / (4a^3b^8c^3 + 40a^6b^5d^3 + 82a^4b^7c^2d - 110a^5b^6c^2d^2 - (2ab^10c^5)/d^2 - (14a^2b^9c^4)/d) - (2ab^8(a + b/x)^{1/2}(a^3d^6 - b^3c^3d^3 + 3ab^2c^2d^4 - 3a^2b^2c^2d^5)^{1/2}) / (4a^3b^8d^3 - 14a^2b^9c^2d^2 + (82a^4b^7d^4)/c - (110a^5b^6d^5)/c^2 + (40a^6b^5d^6)/c^3 - 2ab^10c^2d)\right) * (d^3(ad - bc)^3)^{1/2} * (4ad + bc) / (c^3d^3)$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**(5/2)/(c+d/x)**2,x)`

[Out] Timed out



$$3.244 \quad \int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=237

$$\frac{a^{3/2}(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) \sqrt{bc - ad} (-24a^2d^2 + 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) \sqrt{a + \frac{b}{x}} (-12a^2d^2 + 7abcd + b^2c^2)}{c^4 \cdot 4c^4d^{3/2} \cdot 4c^3d\left(c + \frac{d}{x}\right)}$$

[Out]  $a*(a+b/x)^{(3/2)*x/c/(c+d/x)^2+a^{(3/2)*(-6*a*d+5*b*c)*\arctanh((a+b/x)^{(1/2)/a^{(1/2)})/c^4-1/4*(-24*a^2*d^2+8*a*b*c*d+b^2*c^2)*\arctan(d^{(1/2)*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)})*(-a*d+b*c)^{(1/2)/c^4/d^{(3/2)+1/2*(-3*a*d+b*c)*(-a*d+b*c)*(a+b/x)^{(1/2)/c^2/d/(c+d/x)^2-1/4*(-12*a^2*d^2+7*a*b*c*d+b^2*c^2)*(a+b/x)^{(1/2)/c^3/d/(c+d/x)}$

**Rubi [A]** time = 0.37, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {375, 98, 149, 151, 156, 63, 208, 205}

$$\frac{\sqrt{a + \frac{b}{x}} (-12a^2d^2 + 7abcd + b^2c^2) \sqrt{bc - ad} (-24a^2d^2 + 8abcd + b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) a^{3/2}(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{4c^3d\left(c + \frac{d}{x}\right) \cdot 4c^4d^{3/2} \cdot c^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(5/2)/(c + d/x)^3,x]

[Out]  $((b*c - 3*a*d)*(b*c - a*d)*\text{Sqrt}[a + b/x])/(2*c^2*d*(c + d/x)^2) - ((b^2*c^2 + 7*a*b*c*d - 12*a^2*d^2)*\text{Sqrt}[a + b/x])/(4*c^3*d*(c + d/x)) + (a*(a + b/x)^{(3/2)*x})/(c*(c + d/x)^2) - (\text{Sqrt}[b*c - a*d]*(b^2*c^2 + 8*a*b*c*d - 24*a^2*d^2)*\text{ArcTan}[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(4*c^4*d^{(3/2)}) + (a^{(3/2)*(5*b*c - 6*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/c^4$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 98

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] + Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 2)\*(e + f\*x)^p\*Simp[a\*d\*(d\*e\*(n - 1) + c\*f\*(p + 1)) + b\*c\*(d\*e\*(m - n + 2) - c\*f\*(m + p + 2)) + d\*(a\*d\*f\*(n + p) + b\*(d\*e\*(m + 1) - c\*f\*(m + n + p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 149

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[(b\*g - a\*h)\*(a + b\*x)^(m +

1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] - Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[b\*c\*(f\*g - e\*h)\*(m + 1) + (b\*g - a\*h)\*(d\*e\*n + c\*f\*(p + 1)) + d\*(b\*(f\*g - e\*h)\*(m + 1) + f\*(b\*g - a\*h)\*(n + p + 1))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h)\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 156

Int[(((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{x}\right)^{5/2}}{\left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst}\left(\int \frac{(a + bx)^{5/2}}{x^2(c + dx)^3} dx, x, \frac{1}{x}\right) \\
&= \frac{a\left(a + \frac{b}{x}\right)^{3/2} x}{c\left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(-\frac{1}{2}a(5bc-6ad)-\frac{1}{2}b(2bc-3ad)x\right)}{x(c+dx)^3} dx, x, \frac{1}{x}\right)}{c} \\
&= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b(b^2c^2+6abcd-9a^2d^2)x}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x}\right)}{2c^2d} \\
&= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} + \frac{\text{Subst}\left(\int \frac{a^2d(5bc-6ad)+\frac{1}{2}b(b^2c^2+6abcd-9a^2d^2)x}{x\sqrt{a+bx}(c+dx)^2} dx, x, \frac{1}{x}\right)}{2c^2d} \\
&= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(a^2(5bc-6ad)+\frac{1}{2}b(b^2c^2+6abcd-9a^2d^2))\sqrt{a+\frac{b}{x}}}{2c^2d} \\
&= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{(a^2(5bc-6ad)+\frac{1}{2}b(b^2c^2+6abcd-9a^2d^2))\sqrt{a+\frac{b}{x}}}{2c^2d} \\
&= \frac{(bc-3ad)(bc-ad)\sqrt{a+\frac{b}{x}}}{2c^2d\left(c+\frac{d}{x}\right)^2} - \frac{(b^2c^2+7abcd-12a^2d^2)\sqrt{a+\frac{b}{x}}}{4c^3d\left(c+\frac{d}{x}\right)} + \frac{a\left(a+\frac{b}{x}\right)^{3/2} x}{c\left(c+\frac{d}{x}\right)^2} - \frac{\sqrt{bc-ad}(-24a^2d^2+8abcd+b^2c^2)}{d^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.85, size = 191, normalized size = 0.81

$$\frac{-4a^{3/2}(6ad-5bc)\tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}(2a^2d(2c^2x^2+9cdx+6d^2)-abcd(11cx+7d)+b^2c^2(cx-d))}{d(cx+d)^2} - \frac{\sqrt{bc-ad}(-24a^2d^2+8abcd+b^2c^2)}{d^{3/2}}}{4c^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(5/2)/(c + d/x)^3,x]

[Out] ((c\*Sqrt[a + b/x]\*x\*(b^2\*c^2\*(-d + c\*x) - a\*b\*c\*d\*(7\*d + 11\*c\*x) + 2\*a^2\*d\*(6\*d^2 + 9\*c\*d\*x + 2\*c^2\*x^2)))/(d\*(d + c\*x)^2) - (Sqrt[b\*c - a\*d]\*(b^2\*c^2 + 8\*a\*b\*c\*d - 24\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/d^(3/2) - 4\*a^(3/2)\*(-5\*b\*c + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]/(4\*c^4)

**fricas [A]** time = 1.16, size = 1445, normalized size = 6.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/8*(4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*\sqrt{a}*\log(2*a*x - 2*\sqrt{a})*x*\sqrt{(a*x + b)/x} + b) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x) \\ & *\sqrt{-(b*c - a*d)/d}*\log((2*d*x*\sqrt{-(b*c - a*d)/d}*\sqrt{(a*x + b)/x} + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)* \\ & *\sqrt{(a*x + b)/x)/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*\sqrt{(b*c - a*d)/d}*\arctan(-d*\sqrt{(b*c - a*d)/d}*\sqrt{(a*x + b)/x}/(b*c - a*d)) - 2*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x) \\ & *\sqrt{a}*\log(2*a*x - 2*\sqrt{a})*x*\sqrt{(a*x + b)/x} + b) + (4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*\sqrt{(a*x + b)/x)/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), -1/8*(8*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x)*\sqrt{-a}*\arctan(\sqrt{-a}*\sqrt{(a*x + b)/x}/a) + (b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*\sqrt{-(b*c - a*d)/d}*\log((2*d*x*\sqrt{-(b*c - a*d)/d}*\sqrt{(a*x + b)/x} + b*d - (b*c - 2*a*d)*x)/(c*x + d)) - 2*(4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*\sqrt{(a*x + b)/x)/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3), 1/4*((b^2*c^2*d^2 + 8*a*b*c*d^3 - 24*a^2*d^4 + (b^2*c^4 + 8*a*b*c^3*d - 24*a^2*c^2*d^2)*x^2 + 2*(b^2*c^3*d + 8*a*b*c^2*d^2 - 24*a^2*c*d^3)*x)*\sqrt{(b*c - a*d)/d}*\arctan(-d*\sqrt{(b*c - a*d)/d}*\sqrt{(a*x + b)/x}/(b*c - a*d)) - 4*(5*a*b*c*d^3 - 6*a^2*d^4 + (5*a*b*c^3*d - 6*a^2*c^2*d^2)*x^2 + 2*(5*a*b*c^2*d^2 - 6*a^2*c*d^3)*x) \\ & *\sqrt{-a}*\arctan(\sqrt{-a}*\sqrt{(a*x + b)/x}/a) + (4*a^2*c^3*d*x^3 + (b^2*c^4 - 11*a*b*c^3*d + 18*a^2*c^2*d^2)*x^2 - (b^2*c^3*d + 7*a*b*c^2*d^2 - 12*a^2*c*d^3)*x)*\sqrt{(a*x + b)/x)/(c^6*d*x^2 + 2*c^5*d^2*x + c^4*d^3)] \end{aligned}$$

**giac [B]** time = 0.49, size = 945, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")

[Out] 
$$\begin{aligned} & \sqrt{a*x^2 + b*x}*a^2*\operatorname{sgn}(x)/c^3 - 1/2*(5*a^2*b*c*\operatorname{sgn}(x) - 6*a^3*d*\operatorname{sgn}(x))*\log(\operatorname{abs}(2*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*\sqrt{a} + b))/(\sqrt{a}*c^4) + 1/4*(b^3*c^3*\operatorname{sgn}(x) + 7*a*b^2*c^2*d*\operatorname{sgn}(x) - 32*a^2*b*c*d^2*\operatorname{sgn}(x) + 24*a^3*d^3*\operatorname{sgn}(x))*\arctan(-((\sqrt{a}*x - \sqrt{a*x^2 + b*x})*c + \sqrt{a}*d)/\sqrt{b*c*d - a*d^2}))/(\sqrt{b*c*d - a*d^2}*c^4*d) + 1/4*(\sqrt{a}*b^3*c^3*\arctan(\sqrt{a}*d/\sqrt{b*c*d - a*d^2})) + 7*a^(3/2)*b^2*c^2*d*\arctan(\sqrt{a}*d/\sqrt{b*c*d - a*d^2})) - 32*a^(5/2)*b*c*d^2*\arctan(\sqrt{a}*d/\sqrt{b*c*d - a*d^2})) + 24*a^(7/2)*d^3*\arctan(\sqrt{a}*d/\sqrt{b*c*d - a*d^2})) + 10*\sqrt{b*c*d - a*d^2}*a^2*b*c*d*\log(\operatorname{abs}(b)) - 12*\sqrt{b*c*d - a*d^2}*a^3*d^2*\log(\operatorname{abs}(b)) - \sqrt{b*c*d - a*d^2}*a*b^2*c^2 + 11*\sqrt{b*c*d - a*d^2}*a^2*b*c*d - 10*\sqrt{b*c*d - a*d^2}*a^3*d^2)*\operatorname{sgn}(x)/(\sqrt{b*c*d - a*d^2}*\sqrt{a}*c^4*d) - 1/4*((\sqrt{a})*x - \sqrt{a*x^2 + b*x})^3*\sqrt{a}*b^3*c^4*\operatorname{sgn}(x) - 17*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^3*a^(3/2)*b^2*c^3*d*\operatorname{sgn}(x) + 40*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^3*a^(5/2)*b*c^2*d^2*\operatorname{sgn}(x) - 24*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^3*a^(7/2)*c*d^3*\operatorname{sgn}(x) - 5*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^2*a*b^3*c^3*d*\operatorname{sgn}(x) - 3*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^2*a^2*b^2*c^2*d^2*\operatorname{sgn}(x) + 48*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^2*a^3*b*c*d^3*\operatorname{sgn}(x) - 40*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})^2*a^4*d^4*\operatorname{sgn}(x) - (\sqrt{a}*x - \sqrt{a*x^2 + b*x})*\sqrt{a}*b^4*c^3*d*\operatorname{sgn}(x) - 11*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*a^(3/2)*b^3*c^2*d^2*\operatorname{sgn}(x) + 52*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*a^(5/2)*b^2*c*d^3*\operatorname{sgn}(x) - 40*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*a^(7/2)*b*c*d^2*\operatorname{sgn}(x) + 24*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*a^4*d^3*\operatorname{sgn}(x) - 12*(\sqrt{a}*x - \sqrt{a*x^2 + b*x})*a^5*d^4*\operatorname{sgn}(x) + 10*\sqrt{b*c*d - a*d^2}*a^2*b*c*d*\log(\operatorname{abs}(b)) - 12*\sqrt{b*c*d - a*d^2}*a^3*d^2*\log(\operatorname{abs}(b)) - \sqrt{b*c*d - a*d^2}*a*b^2*c^2 + 11*\sqrt{b*c*d - a*d^2}*a^2*b*c*d - 10*\sqrt{b*c*d - a*d^2}*a^3*d^2)*\operatorname{sgn}(x) \end{aligned}$$

$$(a*x^2 + b*x)*a^{(7/2)}*b*d^4*\text{sgn}(x) - a*b^4*c^2*d^2*\text{sgn}(x) + 11*a^2*b^3*c*d^3*\text{sgn}(x) - 10*a^3*b^2*d^4*\text{sgn}(x)/(((\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))^2*c + 2*(\text{sqrt}(a)*x - \text{sqrt}(a*x^2 + b*x))*\text{sqrt}(a)*d + b*d)^2*\text{sqrt}(a)*c^4*d)$$

**maple [B]** time = 0.07, size = 1638, normalized size = 6.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(5/2)/(c+d/x)^3,x)

[Out] 
$$-1/8*(7*a^{(5/2)}*b^2*c^4*d^2*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^{(1/2)}*a^4*c^3*d^3*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})-36*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(7/2)}*c^3*d^3*x-64*a^{(7/2)}*b*c^2*d^4*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+14*a^{(5/2)}*b^2*c^3*d^3*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+48*((a*d-b*c)/c^2*d)^{(1/2)}*a^4*c^2*d^4*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})+14*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*b*c^3*d^3-20*((a*d-b*c)/c^2*d)^{(1/2)}*a^3*b*c^2*d^4*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})+2*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(3/2)}*b^2*c^4*d^2-2*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(3/2)}*b*c^6*x+2*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(3/2)}*b^2*c^6*x^2+a^{(3/2)}*b^3*c^5*d*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))-6*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(3/2)}*b*c^5*d+12*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(7/2)}*c^5*d*x^3+2*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*b*c^6*x^3-12*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*c^5*d*x-32*a^{(7/2)}*b*c^3*d^3*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+2*a^{(3/2)}*b^3*c^4*d^2*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+24*a^{(9/2)}*d^6*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+30*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*b*c^4*d^2*x-40*((a*d-b*c)/c^2*d)^{(1/2)}*a^3*b*c^3*d^3*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})+18*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*b*c^5*d*x^2-20*((a*d-b*c)/c^2*d)^{(1/2)}*a^3*b*c^4*d^2*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})+4*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(3/2)}*b^2*c^5*d*x+48*a^{(9/2)}*c*d^5*x*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))-24*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(7/2)}*c^2*d^4-32*a^{(7/2)}*b*c*d^5*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+7*a^{(5/2)}*b^2*c^2*d^4*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))+24*((a*d-b*c)/c^2*d)^{(1/2)}*a^4*c*d^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})+24*a^{(9/2)}*c^2*d^4*x^2*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))-8*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*a^{(5/2)}*c^4*d^2+a^{(3/2)}*b^3*c^3*d^3*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d)))*x*((a*x+b)/x)^{(1/2)}/c^5/((a*d-b*c)/c^2*d)^{(1/2)}/a^{(3/2)}/(c*x+d)^2/d^2/((a*x+b)*x)^{(1/2)}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a + \frac{b}{x}\right)^{\frac{5}{2}}}{\left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate((a + b/x)^(5/2)/(c + d/x)^3, x)

**mupad [B]** time = 3.44, size = 1476, normalized size = 6.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(5/2)/(c + d/x)^3, x)

[Out] (atan((b^9\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*5i)/(8\*((5\*a^2\*b^9)/8 + (8\*a^3\*b^8\*d)/c - (159\*a^4\*b^7\*d^2)/(8\*c^2) + (45\*a^5\*b^6\*d^3)/(4\*c^3))) + (a\*b^8\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*8i)/(8\*a^3\*b^8 + (5\*a^2\*b^9\*c)/(8\*d) - (159\*a^4\*b^7\*d)/(8\*c) + (45\*a^5\*b^6\*d^2)/(4\*c^2)) - (a^2\*b^7\*d\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*159i)/(8\*(8\*a^3\*b^8\*c - (159\*a^4\*b^7\*d)/8 + (5\*a^2\*b^9\*c^2)/(8\*d) + (45\*a^5\*b^6\*d^2)/(4\*c))) + (a^3\*b^6\*d^2\*(a + b/x)^(1/2)\*(a^3)^(1/2)\*45i)/(4\*(8\*a^3\*b^8\*c^2 + (45\*a^5\*b^6\*d^2)/4 + (5\*a^2\*b^9\*c^3)/(8\*d) - (159\*a^4\*b^7\*c\*d)/8))\*(6\*a\*d - 5\*b\*c)\*(a^3)^(1/2)\*1i)/c^4 - (((a + b/x)^(3/2)\*(b^4\*c^3 - 2\*4\*a^3\*b\*d^3 + 32\*a^2\*b^2\*c\*d^2 - 9\*a\*b^3\*c^2\*d))/(4\*c^3\*d) - (b\*(a + b/x)^(5/2)\*(b^2\*c^2 - 12\*a^2\*d^2 + 7\*a\*b\*c\*d))/(4\*c^3) + (b\*(a + b/x)^(1/2)\*(12\*a^4\*d^3 - a\*b^3\*c^3 + 14\*a^2\*b^2\*c^2\*d - 25\*a^3\*b\*c\*d^2))/(4\*c^3\*d))/((a + b/x)^2\*(3\*a\*d^2 - 2\*b\*c\*d) - (a + b/x)\*(3\*a^2\*d^2 + b^2\*c^2 - 4\*a\*b\*c\*d) - d^2\*(a + b/x)^3 + a^3\*d^2 + a\*b^2\*c^2 - 2\*a^2\*b\*c\*d) + (log(- (5\*a^2\*b^9\*c^6 + 1728\*a^8\*b^3\*d^6 + 64\*a^3\*b^8\*c^5\*d - 4752\*a^7\*b^4\*c\*d^5 - 59\*a^4\*b^7\*c^4\*d^2 - 1450\*a^5\*b^6\*c^3\*d^3 + 4464\*a^6\*b^5\*c^2\*d^4)/(16\*c^9\*d) - (((a + b/x)^(1/2)\*(b^8\*c^6 + 1152\*a^6\*b^2\*d^6 - 2496\*a^5\*b^3\*c\*d^5 - 15\*a^2\*b^6\*c^4\*d^2 - 400\*a^3\*b^5\*c^3\*d^3 + 1760\*a^4\*b^4\*c^2\*d^4 + 14\*a\*b^7\*c^5\*d))/(8\*c^6\*d) - (((16\*a\*b^5\*c^10\*d^2 - 208\*a^2\*b^4\*c^9\*d^3 + 192\*a^3\*b^3\*c^8\*d^4)/(16\*c^9\*d) - ((64\*b^3\*c^9\*d^3 - 128\*a\*b^2\*c^8\*d^4)\*(a + b/x)^(1/2)\*(d^3\*(a\*d - b\*c)))^(1/2)\*((b^2\*c^2)/8 - 3\*a^2\*d^2 + a\*b\*c\*d))/(8\*c^10\*d^4)\*(d^3\*(a\*d - b\*c)))^(1/2)\*((b^2\*c^2)/8 - 3\*a^2\*d^2 + a\*b\*c\*d))/(c^4\*d^3))\*(d^3\*(a\*d - b\*c))^(1/2)\*((b^2\*c^2)/8 - 3\*a^2\*d^2 + a\*b\*c\*d))/(c^4\*d^3) - (log((((a + b/x)^(1/2)\*(b^8\*c^6 + 1152\*a^6\*b^2\*d^6 - 2496\*a^5\*b^3\*c\*d^5 - 15\*a^2\*b^6\*c^4\*d^2 - 400\*a^3\*b^5\*c^3\*d^3 + 1760\*a^4\*b^4\*c^2\*d^4 + 14\*a\*b^7\*c^5\*d))/(8\*c^6\*d) + (((16\*a\*b^5\*c^10\*d^2 - 208\*a^2\*b^4\*c^9\*d^3 + 192\*a^3\*b^3\*c^8\*d^4)/(16\*c^9\*d) + ((64\*b^3\*c^9\*d^3 - 128\*a\*b^2\*c^8\*d^4)\*(a + b/x)^(1/2)\*(d^3\*(a\*d - b\*c)))^(1/2)\*(b^2\*c^2 - 24\*a^2\*d^2 + 8\*a\*b\*c\*d))/(64\*c^10\*d^4)\*(d^3\*(a\*d - b\*c))^(1/2)\*(b^2\*c^2 - 24\*a^2\*d^2 + 8\*a\*b\*c\*d))/(8\*c^4\*d^3))\*(d^3\*(a\*d - b\*c))^(1/2)\*(b^2\*c^2 - 24\*a^2\*d^2 + 8\*a\*b\*c\*d))/(8\*c^4\*d^3) - (5\*a^2\*b^9\*c^6 + 1728\*a^8\*b^3\*d^6 + 64\*a^3\*b^8\*c^5\*d - 4752\*a^7\*b^4\*c\*d^5 - 59\*a^4\*b^7\*c^4\*d^2 - 1450\*a^5\*b^6\*c^3\*d^3 + 4464\*a^6\*b^5\*c^2\*d^4)/(16\*c^9\*d))\*(d^3\*(a\*d - b\*c))^(1/2)\*(b^2\*c^2 - 24\*a^2\*d^2 + 8\*a\*b\*c\*d))/(8\*c^4\*d^3)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(5/2)/(c+d/x)\*\*3, x)

[Out] Timed out

$$3.245 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=126

$$\frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 9abcd + 3b^2c^2) + \frac{bd(2ad+3bc)}{x}\right)}{3ab^2} + \frac{cx\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{a}$$

[Out]  $-c^2*(-6*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}}{a^{(1/2)}}\right)/a^{(3/2)}-1/3*d*(-4*a^2*d^2+18*a*b*c*d+6*b^2*c^2+b*d*(2*a*d+3*b*c)/x)*(a+b/x)^{(1/2)}/a/b^2+c*(c+d/x)^2*x*(a+b/x)^{(1/2)}/a$

**Rubi [A]** time = 0.09, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {375, 98, 147, 63, 208}

$$\frac{d\sqrt{a + \frac{b}{x}} \left(2(-2a^2d^2 + 9abcd + 3b^2c^2) + \frac{bd(2ad+3bc)}{x}\right)}{3ab^2} - \frac{c^2(bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{cx\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}{a}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(c + \frac{d}{x}\right)^3/\operatorname{Sqrt}[a + b/x], x\right]$

[Out]  $-(d*\operatorname{Sqrt}[a + b/x]*(2*(3*b^2*c^2 + 9*a*b*c*d - 2*a^2*d^2) + (b*d*(3*b*c + 2*a*d))/x))/(3*a*b^2) + (c*\operatorname{Sqrt}[a + b/x]*(c + d/x)^2*x)/a - (c^2*(b*c - 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(3/2)}$

### Rule 63

$\operatorname{Int}\left[\left((a_{.}) + (b_{.})*(x_{.})\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*(x_{.})\right)^{(n_{.})}, x_{\text{Symbol}}\right] \rightarrow \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x\right], x, (a + b*x)^{(1/p)}\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]\right]$

### Rule 98

$\operatorname{Int}\left[\left((a_{.}) + (b_{.})*(x_{.})\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*(x_{.})\right)^{(n_{.})}*\left((e_{.}) + (f_{.})*(x_{.})\right)^{(p_{.})}, x_{\text{Symbol}}\right] \rightarrow \operatorname{Simp}\left[\left((b*c - a*d)*(a + b*x)^{(m+1)}*(c + d*x)^{(n-1)}*(e + f*x)^{(p+1)}\right)/(b*(b*e - a*f)*(m+1)), x\right] + \operatorname{Dist}\left[1/(b*(b*e - a*f)*(m+1)), \operatorname{Int}\left[(a + b*x)^{(m+1)}*(c + d*x)^{(n-2)}*(e + f*x)^p*\operatorname{Simp}\left[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))\right]*x, x\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 1] \&\& (\operatorname{IntegersQ}[2*m, 2*n, 2*p] \|\| \operatorname{IntegersQ}[m, n+p] \|\| \operatorname{IntegersQ}[p, m+n])$

### Rule 147

$\operatorname{Int}\left[\left((a_{.}) + (b_{.})*(x_{.})\right)^{(m_{.})}*\left((c_{.}) + (d_{.})*(x_{.})\right)^{(n_{.})}*\left((e_{.}) + (f_{.})*(x_{.})\right)*\left((g_{.}) + (h_{.})*(x_{.})\right), x_{\text{Symbol}}\right] \rightarrow -\operatorname{Simp}\left[\left((a*d*f*h*(n+2) + b*c*f*h*(m+2) - b*d*(f*g + e*h)*(m+n+3) - b*d*f*h*(m+n+2)*x\right)*(a + b*x)^{(m+1)}*(c + d*x)^{(n+1)}\right)/(b^2*d^2*(m+n+2)*(m+n+3)), x\right] + \operatorname{Dist}\left[(a^2*d^2*f*h*(n+1)*(n+2) + a*b*d*(n+1)*(2*c*f*h*(m+1) - d*(f*g + e*h)*(m+n+3)) + b^2*(c^2*f*h*(m+1)*(m+2) - c*d*(f*g + e*h)*(m+1)*(m+n+3) + d^2*e*g*(m+n+2)*(m+n+3))\right]/(b^2*d^2*(m+n+2)*(m+n+3)), \operatorname{Int}\left[(a + b*x)^m*(c + d*x)^n, x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, g, h, m, n\},$

x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\int \frac{\left(c + \frac{d}{x}\right)^3}{\sqrt{a + \frac{b}{x}}} dx = -\text{Subst}\left(\int \frac{(c + dx)^3}{x^2 \sqrt{a + bx}} dx, x, \frac{1}{x}\right)$$

$$= \frac{c\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 x}{a} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{1}{2}c(bc-6ad) - \frac{1}{2}d(3bc+2ad)x\right)}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{a}$$

$$= -\frac{d\sqrt{a + \frac{b}{x}} \left(2(3b^2c^2 + 9abcd - 2a^2d^2) + \frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc - 6ad)) \text{Su}}{a^3/2}$$

$$= -\frac{d\sqrt{a + \frac{b}{x}} \left(2(3b^2c^2 + 9abcd - 2a^2d^2) + \frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 x}{a} + \frac{(c^2(bc - 6ad)) \text{Su}}{a^3/2}$$

$$= -\frac{d\sqrt{a + \frac{b}{x}} \left(2(3b^2c^2 + 9abcd - 2a^2d^2) + \frac{bd(3bc+2ad)}{x}\right)}{3ab^2} + \frac{c\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2 x}{a} - \frac{c^2(bc - 6ad) \tanh}{a^3/2}$$

**Mathematica [A]** time = 0.15, size = 95, normalized size = 0.75

$$\frac{c^2(6ad - bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a + \frac{b}{x}} (4a^2d^3x - 2abd^2(9cx + d) + 3b^2c^3x^2)}{3ab^2x}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^3/Sqrt[a + b/x], x]

[Out] (Sqrt[a + b/x]\*(4\*a^2\*d^3\*x + 3\*b^2\*c^3\*x^2 - 2\*a\*b\*d^2\*(d + 9\*c\*x)))/(3\*a\*b^2\*x) + (c^2\*(-(b\*c) + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**fricas [A]** time = 0.98, size = 233, normalized size = 1.85

$$\left[ \frac{3(b^3c^3 - 6ab^2c^2d)\sqrt{a}x \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(3ab^2c^3x^2 - 2a^2bd^3 - 2(9a^2bcd^2 - 2a^3d^3)x)\sqrt{\frac{ax+b}{x}}}{6a^2b^2x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="fricas")

[Out] 
$$\left[ -\frac{1}{6} \cdot (3 \cdot (b^3 \cdot c^3 - 6 \cdot a \cdot b^2 \cdot c^2 \cdot d) \cdot \sqrt{a} \cdot x \cdot \log(2 \cdot a \cdot x + 2 \cdot \sqrt{a} \cdot x \cdot \sqrt{(a \cdot x + b)/x}) + b) - 2 \cdot (3 \cdot a \cdot b^2 \cdot c^3 \cdot x^2 - 2 \cdot a^2 \cdot b \cdot d^3 - 2 \cdot (9 \cdot a^2 \cdot b \cdot c \cdot d^2 - 2 \cdot a^3 \cdot d^3) \cdot x) \cdot \sqrt{(a \cdot x + b)/x}) / (a^2 \cdot b^2 \cdot x), \frac{1}{3} \cdot (3 \cdot (b^3 \cdot c^3 - 6 \cdot a \cdot b^2 \cdot c^2 \cdot d) \cdot \sqrt{-a} \cdot x \cdot \arctan(\sqrt{-a} \cdot \sqrt{(a \cdot x + b)/x}) / a + (3 \cdot a \cdot b^2 \cdot c^3 \cdot x^2 - 2 \cdot a^2 \cdot b \cdot d^3 - 2 \cdot (9 \cdot a^2 \cdot b \cdot c \cdot d^2 - 2 \cdot a^3 \cdot d^3) \cdot x) \cdot \sqrt{(a \cdot x + b)/x}) / (a^2 \cdot b^2 \cdot x) \right]$$

**giac** [A] time = 0.20, size = 158, normalized size = 1.25

$$\frac{\frac{3b^2c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a} - \frac{3(b^2c^3-6abc^2d)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a} + \frac{2\left(9b^3cd^2\sqrt{\frac{ax+b}{x}}-3ab^2d^3\sqrt{\frac{ax+b}{x}}+\frac{(ax+b)b^2d^3\sqrt{\frac{ax+b}{x}}}{x}\right)}{b^3}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")

[Out] 
$$-\frac{1}{3} \cdot (3 \cdot b^2 \cdot c^3 \cdot \sqrt{(a \cdot x + b)/x}) / ((a - (a \cdot x + b)/x) \cdot a) - 3 \cdot (b^2 \cdot c^3 - 6 \cdot a \cdot b \cdot c^2 \cdot d) \cdot \arctan(\sqrt{(a \cdot x + b)/x}) / (\sqrt{-a}) / (\sqrt{-a} \cdot a) + 2 \cdot (9 \cdot b^3 \cdot c \cdot d^2 \cdot \sqrt{(a \cdot x + b)/x} - 3 \cdot a \cdot b^2 \cdot d^3 \cdot \sqrt{(a \cdot x + b)/x} + (a \cdot x + b) \cdot b^2 \cdot d^3 \cdot \sqrt{(a \cdot x + b)/x}) / (x) / b^3$$

**maple** [B] time = 0.06, size = 535, normalized size = 4.25

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 3a^3b d^3 x^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - 3a^3b d^3 x^3 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 9a^2b^2c d^2 x^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}}{2\sqrt{a}}\right) \right)}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(1/2),x)

[Out] 
$$\frac{1}{6} \cdot ((a \cdot x + b)/x)^{(1/2)} / x^2 \cdot (3 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a^3 \cdot b \cdot d^3 - 9 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a^2 \cdot b^2 \cdot c \cdot d^2 + 9 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a \cdot b^3 \cdot c^2 \cdot d - 3 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot b^4 \cdot c^3 - 6 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(7/2)} \cdot x^3 \cdot d^3 + 18 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(5/2)} \cdot x^3 \cdot b \cdot c \cdot d^2 + 18 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(3/2)} \cdot x^3 \cdot b^2 \cdot c^2 \cdot d - 3 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a^3 \cdot b \cdot d^3 + 9 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a^2 \cdot b^2 \cdot c \cdot d^2 + 9 \cdot \ln(1/2 \cdot (2 \cdot a \cdot x + b + 2 \cdot (a \cdot x^2 + b \cdot x)^{(1/2)} \cdot a^{(1/2)})) / a^{(1/2)}) \cdot x^3 \cdot a \cdot b^3 \cdot c^2 \cdot d - 6 \cdot a^{(7/2)} \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot x^3 \cdot d^3 + 18 \cdot a^{(5/2)} \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot x^3 \cdot b \cdot c \cdot d^2 - 18 \cdot a^{(3/2)} \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot x^3 \cdot b^2 \cdot c^2 \cdot d + 6 \cdot a^{(1/2)} \cdot ((a \cdot x + b) \cdot x)^{(1/2)} \cdot x^3 \cdot b^3 \cdot c^3 + 12 \cdot (a \cdot x^2 + b \cdot x)^{(3/2)} \cdot a^{(5/2)} \cdot x \cdot d^3 - 36 \cdot (a \cdot x^2 + b \cdot x)^{(3/2)} \cdot a^{(3/2)} \cdot x \cdot b \cdot c \cdot d^2 - 4 \cdot d^3 \cdot (a \cdot x^2 + b \cdot x)^{(3/2)} \cdot b \cdot a^{(3/2)}) / ((a \cdot x + b) \cdot x)^{(1/2)} / b^3 / a^{(3/2)}$$

**maxima** [A] time = 1.33, size = 166, normalized size = 1.32

$$\frac{1}{2} c^3 \left( \frac{2 \sqrt{a + \frac{b}{x}} b}{\left(a + \frac{b}{x}\right) a - a^2} + \frac{b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{a^{\frac{3}{2}}} \right) - \frac{2}{3} d^3 \left( \frac{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}}{b^2} - \frac{3 \sqrt{a + \frac{b}{x}} a}{b^2} \right) - \frac{3 c^2 d \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{\sqrt{a}} - \frac{6 \sqrt{a + \frac{b}{x}} c d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{2}c^3(2\sqrt{a + b/x})b/((a + b/x)a - a^2) + b\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{3/2} - \frac{2}{3}d^3((a + b/x)^{3/2}/b^2 - 3\sqrt{a + b/x}a/b^2) - 3c^2d\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/\sqrt{a} - 6\sqrt{a + b/x}cd^2/b$

**mupad [B]** time = 1.73, size = 107, normalized size = 0.85

$$\sqrt{a + \frac{b}{x}} \left( \frac{6ad^3 - 6bcd^2}{b^2} - \frac{4ad^3}{b^2} \right) - \frac{2d^3 \left( a + \frac{b}{x} \right)^{3/2}}{3b^2} + \frac{c^3 x \sqrt{a + \frac{b}{x}}}{a} - \frac{c^2 \operatorname{atan} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) (6ad - bc) \operatorname{li}}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d/x)^3/(a + b/x)^(1/2), x)`

[Out]  $(a + b/x)^{1/2} * ((6ad^3 - 6bcd^2)/b^2 - (4ad^3)/b^2) - (2d^3(a + b/x)^{3/2})/(3b^2) + (c^3x(a + b/x)^{1/2})/a - (c^2 \operatorname{atan}(((a + b/x)^{1/2}) * 1i)/a^{1/2}) * (6ad - bc) * 1i/a^{3/2}$

**sympy [A]** time = 90.00, size = 386, normalized size = 3.06

$$\frac{4a^{\frac{7}{2}}b^{\frac{3}{2}}d^3x^2\sqrt{\frac{ax}{b} + 1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}} + 3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} + \frac{2a^{\frac{5}{2}}b^{\frac{5}{2}}d^3x\sqrt{\frac{ax}{b} + 1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}} + 3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{2a^{\frac{3}{2}}b^{\frac{7}{2}}d^3\sqrt{\frac{ax}{b} + 1}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}} + 3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^4bd^3x^{\frac{5}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}} + 3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} - \frac{4a^3b^2d^3x^{\frac{3}{2}}}{3a^{\frac{5}{2}}b^3x^{\frac{5}{2}} + 3a^{\frac{3}{2}}b^4x^{\frac{3}{2}}} + 3ca$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d/x)**3/(a+b/x)**(1/2), x)`

[Out]  $4a^{7/2}b^{3/2}d^3x^2\sqrt{ax/b + 1}/(3a^{5/2}b^3x^{5/2} + 3a^{3/2}b^4x^{3/2}) + 2a^{5/2}b^{5/2}d^3x\sqrt{ax/b + 1}/(3a^{5/2}b^3x^{5/2} + 3a^{3/2}b^4x^{3/2}) - 2a^{3/2}b^{7/2}d^3\sqrt{ax/b + 1}/(3a^{5/2}b^3x^{5/2} + 3a^{3/2}b^4x^{3/2}) - 4a^4bd^3x^{5/2}/(3a^{5/2}b^3x^{5/2} + 3a^{3/2}b^4x^{3/2}) - 4a^3b^2d^3x^{3/2}/(3a^{5/2}b^3x^{5/2} + 3a^{3/2}b^4x^{3/2}) + 3c*d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**3*sqrt(x)*sqrt(ax/b + 1)/a - 6*c**2*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**3*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a^{3/2}$

$$3.246 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\sqrt{a + \frac{b}{x}}} dx$$

Optimal. Leaf size=73

$$-\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \sqrt{a + \frac{b}{x}}}{b}$$

[Out]  $-c*(-4*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(3/2)}-2*d^2*(a+b/x)^{(1/2)}/b+c^2*x*(a+b/x)^{(1/2)}/a$

**Rubi [A]** time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {375, 89, 80, 63, 208}

$$-\frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} - \frac{2d^2 \sqrt{a + \frac{b}{x}}}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^2/Sqrt[a + b/x], x]

[Out]  $(-2*d^2*\operatorname{Sqrt}[a + b/x])/b + (c^2*\operatorname{Sqrt}[a + b/x]*x)/a - (c*(b*c - 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(3/2)}$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 80

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] + Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(d\*f\*(n + p + 2)), Int[(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

#### Rule 89

Int[((a\_.) + (b\_.)\*(x\_))^2\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[((b\*c - a\*d)^2\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d^2\*(d\*e - c\*f)\*(n + 1)), x] - Dist[1/(d^2\*(d\*e - c\*f)\*(n + 1)), Int[(c + d\*x)^(n + 1)\*(e + f\*x)^p\*Simp[a^2\*d^2\*f\*(n + p + 2) + b^2\*c\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - 2\*a\*b\*d\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - b^2\*d\*(d\*e - c\*f)\*(n + 1)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

#### Rule 208

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

## Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

## Rubi steps

$$\begin{aligned}
\int \frac{\left(c + \frac{d}{x}\right)^2}{\sqrt{a + \frac{b}{x}}} dx &= -\text{Subst}\left(\int \frac{(c + dx)^2}{x^2 \sqrt{a + bx}} dx, x, \frac{1}{x}\right) \\
&= \frac{c^2 \sqrt{a + \frac{b}{x}}}{a} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(bc - 4ad) + ad^2 x}{x \sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{2d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c^2 \sqrt{a + \frac{b}{x}}}{a} + \frac{(c(bc - 4ad)) \text{Subst}\left(\int \frac{1}{x \sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{2a} \\
&= -\frac{2d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c^2 \sqrt{a + \frac{b}{x}}}{a} + \frac{(c(bc - 4ad)) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{ab} \\
&= -\frac{2d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c^2 \sqrt{a + \frac{b}{x}}}{a} - \frac{c(bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

**Mathematica** [A] time = 0.09, size = 66, normalized size = 0.90

$$\frac{c(4ad - bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{a + \frac{b}{x}} (bc^2 x - 2ad^2)}{ab}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^2/Sqrt[a + b/x], x]
```

```
[Out] (Sqrt[a + b/x]*(-2*a*d^2 + b*c^2*x))/(a*b) + (c*(-(b*c) + 4*a*d)*ArcTanh[Sq
rt[a + b/x]/Sqrt[a]])/a^(3/2)
```

**fricas** [A] time = 0.93, size = 158, normalized size = 2.16

$$\left[ \frac{(b^2 c^2 - 4abcd)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(abc^2x - 2a^2d^2)\sqrt{\frac{ax+b}{x}}}{2a^2b}, \frac{(b^2 c^2 - 4abcd)\sqrt{-a} \arctan\left(\frac{\sqrt{-a}}{\sqrt{a + \frac{b}{x}}}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^2/(a+b/x)^(1/2), x, algorithm="fricas")
```

```
[Out] [-1/2*((b^2*c^2 - 4*a*b*c*d)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)
/x) + b) - 2*(a*b*c^2*x - 2*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b), ((b^2*c^2
- 4*a*b*c*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (a*b*c^2*x - 2
*a^2*d^2)*sqrt((a*x + b)/x))/(a^2*b)]
```

**giac** [A] time = 0.18, size = 99, normalized size = 1.36

$$\frac{\frac{b^2 c^2 \sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} + 2 d^2 \sqrt{\frac{ax+b}{x}} - \frac{(b^2 c^2 - 4abcd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="giac")

[Out]  $-(b^2 c^2 \sqrt{(a*x + b)/x}) / ((a - (a*x + b)/x) * a) + 2*d^2 \sqrt{(a*x + b)/x} - (b^2 c^2 - 4*a*b*c*d) * \arctan(\sqrt{(a*x + b)/x} / \sqrt{-a}) / (\sqrt{-a} * a) / b$

**maple** [B] time = 0.06, size = 348, normalized size = 4.77

$$\frac{\sqrt{\frac{ax+b}{x}} \left( a^2 b d^2 x^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - a^2 b d^2 x^2 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 2a b^2 c d x^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+b/x)^(1/2),x)

[Out]  $-1/2*((a*x+b)/x)^{(1/2)}/x*(\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}) * x^2*a^2*b*d^2-2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}) * x^2*a*b^2*c*d+\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}) * x^2*b^3*c^2-2*(a*x^2+b*x)^{(1/2)}*a^{(5/2)} * x^2*d^2-4*(a*x^2+b*x)^{(1/2)}*a^{(3/2)} * x^2*b*c*d-\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}) * x^2*a^2*b*d^2-2*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)}) * x^2*a*b^2*c*d-2*a^{(5/2)} * ((a*x+b)*x)^{(1/2)} * x^2*d^2+4*a^{(3/2)} * ((a*x+b)*x)^{(1/2)} * x^2*b*c*d-2*a^{(1/2)} * ((a*x+b)*x)^{(1/2)} * x^2*b^2*c^2+4*(a*x^2+b*x)^{(3/2)} * a^{(3/2)} * d^2)/((a*x+b)*x)^{(1/2)}/b^2/a^{(3/2)}$

**maxima** [B] time = 1.24, size = 129, normalized size = 1.77

$$\frac{1}{2} c^2 \left( \frac{2 \sqrt{a + \frac{b}{x}} b}{\left(a + \frac{b}{x}\right)a - a^2} + \frac{b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{a^{\frac{3}{2}}} \right) - \frac{2 c d \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{\sqrt{a}} - \frac{2 \sqrt{a + \frac{b}{x}} d^2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")

[Out]  $1/2*c^2*(2*\sqrt{a + b/x}*b/((a + b/x)*a - a^2) + b*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(3/2)}) - 2*c*d*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/\sqrt{a} - 2*\sqrt{a + b/x}*d^2/b$

**mupad** [B] time = 1.62, size = 63, normalized size = 0.86

$$\frac{c^2 x \sqrt{a + \frac{b}{x}}}{a} - \frac{2 d^2 \sqrt{a + \frac{b}{x}}}{b} + \frac{c \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) (4 a d - b c)}{a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^2/(a + b/x)^(1/2),x)

[Out]  $(c^2*x*(a + b/x)^{(1/2)})/a - (2*d^2*(a + b/x)^{(1/2)})/b + (c*\operatorname{atanh}((a + b/x)^{(1/2)})/a^{(1/2)})*(4*a*d - b*c)/a^{(3/2)}$

**sympy** [A] time = 83.77, size = 114, normalized size = 1.56

$$d^2 \left( \begin{cases} -\frac{1}{\sqrt{a}x} & \text{for } b = 0 \\ -\frac{2\sqrt{a+\frac{b}{x}}}{b} & \text{otherwise} \end{cases} \right) + \frac{\sqrt{b}c^2\sqrt{x}\sqrt{\frac{ax}{b}+1}}{a} - \frac{4cd \operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a}}\sqrt{a+\frac{b}{x}}}\right)}{a\sqrt{-\frac{1}{a}}} - \frac{bc^2 \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d/x)**2/(a+b/x)**(1/2),x)`

[Out] `d**2*Piecewise((-1/(sqrt(a)*x), Eq(b, 0)), (-2*sqrt(a + b/x)/b, True)) + sqrt(b)*c**2*sqrt(x)*sqrt(a*x/b + 1)/a - 4*c*d*atan(1/(sqrt(-1/a)*sqrt(a + b/x)))/(a*sqrt(-1/a)) - b*c**2*asinh(sqrt(a)*sqrt(x)/sqrt(b))/a**(3/2)`

$$3.247 \quad \int \frac{c + \frac{d}{x}}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=51

$$\frac{cx\sqrt{a + \frac{b}{x}}}{a} - \frac{(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out]  $-(-2*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(3/2)}+c*x*(a+b/x)^{(1/2)}/a$

**Rubi [A]** time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {375, 78, 63, 208}

$$\frac{cx\sqrt{a + \frac{b}{x}}}{a} - \frac{(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/Sqrt[a + b/x], x]

[Out]  $(c*\operatorname{Sqrt}[a + b/x]*x)/a - ((b*c - 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(3/2)}$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q]/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{c + \frac{d}{x}}{\sqrt{a + \frac{b}{x}}} dx &= -\text{Subst} \left( \int \frac{c + dx}{x^2 \sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\
&= \frac{c\sqrt{a + \frac{b}{x}}}{a} - \frac{\left(-\frac{bc}{2} + ad\right) \text{Subst} \left( \int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x} \right)}{a} \\
&= \frac{c\sqrt{a + \frac{b}{x}}}{a} - \frac{\left(2\left(-\frac{bc}{2} + ad\right)\right) \text{Subst} \left( \int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{ab} \\
&= \frac{c\sqrt{a + \frac{b}{x}}}{a} - \frac{(bc - 2ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2}}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 53, normalized size = 1.04

$$\frac{2\left(ad - \frac{bc}{2}\right) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2}} + \frac{cx\sqrt{a + \frac{b}{x}}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/Sqrt[a + b/x], x]

[Out] (c\*Sqrt[a + b/x]\*x)/a + (2\*(-1/2\*(b\*c) + a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**fricas** [A] time = 0.98, size = 115, normalized size = 2.25

$$\left[ \frac{2acx\sqrt{\frac{ax+b}{x}} - (bc - 2ad)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right)}{2a^2}, \frac{acx\sqrt{\frac{ax+b}{x}} + (bc - 2ad)\sqrt{-a} \arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(1/2), x, algorithm="fricas")

[Out] [1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x) - (b\*c - 2\*a\*d)\*sqrt(a)\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b))/a^2, (a\*c\*x\*sqrt((a\*x + b)/x) + (b\*c - 2\*a\*d)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a))/a^2]

**giac** [A] time = 0.20, size = 78, normalized size = 1.53

$$\frac{\frac{b^2c\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a} - \frac{(b^2c - 2abd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(1/2), x, algorithm="giac")

[Out] -(b^2\*c\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a) - (b^2\*c - 2\*a\*b\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a)/b



**maple [B]** time = 0.06, size = 173, normalized size = 3.39

$$\frac{\sqrt{\frac{ax+b}{x}} \left( abd \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) + abd \ln \left( \frac{2ax+b+2\sqrt{ax^2+bx} \sqrt{a}}{2\sqrt{a}} \right) - b^2 c \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) + 2\sqrt{ax^2+bx} \right)}{2\sqrt{(ax+b)x} a^{\frac{3}{2}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)/(a+b/x)^(1/2), x)

[Out]  $\frac{1}{2} * ((a*x+b)/x)^{(1/2)} * x * (2*a^{(3/2)} * (a*x^2+b*x)^{(1/2)} * d - 2*a^{(3/2)} * ((a*x+b)*x)^{(1/2)} * d + 2*a^{(1/2)} * ((a*x+b)*x)^{(1/2)} * b * c + \ln(1/2 * (2*a*x+b+2*((a*x+b)*x)^{(1/2)} * a^{(1/2)}) / a^{(1/2)}) * a * b * d - \ln(1/2 * (2*a*x+b+2*((a*x+b)*x)^{(1/2)} * a^{(1/2)}) / a^{(1/2)}) * b^2 * c + \ln(1/2 * (2*a*x+b+2*(a*x^2+b*x)^{(1/2)} * a^{(1/2)}) / a^{(1/2)}) * a * b * d) / (((a*x+b)*x)^{(1/2)} / b / a^{(3/2)})$

**maxima [B]** time = 1.30, size = 109, normalized size = 2.14

$$\frac{1}{2} c \left( \frac{2 \sqrt{a + \frac{b}{x}} b}{\left(a + \frac{b}{x}\right) a - a^2} + \frac{b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{3}{2}}} \right) - \frac{d \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(1/2), x, algorithm="maxima")

[Out]  $\frac{1}{2} * c * (2 * \sqrt{a + b/x} * b / ((a + b/x) * a - a^2) + b * \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a}))) / a^{(3/2)} - d * \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a})) / \sqrt{a}$

**mupad [B]** time = 1.98, size = 88, normalized size = 1.73

$$\frac{2 d \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{\sqrt{a}} + \frac{2 c x \left( \frac{3 \sqrt{b} \sqrt{b + a x}}{2 a x} + \frac{b^{3/2} \operatorname{asin} \left( \frac{\sqrt{a} \sqrt{x} 1i}{\sqrt{b}} \right) 3i}{2 a^{3/2} x^{3/2}} \right) \sqrt{\frac{a x}{b} + 1}}{3 \sqrt{a + \frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)/(a + b/x)^(1/2), x)

[Out]  $(2 * d * \operatorname{atanh}((a + b/x)^{(1/2)} / a^{(1/2)})) / a^{(1/2)} + (2 * c * x * ((3 * b^{(1/2)} * (b + a * x)^{(1/2)}) / (2 * a * x) + (b^{(3/2)} * \operatorname{asin}((a^{(1/2)} * x^{(1/2)} * 1i) / b^{(1/2)}) * 3i) / (2 * a^{(3/2)} * x^{(3/2)})) * ((a * x) / b + 1)^{(1/2)}) / (3 * (a + b/x)^{(1/2)})$

**sympy [A]** time = 60.25, size = 82, normalized size = 1.61

$$\frac{\sqrt{b} c \sqrt{x} \sqrt{\frac{ax}{b} + 1}}{a} - \frac{2 d \operatorname{atan} \left( \frac{1}{\sqrt{-\frac{1}{a}} \sqrt{a + \frac{b}{x}}} \right)}{a \sqrt{-\frac{1}{a}}} - \frac{b c \operatorname{asinh} \left( \frac{\sqrt{a} \sqrt{x}}{\sqrt{b}} \right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(1/2), x)

[Out]  $\sqrt{b} * c * \sqrt{x} * \sqrt{a * x / b + 1} / a - 2 * d * \operatorname{atan}(1 / (\sqrt{-1/a} * \sqrt{a + b/x})) / (a * \sqrt{-1/a}) - b * c * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / a^{(3/2)}$

$$3.248 \quad \int \frac{1}{\sqrt{a + \frac{b}{x}}} dx$$

**Optimal.** Leaf size=43

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

[Out]  $-b \operatorname{arctanh}\left(\frac{(a+b/x)^{1/2}}{a^{1/2}}\right)/a^{3/2} + x(a+b/x)^{1/2}/a$

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {242, 51, 63, 208}

$$\frac{x\sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b/x], x]

[Out] (Sqrt[a + b/x]\*x)/a - (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 242

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a + \frac{b}{x}}} dx &= -\text{Subst} \left( \int \frac{1}{x^2 \sqrt{a + bx}} dx, x, \frac{1}{x} \right) \\
&= \frac{\sqrt{a + \frac{b}{x}}}{a} + \frac{b \text{Subst} \left( \int \frac{1}{x \sqrt{a + bx}} dx, x, \frac{1}{x} \right)}{2a} \\
&= \frac{\sqrt{a + \frac{b}{x}}}{a} + \frac{\text{Subst} \left( \int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{a} \\
&= \frac{\sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.02, size = 43, normalized size = 1.00

$$\frac{x \sqrt{a + \frac{b}{x}}}{a} - \frac{b \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b/x], x]

[Out] (Sqrt[a + b/x]\*x)/a - (b\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)

**fricas [A]** time = 0.93, size = 98, normalized size = 2.28

$$\left[ \frac{2ax\sqrt{\frac{ax+b}{x}} + \sqrt{a}b \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right)}{2a^2}, \frac{ax\sqrt{\frac{ax+b}{x}} + \sqrt{-a}b \arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(1/2), x, algorithm="fricas")

[Out] [1/2\*(2\*a\*x\*sqrt((a\*x + b)/x) + sqrt(a)\*b\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b))/a^2, (a\*x\*sqrt((a\*x + b)/x) + sqrt(-a)\*b\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a))/a^2]

**giac [B]** time = 0.24, size = 71, normalized size = 1.65

$$-\frac{b \log(|b|) \text{sgn}(x)}{2a^{\frac{3}{2}}} + \frac{b \log\left(\left| -2\left(\sqrt{a}x - \sqrt{ax^2 + bx}\right)\sqrt{a} - b \right|\right)}{2a^{\frac{3}{2}} \text{sgn}(x)} + \frac{\sqrt{ax^2 + bx}}{a \text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(1/2), x, algorithm="giac")

[Out] -1/2\*b\*log(abs(b))\*sgn(x)/a^(3/2) + 1/2\*b\*log(abs(-2\*(sqrt(a)\*x - sqrt(a\*x^2 + b\*x))\*sqrt(a) - b))/(a^(3/2)\*sgn(x)) + sqrt(a\*x^2 + b\*x)/(a\*sgn(x))

**maple [A]** time = 0.05, size = 71, normalized size = 1.65

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -b \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) + 2\sqrt{(ax+b)x} \sqrt{a} \right) x}{2\sqrt{(ax+b)x} a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b/x)^(1/2),x)`

[Out]  $1/2*((a*x+b)/x)^{(1/2)}*x*(2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}-b*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)))/a^{(1/2)}))/((a*x+b)*x)^{(1/2)}/a^{(3/2)}$

**maxima** [A] time = 1.21, size = 67, normalized size = 1.56

$$\frac{\sqrt{a + \frac{b}{x}} b}{\left(a + \frac{b}{x}\right) a - a^2} + \frac{b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{2 a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b/x)^(1/2),x, algorithm="maxima")`

[Out]  $\sqrt{a + b/x} * b / ((a + b/x) * a - a^2) + 1/2 * b * \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a})) / a^{(3/2)}$

**mupad** [B] time = 1.44, size = 66, normalized size = 1.53

$$\frac{2x \left( \frac{3\sqrt{b}\sqrt{b+ax}}{2ax} + \frac{b^{3/2} \operatorname{asin}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right) 3i}{2a^{3/2}x^{3/2}} \right) \sqrt{\frac{ax}{b} + 1}}{3\sqrt{a + \frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b/x)^(1/2),x)`

[Out]  $(2*x*((3*b^{(1/2)}*(b + a*x)^{(1/2)})/(2*a*x) + (b^{(3/2)}*asin((a^{(1/2)}*x^{(1/2)}*1i)/b^{(1/2)})*3i)/(2*a^{(3/2)}*x^{(3/2)}))*((a*x)/b + 1)^{(1/2)}/(3*(a + b/x)^{(1/2)})$

**sympy** [A] time = 3.09, size = 44, normalized size = 1.02

$$\frac{\sqrt{b}\sqrt{x}\sqrt{\frac{ax}{b} + 1}}{a} - \frac{b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b/x)**(1/2),x)`

[Out]  $\sqrt{b} * \sqrt{x} * \sqrt{a*x/b + 1} / a - b * \operatorname{asinh}(\sqrt{a} * \sqrt{x} / \sqrt{b}) / a^{(3/2)}$

$$3.249 \quad \int \frac{1}{\sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)} dx$$

Optimal. Leaf size=108

$$-\frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2} - \frac{2d^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} + \frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

[Out]  $-(2*a*d+b*c)*\operatorname{arctanh}\left(\frac{(a+b/x)^{(1/2)}}{a^{(1/2)}}\right)/a^{(3/2)}/c^2-2*d^{(3/2)}*\operatorname{arctan}\left(\frac{d^{(1/2)}*(a+b/x)^{(1/2)}}{(-a*d+b*c)^{(1/2)}}\right)/c^2/(-a*d+b*c)^{(1/2)}+x*(a+b/x)^{(1/2)}/a/c$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 103, 156, 63, 208, 205}

$$-\frac{(2ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}c^2} - \frac{2d^{3/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2\sqrt{bc-ad}} + \frac{x\sqrt{a+\frac{b}{x}}}{ac}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)), x]

[Out]  $(\operatorname{Sqrt}[a + b/x]*x)/(a*c) - (2*d^{(3/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/(\operatorname{Sqrt}[b*c - a*d])])/(c^2*\operatorname{Sqrt}[b*c - a*d]) - ((b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/(\operatorname{Sqrt}[a])])/(a^{(3/2)}*c^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 156

Int((((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} dx &= -\text{Subst} \left( \int \frac{1}{x^2 \sqrt{a + bx} (c + dx)} dx, x, \frac{1}{x} \right) \\
 &= \frac{\sqrt{a + \frac{b}{x}}}{ac} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(bc+2ad) + \frac{bdx}{2}}{x \sqrt{a+bx} (c+dx)} dx, x, \frac{1}{x} \right)}{ac} \\
 &= \frac{\sqrt{a + \frac{b}{x}}}{ac} - \frac{d^2 \text{Subst} \left( \int \frac{1}{\sqrt{a+bx} (c+dx)} dx, x, \frac{1}{x} \right)}{c^2} + \frac{(bc + 2ad) \text{Subst} \left( \int \frac{1}{x \sqrt{a+bx}} dx, x, \frac{1}{x} \right)}{2ac^2} \\
 &= \frac{\sqrt{a + \frac{b}{x}}}{ac} - \frac{(2d^2) \text{Subst} \left( \int \frac{1}{c - \frac{ad}{b} + \frac{dx^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{bc^2} + \frac{(bc + 2ad) \text{Subst} \left( \int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \frac{1}{x} \right)}{abc^2} \\
 &= \frac{\sqrt{a + \frac{b}{x}}}{ac} - \frac{2d^{3/2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc-ad}} \right)}{c^2 \sqrt{bc-ad}} - \frac{(bc + 2ad) \tanh^{-1} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right)}{a^{3/2} c^2}
 \end{aligned}$$

**Mathematica [A]** time = 0.25, size = 104, normalized size = 0.96

$$\frac{(2ad+bc) \tanh^{-1} \left( \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}} \right) - \frac{2d^{3/2} \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}} \right)}{\sqrt{bc-ad}} + \frac{cx \sqrt{a+\frac{b}{x}}}{a}}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x]\*(c + d/x)),x]

[Out] ((c\*Sqrt[a + b/x]\*x)/a - (2\*d^(3/2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/Sqrt[b\*c - a\*d] - ((b\*c + 2\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(3/2)/c^2

**fricas [A]** time = 0.99, size = 542, normalized size = 5.02

$$\left[ \frac{2 a^2 d \sqrt{-\frac{d}{bc-ad}} \log \left( -\frac{2 (bc-ad)x \sqrt{-\frac{d}{bc-ad}} \sqrt{\frac{ax+b}{x}} - bd + (bc-2ad)x}{cx+d} \right) + 2 acx \sqrt{\frac{ax+b}{x}} + (bc + 2ad) \sqrt{a} \log \left( 2 ax - 2 \sqrt{a} x \sqrt{\frac{ax+b}{x}} \right)}{2 a^2 c^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="fricas")

```
[Out] [1/2*(2*a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d)))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d) + 2*a*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a^2*c^2), (a^2*d*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d)))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d) + a*c*x*sqrt((a*x + b)/x) + (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2), -1/2*(4*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*a*c*x*sqrt((a*x + b)/x) - (b*c + 2*a*d)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b))/(a^2*c^2), -(2*a^2*d*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - a*c*x*sqrt((a*x + b)/x) - (b*c + 2*a*d)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a))/(a^2*c^2)]
```

**giac** [A] time = 0.18, size = 134, normalized size = 1.24

$$-b^2 \left( \frac{2d^2 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{\sqrt{bcd-ad^2} b^2 c^2} + \frac{\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right) abc} - \frac{(bc + 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} ab^2 c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)/(a+b/x)^(1/2),x, algorithm="giac")
```

```
[Out] -b^2*(2*d^2*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/(sqrt(b*c*d - a*d^2)*b^2*c^2) + sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a*b*c) - (b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a*b^2*c^2)
```

**maple** [B] time = 0.06, size = 228, normalized size = 2.11

$$\frac{\left( 2a^3 d^2 \ln\left(\frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}} \sqrt{(ax+b)x} c}{cx+d}\right) + 2\sqrt{\frac{(ad-bc)d}{c^2}} acd \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) + \sqrt{\frac{(ad-bc)d}{c^2}} b c^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) \right)}{2\sqrt{\frac{(ad-bc)d}{c^2}} \sqrt{(ax+b)x} a^{\frac{3}{2}} c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*(2*a^(3/2)*d^2*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(1/2)*c^2+2*((a*d-b*c)/c^2*d)^(1/2)*a*c*d*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+((a*d-b*c)/c^2*d)^(1/2)*b*c^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*x*((a*x+b)/x)^(1/2)/((a*d-b*c)/c^2*d)^(1/2)/c^3/a^(3/2)/((a*x+b)*x)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(a + b/x)*(c + d/x)), x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)), x)
```

**mupad [B]** time = 1.98, size = 1183, normalized size = 10.95

$$\frac{x \sqrt{a + \frac{b}{x}}}{ac} \operatorname{atanh} \left( \frac{12b^4 d^4 \sqrt{a + \frac{b}{x}}}{\sqrt{a^3} \left( \frac{12b^4 d^4}{a} + \frac{10b^5 c d^3}{a^2} + \frac{2b^6 c^2 d^2}{a^3} \right)} + \frac{10b^5 d^3 \sqrt{a + \frac{b}{x}}}{\sqrt{a^3} \left( \frac{10b^5 d^3}{a} + \frac{12b^4 d^4}{c} + \frac{2b^6 c d^2}{a^2} \right)} + \frac{2b^6 d^2 \sqrt{a + \frac{b}{x}}}{\sqrt{a^3} \left( \frac{2b^6 d^2}{a} + \frac{10b^5 d^3}{c} + \frac{12ab^4 d^4}{c^2} \right)} \right) (2ad + b^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b/x)^(1/2)*(c + d/x)),x)`

[Out]  $(x*(a + b/x)^{(1/2)})/(a*c) - (\operatorname{atan}(\frac{(((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^{(1/2)})*(a*d^4 - b*c*d^3)^{(1/2)})/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d) - (2*(a + b/x)^{(1/2)}*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^{(1/2)}*i)/(b*c^3 - a*c^2*d) - (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^{(1/2)}*(a*d^4 - b*c*d^3)^{(1/2)})/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d) + (2*(a + b/x)^{(1/2)}*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^{(1/2)}*i)/(b*c^3 - a*c^2*d))/(((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) - (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^{(1/2)}*(a*d^4 - b*c*d^3)^{(1/2)})/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d) - (2*(a + b/x)^{(1/2)}*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d) - (4*(2*a*b^3*d^5 + b^4*c*d^4))/(a^2*c^3) + (((((2*(2*a*b^4*c^5*d^2 + 2*a^2*b^3*c^4*d^3))/(a^2*c^3) + (2*(4*a^2*b^3*c^5*d^2 - 8*a^3*b^2*c^4*d^3)*(a + b/x)^{(1/2)}*(a*d^4 - b*c*d^3)^{(1/2)})/(a^2*c^2*(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d) + (2*(a + b/x)^{(1/2)}*(8*a^2*b^2*d^5 + b^4*c^2*d^3 + 4*a*b^3*c*d^4))/(a^2*c^2))*(a*d^4 - b*c*d^3)^{(1/2)})/(b*c^3 - a*c^2*d)))*(a*d^4 - b*c*d^3)^{(1/2)}*i)/(b*c^3 - a*c^2*d) - (\operatorname{atanh}(\frac{12*b^4*d^4*(a + b/x)^{(1/2)}}{(a^3)^{(1/2)}*((12*b^4*d^4)/a + (10*b^5*c*d^3)/a^2 + (2*b^6*c^2*d^2)/a^3)} + \frac{10*b^5*d^3*(a + b/x)^{(1/2)}}{(a^3)^{(1/2)}*((10*b^5*d^3)/a + (12*b^4*d^4)/c + (2*b^6*c*d^2)/a^2)} + \frac{2*b^6*d^2*(a + b/x)^{(1/2)}}{(a^3)^{(1/2)}*((2*b^6*d^2)/a + (10*b^5*d^3)/c + (12*a*b^4*d^4)/c^2)}))*(2*a*d + b*c))/(c^2*(a^3)^{(1/2)})$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + \frac{b}{x}} (cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(c+d/x)/(a+b/x)**(1/2),x)`

[Out] `Integral(x/(sqrt(a + b/x)*(c*x + d)), x)`



$$3.250 \quad \int \frac{1}{\sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=172

$$\frac{(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) d^{3/2}(5bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{d\sqrt{a+\frac{b}{x}}(bc - 2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc - ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}}{a^{3/2}c^3 - c^3(bc - ad)^{3/2}}$$

[Out]  $-d^{3/2}*(-4*a*d+5*b*c)*\arctan(d^{1/2}*(a+b/x)^{1/2}/(-a*d+b*c)^{1/2})/c^3/(-a*d+b*c)^{3/2}-(4*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{1/2}/a^{1/2})/a^{3/2}/c^3+d*(-2*a*d+b*c)*(a+b/x)^{1/2}/a/c^2/(-a*d+b*c)/(c+d/x)+x*(a+b/x)^{1/2}/a/c/(c+d/x)$

**Rubi [A]** time = 0.22, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 103, 151, 156, 63, 208, 205}

$$\frac{(4ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) d^{3/2}(5bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{d\sqrt{a+\frac{b}{x}}(bc - 2ad)}{ac^2\left(c+\frac{d}{x}\right)(bc - ad)} + \frac{x\sqrt{a+\frac{b}{x}}}{ac\left(c+\frac{d}{x}\right)}}{a^{3/2}c^3 - c^3(bc - ad)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)^2), x]

[Out]  $(d*(b*c - 2*a*d)*\operatorname{Sqrt}[a + b/x])/(a*c^2*(b*c - a*d)*(c + d/x)) + (\operatorname{Sqrt}[a + b/x]*x)/(a*c*(c + d/x)) - (d^{3/2}*(5*b*c - 4*a*d)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(c^3*(b*c - a*d)^{3/2}) - ((b*c + 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(a^{3/2}*c^3)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.)\*((g\_.) + (h\_.)\*(x\_.)), x\_Symbol] :> Simp[(b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x]

, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 156

Int[(((e\_) + (f\_)\*(x\_))^(p\_)\*((g\_) + (h\_)\*(x\_)))/(((a\_) + (b\_)\*(x\_))\*((c\_) + (d\_)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} dx &= -\text{Subst} \left( \int \frac{1}{x^2 \sqrt{a + bx} (c + dx)^2} dx, x, \frac{1}{x} \right) \\ &= \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(bc + 4ad) + \frac{3bdx}{2}}{x \sqrt{a + bx} (c + dx)^2} dx, x, \frac{1}{x} \right)}{ac} \\ &= \frac{d(bc - 2ad) \sqrt{a + \frac{b}{x}}}{ac^2(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)} - \frac{\text{Subst} \left( \int \frac{-\frac{1}{2}(bc - ad)(bc + 4ad) - \frac{1}{2}bd(bc - 2ad)x}{x \sqrt{a + bx} (c + dx)} dx, x, \frac{1}{x} \right)}{ac^2(bc - ad)} \\ &= \frac{d(bc - 2ad) \sqrt{a + \frac{b}{x}}}{ac^2(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)} - \frac{(d^2(5bc - 4ad)) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx} (c + dx)} dx, x, \frac{1}{x} \right)}{2c^3(bc - ad)} + \dots \\ &= \frac{d(bc - 2ad) \sqrt{a + \frac{b}{x}}}{ac^2(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)} - \frac{(d^2(5bc - 4ad)) \text{Subst} \left( \int \frac{1}{c - \frac{ad}{b} + \frac{dx^2}{b}} dx, x, \sqrt{a + \frac{b}{x}} \right)}{bc^3(bc - ad)} \\ &= \frac{d(bc - 2ad) \sqrt{a + \frac{b}{x}}}{ac^2(bc - ad) \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)} - \frac{d^{3/2}(5bc - 4ad) \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{c^3(bc - ad)^{3/2}} - \frac{(bc + 4ad) \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{a^{3/2}c} \end{aligned}$$

**Mathematica [A]** time = 0.78, size = 150, normalized size = 0.87

$$\frac{ad^{3/2}(4ad-5bc) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + \frac{cx\sqrt{a+\frac{b}{x}}(bc(cx+d)-ad(cx+2d))}{(cx+d)(bc-ad)} - \frac{(4ad+bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{\sqrt{a}}}{ac^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a + b/x]*(c + d/x)^2), x]
[Out] ((c*Sqrt[a + b/x]*x*(b*c*(d + c*x) - a*d*(2*d + c*x)))/((b*c - a*d)*(d + c*x)) + (a*d^(3/2)*(-5*b*c + 4*a*d)*ArcTan[(Sqrt[d]*Sqrt[a + b/x])/Sqrt[b*c - a*d]])/(b*c - a*d)^(3/2) - ((b*c + 4*a*d)*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/Sqrt[a]/(a*c^3)
```

**fricas [A]** time = 1.07, size = 1163, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2), x, algorithm="fricas")
[Out] [1/2*((b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x), -1/2*(2*(5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x), 1/2*(2*(b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(-d/(b*c - a*d))*log(-2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d) + 2*((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x), -((5*a^2*b*c*d^2 - 4*a^3*d^3 + (5*a^2*b*c^2*d - 4*a^3*c*d^2)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - (b^2*c^2*d + 3*a*b*c*d^2 - 4*a^2*d^3 + (b^2*c^3 + 3*a*b*c^2*d - 4*a^2*c*d^2)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - ((a*b*c^3 - a^2*c^2*d)*x^2 + (a*b*c^2*d - 2*a^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^2*b*c^4*d - a^3*c^3*d^2 + (a^2*b*c^5 - a^3*c^4*d)*x)]
```

**giac [A]** time = 0.22, size = 300, normalized size = 1.74

$$-b^3 \left( \frac{(5bcd^2 - 4ad^3) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^4c^4 - ab^3c^3d)\sqrt{bcd - ad^2}} + \frac{b^2c^2\sqrt{\frac{ax+b}{x}} - 2abcd\sqrt{\frac{ax+b}{x}} + 2a^2d^2\sqrt{\frac{ax+b}{x}} + \frac{(ax+b)bcd\sqrt{\frac{ax+b}{x}}}{x} - \frac{2(ax+b)d}{x^2}}{(ab^3c^3 - a^2b^2c^2d)\left(abc - a^2d - \frac{(ax+b)bc}{x} + \frac{2(ax+b)ad}{x} - \frac{(ax+b)^2d}{x^2}\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2), x, algorithm="giac")
```

```
[Out] -b^3*((5*b*c*d^2 - 4*a*d^3)*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))
/(b^4*c^4 - a*b^3*c^3*d)*sqrt(b*c*d - a*d^2)) + (b^2*c^2*sqrt((a*x + b)/x)
- 2*a*b*c*d*sqrt((a*x + b)/x) + 2*a^2*d^2*sqrt((a*x + b)/x) + (a*x + b)*b*
c*d*sqrt((a*x + b)/x)/x - 2*(a*x + b)*a*d^2*sqrt((a*x + b)/x)/((a*b^3*c^
3 - a^2*b^2*c^2*d)*(a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (
a*x + b)^2*d/x^2)) - (b*c + 4*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt
(-a)*a*b^3*c^3))
```

**maple [B]** time = 0.07, size = 1135, normalized size = 6.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^2/(a+b/x)^(1/2),x)
```

```
[Out] -1/2*((a*x+b)/x)^(1/2)*x*(4*a^(9/2)*c*d^4*x*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+2*((a*x+b)*x)^(1/2)*((a*d-b
*c)/c^2*d)^(1/2)*a^(7/2)*c^4*d*x^2+4*a^(9/2)*d^5*ln((-2*a*d*x+b*c*x-b*d+2*((
a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*((a*x+b)*x)^(1/2)*((
a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^3*d^2*x-9*a^(7/2)*b*c^2*d^3*x*ln((-2*a*d*x+
b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-4*((a*x+b
)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(7/2)*c^2*d^3-9*a^(7/2)*b*c*d^4*ln((-2
*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))-2*
((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*c^4*d+6*((a*x+b)*x)^(1/2)
*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^4*d*x+5*a^(5/2)*b^2*c^3*d^2*x*ln((-2*a
*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))+6*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*a^(5/2)*b*c^3*d^2+5*a^(5/2)*b^2*c^2
*d^3*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/
(c*x+d))-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^5-2*a^
(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^4*d+4*((a*d-b*c)/c^2*
d)^(1/2)*a^4*c^2*d^3*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
)-7*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^3*d^2*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(
1/2)*a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a
^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4*d+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c
^5*x*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))+4*((a*d-b*c)/c^2
*d)^(1/2)*a^4*c*d^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))-7
*((a*d-b*c)/c^2*d)^(1/2)*a^3*b*c^2*d^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*
a^(1/2))/a^(1/2))+2*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2)
))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^2+((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^4*d*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))/c^4/((a*x+b)*x)^(1/2)
)/(a*d-b*c)^2/(c*x+d)/a^(5/2)/((a*d-b*c)/c^2*d)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c+d/x)^2/(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a + b/x)*(c + d/x)^2), x)
```

**mupad [B]** time = 3.54, size = 3813, normalized size = 22.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(1/2)*(c + d/x)^2),x)
```



$$7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) + ((d^3*(a*d - b*c)^3)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)) + ((d^3*(a*d - b*c)^3)^{(1/2)}*(4*a*d - 5*b*c)*((2*(a + b/x)^{(1/2)}*(32*a^4*b^2*d^7 + b^6*c^4*d^3 + 6*a*b^5*c^3*d^4 - 64*a^3*b^3*c*d^6 + 26*a^2*b^4*c^2*d^5)))/(a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d) - ((d^3*(a*d - b*c)^3)^{(1/2)}*(4*a*d - 5*b*c)*((4*a*b^6*c^9*d^2 + 4*a^2*b^5*c^8*d^3 - 16*a^3*b^4*c^7*d^4 + 8*a^4*b^3*c^6*d^5)/(a^2*b^2*c^8 + a^4*c^6*d^2 - 2*a^3*b*c^7*d) - ((d^3*(a*d - b*c)^3)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d - 5*b*c)*(4*a^2*b^5*c^9*d^2 - 16*a^3*b^4*c^8*d^3 + 20*a^4*b^3*c^7*d^4 - 8*a^5*b^2*c^6*d^5))/((a^2*b^2*c^6 + a^4*c^4*d^2 - 2*a^3*b*c^5*d)*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))/((2*(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)))*((d^3*(a*d - b*c)^3)^{(1/2)}*(4*a*d - 5*b*c)*1i)/(b^3*c^6 - a^3*c^3*d^3 + 3*a^2*b*c^4*d^2 - 3*a*b^2*c^5*d)$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)\*\*2/(a+b/x)\*\*(1/2),x)

[Out] Timed out

$$3.251 \quad \int \frac{1}{\sqrt{a+\frac{b}{x}} \left(c+\frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=250

$$\frac{(6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) d^{3/2} (24a^2d^2 - 56abcd + 35b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{a^{3/2}c^4 - 4c^4(bc-ad)^{5/2} + 4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

[Out]  $-1/4*d^{(3/2)}*(24*a^2*d^2-56*a*b*c*d+35*b^2*c^2)*\arctan(d^{(1/2)}*(a+b/x)^{(1/2)}/(-a*d+b*c)^{(1/2)})/c^4/(-a*d+b*c)^{(5/2)}-(6*a*d+b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(3/2)}/c^4+1/2*d*(-3*a*d+2*b*c)*(a+b/x)^{(1/2)}/a/c^2/(-a*d+b*c)/(c+d/x)^2+1/4*d*(-4*a*d+b*c)*(-3*a*d+4*b*c)*(a+b/x)^{(1/2)}/a/c^3/(-a*d+b*c)^2/(c+d/x)+x*(a+b/x)^{(1/2)}/a/c/(c+d/x)^2$

**Rubi [A]** time = 0.40, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 103, 151, 156, 63, 208, 205}

$$\frac{d^{3/2} (24a^2d^2 - 56abcd + 35b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) (6ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) + d\sqrt{a+\frac{b}{x}}(bc-4ad)(4bc-3ad)}{4c^4(bc-ad)^{5/2} - a^{3/2}c^4 + 4ac^3\left(c+\frac{d}{x}\right)(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x]\*(c + d/x)^3), x]

[Out]  $(d*(2*b*c - 3*a*d)*\operatorname{Sqrt}[a + b/x])/(2*a*c^2*(b*c - a*d)*(c + d/x)^2) + (d*(b*c - 4*a*d)*(4*b*c - 3*a*d)*\operatorname{Sqrt}[a + b/x])/(4*a*c^3*(b*c - a*d)^2*(c + d/x)) + (\operatorname{Sqrt}[a + b/x]*x)/(a*c*(c + d/x)^2) - (d^{(3/2)}*(35*b^2*c^2 - 56*a*b*c*d + 24*a^2*d^2)*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/\operatorname{Sqrt}[b*c - a*d]])/(4*c^4*(b*c - a*d)^{(5/2)}) - ((b*c + 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(a^{(3/2)}*c^4)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)),

$x] + \text{Dist}[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n*(e + f*x)^p*\text{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 156

$\text{Int}[(((e_.) + (f_.)*(x_.))^{(p_.)}*((g_.) + (h_.)*(x_.)))/(((a_.) + (b_.)*(x_.))*(c_.) + (d_.)*(x_.)), x\_Symbol] := \text{Dist}[(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

$\text{Int}[((a_) + (b_.)*(x_)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

$\text{Int}[((a_) + (b_.)*(x_)^{(n_.)})^{(p_.)}*((c_) + (d_.)*(x_)^{(n_.)})^{(q_.)}, x\_Symbol] := -\text{Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] /;$  FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps



$$\begin{aligned}
\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst} \left( \int \frac{1}{x^2 \sqrt{a + bx} (c + dx)^3} dx, x, \frac{1}{x} \right) \\
&= \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(bc+6ad) + \frac{5bdx}{2}}{x \sqrt{a+bx} (c+dx)^3} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{d(2bc - 3ad) \sqrt{a + \frac{b}{x}}}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst} \left( \int \frac{-(bc-ad)(bc+6ad) - \frac{3}{2}bd(2bc-3ad)x}{x \sqrt{a+bx} (c+dx)^2} dx, x, \frac{1}{x} \right)}{2ac^2(bc - ad)} \\
&= \frac{d(2bc - 3ad) \sqrt{a + \frac{b}{x}}}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} + \frac{d(bc - 4ad)(4bc - 3ad) \sqrt{a + \frac{b}{x}}}{4ac^3(bc - ad)^2 \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst} \left( \int \right)}{2ac^2(bc - ad)} \\
&= \frac{d(2bc - 3ad) \sqrt{a + \frac{b}{x}}}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} + \frac{d(bc - 4ad)(4bc - 3ad) \sqrt{a + \frac{b}{x}}}{4ac^3(bc - ad)^2 \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} + \frac{(bc + 6ad)}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} \\
&= \frac{d(2bc - 3ad) \sqrt{a + \frac{b}{x}}}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} + \frac{d(bc - 4ad)(4bc - 3ad) \sqrt{a + \frac{b}{x}}}{4ac^3(bc - ad)^2 \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} + \frac{(bc + 6ad)}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} \\
&= \frac{d(2bc - 3ad) \sqrt{a + \frac{b}{x}}}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2} + \frac{d(bc - 4ad)(4bc - 3ad) \sqrt{a + \frac{b}{x}}}{4ac^3(bc - ad)^2 \left(c + \frac{d}{x}\right)} + \frac{\sqrt{a + \frac{b}{x}}}{ac \left(c + \frac{d}{x}\right)^2} + \frac{d^{3/2} (35b^2)}{2ac^2(bc - ad) \left(c + \frac{d}{x}\right)^2}
\end{aligned}$$

**Mathematica [A]** time = 1.75, size = 216, normalized size = 0.86

$$\frac{cx \sqrt{a + \frac{b}{x}} (2a^2 d^2 (2c^2 x^2 + 9cdx + 6d^2) - abcd(8c^2 x^2 + 29cdx + 19d^2) + 4b^2 c^2 (cx + d)^2)}{(cx + d)^2 (bc - ad)^2} - \frac{ad^{3/2} (24a^2 d^2 - 56abcd + 35b^2 c^2) \tan^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right)}{(bc - ad)^{5/2}} - \frac{4(6ad + bc)}{4ac^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x]\*(c + d/x)^3), x]

[Out] ((c\*Sqrt[a + b/x]\*x\*(4\*b^2\*c^2\*(d + c\*x)^2 + 2\*a^2\*d^2\*(6\*d^2 + 9\*c\*d\*x + 2\*c^2\*x^2) - a\*b\*c\*d\*(19\*d^2 + 29\*c\*d\*x + 8\*c^2\*x^2)))/((b\*c - a\*d)^2\*(d + c\*x)^2) - (a\*d^(3/2)\*(35\*b^2\*c^2 - 56\*a\*b\*c\*d + 24\*a^2\*d^2)\*ArcTan[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]])/(b\*c - a\*d)^(5/2) - (4\*(b\*c + 6\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]]/Sqrt[a])/((4\*a\*c^4))

**fricas [B]** time = 1.94, size = 2307, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2), x, algorithm="fricas")

```
[Out] [1/8*(4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), 1/8*(8*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 2*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x), -1/4*((35*a^2*b^2*c^2*d^3 - 56*a^3*b*c*d^4 + 24*a^4*d^5 + (35*a^2*b^2*c^4*d - 56*a^3*b*c^3*d^2 + 24*a^4*c^2*d^3)*x^2 + 2*(35*a^2*b^2*c^3*d^2 - 56*a^3*b*c^2*d^3 + 24*a^4*c*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 4*(b^3*c^3*d^2 + 4*a*b^2*c^2*d^3 - 11*a^2*b*c*d^4 + 6*a^3*d^5 + (b^3*c^5 + 4*a*b^2*c^4*d - 11*a^2*b*c^3*d^2 + 6*a^3*c^2*d^3)*x^2 + 2*(b^3*c^4*d + 4*a*b^2*c^3*d^2 - 11*a^2*b*c^2*d^3 + 6*a^3*c*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (4*(a*b^2*c^5 - 2*a^2*b*c^4*d + a^3*c^3*d^2)*x^3 + (8*a*b^2*c^4*d - 29*a^2*b*c^3*d^2 + 18*a^3*c^2*d^3)*x^2 + (4*a*b^2*c^3*d^2 - 19*a^2*b*c^2*d^3 + 12*a^3*c*d^4)*x)*sqrt((a*x + b)/x))/(a^2*b^2*c^6*d^2 - 2*a^3*b*c^5*d^3 + a^4*c^4*d^4 + (a^2*b^2*c^8 - 2*a^3*b*c^7*d + a^4*c^6*d^2)*x^2 + 2*(a^2*b^2*c^7*d - 2*a^3*b*c^6*d^2 + a^4*c^5*d^3)*x)]
```

**giac** [A] time = 0.29, size = 352, normalized size = 1.41

$$-\frac{1}{4} b^4 \left( \frac{(35 b^2 c^2 d^2 - 56 a b c d^3 + 24 a^2 d^4) \arctan\left(\frac{d \sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^6 c^6 - 2 a b^5 c^5 d + a^2 b^4 c^4 d^2) \sqrt{bcd - ad^2}} + \frac{13 b^2 c^2 d^2 \sqrt{\frac{ax+b}{x}} - 21 a b c d^3 \sqrt{\frac{ax+b}{x}} + 8 a^2 d^4 \sqrt{\frac{ax+b}{x}}}{(b^5 c^5 - 2 a b^4 c^4 d + a^2 b^3 c^3 d^2) (bc - ad^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="giac")

```
[Out] -1/4*b^4*((35*b^2*c^2*d^2 - 56*a*b*c*d^3 + 24*a^2*d^4)*arctan(d*sqrt((a*x +
b)/x)/sqrt(b*c*d - a*d^2))/((b^6*c^6 - 2*a*b^5*c^5*d + a^2*b^4*c^4*d^2)*sq
rt(b*c*d - a*d^2)) + (13*b^2*c^2*d^2*sqrt((a*x + b)/x) - 21*a*b*c*d^3*sqrt(
(a*x + b)/x) + 8*a^2*d^4*sqrt((a*x + b)/x) + 11*(a*x + b)*b*c*d^3*sqrt((a*x
+ b)/x)/x - 8*(a*x + b)*a*d^4*sqrt((a*x + b)/x)/x)/((b^5*c^5 - 2*a*b^4*c^4
*d + a^2*b^3*c^3*d^2)*(b*c - a*d + (a*x + b)*d/x)^2) + 4*sqrt((a*x + b)/x)/
((a - (a*x + b)/x)*a*b^3*c^3) - 4*(b*c + 6*a*d)*arctan(sqrt((a*x + b)/x)/sq
rt(-a))/(sqrt(-a)*a*b^4*c^4)
```

**maple [B]** time = 0.07, size = 2269, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c+d/x)^3/(a+b/x)^(1/2), x)
```

```
[Out] -1/8*((a*x+b)/x)^(1/2)*x*(60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
)))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d^4-12*a^2*ln(1/2*(2*a*x+b+2*((
a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4*d^3+12*a^
(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^5*d^2-12*a^(7/2)*((a*
x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^5*d^2-80*a^(9/2)*ln((-2*a*d*x+b*c
*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*c^3*d^
4+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(
1/2)*c)/(c*x+d))*x^2*b^2*c^4*d^3-35*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-
b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b^3*c^5*d^2+24*a^5*ln(1
/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x
^2*c^3*d^4+18*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b*c^5*d^2-3
6*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*c^3*d^4-160*a^(9/2)*l
n((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d
))*x*b*c^2*d^5-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((
a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^7+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^
2*d)^(1/2)*b^3*c^5*d^2-4*a*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(
1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^5*d^2+182*a^(7/2)*ln((-2*a*d*x+b*c*x-b*
d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*c^3*d^4-70*
a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*
c)/(c*x+d))*x*b^3*c^4*d^3+48*a^5*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2
))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*c^2*d^5+62*a^(7/2)*((a*x+b)*x)^(1/2)*
((a*d-b*c)/c^2*d)^(1/2)*b*c^3*d^4-46*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c
^2*d)^(1/2)*b^2*c^4*d^3-68*a^4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))
)/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b*c^2*d^5+8*a^(3/2)*((a*x+b)*x)^(1/2)*((a
*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^7+24*a^(11/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d
-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*d^7-92*a^(5/2)*((a*x+b)*x)
^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d^2-136*a^4*ln(1/2*(2*a*x+b+2*((a*
x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d^4+120*a^3
*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1
/2)*x*b^2*c^4*d^3-24*a^2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/
2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^5*d^2+60*a^3*ln(1/2*(2*a*x+b+2*((a*x+b)
*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^5*d^2-12*a^2*
ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/
2)*x^2*b^3*c^6*d+102*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*
c^4*d^3+22*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^6*d+18*a
^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^5*d^2-46*a^(5/2)*((
a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^6*d-68*a^4*ln(1/2*(2*a*x
+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^4*
d^3-22*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^6*d-8*a*ln
(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)
*x*b^4*c^6*d+16*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^6
*d-24*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*c^2*d^5-80*a^(9/2)*
ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+
d))*b*c*d^6+91*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a
```

$(x+b)x^{1/2}c/(cxd) * b^2c^2d^5 - 35a^{5/2} * \ln((-2adx+bcx-bd+2((a-d-bc)/c^2d)^{1/2} * ((ax+b)x)^{1/2}c/(cxd)) * b^3c^3d^4 + 24a^5 * \ln(1/2 * (2ax+b+2((ax+b)x)^{1/2}a^{1/2})/a^{1/2}) * ((a-d-bc)/c^2d)^{1/2} * cd^6 + 24a^{11/2} * \ln((-2adx+bcx-bd+2((a-d-bc)/c^2d)^{1/2} * ((ax+b)x)^{1/2}c/(cxd)) * x^2c^2d^5 - 8a^{7/2} * ((ax+b)x)^{3/2} * ((a-d-bc)/c^2d)^{1/2} * c^4d^3 + 48a^{11/2} * \ln((-2adx+bcx-bd+2((a-d-bc)/c^2d)^{1/2} * ((ax+b)x)^{1/2}c/(cxd)) * xc^4d^6/c^5 / ((ax+b)x)^{1/2} / (a-d-bc)^3 / (cxd)^2 / a^{5/2} / ((a-d-bc)/c^2d)^{1/2}$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)^3/(a+b/x)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(a + b/x)\*(c + d/x)^3), x)

**mupad** [B] time = 5.48, size = 2890, normalized size = 11.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(1/2)\*(c + d/x)^3),x)

[Out]  $(\log((d^3(a-d-bc)^5)^{1/2} * (a + b/x)^{1/2}) - a^3d^4 + b^3c^3d - 3ab^2c^2d^2 + 3a^2b^2cd^3) * (d^3(a-d-bc)^5)^{1/2} * (3a^2d^2 + (35b^2c^2)/8 - 7abc^2d) / (b^5c^9 - a^5c^4d^5 + 5a^4b^2c^5d^4 + 10a^2b^3c^7d^2 - 10a^3b^2c^6d^3 - 5ab^4c^8d) - ((b(a + b/x)^{5/2} * (12a^2d^4 + 4b^2c^2d^2 - 19abc^2d^3)) / (4a^3c^3(a-d-bc)^2) - ((a + b/x)^{1/2} * (4b^4c^3 - 12a^3b^2d^3 + 25a^2b^2c^2d^2 - 12ab^3c^2d)) / (4a^3c^3(a-d-bc)) + (d(a + b/x)^{3/2} * (8b^4c^3 - 24a^3b^2d^3 + 56a^2b^2c^2d^2 - 37ab^3c^2d)) / (4c^3(a^2d - abc) * (a-d-bc))) / ((a + b/x)^2 * (3a^2d^2 - 2b^2cd) - (a + b/x) * (3a^2d^2 + b^2c^2 - 4abc^2d) - d^2 * (a + b/x)^3 + a^3d^2 + ab^2c^2 - 2a^2bcd) - (\log((d^3(a-d-bc)^5)^{1/2} * (a + b/x)^{1/2}) + a^3d^4 - b^3c^3d + 3ab^2c^2d^2 - 3a^2b^2cd^3) * (d^3(a-d-bc)^5)^{1/2} * (24a^2d^2 + 35b^2c^2 - 56abc^2d) / (8 * (b^5c^9 - a^5c^4d^5 + 5a^4b^2c^5d^4 + 10a^2b^3c^7d^2 - 10a^3b^2c^6d^3 - 5ab^4c^8d)) - (\operatorname{atan}((((a + b/x)^{1/2} * (1152a^6b^2d^9 + 16b^8c^6d^3 + 128a^5b^7c^5d^4 - 4800a^5b^3cd^8 + 1129a^2b^6c^4d^5 - 5136a^3b^5c^3d^6 + 7520a^4b^4c^2d^7))) / (8 * (a^2b^4c^10 + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2))) - (((4ab^8c^13d^2 + 4a^2b^7c^12d^3 - 45a^3b^6c^11d^4 + 74a^4b^5c^10d^5 - 49a^5b^4c^9d^6 + 12a^6b^3c^8d^7) / (a^2b^4c^13 + a^6c^9d^4 - 4a^3b^3c^12d - 4a^5b^2c^10d^3 + 6a^4b^2c^11d^2) - ((a + b/x)^{1/2} * (6ad + bc) * (64a^2b^7c^13d^2 - 384a^3b^6c^12d^3 + 896a^4b^5c^11d^4 - 1024a^5b^4c^10d^5 + 576a^6b^3c^9d^6 - 128a^7b^2c^8d^7)) / (16c^4 * (a^3)^{1/2} * (a^2b^4c^10 + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2))) * (6ad + bc) / (2c^4 * (a^3)^{1/2})) * (6ad + bc) * i) / (2c^4 * (a^3)^{1/2}) + (((a + b/x)^{1/2} * (1152a^6b^2d^9 + 16b^8c^6d^3 + 128a^5b^7c^5d^4 - 4800a^5b^3cd^8 + 1129a^2b^6c^4d^5 - 5136a^3b^5c^3d^6 + 7520a^4b^4c^2d^7)) / (8 * (a^2b^4c^10 + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2))) + (((4ab^8c^13d^2 + 4a^2b^7c^12d^3 - 45a^3b^6c^11d^4 + 74a^4b^5c^10d^5 - 49a^5b^4c^9d^6 + 12a^6b^3c^8d^7) / (a^2b^4c^13 + a^6c^9d^4 - 4a^3b^3c^12d - 4a^5b^2c^10d^3 + 6a^4b^2c^11d^2) + ((a + b/x)^{1/2} * (6ad + bc) * (64a^2b^7c^13d^2 - 384a^3b^6c^12d^3 + 896a^4b^5c^11d^4 - 1024a^5b^4c^10d^5 + 576a^6b^3c^9d^6 - 128a^7b^2c^8d^7))$

$$\frac{((16c^4(a^3)^{1/2}(a^2b^4c^{10} + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2)) * (6ad + bc)) / (2c^4(a^3)^{1/2})) * (6ad + bc) * 1i) / (2c^4(a^3)^{1/2})) / (((216a^5b^3d^9 + (35b^8c^5d^4)/2 - (49ab^7c^4d^5)/8 - 810a^4b^4c^8d^8 - (1877a^2b^6c^3d^6)/4 + 1044a^3b^5c^2d^7) / (a^2b^4c^{13} + a^6c^9d^4 - 4a^3b^3c^{12}d - 4a^5b^2c^{10}d^3 + 6a^4b^2c^{11}d^2) + (((a + b/x)^{1/2} * (1152a^6b^2d^9 + 16b^8c^6d^3 + 128ab^7c^5d^4 - 4800a^5b^3c^8d^8 + 1129a^2b^6c^4d^5 - 5136a^3b^5c^3d^6 + 7520a^4b^4c^2d^7)) / (8(a^2b^4c^{10} + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^8d^2)) - (((4ab^8c^{13}d^2 + 4a^2b^7c^{12}d^3 - 45a^3b^6c^{11}d^4 + 74a^4b^5c^{10}d^5 - 49a^5b^4c^9d^6 + 12a^6b^3c^8d^7) / (a^2b^4c^{13} + a^6c^9d^4 - 4a^3b^3c^{12}d - 4a^5b^2c^{10}d^3 + 6a^4b^2c^{11}d^2) - ((a + b/x)^{1/2} * (6ad + bc) * (64a^2b^7c^{13}d^2 - 384a^3b^6c^{12}d^3 + 896a^4b^5c^{11}d^4 - 1024a^5b^4c^{10}d^5 + 576a^6b^3c^9d^6 - 128a^7b^2c^8d^7)) / (16c^4(a^3)^{1/2}(a^2b^4c^{10} + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2)) * (6ad + bc)) / (2c^4(a^3)^{1/2})) * (6ad + bc) / (2c^4(a^3)^{1/2})) - (((a + b/x)^{1/2} * (1152a^6b^2d^9 + 16b^8c^6d^3 + 128ab^7c^5d^4 - 4800a^5b^3c^8d^8 + 1129a^2b^6c^4d^5 - 5136a^3b^5c^3d^6 + 7520a^4b^4c^2d^7)) / (8(a^2b^4c^{10} + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^8d^2)) + (((4ab^8c^{13}d^2 + 4a^2b^7c^{12}d^3 - 45a^3b^6c^{11}d^4 + 74a^4b^5c^{10}d^5 - 49a^5b^4c^9d^6 + 12a^6b^3c^8d^7) / (a^2b^4c^{13} + a^6c^9d^4 - 4a^3b^3c^{12}d - 4a^5b^2c^{10}d^3 + 6a^4b^2c^{11}d^2) + ((a + b/x)^{1/2} * (6ad + bc) * (64a^2b^7c^{13}d^2 - 384a^3b^6c^{12}d^3 + 896a^4b^5c^{11}d^4 - 1024a^5b^4c^{10}d^5 + 576a^6b^3c^9d^6 - 128a^7b^2c^8d^7)) / (16c^4(a^3)^{1/2}(a^2b^4c^{10} + a^6c^6d^4 - 4a^3b^3c^9d - 4a^5b^2c^7d^3 + 6a^4b^2c^8d^2)) * (6ad + bc)) / (2c^4(a^3)^{1/2})) * (6ad + bc) / (2c^4(a^3)^{1/2})) * (6ad + bc) * 1i) / (c^4(a^3)^{1/2}))$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c+d/x)\*\*3/(a+b/x)\*\*(1/2),x)

[Out] Timed out

$$3.252 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=132

$$-\frac{3c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{(bc - 2ad)(2a^2d^2 - 2abcd + 3b^2c^2) - \frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\sqrt{a + \frac{b}{x}}}$$

[Out]  $-3c^2(-2ad+bc) \operatorname{arctanh}\left(\frac{\sqrt{a+b/x}}{\sqrt{a}}\right)/a^{5/2} + ((-2ad+bc)(2a^2d^2 - 2abcd + 3b^2c^2) - abd^2(2ad+bc)/x)/a^2b^2\sqrt{a+b/x} + cx(c+d/x)^2/a\sqrt{a+b/x}$

**Rubi [A]** time = 0.10, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {375, 98, 146, 63, 208}

$$\frac{(bc - 2ad)(2a^2d^2 - 2abcd + 3b^2c^2) - \frac{abd^2(2ad+bc)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} - \frac{3c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{cx\left(c + \frac{d}{x}\right)^2}{a\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(c + \frac{d}{x}\right)^3/\left(a + \frac{b}{x}\right)^{3/2}, x\right]$

[Out]  $((b*c - 2*a*d)*(3*b^2*c^2 - 2*a*b*c*d + 2*a^2*d^2) - (a*b*d^2*(b*c + 2*a*d))/x)/(a^2*b^2*\operatorname{Sqrt}[a + b/x]) + (c*(c + d/x)^2*x)/(a*\operatorname{Sqrt}[a + b/x]) - (3*c^2*(b*c - 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{5/2}$

### Rule 63

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left((c_.) + (d_.)*(x_.)\right)^{(n_.)}, x\_Symbol\right] :> \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{(p*(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x\right], x, (a + b*x)^{(1/p)}\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, x\} \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 98

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left((c_.) + (d_.)*(x_.)\right)^{(n_.)}\left((e_.) + (f_.)*(x_.)\right)^{(p_.)}, x\_Symbol\right] :> \operatorname{Simp}\left[\left((b*c - a*d)*(a + b*x)^{(m+1)}(c + d*x)^{(n-1)}(e + f*x)^{(p+1)}\right)/(b*(b*e - a*f)*(m+1)), x\right] + \operatorname{Dist}\left[1/(b*(b*e - a*f)*(m+1)), \operatorname{Int}\left[(a + b*x)^{(m+1)}(c + d*x)^{(n-2)}(e + f*x)^p \operatorname{Simp}[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))]*x, x\right], x\right] /; \operatorname{FreeQ}\{a, b, c, d, e, f, p\}, x\} \&\& \operatorname{LtQ}[m, -1] \&\& \operatorname{GtQ}[n, 1] \&\& (\operatorname{IntegersQ}[2*m, 2*n, 2*p] \|\| \operatorname{IntegersQ}[m, n+p] \|\| \operatorname{IntegersQ}[p, m+n])$

### Rule 146

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left((c_.) + (d_.)*(x_.)\right)^{(n_.)}\left((e_.) + (f_.)*(x_.)\right)*\left((g_.) + (h_.)*(x_.)\right), x\_Symbol\right] :> \operatorname{Simp}\left[\left((a^2*d*f*h*(n+2) + b^2*d*e*g*(m+n+3) + a*b*(c*f*h*(m+1) - d*(f*g + e*h)*(m+n+3)) + b*f*h*(b*c - a*d)*(m+1)*x\right)*(a + b*x)^{(m+1)}(c + d*x)^{(n+1)}\right)/(b^2*d*(b*c - a*d)*(m+1)*(m+n+3)), x\right] - \operatorname{Dist}\left[\left(a^2*d^2*f*h*(n+1)*(n+2) + a*b*d*(n+1)*(2*c*f*h*(m+1) - d*(f*g + e*h)*(m+n+3)) + b^2*(c^2*f*h*(m+1)*(m+2)\right)\right]$

```
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{3/2}} dx = -\text{Subst}\left(\int \frac{(c + dx)^3}{x^2(a + bx)^{3/2}} dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\sqrt{a + \frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{(c+dx)\left(\frac{3}{2}c(bc-2ad)-\frac{1}{2}d(bc+2ad)x\right)}{x(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{(bc - 2ad)(3b^2c^2 - 2abcd + 2a^2d^2) - \frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\sqrt{a + \frac{b}{x}}} + \frac{(3c^2(bc - 2ad)) \text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{2a^2}$$

$$= \frac{(bc - 2ad)(3b^2c^2 - 2abcd + 2a^2d^2) - \frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\sqrt{a + \frac{b}{x}}} + \frac{(3c^2(bc - 2ad)) \text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{a^2}$$

$$= \frac{(bc - 2ad)(3b^2c^2 - 2abcd + 2a^2d^2) - \frac{abd^2(bc+2ad)}{x}}{a^2b^2\sqrt{a + \frac{b}{x}}} + \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\sqrt{a + \frac{b}{x}}} - \frac{3c^2(bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}$$

**Mathematica [C]** time = 0.07, size = 92, normalized size = 0.70

$$\frac{a\left(-4a^2d^3x - 2abd^2(d - 3cx) + b^2c^3x^2\right) + 3b^2c^2x(bc - 2ad) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax} + 1\right)}{a^2b^2x\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d/x)^3/(a + b/x)^(3/2), x]
```

```
[Out] (a*(-4*a^2*d^3*x + b^2*c^3*x^2 - 2*a*b*d^2*(d - 3*c*x)) + 3*b^2*c^2*(b*c -
2*a*d)*x*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])/(a^2*b^2*Sqrt[a + b/
x])*x
```

**fricas** [A] time = 0.98, size = 336, normalized size = 2.55

$$\frac{3(b^4c^3 - 2ab^3c^2d + (ab^3c^3 - 2a^2b^2c^2d)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(a^2b^2c^3x^2 - 2a^3bd^3 + (3ab^3c^3 - 2a^2b^2c^2d)x)\sqrt{a}}{2(a^4b^2x + a^3b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="fricas")

[Out]  $[-1/2*(3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*\text{sqrt}(a) * \log(2*a*x + 2*\text{sqrt}(a)*x*\text{sqrt}((a*x + b)/x) + b) - 2*(a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*\text{sqrt}((a*x + b)/x))/(a^4*b^2*x + a^3*b^3), (3*(b^4*c^3 - 2*a*b^3*c^2*d + (a*b^3*c^3 - 2*a^2*b^2*c^2*d)*x)*\text{sqrt}(-a)*\text{arctan}(\text{sqrt}(-a)*\text{sqrt}((a*x + b)/x)/a) + (a^2*b^2*c^3*x^2 - 2*a^3*b*d^3 + (3*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 4*a^4*d^3)*x)*\text{sqrt}((a*x + b)/x))/(a^4*b^2*x + a^3*b^3)]$

**giac** [A] time = 0.21, size = 222, normalized size = 1.68

$$\frac{2d^3\sqrt{\frac{ax+b}{x}}}{b} - \frac{3(b^2c^3 - 2abc^2d)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} - \frac{2ab^3c^3 - 6a^2b^2c^2d + 6a^3bcd^2 - 2a^4d^3 - \frac{3(ax+b)b^3c^3}{x} + \frac{6(ax+b)ab^2c^2d}{x} - \frac{6(ax+b)a^2bcd^2}{x} + \frac{2(ax+b)a^3d^3}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(3/2),x, algorithm="giac")

[Out]  $-(2*d^3*\text{sqrt}((a*x + b)/x)/b - 3*(b^2*c^3 - 2*a*b*c^2*d)*\text{arctan}(\text{sqrt}((a*x + b)/x)/\text{sqrt}(-a))/(\text{sqrt}(-a)*a^2) - (2*a*b^3*c^3 - 6*a^2*b^2*c^2*d + 6*a^3*b*c*d^2 - 2*a^4*d^3 - 3*(a*x + b)*b^3*c^3/x + 6*(a*x + b)*a*b^2*c^2*d/x - 6*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x)/((a*\text{sqrt}((a*x + b)/x) - (a*x + b)*\text{sqrt}((a*x + b)/x)/x)*a^2*b)/b$

**maple** [B] time = 0.07, size = 969, normalized size = 7.34

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 3a^4b^2cd^2x^4 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - 3a^4b^2cd^2x^4 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx}\sqrt{a}}{2\sqrt{a}}\right) - 6a^3b^3c^2d^2x^4 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}}{2\sqrt{a}}\right) \right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(3/2),x)

[Out]  $-1/2*((a*x+b)/x)^(1/2)/x/a^(5/2)*(3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^2*b^6*c^3+4*(a*x^2+b*x)^(3/2)*a^(9/2)*x^2*d^3-4*a^(9/2)*((a*x+b)*x)^(3/2)*x^2*d^3+4*(a*x^2+b*x)^(3/2)*a^(5/2)*b^2*d^3+6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2-12*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^3*a^2*b^4*c^2*d-6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^4*a^3*b^3*c^2*d-3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2+3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^4*a^4*b^2*c*d^2-6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2))*a^(1/2))/a^(1/2))*x^3*a^3*b^3*c*d^2+12*a^(3/2)*((a*x+b)*x)^(1/2)*x^2*b^4*c^2*d-6*(a*x^2+b*x)^(1/2)*a^(9/2)*x^4*b*c*d^2-6*a^(9/2)*((a*x+b)*x)^(1/2)*x^4*b*c*d^2+12*a^(7/2)*((a*x+b)*x)^(1/2)*x^4*b^2*c^2*d-12*$



$a^2 x^2 + b x)^{1/2} a^{7/2} x^3 b^2 c^2 d^2 + 12 a^{7/2} ((a x + b) x)^{3/2} x^2 b^2 c^2 d^2 - 12 a^{5/2} ((a x + b) x)^{3/2} x^2 b^2 c^2 d - 12 a^{7/2} ((a x + b) x)^{1/2} x^3 b^2 c^2 d^2 + 24 a^{5/2} ((a x + b) x)^{1/2} x^3 b^3 c^2 d - 6 (a^2 x^2 + b x)^{1/2} a^{5/2} x^2 b^3 c^2 d^2 - 6 \ln(1/2 (2 a x + b + 2 ((a x + b) x)^{1/2} a^{1/2})) / a^{1/2} x^2 a b^5 c^2 d - 3 \ln(1/2 (2 a x + b + 2 (a^2 x^2 + b x)^{1/2} a^{1/2})) / a^{1/2} x^2 a^2 b^4 c^2 d + 3 \ln(1/2 (2 a x + b + 2 ((a x + b) x)^{1/2} a^{1/2})) / a^{1/2} x^2 a^2 b^4 c^2 d - 6 a^{5/2} ((a x + b) x)^{1/2} x^2 b^3 c^2 d + 8 (a^2 x^2 + b x)^{3/2} a^{7/2} x b^3 d^3 - 12 a^{3/2} ((a x + b) x)^{1/2} x^3 b^4 c^3 + 4 a^{3/2} ((a x + b) x)^{3/2} x^2 b^3 c^3 - 6 a^{5/2} ((a x + b) x)^{1/2} x^4 b^3 c^3 + 3 \ln(1/2 (2 a x + b + 2 ((a x + b) x)^{1/2} a^{1/2})) / a^{1/2} x^4 a^2 b^4 c^3 + 6 \ln(1/2 (2 a x + b + 2 ((a x + b) x)^{1/2} a^{1/2})) / a^{1/2} x^3 a b^5 c^3 - 6 a^{1/2} ((a x + b) x)^{1/2} x^2 b^5 c^3 / ((a x + b) x)^{1/2} / b^3 / (a x + b)^2$

**maxima** [A] time = 1.26, size = 200, normalized size = 1.52

$$\frac{1}{2} c^3 \left( \frac{2 \left( 3 \left( a + \frac{b}{x} \right) b - 2 a b \right)}{\left( a + \frac{b}{x} \right)^2 a^2 - \sqrt{a + \frac{b}{x}} a^3} + \frac{3 b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^2} \right) - 3 c^2 d \left( \frac{\log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^2} + \frac{2}{\sqrt{a + \frac{b}{x}} a} \right) - 2 d^3 \left( \frac{\sqrt{a + \frac{b}{x}}}{b^2} + \sqrt{a + \frac{b}{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(3/2), x, algorithm="maxima")

[Out]  $\frac{1}{2} c^3 (2 (3 (a + b/x) b - 2 a b) / ((a + b/x)^{3/2} a^2 - \sqrt{a + b/x} a^3) + 3 b \log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a}))) / a^{5/2} - 3 c^2 d (\log((\sqrt{a + b/x} - \sqrt{a}) / (\sqrt{a + b/x} + \sqrt{a}))) / a^{3/2} + 2 / (\sqrt{a + b/x} a) - 2 d^3 (\sqrt{a + b/x} / b^2 + a / (\sqrt{a + b/x} b^2)) + 6 c d^2 / (\sqrt{a + b/x} b)$

**mupad** [B] time = 1.91, size = 172, normalized size = 1.30

$$\frac{2 (a^3 d^3 - 3 a^2 b c d^2 + 3 a b^2 c^2 d - b^3 c^3)}{a} - \frac{(a + \frac{b}{x}) (2 a^3 d^3 - 6 a^2 b c d^2 + 6 a b^2 c^2 d - 3 b^3 c^3)}{a^2} - \frac{2 d^3 \sqrt{a + \frac{b}{x}}}{b^2} + \frac{3 c^2 \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) (2 a d - b^2)}{a^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^3/(a + b/x)^(3/2), x)

[Out]  $((2 (a^3 d^3 - b^3 c^3 + 3 a b^2 c^2 d - 3 a^2 b^2 c^2 d)) / a - ((a + b/x) (2 a^3 d^3 - 3 b^3 c^3 + 6 a b^2 c^2 d - 6 a^2 b^2 c^2 d)) / a^2) / (b^2 (a + b/x)^{3/2} - a b^2 (a + b/x)^{1/2}) - (2 d^3 (a + b/x)^{1/2}) / b^2 + (3 c^2 \operatorname{atanh}(a + b/x)^{1/2} / a^{1/2}) (2 a d - b^2) / a^{5/2}$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c x + d)^3}{x^3 \left( a + \frac{b}{x} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*3/(a+b/x)\*\*(3/2), x)

[Out] Integral((c\*x + d)\*\*3/(x\*\*3\*(a + b/x)\*\*(3/2)), x)

$$3.253 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=94

$$-\frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a^2d^2 + bc(3bc - 4ad)}{a^2b\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}}$$

[Out]  $-c*(-4*a*d+3*b*c)*\operatorname{arctanh}\left(\frac{\sqrt{a+b/x}}{\sqrt{a}}\right)/a^{5/2}+(2*a^2*d^2+b*c*(-4*a*d+3*b*c))/a^2/b/\sqrt{a+b/x}+c^2*x/a/\sqrt{a+b/x}$

**Rubi [A]** time = 0.08, antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {375, 89, 78, 63, 208}

$$\frac{\frac{c(3bc-4ad)}{a^2} + \frac{2d^2}{b}}{\sqrt{a + \frac{b}{x}}} - \frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)^2/(a + b/x)^(3/2), x]

[Out]  $((2*d^2)/b + (c*(3*b*c - 4*a*d))/a^2)/\operatorname{Sqrt}[a + b/x] + (c^2*x)/(a*\operatorname{Sqrt}[a + b/x]) - (c*(3*b*c - 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{5/2}$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

### Rule 89

Int[((a\_.) + (b\_.)\*(x\_))^2\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[((b\*c - a\*d)^2\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d^2\*(d\*e - c\*f)\*(n + 1)), x] - Dist[1/(d^2\*(d\*e - c\*f)\*(n + 1)), Int[(c + d\*x)^(n + 1)\*(e + f\*x)^p\*Simp[a^2\*d^2\*f\*(n + p + 2) + b^2\*c\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - 2\*a\*b\*d\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - b^2\*d\*(d\*e - c\*f)\*(n + 1)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{3/2}} dx &= -\text{Subst}\left(\int \frac{(c + dx)^2}{x^2(a + bx)^{3/2}} dx, x, \frac{1}{x}\right) \\ &= \frac{c^2x}{a\sqrt{a + \frac{b}{x}}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(3bc - 4ad) + ad^2x}{x(a + bx)^{3/2}} dx, x, \frac{1}{x}\right)}{a} \\ &= \frac{\frac{2d^2}{b} + \frac{c(3bc - 4ad)}{a^2}}{\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}} + \frac{(c(3bc - 4ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{2a^2} \\ &= \frac{\frac{2d^2}{b} + \frac{c(3bc - 4ad)}{a^2}}{\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}} + \frac{(c(3bc - 4ad)) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{a^2b} \\ &= \frac{\frac{2d^2}{b} + \frac{c(3bc - 4ad)}{a^2}}{\sqrt{a + \frac{b}{x}}} + \frac{c^2x}{a\sqrt{a + \frac{b}{x}}} - \frac{c(3bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 81, normalized size = 0.86

$$\frac{c(4ad - 3bc) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{2a^2d^2 + abc(cx - 4d) + 3b^2c^2}{a^2b\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^2/(a + b/x)^(3/2), x]

[Out] (3\*b^2\*c^2 + 2\*a^2\*d^2 + a\*b\*c\*(-4\*d + c\*x))/(a^2\*b\*Sqrt[a + b/x]) + (c\*(-3\*b\*c + 4\*a\*d)\*ArcTanh[Sqrt[a + b/x]/Sqrt[a]])/a^(5/2)

**fricas [A]** time = 0.89, size = 272, normalized size = 2.89

$$\frac{(3b^3c^2 - 4ab^2cd + (3ab^2c^2 - 4a^2bcd)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(a^2bc^2x^2 + (3ab^2c^2 - 4a^2bcd)x - 3b^3c^2)}{2(a^4bx + a^3b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="fricas")

[Out] [-1/2\*((3\*b^3\*c^2 - 4\*a\*b^2\*c\*d + (3\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(a^2\*b\*c^2\*x^2 + (3\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d + 2\*a^3\*d^2)\*x)\*sqrt((a\*x + b)/x))/(a^4\*b\*x + a^3\*b^2), ((3\*b^3\*c^2 - 4\*a\*b^2\*c\*d + (3\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (a^2\*b\*c^2\*x^2 + (3\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d + 2\*a^3\*d^2)\*x)\*sqrt((a\*x + b)/x))/(a^4\*b\*x + a^3\*b^2)]

**giac** [A] time = 0.23, size = 160, normalized size = 1.70

$$\frac{(3b^2c^2 - 4abcd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right) + \frac{2ab^2c^2 - 4a^2bcd + 2a^3d^2 - \frac{3(ax+b)b^2c^2}{x} + \frac{4(ax+b)abcd}{x} - \frac{2(ax+b)a^2d^2}{x}}{\sqrt{-a}a^2}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2} \cdot b$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="giac")

[Out] ((3\*b^2\*c^2 - 4\*a\*b\*c\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2) + (2\*a\*b^2\*c^2 - 4\*a^2\*b\*c\*d + 2\*a^3\*d^2 - 3\*(a\*x + b)\*b^2\*c^2/x + 4\*(a\*x + b)\*a\*b\*c\*d/x - 2\*(a\*x + b)\*a^2\*d^2/x)/((a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x)\*a^2)/b

**maple** [B] time = 0.06, size = 789, normalized size = 8.39

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -a^4b d^2x^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) + a^4b d^2x^2 \ln\left(\frac{2ax+b+2\sqrt{ax^2+bx} \sqrt{a}}{2\sqrt{a}}\right) + 4a^3b^2cd x^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+b/x)^(3/2),x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*x\*(ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2)))\*a^2\*b^3\*d^2+2\*a^(9/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*d^2+2\*a^(9/2)\*(a\*x^2+b\*x)^(1/2)\*x^2\*d^2-4\*a^(3/2)\*((a\*x+b)\*x)^(3/2)\*b^2\*c^2+2\*a^(5/2)\*((a\*x+b)\*x)^(1/2)\*b^2\*d^2+2\*a^(5/2)\*(a\*x^2+b\*x)^(1/2)\*b^2\*d^2-ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*a^2\*b^3\*d^2+6\*a^(1/2)\*((a\*x+b)\*x)^(1/2)\*b^4\*c^2-16\*a^(5/2)\*((a\*x+b)\*x)^(1/2)\*x\*b^2\*c\*d+8\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x\*a^2\*b^3\*c\*d+4\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a^3\*b^2\*c\*d-8\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b\*c\*d-3\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*b^5\*c^2-4\*a^(7/2)\*((a\*x+b)\*x)^(3/2)\*d^2-2\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x\*a^3\*b^2\*d^2-6\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x\*a\*b^4\*c^2+2\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x\*a^3\*b^2\*d^2+4\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*a\*b^4\*c\*d+4\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*x\*b\*d^2+12\*a^(3/2)\*((a\*x+b)\*x)^(1/2)\*x\*b^3\*c^2+4\*a^(7/2)\*(a\*x^2+b\*x)^(1/2)\*x\*b\*d^2-8\*a^(3/2)\*((a\*x+b)\*x)^(1/2)\*b^3\*c\*d+6\*a^(5/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b^2\*c^2+8\*a^(5/2)\*((a\*x+b)\*x)^(3/2)\*b\*c\*d-ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a^4\*b\*d^2-3\*ln(1/2\*(2\*a\*x+b+2\*(a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a^2\*b^3\*c^2+ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a^4\*b\*d^2/a^(5/2)/((a\*x+b)\*x)^(1/2)/b^2/(a\*x+b)^2

**maxima [A]** time = 1.16, size = 164, normalized size = 1.74

$$\frac{1}{2}c^2 \left( \frac{2 \left( 3 \left( a + \frac{b}{x} \right) b - 2ab \right)}{\left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^2 - \sqrt{a + \frac{b}{x}} a^3} + \frac{3b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{5}{2}}} \right) - 2cd \left( \frac{\log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a + \frac{b}{x}} a} \right) + \frac{2d^2}{\sqrt{a + \frac{b}{x}} b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(3/2),x, algorithm="maxima")

[Out] 1/2\*c^2\*(2\*(3\*(a + b/x)\*b - 2\*a\*b)/((a + b/x)^(3/2)\*a^2 - sqrt(a + b/x)\*a^3) + 3\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2)) - 2\*c\*d\*(log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(3/2) + 2/(sqrt(a + b/x)\*a)) + 2\*d^2/(sqrt(a + b/x)\*b)

**mupad [B]** time = 1.83, size = 120, normalized size = 1.28

$$\frac{c \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) (4ad - 3bc)}{a^{5/2}} - \frac{2(a^2 d^2 - 2abcd + b^2 c^2)}{a} - \frac{\left( a + \frac{b}{x} \right) (2a^2 d^2 - 4abcd + 3b^2 c^2)}{a^2}}{b \left( a + \frac{b}{x} \right)^{3/2} - ab \sqrt{a + \frac{b}{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^2/(a + b/x)^(3/2),x)

[Out] (c\*atanh((a + b/x)^(1/2)/a^(1/2))\*(4\*a\*d - 3\*b\*c))/a^(5/2) - ((2\*(a^2\*d^2 + b^2\*c^2 - 2\*a\*b\*c\*d))/a - ((a + b/x)\*(2\*a^2\*d^2 + 3\*b^2\*c^2 - 4\*a\*b\*c\*d))/a^2)/(b\*(a + b/x)^(3/2) - a\*b\*(a + b/x)^(1/2))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx + d)^2}{x^2 \left( a + \frac{b}{x} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2/(a+b/x)\*\*(3/2),x)

[Out] Integral((c\*x + d)\*\*2/(x\*\*2\*(a + b/x)\*\*(3/2)), x)

$$3.254 \quad \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

Optimal. Leaf size=76

$$-\frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3bc - 2ad}{a^2 \sqrt{a + \frac{b}{x}}} + \frac{cx}{a \sqrt{a + \frac{b}{x}}}$$

[Out]  $-(2ad + 3bc) \operatorname{arctanh}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right) / a^{5/2} + (-2ad + 3bc) / a^2 / \left(a + \frac{b}{x}\right)^{1/2} + cx / a \sqrt{a + \frac{b}{x}}$

**Rubi [A]** time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {375, 78, 51, 63, 208}

$$\frac{3bc - 2ad}{a^2 \sqrt{a + \frac{b}{x}}} - \frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{cx}{a \sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/(a + b/x)^(3/2), x]

[Out]  $(3bc - 2ad) / (a^2 \sqrt{a + \frac{b}{x}}) + (cx) / (a \sqrt{a + \frac{b}{x}}) - ((3bc - 2ad) \operatorname{ArcTanh}[\sqrt{a + \frac{b}{x}} / \sqrt{a}]) / a^{5/2}$

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] ] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] ] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n]))))

#### Rule 208

$\text{Int}[(a_ + (b_.)*(x_)^2)^{-1}, x\_Symbol] \text{ :> Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ /; FreeQ}\{a, b\}, x\} \ \&\& \ \text{NegQ}[a/b]$

### Rule 375

$\text{Int}[(a_ + (b_.)*(x_)^{n_})^{p_}*(c_ + (d_.)*(x_)^{n_})^{q_}, x\_Symbol] \text{ :> -Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] \text{ /; FreeQ}\{a, b, c, d, p, q\}, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{ILtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{3/2}} dx &= -\text{Subst}\left(\int \frac{c + dx}{x^2(a + bx)^{3/2}} dx, x, \frac{1}{x}\right) \\ &= \frac{cx}{a\sqrt{a + \frac{b}{x}}} - \frac{\left(-\frac{3bc}{2} + ad\right) \text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{a} \\ &= \frac{3bc - 2ad}{a^2\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\sqrt{a + \frac{b}{x}}} + \frac{(3bc - 2ad) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2a^2} \\ &= \frac{3bc - 2ad}{a^2\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\sqrt{a + \frac{b}{x}}} + \frac{(3bc - 2ad) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{a^2b} \\ &= \frac{3bc - 2ad}{a^2\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\sqrt{a + \frac{b}{x}}} - \frac{(3bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} \end{aligned}$$

**Mathematica [C]** time = 0.02, size = 48, normalized size = 0.63

$$\frac{(3bc - 2ad) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax} + 1\right) + acx}{a^2\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/(a + b/x)^(3/2), x]

[Out] (a\*c\*x + (3\*b\*c - 2\*a\*d)\*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a\*x)])/(a^2\*Sqrt[a + b/x])

**fricas [A]** time = 1.05, size = 210, normalized size = 2.76

$$\left[ \frac{(3b^2c - 2abd + (3abc - 2a^2d)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(a^2cx^2 + (3abc - 2a^2d)x)\sqrt{\frac{ax+b}{x}}}{2(a^4x + a^3b)}, \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(3/2), x, algorithm="fricas")

[Out]  $[-1/2*((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*\sqrt{a}*\log(2*a*x + 2*\sqrt{a}*x*\sqrt{(a*x + b)/x} + b) - 2*(a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*\sqrt{(a*x + b)/x})/(a^4*x + a^3*b), ((3*b^2*c - 2*a*b*d + (3*a*b*c - 2*a^2*d)*x)*\sqrt{-a}*\arctan(\sqrt{-a}*\sqrt{(a*x + b)/x}/a) + (a^2*c*x^2 + (3*a*b*c - 2*a^2*d)*x)*\sqrt{(a*x + b)/x})/(a^4*x + a^3*b)]$

**giac [A]** time = 0.20, size = 127, normalized size = 1.67

$$\frac{(3b^2c - 2abd) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right) + \frac{2ab^2c - 2a^2bd - \frac{3(ax+b)b^2c}{x} + \frac{2(ax+b)abd}{x}}{\sqrt{-a}a^2}}{\frac{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)a^2}{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="giac")

[Out]  $((3*b^2*c - 2*a*b*d)*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^2) + (2*a*b^2*c - 2*a^2*b*d - 3*(a*x + b)*b^2*c/x + 2*(a*x + b)*a*b*d/x)/((a*\sqrt{(a*x + b)/x} - (a*x + b)*\sqrt{(a*x + b)/x}/x)*a^2))/b$

**maple [B]** time = 0.06, size = 387, normalized size = 5.09

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 2a^3bdx^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - 3a^2b^2cx^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) + 4a^2b^2dx \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)/(a+b/x)^(3/2),x)

[Out]  $1/2*((a*x+b)/x)^(1/2)*x/a^(5/2)*(2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^2*a^3*b*d-3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^2*a^2*b^2*c-4*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*d+6*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b*c+4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x*a^2*b^2*d-6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x*a*b^3*c+4*a^(5/2)*((a*x+b)*x)^(3/2)*d-4*a^(3/2)*((a*x+b)*x)^(3/2)*b*c-8*a^(5/2)*((a*x+b)*x)^(1/2)*x*b*d+12*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^2*c+2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*a*b^3*d-3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*b^4*c-4*a^(3/2)*((a*x+b)*x)^(1/2)*b^2*d+6*a^(1/2)*((a*x+b)*x)^(1/2)*b^3*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^2$

**maxima [B]** time = 1.37, size = 144, normalized size = 1.89

$$\frac{1}{2}c \left( \frac{2 \left( 3 \left( a + \frac{b}{x} \right) b - 2ab \right)}{\left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^2 - \sqrt{a + \frac{b}{x}} a^3} + \frac{3b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{a^{\frac{5}{2}}} \right) - d \left( \frac{\log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{a + \frac{b}{x}} a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(3/2),x, algorithm="maxima")

[Out]  $1/2*c*(2*(3*(a + b/x)*b - 2*a*b)/((a + b/x)^(3/2)*a^2 - \sqrt{a + b/x}*a^3) + 3*b*log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^(5/2)) - d*(log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^(3/2) + 2/(\sqrt{a + b/x}*a))$



**mupad [B]** time = 2.44, size = 71, normalized size = 0.93

$$\frac{2d \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{2d}{a\sqrt{a+\frac{b}{x}}} + \frac{2cx\left(\frac{ax}{b}+1\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{7}{2}; -\frac{ax}{b}\right)}{5\left(a+\frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)/(a + b/x)^(3/2), x)

[Out] (2\*d\*atanh((a + b/x)^(1/2)/a^(1/2)))/a^(3/2) - (2\*d)/(a\*(a + b/x)^(1/2)) + (2\*c\*x\*((a\*x)/b + 1)^(3/2)\*hypergeom([3/2, 5/2], 7/2, -(a\*x)/b))/(5\*(a + b/x)^(3/2))

**sympy [B]** time = 81.34, size = 224, normalized size = 2.95

$$c\left(\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b}+1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b}+1}} - \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}\right) + d\left(-\frac{2a^3x\sqrt{1+\frac{b}{ax}}}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} - \frac{a^3x\log\left(\frac{b}{ax}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b} + \frac{2a^3x\log\left(\sqrt{1+\frac{b}{ax}}\right)}{a^{\frac{9}{2}}x+a^{\frac{7}{2}}b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(3/2), x)

[Out] c\*(x\*\*(3/2)/(a\*sqrt(b)\*sqrt(a\*x/b + 1)) + 3\*sqrt(b)\*sqrt(x)/(a\*\*2\*sqrt(a\*x/b + 1)) - 3\*b\*asinh(sqrt(a)\*sqrt(x)/sqrt(b))/a\*\*(5/2)) + d\*(-2\*a\*\*3\*x\*sqrt(1 + b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) - a\*\*3\*x\*log(b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) + 2\*a\*\*3\*x\*log(sqrt(1 + b/(a\*x)) + 1)/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) - a\*\*2\*b\*log(b/(a\*x))/(a\*\*(9/2)\*x + a\*\*(7/2)\*b) + 2\*a\*\*2\*b\*log(sqrt(1 + b/(a\*x)) + 1)/(a\*\*(9/2)\*x + a\*\*(7/2)\*b))

$$3.255 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=60

$$-\frac{3b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{3b}{a^2 \sqrt{a + \frac{b}{x}}} + \frac{x}{a \sqrt{a + \frac{b}{x}}}$$

[Out]  $-3*b*\operatorname{arctanh}\left(\frac{\sqrt{a+b/x}}{\sqrt{a}}\right)/a^{5/2}+3*b/a^2/\sqrt{a+b/x}+x/a/\sqrt{a+b/x}$

**Rubi [A]** time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {242, 51, 63, 208}

$$\frac{3x\sqrt{a + \frac{b}{x}}}{a^2} - \frac{3b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{2x}{a\sqrt{a + \frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(-3/2), x]

[Out]  $(-2*x)/(a*\operatorname{Sqrt}[a + b/x]) + (3*\operatorname{Sqrt}[a + b/x]*x)/a^2 - (3*b*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{5/2}$

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] ] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] ] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 242

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2}} dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} - \frac{3\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{a} \\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}}{a^2} + \frac{(3b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2a^2} \\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}}{a^2} + \frac{3\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+\frac{b}{x}}\right)}{a^2} \\
&= -\frac{2x}{a\sqrt{a+\frac{b}{x}}} + \frac{3\sqrt{a+\frac{b}{x}}}{a^2} - \frac{3b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.01, size = 36, normalized size = 0.60

$$\frac{2b {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{a+\frac{b}{x}}{a}\right)}{a^2\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(-3/2), x]

[Out] (2\*b\*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/x)/a])/(a^2\*Sqrt[a + b/x])

**fricas [A]** time = 0.77, size = 156, normalized size = 2.60

$$\left[ \frac{3(abx + b^2)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(a^2x^2 + 3abx)\sqrt{\frac{ax+b}{x}}}{2(a^4x + a^3b)}, \frac{3(abx + b^2)\sqrt{-a} \arctan\left(\frac{\sqrt{-a}\sqrt{\frac{ax+b}{x}}}{a}\right)}{a^4x + a^3b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2), x, algorithm="fricas")

[Out] [1/2\*(3\*(a\*b\*x + b^2)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(a^2\*x^2 + 3\*a\*b\*x)\*sqrt((a\*x + b)/x))/(a^4\*x + a^3\*b), (3\*(a\*b\*x + b^2)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (a^2\*x^2 + 3\*a\*b\*x)\*sqrt((a\*x + b)/x))/(a^4\*x + a^3\*b)]

**giac [A]** time = 0.16, size = 86, normalized size = 1.43

$$b \left( \frac{3 \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a^2} + \frac{2a - \frac{3(ax+b)}{x}}{\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right) a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2),x, algorithm="giac")

[Out]  $b*(3*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^2) + (2*a - 3*(a*x + b)/x)/((a*\sqrt{(a*x + b)/x} - (a*x + b)*\sqrt{(a*x + b)/x}/x)*a^2))$

**maple** [B] time = 0.06, size = 198, normalized size = 3.30

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -3a^2b x^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) - 6ab^2x \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) + 6\sqrt{(ax+b)x} a^{\frac{5}{2}}x^2 - 3b^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}}\right) \right)}{2\sqrt{(ax+b)x} (ax+b)^2 a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2),x)

[Out]  $1/2*((a*x+b)/x)^{(1/2)}*x*(6*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*x^2-4*a^{(3/2)}*((a*x+b)*x)^{(3/2)}+12*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*x*b-3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)})*x^2*a^2*b-6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)})*x*a*b^2+6*a^{(1/2)}*((a*x+b)*x)^{(1/2)}*b^2-3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}))/a^{(1/2)})*b^3)/a^{(5/2)}/((a*x+b)*x)^{(1/2)}/(a*x+b)^2$

**maxima** [A] time = 1.18, size = 85, normalized size = 1.42

$$\frac{3\left(a + \frac{b}{x}\right)b - 2ab}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}}a^2 - \sqrt{a + \frac{b}{x}}a^3} + \frac{3b \log\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{2a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2),x, algorithm="maxima")

[Out]  $(3*(a + b/x)*b - 2*a*b)/((a + b/x)^{(3/2)}*a^2 - \sqrt{a + b/x}*a^3) + 3/2*b*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(5/2)}$

**mupad** [B] time = 1.87, size = 34, normalized size = 0.57

$$\frac{2x\left(\frac{ax}{b} + 1\right)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{7}{2}; -\frac{ax}{b}\right)}{5\left(a + \frac{b}{x}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/x)^(3/2),x)

[Out]  $(2*x*((a*x)/b + 1)^{(3/2)}*\text{hypergeom}([3/2, 5/2], 7/2, -(a*x)/b))/(5*(a + b/x)^{(3/2)})$

**sympy** [A] time = 4.79, size = 71, normalized size = 1.18

$$\frac{x^{\frac{3}{2}}}{a\sqrt{b}\sqrt{\frac{ax}{b} + 1}} + \frac{3\sqrt{b}\sqrt{x}}{a^2\sqrt{\frac{ax}{b} + 1}} - \frac{3b \operatorname{asinh}\left(\frac{\sqrt{a}\sqrt{x}}{\sqrt{b}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2),x)

[Out]  $x^{(3/2)}/(a*\sqrt{b}*\sqrt{a*x/b + 1}) + 3*\sqrt{b}*\sqrt{x}/(a**2*\sqrt{a*x/b + 1}) - 3*b*\operatorname{asinh}(\sqrt{a}*\sqrt{x}/\sqrt{b})/a^{(5/2)}$

$$3.256 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} dx$$

**Optimal.** Leaf size=147

$$-\frac{(2ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2} + \frac{b(3bc - ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc - ad)} + \frac{2d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{3/2}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

[Out]  $2*d^{5/2}*arctan(d^{1/2}*(a+b/x)^{1/2}/(-a*d+b*c)^{1/2})/c^2/(-a*d+b*c)^{3/2} - (2*a*d+3*b*c)*arctanh((a+b/x)^{1/2}/a^{1/2})/a^{5/2}/c^2+b*(-a*d+3*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^{1/2}+x/a/c/(a+b/x)^{1/2}$

**Rubi [A]** time = 0.19, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 103, 152, 156, 63, 208, 205}

$$-\frac{(2ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^2} + \frac{b(3bc - ad)}{a^2c\sqrt{a+\frac{b}{x}}(bc - ad)} + \frac{2d^{5/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{3/2}} + \frac{x}{ac\sqrt{a+\frac{b}{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)), x]

[Out]  $(b*(3*b*c - a*d))/(a^2*c*(b*c - a*d)*\text{Sqrt}[a + b/x]) + x/(a*c*\text{Sqrt}[a + b/x]) + (2*d^{5/2}*ArcTan[(\text{Sqrt}[d]*\text{Sqrt}[a + b/x])/\text{Sqrt}[b*c - a*d]])/(c^2*(b*c - a*d)^{3/2}) - ((3*b*c + 2*a*d)*ArcTanh[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/(a^{5/2}*c^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 152

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ

ersQ[2\*m, 2\*n, 2\*p]

### Rule 156

Int[(((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*  
((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e +  
f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c  
+ d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a  
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/  
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol  
] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,  
b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned} \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}(c+dx)} dx, x, \frac{1}{x}\right) \\ &= \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3bc+2ad) + \frac{3bdx}{2}}{x(a+bx)^{3/2}(c+dx)} dx, x, \frac{1}{x}\right)}{ac} \\ &= \frac{b(3bc - ad)}{a^2c(bc - ad)\sqrt{a + \frac{b}{x}}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{2\text{Subst}\left(\int \frac{\frac{1}{4}(bc-ad)(3bc+2ad) + \frac{1}{4}bd(3bc-ad)x}{x\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{a^2c(bc - ad)} \\ &= \frac{b(3bc - ad)}{a^2c(bc - ad)\sqrt{a + \frac{b}{x}}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{d^3\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{c^2(bc - ad)} + \frac{(3bc + 2ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{c^2(bc - ad)} \\ &= \frac{b(3bc - ad)}{a^2c(bc - ad)\sqrt{a + \frac{b}{x}}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{(2d^3)\text{Subst}\left(\int \frac{1}{c - \frac{ad}{b} + \frac{dx^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{bc^2(bc - ad)} + \frac{(3bc + 2ad)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}(c+dx)} dx, x, \frac{1}{x}\right)}{c^2(bc - ad)} \\ &= \frac{b(3bc - ad)}{a^2c(bc - ad)\sqrt{a + \frac{b}{x}}} + \frac{x}{ac\sqrt{a + \frac{b}{x}}} + \frac{2d^{5/2}\tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{3/2}} - \frac{(3bc + 2ad)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{c+dx}}\right)}{a^{5/2}c^2} \end{aligned}$$

**Mathematica [C]** time = 0.07, size = 106, normalized size = 0.72

$$\frac{(ad - bc) \left( (2ad + 3bc) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax} + 1 \right) + acx \right) - 2a^2d^2 {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{d \left( a + \frac{b}{x} \right)}{ad - bc} \right)}{a^2c^2 \sqrt{a + \frac{b}{x}} (ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)\*(c + d/x)),x]

[Out] (-2\*a^2\*d^2\*Hypergeometric2F1[-1/2, 1, 1/2, (d\*(a + b/x))/(-(b\*c) + a\*d)] + (- (b\*c) + a\*d)\*(a\*c\*x + (3\*b\*c + 2\*a\*d)\*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a\*x)])))/(a^2\*c^2\*(-(b\*c) + a\*d)\*Sqrt[a + b/x])

**fricas [B]** time = 1.17, size = 1075, normalized size = 7.31

$$\left[ \frac{(3b^3c^2 - ab^2cd - 2a^2bd^2 + (3ab^2c^2 - a^2bcd - 2a^3d^2)x)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(a^4d^2x + a^3bd^2)}{2(a^3b^2c^3 - a^4bc^2d + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="fricas")

[Out] [1/2\*((3\*b^3\*c^2 - a\*b^2\*c\*d - 2\*a^2\*b\*d^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d - 2\*a^3\*d^2)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(a^4\*d^2\*x + a^3\*b\*d^2)\*sqrt(-d/(b\*c - a\*d))\*log(-(2\*(b\*c - a\*d)\*x\*sqrt(-d/(b\*c - a\*d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) + 2\*((a^2\*b\*c^2 - a^3\*c\*d)\*x^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^3\*b^2\*c^3 - a^4\*b\*c^2\*d + (a^4\*b\*c^3 - a^5\*c^2\*d)\*x), ((3\*b^3\*c^2 - a\*b^2\*c\*d - 2\*a^2\*b\*d^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d - 2\*a^3\*d^2)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) - (a^4\*d^2\*x + a^3\*b\*d^2)\*sqrt(-d/(b\*c - a\*d))\*log(-(2\*(b\*c - a\*d)\*x\*sqrt(-d/(b\*c - a\*d))\*sqrt((a\*x + b)/x) - b\*d + (b\*c - 2\*a\*d)\*x)/(c\*x + d)) + ((a^2\*b\*c^2 - a^3\*c\*d)\*x^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^3\*b^2\*c^3 - a^4\*b\*c^2\*d + (a^4\*b\*c^3 - a^5\*c^2\*d)\*x), 1/2\*(4\*(a^4\*d^2\*x + a^3\*b\*d^2)\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*x\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)/(a\*d\*x + b\*d)) + (3\*b^3\*c^2 - a\*b^2\*c\*d - 2\*a^2\*b\*d^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d - 2\*a^3\*d^2)\*x)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*((a^2\*b\*c^2 - a^3\*c\*d)\*x^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^3\*b^2\*c^3 - a^4\*b\*c^2\*d + (a^4\*b\*c^3 - a^5\*c^2\*d)\*x), (2\*(a^4\*d^2\*x + a^3\*b\*d^2)\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*x\*sqrt(d/(b\*c - a\*d))\*sqrt((a\*x + b)/x)/(a\*d\*x + b\*d)) + (3\*b^3\*c^2 - a\*b^2\*c\*d - 2\*a^2\*b\*d^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d - 2\*a^3\*d^2)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + ((a^2\*b\*c^2 - a^3\*c\*d)\*x^2 + (3\*a\*b^2\*c^2 - a^2\*b\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^3\*b^2\*c^3 - a^4\*b\*c^2\*d + (a^4\*b\*c^3 - a^5\*c^2\*d)\*x)]

**giac [A]** time = 0.21, size = 200, normalized size = 1.36

$$\left( \frac{2d^3 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^3c^3 - ab^2c^2d)\sqrt{bcd - ad^2}} + \frac{2abc - \frac{3(ax+b)bc}{x} + \frac{(ax+b)ad}{x}}{(a^2b^2c^2 - a^3bcd)\left(a\sqrt{\frac{ax+b}{x}} - \frac{(ax+b)\sqrt{\frac{ax+b}{x}}}{x}\right)} + \frac{(3bc + 2ad) \arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a} a^2 b^2 c^2} \right) b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x),x, algorithm="giac")

```
[Out] (2*d^3*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^3*c^3 - a*b^2*c^2*d)*sqrt(b*c*d - a*d^2)) + (2*a*b*c - 3*(a*x + b)*b*c/x + (a*x + b)*a*d/x)/((a^2*b^2*c^2 - a^3*b*c*d)*(a*sqrt((a*x + b)/x) - (a*x + b)*sqrt((a*x + b)/x)/x)) + (3*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/((sqrt(-a)*a^2*b^2*c^2))*b^2
```

**maple [B]** time = 0.07, size = 962, normalized size = 6.54

$$\left( 2a^{\frac{9}{2}}d^3x^2 \ln \left( \frac{-2adx+bcx-bd+2\sqrt{\frac{(ad-bc)d}{c^2}} \sqrt{(ax+b)x} c}{cx+d} \right) + 2\sqrt{\frac{(ad-bc)d}{c^2}} a^4c d^2x^2 \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) + \sqrt{\frac{(ad-bc)d}{c^2}} a^3b c^2d \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b/x)^(3/2)/(c+d/x), x)
```

```
[Out] -1/2*(2*a^(9/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*d^3-2*a^(7/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x^2*c^2*d+4*a^(7/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^3+6*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x^2*b*c^3-4*a^(5/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b*c^2*d+2*a^(5/2)*ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^2*d^3-4*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(3/2)*b*c^3+12*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*x*b^2*c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^4*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*b*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b^2*c^3-2*a^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^2*c^2*d+4*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^3*b*c*d^2+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^2*b^2*c^2*d-6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a*b^3*c^3+6*a^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*b^3*c^3+2*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*b^2*c*d^2+ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^3*c^2*d-3*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3)*x*((a*x+b)/x)^(1/2)/a^(5/2)/(a*x+b)^2/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)/((a*x+b)*x)^(1/2)
```

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)^(3/2)/(c+d/x), x, algorithm="maxima")
```

```
[Out] integrate(1/((a + b/x)^(3/2)*(c + d/x)), x)
```

**mupad [B]** time = 2.68, size = 3000, normalized size = 20.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b/x)^(3/2)*(c + d/x)), x)
```



```

[Out] (atan((((d^5*(a*d - b*c)^3)^(1/2))*((a + b/x)^(1/2))*(18*a^6*b^9*c^10*d^3 - 6
6*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c
^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) +
((d^5*(a*d - b*c)^3)^(1/2))*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^12*d^2 - 13
2*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6 + 4*a^14*b
^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2))*(a + b/x)^(1/2))*(8*a^10*b^8*c^13*d^
2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 +
200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8)))/(c^2*(a*
d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) + ((d^5*(a*d - b
*c)^3)^(1/2))*((a + b/x)^(1/2))*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9*d^4 + 6
8*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c
^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d - b*c)^3)
^(1/2))*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^10*d^4 -
128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 + ((d^5*(a
*d - b*c)^3)^(1/2))*(a + b/x)^(1/2))*(8*a^10*b^8*c^13*d^2 - 56*a^11*b^7*c^12*
d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*c^9*d^6
- 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8)))/(c^2*(a*d - b*c)^3)))/(c^2*(a
*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3))/(36*a^6*b^8*c^7*d^5 - 96*a^7*b^7*c^6
*d^6 + 64*a^8*b^6*c^5*d^7 + 24*a^9*b^5*c^4*d^8 - 36*a^10*b^4*c^3*d^9 + 8*a^
11*b^3*c^2*d^10 - ((d^5*(a*d - b*c)^3)^(1/2))*((a + b/x)^(1/2))*(18*a^6*b^9*c
^10*d^3 - 66*a^7*b^8*c^9*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62
*a^10*b^5*c^6*d^7 - 2*a^11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*
c^3*d^10) + ((d^5*(a*d - b*c)^3)^(1/2))*(64*a^9*b^8*c^11*d^3 - 12*a^8*b^9*c^
12*d^2 - 132*a^10*b^7*c^10*d^4 + 128*a^11*b^6*c^9*d^5 - 52*a^12*b^5*c^8*d^6
+ 4*a^14*b^3*c^6*d^8 + ((d^5*(a*d - b*c)^3)^(1/2))*(a + b/x)^(1/2))*(8*a^10*
b^8*c^13*d^2 - 56*a^11*b^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*
c^10*d^5 + 200*a^14*b^4*c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8)
))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3) + ((d^5*
(a*d - b*c)^3)^(1/2))*((a + b/x)^(1/2))*(18*a^6*b^9*c^10*d^3 - 66*a^7*b^8*c^9
*d^4 + 68*a^8*b^7*c^8*d^5 + 20*a^9*b^6*c^7*d^6 - 62*a^10*b^5*c^6*d^7 - 2*a^
11*b^4*c^5*d^8 + 40*a^12*b^3*c^4*d^9 - 16*a^13*b^2*c^3*d^10) + ((d^5*(a*d -
b*c)^3)^(1/2))*(12*a^8*b^9*c^12*d^2 - 64*a^9*b^8*c^11*d^3 + 132*a^10*b^7*c^
10*d^4 - 128*a^11*b^6*c^9*d^5 + 52*a^12*b^5*c^8*d^6 - 4*a^14*b^3*c^6*d^8 +
((d^5*(a*d - b*c)^3)^(1/2))*(a + b/x)^(1/2))*(8*a^10*b^8*c^13*d^2 - 56*a^11*b
^7*c^12*d^3 + 160*a^12*b^6*c^11*d^4 - 240*a^13*b^5*c^10*d^5 + 200*a^14*b^4*
c^9*d^6 - 88*a^15*b^3*c^8*d^7 + 16*a^16*b^2*c^7*d^8)))/(c^2*(a*d - b*c)^3))
)/(c^2*(a*d - b*c)^3)))/(c^2*(a*d - b*c)^3))*1i)/(c^2*(a*d - b*c)^3) - (atanh((54*a^5*b^11*c^10*d^2*(a + b/x)^(1/2)))/((a^5)^(
1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 +
124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b
^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (216*a^6*b^10*c^9*d^3*(a + b/x)^(1/2))
)/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^
8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 1
10*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (234*a^7*b^9*c^8*d^4*(a + b/x)
^(1/2))/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5
*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*
d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (124*a^8*b^8*c^7*d^5*(a
+ b/x)^(1/2))/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9*d^3 +
234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 120*a^8*b
^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) - (366*a^9*b^7*c^6
*d^6*(a + b/x)^(1/2))/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*b^10*c^9
*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*d^6 + 12
0*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (120*a^10
*b^6*c^5*d^7*(a + b/x)^(1/2))/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 - 216*a^4*
b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*b^7*c^6*
d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d^9)) + (
110*a^11*b^5*c^4*d^8*(a + b/x)^(1/2))/((a^5)^(1/2))*(54*a^3*b^11*c^10*d^2 -
216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 366*a^7*
b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^4*c^3*d

```

```

^9)) - (60*a^12*b^4*c^3*d^9*(a + b/x)^(1/2))/((a^5)^(1/2)*(54*a^3*b^11*c^10
*d^2 - 216*a^4*b^10*c^9*d^3 + 234*a^5*b^9*c^8*d^4 + 124*a^6*b^8*c^7*d^5 - 3
66*a^7*b^7*c^6*d^6 + 120*a^8*b^6*c^5*d^7 + 110*a^9*b^5*c^4*d^8 - 60*a^10*b^
4*c^3*d^9))*(2*a*d + 3*b*c))/(c^2*(a^5)^(1/2)) - ((2*b^2)/(a^2*d - a*b*c)
+ (b*(a + b/x)*(a*d - 3*b*c))/(a^2*c*(a*d - b*c)))/(a*(a + b/x)^(1/2) - (a
+ b/x)^(3/2))

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} (cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x)**(3/2)/(c+d/x),x)
```

```
[Out] Integral(x/((a + b/x)**(3/2)*(c*x + d)), x)
```

$$3.257 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=224

$$\frac{(4ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3} + \frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2} + \frac{d^{5/2}(7bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}} + \frac{d(bc - 2a^2)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

[Out]  $d^{5/2}(-4ad+7bc)\operatorname{arctan}\left(\frac{d^{1/2}(a+b/x)^{1/2}}{(-ad+bc)^{1/2}}\right)/c^3/(-ad+bc)^{5/2} - (4ad+3bc)\operatorname{arctanh}\left(\frac{(a+b/x)^{1/2}}{a^{1/2}}\right)/a^{5/2}/c^3 + b(2a^2d^2 - 2abcd + 3b^2c^2)/a^2/c^2/(-ad+bc)^2/(a+b/x)^{1/2} + d(-2ad+bc)/a/c^2/(-ad+bc)/(c+d/x)/(a+b/x)^{1/2} + x/a/c/(c+d/x)/(a+b/x)^{1/2}$

**Rubi [A]** time = 0.32, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b(2a^2d^2 - 2abcd + 3b^2c^2)}{a^2c^2\sqrt{a+\frac{b}{x}}(bc-ad)^2} - \frac{(4ad + 3bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^3} + \frac{d^{5/2}(7bc - 4ad) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^3(bc-ad)^{5/2}} + \frac{d(bc - 2a^2)}{ac^2\sqrt{a+\frac{b}{x}}\left(c+\frac{d}{x}\right)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)^2), x]

[Out]  $(b(3b^2c^2 - 2abc*d + 2a^2d^2))/(a^2c^2(bc - ad)^2\sqrt{a + b/x}) + (d(bc - 2a^2))/(a^2c^2(bc - ad)\sqrt{a + b/x}(c + d/x)) + x/(ac^2\sqrt{a + b/x}(c + d/x)) + (d^{5/2}(7bc - 4ad)\operatorname{ArcTan}[\sqrt{d}\sqrt{a + b/x}/\sqrt{bc - ad}])/(c^3(bc - ad)^{5/2}) - ((3bc + 4ad)\operatorname{ArcTanh}[\sqrt{a + b/x}/\sqrt{a}])/(a^{5/2}c^3)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] :> Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g

$- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]$   
 $, x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 152

$\text{Int}[\frac{(a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}*((e_.) + (f_.)*(x_.)^{(p_.)}*((g_.) + (h_.)*(x_.)^{(n_.)}))}{(b*g - a*h)*(a + b*x)^{(m+1)*(c + d*x)^{(n+1)*(e + f*x)^{(p+1)}}}{(m+1)*(b*c - a*d)*(b*e - a*f)}, x]$   
 $+ \text{Dist}[1/((m+1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{(m+1)*(c + d*x)^n*(e + f*x)^p*\text{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m+1) - (b*g - a*h)*(d*e*(n+1) + c*f*(p+1)) - d*f*(b*g - a*h)*(m+n+p+3)*x, x], x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2\*m, 2\*n, 2\*p]

### Rule 156

$\text{Int}[\frac{((e_.) + (f_.)*(x_.)^{(p_.)}*((g_.) + (h_.)*(x_.)^{(n_.)}))/((a_.) + (b_.)*(x_.)^{(m_.)}*((c_.) + (d_.)*(x_.)^{(n_.)}))}{(b*g - a*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(a + b*x), x], x] - \text{Dist}[(d*g - c*h)/(b*c - a*d), \text{Int}[(e + f*x)^p/(c + d*x), x], x] /;$  FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[a/b, 2]*\text{ArcTan}[x/\text{Rt}[a/b, 2]])/a, x] /;$  FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

$\text{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$  FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

$\text{Int}[(a_.) + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x\_Symbol] := -\text{Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] /;$  FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} dx &= -\text{Subst} \left( \int \frac{1}{x^2(a + bx)^{3/2}(c + dx)^2} dx, x, \frac{1}{x} \right) \\
&= \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(3bc+4ad) + \frac{5bdx}{2}}{x(a+bx)^{3/2}(c+dx)^2} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} - \frac{\text{Subst} \left( \int \frac{-\frac{1}{2}(bc-ad)(3bc+4ad) - \frac{3}{2}bdx}{x(a+bx)^{3/2}(c+dx)^2} dx, x, \frac{1}{x} \right)}{ac^2(bc - ad)} \\
&= \frac{b(3b^2c^2 - 2abcd + 2a^2d^2)}{a^2c^2(bc - ad)^2\sqrt{a + \frac{b}{x}}} + \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} - \frac{b(3b^2c^2 - 2abcd + 2a^2d^2)}{a^2c^2(bc - ad)^2\sqrt{a + \frac{b}{x}}} + \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{b(3b^2c^2 - 2abcd + 2a^2d^2)}{a^2c^2(bc - ad)^2\sqrt{a + \frac{b}{x}}} + \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{b(3b^2c^2 - 2abcd + 2a^2d^2)}{a^2c^2(bc - ad)^2\sqrt{a + \frac{b}{x}}} + \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{b(3b^2c^2 - 2abcd + 2a^2d^2)}{a^2c^2(bc - ad)^2\sqrt{a + \frac{b}{x}}} + \frac{d(bc - 2ad)}{ac^2(bc - ad)\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)} + \frac{x}{ac\sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)}
\end{aligned}$$

**Mathematica [C]** time = 0.14, size = 164, normalized size = 0.73

$$\frac{(bc - ad) \left( (cx + d) \left( -4a^2d^2 + abcd + 3b^2c^2 \right) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax} + 1 \right) + acx(bc(cx + d) - ad(cx + 2d)) \right) + a^2d^2(cx + d)}{a^2c^3\sqrt{a + \frac{b}{x}}(cx + d)(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)\*(c + d/x)^2), x]

[Out] (a^2\*d^2\*(7\*b\*c - 4\*a\*d)\*(d + c\*x)\*Hypergeometric2F1[-1/2, 1, 1/2, (d\*(a + b/x))/(-b\*c) + a\*d]) + (b\*c - a\*d)\*(a\*c\*x\*(b\*c\*(d + c\*x) - a\*d\*(2\*d + c\*x)) + (3\*b^2\*c^2 + a\*b\*c\*d - 4\*a^2\*d^2)\*(d + c\*x)\*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a\*x)]))/(a^2\*c^3\*(b\*c - a\*d)^2\*Sqrt[a + b/x]\*(d + c\*x))

**fricas [B]** time = 2.00, size = 2321, normalized size = 10.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="fricas")

[Out] [1/2\*((3\*b^4\*c^3\*d - 2\*a\*b^3\*c^2\*d^2 - 5\*a^2\*b^2\*c\*d^3 + 4\*a^3\*b\*d^4 + (3\*a\*b^3\*c^4 - 2\*a^2\*b^2\*c^3\*d - 5\*a^3\*b\*c^2\*d^2 + 4\*a^4\*c\*d^3)\*x^2 + (3\*b^4\*c^

```

4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d)))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d)))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), 1/2*(2*(3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d)))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x), ((7*a^3*b^2*c*d^3 - 4*a^4*b*d^4 + (7*a^4*b*c^2*d^2 - 4*a^5*c*d^3)*x^2 + (7*a^3*b^2*c^2*d^2 + 3*a^4*b*c*d^3 - 4*a^5*d^4)*x)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d)))*sqrt((a*x + b)/x)/(a*d*x + b*d)) + (3*b^4*c^3*d - 2*a*b^3*c^2*d^2 - 5*a^2*b^2*c*d^3 + 4*a^3*b*d^4 + (3*a*b^3*c^4 - 2*a^2*b^2*c^3*d - 5*a^3*b*c^2*d^2 + 4*a^4*c*d^3)*x^2 + (3*b^4*c^4 + a*b^3*c^3*d - 7*a^2*b^2*c^2*d^2 - a^3*b*c*d^3 + 4*a^4*d^4)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + ((a^2*b^2*c^4 - 2*a^3*b*c^3*d + a^4*c^2*d^2)*x^3 + (3*a*b^3*c^4 - a^2*b^2*c^3*d - a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x^2 + (3*a*b^3*c^3*d - 2*a^2*b^2*c^2*d^2 + 2*a^3*b*c^2*d^2 + 2*a^4*c*d^3)*x)*sqrt((a*x + b)/x))/(a^3*b^3*c^5*d - 2*a^4*b^2*c^4*d^2 + a^5*b*c^3*d^3 + (a^4*b^2*c^6 - 2*a^5*b*c^5*d + a^6*c^4*d^2)*x^2 + (a^3*b^3*c^6 - a^4*b^2*c^5*d - a^5*b*c^4*d^2 + a^6*c^3*d^3)*x)]

```

**giac [B]** time = 0.26, size = 424, normalized size = 1.89

$$b^3 \left( \frac{(7bcd^3 - 4ad^4) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^5c^5 - 2ab^4c^4d + a^2b^3c^3d^2)\sqrt{bcd-ad^2}} + \frac{2ab^3c^3 - 2a^2b^2c^2d - \frac{3(ax+b)b^3c^3}{x} + \frac{7(ax+b)ab^2c^2d}{x} - \frac{3(ax+b)a^2bcd^2}{x} + \frac{2(a^2b^4c^4 - 2a^3b^3c^3d + a^4b^2c^2d^2)\left(abc\sqrt{\frac{ax+b}{x}} - a^2d\sqrt{\frac{ax+b}{x}} - \frac{a^3d^2}{x}\right)}{(a^2b^4c^4 - 2a^3b^3c^3d + a^4b^2c^2d^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="giac")

[Out] b^3\*((7\*b\*c\*d^3 - 4\*a\*d^4)\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^5\*c^5 - 2\*a\*b^4\*c^4\*d + a^2\*b^3\*c^3\*d^2)\*sqrt(b\*c\*d - a\*d^2)) + (2\*a\*b^3\*c^3 - 2\*a^2\*b^2\*c^2\*d - 3\*(a\*x + b)\*b^3\*c^3/x + 7\*(a\*x + b)\*a\*b^2\*c^2\*d/x

$$- 3*(a*x + b)*a^2*b*c*d^2/x + 2*(a*x + b)*a^3*d^3/x - 3*(a*x + b)^2*b^2*c^2*d/x^2 + 2*(a*x + b)^2*a*b*c*d^2/x^2 - 2*(a*x + b)^2*a^2*d^3/x^2)/((a^2*b^4*c^4 - 2*a^3*b^3*c^3*d + a^4*b^2*c^2*d^2)*(a*b*c*\sqrt{(a*x + b)/x} - a^2*d*\sqrt{(a*x + b)/x} - (a*x + b)*b*c*\sqrt{(a*x + b)/x}/x + 2*(a*x + b)*a*d*\sqrt{(a*x + b)/x}/x - (a*x + b)^2*d*\sqrt{(a*x + b)/x}/x^2)) + (3*b*c + 4*a*d)*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^2*b^3*c^3)$$

**maple [B]** time = 0.07, size = 3119, normalized size = 13.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2)/(c+d/x)^2,x)

[Out] 
$$-1/2*((a*x+b)/x)^{(1/2)}*x*(4*a^{(15/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^2*d^6+4*a^{(11/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*b^2*d^6-6*a^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^5*c^6+2*a^{(7/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^2*c^4*d^2-4*a^{(5/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^3*c^5*d-3*a*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^6*c^6+6*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^5*c^5*d-3*a*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^6*c^5*d+2*a^{(13/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^4*c^4*d^2-2*a^{(11/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*c^4*d^2-2*a^{(13/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*c^3*d^3+6*a^{(7/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^3*c^6-11*a^{(13/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^3*b*c^2*d^4+7*a^{(11/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^3*b^2*c^3*d^3+4*a^7*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*c^2*d^4-3*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^4*c^6-4*a^{(5/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^3*c^6-4*a^{(13/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*c^2*d^4+12*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^4*c^6-3*a^{(13/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^2*b*c*d^5-18*a^{(11/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x*b^2*c*d^5+3*a^{(9/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x*b^3*c^2*d^4+7*a^{(7/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x*b^4*c^3*d^3-4*a^{(9/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^2*c^2*d^4+8*a^{(7/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^3*c^3*d^3-10*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*b^4*c^4*d^2+4*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^2*c*d^5-9*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^3*c^2*d^4+3*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^4*c^3*d^3+5*a^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*b^5*c^4*d^2-15*a^{(11/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^2*b^2*c^2*d^4+14*a^{(9/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^2*b^3*c^3*d^3+4*a^7*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*c*d^5+6*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^5*c^6+13*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^4*c^4*d^2-8*a^{(11/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b*c^2*d^4+14*a^{(9/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^2*c^3*d^3-12*a^{(7/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^3*c^4*d^2+2*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^4*c^5*d+8*a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b*c*d^5-14*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^2*c^2*$$

$$d^4+11*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^4*d^2+7*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^4*c^5*d-a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^2*d^4-15*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^3*d^3+8*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^4*d^2-14*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^5*d-10*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^5*d-9*a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d^3+3*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4*d^2+5*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^3*c^5*d-4*a^(9/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^4*d^2+4*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^5*d+4*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^3*d^3+12*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^4*d^2-a^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^5*c^5*d-3*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^3*d^3+4*a^(15/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*c*d^5+8*a^(13/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b*d^6-11*a^(9/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*c*d^5+7*a^(7/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^4*c^2*d^4/a^(7/2)/c^4/((a*x+b)*x)^(1/2)/(a*d-b*c)^3/(c*x+d)/((a*d-b*c)/c^2*d)^(1/2)/(a*x+b)^2$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^2,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^2), x)

**mupad** [B] time = 6.20, size = 4274, normalized size = 19.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(3/2)\*(c + d/x)^2), x)

[Out] ((2\*b^3)/(a^2\*d - a\*b\*c) + (b\*(a + b/x)^2\*(2\*a^2\*d^3 + 3\*b^2\*c^2\*d - 2\*a\*b\*c\*d^2))/(c^2\*(a^2\*d - a\*b\*c)^2) - (b\*(a + b/x)\*(2\*a\*d - b\*c)\*(a^2\*d^2 + 3\*b^2\*c^2 - a\*b\*c\*d))/(c^2\*(a^2\*d - a\*b\*c)^2))/(d\*(a + b/x)^(5/2) + (a + b/x)^(1/2)\*(a^2\*d - a\*b\*c) - (a + b/x)^(3/2)\*(2\*a\*d - b\*c)) + (atan((a^13\*b^11\*c^11\*d^3\*(a + b/x)^(1/2)\*35i - a^12\*b^12\*c^12\*d^2\*(a + b/x)^(1/2)\*441i - a^10\*b^14\*c^14\*(a + b/x)^(1/2)\*27i + a^14\*b^10\*c^10\*d^4\*(a + b/x)^(1/2)\*1694i - a^15\*b^9\*c^9\*d^5\*(a + b/x)^(1/2)\*3073i + a^16\*b^8\*c^8\*d^6\*(a + b/x)^(1/2)\*1316i + a^17\*b^7\*c^7\*d^7\*(a + b/x)^(1/2)\*2561i - a^18\*b^6\*c^6\*d^8\*(a + b/x)^(1/2)\*4375i + a^19\*b^5\*c^5\*d^9\*(a + b/x)^(1/2)\*2996i - a^20\*b^4\*c^4\*d^10\*(a + b/x)^(1/2)\*1015i + a^21\*b^3\*c^3\*d^11\*(a + b/x)^(1/2)\*140i + a^11\*b^13\*c^13\*d\*(a + b/x)^(1/2)\*189i)/(a^5\*(a^5)^(1/2)\*(a^5\*(2561\*b^7\*c^7\*d^7 - 4375\*a\*b^6\*c^6\*d^8 + 2996\*a^2\*b^5\*c^5\*d^9 - 1015\*a^3\*b^4\*c^4\*d^10 + 140\*a^4\*b^3\*c^3\*d^11) - 441\*b^12\*c^12\*d^2 + 35\*a\*b^11\*c^11\*d^3 + 1694\*a^2\*b^10\*c^10\*d^4 - 3073\*a^3\*b^9\*c^9\*d^5 + 1316\*a^4\*b^8\*c^8\*d^6) - 27\*a^3\*b^14\*c^14 + 189\*a^4\*b^13\*c^13\*d))\*(4\*a\*d + 3\*b\*c)\*i)/(c^3\*(a^5)^(1/2)) - (atan(((d^5





$$\begin{aligned}
& 7*c^{11}*d^{10} + 5960*a^{14}*b^6*c^{10}*d^{11} - 4768*a^{15}*b^5*c^9*d^{12} + 2228*a^{16}* \\
& b^4*c^8*d^{13} - 576*a^{17}*b^3*c^7*d^{14} + 64*a^{18}*b^2*c^6*d^{15}) + ((d^5*(a*d - \\
& b*c)^5)^{(1/2)}*(4*a*d - 7*b*c)*(12*a^8*b^{14}*c^{21}*d^2 - 116*a^9*b^{13}*c^{20}*d^ \\
& 3 + 484*a^{10}*b^{12}*c^{19}*d^4 - 1128*a^{11}*b^{11}*c^{18}*d^5 + 1560*a^{12}*b^{10}*c^{17}* \\
& d^6 - 1176*a^{13}*b^9*c^{16}*d^7 + 168*a^{14}*b^8*c^{15}*d^8 + 576*a^{15}*b^7*c^{14}*d^ \\
& 9 - 612*a^{16}*b^6*c^{13}*d^{10} + 300*a^{17}*b^5*c^{12}*d^{11} - 76*a^{18}*b^4*c^{11}*d^{12} \\
& + 8*a^{19}*b^3*c^{10}*d^{13} + ((d^5*(a*d - b*c)^5)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d \\
& - 7*b*c)*(8*a^{10}*b^{13}*c^{23}*d^2 - 96*a^{11}*b^{12}*c^{22}*d^3 + 520*a^{12}*b^{11}*c^{2 \\
& 1}*d^4 - 1680*a^{13}*b^{10}*c^{20}*d^5 + 3600*a^{14}*b^9*c^{19}*d^6 - 5376*a^{15}*b^8*c^ \\
& 18*d^7 + 5712*a^{16}*b^7*c^{17}*d^8 - 4320*a^{17}*b^6*c^{16}*d^9 + 2280*a^{18}*b^5*c^ \\
& 15*d^{10} - 800*a^{19}*b^4*c^{14}*d^{11} + 168*a^{20}*b^3*c^{13}*d^{12} - 16*a^{21}*b^2*c^{1 \\
& 2}*d^{13}))/((2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - \\
& 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))/((2*(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b \\
& *c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)))/((2*( \\
& b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b*c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^ \\
& 5*d^3 - 5*a*b^4*c^7*d)) - 126*a^6*b^{13}*c^{14}*d^5 + 744*a^7*b^{12}*c^{13}*d^6 - \\
& 1742*a^8*b^{11}*c^{12}*d^7 + 1756*a^9*b^{10}*c^{11}*d^8 + 322*a^{10}*b^9*c^{10}*d^9 - 3 \\
& 248*a^{11}*b^8*c^9*d^{10} + 4606*a^{12}*b^7*c^8*d^{11} - 3668*a^{13}*b^6*c^7*d^{12} + 1 \\
& 804*a^{14}*b^5*c^6*d^{13} - 512*a^{15}*b^4*c^5*d^{14} + 64*a^{16}*b^3*c^4*d^{15}))*((d^5 \\
& *(a*d - b*c)^5)^{(1/2)}*(4*a*d - 7*b*c)*1i)/(b^5*c^8 - a^5*c^3*d^5 + 5*a^4*b* \\
& c^4*d^4 + 10*a^2*b^3*c^6*d^2 - 10*a^3*b^2*c^5*d^3 - 5*a*b^4*c^7*d)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*2,x)

[Out] Timed out

$$3.258 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=320

$$\frac{3(2ad + bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}c^4} + \frac{3d^{5/2}(8a^2d^2 - 24abcd + 21b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc - ad)^{7/2}} + \frac{d(12a^2d^2 - 21abcd + 4b^2c^2)}{4ac^3\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)(bc - ad)}$$

[Out]  $\frac{3}{4}d^{5/2}(8a^2d^2 - 24abcd + 21b^2c^2) \arctan\left(\frac{d^{1/2}(a+b/x)^{1/2}}{(-a*d+b*c)^{1/2}}\right)/c^4/(-a*d+b*c)^{7/2} - 3(2*a*d+b*c) \operatorname{arctanh}\left((a+b/x)^{1/2}/a^{1/2}\right)/a^{5/2}/c^4 + 3/4*b*(-a*d+2*b*c)*(4*a^2*d^2 - a*b*c*d + 2*b^2*c^2)/a^2/c^3/(-a*d+b*c)^3/(a+b/x)^{1/2} + 1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(c+d/x)^2/(a+b/x)^{1/2} + 1/4*d*(12*a^2*d^2 - 21*a*b*c*d + 4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(c+d/x)/(a+b/x)^{1/2} + x/a/c/(c+d/x)^2/(a+b/x)^{1/2}$

**Rubi [A]** time = 0.52, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d(12a^2d^2 - 21abcd + 4b^2c^2)}{4ac^3\sqrt{a + \frac{b}{x}}\left(c + \frac{d}{x}\right)(bc - ad)^2} + \frac{3b(2bc - ad)(4a^2d^2 - abcd + 2b^2c^2)}{4a^2c^3\sqrt{a + \frac{b}{x}}(bc - ad)^3} + \frac{3d^{5/2}(8a^2d^2 - 24abcd + 21b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{4c^4(bc - ad)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(3/2)\*(c + d/x)^3), x]

[Out]  $(3*b*(2*b*c - a*d)*(2*b^2*c^2 - a*b*c*d + 4*a^2*d^2))/(4*a^2*c^3*(b*c - a*d)^3*\sqrt{a + b/x}) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*\sqrt{a + b/x}*(c + d/x)^2) + (d*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*\sqrt{a + b/x}*(c + d/x)) + x/(a*c*\sqrt{a + b/x}*(c + d/x)^2) + (3*d^{5/2}*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*\operatorname{ArcTan}[\sqrt{d}*\sqrt{a + b/x}]/\sqrt{b*c - a*d}]/(4*c^4*(b*c - a*d)^{7/2}) - (3*(b*c + 2*a*d)*\operatorname{ArcTanh}[\sqrt{a + b/x}/\sqrt{a}])/(a^{5/2}*c^4)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]
```

### Rule 152

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]
```

### Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e + f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst} \left( \int \frac{1}{x^2(a + bx)^{3/2}(c + dx)^3} dx, x, \frac{1}{x} \right) \\
&= \frac{x}{ac \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst} \left( \int \frac{\frac{3}{2}(bc+2ad) + \frac{7bdx}{2}}{x(a+bx)^{3/2}(c+dx)^3} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{x}{ac \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst} \left( \int \frac{-3(bc-ad)(bc+2ad) - \frac{5}{2}}{x(a+bx)^{3/2}(c+dx)^3} dx, x, \frac{1}{x} \right)}{2ac^2(bc - ad)} \\
&= \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 21abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{x}{ac \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} \\
&= \frac{3b(2bc - ad)(2b^2c^2 - abcd + 4a^2d^2)}{4a^2c^3(bc - ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 21abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} \\
&= \frac{3b(2bc - ad)(2b^2c^2 - abcd + 4a^2d^2)}{4a^2c^3(bc - ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 21abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} \\
&= \frac{3b(2bc - ad)(2b^2c^2 - abcd + 4a^2d^2)}{4a^2c^3(bc - ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 21abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} \\
&= \frac{3b(2bc - ad)(2b^2c^2 - abcd + 4a^2d^2)}{4a^2c^3(bc - ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 21abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}
\end{aligned}$$

**Mathematica [C]** time = 0.32, size = 239, normalized size = 0.75

$$\frac{(cx + d) \left( 2(cx + d) \left( \frac{3}{4}a^2d^2 (8a^2d^2 - 24abcd + 21b^2c^2) {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{d(a + \frac{b}{x})}{ad - bc} \right) + 3(2ad + bc)(bc - ad)^3 {}_2F_1 \left( -\frac{1}{2}, 1; \frac{1}{2}; \frac{d(a + \frac{b}{x})}{ad - bc} \right) \right) + 2a^2c^4 \sqrt{a + \frac{b}{x}} (cx + d) \right)}{4a^2c^3(bc - ad)^3 \sqrt{a + \frac{b}{x}} \left(c + \frac{d}{x}\right)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(3/2)\*(c + d/x)^3), x]

[Out]  $(-(a*c^2*d*(b*c - a*d)^2*(-2*b*c + 3*a*d)*x^2) + 2*a*c^3*(b*c - a*d)^3*x^3 + (d + c*x)*(-1/2*(a*c*d*(-(b*c) + a*d)*(4*b^2*c^2 - 21*a*b*c*d + 12*a^2*d^2)*x) + 2*(d + c*x)*((3*a^2*d^2*(21*b^2*c^2 - 24*a*b*c*d + 8*a^2*d^2)*Hypergeometric2F1[-1/2, 1, 1/2, (d*(a + b/x))/(-(b*c) + a*d)]/4 + 3*(b*c - a*d)^3*(b*c + 2*a*d)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a*x)])))/(2*a^2*c^4*(b*c - a*d)^3*sqrt[a + b/x]*(d + c*x)^2)$

**fricas [B]** time = 5.03, size = 4093, normalized size = 12.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/8*(12*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 \\ & - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - \\ & a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6) \\ & *x)*\sqrt{a}*\log(2*a*x - 2*\sqrt{a}*x*\sqrt{(a*x + b)/x} + b) - 3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40 \\ & *a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*\sqrt{-d/(b*c - a*d)}*\log(-(2*(b*c - a*d) \\ & *x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*\sqrt{(a*x + b)/x)/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4) \\ & *x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/8*(24*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*\sqrt{-a}*\arctan(\sqrt{-a}*\sqrt{(a*x + b)/x})/a - 3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*\sqrt{-d/(b*c - a*d)}*\log(-(2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*\sqrt{(a*x + b)/x)/(a^3*b^4*c^7*d^2 - 3*a^4*b^3*c^6*d^3 + 3*a^5*b^2*c^5*d^4 - a^6*b*c^4*d^5 + (a^4*b^3*c^9 - 3*a^5*b^2*c^8*d + 3*a^6*b*c^7*d^2 - a^7*c^6*d^3)*x^3 + (a^3*b^4*c^9 - a^4*b^3*c^8*d - 3*a^5*b^2*c^7*d^2 + 5*a^6*b*c^6*d^3 - 2*a^7*c^5*d^4)*x^2 + (2*a^3*b^4*c^8*d - 5*a^4*b^3*c^7*d^2 + 3*a^5*b^2*c^6*d^3 + a^6*b*c^5*d^4 - a^7*c^4*d^5)*x), 1/4*(3*(21*a^3*b^3*c^2*d^4 - 24*a^4*b^2*c*d^5 + 8*a^5*b*d^6 + (21*a^4*b^2*c^4*d^2 - 24*a^5*b*c^3*d^3 + 8*a^6*c^2*d^4)*x^3 + (21*a^3*b^3*c^4*d^2 + 18*a^4*b^2*c^3*d^3 - 40*a^5*b*c^2*d^4 + 16*a^6*c*d^5)*x^2 + (42*a^3*b^3*c^3*d^3 - 27*a^4*b^2*c^2*d^4 - 8*a^5*b*c*d^5 + 8*a^6*d^6)*x)*\sqrt{d/(b*c - a*d)}*\arctan(-(b*c - a*d)*x*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x})/(a*d*x + b*d)) + 6*(b^5*c^4*d^2 - a*b^4*c^3*d^3 - 3*a^2*b^3*c^2*d^4 + 5*a^3*b^2*c*d^5 - 2*a^4*b*d^6 + (a*b^4*c^6 - a^2*b^3*c^5*d - 3*a^3*b^2*c^4*d^2 + 5*a^4*b*c^3*d^3 - 2*a^5*c^2*d^4)*x^3 + (b^5*c^6 + a*b^4*c^5*d - 5*a^2*b^3*c^4*d^2 - a^3*b^2*c^3*d^3 + 8*a^4*b*c^2*d^4 - 4*a^5*c*d^5)*x^2 + (2*b^5*c^5*d - a*b^4*c^4*d^2 - 7*a^2*b^3*c^3*d^3 + 7*a^3*b^2*c^2*d^4 + a^4*b*c*d^5 - 2*a^5*d^6)*x)*\sqrt{a}*\log(2*a*x - 2*\sqrt{a}*x*\sqrt{(a*x + b)/x} + b) + (4*(a^2*b^3*c^6 - 3*a^3*b^2*c^5*d + 3*a^4*b*c^4*d^2 - a^5*c^3*d^3)*x^4 + (12*a*b^4*c^6 - 4*a^2*b^3*c^5*d - 12*a^3*b^2*c^4*d^2 + 37*a^4*b*c^3*d^3 - 18*a^5*c^2*d^4)*x^3 + (24*a*b^4*c^5*d - 20*a^2*b^3*c^4*d^2 + 29*a^3*b^2*c^3*d^3 + 9*a^4*b*c^2*d^4 - 12*a^5*c*d^5)*x^2 + 3*(4*a*b^4*c^4*d^2 - 4*a^2*b^3*c^3*d^3 + 9*a^3*b^2*c^2*d^4 - 4*a^4*b*c*d^5)*x)*\sqrt{(a*x +$$

b)/x))/((a^3\*b^4\*c^7\*d^2 - 3\*a^4\*b^3\*c^6\*d^3 + 3\*a^5\*b^2\*c^5\*d^4 - a^6\*b\*c^4\*d^5 + (a^4\*b^3\*c^9 - 3\*a^5\*b^2\*c^8\*d + 3\*a^6\*b\*c^7\*d^2 - a^7\*c^6\*d^3)\*x^3 + (a^3\*b^4\*c^9 - a^4\*b^3\*c^8\*d - 3\*a^5\*b^2\*c^7\*d^2 + 5\*a^6\*b\*c^6\*d^3 - 2\*a^7\*c^5\*d^4)\*x^2 + (2\*a^3\*b^4\*c^8\*d - 5\*a^4\*b^3\*c^7\*d^2 + 3\*a^5\*b^2\*c^6\*d^3 + a^6\*b\*c^5\*d^4 - a^7\*c^4\*d^5)\*x), 1/4\*(3\*(21\*a^3\*b^3\*c^2\*d^4 - 24\*a^4\*b^2\*c\*d^5 + 8\*a^5\*b\*d^6 + (21\*a^4\*b^2\*c^4\*d^2 - 24\*a^5\*b\*c^3\*d^3 + 8\*a^6\*c^2\*d^4)\*x^3 + (21\*a^3\*b^3\*c^4\*d^2 + 18\*a^4\*b^2\*c^3\*d^3 - 40\*a^5\*b\*c^2\*d^4 + 16\*a^6\*c\*d^5)\*x^2 + (42\*a^3\*b^3\*c^3\*d^3 - 27\*a^4\*b^2\*c^2\*d^4 - 8\*a^5\*b\*c\*d^5 + 8\*a^6\*d^6)\*x)\*sqrt(d/(b\*c - a\*d))\*arctan(-(b\*c - a\*d)\*x\*sqrt(d/(b\*c - a\*d)))\*sqrt((a\*x + b)/x)/(a\*d\*x + b\*d)) + 12\*(b^5\*c^4\*d^2 - a\*b^4\*c^3\*d^3 - 3\*a^2\*b^3\*c^2\*d^4 + 5\*a^3\*b^2\*c\*d^5 - 2\*a^4\*b\*d^6 + (a\*b^4\*c^6 - a^2\*b^3\*c^5\*d - 3\*a^3\*b^2\*c^4\*d^2 + 5\*a^4\*b\*c^3\*d^3 - 2\*a^5\*c^2\*d^4)\*x^3 + (b^5\*c^6 + a\*b^4\*c^5\*d - 5\*a^2\*b^3\*c^4\*d^2 - a^3\*b^2\*c^3\*d^3 + 8\*a^4\*b\*c^2\*d^4 - 4\*a^5\*c\*d^5)\*x^2 + (2\*b^5\*c^5\*d - a\*b^4\*c^4\*d^2 - 7\*a^2\*b^3\*c^3\*d^3 + 7\*a^3\*b^2\*c^2\*d^4 + a^4\*b\*c\*d^5 - 2\*a^5\*d^6)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (4\*(a^2\*b^3\*c^6 - 3\*a^3\*b^2\*c^5\*d + 3\*a^4\*b\*c^4\*d^2 - a^5\*c^3\*d^3)\*x^4 + (12\*a\*b^4\*c^6 - 4\*a^2\*b^3\*c^5\*d - 12\*a^3\*b^2\*c^4\*d^2 + 37\*a^4\*b\*c^3\*d^3 - 18\*a^5\*c^2\*d^4)\*x^3 + (24\*a\*b^4\*c^5\*d - 20\*a^2\*b^3\*c^4\*d^2 + 29\*a^3\*b^2\*c^3\*d^3 + 9\*a^4\*b\*c^2\*d^4 - 12\*a^5\*c\*d^5)\*x^2 + 3\*(4\*a\*b^4\*c^4\*d^2 - 4\*a^2\*b^3\*c^3\*d^3 + 9\*a^3\*b^2\*c^2\*d^4 - 4\*a^4\*b\*c\*d^5)\*x)\*sqrt((a\*x + b)/x))/(a^3\*b^4\*c^7\*d^2 - 3\*a^4\*b^3\*c^6\*d^3 + 3\*a^5\*b^2\*c^5\*d^4 - a^6\*b\*c^4\*d^5 + (a^4\*b^3\*c^9 - 3\*a^5\*b^2\*c^8\*d + 3\*a^6\*b\*c^7\*d^2 - a^7\*c^6\*d^3)\*x^3 + (a^3\*b^4\*c^9 - a^4\*b^3\*c^8\*d - 3\*a^5\*b^2\*c^7\*d^2 + 5\*a^6\*b\*c^6\*d^3 - 2\*a^7\*c^5\*d^4)\*x^2 + (2\*a^3\*b^4\*c^8\*d - 5\*a^4\*b^3\*c^7\*d^2 + 3\*a^5\*b^2\*c^6\*d^3 + a^6\*b\*c^5\*d^4 - a^7\*c^4\*d^5)\*x)]

**giac [A]** time = 0.31, size = 516, normalized size = 1.61

$$\frac{1}{4} b^4 \left( \frac{3(21 b^2 c^2 d^3 - 24 a b c d^4 + 8 a^2 d^5) \arctan\left(\frac{d \sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^7 c^7 - 3 a b^6 c^6 d + 3 a^2 b^5 c^5 d^2 - a^3 b^4 c^4 d^3) \sqrt{bcd - ad^2}} + \frac{4 \left( 2 a b^3 c^3 - \frac{3(ax+b)b^3 c^3}{x} + \frac{3(ax+b)ab^2 c^2 d}{x} - \frac{3(ax+b)a^2 b c d^2}{x} \right)}{(a^2 b^6 c^6 - 3 a^3 b^5 c^5 d + 3 a^4 b^4 c^4 d^2 - a^5 b^3 c^3 d^3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="giac")

[Out] 1/4\*b^4\*(3\*(21\*b^2\*c^2\*d^3 - 24\*a\*b\*c\*d^4 + 8\*a^2\*d^5)\*arctan(d\*sqrt((a\*x + b)/x)/sqrt(b\*c\*d - a\*d^2))/((b^7\*c^7 - 3\*a\*b^6\*c^6\*d + 3\*a^2\*b^5\*c^5\*d^2 - a^3\*b^4\*c^4\*d^3)\*sqrt(b\*c\*d - a\*d^2)) + 4\*(2\*a\*b^3\*c^3 - 3\*(a\*x + b)\*b^3\*c^3/x + 3\*(a\*x + b)\*a\*b^2\*c^2\*d/x - 3\*(a\*x + b)\*a^2\*b\*c\*d^2/x + (a\*x + b)\*a^3\*d^3/x)/((a^2\*b^6\*c^6 - 3\*a^3\*b^5\*c^5\*d + 3\*a^4\*b^4\*c^4\*d^2 - a^5\*b^3\*c^3\*d^3)\*(a\*sqrt((a\*x + b)/x) - (a\*x + b)\*sqrt((a\*x + b)/x)/x) + (17\*b^2\*c^2\*d^3\*sqrt((a\*x + b)/x) - 25\*a\*b\*c\*d^4\*sqrt((a\*x + b)/x) + 8\*a^2\*d^5\*sqrt((a\*x + b)/x) + 15\*(a\*x + b)\*b\*c\*d^4\*sqrt((a\*x + b)/x)/x - 8\*(a\*x + b)\*a\*d^5\*sqrt((a\*x + b)/x)/x)/((b^6\*c^6 - 3\*a\*b^5\*c^5\*d + 3\*a^2\*b^4\*c^4\*d^2 - a^3\*b^3\*c^3\*d^3)\*(b\*c - a\*d + (a\*x + b)\*d/x)^2) + 12\*(b\*c + 2\*a\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^2\*b^4\*c^4)

**maple [B]** time = 0.08, size = 5158, normalized size = 16.12

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(3/2)/(c+d/x)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{3}{2}} \left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(3/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(3/2)\*(c + d/x)^3), x)

**mupad** [B] time = 9.49, size = 8936, normalized size = 27.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(3/2)\*(c + d/x)^3),x)

[Out] ((2\*b^4)/(a^2\*d - a\*b\*c) + (b\*(a + b/x)\*(12\*a^4\*d^4 + 12\*b^4\*c^4 + 24\*a^2\*b^2\*c^2\*d^2 - 40\*a\*b^3\*c^3\*d - 33\*a^3\*b\*c\*d^3))/(4\*a\*c^3\*(a^2\*d - a\*b\*c)\*(a\*d - b\*c)) + (3\*b\*(a + b/x)^3\*(4\*a^3\*d^5 - 4\*b^3\*c^3\*d^2 + 4\*a\*b^2\*c^2\*d^3 - 9\*a^2\*b\*c\*d^4))/(4\*a\*c^3\*(a^2\*d - a\*b\*c)\*(a\*d - b\*c)^2) - (b\*(a + b/x)^2\*(24\*a^4\*d^5 + 24\*b^4\*c^4\*d - 56\*a\*b^3\*c^3\*d^2 + 65\*a^2\*b^2\*c^2\*d^3 - 72\*a^3\*b\*c\*d^4))/(4\*a\*c^3\*(a^2\*d - a\*b\*c)\*(a\*d - b\*c)^2))/((a + b/x)^(3/2)\*(3\*a^2\*d^2 + b^2\*c^2 - 4\*a\*b\*c\*d) - (a + b/x)^(5/2)\*(3\*a\*d^2 - 2\*b\*c\*d) + d^2\*(a + b/x)^(7/2) - (a + b/x)^(1/2)\*(a^3\*d^2 + a\*b^2\*c^2 - 2\*a^2\*b\*c\*d)) + (atan((((a + b/x)^(1/2)\*(18432\*a^6\*b^19\*c^26\*d^3 - 202752\*a^7\*b^18\*c^25\*d^4 + 903168\*a^8\*b^17\*c^24\*d^5 - 1751040\*a^9\*b^16\*c^23\*d^6 - 137088\*a^10\*b^15\*c^22\*d^7 + 6007680\*a^11\*b^14\*c^21\*d^8 + 1276416\*a^12\*b^13\*c^20\*d^9 - 65382912\*a^13\*b^12\*c^19\*d^10 + 216610560\*a^14\*b^11\*c^18\*d^11 - 407418624\*a^15\*b^10\*c^17\*d^12 + 521961984\*a^16\*b^9\*c^16\*d^13 - 482904576\*a^17\*b^8\*c^15\*d^14 + 328809600\*a^18\*b^7\*c^14\*d^15 - 164257920\*a^19\*b^6\*c^13\*d^16 + 58816512\*a^20\*b^5\*c^12\*d^17 - 14340096\*a^21\*b^4\*c^11\*d^18 + 2138112\*a^22\*b^3\*c^10\*d^19 - 147456\*a^23\*b^2\*c^9\*d^20) - (3\*(d^5\*(a\*d - b\*c)^7)^(1/2)\*(8\*a^2\*d^2 + 21\*b^2\*c^2 - 24\*a\*b\*c\*d)\*(12288\*a^8\*b^19\*c^30\*d^2 - 172032\*a^9\*b^18\*c^29\*d^3 + 1081344\*a^10\*b^17\*c^28\*d^4 - 3996672\*a^11\*b^16\*c^27\*d^5 + 9449472\*a^12\*b^15\*c^26\*d^6 - 14112768\*a^13\*b^14\*c^25\*d^7 + 10407936\*a^14\*b^13\*c^24\*d^8 + 6454272\*a^15\*b^12\*c^23\*d^9 - 30007296\*a^16\*b^11\*c^22\*d^10 + 45551616\*a^17\*b^10\*c^21\*d^11 - 44064768\*a^18\*b^9\*c^20\*d^12 + 30096384\*a^19\*b^8\*c^19\*d^13 - 14831616\*a^20\*b^7\*c^18\*d^14 + 5203968\*a^21\*b^6\*c^17\*d^15 - 1241088\*a^22\*b^5\*c^16\*d^16 + 181248\*a^23\*b^4\*c^15\*d^17 - 12288\*a^24\*b^3\*c^14\*d^18 - (3\*(d^5\*(a\*d - b\*c)^7)^(1/2)\*(a + b/x)^(1/2)\*(8\*a^2\*d^2 + 21\*b^2\*c^2 - 24\*a\*b\*c\*d)\*(8192\*a^10\*b^18\*c^33\*d^2 - 139264\*a^11\*b^17\*c^32\*d^3 + 1105920\*a^12\*b^16\*c^31\*d^4 - 5447680\*a^13\*b^15\*c^30\*d^5 + 18636800\*a^14\*b^14\*c^29\*d^6 - 46964736\*a^15\*b^13\*c^28\*d^7 + 90202112\*a^16\*b^12\*c^27\*d^8 - 134717440\*a^17\*b^11\*c^26\*d^9 + 158146560\*a^18\*b^10\*c^25\*d^10 - 146432000\*a^19\*b^9\*c^24\*d^11 + 106602496\*a^20\*b^8\*c^23\*d^12 - 60383232\*a^21\*b^7\*c^22\*d^13 + 26091520\*a^22\*b^6\*c^21\*d^14 - 8314880\*a^23\*b^5\*c^20\*d^15 + 1843200\*a^24\*b^4\*c^19\*d^16 - 253952\*a^25\*b^3\*c^18\*d^17 + 16384\*a^26\*b^2\*c^17\*d^18)))/(8\*(b^7\*c^11 - a^7\*c^4\*d^7 + 7\*a^6\*b\*c^5\*d^6 + 21\*a^2\*b^5\*c^9\*d^2 - 35\*a^3\*b^4\*c^8\*d^3 + 35\*a^4\*b^3\*c^7\*d^4 - 21\*a^5\*b^2\*c^6\*d^5 - 7\*a\*b^6\*c^10\*d)))/(8\*(b^7\*c^11 - a^7\*c^4\*d^7 + 7\*a^6\*b\*c^5\*d^6 + 21\*a^2\*b^5\*c^9\*d^2 - 35\*a^3\*b^4\*c^8\*d^3 + 35\*a^4\*b^3\*c^7\*d^4 - 21\*a^5\*b^2\*c^6\*d^5 - 7\*a\*b^6\*c^10\*d)))\*(d^5\*(a\*d - b\*c)^7)^(1/2)\*(8\*a^2\*d^2 + 21\*b^2\*c^2 - 24\*a\*b\*c\*d)\*3i)/(8\*(b^7\*c^11 - a^7\*c^4\*d^7 + 7\*a^6\*b\*c^5\*d^6 + 21\*a^2\*b^5\*c^9\*d^2 - 35\*a^3\*b^4\*c^8\*d^3 + 35\*a^4\*b^3\*c^7\*d^4 - 21\*a^5\*b^2\*c^6\*d^5 - 7\*a\*b^6\*c^10\*d)) + (((a + b/x)^(1/2)\*(18432\*a^6\*b^19\*c^26\*d^3 - 202752\*a^7\*b^18\*c^25\*d^4 + 903168\*a^8\*b^17\*c^24\*d^5 - 1751040\*a^9\*b^16\*c^23\*d^6 - 137088\*a^10\*b^15\*c^22\*d^7 + 6007680\*a^11\*b^14\*c^21\*d^8 + 1276416\*a^12\*b^13\*c^20\*d^9 - 65382912\*a^13\*b^12\*c^19\*d^10 + 216610560\*a^14\*b^11



$$\begin{aligned}
& 1*c^{18}*d^{11} - 407418624*a^{15}*b^{10}*c^{17}*d^{12} + 521961984*a^{16}*b^9*c^{16}*d^{13} \\
& - 482904576*a^{17}*b^8*c^{15}*d^{14} + 328809600*a^{18}*b^7*c^{14}*d^{15} - 164257920*a \\
& ^{19}*b^6*c^{13}*d^{16} + 58816512*a^{20}*b^5*c^{12}*d^{17} - 14340096*a^{21}*b^4*c^{11}*d^{18} + 2138112*a^{22}*b^3*c^{10}*d^{19} - 147456*a^{23}*b^2*c^9*d^{20}) + (3*(d^5*(a*d \\
& - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^{10}*b^{17}*c^{28}*d^4 - 3996672*a^{11}* \\
& b^{16}*c^{27}*d^5 + 9449472*a^{12}*b^{15}*c^{26}*d^6 - 14112768*a^{13}*b^{14}*c^{25}*d^7 + 10407936*a^{14}*b^{13}*c^{24}*d^8 + 6454272*a^{15}*b^{12}*c^{23}*d^9 - 30007296*a^{16}*b^{11}*c^{22}*d^{10} + 45551616*a^{17}*b^{10}*c^{21}*d^{11} - 44064768*a^{18}*b^9*c^{20}*d^{12} + \\
& 30096384*a^{19}*b^8*c^{19}*d^{13} - 14831616*a^{20}*b^7*c^{18}*d^{14} + 5203968*a^{21}*b^6*c^{17}*d^{15} - 1241088*a^{22}*b^5*c^{16}*d^{16} + 181248*a^{23}*b^4*c^{15}*d^{17} - 122 \\
& 88*a^{24}*b^3*c^{14}*d^{18} + (3*(d^5*(a*d - b*c)^7)^{(1/2)}*(a + b/x)^{(1/2)}*(8*a^2 \\
& *d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^{10}*b^{18}*c^{33}*d^2 - 139264*a^{11}*b^{17} \\
& *c^{32}*d^3 + 1105920*a^{12}*b^{16}*c^{31}*d^4 - 5447680*a^{13}*b^{15}*c^{30}*d^5 + 18636 \\
& 800*a^{14}*b^{14}*c^{29}*d^6 - 46964736*a^{15}*b^{13}*c^{28}*d^7 + 90202112*a^{16}*b^{12}*c^{27}*d^8 - 134717440*a^{17}*b^{11}*c^{26}*d^9 + 158146560*a^{18}*b^{10}*c^{25}*d^{10} - 14 \\
& 6432000*a^{19}*b^9*c^{24}*d^{11} + 106602496*a^{20}*b^8*c^{23}*d^{12} - 60383232*a^{21}*b^7*c^{22}*d^{13} + 26091520*a^{22}*b^6*c^{21}*d^{14} - 8314880*a^{23}*b^5*c^{20}*d^{15} + 1 \\
& 843200*a^{24}*b^4*c^{19}*d^{16} - 253952*a^{25}*b^3*c^{18}*d^{17} + 16384*a^{26}*b^2*c^{17} \\
& *d^{18}))/ (8*(b^7*c^{11} - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - \\
& 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^1 \\
& 0*d)))/ (8*(b^7*c^{11} - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - \\
& 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^1 \\
& 0*d)))*(d^5*(a*d - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*3i)/ \\
& (8*(b^7*c^{11} - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3* \\
& b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^{10}*d)))/ ( \\
& 290304*a^6*b^{18}*c^{21}*d^5 - 2654208*a^7*b^{17}*c^{20}*d^6 + 10675584*a^8*b^{16}*c^{19} \\
& *d^7 - 23497344*a^9*b^{15}*c^{18}*d^8 + 23604480*a^{10}*b^{14}*c^{17}*d^9 + 2473113 \\
& 6*a^{11}*b^{13}*c^{16}*d^{10} - 148172544*a^{12}*b^{12}*c^{15}*d^{11} + 320101632*a^{13}*b^{11} \\
& *c^{14}*d^{12} - 452086272*a^{14}*b^{10}*c^{13}*d^{13} + 459302400*a^{15}*b^9*c^{12}*d^{14} - \\
& 343108224*a^{16}*b^8*c^{11}*d^{15} + 187373952*a^{17}*b^7*c^{10}*d^{16} - 72873216*a^{18} \\
& *b^6*c^9*d^{17} + 19132416*a^{19}*b^5*c^8*d^{18} - 3041280*a^{20}*b^4*c^7*d^{19} + 2 \\
& 21184*a^{21}*b^3*c^6*d^{20} - (3*((a + b/x)^{(1/2)}*(18432*a^6*b^{19}*c^{26}*d^3 - 20 \\
& 2752*a^7*b^{18}*c^{25}*d^4 + 903168*a^8*b^{17}*c^{24}*d^5 - 1751040*a^9*b^{16}*c^{23}*d^6 - \\
& 137088*a^{10}*b^{15}*c^{22}*d^7 + 6007680*a^{11}*b^{14}*c^{21}*d^8 + 1276416*a^{12}* \\
& b^{13}*c^{20}*d^9 - 65382912*a^{13}*b^{12}*c^{19}*d^{10} + 216610560*a^{14}*b^{11}*c^{18}*d^{11} \\
& - 407418624*a^{15}*b^{10}*c^{17}*d^{12} + 521961984*a^{16}*b^9*c^{16}*d^{13} - 48290457 \\
& 6*a^{17}*b^8*c^{15}*d^{14} + 328809600*a^{18}*b^7*c^{14}*d^{15} - 164257920*a^{19}*b^6*c^{13} \\
& *d^{16} + 58816512*a^{20}*b^5*c^{12}*d^{17} - 14340096*a^{21}*b^4*c^{11}*d^{18} + 21381 \\
& 12*a^{22}*b^3*c^{10}*d^{19} - 147456*a^{23}*b^2*c^9*d^{20}) - (3*(d^5*(a*d - b*c)^7)^{(1/2)} \\
& *(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(12288*a^8*b^{19}*c^{30}*d^2 - 1720 \\
& 32*a^9*b^{18}*c^{29}*d^3 + 1081344*a^{10}*b^{17}*c^{28}*d^4 - 3996672*a^{11}*b^{16}*c^{27} \\
& *d^5 + 9449472*a^{12}*b^{15}*c^{26}*d^6 - 14112768*a^{13}*b^{14}*c^{25}*d^7 + 10407936*a \\
& ^{14}*b^{13}*c^{24}*d^8 + 6454272*a^{15}*b^{12}*c^{23}*d^9 - 30007296*a^{16}*b^{11}*c^{22}*d^{10} + 45551616*a^{17}*b^{10}*c^{21}*d^{11} - 44064768*a^{18}*b^9*c^{20}*d^{12} + 30096384* \\
& a^{19}*b^8*c^{19}*d^{13} - 14831616*a^{20}*b^7*c^{18}*d^{14} + 5203968*a^{21}*b^6*c^{17}*d^{15} - 1241088*a^{22}*b^5*c^{16}*d^{16} + 181248*a^{23}*b^4*c^{15}*d^{17} - 12288*a^{24}*b^3 \\
& *c^{14}*d^{18} - (3*(d^5*(a*d - b*c)^7)^{(1/2)}*(a + b/x)^{(1/2)}*(8*a^2*d^2 + 21* \\
& b^2*c^2 - 24*a*b*c*d)*(8192*a^{10}*b^{18}*c^{33}*d^2 - 139264*a^{11}*b^{17}*c^{32}*d^3 \\
& + 1105920*a^{12}*b^{16}*c^{31}*d^4 - 5447680*a^{13}*b^{15}*c^{30}*d^5 + 18636800*a^{14}*b^{14} \\
& *c^{29}*d^6 - 46964736*a^{15}*b^{13}*c^{28}*d^7 + 90202112*a^{16}*b^{12}*c^{27}*d^8 - \\
& 134717440*a^{17}*b^{11}*c^{26}*d^9 + 158146560*a^{18}*b^{10}*c^{25}*d^{10} - 146432000*a^{19} \\
& *b^9*c^{24}*d^{11} + 106602496*a^{20}*b^8*c^{23}*d^{12} - 60383232*a^{21}*b^7*c^{22}*d^{13} + 26091520*a^{22} \\
& *b^6*c^{21}*d^{14} - 8314880*a^{23}*b^5*c^{20}*d^{15} + 1843200*a^{24} \\
& *b^4*c^{19}*d^{16} - 253952*a^{25}*b^3*c^{18}*d^{17} + 16384*a^{26}*b^2*c^{17}*d^{18}))/ (8 \\
& *(b^7*c^{11} - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4 \\
& *c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^{10}*d)))/ (8 \\
& *(b^7*c^{11} - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4 \\
& *c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^{10}*d)))*(d^
\end{aligned}$$

$$\begin{aligned}
& 5*(a*d - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 \\
& - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + \\
& 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a*b^6*c^10*d)) + (3*((a + b/x) \\
& ^{(1/2)}*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17 \\
& *c^24*d^5 - 1751040*a^9*b^16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 600768 \\
& 0*a^11*b^14*c^21*d^8 + 1276416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19 \\
& *d^10 + 216610560*a^14*b^11*c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521 \\
& 961984*a^16*b^9*c^16*d^13 - 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7 \\
& *c^14*d^15 - 164257920*a^19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - \\
& 14340096*a^21*b^4*c^11*d^18 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2 \\
& *c^9*d^20) + (3*(d^5*(a*d - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b* \\
& c*d)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17 \\
& *c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 1411 \\
& 2768*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23 \\
& *d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 440 \\
& 64768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18 \\
& *d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 18124 \\
& 8*a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 + (3*(d^5*(a*d - b*c)^7)^{(1 \\
& /2)}*(a + b/x)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - 24*a*b*c*d)*(8192*a^10*b^18*c^ \\
& ^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^16*c^31*d^4 - 5447680* \\
& a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 46964736*a^15*b^13*c^28* \\
& d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^11*c^26*d^9 + 15814656 \\
& 0*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 + 106602496*a^20*b^8*c^ \\
& ^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^22*b^6*c^21*d^14 - 8314 \\
& 880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 - 253952*a^25*b^3*c^18* \\
& d^17 + 16384*a^26*b^2*c^17*d^18))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5* \\
& d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5 \\
& *b^2*c^6*d^5 - 7*a*b^6*c^10*d)))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5* \\
& d^6 + 21*a^2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5 \\
& *b^2*c^6*d^5 - 7*a*b^6*c^10*d))*((d^5*(a*d - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21* \\
& b^2*c^2 - 24*a*b*c*d))/(8*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^ \\
& 2*b^5*c^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^ \\
& 5 - 7*a*b^6*c^10*d)))*((d^5*(a*d - b*c)^7)^{(1/2)}*(8*a^2*d^2 + 21*b^2*c^2 - \\
& 24*a*b*c*d)*3i)/(4*(b^7*c^11 - a^7*c^4*d^7 + 7*a^6*b*c^5*d^6 + 21*a^2*b^5*c^ \\
& ^9*d^2 - 35*a^3*b^4*c^8*d^3 + 35*a^4*b^3*c^7*d^4 - 21*a^5*b^2*c^6*d^5 - 7*a \\
& *b^6*c^10*d)) + (atan((((2*a*d + b*c)*((a + b/x)^{(1/2)}*(18432*a^6*b^19*c^26 \\
& *d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^17*c^24*d^5 - 1751040*a^9*b^ \\
& 16*c^23*d^6 - 137088*a^10*b^15*c^22*d^7 + 6007680*a^11*b^14*c^21*d^8 + 1276 \\
& 416*a^12*b^13*c^20*d^9 - 65382912*a^13*b^12*c^19*d^10 + 216610560*a^14*b^11 \\
& *c^18*d^11 - 407418624*a^15*b^10*c^17*d^12 + 521961984*a^16*b^9*c^16*d^13 - \\
& 482904576*a^17*b^8*c^15*d^14 + 328809600*a^18*b^7*c^14*d^15 - 164257920*a^ \\
& 19*b^6*c^13*d^16 + 58816512*a^20*b^5*c^12*d^17 - 14340096*a^21*b^4*c^11*d^1 \\
& 8 + 2138112*a^22*b^3*c^10*d^19 - 147456*a^23*b^2*c^9*d^20) - (3*(2*a*d + b* \\
& c)*(12288*a^8*b^19*c^30*d^2 - 172032*a^9*b^18*c^29*d^3 + 1081344*a^10*b^17* \\
& c^28*d^4 - 3996672*a^11*b^16*c^27*d^5 + 9449472*a^12*b^15*c^26*d^6 - 141127 \\
& 68*a^13*b^14*c^25*d^7 + 10407936*a^14*b^13*c^24*d^8 + 6454272*a^15*b^12*c^23 \\
& *d^9 - 30007296*a^16*b^11*c^22*d^10 + 45551616*a^17*b^10*c^21*d^11 - 44064 \\
& 768*a^18*b^9*c^20*d^12 + 30096384*a^19*b^8*c^19*d^13 - 14831616*a^20*b^7*c^18 \\
& *d^14 + 5203968*a^21*b^6*c^17*d^15 - 1241088*a^22*b^5*c^16*d^16 + 181248* \\
& a^23*b^4*c^15*d^17 - 12288*a^24*b^3*c^14*d^18 - (3*(a + b/x)^{(1/2)}*(2*a*d + \\
& b*c)*(8192*a^10*b^18*c^33*d^2 - 139264*a^11*b^17*c^32*d^3 + 1105920*a^12*b^ \\
& ^16*c^31*d^4 - 5447680*a^13*b^15*c^30*d^5 + 18636800*a^14*b^14*c^29*d^6 - 4 \\
& 6964736*a^15*b^13*c^28*d^7 + 90202112*a^16*b^12*c^27*d^8 - 134717440*a^17*b^ \\
& ^11*c^26*d^9 + 158146560*a^18*b^10*c^25*d^10 - 146432000*a^19*b^9*c^24*d^11 \\
& + 106602496*a^20*b^8*c^23*d^12 - 60383232*a^21*b^7*c^22*d^13 + 26091520*a^ \\
& 22*b^6*c^21*d^14 - 8314880*a^23*b^5*c^20*d^15 + 1843200*a^24*b^4*c^19*d^16 \\
& - 253952*a^25*b^3*c^18*d^17 + 16384*a^26*b^2*c^17*d^18))/(2*c^4*(a^5)^{(1/2)} \\
& )))/(2*c^4*(a^5)^{(1/2)))*3i)/(2*c^4*(a^5)^{(1/2)) + ((2*a*d + b*c)*((a + b/x) \\
& )^{(1/2)}*(18432*a^6*b^19*c^26*d^3 - 202752*a^7*b^18*c^25*d^4 + 903168*a^8*b^
\end{aligned}$$

$$\begin{aligned}
& 17c^{24}d^5 - 1751040a^9b^{16}c^{23}d^6 - 137088a^{10}b^{15}c^{22}d^7 + 6007680a^{11}b^{14}c^{21}d^8 + 1276416a^{12}b^{13}c^{20}d^9 - 65382912a^{13}b^{12}c^{19}d^{10} + 216610560a^{14}b^{11}c^{18}d^{11} - 407418624a^{15}b^{10}c^{17}d^{12} + 521961984a^{16}b^9c^{16}d^{13} - 482904576a^{17}b^8c^{15}d^{14} + 328809600a^{18}b^7c^{14}d^{15} - 164257920a^{19}b^6c^{13}d^{16} + 58816512a^{20}b^5c^{12}d^{17} - 14340096a^{21}b^4c^{11}d^{18} + 2138112a^{22}b^3c^{10}d^{19} - 147456a^{23}b^2c^9d^{20} + (3(2ad + bc))(12288a^8b^{19}c^{30}d^2 - 172032a^9b^{18}c^{29}d^3 + 1081344a^{10}b^{17}c^{28}d^4 - 3996672a^{11}b^{16}c^{27}d^5 + 9449472a^{12}b^{15}c^{26}d^6 - 14112768a^{13}b^{14}c^{25}d^7 + 10407936a^{14}b^{13}c^{24}d^8 + 6454272a^{15}b^{12}c^{23}d^9 - 30007296a^{16}b^{11}c^{22}d^{10} + 45551616a^{17}b^{10}c^{21}d^{11} - 44064768a^{18}b^9c^{20}d^{12} + 30096384a^{19}b^8c^{19}d^{13} - 14831616a^{20}b^7c^{18}d^{14} + 5203968a^{21}b^6c^{17}d^{15} - 1241088a^{22}b^5c^{16}d^{16} + 181248a^{23}b^4c^{15}d^{17} - 12288a^{24}b^3c^{14}d^{18} + (3(a + b/x)^{1/2})(2ad + bc)(8192a^{10}b^{18}c^{33}d^2 - 139264a^{11}b^{17}c^{32}d^3 + 1105920a^{12}b^{16}c^{31}d^4 - 5447680a^{13}b^{15}c^{30}d^5 + 18636800a^{14}b^{14}c^{29}d^6 - 46964736a^{15}b^{13}c^{28}d^7 + 90202112a^{16}b^{12}c^{27}d^8 - 134717440a^{17}b^{11}c^{26}d^9 + 158146560a^{18}b^{10}c^{25}d^{10} - 146432000a^{19}b^9c^{24}d^{11} + 106602496a^{20}b^8c^{23}d^{12} - 60383232a^{21}b^7c^{22}d^{13} + 26091520a^{22}b^6c^{21}d^{14} - 8314880a^{23}b^5c^{20}d^{15} + 1843200a^{24}b^4c^{19}d^{16} - 253952a^{25}b^3c^{18}d^{17} + 16384a^{26}b^2c^{17}d^{18}))/((2c^4(a^5)^{1/2}))/((2c^4(a^5)^{1/2}))*3i)/((2c^4(a^5)^{1/2}))/((290304a^6b^{18}c^{21}d^5 - 2654208a^7b^{17}c^{20}d^6 + 10675584a^8b^{16}c^{19}d^7 - 23497344a^9b^{15}c^{18}d^8 + 23604480a^{10}b^{14}c^{17}d^9 + 24731136a^{11}b^{13}c^{16}d^{10} - 148172544a^{12}b^{12}c^{15}d^{11} + 320101632a^{13}b^{11}c^{14}d^{12} - 452086272a^{14}b^{10}c^{13}d^{13} + 459302400a^{15}b^9c^{12}d^{14} - 343108224a^{16}b^8c^{11}d^{15} + 187373952a^{17}b^7c^{10}d^{16} - 72873216a^{18}b^6c^9d^{17} + 19132416a^{19}b^5c^8d^{18} - 3041280a^{20}b^4c^7d^{19} + 221184a^{21}b^3c^6d^{20} - (3(2ad + bc))(a + b/x)^{1/2})(18432a^6b^{19}c^{26}d^3 - 202752a^7b^{18}c^{25}d^4 + 903168a^8b^{17}c^{24}d^5 - 1751040a^9b^{16}c^{23}d^6 - 137088a^{10}b^{15}c^{22}d^7 + 6007680a^{11}b^{14}c^{21}d^8 + 1276416a^{12}b^{13}c^{20}d^9 - 65382912a^{13}b^{12}c^{19}d^{10} + 216610560a^{14}b^{11}c^{18}d^{11} - 407418624a^{15}b^{10}c^{17}d^{12} + 521961984a^{16}b^9c^{16}d^{13} - 482904576a^{17}b^8c^{15}d^{14} + 328809600a^{18}b^7c^{14}d^{15} - 164257920a^{19}b^6c^{13}d^{16} + 58816512a^{20}b^5c^{12}d^{17} - 14340096a^{21}b^4c^{11}d^{18} + 2138112a^{22}b^3c^{10}d^{19} - 147456a^{23}b^2c^9d^{20}) - (3(2ad + bc))(12288a^8b^{19}c^{30}d^2 - 172032a^9b^{18}c^{29}d^3 + 1081344a^{10}b^{17}c^{28}d^4 - 3996672a^{11}b^{16}c^{27}d^5 + 9449472a^{12}b^{15}c^{26}d^6 - 14112768a^{13}b^{14}c^{25}d^7 + 10407936a^{14}b^{13}c^{24}d^8 + 6454272a^{15}b^{12}c^{23}d^9 - 30007296a^{16}b^{11}c^{22}d^{10} + 45551616a^{17}b^{10}c^{21}d^{11} - 44064768a^{18}b^9c^{20}d^{12} + 30096384a^{19}b^8c^{19}d^{13} - 14831616a^{20}b^7c^{18}d^{14} + 5203968a^{21}b^6c^{17}d^{15} - 1241088a^{22}b^5c^{16}d^{16} + 181248a^{23}b^4c^{15}d^{17} - 12288a^{24}b^3c^{14}d^{18} - (3(a + b/x)^{1/2})(2ad + bc)(8192a^{10}b^{18}c^{33}d^2 - 139264a^{11}b^{17}c^{32}d^3 + 1105920a^{12}b^{16}c^{31}d^4 - 5447680a^{13}b^{15}c^{30}d^5 + 18636800a^{14}b^{14}c^{29}d^6 - 46964736a^{15}b^{13}c^{28}d^7 + 90202112a^{16}b^{12}c^{27}d^8 - 134717440a^{17}b^{11}c^{26}d^9 + 158146560a^{18}b^{10}c^{25}d^{10} - 146432000a^{19}b^9c^{24}d^{11} + 106602496a^{20}b^8c^{23}d^{12} - 60383232a^{21}b^7c^{22}d^{13} + 26091520a^{22}b^6c^{21}d^{14} - 8314880a^{23}b^5c^{20}d^{15} + 1843200a^{24}b^4c^{19}d^{16} - 253952a^{25}b^3c^{18}d^{17} + 16384a^{26}b^2c^{17}d^{18}))/((2c^4(a^5)^{1/2}))/((2c^4(a^5)^{1/2}))/((2c^4(a^5)^{1/2})) + (3(2ad + bc))(a + b/x)^{1/2})(18432a^6b^{19}c^{26}d^3 - 202752a^7b^{18}c^{25}d^4 + 903168a^8b^{17}c^{24}d^5 - 1751040a^9b^{16}c^{23}d^6 - 137088a^{10}b^{15}c^{22}d^7 + 6007680a^{11}b^{14}c^{21}d^8 + 1276416a^{12}b^{13}c^{20}d^9 - 65382912a^{13}b^{12}c^{19}d^{10} + 216610560a^{14}b^{11}c^{18}d^{11} - 407418624a^{15}b^{10}c^{17}d^{12} + 521961984a^{16}b^9c^{16}d^{13} - 482904576a^{17}b^8c^{15}d^{14} + 328809600a^{18}b^7c^{14}d^{15} - 164257920a^{19}b^6c^{13}d^{16} + 58816512a^{20}b^5c^{12}d^{17} - 14340096a^{21}b^4c^{11}d^{18} + 2138112a^{22}b^3c^{10}d^{19} - 147456a^{23}b^2c^9d^{20}) + (3(2ad + bc))(12288a^8b^{19}c^{30}d^2 - 172032a^9b^{18}c^{29}d^3 + 1081344a^{10}b^{17}c^{28}d^4 - 3996672a^{11}b^{16}c^{27}d^5 +
\end{aligned}$$

$$\begin{aligned}
& 9449472*a^{12}*b^{15}*c^{26}*d^6 - 14112768*a^{13}*b^{14}*c^{25}*d^7 + 10407936*a^{14}*b^{13}*c^{24}*d^8 + 6454272*a^{15}*b^{12}*c^{23}*d^9 - 30007296*a^{16}*b^{11}*c^{22}*d^{10} + \\
& 45551616*a^{17}*b^{10}*c^{21}*d^{11} - 44064768*a^{18}*b^9*c^{20}*d^{12} + 30096384*a^{19}*b^8*c^{19}*d^{13} - 14831616*a^{20}*b^7*c^{18}*d^{14} + 5203968*a^{21}*b^6*c^{17}*d^{15} - \\
& 1241088*a^{22}*b^5*c^{16}*d^{16} + 181248*a^{23}*b^4*c^{15}*d^{17} - 12288*a^{24}*b^3*c^{14}*d^{18} + (3*(a + b/x)^{(1/2)}*(2*a*d + b*c)*(8192*a^{10}*b^{18}*c^{33}*d^2 - 139264 \\
& *a^{11}*b^{17}*c^{32}*d^3 + 1105920*a^{12}*b^{16}*c^{31}*d^4 - 5447680*a^{13}*b^{15}*c^{30}*d^5 + 18636800*a^{14}*b^{14}*c^{29}*d^6 - 46964736*a^{15}*b^{13}*c^{28}*d^7 + 90202112*a^{16}*b^{12}*c^{27}*d^8 - \\
& 134717440*a^{17}*b^{11}*c^{26}*d^9 + 158146560*a^{18}*b^{10}*c^{25}*d^{10} - 146432000*a^{19}*b^9*c^{24}*d^{11} + 106602496*a^{20}*b^8*c^{23}*d^{12} - 60383232*a^{21}*b^7*c^{22}*d^{13} + \\
& 26091520*a^{22}*b^6*c^{21}*d^{14} - 8314880*a^{23}*b^5*c^{20}*d^{15} + 1843200*a^{24}*b^4*c^{19}*d^{16} - 253952*a^{25}*b^3*c^{18}*d^{17} + 16384*a^{26}*b^2*c^{17}*d^{18}))/ \\
& (2*c^4*(a^5)^{(1/2)}))/((2*c^4*(a^5)^{(1/2)}))/((2*c^4*(a^5)^{(1/2)}))/((2*c^4*(a^5)^{(1/2)}))* \\
& (2*a*d + b*c)*3i)/(c^4*(a^5)^{(1/2)})
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(3/2)/(c+d/x)\*\*3,x)

[Out] Timed out

$$3.259 \quad \int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=143

$$\frac{c^2(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{(bc - ad)(-4a^3d^2x - 2a^2bd(5cx + 3d) + ab^2c(20cx - 3d) + 15b^3c^2)}{3a^3b^2x\left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx\left(c + \frac{d}{x}\right)}{a\left(a + \frac{b}{x}\right)^{3/2}}$$

[Out]  $c*(c+d/x)^2*x/a/(a+b/x)^{(3/2)}+1/3*(-a*d+b*c)*(15*b^3*c^2-4*a^3*d^2*x-2*a^2*b*d*(5*c*x+3*d)+a*b^2*c*(20*c*x-3*d))/a^3/b^2/(a+b/x)^{(3/2)}/x-c^2*(-6*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(7/2)}$

**Rubi [A]** time = 0.15, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$ , Rules used = {375, 98, 145, 63, 208}

$$\frac{(bc - ad)(-2a^2bd(5cx + 3d) - 4a^3d^2x + ab^2c(20cx - 3d) + 15b^3c^2)}{3a^3b^2x\left(a + \frac{b}{x}\right)^{3/2}} - \frac{c^2(5bc - 6ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{cx\left(c + \frac{d}{x}\right)}{a\left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\left(c + \frac{d}{x}\right)^3/\left(a + \frac{b}{x}\right)^{(5/2)}, x\right]$

[Out]  $(c*(c + d/x)^2*x)/(a*(a + b/x)^{(3/2)}) + ((b*c - a*d)*(15*b^3*c^2 - 4*a^3*d^2*x - 2*a^2*b*d*(3*d + 5*c*x) + a*b^2*c*(-3*d + 20*c*x)))/(3*a^3*b^2*(a + b/x)^{(3/2)*x} - (c^2*(5*b*c - 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(7/2)})$

### Rule 63

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left(\left(c_.) + (d_.)*(x_.)\right)^{(n_.)}\right), x\_Symbol] :> \operatorname{With}\left[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}\left[p/b, \operatorname{Subst}\left[\operatorname{Int}\left[x^{(p*(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n, x\right], x, (a + b*x)^{(1/p)}\right], x\right] /; \operatorname{FreeQ}\left[\{a, b, c, d\}, x\right] \&\& \operatorname{NeQ}\left[b*c - a*d, 0\right] \&\& \operatorname{LtQ}\left[-1, m, 0\right] \&\& \operatorname{LeQ}\left[-1, n, 0\right] \&\& \operatorname{LeQ}\left[\operatorname{Denominator}[n], \operatorname{Denominator}[m]\right] \&\& \operatorname{IntLinearQ}\left[a, b, c, d, m, n, x\right]$

### Rule 98

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left(\left(c_.) + (d_.)*(x_.)\right)^{(n_.)}\left(\left(e_.) + (f_.)*(x_.)\right)^{(p_.)}\right), x\_Symbol] :> \operatorname{Simp}\left[\left((b*c - a*d)*(a + b*x)^{(m+1)}(c + d*x)^{(n-1)}(e + f*x)^{(p+1)}\right)/(b*(b*e - a*f)*(m+1)), x\right] + \operatorname{Dist}\left[1/(b*(b*e - a*f)*(m+1)), \operatorname{Int}\left[(a + b*x)^{(m+1)}(c + d*x)^{(n-2)}(e + f*x)^p \operatorname{Simp}\left[a*d*(d*e*(n-1) + c*f*(p+1)) + b*c*(d*e*(m-n+2) - c*f*(m+p+2)) + d*(a*d*f*(n+p) + b*(d*e*(m+1) - c*f*(m+n+p+1))\right]*x, x\right], x\right] /; \operatorname{FreeQ}\left[\{a, b, c, d, e, f, p\}, x\right] \&\& \operatorname{LtQ}\left[m, -1\right] \&\& \operatorname{GtQ}\left[n, 1\right] \&\& (\operatorname{IntegersQ}\left[2*m, 2*n, 2*p\right] \|\| \operatorname{IntegersQ}\left[m, n+p\right] \|\| \operatorname{IntegersQ}\left[p, m+n\right])$

### Rule 145

$\operatorname{Int}\left[\left((a_.) + (b_.)*(x_.)\right)^{(m_.)}\left(\left(c_.) + (d_.)*(x_.)\right)^{(n_.)}\left(\left(e_.) + (f_.)*(x_.)\right)^p\left(\left(g_.) + (h_.)*(x_.)\right)\right), x\_Symbol] :> \operatorname{Simp}\left[\left((b^3*c*e*g*(m+2) - a^3*d*f*h*(n+2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m+n+3)) - a*b^2*(c*(f*g + e*h) + d*e*g*(2*m+n+4)) + b*(a^2*d*f*h*(m-n) - a*b*(2*c*f*h*(m+1) - d*(f*g + e*h)*(n+1)) + b^2*(c*(f*g + e*h)*(m+1) - d*e*g*(m+n+2))\right)*x\right]$

```

a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
+ Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1)
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] && !LtQ[n, -2]))

```

Rule 208

```

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

Rule 375

```

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && !LtQ[n, 0]

```

Rubi steps

$$\int \frac{\left(c + \frac{d}{x}\right)^3}{\left(a + \frac{b}{x}\right)^{5/2}} dx = -\text{Subst}\left(\int \frac{(c + dx)^3}{x^2(a + bx)^{5/2}} dx, x, \frac{1}{x}\right)$$

$$= \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}c(5bc - 6ad) + \frac{1}{2}d(bc - 2ad)x\right)}{x(a + bx)^{5/2}} dx, x, \frac{1}{x}\right)}{a}$$

$$= \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(bc - ad)\left(15b^3c^2 - 4a^3d^2x - ab^2c(3d - 20cx) - 2a^2bd(3d + 5cx)\right)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x} + \frac{c^2(5bc - 6ad)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x}$$

$$= \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(bc - ad)\left(15b^3c^2 - 4a^3d^2x - ab^2c(3d - 20cx) - 2a^2bd(3d + 5cx)\right)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x} + \frac{c^2(5bc - 6ad)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x}$$

$$= \frac{c\left(c + \frac{d}{x}\right)^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(bc - ad)\left(15b^3c^2 - 4a^3d^2x - ab^2c(3d - 20cx) - 2a^2bd(3d + 5cx)\right)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x} - \frac{c^2(5bc - 6ad)}{3a^3b^2\left(a + \frac{b}{x}\right)^{3/2}x}$$

**Mathematica [A]** time = 0.43, size = 145, normalized size = 1.01

$$\frac{\frac{4a^5d^3x}{b^2} + \frac{6a^4d^2(cx+d)}{b} + 3a^3c^2x(cx - 8d) + 2a^2bc^2(10cx - 9d) + 15ab^2c^3 + 3ac^2\sqrt{\frac{b}{ax}} + 1(ax + b)(6ad - 5bc)\tanh^{-1}\left(\frac{ax + b}{\sqrt{a + \frac{b}{x}}}\right)}{3a^4\sqrt{a + \frac{b}{x}}(ax + b)}$$

Antiderivative was successfully verified.

```

[In] Integrate[(c + d/x)^3/(a + b/x)^(5/2), x]
[Out] (15*a*b^2*c^3 + (4*a^5*d^3*x)/b^2 + 3*a^3*c^2*x*(-8*d + c*x) + (6*a^4*d^2*(
d + c*x))/b + 2*a^2*b*c^2*(-9*d + 10*c*x) + 3*a*c^2*(-5*b*c + 6*a*d)*Sqrt[1

```

+ b/(a\*x)]\*(b + a\*x)\*ArcTanh[Sqrt[1 + b/(a\*x)])]/(3\*a^4\*Sqrt[a + b/x]\*(b + a\*x))

**fricas** [A] time = 0.98, size = 483, normalized size = 3.38

$$\frac{3(5b^5c^3 - 6ab^4c^2d + (5a^2b^3c^3 - 6a^3b^2c^2d)x^2 + 2(5ab^4c^3 - 6a^2b^3c^2d)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right)}{6(a^6b^2x^2 + \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="fricas")

[Out] [-1/6\*(3\*(5\*b^5\*c^3 - 6\*a\*b^4\*c^2\*d + (5\*a^2\*b^3\*c^3 - 6\*a^3\*b^2\*c^2\*d)\*x^2 + 2\*(5\*a\*b^4\*c^3 - 6\*a^2\*b^3\*c^2\*d)\*x)\*sqrt(a)\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(3\*a^3\*b^2\*c^3\*x^3 + 2\*(10\*a^2\*b^3\*c^3 - 12\*a^3\*b^2\*c^2\*d + 3\*a^4\*b\*c\*d^2 + 2\*a^5\*d^3)\*x^2 + 3\*(5\*a\*b^4\*c^3 - 6\*a^2\*b^3\*c^2\*d + 2\*a^4\*b\*d^3)\*x)\*sqrt((a\*x + b)/x))/(a^6\*b^2\*x^2 + 2\*a^5\*b^3\*x + a^4\*b^4), 1/3\*(3\*(5\*b^5\*c^3 - 6\*a\*b^4\*c^2\*d + (5\*a^2\*b^3\*c^3 - 6\*a^3\*b^2\*c^2\*d)\*x^2 + 2\*(5\*a\*b^4\*c^3 - 6\*a^2\*b^3\*c^2\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (3\*a^3\*b^2\*c^3\*x^3 + 2\*(10\*a^2\*b^3\*c^3 - 12\*a^3\*b^2\*c^2\*d + 3\*a^4\*b\*c\*d^2 + 2\*a^5\*d^3)\*x^2 + 3\*(5\*a\*b^4\*c^3 - 6\*a^2\*b^3\*c^2\*d + 2\*a^4\*b\*d^3)\*x)\*sqrt((a\*x + b)/x))/(a^6\*b^2\*x^2 + 2\*a^5\*b^3\*x + a^4\*b^4)]

**giac** [A] time = 0.26, size = 203, normalized size = 1.42

$$\frac{3b^2c^3\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3} - \frac{3(5b^2c^3-6abc^2d)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{2\left(ab^3c^3-3a^2b^2c^2d+3a^3bcd^2-a^4d^3+\frac{6(ax+b)b^3c^3}{x}-\frac{9(ax+b)ab^2c^2d}{x}+\frac{3(ax+b)a^3d^3}{x}\right)x}{(ax+b)a^3b\sqrt{\frac{ax+b}{x}}}$$

3b

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="giac")

[Out] -1/3\*(3\*b^2\*c^3\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3) - 3\*(5\*b^2\*c^3 - 6\*a\*b\*c^2\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3) - 2\*(a\*b^3\*c^3 - 3\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2 - a^4\*d^3 + 6\*(a\*x + b)\*b^3\*c^3/x - 9\*(a\*x + b)\*a\*b^2\*c^2\*d/x + 3\*(a\*x + b)\*a^3\*d^3/x)\*x/((a\*x + b)\*a^3\*b\*sqrt((a\*x + b)/x)))/b

**maple** [B] time = 0.06, size = 1150, normalized size = 8.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^3/(a+b/x)^(5/2),x)

[Out] -1/6\*((a\*x+b)/x)^(1/2)\*x/a^(7/2)\*(3\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*a^3\*b^4\*d^3-3\*ln(1/2\*(2\*a\*x+b+2\*(a\*x^2+b\*x)^(1/2)\*a^(1/2))/a^(1/2))\*a^3\*b^4\*d^3-30\*a^(1/2)\*((a\*x+b)\*x)^(1/2)\*b^6\*c^3-6\*(a\*x^2+b\*x)^(1/2)\*a^(13/2)\*x^3\*d^3-6\*a^(13/2)\*((a\*x+b)\*x)^(1/2)\*x^3\*d^3+12\*a^(11/2)\*((a\*x+b)\*x)^(3/2)\*x\*d^3+16\*a^(9/2)\*((a\*x+b)\*x)^(3/2)\*b\*d^3+20\*a^(3/2)\*((a\*x+b)\*x)^(3/2)\*b^4\*c^3-6\*(a\*x^2+b\*x)^(1/2)\*a^(7/2)\*b^3\*d^3-6\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*b^3\*d^3+36\*a^(9/2)\*((a\*x+b)\*x)^(1/2)\*x^3\*b^2\*c^2\*d-36\*a^(7/2)\*((a\*x+b)\*x)^(3/2)\*x\*b^2\*c^2\*d+108\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b^3\*c^2\*d-54\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x\*a^2\*b^5\*c^2\*d-54\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2))/a^(1/2))\*x^2\*a^3\*b^4\*c^2\*d-18\*ln(1/2\*(2

$$\begin{aligned}
& a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x^3*a^4*b^3*c^2*d+108*a^{(5/2)}* \\
& ((a*x+b)*x)^{(1/2)}*x*b^4*c^2*d+15*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)} \\
& )/a^{(1/2)})*b^7*c^3+9*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})* \\
& x*a^4*b^3*d^3+45*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x*a* \\
& b^6*c^3-9*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x*a^4*b^3*d \\
& ^3-18*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*a*b^6*c^2*d-18* \\
& (a*x^2+b*x)^{(1/2)}*a^{(9/2)}*x*b^2*d^3-12*a^{(7/2)}*((a*x+b)*x)^{(3/2)}*b^2*c*d^2- \\
& 24*a^{(5/2)}*((a*x+b)*x)^{(3/2)}*b^3*c^2*d-18*a^{(9/2)}*((a*x+b)*x)^{(1/2)}*x*b^2*d \\
& ^3-90*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*x*b^5*c^3+36*a^{(3/2)}*((a*x+b)*x)^{(1/2)}*b^5* \\
& c^2*d-90*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*x^2*b^4*c^3+24*a^{(5/2)}*((a*x+b)*x)^{(3/2)} \\
& *x*b^3*c^3-18*a^{(11/2)}*((a*x+b)*x)^{(1/2)}*x^2*b*d^3-18*(a*x^2+b*x)^{(1/2)}*a^{( \\
& 11/2)}*x^2*b*d^3-30*a^{(7/2)}*((a*x+b)*x)^{(1/2)}*x^3*b^3*c^3+3*\ln(1/2*(2*a*x+b+ \\
& 2*((a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x^3*a^6*b*d^3+15*\ln(1/2*(2*a*x+b+2*(( \\
& a*x+b)*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x^3*a^3*b^4*c^3-3*\ln(1/2*(2*a*x+b+2*(a*x^ \\
& 2+b*x)^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x^3*a^6*b*d^3+9*\ln(1/2*(2*a*x+b+2*((a*x+b)*x \\
& )^{(1/2)}*a^{(1/2)}/a^{(1/2)})*x^2*a^5*b^2*d^3+45*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^ \\
& (1/2)*a^{(1/2)}/a^{(1/2)})*x^2*a^2*b^5*c^3-9*\ln(1/2*(2*a*x+b+2*(a*x^2+b*x)^{(1/ \\
& 2)*a^{(1/2)}/a^{(1/2)})*x^2*a^5*b^2*d^3)/((a*x+b)*x)^{(1/2)}/b^3/(a*x+b)^3
\end{aligned}$$

**maxima [A]** time = 1.34, size = 228, normalized size = 1.59

$$\frac{1}{6}c^3 \left( \frac{2 \left( 15 \left( a + \frac{b}{x} \right)^2 b - 10 \left( a + \frac{b}{x} \right) ab - 2 a^2 b \right)}{\left( a + \frac{b}{x} \right)^{\frac{5}{2}} a^3 - \left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^4} + \frac{15 b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{7}{2}}} \right) - c^2 d \left( \frac{3 \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{5}{2}}} + \frac{2 \left( 4 a + \frac{3 b}{x} \right)}{\left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^2} \right) + \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^3/(a+b/x)^(5/2),x, algorithm="maxima")

[Out]  $\frac{1}{6}c^3(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^{(5/2)}*a^3 - (a + b/x)^{(3/2)}*a^4) + 15*b*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(7/2)} - c^2*d*(3*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(5/2)} + 2*(4*a + 3*b/x)/((a + b/x)^{(3/2)}*a^2)) + 2/3*d^3*(3/(\sqrt{a + b/x}*b^2) - a/((a + b/x)^{(3/2)}*b^2)) + 2*c*d^2/((a + b/x)^{(3/2)}*b)$

**mupad [B]** time = 2.05, size = 194, normalized size = 1.36

$$\frac{2(a^3 d^3 - 3 a^2 b c d^2 + 3 a b^2 c^2 d - b^3 c^3)}{3 a} + \frac{\left(a + \frac{b}{x}\right)^2 (2 a^3 d^3 - 6 a b^2 c^2 d + 5 b^3 c^3)}{a^3} - \frac{2\left(a + \frac{b}{x}\right) (4 a^3 d^3 - 3 a^2 b c d^2 - 6 a b^2 c^2 d + 5 b^3 c^3)}{3 a^2} + \frac{c^2 \operatorname{atanh}\left(\frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}}\right)}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} - a b^2 \left(a + \frac{b}{x}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^3/(a + b/x)^(5/2),x)

[Out]  $((2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a) + ((a + b/x)^2*(2*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d))/a^3 - (2*(a + b/x)*(4*a^3*d^3 + 5*b^3*c^3 - 6*a*b^2*c^2*d - 3*a^2*b*c*d^2))/(3*a^2))/((b^2*(a + b/x)^{(5/2)} - a*b^2*(a + b/x)^{(3/2)}) + (c^2*atanh((a + b/x)^{(1/2)}/a^{(1/2)})*(6*a*d - 5*b*c))/a^{(7/2)}$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx + d)^3}{x^3 \left(a + \frac{b}{x}\right)^{\frac{5}{2}}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)**3/(a+b/x)**(5/2),x)
```

```
[Out] Integral((c*x + d)**3/(x**3*(a + b/x)**(5/2)), x)
```

$$3.260 \quad \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=122

$$-\frac{c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{2a^2 d^2 + bc(5bc - 4ad)}{3a^2 b \left(a + \frac{b}{x}\right)^{3/2}} + \frac{c^2 x}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

[Out]  $1/3*(2*a^2*d^2+b*c*(-4*a*d+5*b*c))/a^2/b/(a+b/x)^{(3/2)}+c^2*x/a/(a+b/x)^{(3/2)}-c*(-4*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(7/2)}+c*(-4*a*d+5*b*c)/a^3/(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 118, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$ , Rules used = {375, 89, 78, 51, 63, 208}

$$\frac{\frac{c(5bc-4ad)}{a^2} + \frac{2d^2}{b}}{3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} - \frac{c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{c^2 x}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(c + d/x)^2/(a + b/x)^{(5/2)}, x]$

[Out]  $((2*d^2)/b + (c*(5*b*c - 4*a*d))/a^2)/(3*(a + b/x)^{(3/2)}) + (c*(5*b*c - 4*a*d))/(a^3*\operatorname{Sqrt}[a + b/x]) + (c^2*x)/(a*(a + b/x)^{(3/2)}) - (c*(5*b*c - 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(7/2)}$

### Rule 51

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{Simp}[(a + b*x)^{(m + 1)}*(c + d*x)^{(n + 1)}]/((b*c - a*d)*(m + 1)), x] - \operatorname{Dist}[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), \operatorname{Int}[(a + b*x)^{(m + 1)}*(c + d*x)^n, x] /; \operatorname{FreeQ}\{a, b, c, d, n\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[m, -1] \&\& !(\operatorname{LtQ}[n, -1] \&\& (\operatorname{EqQ}[a, 0] \mid\mid (\operatorname{NeQ}[c, 0] \&\& \operatorname{LtQ}[m - n, 0] \&\& \operatorname{IntegerQ}[n]))) \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)}*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m + 1) - 1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /; \operatorname{FreeQ}\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

### Rule 78

$\operatorname{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] \rightarrow -\operatorname{Simp}[(b*e - a*f)*(c + d*x)^{(n + 1)}*(e + f*x)^{(p + 1)}]/(f*(p + 1)*(c*f - d*e)), x] - \operatorname{Dist}[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1))]/(f*(p + 1)*(c*f - d*e)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^{(p + 1)}, x], x] /; \operatorname{FreeQ}\{a, b, c, d, e, f, n\}, x] \&\& \operatorname{LtQ}[p, -1] \&\& (!\operatorname{LtQ}[n, -1] \mid\mid \operatorname{IntegerQ}[p] \mid\mid !(\operatorname{IntegerQ}[n] \mid\mid !(\operatorname{EqQ}[e, 0] \mid\mid !(\operatorname{EqQ}[c, 0] \mid\mid \operatorname{LtQ}[p, n]))))$

Rule 89

Int[((a\_.) + (b\_.)\*(x\_))<sup>2</sup>\*((c\_.) + (d\_.)\*(x\_))<sup>(n\_.)</sup>\*((e\_.) + (f\_.)\*(x\_))<sup>(p\_.)</sup>, x\_Symbol] := Simp[((b\*c - a\*d)<sup>2</sup>\*(c + d\*x)<sup>(n + 1)</sup>\*(e + f\*x)<sup>(p + 1)</sup>)/(d<sup>2</sup>\*(d\*e - c\*f)\*(n + 1)), x] - Dist[1/(d<sup>2</sup>\*(d\*e - c\*f)\*(n + 1)), Int[(c + d\*x)<sup>(n + 1)</sup>\*(e + f\*x)<sup>p</sup>\*Simp[a<sup>2</sup>\*d<sup>2</sup>\*f\*(n + p + 2) + b<sup>2</sup>\*c\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - 2\*a\*b\*d\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - b<sup>2</sup>\*d\*(d\*e - c\*f)\*(n + 1)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p, 1])))

Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)<sup>(-1)</sup>, x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))<sup>(p\_.)</sup>\*((c\_) + (d\_.)\*(x\_)^(n\_))<sup>(q\_.)</sup>, x\_Symbol] := -Subst[Int[((a + b/x<sup>n</sup>)<sup>p</sup>\*(c + d/x<sup>n</sup>)<sup>q</sup>]/x<sup>2</sup>, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(c + \frac{d}{x}\right)^2}{\left(a + \frac{b}{x}\right)^{5/2}} dx &= -\text{Subst}\left(\int \frac{(c + dx)^2}{x^2(a + bx)^{5/2}} dx, x, \frac{1}{x}\right) \\
 &= \frac{c^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}c(5bc - 4ad) + ad^2 x}{x(a + bx)^{5/2}} dx, x, \frac{1}{x}\right)}{a} \\
 &= \frac{\frac{2d^2}{b} + \frac{c(5bc - 4ad)}{a^2}}{3\left(a + \frac{b}{x}\right)^{3/2}} + \frac{c^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(c(5bc - 4ad)) \text{Subst}\left(\int \frac{1}{x(a + bx)^{3/2}} dx, x, \frac{1}{x}\right)}{2a^2} \\
 &= \frac{\frac{2d^2}{b} + \frac{c(5bc - 4ad)}{a^2}}{3\left(a + \frac{b}{x}\right)^{3/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{c^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(c(5bc - 4ad)) \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx}} dx, x, \frac{1}{x}\right)}{2a^3} \\
 &= \frac{\frac{2d^2}{b} + \frac{c(5bc - 4ad)}{a^2}}{3\left(a + \frac{b}{x}\right)^{3/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{c^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(c(5bc - 4ad)) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{a^3 b} \\
 &= \frac{\frac{2d^2}{b} + \frac{c(5bc - 4ad)}{a^2}}{3\left(a + \frac{b}{x}\right)^{3/2}} + \frac{c(5bc - 4ad)}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{c^2 x}{a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{c(5bc - 4ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
 \end{aligned}$$

**Mathematica [C]** time = 0.08, size = 97, normalized size = 0.80

$$\frac{ax(2a^2d^2 + abc(3cx - 4d) + 5b^2c^2) + 3bc(ax + b)(5bc - 4ad) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b}{ax} + 1\right)}{3a^3b\sqrt{a + \frac{b}{x}}(ax + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)^2/(a + b/x)^(5/2),x]

[Out] (a\*x\*(5\*b^2\*c^2 + 2\*a^2\*d^2 + a\*b\*c\*(-4\*d + 3\*c\*x)) + 3\*b\*c\*(5\*b\*c - 4\*a\*d) \* (b + a\*x)\*Hypergeometric2F1[-1/2, 1, 1/2, 1 + b/(a\*x)])/(3\*a^3\*b\*Sqrt[a + b/x]\*(b + a\*x))

**fricas** [A] time = 1.01, size = 407, normalized size = 3.34

$$\frac{3(5b^4c^2 - 4ab^3cd + (5a^2b^2c^2 - 4a^3bcd)x^2 + 2(5ab^3c^2 - 4a^2b^2cd)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(3a^2b^2c^2 - 4a^3bcd)}{6(a^6bx^2 + 2a^5b^2x + a^4b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="fricas")

[Out] [-1/6\*(3\*(5\*b^4\*c^2 - 4\*a\*b^3\*c\*d + (5\*a^2\*b^2\*c^2 - 4\*a^3\*b\*c\*d)\*x^2 + 2\*(5\*a\*b^3\*c^2 - 4\*a^2\*b^2\*c\*d)\*x)\*sqrt(a)\*log(2\*a\*x + 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) - 2\*(3\*a^3\*b\*c^2\*x^3 + 2\*(10\*a^2\*b^2\*c^2 - 8\*a^3\*b\*c\*d + a^4\*d^2)\*x^2 + 3\*(5\*a\*b^3\*c^2 - 4\*a^2\*b^2\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^6\*b\*x^2 + 2\*a^5\*b^2\*x + a^4\*b^3), 1/3\*(3\*(5\*b^4\*c^2 - 4\*a\*b^3\*c\*d + (5\*a^2\*b^2\*c^2 - 4\*a^3\*b\*c\*d)\*x^2 + 2\*(5\*a\*b^3\*c^2 - 4\*a^2\*b^2\*c\*d)\*x)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (3\*a^3\*b\*c^2\*x^3 + 2\*(10\*a^2\*b^2\*c^2 - 8\*a^3\*b\*c\*d + a^4\*d^2)\*x^2 + 3\*(5\*a\*b^3\*c^2 - 4\*a^2\*b^2\*c\*d)\*x)\*sqrt((a\*x + b)/x))/(a^6\*b\*x^2 + 2\*a^5\*b^2\*x + a^4\*b^3)]

**giac** [A] time = 0.21, size = 163, normalized size = 1.34

$$\frac{\frac{3b^2c^2\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3} - \frac{3(5b^2c^2-4abcd)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{2\left(ab^2c^2-2a^2bcd+a^3d^2+\frac{6(ax+b)b^2c^2}{x}-\frac{6(ax+b)abcd}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(5/2),x, algorithm="giac")

[Out] -1/3\*(3\*b^2\*c^2\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3) - 3\*(5\*b^2\*c^2 - 4\*a\*b\*c\*d)\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3) - 2\*(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2 + 6\*(a\*x + b)\*b^2\*c^2/x - 6\*(a\*x + b)\*a\*b\*c\*d/x)\*x/((a\*x + b)\*a^3\*sqrt((a\*x + b)/x))/b

**maple** [B] time = 0.06, size = 588, normalized size = 4.82

$$\frac{\sqrt{\frac{ax+b}{x}} \left( -12a^4bcdx^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) + 15a^3b^2c^2x^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - 36a^3b^2cdx^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) \right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^2/(a+b/x)^(5/2),x)

[Out] -1/6\*((a\*x+b)/x)^(1/2)\*x\*(24\*a^(9/2)\*((a\*x+b)\*x)^(1/2)\*x^3\*c\*d-30\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*x^3\*b\*c^2-24\*a^(7/2)\*((a\*x+b)\*x)^(3/2)\*x\*c\*d+72\*((a\*x+b)\*x)^(1/2)\*a^(7/2)\*b\*c\*d\*x^2-4\*((a\*x+b)\*x)^(3/2)\*a^(7/2)\*d^2+24\*a^(5/2)\*((a\*x+b)\*x)^(3/2)\*x\*b\*c^2-90\*((a\*x+b)\*x)^(1/2)\*a^(5/2)\*b^2\*c^2\*x^2-16\*((a\*x+b)\*x)^(3/2)\*a^(5/2)\*b\*c\*d+72\*((a\*x+b)\*x)^(1/2)\*a^(5/2)\*b^2\*c\*d\*x-12\*ln(1/2\*(2\*a\*

$$x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)})*x^3*a^4*b*c*d+15*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)})*x^3*a^3*b^2*c^2+20*((a*x+b)*x)^{(3/2)}*a^{(3/2)}*b^2*c^2-90*((a*x+b)*x)^{(1/2)}*a^{(3/2)}*b^3*c^2*x-36*a^3*b^2*c*d*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))+45*a^2*b^3*c^2*x^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))+24*((a*x+b)*x)^{(1/2)}*a^{(3/2)}*b^3*c*d-36*a^2*b^3*c*d*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))+45*a*b^4*c^2*x*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))-30*((a*x+b)*x)^{(1/2)}*a^{(1/2)}*b^4*c^2-12*a*b^4*c*d*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))+15*b^5*c^2*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2))/a^{(1/2)}))/a^{(7/2)}/((a*x+b)*x)^{(1/2)}/b/(a*x+b)^3$$

**maxima [A]** time = 1.31, size = 190, normalized size = 1.56

$$\frac{1}{6}c^2 \left( \frac{2 \left( 15 \left( a + \frac{b}{x} \right)^2 b - 10 \left( a + \frac{b}{x} \right) ab - 2 a^2 b \right)}{\left( a + \frac{b}{x} \right)^{\frac{5}{2}} a^3 - \left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^4} + \frac{15 b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{7}{2}}} \right) - \frac{2}{3} cd \left( \frac{3 \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{a^{\frac{5}{2}}} + \frac{2 \left( 4 a + \frac{3 b}{x} \right)}{\left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^2/(a+b/x)^(5/2), x, algorithm="maxima")

[Out] 1/6\*c^2\*(2\*(15\*(a + b/x)^2\*b - 10\*(a + b/x)\*a\*b - 2\*a^2\*b)/((a + b/x)^(5/2)\*a^3 - (a + b/x)^(3/2)\*a^4) + 15\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)) - 2/3\*c\*d\*(3\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(5/2) + 2\*(4\*a + 3\*b/x)/((a + b/x)^(3/2)\*a^2)) + 2/3\*d^2/((a + b/x)^(3/2)\*b)

**mupad [B]** time = 2.22, size = 144, normalized size = 1.18

$$\frac{2 \left( a + \frac{b}{x} \right) (a^2 d^2 + 4 a b c d - 5 b^2 c^2)}{3 a^2} - \frac{2 (a^2 d^2 - 2 a b c d + b^2 c^2)}{3 a} + \frac{b \left( a + \frac{b}{x} \right)^2 (5 b c^2 - 4 a c d)}{a^3} + \frac{c \operatorname{atanh} \left( \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}} \right) (4 a d - 5 b c)}{a^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)^2/(a + b/x)^(5/2), x)

[Out] ((2\*(a + b/x)\*(a^2\*d^2 - 5\*b^2\*c^2 + 4\*a\*b\*c\*d))/(3\*a^2) - (2\*(a^2\*d^2 + b^2\*c^2 - 2\*a\*b\*c\*d))/(3\*a) + (b\*(a + b/x)^2\*(5\*b\*c^2 - 4\*a\*c\*d))/a^3)/(b\*(a + b/x)^(5/2) - a\*b\*(a + b/x)^(3/2)) + (c\*atanh((a + b/x)^(1/2)/a^(1/2))\*(4\*a\*d - 5\*b\*c))/a^(7/2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(cx + d)^2}{x^2 \left( a + \frac{b}{x} \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*2/(a+b/x)\*\*(5/2), x)

[Out] Integral((c\*x + d)\*\*2/(x\*\*2\*(a + b/x)\*\*(5/2)), x)

$$3.261 \quad \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=103

$$-\frac{(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5bc - 2ad}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{5bc - 2ad}{3a^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

[Out]  $1/3*(-2*a*d+5*b*c)/a^2/(a+b/x)^{(3/2)}+c*x/a/(a+b/x)^{(3/2)}-(-2*a*d+5*b*c)*\text{arc tanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(7/2)}+(-2*a*d+5*b*c)/a^3/(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {375, 78, 51, 63, 208}

$$\frac{5bc - 2ad}{a^3 \sqrt{a + \frac{b}{x}}} + \frac{5bc - 2ad}{3a^2 \left(a + \frac{b}{x}\right)^{3/2}} - \frac{(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{cx}{a \left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c + d/x)/(a + b/x)^(5/2), x]

[Out]  $(5*b*c - 2*a*d)/(3*a^2*(a + b/x)^{(3/2)}) + (5*b*c - 2*a*d)/(a^3*\text{Sqrt}[a + b/x]) + (c*x)/(a*(a + b/x)^{(3/2)}) - ((5*b*c - 2*a*d)*\text{ArcTanh}[\text{Sqrt}[a + b/x]/\text{Sqrt}[a]])/a^{(7/2)}$

### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n])))

### Rule 208

$\text{Int}[(a + b \cdot x) \cdot x^{-2}, x\_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b]$

### Rule 375

$\text{Int}[(a + b \cdot x)^n \cdot (c + d \cdot x)^q, x\_Symbol] \rightarrow -\text{Subst}[\text{Int}[(a + b/x)^n \cdot (c + d/x)^q / x^2, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, p, q, x\} \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{ILtQ}[n, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{c + \frac{d}{x}}{\left(a + \frac{b}{x}\right)^{5/2}} dx &= -\text{Subst}\left(\int \frac{c + dx}{x^2(a + bx)^{5/2}} dx, x, \frac{1}{x}\right) \\ &= \frac{cx}{a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{\left(-\frac{5bc}{2} + ad\right) \text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, \frac{1}{x}\right)}{a} \\ &= \frac{5bc - 2ad}{3a^2\left(a + \frac{b}{x}\right)^{3/2}} + \frac{cx}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(5bc - 2ad) \text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{2a^2} \\ &= \frac{5bc - 2ad}{3a^2\left(a + \frac{b}{x}\right)^{3/2}} + \frac{5bc - 2ad}{a^3\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(5bc - 2ad) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2a^3} \\ &= \frac{5bc - 2ad}{3a^2\left(a + \frac{b}{x}\right)^{3/2}} + \frac{5bc - 2ad}{a^3\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\left(a + \frac{b}{x}\right)^{3/2}} + \frac{(5bc - 2ad) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{a^3b} \\ &= \frac{5bc - 2ad}{3a^2\left(a + \frac{b}{x}\right)^{3/2}} + \frac{5bc - 2ad}{a^3\sqrt{a + \frac{b}{x}}} + \frac{cx}{a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{(5bc - 2ad) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} \end{aligned}$$

**Mathematica [C]** time = 0.04, size = 60, normalized size = 0.58

$$\frac{x \left( (5bc - 2ad) {}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b}{ax} + 1\right) + 3acx \right)}{3a^2 \sqrt{a + \frac{b}{x}} (ax + b)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d/x)/(a + b/x)^(5/2), x]

[Out] (x\*(3\*a\*c\*x + (5\*b\*c - 2\*a\*d)\*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a\*x)]))/(3\*a^2\*Sqrt[a + b/x]\*(b + a\*x))

**fricas [A]** time = 0.92, size = 331, normalized size = 3.21

$$\left[ \frac{3(5b^3c - 2ab^2d + (5a^2bc - 2a^3d)x^2 + 2(5ab^2c - 2a^2bd)x)\sqrt{a} \log\left(2ax + 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) - 2(3a^3cx^3)}{6(a^6x^2 + 2a^5bx + a^4b^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(a)*log(2*a*x + 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2), 1/3*(3*(5*b^3*c - 2*a*b^2*d + (5*a^2*b*c - 2*a^3*d)*x^2 + 2*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + (3*a^3*c*x^3 + 4*(5*a^2*b*c - 2*a^3*d)*x^2 + 3*(5*a*b^2*c - 2*a^2*b*d)*x)*sqrt((a*x + b)/x))/(a^6*x^2 + 2*a^5*b*x + a^4*b^2)]
```

**giac** [A] time = 0.22, size = 145, normalized size = 1.41

$$\frac{\frac{3b^2c\sqrt{\frac{ax+b}{x}}}{\left(a-\frac{ax+b}{x}\right)a^3} - \frac{3(5b^2c-2abd)\arctan\left(\frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3} - \frac{2\left(ab^2c-a^2bd+\frac{6(ax+b)b^2c}{x}-\frac{3(ax+b)abd}{x}\right)x}{(ax+b)a^3\sqrt{\frac{ax+b}{x}}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*c*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3) - 3*(5*b^2*c - 2*a*b*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3) - 2*(a*b^2*c - a^2*b*d + 6*(a*x + b)*b^2*c/x - 3*(a*x + b)*a*b*d/x)*x/((a*x + b)*a^3*sqrt((a*x + b)/x))/b
```

**maple** [B] time = 0.06, size = 541, normalized size = 5.25

$$\frac{\sqrt{\frac{ax+b}{x}} \left( 6a^4bdx^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) - 15a^3b^2cx^3 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) + 18a^3b^2dx^2 \ln\left(\frac{2ax+b+2\sqrt{(ax+b)x}\sqrt{a}}{2\sqrt{a}}\right) \right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c+d/x)/(a+b/x)^(5/2),x)
```

```
[Out] 1/6*((a*x+b)/x)^(1/2)*x/a^(7/2)*(6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^3*a^4*b*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^3*a^3*b^2*c-12*a^(9/2)*((a*x+b)*x)^(1/2)*x^3*d+30*a^(7/2)*((a*x+b)*x)^(1/2)*x^3*b*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^2*a^3*b^2*d-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x^2*a^2*b^3*c+12*a^(7/2)*((a*x+b)*x)^(3/2)*x*d-24*a^(5/2)*((a*x+b)*x)^(3/2)*x*b*c-36*a^(7/2)*((a*x+b)*x)^(1/2)*x^2*b*d+90*a^(5/2)*((a*x+b)*x)^(1/2)*x^2*b^2*c+18*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x*a^2*b^3*d-45*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*x*a*b^4*c+8*a^(5/2)*((a*x+b)*x)^(3/2)*b*d-20*a^(3/2)*((a*x+b)*x)^(3/2)*b^2*c-36*a^(5/2)*((a*x+b)*x)^(1/2)*x*b^2*d+90*a^(3/2)*((a*x+b)*x)^(1/2)*x*b^3*c+6*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*a*b^4*d-15*ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2)))/a^(1/2))*b^5*c-12*a^(3/2)*((a*x+b)*x)^(1/2)*b^3*d+30*a^(1/2)*((a*x+b)*x)^(1/2)*b^4*c)/((a*x+b)*x)^(1/2)/b/(a*x+b)^3
```

**maxima** [A] time = 1.25, size = 170, normalized size = 1.65

$$\frac{1}{6}c \left( \frac{2 \left( 15 \left( a + \frac{b}{x} \right)^2 b - 10 \left( a + \frac{b}{x} \right) ab - 2 a^2 b \right)}{\left( a + \frac{b}{x} \right)^{\frac{5}{2}} a^3 - \left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^4} + \frac{15 b \log \left( \frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}} \right)}{a^{\frac{7}{2}}} \right) - \frac{1}{3}d \left( \frac{3 \log \left( \frac{\sqrt{a+\frac{b}{x}}-\sqrt{a}}{\sqrt{a+\frac{b}{x}}+\sqrt{a}} \right)}{a^{\frac{5}{2}}} + \frac{2 \left( 4 a + \frac{3 b}{x} \right)}{\left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^2} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)^(5/2),x, algorithm="maxima")

[Out]  $\frac{1}{6}c*(2*(15*(a + b/x)^2*b - 10*(a + b/x)*a*b - 2*a^2*b)/((a + b/x)^{(5/2)}*a^3 - (a + b/x)^{(3/2)}*a^4) + 15*b*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(7/2)} - 1/3*d*(3*\log((\sqrt{a + b/x} - \sqrt{a})/(\sqrt{a + b/x} + \sqrt{a}))/a^{(5/2)} + 2*(4*a + 3*b/x)/((a + b/x)^{(3/2)}*a^2))$

**mupad [B]** time = 2.91, size = 87, normalized size = 0.84

$$\frac{2d \operatorname{atanh}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{5/2}} - \frac{\frac{2d}{3a} + \frac{2d\left(a+\frac{b}{x}\right)}{a^2}}{\left(a+\frac{b}{x}\right)^{3/2}} + \frac{2cx\left(\frac{ax}{b}+1\right)^{5/2} {}_2F_1\left(\frac{5}{2}, \frac{7}{2}; \frac{9}{2}; -\frac{ax}{b}\right)}{7\left(a+\frac{b}{x}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d/x)/(a + b/x)^(5/2),x)

[Out]  $(2*d*\operatorname{atanh}((a + b/x)^{(1/2)}/a^{(1/2)}))/a^{(5/2)} - ((2*d)/(3*a) + (2*d*(a + b/x))/a^2)/(a + b/x)^{(3/2)} + (2*c*x*((a*x)/b + 1)^{(5/2)}*\operatorname{hypergeom}([5/2, 7/2], 9/2, -(a*x)/b))/7*(a + b/x)^{(5/2)}$

**sympy [B]** time = 155.82, size = 1479, normalized size = 14.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)/(a+b/x)\*\*(5/2),x)

[Out]  $c*(6*a**17*x**4*\sqrt{1 + b/(a*x)})/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 46*a**16*b*x**3*\sqrt{1 + b/(a*x)})/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15*a**16*b*x**3*\log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**16*b*x**3*\log(\sqrt{1 + b/(a*x)} + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 70*a**15*b**2*x**2*\sqrt{1 + b/(a*x)})/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**15*b**2*x**2*\log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**15*b**2*x**2*\log(\sqrt{1 + b/(a*x)} + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 30*a**14*b**3*x*\sqrt{1 + b/(a*x)})/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 45*a**14*b**3*x*\log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 90*a**14*b**3*x*\log(\sqrt{1 + b/(a*x)} + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + 15*a**13*b**4*\log(b/(a*x))/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) - 30*a**13*b**4*\log(\sqrt{1 + b/(a*x)} + 1)/(6*a**(39/2)*x**3 + 18*a**(37/2)*b*x**2 + 18*a**(35/2)*b**2*x + 6*a**(33/2)*b**3) + d*(-8*a**7*x**3*\sqrt{1 + b/(a*x)})/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 3*a**7*x**3*\log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 6*a**7*x**3*\log(\sqrt{1 + b/(a*x)} + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 9*a**6*b*x**2*\log(b/(a*x))/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) + 18*a**6*b*x**2*\log(\sqrt{1 + b/(a*x)} + 1)/(3*a**(19/2)*x**3 + 9*a**(17/2)*b*x**2 + 9*a**(15/2)*b**2*x + 3*a**(13/2)*b**3) - 6*a**5*b**2*x*\sqrt{1 + b/(a*x)})/(3*a**(19/2)*$

$$\begin{aligned}
& x^3 + 9a^{17/2}bx^2 + 9a^{15/2}b^2x + 3a^{13/2}b^3 - 9a^5 \\
& b^2x \log(b/(ax)) / (3a^{19/2}x^3 + 9a^{17/2}bx^2 + 9a^{15/2}b^2x \\
& + 3a^{13/2}b^3) + 18a^5b^2x \log(\sqrt{1 + b/(ax)} + 1) / (3a^{19/2}x^3 \\
& + 9a^{17/2}bx^2 + 9a^{15/2}b^2x + 3a^{13/2}b^3) \\
& - 3a^4b^3 \log(b/(ax)) / (3a^{19/2}x^3 + 9a^{17/2}bx^2 + 9a^{15/2}b^2x \\
& + 3a^{13/2}b^3) + 6a^4b^3 \log(\sqrt{1 + b/(ax)} + 1) / (3a^{19/2}x^3 \\
& + 9a^{17/2}bx^2 + 9a^{15/2}b^2x + 3a^{13/2}b^3)
\end{aligned}$$

$$3.262 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2}} dx$$

**Optimal.** Leaf size=79

$$-\frac{5b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} + \frac{5b}{a^3 \sqrt{a+\frac{b}{x}}} + \frac{5b}{3a^2 \left(a+\frac{b}{x}\right)^{3/2}} + \frac{x}{a \left(a+\frac{b}{x}\right)^{3/2}}$$

[Out]  $5/3*b/a^2/(a+b/x)^{(3/2)}+x/a/(a+b/x)^{(3/2)}-5*b*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(7/2)}+5*b/a^3/(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 4, integrand size = 11,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$ , Rules used = {242, 51, 63, 208}

$$\frac{5x\sqrt{a+\frac{b}{x}}}{a^3} - \frac{10x}{3a^2\sqrt{a+\frac{b}{x}}} - \frac{5b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}} - \frac{2x}{3a\left(a+\frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x)^(-5/2), x]

[Out]  $(-2*x)/(3*a*(a + b/x)^{(3/2)}) - (10*x)/(3*a^2*\operatorname{Sqrt}[a + b/x]) + (5*\operatorname{Sqrt}[a + b/x]*x)/a^3 - (5*b*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/a^{(7/2)}$

#### Rule 51

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[ ((a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1))/((b\*c - a\*d)\*(m + 1)), x] - Dist[(d\*(m + n + 2))/((b\*c - a\*d)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 242

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2}} dx &= -\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{5/2}} dx, x, \frac{1}{x}\right) \\
&= -\frac{2x}{3a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{5\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}} dx, x, \frac{1}{x}\right)}{3a} \\
&= -\frac{2x}{3a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a + \frac{b}{x}}} - \frac{5\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{a^2} \\
&= -\frac{2x}{3a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a + \frac{b}{x}}} + \frac{5\sqrt{a + \frac{b}{x}}x}{a^3} + \frac{(5b)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{x}\right)}{2a^3} \\
&= -\frac{2x}{3a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a + \frac{b}{x}}} + \frac{5\sqrt{a + \frac{b}{x}}x}{a^3} + \frac{5\text{Subst}\left(\int \frac{1}{\frac{-a}{-b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{x}}\right)}{a^3} \\
&= -\frac{2x}{3a\left(a + \frac{b}{x}\right)^{3/2}} - \frac{10x}{3a^2\sqrt{a + \frac{b}{x}}} + \frac{5\sqrt{a + \frac{b}{x}}x}{a^3} - \frac{5b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}}
\end{aligned}$$

**Mathematica [C]** time = 0.02, size = 38, normalized size = 0.48

$$\frac{2b {}_2F_1\left(-\frac{3}{2}, 2; -\frac{1}{2}; \frac{a + \frac{b}{x}}{a}\right)}{3a^2\left(a + \frac{b}{x}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x)^(-5/2), x]

[Out] (2\*b\*Hypergeometric2F1[-3/2, 2, -1/2, (a + b/x)/a])/(3\*a^2\*(a + b/x)^(3/2))

**fricas [A]** time = 0.59, size = 225, normalized size = 2.85

$$\left[ \frac{15(a^2bx^2 + 2ab^2x + b^3)\sqrt{a} \log\left(2ax - 2\sqrt{a}x\sqrt{\frac{ax+b}{x}} + b\right) + 2(3a^3x^3 + 20a^2bx^2 + 15ab^2x)\sqrt{\frac{ax+b}{x}}}{6(a^6x^2 + 2a^5bx + a^4b^2)}, \frac{15(a^2bx^2 + 2ab^2x + b^3)\sqrt{a} \operatorname{arctan}\left(\sqrt{\frac{ax+b}{x}}\right)}{6(a^6x^2 + 2a^5bx + a^4b^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2), x, algorithm="fricas")

[Out] [1/6\*(15\*(a^2\*b\*x^2 + 2\*a\*b^2\*x + b^3)\*sqrt(a)\*log(2\*a\*x - 2\*sqrt(a)\*x\*sqrt((a\*x + b)/x) + b) + 2\*(3\*a^3\*x^3 + 20\*a^2\*b\*x^2 + 15\*a\*b^2\*x)\*sqrt((a\*x + b)/x))/(a^6\*x^2 + 2\*a^5\*b\*x + a^4\*b^2), 1/3\*(15\*(a^2\*b\*x^2 + 2\*a\*b^2\*x + b^3)\*sqrt(-a)\*arctan(sqrt(-a)\*sqrt((a\*x + b)/x)/a) + (3\*a^3\*x^3 + 20\*a^2\*b\*x^2 + 15\*a\*b^2\*x)\*sqrt((a\*x + b)/x))/(a^6\*x^2 + 2\*a^5\*b\*x + a^4\*b^2)]

**giac** [A] time = 0.24, size = 98, normalized size = 1.24

$$\frac{1}{3} b \left( \frac{2 \left( a + \frac{6(ax+b)}{x} \right) x}{(ax+b)a^3 \sqrt{\frac{ax+b}{x}}} + \frac{15 \arctan \left( \frac{\sqrt{\frac{ax+b}{x}}}{\sqrt{-a}} \right)}{\sqrt{-a} a^3} - \frac{3 \sqrt{\frac{ax+b}{x}}}{\left( a - \frac{ax+b}{x} \right) a^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2),x, algorithm="giac")

[Out] 1/3\*b\*(2\*(a + 6\*(a\*x + b)/x)\*x/((a\*x + b)\*a^3\*sqrt((a\*x + b)/x)) + 15\*arctan(sqrt((a\*x + b)/x)/sqrt(-a))/(sqrt(-a)\*a^3) - 3\*sqrt((a\*x + b)/x)/((a - (a\*x + b)/x)\*a^3)

**maple** [B] time = 0.07, size = 271, normalized size = 3.43

$$\sqrt{\frac{ax+b}{x}} \left( -15a^3 b x^3 \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) - 45a^2 b^2 x^2 \ln \left( \frac{2ax+b+2\sqrt{(ax+b)x} \sqrt{a}}{2\sqrt{a}} \right) + 30\sqrt{(ax+b)x} a^{\frac{7}{2}} x^3 - 45a b^3 x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(5/2),x)

[Out] 1/6\*((a\*x+b)/x)^(1/2)\*x\*(30\*a^(7/2)\*((a\*x+b)\*x)^(1/2)\*x^3-24\*a^(5/2)\*((a\*x+b)\*x)^(3/2)\*x+90\*a^(5/2)\*((a\*x+b)\*x)^(1/2)\*x^2\*b-15\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*x^3\*a^3\*b-20\*b\*a^(3/2)\*((a\*x+b)\*x)^(3/2)+90\*a^(3/2)\*((a\*x+b)\*x)^(1/2)\*x\*b^2-45\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*x^2\*a^2\*b^2-45\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*x\*a\*b^3+30\*a^(1/2)\*((a\*x+b)\*x)^(1/2)\*b^3-15\*ln(1/2\*(2\*a\*x+b+2\*((a\*x+b)\*x)^(1/2)\*a^(1/2)))/a^(1/2))\*b^4)/a^(7/2)/((a\*x+b)\*x)^(1/2)/(a\*x+b)^3

**maxima** [A] time = 1.29, size = 101, normalized size = 1.28

$$\frac{15 \left( a + \frac{b}{x} \right)^2 b - 10 \left( a + \frac{b}{x} \right) a b - 2 a^2 b}{3 \left( \left( a + \frac{b}{x} \right)^{\frac{5}{2}} a^3 - \left( a + \frac{b}{x} \right)^{\frac{3}{2}} a^4 \right)} + \frac{5 b \log \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{a + \frac{b}{x}} + \sqrt{a}} \right)}{2 a^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2),x, algorithm="maxima")

[Out] 1/3\*(15\*(a + b/x)^2\*b - 10\*(a + b/x)\*a\*b - 2\*a^2\*b)/((a + b/x)^(5/2)\*a^3 - (a + b/x)^(3/2)\*a^4) + 5/2\*b\*log((sqrt(a + b/x) - sqrt(a))/(sqrt(a + b/x) + sqrt(a)))/a^(7/2)

**mupad** [B] time = 1.72, size = 34, normalized size = 0.43

$$\frac{2x \left( \frac{ax}{b} + 1 \right)^{5/2} {}_2F_1 \left( \frac{5}{2}, \frac{7}{2}; \frac{9}{2}; -\frac{ax}{b} \right)}{7 \left( a + \frac{b}{x} \right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/x)^(5/2),x)

[Out]  $(2*x*((a*x)/b + 1)^{(5/2)}*hypergeom([5/2, 7/2], 9/2, -(a*x)/b))/(7*(a + b/x)^{(5/2)})$

**sympy [B]** time = 7.92, size = 774, normalized size = 9.80

$$\frac{6a^{17}x^4\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{46a^{16}bx^3\sqrt{1+\frac{b}{ax}}}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3} + \frac{15a^{16}bx^3\log\left(\frac{b}{ax}\right)}{6a^{\frac{39}{2}}x^3+18a^{\frac{37}{2}}bx^2+18a^{\frac{35}{2}}b^2x+6a^{\frac{33}{2}}b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2),x)

[Out]  $6*a^{17}*x^{14}*sqrt(1 + b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 46*a^{16}*b*x^{30}*sqrt(1 + b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 15*a^{16}*b*x^{30}*log(b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) - 30*a^{16}*b*x^{30}*log(sqrt(1 + b/(a*x)) + 1)/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 70*a^{15}*b^2*x^{30}*sqrt(1 + b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 45*a^{15}*b^2*x^{30}*log(b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) - 90*a^{15}*b^2*x^{30}*log(sqrt(1 + b/(a*x)) + 1)/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 30*a^{14}*b^3*x^{30}*sqrt(1 + b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 45*a^{14}*b^3*x^{30}*log(b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) - 90*a^{14}*b^3*x^{30}*log(sqrt(1 + b/(a*x)) + 1)/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) + 15*a^{13}*b^4*x^{30}*log(b/(a*x))/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30}) - 30*a^{13}*b^4*x^{30}*log(sqrt(1 + b/(a*x)) + 1)/(6*a^{39/2}*x^{33} + 18*a^{37/2}*b*x^{32} + 18*a^{35/2}*b^2*x^{31} + 6*a^{33/2}*b^3*x^{30})$

$$3.263 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)} dx$$

**Optimal.** Leaf size=201

$$\frac{(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2} + \frac{b(5bc - 3ad)}{3a^2c\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)} + \frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c\sqrt{a + \frac{b}{x}}(bc - ad)^2} - \frac{2d^{7/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{5/2}} + \dots$$

[Out]  $1/3*b*(-3*a*d+5*b*c)/a^2/c/(-a*d+b*c)/(a+b/x)^{(3/2)}+x/a/c/(a+b/x)^{(3/2)}-2*d^{(7/2)}*\arctan(d^{(1/2)}*(a+b/x)^{(1/2)/(-a*d+b*c)^{(1/2)})/c^2/(-a*d+b*c)^{(5/2)}-(2*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)/a^{(1/2)})/a^{(7/2)}/c^2+b*(a^2*d^2-8*a*b*c*d+5*b^2*c^2)/a^3/c/(-a*d+b*c)^2/(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$ , Rules used = {375, 103, 152, 156, 63, 208, 205}

$$\frac{b(a^2d^2 - 8abcd + 5b^2c^2)}{a^3c\sqrt{a + \frac{b}{x}}(bc - ad)^2} - \frac{(2ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^2} + \frac{b(5bc - 3ad)}{3a^2c\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)} - \frac{2d^{7/2} \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right)}{c^2(bc - ad)^{5/2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)), x]

[Out]  $(b*(5*b*c - 3*a*d))/(3*a^2*c*(b*c - a*d)*(a + b/x)^{(3/2)}) + (b*(5*b^2*c^2 - 8*a*b*c*d + a^2*d^2))/(a^3*c*(b*c - a*d)^2*\operatorname{Sqrt}[a + b/x]) + x/(a*c*(a + b/x)^{(3/2)}) - (2*d^{(7/2)}*\operatorname{ArcTan}[(\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x])/(\operatorname{Sqrt}[b*c - a*d])]/(c^2*(b*c - a*d)^{(5/2)}) - ((5*b*c + 2*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(a^{(7/2)}*c^2)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 152

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d

```
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]
```

### Rule 156

```
Int[(((e_.) + (f_.)*(x_))^(p_))*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

### Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_))*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)} dx &= -\text{Subst} \left( \int \frac{1}{x^2(a + bx)^{5/2}(c + dx)} dx, x, \frac{1}{x} \right) \\
&= \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(5bc+2ad) + \frac{5bdx}{2}}{x(a+bx)^{5/2}(c+dx)} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{b(5bc - 3ad)}{3a^2c(bc - ad) \left(a + \frac{b}{x}\right)^{3/2}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} + \frac{2 \text{Subst} \left( \int \frac{\frac{3}{4}(bc-ad)(5bc+2ad) + \frac{3}{4}bd(5bc-3ad)x}{x(a+bx)^{3/2}(c+dx)} \right)}{3a^2c(bc - ad)} \\
&= \frac{b(5bc - 3ad)}{3a^2c(bc - ad) \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2 - 8abcd + a^2d^2)}{a^3c(bc - ad)^2 \sqrt{a + \frac{b}{x}}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} + \frac{4 \text{Subst} \left( \int \right)}{c^2} \\
&= \frac{b(5bc - 3ad)}{3a^2c(bc - ad) \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2 - 8abcd + a^2d^2)}{a^3c(bc - ad)^2 \sqrt{a + \frac{b}{x}}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} - \frac{d^4 \text{Subst} \left( \int \right)}{c^2} \\
&= \frac{b(5bc - 3ad)}{3a^2c(bc - ad) \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2 - 8abcd + a^2d^2)}{a^3c(bc - ad)^2 \sqrt{a + \frac{b}{x}}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} - \frac{(2d^4) \text{Subst} \left( \int \right)}{c^2} \\
&= \frac{b(5bc - 3ad)}{3a^2c(bc - ad) \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(5b^2c^2 - 8abcd + a^2d^2)}{a^3c(bc - ad)^2 \sqrt{a + \frac{b}{x}}} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2}} - \frac{2d^{7/2} \tan^{-1} \left( \right)}{c^2(bc - ad)}
\end{aligned}$$

**Mathematica [C]** time = 0.08, size = 118, normalized size = 0.59

$$\frac{x \left( (ad - bc) \left( (2ad + 5bc) {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; \frac{b}{ax} + 1 \right) + 3acx \right) - 2a^2d^2 {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; \frac{d \left( a + \frac{b}{x} \right)}{ad - bc} \right) \right)}{3a^2c^2 \sqrt{a + \frac{b}{x}} (ax + b)(ad - bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(5/2)\*(c + d/x)),x]

[Out] (x\*(-2\*a^2\*d^2\*Hypergeometric2F1[-3/2, 1, -1/2, (d\*(a + b/x))/(-(b\*c) + a\*d)] + (-(b\*c) + a\*d)\*(3\*a\*c\*x + (5\*b\*c + 2\*a\*d)\*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a\*x)])))/(3\*a^2\*c^2\*(-(b\*c) + a\*d)\*Sqrt[a + b/x]\*(b + a\*x))

**fricas [B]** time = 5.60, size = 1990, normalized size = 9.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="fricas")

[Out] [1/6\*(3\*(5\*b^5\*c^3 - 8\*a\*b^4\*c^2\*d + a^2\*b^3\*c\*d^2 + 2\*a^3\*b^2\*d^3 + (5\*a^2\*b^3\*c^3 - 8\*a^3\*b^2\*c^2\*d + a^4\*b\*c\*d^2 + 2\*a^5\*d^3)\*x^2 + 2\*(5\*a\*b^4\*c^3

```

- 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) + 6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x), 1/3*(3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) + 3*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(-d/(b*c - a*d))*log(-(2*(b*c - a*d)*x*sqrt(-d/(b*c - a*d))*sqrt((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x), -1/6*(12*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(a)*log(2*a*x - 2*sqrt(a)*x*sqrt((a*x + b)/x) + b) - 2*(3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x), -1/3*(6*(a^6*d^3*x^2 + 2*a^5*b*d^3*x + a^4*b^2*d^3)*sqrt(d/(b*c - a*d))*arctan(-(b*c - a*d)*x*sqrt(d/(b*c - a*d))*sqrt((a*x + b)/x)/(a*d*x + b*d)) - 3*(5*b^5*c^3 - 8*a*b^4*c^2*d + a^2*b^3*c*d^2 + 2*a^3*b^2*d^3 + (5*a^2*b^3*c^3 - 8*a^3*b^2*c^2*d + a^4*b*c*d^2 + 2*a^5*d^3)*x^2 + 2*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2 + 2*a^4*b*d^3)*x)*sqrt(-a)*arctan(sqrt(-a)*sqrt((a*x + b)/x)/a) - (3*(a^3*b^2*c^3 - 2*a^4*b*c^2*d + a^5*c*d^2)*x^3 + 2*(10*a^2*b^3*c^3 - 16*a^3*b^2*c^2*d + 3*a^4*b*c*d^2)*x^2 + 3*(5*a*b^4*c^3 - 8*a^2*b^3*c^2*d + a^3*b^2*c*d^2)*x)*sqrt((a*x + b)/x))/(a^4*b^4*c^4 - 2*a^5*b^3*c^3*d + a^6*b^2*c^2*d^2 + (a^6*b^2*c^4 - 2*a^7*b*c^3*d + a^8*c^2*d^2)*x^2 + 2*(a^5*b^3*c^4 - 2*a^6*b^2*c^3*d + a^7*b*c^2*d^2)*x)]

```

**giac** [A] time = 0.22, size = 247, normalized size = 1.23

$$-\frac{1}{3} \left( \frac{6d^4 \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^4c^4 - 2ab^3c^3d + a^2b^2c^2d^2)\sqrt{bcd-ad^2}} - \frac{2\left(abc - a^2d + \frac{6(ax+b)bc}{x} - \frac{9(ax+b)ad}{x}\right)x}{(a^3b^2c^2 - 2a^4bcd + a^5d^2)(ax+b)\sqrt{\frac{ax+b}{x}}} + \frac{3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3bc} - \frac{3\sqrt{\frac{ax+b}{x}}}{\left(a - \frac{ax+b}{x}\right)a^3bc} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x),x, algorithm="giac")

```

[Out] -1/3*(6*d^4*arctan(d*sqrt((a*x + b)/x)/sqrt(b*c*d - a*d^2))/((b^4*c^4 - 2*a*b^3*c^3*d + a^2*b^2*c^2*d^2)*sqrt(b*c*d - a*d^2)) - 2*(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 9*(a*x + b)*a*d/x)*x/((a^3*b^2*c^2 - 2*a^4*b*c*d + a^5*d^2)*(a*x + b)*sqrt((a*x + b)/x)) + 3*sqrt((a*x + b)/x)/((a - (a*x + b)/x)*a^3*b*c) - 3*(5*b*c + 2*a*d)*arctan(sqrt((a*x + b)/x)/sqrt(-a))/(sqrt(-a)*a^3*b^2*c^2))*b^2

```

**maple** [B] time = 0.07, size = 1767, normalized size = 8.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a+b/x)^(5/2)/(c+d/x), x)`

[Out] 
$$-1/6*(15*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*b^6*c^4+6*a^(13/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2))*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^3*d^4+6*a^(7/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2))*((a*x+b)*x)^(1/2)*c)/(c*x+d))*b^3*d^4+3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^2*b^4*c^2*d^2+45*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^2*b^4*c^4+6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^6*c*d^3+15*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^3*b^3*c^4-30*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b^2*c^4+24*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^4-90*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^3*c^4-32*a^(5/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^2*c^3*d-90*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^4*c^4-6*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^2*d^2+48*a^(3/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^4*c^3*d-6*a^(11/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*c^2*d^2+45*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^b^5*c^4+6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a^3*b^3*c*d^3-24*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*a*b^5*c^3*d+144*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b^2*c^3*d-18*a^(7/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^2*c^2*d^2+144*a^(5/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b^3*c^3*d-18*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^2*b*c^2*d^2+48*a^(9/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*x^3*b*c^3*d-36*a^(7/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*x*b*c^3*d+3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^5*b*c^2*d^2-24*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^3*a^4*b^2*c^3*d+18*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^5*b*c*d^3+9*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^4*b^2*c^2*d^2-72*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x^2*a^3*b^3*c^3*d+18*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^4*b^2*c*d^3+9*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^(1/2)*a^(1/2))/a^(1/2))*((a*d-b*c)/c^2*d)^(1/2)*x*a^2*b^4*c^3*d+18*a^(11/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2))*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x^2*b*d^4-30*a^(1/2)*((a*x+b)*x)^(1/2)*((a*d-b*c)/c^2*d)^(1/2)*b^5*c^4+20*a^(3/2)*((a*x+b)*x)^(3/2)*((a*d-b*c)/c^2*d)^(1/2)*b^3*c^4+18*a^(9/2)*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2))*((a*x+b)*x)^(1/2)*c)/(c*x+d))*x*b^2*d^4)*x*((a*x+b)/x)^(1/2)/a^(7/2)/(a*x+b)^3/((a*d-b*c)/c^2*d)^(1/2)/c^3/(a*d-b*c)^2/((a*x+b)*x)^(1/2)$$

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} \left(c + \frac{d}{x}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b/x)^(5/2)/(c+d/x), x, algorithm="maxima")`

[Out] `integrate(1/((a + b/x)^(5/2)*(c + d/x)), x)`

**mupad** [B] time = 4.62, size = 5387, normalized size = 26.80

result too large to display





```

c^9*d^10 - 228*a^21*b^5*c^8*d^11 + 4*a^22*b^4*c^7*d^12 + 4*a^23*b^3*c^6*d^1
3 + ((d^7*(a*d - b*c)^5)^(1/2)*(a + b/x)^(1/2)*(8*a^15*b^13*c^18*d^2 - 96*a
^16*b^12*c^17*d^3 + 520*a^17*b^11*c^16*d^4 - 1680*a^18*b^10*c^15*d^5 + 3600
*a^19*b^9*c^14*d^6 - 5376*a^20*b^8*c^13*d^7 + 5712*a^21*b^7*c^12*d^8 - 4320
*a^22*b^6*c^11*d^9 + 2280*a^23*b^5*c^10*d^10 - 800*a^24*b^4*c^9*d^11 + 168*
a^25*b^3*c^8*d^12 - 16*a^26*b^2*c^7*d^13))/(c^2*(a*d - b*c)^5)))/(c^2*(a*d
- b*c)^5)))/(c^2*(a*d - b*c)^5) + 100*a^9*b^12*c^11*d^6 - 720*a^10*b^11*c^1
0*d^7 + 2176*a^11*b^10*c^9*d^8 - 3528*a^12*b^9*c^8*d^9 + 3192*a^13*b^8*c^7*
d^10 - 1400*a^14*b^7*c^6*d^11 + 264*a^16*b^5*c^4*d^13 - 92*a^17*b^4*c^3*d^1
4 + 8*a^18*b^3*c^2*d^15))*(d^7*(a*d - b*c)^5)^(1/2)*2i)/(c^2*(a*d - b*c)^5)

```

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} (cx + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x), x)

[Out] Integral(x/((a + b/x)\*\*(5/2)\*(c\*x + d)), x)

$$3.264 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2} dx$$

**Optimal.** Leaf size=287

$$\frac{(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} + \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^2} + \frac{b(bc - 2ad)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2\sqrt{a + \frac{b}{x}}(bc - ad)^3} - \frac{d^{7/2}(9bc - 4ad)}{c^3(bc - ad)}$$

[Out]  $1/3*b*(6*a^2*d^2-6*a*b*c*d+5*b^2*c^2)/a^2/c^2/(-a*d+b*c)^2/(a+b/x)^(3/2)+d*(-2*a*d+b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^(3/2)/(c+d/x)+x/a/c/(a+b/x)^(3/2)/(c+d/x)-d^(7/2)*(-4*a*d+9*b*c)*\arctan(d^(1/2)*(a+b/x)^(1/2)/(-a*d+b*c)^(1/2))/c^3/(-a*d+b*c)^(7/2)-(4*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^(1/2)/a^(1/2))/a^(7/2)/c^3+b*(-2*a*d+b*c)*(a^2*d^2-a*b*c*d+5*b^2*c^2)/a^3/c^2/(-a*d+b*c)^3/(a+b/x)^(1/2)$

**Rubi [A]** time = 0.45, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{b(bc - 2ad)(a^2d^2 - abcd + 5b^2c^2)}{a^3c^2\sqrt{a + \frac{b}{x}}(bc - ad)^3} + \frac{b(6a^2d^2 - 6abcd + 5b^2c^2)}{3a^2c^2\left(a + \frac{b}{x}\right)^{3/2}(bc - ad)^2} - \frac{(4ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right)}{a^{7/2}c^3} - \frac{d^{7/2}(9bc - 4ad)}{c^3(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)^2), x]

[Out]  $(b*(5*b^2*c^2 - 6*a*b*c*d + 6*a^2*d^2))/(3*a^2*c^2*(b*c - a*d)^2*(a + b/x)^(3/2)) + (b*(b*c - 2*a*d)*(5*b^2*c^2 - a*b*c*d + a^2*d^2))/(a^3*c^2*(b*c - a*d)^3*\operatorname{Sqrt}[a + b/x]) + (d*(b*c - 2*a*d))/(a*c^2*(b*c - a*d)*(a + b/x)^(3/2)*(c + d/x)) + x/(a*c*(a + b/x)^(3/2)*(c + d/x)) - (d^(7/2)*(9*b*c - 4*a*d)*\operatorname{ArcTan}[\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[b*c - a*d]])/(c^3*(b*c - a*d)^(7/2)) - ((5*b*c + 4*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(a^(7/2)*c^3)$

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m +

```

1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^(n)*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
erQ[m]

```

### Rule 152

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^(n)*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
- a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && Integ
ersQ[2*m, 2*n, 2*p]

```

### Rule 156

```

Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_Symbol] := Dist[(b*g - a*h)/(b*c - a*d), Int[(e +
f*x)^p/(a + b*x), x], x] - Dist[(d*g - c*h)/(b*c - a*d), Int[(e + f*x)^p/(c
+ d*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

```

### Rule 205

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

```

### Rule 208

```

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

```

### Rule 375

```

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol
] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a,
b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]

```

### Rubi steps



$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^2} dx &= -\text{Subst} \left( \int \frac{1}{x^2(a+bx)^{5/2}(c+dx)^2} dx, x, \frac{1}{x} \right) \\
&= \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(5bc+4ad) + \frac{7bdx}{2}}{x(a+bx)^{5/2}(c+dx)^2} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{d(bc-2ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} - \frac{\text{Subst} \left( \int \frac{-\frac{1}{2}(bc-ad)(5bc+4ad)}{x(a+bx)^{5/2}} dx, x, \frac{1}{x} \right)}{ac^2(bc-ad)} \\
&= \frac{b(5b^2c^2 - 6abcd + 6a^2d^2)}{3a^2c^2(bc-ad)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{d(bc-2ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} \\
&= \frac{b(5b^2c^2 - 6abcd + 6a^2d^2)}{3a^2c^2(bc-ad)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2 - abcd + a^2d^2)}{a^3c^2(bc-ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(bc-ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)} \\
&= \frac{b(5b^2c^2 - 6abcd + 6a^2d^2)}{3a^2c^2(bc-ad)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2 - abcd + a^2d^2)}{a^3c^2(bc-ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(bc-ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)} \\
&= \frac{b(5b^2c^2 - 6abcd + 6a^2d^2)}{3a^2c^2(bc-ad)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2 - abcd + a^2d^2)}{a^3c^2(bc-ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(bc-ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)} \\
&= \frac{b(5b^2c^2 - 6abcd + 6a^2d^2)}{3a^2c^2(bc-ad)^2 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(bc-2ad)(5b^2c^2 - abcd + a^2d^2)}{a^3c^2(bc-ad)^3 \sqrt{a + \frac{b}{x}}} + \frac{d(bc-ad)}{ac^2(bc-ad) \left(a + \frac{b}{x}\right)}
\end{aligned}$$

**Mathematica [C]** time = 0.19, size = 178, normalized size = 0.62

$$\frac{x \left( (ad-bc) \left( 3acx(ad(cx+2d) - bc(cx+d)) - (cx+d) (-4a^2d^2 - abcd + 5b^2c^2) {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; \frac{b}{ax} + 1 \right) \right) + a^2d \right)}{3a^2c^3 \sqrt{a + \frac{b}{x}} (ax+b)(cx+d)(bc-ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(5/2)\*(c + d/x)^2), x]

[Out] (x\*(a^2\*d^2\*(9\*b\*c - 4\*a\*d)\*(d + c\*x)\*Hypergeometric2F1[-3/2, 1, -1/2, (d\*(a + b/x))/(-b\*c) + a\*d]) + (-b\*c) + a\*d)\*(3\*a\*c\*x\*(-b\*c\*(d + c\*x)) + a\*d\*(2\*d + c\*x)) - (5\*b^2\*c^2 - a\*b\*c\*d - 4\*a^2\*d^2)\*(d + c\*x)\*Hypergeometric2F1[-3/2, 1, -1/2, 1 + b/(a\*x)])))/(3\*a^2\*c^3\*(b\*c - a\*d)^2\*Sqrt[a + b/x]\*(b + a\*x)\*(d + c\*x))

**fricas [B]** time = 5.60, size = 3887, normalized size = 13.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/6*(3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 \\ & - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 \\ & + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5 \\ & *a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b \\ & ^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2 \\ & *c*d^4 - 8*a^5*b*d^5)*x)*\sqrt{a}*\log(2*a*x - 2*\sqrt{a}*x*\sqrt{(a*x + b)/x} \\ & + b) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a^6*b*c^2*d^3 - 4*a^7*c*d^4) \\ & *x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7*d^5)*x^2 + (9*a^4*b^3*c^2* \\ & d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*\sqrt{-d/(b*c - a*d)}*\log(-(2*(b*c \\ & - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} - b*d + (b*c - 2*a*d)*x)/(c \\ & *x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a^6*c^2* \\ & d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + 3*a^5*b \\ & *c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35*a^3*b^3 \\ & *c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4*d - 11 \\ & *a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*\sqrt{(a*x + b)/x} \\ & )/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2*c^3*d^4 \\ & + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x^3 + (2 \\ & *a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - a^9*c^ \\ & 3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^7*b^2*c \\ & ^4*d^3 - 2*a^8*b*c^3*d^4)*x), -1/6*(6*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9 \\ & *a^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a \\ & ^7*d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*\sqrt{ \\ & d/(b*c - a*d)}*\arctan(-(b*c - a*d)*x*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x}/ \\ & (a*d*x + b*d)) - 3*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 + 3*a^2*b^4*c^2*d^3 + 7* \\ & a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 11*a^3*b^3*c^4*d + 3*a^4*b \\ & ^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + (10*a*b^5*c^5 - 17*a^2*b^ \\ & 4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - a^5*b*c*d^4 - 4*a^6*d^5) \\ & *x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d^2 + 13*a^3*b^3*c^2*d^3 + \\ & 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*\sqrt{a}*\log(2*a*x - 2*\sqrt{a}*x*\sqrt{(a \\ & *x + b)/x} + b) - 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + 3*a^5*b*c^3*d^2 - a \\ & ^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + 9*a^4*b^2*c^3*d^2 + \\ & 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13*a^2*b^4*c^4*d - 35* \\ & a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)*x^2 + 3*(5*a*b^5*c^4 \\ & *d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2*c*d^4)*x)*\sqrt{(a*x \\ & + b)/x} )/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^3*c^4*d^3 - a^7*b^2* \\ & c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5*d^2 - a^9*c^4*d^3)*x \\ & ^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d^2 + a^8*b*c^4*d^3 - \\ & a^9*c^3*d^4)*x^2 + (a^4*b^5*c^7 - a^5*b^4*c^6*d - 3*a^6*b^3*c^5*d^2 + 5*a^ \\ & 7*b^2*c^4*d^3 - 2*a^8*b*c^3*d^4)*x), 1/6*(6*(5*b^6*c^4*d - 11*a*b^5*c^3*d^2 \\ & + 3*a^2*b^4*c^2*d^3 + 7*a^3*b^3*c*d^4 - 4*a^4*b^2*d^5 + (5*a^2*b^4*c^5 - 1 \\ & 1*a^3*b^3*c^4*d + 3*a^4*b^2*c^3*d^2 + 7*a^5*b*c^2*d^3 - 4*a^6*c*d^4)*x^3 + \\ & (10*a*b^5*c^5 - 17*a^2*b^4*c^4*d - 5*a^3*b^3*c^3*d^2 + 17*a^4*b^2*c^2*d^3 - \\ & a^5*b*c*d^4 - 4*a^6*d^5)*x^2 + (5*b^6*c^5 - a*b^5*c^4*d - 19*a^2*b^4*c^3*d \\ & ^2 + 13*a^3*b^3*c^2*d^3 + 10*a^4*b^2*c*d^4 - 8*a^5*b*d^5)*x)*\sqrt{-a}*\arcta \\ & n(\sqrt{-a}*\sqrt{(a*x + b)/x}/a) + 3*(9*a^4*b^3*c*d^4 - 4*a^5*b^2*d^5 + (9*a \\ & ^6*b*c^2*d^3 - 4*a^7*c*d^4)*x^3 + (18*a^5*b^2*c^2*d^3 + a^6*b*c*d^4 - 4*a^7 \\ & *d^5)*x^2 + (9*a^4*b^3*c^2*d^3 + 14*a^5*b^2*c*d^4 - 8*a^6*b*d^5)*x)*\sqrt{-d \\ & /(b*c - a*d)}*\log(-(2*(b*c - a*d)*x*\sqrt{-d/(b*c - a*d)}*\sqrt{(a*x + b)/x} \\ & - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(3*(a^3*b^3*c^5 - 3*a^4*b^2*c^4*d + \\ & 3*a^5*b*c^3*d^2 - a^6*c^2*d^3)*x^4 + (20*a^2*b^4*c^5 - 41*a^3*b^3*c^4*d + \\ & 9*a^4*b^2*c^3*d^2 + 3*a^5*b*c^2*d^3 - 6*a^6*c*d^4)*x^3 + (15*a*b^5*c^5 - 13 \\ & *a^2*b^4*c^4*d - 35*a^3*b^3*c^3*d^2 + 15*a^4*b^2*c^2*d^3 - 12*a^5*b*c*d^4)* \\ & x^2 + 3*(5*a*b^5*c^4*d - 11*a^2*b^4*c^3*d^2 + 3*a^3*b^3*c^2*d^3 - 2*a^4*b^2 \\ & *c*d^4)*x)*\sqrt{(a*x + b)/x} )/(a^4*b^5*c^6*d - 3*a^5*b^4*c^5*d^2 + 3*a^6*b^ \\ & 3*c^4*d^3 - a^7*b^2*c^3*d^4 + (a^6*b^3*c^7 - 3*a^7*b^2*c^6*d + 3*a^8*b*c^5* \\ & d^2 - a^9*c^4*d^3)*x^3 + (2*a^5*b^4*c^7 - 5*a^6*b^3*c^6*d + 3*a^7*b^2*c^5*d \end{aligned}$$

$$\begin{aligned} &^2 + a^8 b^3 c^4 d^3 - a^9 c^3 d^4) x^2 + (a^4 b^5 c^7 - a^5 b^4 c^6 d - 3 a^6 b^3 c^5 d^2 + 5 a^7 b^2 c^4 d^3 - 2 a^8 b^3 c^3 d^4) x, \\ &-1/3 * (3 * (9 a^4 b^3 c^3 d^4 - 4 a^5 b^2 c^2 d^5 + (9 a^6 b^2 c^2 d^3 - 4 a^7 c^2 d^4) x^3 + (18 a^5 b^2 c^2 d^3 + a^6 b^3 c^2 d^4 - 4 a^7 d^5) x^2 + (9 a^4 b^3 c^2 d^3 + 14 a^5 b^2 c^2 d^4 - 8 a^6 b^3 d^5) x) * \sqrt{d/(b*c - a*d)} * \arctan(-(b*c - a*d) * x * \sqrt{d/(b*c - a*d)}) * \sqrt{(a*x + b)/x} / (a*d*x + b*d)) - 3 * (5 b^6 c^4 d - 11 a b^5 c^3 d^2 + 3 a^2 b^4 c^2 d^3 + 7 a^3 b^3 c^2 d^4 - 4 a^4 b^2 c^2 d^5 + (5 a^2 b^4 c^5 - 11 a^3 b^3 c^4 d + 3 a^4 b^2 c^3 d^2 + 7 a^5 b^2 c^2 d^3 - 4 a^6 c^2 d^4) x^3 + (10 a b^5 c^5 - 17 a^2 b^4 c^4 d - 5 a^3 b^3 c^3 d^2 + 17 a^4 b^2 c^2 d^3 - a^5 b^3 c^2 d^4 - 4 a^6 d^5) x^2 + (5 b^6 c^5 - a b^5 c^4 d - 19 a^2 b^4 c^3 d^2 + 13 a^3 b^3 c^2 d^3 + 10 a^4 b^2 c^2 d^4 - 8 a^5 b^3 d^5) x) * \sqrt{-a} * \arctan(\sqrt{-a} * \sqrt{(a*x + b)/x} / a) - (3 * (a^3 b^3 c^5 - 3 a^4 b^2 c^4 d + 3 a^5 b^2 c^3 d^2 - a^6 c^2 d^3) x^4 + (20 a^2 b^4 c^5 - 41 a^3 b^3 c^4 d + 9 a^4 b^2 c^3 d^2 + 3 a^5 b^2 c^2 d^3 - 6 a^6 c^2 d^4) x^3 + (15 a b^5 c^5 - 13 a^2 b^4 c^4 d - 35 a^3 b^3 c^3 d^2 + 15 a^4 b^2 c^2 d^3 - 12 a^5 b^3 c^2 d^4) x^2 + 3 * (5 a b^5 c^4 d - 11 a^2 b^4 c^3 d^2 + 3 a^3 b^3 c^2 d^3 - 2 a^4 b^2 c^2 d^4) x) * \sqrt{(a*x + b)/x}) / (a^4 b^5 c^6 d - 3 a^5 b^4 c^5 d^2 + 3 a^6 b^3 c^4 d^3 - a^7 b^2 c^3 d^4 + (a^6 b^3 c^7 - 3 a^7 b^2 c^6 d + 3 a^8 b^2 c^5 d^2 - a^9 c^4 d^3) x^3 + (2 a^5 b^4 c^7 - 5 a^6 b^3 c^6 d + 3 a^7 b^2 c^5 d^2 + a^8 b^2 c^4 d^3 - a^9 c^3 d^4) x^2 + (a^4 b^5 c^7 - a^5 b^4 c^6 d - 3 a^6 b^3 c^5 d^2 + 5 a^7 b^2 c^4 d^3 - 2 a^8 b^2 c^3 d^4) x) ] \end{aligned}$$

**giac [B]** time = 0.33, size = 576, normalized size = 2.01

$$-\frac{1}{3} b^3 \left( \frac{3 (9 b c d^4 - 4 a d^5) \arctan\left(\frac{d \sqrt{\frac{a x + b}{x}}}{\sqrt{b c d - a d^2}}\right)}{(b^6 c^6 - 3 a b^5 c^5 d + 3 a^2 b^4 c^4 d^2 - a^3 b^3 c^3 d^3) \sqrt{b c d - a d^2}} - \frac{2 \left( a b c - a^2 d + \frac{6 (a x + b) b c}{x} - \frac{12 (a x + b) a d}{x} \right) x}{(a^3 b^3 c^3 - 3 a^4 b^2 c^2 d + 3 a^5 b c d^2 - a^6 d^3) (a x + b)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="giac")

[Out] 
$$-1/3 * b^3 * (3 * (9 b^3 c^3 d^4 - 4 a^4 d^5) * \arctan(d * \sqrt{(a*x + b)/x} / \sqrt{b*c*d - a*d^2}) / ((b^6*c^6 - 3*a*b^5*c^5*d + 3*a^2*b^4*c^4*d^2 - a^3*b^3*c^3*d^3) * \sqrt{b*c*d - a*d^2}) - 2 * (a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 12*(a*x + b)*a*d/x) * x / ((a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3) * (a*x + b) * \sqrt{(a*x + b)/x}) + 3 * (b^4*c^4 * \sqrt{(a*x + b)/x} - 4*a*b^3*c^3*d * \sqrt{(a*x + b)/x} + 6*a^2*b^2*c^2*d^2 * \sqrt{(a*x + b)/x} - 4*a^3*b*c*d^3 * \sqrt{(a*x + b)/x} + 2*a^4*d^4 * \sqrt{(a*x + b)/x} + (a*x + b)*b^3*c^3*d * \sqrt{(a*x + b)/x} / x - 3*(a*x + b)*a*b^2*c^2*d^2 * \sqrt{(a*x + b)/x} / x + 3*(a*x + b)*a^2*b*c*d^3 * \sqrt{(a*x + b)/x} / x - 2*(a*x + b)*a^3*d^4 * \sqrt{(a*x + b)/x} / x) / ((a^3*b^5*c^5 - 3*a^4*b^4*c^4*d + 3*a^5*b^3*c^3*d^2 - a^6*b^2*c^2*d^3) * (a*b*c - a^2*d - (a*x + b)*b*c/x + 2*(a*x + b)*a*d/x - (a*x + b)^2*d/x^2)) - 3 * (5*b*c + 4*a*d) * \arctan(\sqrt{(a*x + b)/x} / \sqrt{-a}) / (\sqrt{-a} * a^3 * b^3 * c^3)$$

**maple [B]** time = 0.08, size = 4644, normalized size = 16.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(5/2)/(c+d/x)^2,x)

[Out] 
$$-1/6 * ((a*x+b)/x)^(1/2) * x * (12*a^(19/2) * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)) * ((a*x+b)*x)^(1/2) * c) / (c*x+d)) * x^3 * d^7 + 12*a^(13/2) * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^(1/2)) * ((a*x+b)*x)^(1/2) * c) / (c*x+d)) * b^3 * d^7 + 30*a^(9/2) * ((a*x+b)*x)^(1/2) * ((a*d-b*c)/c^2*d)^(1/2) * b^4 * c^3 * d^4 - 84*a^(7/2) * ((a*x+b)*x)^(1/2) * ((a*d-b*c)/c^2*d)^(1/2) * b^5 * c^4 * d^3 + 96*a^(5/2) * ((a*x+b)*x)^(1/2) * ((a*d-b*c)/c^2*d)^(1/2) * b^6 * c^5 * d^2 + 12*a^6 * \ln(1/2 * (2*a*x+b+2*((a*x+$$

$$\begin{aligned}
& b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^3 * c * d^6 - 33 * a^5 * \ln(1 \\
& / 2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * b \\
& ^4 * c^2 * d^5 + 12 * a^4 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a \\
& * d - b * c) / c^2 * d)^{(1/2)} * b^5 * c^3 * d^4 - 6 * a^{(17/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^ \\
& ^2 * d)^{(1/2)} * x^4 * c^3 * d^4 - 30 * a^{(9/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} \\
& * x^4 * b^4 * c^7 - 39 * a^{(17/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ( \\
& (a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^4 * b * c^2 * d^5 + 27 * a^{(15/2)} * \ln((-2 * a * d * x + b * c * x - b \\
& * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^4 * b^2 * c^3 * d^4 + \\
& 12 * a^9 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 \\
& * d)^{(1/2)} * x^4 * c^2 * d^5 + 15 * a^4 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a \\
& ^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b^5 * c^7 + 24 * a^{(7/2)} * ((a * x + b) * x)^{(3/2)} * (( \\
& a * d - b * c) / c^2 * d)^{(1/2)} * x^2 * b^4 * c^7 - 12 * a^{(17/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / \\
& c^2 * d)^{(1/2)} * x^3 * c^2 * d^5 - 90 * a^{(7/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} \\
& * x^3 * b^5 * c^7 - 3 * a^{(17/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ( \\
& (a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^3 * b * c * d^6 - 90 * a^{(15/2)} * \ln((-2 * a * d * x + b * c * x - b * \\
& d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^3 * b^2 * c^2 * d^5 + 8 \\
& 1 * a^{(13/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/ \\
& 2)} * c) / (c * x + d)) * x^3 * b^3 * c^3 * d^4 + 12 * a^9 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a \\
& ^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^3 * c * d^6 + 45 * a^3 * \ln(1/2 * (2 * a * x + b + 2 \\
& * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^3 * b^6 * c^7 + 20 \\
& * a^{(5/2)} * ((a * x + b) * x)^{(3/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * x * b^5 * c^7 - 90 * a^{(5/2)} * ((a \\
& * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^2 * b^6 * c^7 - 81 * a^{(15/2)} * \ln((-2 * a * d * x \\
& + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^2 * b^2 * \\
& c * d^6 - 36 * a^{(13/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) \\
& * x)^{(1/2)} * c) / (c * x + d)) * x^2 * b^3 * c^2 * d^5 + 81 * a^{(11/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * \\
& ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x^2 * b^4 * c^3 * d^4 + 45 * a^ \\
& 2 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} \\
& * x^2 * b^7 * c^7 + 38 * a^{(9/2)} * ((a * x + b) * x)^{(3/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^3 * c \\
& ^4 * d^3 - 64 * a^{(7/2)} * ((a * x + b) * x)^{(3/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^4 * c^5 * d^2 + 20 * \\
& a^{(5/2)} * ((a * x + b) * x)^{(3/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^5 * c^6 * d - 105 * a^{(13/2)} * \ln \\
& ((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d) \\
& ) * x * b^3 * c * d^6 + 42 * a^{(11/2)} * \ln((-2 * a * d * x + b * c * x - b * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * \\
& ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x * b^4 * c^2 * d^5 + 27 * a^{(9/2)} * \ln((-2 * a * d * x + b * c * x - b \\
& * d + 2 * ((a * d - b * c) / c^2 * d)^{(1/2)} * ((a * x + b) * x)^{(1/2)} * c) / (c * x + d)) * x * b^5 * c^3 * d^4 - 12 \\
& * a^{(11/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^3 * c^2 * d^5 + 15 * a * \ln(1/2 \\
& * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x * b \\
& ^8 * c^7 - 30 * a^{(3/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^7 * c^6 * d + 15 * a * \\
& \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} \\
& * b^8 * c^6 * d + 42 * a^3 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ( \\
& (a * d - b * c) / c^2 * d)^{(1/2)} * b^6 * c^4 * d^3 - 48 * a^2 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/ \\
& 2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * b^7 * c^5 * d^2 + 6 * a^{(17/2)} * ((a * x + b) \\
& * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^5 * c^4 * d^3 - 6 * a^{(15/2)} * ((a * x + b) * x)^{(3/2)} \\
& * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^3 * c^4 * d^3 - 30 * a^{(3/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * c) \\
& ) / c^2 * d)^{(1/2)} * x * b^7 * c^7 - 102 * a^3 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} \\
& ) / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x * b^6 * c^5 * d^2 - 3 * a^2 * \ln(1/2 * (2 * a * x + b + 2 * ( \\
& (a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x * b^7 * c^6 * d + 36 * a \\
& ^8 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} \\
& * x^2 * b * c * d^6 - 63 * a^7 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ( \\
& (a * d - b * c) / c^2 * d)^{(1/2)} * x^2 * b^2 * c^2 * d^5 + 12 * a^{(15/2)} * ((a * x + b) * x)^{(1/2)} * ( \\
& (a * d - b * c) / c^2 * d)^{(1/2)} * x^3 * b * c^3 * d^4 + 24 * a^{(13/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b * \\
& c) / c^2 * d)^{(1/2)} * x^3 * b^2 * c^4 * d^3 - 33 * a^8 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * \\
& a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b * c^3 * d^4 + 12 * a^7 * \ln(1/2 * (2 * a * \\
& x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b^2 * c \\
& ^4 * d^3 + 42 * a^6 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} * a^{(1/2)} / a^{(1/2)}) * ((a * d - b \\
& * c) / c^2 * d)^{(1/2)} * x^4 * b^3 * c^5 * d^2 - 48 * a^5 * \ln(1/2 * (2 * a * x + b + 2 * ((a * x + b) * x)^{(1/2)} \\
& * a^{(1/2)} / a^{(1/2)}) * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b^4 * c^6 * d + 48 * a^{(15/2)} * ((a * x + \\
& b) * x)^{(1/2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b * c^4 * d^3 - 84 * a^{(13/2)} * ((a * x + b) * x)^{(1/ \\
& 2)} * ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b^2 * c^5 * d^2 + 96 * a^{(11/2)} * ((a * x + b) * x)^{(1/2)} * \\
& ((a * d - b * c) / c^2 * d)^{(1/2)} * x^4 * b^3 * c^6 * d - 36 * a^{(13/2)} * ((a * x + b) * x)^{(1/2)} * ((a * d - b
\end{aligned}$$

$$\begin{aligned} & *c)/c^2*d)^{(1/2)}*x*b^2*c^2*d^5-63*a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a \\ & ^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^3*c^3*d^4+162*a^5*\ln(1/2*(2* \\ & a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^4 \\ & *c^4*d^3-18*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d \\ & -b*c)/c^2*d)^{(1/2)}*x^2*b^5*c^5*d^2-99*a^3*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/ \\ & 2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^6*c^6*d+198*a^{(7/2)}*((a* \\ & x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b^5*c^6*d-28*a^{(9/2)}*((a*x+b)*x)^{( \\ & 3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^3*c^5*d^2-40*a^{(7/2)}*((a*x+b)*x)^{(3/2)}*(( \\ & a*d-b*c)/c^2*d)^{(1/2)}*x*b^4*c^6*d-36*a^{(15/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/ \\ & c^2*d)^{(1/2)}*x^2*b*c^2*d^5+72*a^{(13/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{( \\ & 1/2)}*x^2*b^2*c^3*d^4-156*a^{(11/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2 \\ & )} *x^2*b^3*c^4*d^3+36*a^{(9/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^2* \\ & b^4*c^5*d^2+78*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*(( \\ & a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^4*c^5*d^2-129*a^4*\ln(1/2*(2*a*x+b+2*((a*x+b)*x) \\ & ^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^5*c^6*d+30*a^{(11/2)}* \\ & ((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^2*c^4*d^3+3*a^8*\ln(1/2*(2*a*x \\ & +b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b*c^2* \\ & d^5-87*a^7*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c) \\ & /c^2*d)^{(1/2)}*x^3*b^2*c^3*d^4+78*a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^ \\ & (1/2))/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^3*c^4*d^3-156*a^{(11/2)}*((a*x+ \\ & b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^3*c^5*d^2+258*a^{(9/2)}*((a*x+b)*x) \\ & ^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x^3*b^4*c^6*d-18*a^{(13/2)}*((a*x+b)*x)^{(3/2)}* \\ & ((a*d-b*c)/c^2*d)^{(1/2)}*x^2*b*c^4*d^3+48*a^{(11/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b \\ & *c)/c^2*d)^{(1/2)}*x^2*b^2*c^5*d^2-72*a^{(9/2)}*((a*x+b)*x)^{(3/2)}*((a*d-b*c)/c^ \\ & 2*d)^{(1/2)}*x^2*b^3*c^6*d+84*a^{(11/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1 \\ & /2)}*x*b^3*c^3*d^4-222*a^{(9/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b \\ & ^4*c^4*d^3+204*a^{(7/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^5*c^5* \\ & d^2+6*a^{(5/2)}*((a*x+b)*x)^{(1/2)}*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^6*c^6*d+36*a^7* \\ & \ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/ \\ & 2)}*x*b^2*c*d^6-87*a^6*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)}) \\ & *((a*d-b*c)/c^2*d)^{(1/2)}*x*b^3*c^2*d^5+3*a^5*\ln(1/2*(2*a*x+b+2*((a*x+b)*x)^{( \\ & 1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x*b^4*c^3*d^4+138*a^4*\ln(1/ \\ & 2*(2*a*x+b+2*((a*x+b)*x)^{(1/2)}*a^{(1/2)})/a^{(1/2)})*((a*d-b*c)/c^2*d)^{(1/2)}*x* \\ & b^5*c^4*d^3+12*a^{(19/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*(( \\ & a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^4*c*d^6+36*a^{(17/2)}*\ln((-2*a*d*x+b*c*x-b*d+2* \\ & ((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*x^2*b*d^7+36*a^{(15/2)} \\ & * \ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x \\ & +d))*x*b^2*d^7-39*a^{(11/2)}*\ln((-2*a*d*x+b*c*x-b*d+2*((a*d-b*c)/c^2*d)^{(1/2)} \\ & *((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*b^4*c*d^6+27*a^{(9/2)}*\ln((-2*a*d*x+b*c*x-b*d+ \\ & 2*((a*d-b*c)/c^2*d)^{(1/2)}*((a*x+b)*x)^{(1/2)}*c)/(c*x+d))*b^5*c^2*d^5/a^{(9/2 \\ & )}/c^4/((a*x+b)*x)^{(1/2)}/(a*d-b*c)^4/(c*x+d)/((a*d-b*c)/c^2*d)^{(1/2)}/(a*x+b) \\ & ^3 \end{aligned}$$

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} \left(c + \frac{d}{x}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^2,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(5/2)\*(c + d/x)^2), x)

**mupad [B]** time = 8.73, size = 5789, normalized size = 20.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x)^(5/2)\*(c + d/x)^2),x)

[Out] 
$$\begin{aligned} & \left( \frac{(2b^3)/(3(a^2d - abc)) + (10b^3(a + b/x)(2ad - bc))/(3(a^2d - abc)^2) - (b(a + b/x)^2(6a^4d^4 + 15b^4c^4 + 64a^2b^2c^2d^2 - 58ab^3c^3d - 12a^3b^2cd^3))/(3c^2(a^2d - abc)^3) + (b(a + b/x)^3(2ad - bc)(a^2d^3 + 5b^2c^2d - abc^2d^2))/(c^2(a^2d - abc)^3)}{(d(a + b/x)^{7/2} + (a + b/x)^{3/2}(a^2d - abc) - (a + b/x)^{5/2}(2ad - bc)) + (\operatorname{atan}((a^{15}b^{19}c^{19}(a + b/x)^{1/2})^{125i} + a^{17}b^{17}c^{17}d^2(a + b/x)^{1/2})^{10440i} - a^{18}b^{16}c^{16}d^3(a + b/x)^{1/2})^{37776i} + a^{19}b^{15}c^{15}d^4(a + b/x)^{1/2})^{87276i} - a^{20}b^{14}c^{14}d^5(a + b/x)^{1/2})^{126720i} + a^{21}b^{13}c^{13}d^6(a + b/x)^{1/2})^{91560i} + a^{22}b^{12}c^{12}d^7(a + b/x)^{1/2})^{40965i} - a^{23}b^{11}c^{11}d^8(a + b/x)^{1/2})^{184563i} + a^{24}b^{10}c^{10}d^9(a + b/x)^{1/2})^{212608i} - a^{25}b^9c^9d^{10}(a + b/x)^{1/2})^{107740i} - a^{26}b^8c^8d^{11}(a + b/x)^{1/2})^{19530i} + a^{27}b^7c^7d^{12}(a + b/x)^{1/2})^{71070i} - a^{28}b^6c^6d^{13}(a + b/x)^{1/2})^{52836i} + a^{29}b^5c^5d^{14}(a + b/x)^{1/2})^{20916i} - a^{30}b^4c^4d^{15}(a + b/x)^{1/2})^{4515i} + a^{31}b^3c^3d^{16}(a + b/x)^{1/2})^{420i} - a^{16}b^{18}c^{18}d(a + b/x)^{1/2})^{1700i}}{(a^7(a^7)^{1/2})(a^7(212608b^{10}c^{10}d^9 - 107740ab^9c^9d^{10} - 19530a^2b^8c^8d^{11} + 71070a^3b^7c^7d^{12} - 52836a^4b^6c^6d^{13} + 20916a^5b^5c^5d^{14} - 4515a^6b^4c^4d^{15} + 420a^7b^3c^3d^{16}) + 10440b^{17}c^{17}d^2 - 37776ab^{16}c^{16}d^3 + 87276a^2b^{15}c^{15}d^4 - 126720a^3b^{14}c^{14}d^5 + 91560a^4b^{13}c^{13}d^6 + 40965a^5b^{12}c^{12}d^7 - 184563a^6b^{11}c^{11}d^8) + 125a^5b^{19}c^{19} - 1700a^6b^{18}c^{18}d)} \right) \\ & \cdot (4ad + 5bc) \cdot i / (c^3(a^7)^{1/2}) - (\operatorname{atan}(((d^7(ad - bc)^7)^{1/2}) \cdot ((a + b/x)^{1/2}) \cdot (670a^{10}b^{18}c^{22}d^4 - 50a^9b^{19}c^{23}d^3 - 4082a^{11}b^{17}c^{21}d^5 + 14830a^{12}b^{16}c^{20}d^6 - 35210a^{13}b^{15}c^{19}d^7 + 55510a^{14}b^{14}c^{18}d^8 - 53852a^{15}b^{13}c^{17}d^9 + 19048a^{16}b^{12}c^{16}d^{10} + 25730a^{17}b^{11}c^{15}d^{11} - 39550a^{18}b^{10}c^{14}d^{12} + 10670a^{19}b^9c^{13}d^{13} + 29414a^{20}b^8c^{12}d^{14} - 45430a^{21}b^7c^{11}d^{15} + 34490a^{22}b^6c^{10}d^{16} - 16240a^{23}b^5c^9d^{17} + 4820a^{24}b^4c^8d^{18} - 832a^{25}b^3c^7d^{19} + 64a^{26}b^2c^6d^{20}) - ((d^7(ad - bc)^7)^{1/2}) \cdot (4ad - 9bc) \cdot (304a^{13}b^{18}c^{25}d^3 - 20a^{12}b^{19}c^{26}d^2 - 2144a^{14}b^{17}c^{24}d^4 + 9280a^{15}b^{16}c^{23}d^5 - 27476a^{16}b^{15}c^{22}d^6 + 58688a^{17}b^{14}c^{21}d^7 - 92840a^{18}b^{13}c^{20}d^8 + 109648a^{19}b^{12}c^{19}d^9 - 95700a^{20}b^{11}c^{18}d^{10} + 59312a^{21}b^{10}c^{17}d^{11} - 23056a^{22}b^9c^{16}d^{12} + 2528a^{23}b^8c^{15}d^{13} + 2996a^{24}b^7c^{14}d^{14} - 2080a^{25}b^6c^{13}d^{15} + 664a^{26}b^5c^{12}d^{16} - 112a^{27}b^4c^{11}d^{17} + 8a^{28}b^3c^{10}d^{18} + 8a^{28}b^3c^{10}d^{18} + ((d^7(ad - bc)^7)^{1/2}) \cdot (a + b/x)^{1/2} \cdot (4ad - 9bc) \cdot (8a^{15}b^{18}c^{28}d^2 - 136a^{16}b^{17}c^{27}d^3 + 1080a^{17}b^{16}c^{26}d^4 - 5320a^{18}b^{15}c^{25}d^5 + 18200a^{19}b^{14}c^{24}d^6 - 45864a^{20}b^{13}c^{23}d^7 + 88088a^{21}b^{12}c^{22}d^8 - 131560a^{22}b^{11}c^{21}d^9 + 154440a^{23}b^{10}c^{20}d^{10} - 143000a^{24}b^9c^{19}d^{11} + 104104a^{25}b^8c^{18}d^{12} - 58968a^{26}b^7c^{17}d^{13} + 25480a^{27}b^6c^{16}d^{14} - 8120a^{28}b^5c^{15}d^{15} + 1800a^{29}b^4c^{14}d^{16} - 248a^{30}b^3c^{13}d^{17} + 16a^{31}b^2c^{12}d^{18}))/((2(b^7c^{10} - a^7c^3d^7 + 7a^6b^3c^4d^6 + 21a^2b^5c^8d^2 - 35a^3b^4c^7d^3 + 35a^4b^3c^6d^4 - 21a^5b^2c^5d^5 - 7a^6b^2c^9d^9)))/(2(b^7c^{10} - a^7c^3d^7 + 7a^6b^3c^4d^6 + 21a^2b^5c^8d^2 - 35a^3b^4c^7d^3 + 35a^4b^3c^6d^4 - 21a^5b^2c^5d^5 - 7a^6b^2c^9d^9)) \cdot (4ad - 9bc) \cdot i) / (2(b^7c^{10} - a^7c^3d^7 + 7a^6b^3c^4d^6 + 21a^2b^5c^8d^2 - 35a^3b^4c^7d^3 + 35a^4b^3c^6d^4 - 21a^5b^2c^5d^5 - 7a^6b^2c^9d^9)) + ((d^7(ad - bc)^7)^{1/2}) \cdot ((a + b/x)^{1/2}) \cdot (670a^{10}b^{18}c^{22}d^4 - 50a^9b^{19}c^{23}d^3 - 4082a^{11}b^{17}c^{21}d^5 + 14830a^{12}b^{16}c^{20}d^6 - 35210a^{13}b^{15}c^{19}d^7 + 55510a^{14}b^{14}c^{18}d^8 - 53852a^{15}b^{13}c^{17}d^9 + 19048a^{16}b^{12}c^{16}d^{10} + 25730a^{17}b^{11}c^{15}d^{11} - 39550a^{18}b^{10}c^{14}d^{12} + 10670a^{19}b^9c^{13}d^{13} + 29414a^{20}b^8c^{12}d^{14} - 45430a^{21}b^7c^{11}d^{15} + 34490a^{22}b^6c^{10}d^{16} - 16240a^{23}b^5c^9d^{17} + 4820a^{24}b^4c^8d^{18} - 832a^{25}b^3c^7d^{19} + 64a^{26}b^2c^6d^{20}) - ((d^7(ad - bc)^7)^{1/2}) \cdot (4ad - 9bc) \cdot (20a^{12}b^{19}c^{26}d^2 - 304a^{13}b^{18}c^{25}d^3 + 2144a^{14}b^{17}c^{24}d^4 - 9280a^{15}b^{16}c^{23}d^5 + 27476a^{16}b^{15}c^{22}d^6 - 58688a^{17}b^{14}c^{21}d^7 + 92840a^{18}b^{13}c^{20}d^8 - 10$$

$$\begin{aligned}
& 9648a^{19}b^{12}c^{19}d^9 + 95700a^{20}b^{11}c^{18}d^{10} - 59312a^{21}b^{10}c^{17}d^{11} + 23056a^{22}b^9c^{16}d^{12} - 2528a^{23}b^8c^{15}d^{13} - 2996a^{24}b^7c^{14}d^{14} + 2080a^{25}b^6c^{13}d^{15} - 664a^{26}b^5c^{12}d^{16} + 112a^{27}b^4c^{11}d^{17} - 8a^{28}b^3c^{10}d^{18} + ((d^7(a*d - b*c)^7)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d - 9*b*c)*(8*a^{15}b^{18}c^{28}d^2 - 136*a^{16}b^{17}c^{27}d^3 + 1080*a^{17}b^{16}c^{26}d^4 - 5320*a^{18}b^{15}c^{25}d^5 + 18200*a^{19}b^{14}c^{24}d^6 - 45864*a^{20}b^{13}c^{23}d^7 + 88088*a^{21}b^{12}c^{22}d^8 - 131560*a^{22}b^{11}c^{21}d^9 + 154440*a^{23}b^{10}c^{20}d^{10} - 143000*a^{24}b^9c^{19}d^{11} + 104104*a^{25}b^8c^{18}d^{12} - 58968*a^{26}b^7c^{17}d^{13} + 25480*a^{27}b^6c^{16}d^{14} - 8120*a^{28}b^5c^{15}d^{15} + 1800*a^{29}b^4c^{14}d^{16} - 248*a^{30}b^3c^{13}d^{17} + 16*a^{31}b^2c^{12}d^{18}))/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c)*i)/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/(4880*a^{10}b^{16}c^{17}d^7 - 450*a^9b^{17}c^{18}d^6 - 23428*a^{11}b^{15}c^{16}d^8 + 65234*a^{12}b^{14}c^{15}d^9 - 115136*a^{13}b^{13}c^{14}d^{10} + 129800*a^{14}b^{12}c^{13}d^{11} - 83040*a^{15}b^{11}c^{12}d^{12} + 5916*a^{16}b^{10}c^{11}d^{13} + 45702*a^{17}b^9c^{10}d^{14} - 51528*a^{18}b^8c^9d^{15} + 32500*a^{19}b^7c^8d^{16} - 13790*a^{20}b^6c^7d^{17} + 4012*a^{21}b^5c^6d^{18} - 736*a^{22}b^4c^5d^{19} + 64*a^{23}b^3c^4d^{20} + ((d^7*(a*d - b*c)^7)^{(1/2)}*((a + b/x)^{(1/2)}*(670*a^{10}b^{18}c^{22}d^4 - 50*a^9b^{19}c^{23}d^3 - 4082*a^{11}b^{17}c^{21}d^5 + 14830*a^{12}b^{16}c^{20}d^6 - 35210*a^{13}b^{15}c^{19}d^7 + 55510*a^{14}b^{14}c^{18}d^8 - 53852*a^{15}b^{13}c^{17}d^9 + 19048*a^{16}b^{12}c^{16}d^{10} + 25730*a^{17}b^{11}c^{15}d^{11} - 39550*a^{18}b^{10}c^{14}d^{12} + 10670*a^{19}b^9c^{13}d^{13} + 29414*a^{20}b^8c^{12}d^{14} - 45430*a^{21}b^7c^{11}d^{15} + 34490*a^{22}b^6c^{10}d^{16} - 16240*a^{23}b^5c^9d^{17} + 4820*a^{24}b^4c^8d^{18} - 832*a^{25}b^3c^7d^{19} + 64*a^{26}b^2c^6d^{20}) - ((d^7*(a*d - b*c)^7)^{(1/2)}*(4*a*d - 9*b*c)*(304*a^{13}b^{18}c^{25}d^3 - 20*a^{12}b^{19}c^{26}d^2 - 2144*a^{14}b^{17}c^{24}d^4 + 9280*a^{15}b^{16}c^{23}d^5 - 27476*a^{16}b^{15}c^{22}d^6 + 58688*a^{17}b^{14}c^{21}d^7 - 92840*a^{18}b^{13}c^{20}d^8 + 109648*a^{19}b^{12}c^{19}d^9 - 95700*a^{20}b^{11}c^{18}d^{10} + 59312*a^{21}b^{10}c^{17}d^{11} - 23056*a^{22}b^9c^{16}d^{12} + 2528*a^{23}b^8c^{15}d^{13} + 2996*a^{24}b^7c^{14}d^{14} - 2080*a^{25}b^6c^{13}d^{15} + 664*a^{26}b^5c^{12}d^{16} - 112*a^{27}b^4c^{11}d^{17} + 8*a^{28}b^3c^{10}d^{18} + ((d^7*(a*d - b*c)^7)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d - 9*b*c)*(8*a^{15}b^{18}c^{28}d^2 - 136*a^{16}b^{17}c^{27}d^3 + 1080*a^{17}b^{16}c^{26}d^4 - 5320*a^{18}b^{15}c^{25}d^5 + 18200*a^{19}b^{14}c^{24}d^6 - 45864*a^{20}b^{13}c^{23}d^7 + 88088*a^{21}b^{12}c^{22}d^8 - 131560*a^{22}b^{11}c^{21}d^9 + 154440*a^{23}b^{10}c^{20}d^{10} - 143000*a^{24}b^9c^{19}d^{11} + 104104*a^{25}b^8c^{18}d^{12} - 58968*a^{26}b^7c^{17}d^{13} + 25480*a^{27}b^6c^{16}d^{14} - 8120*a^{28}b^5c^{15}d^{15} + 1800*a^{29}b^4c^{14}d^{16} - 248*a^{30}b^3c^{13}d^{17} + 16*a^{31}b^2c^{12}d^{18}))/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/((b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)) - ((d^7*(a*d - b*c)^7)^{(1/2)}*((a + b/x)^{(1/2)}*(670*a^{10}b^{18}c^{22}d^4 - 50*a^9b^{19}c^{23}d^3 - 4082*a^{11}b^{17}c^{21}d^5 + 14830*a^{12}b^{16}c^{20}d^6 - 35210*a^{13}b^{15}c^{19}d^7 + 55510*a^{14}b^{14}c^{18}d^8 - 53852*a^{15}b^{13}c^{17}d^9 + 19048*a^{16}b^{12}c^{16}d^{10} + 25730*a^{17}b^{11}c^{15}d^{11} - 39550*a^{18}b^{10}c^{14}d^{12} + 10670*a^{19}b^9c^{13}d^{13} + 29414*a^{20}b^8c^{12}d^{14} - 45430*a^{21}b^7c^{11}d^{15} + 34490*a^{22}b^6c^{10}d^{16} - 16240*a^{23}b^5c^9d^{17} + 4820*a^{24}b^4c^8d^{18} - 832*a^{25}b^3c^7d^{19} + 64*a^{26}b^2c^6d^{20}) - ((d^7*(a*d - b*c)^7)^{(1/2)}*(4*a*d - 9*b*c)*(20*a^{12}b^{19}c^{26}d^2 - 304*a^{13}b^{18}c^{25}d^3 + 2144*a^{14}b^{17}c^{24}d^4 - 9280*a^{15}b^{16}c^{23}d^5 + 27476*a^{16}b^{15}c^{22}d^6 - 58688*a^{17}b^{14}c^{21}d^7 + 92840*a^{18}b^{13}c^{20}d^8 - 109648*a^{19}b^{12}c^{19}d^9 + 95700*a^{20}b^{11}c^{18}d^{10} - 59312*a^{21}b^{10}c^{17}d^{11}
\end{aligned}$$

$$\begin{aligned}
& 17*d^{11} + 23056*a^{22}*b^9*c^{16}*d^{12} - 2528*a^{23}*b^8*c^{15}*d^{13} - 2996*a^{24}*b^7*c^{14}*d^{14} + 2080*a^{25}*b^6*c^{13}*d^{15} - 664*a^{26}*b^5*c^{12}*d^{16} + 112*a^{27}*b^4*c^{11}*d^{17} - 8*a^{28}*b^3*c^{10}*d^{18} + ((d^7*(a*d - b*c)^7)^{(1/2)}*(a + b/x)^{(1/2)}*(4*a*d - 9*b*c)*(8*a^{15}*b^{18}*c^{28}*d^2 - 136*a^{16}*b^{17}*c^{27}*d^3 + 1080*a^{17}*b^{16}*c^{26}*d^4 - 5320*a^{18}*b^{15}*c^{25}*d^5 + 18200*a^{19}*b^{14}*c^{24}*d^6 - 45864*a^{20}*b^{13}*c^{23}*d^7 + 88088*a^{21}*b^{12}*c^{22}*d^8 - 131560*a^{22}*b^{11}*c^{21}*d^9 + 154440*a^{23}*b^{10}*c^{20}*d^{10} - 143000*a^{24}*b^9*c^{19}*d^{11} + 104104*a^{25}*b^8*c^{18}*d^{12} - 58968*a^{26}*b^7*c^{17}*d^{13} + 25480*a^{27}*b^6*c^{16}*d^{14} - 8120*a^{28}*b^5*c^{15}*d^{15} + 1800*a^{29}*b^4*c^{14}*d^{16} - 248*a^{30}*b^3*c^{13}*d^{17} + 16*a^{31}*b^2*c^{12}*d^{18}))/((2*(b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))/(2*(b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(4*a*d - 9*b*c))/((2*(b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)))*(d^7*(a*d - b*c)^7)^{(1/2)}*(4*a*d - 9*b*c)*i)/(b^7*c^{10} - a^7*c^3*d^7 + 7*a^6*b*c^4*d^6 + 21*a^2*b^5*c^8*d^2 - 35*a^3*b^4*c^7*d^3 + 35*a^4*b^3*c^6*d^4 - 21*a^5*b^2*c^5*d^5 - 7*a*b^6*c^9*d)
\end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x)\*\*2,x)

[Out] Timed out



$$3.265 \quad \int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3} dx$$

**Optimal.** Leaf size=409

$$\frac{(6ad + 5bc) \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{x}}}{\sqrt{a}}\right) d^{7/2} (24a^2d^2 - 88abcd + 99b^2c^2) \tan^{-1}\left(\frac{\sqrt{d}\sqrt{a+\frac{b}{x}}}{\sqrt{bc-ad}}\right) + d(12a^2d^2 - 23abcd + 4b^2c^2)}{a^{7/2}c^4 - 4c^4(bc - ad)^{9/2} + 4ac^3\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)}$$

[Out]  $1/12*b*(-36*a^3*d^3+87*a^2*b*c*d^2-36*a*b^2*c^2*d+20*b^3*c^3)/a^2/c^3/(-a*d+b*c)^3/(a+b/x)^{(3/2)}+1/2*d*(-3*a*d+2*b*c)/a/c^2/(-a*d+b*c)/(a+b/x)^{(3/2)}/(c+d/x)^2+1/4*d*(12*a^2*d^2-23*a*b*c*d+4*b^2*c^2)/a/c^3/(-a*d+b*c)^2/(a+b/x)^{(3/2)}/(c+d/x)+x/a/c/(a+b/x)^{(3/2)}/(c+d/x)^2-1/4*d^{(7/2)}*(24*a^2*d^2-88*a*b*c*d+99*b^2*c^2)*\arctan(d^{(1/2)}*(a+b/x)^{(1/2)}/(-a*d+b*c)^{(1/2)})/c^4/(-a*d+b*c)^{(9/2)}-(6*a*d+5*b*c)*\operatorname{arctanh}((a+b/x)^{(1/2)}/a^{(1/2)})/a^{(7/2)}/c^4+1/4*b*(12*a^4*d^4-35*a^3*b*c*d^3+24*a^2*b^2*c^2*d^2-56*a*b^3*c^3*d+20*b^4*c^4)/a^3/c^3/(-a*d+b*c)^4/(a+b/x)^{(1/2)}$

**Rubi [A]** time = 0.70, antiderivative size = 409, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$ , Rules used = {375, 103, 151, 152, 156, 63, 208, 205}

$$\frac{d(12a^2d^2 - 23abcd + 4b^2c^2)}{4ac^3\left(a + \frac{b}{x}\right)^{3/2}\left(c + \frac{d}{x}\right)(bc - ad)^2} + \frac{b(24a^2b^2c^2d^2 - 35a^3bcd^3 + 12a^4d^4 - 56ab^3c^3d + 20b^4c^4)}{4a^3c^3\sqrt{a + \frac{b}{x}}(bc - ad)^4} + \frac{b(87a^2bcd^2 - 35a^3cd^3)}{12a^2c^3}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b/x)^(5/2)\*(c + d/x)^3), x]

[Out]  $(b*(20*b^3*c^3 - 36*a*b^2*c^2*d + 87*a^2*b*c*d^2 - 36*a^3*d^3))/(12*a^2*c^3*(b*c - a*d)^3*(a + b/x)^{(3/2)}) + (b*(20*b^4*c^4 - 56*a*b^3*c^3*d + 24*a^2*b^2*c^2*d^2 - 35*a^3*b*c*d^3 + 12*a^4*d^4))/(4*a^3*c^3*(b*c - a*d)^4*\operatorname{Sqrt}[a + b/x]) + (d*(2*b*c - 3*a*d))/(2*a*c^2*(b*c - a*d)*(a + b/x)^{(3/2)}*(c + d/x)^2) + (d*(4*b^2*c^2 - 23*a*b*c*d + 12*a^2*d^2))/(4*a*c^3*(b*c - a*d)^2*(a + b/x)^{(3/2)}*(c + d/x)) + x/(a*c*(a + b/x)^{(3/2)}*(c + d/x)^2) - (d^{(7/2)}*(99*b^2*c^2 - 88*a*b*c*d + 24*a^2*d^2)*\operatorname{ArcTan}[\operatorname{Sqrt}[d]*\operatorname{Sqrt}[a + b/x]]/\operatorname{Sqrt}[b*c - a*d])/(4*c^4*(b*c - a*d)^{(9/2)}) - ((5*b*c + 6*a*d)*\operatorname{ArcTanh}[\operatorname{Sqrt}[a + b/x]/\operatorname{Sqrt}[a]])/(a^{(7/2)}*c^4)$

**Rule 63**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

**Rule 103**

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[

m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 151

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

### Rule 152

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)), x\_Symbol] := Simp[((b\*g - a\*h)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[(a\*d\*f\*g - b\*(d\*e + c\*f)\*g + b\*c\*e\*h]\*(m + 1) - (b\*g - a\*h)\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - d\*f\*(b\*g - a\*h)\*(m + n + p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[2\*m, 2\*n, 2\*p]

### Rule 156

Int[(((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/(((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))), x\_Symbol] := Dist[(b\*g - a\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(a + b\*x), x], x] - Dist[(d\*g - c\*h)/(b\*c - a\*d), Int[(e + f\*x)^p/(c + d\*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]

### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a + \frac{b}{x}\right)^{5/2} \left(c + \frac{d}{x}\right)^3} dx &= -\text{Subst} \left( \int \frac{1}{x^2(a + bx)^{5/2}(c + dx)^3} dx, x, \frac{1}{x} \right) \\
&= \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} + \frac{\text{Subst} \left( \int \frac{\frac{1}{2}(5bc+6ad) + \frac{9bdx}{2}}{x(a+bx)^{5/2}(c+dx)^3} dx, x, \frac{1}{x} \right)}{ac} \\
&= \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} + \frac{x}{ac \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} - \frac{\text{Subst} \left( \int \frac{-(bc-ad)(5bc+6ad)}{x(a+bx)^{5/2}(c+dx)^3} dx, x, \frac{1}{x} \right)}{2ac^2} \\
&= \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} + \frac{d(4b^2c^2 - 23abcd + 12a^2d^2)}{4ac^3(bc - ad)^2 \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)} + \frac{1}{ac \left(a + \frac{b}{x}\right)} \\
&= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc - ad)^3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{d(2bc - 3ad)}{2ac^2(bc - ad) \left(a + \frac{b}{x}\right)^{3/2} \left(c + \frac{d}{x}\right)^2} + \frac{1}{ac \left(a + \frac{b}{x}\right)} \\
&= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc - ad)^3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2)}{4a^3c^3(bc - ad)^4 \sqrt{\left(a + \frac{b}{x}\right)}} \\
&= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc - ad)^3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2)}{4a^3c^3(bc - ad)^4 \sqrt{\left(a + \frac{b}{x}\right)}} \\
&= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc - ad)^3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2)}{4a^3c^3(bc - ad)^4 \sqrt{\left(a + \frac{b}{x}\right)}} \\
&= \frac{b(20b^3c^3 - 36ab^2c^2d + 87a^2bcd^2 - 36a^3d^3)}{12a^2c^3(bc - ad)^3 \left(a + \frac{b}{x}\right)^{3/2}} + \frac{b(20b^4c^4 - 56ab^3c^3d + 24a^2b^2c^2d^2)}{4a^3c^3(bc - ad)^4 \sqrt{\left(a + \frac{b}{x}\right)}}
\end{aligned}$$

**Mathematica [C]** time = 0.39, size = 239, normalized size = 0.58

$$\frac{(cx + d) \left( 2(cx + d) \left( \frac{1}{4}a^2d^2 (24a^2d^2 - 88abcd + 99b^2c^2) {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; \frac{d(a + \frac{b}{x})}{ad - bc} \right) + (6ad + 5bc)(bc - ad)^3 {}_2F_1 \left( -\frac{3}{2}, 1; -\frac{1}{2}; \frac{d(a + \frac{b}{x})}{ad - bc} \right) \right)}{6a^2c^4 \left( a + \frac{b}{x} \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b/x)^(5/2)\*(c + d/x)^3),x]

[Out] (-3\*a\*c^2\*d\*(b\*c - a\*d)^2\*(-2\*b\*c + 3\*a\*d)\*x^2 + 6\*a\*c^3\*(b\*c - a\*d)^3\*x^3 + (d + c\*x)\*((-3\*a\*c\*d\*(-(b\*c) + a\*d)\*(4\*b^2\*c^2 - 23\*a\*b\*c\*d + 12\*a^2\*d^2)\*x)/2 + 2\*(d + c\*x)\*((a^2\*d^2\*(99\*b^2\*c^2 - 88\*a\*b\*c\*d + 24\*a^2\*d^2)\*Hypergeometric2F1[-3/2, 1, -1/2, (d\*(a + b/x))/(-(b\*c) + a\*d)])/4 + (b\*c - a\*d)^3

$(5*b*c + 6*a*d)*\text{Hypergeometric2F1}[-3/2, 1, -1/2, 1 + b/(a*x)]/((6*a^2*c^4*(b*c - a*d)^3*(a + b/x)^{(3/2)}*(d + c*x)^2)$

**fricas [B]** time = 14.87, size = 6171, normalized size = 15.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="fricas")

[Out]  $[1/24*(12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*\text{sqrt}(a)*\text{log}(2*a*x - 2*\text{sqrt}(a)*x*\text{sqrt}((a*x + b)/x) + b) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*\text{sqrt}(-d/(b*c - a*d))*\text{log}(-(2*(b*c - a*d)*x*\text{sqrt}(-d/(b*c - a*d))*\text{sqrt}((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 4*8*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*\text{sqrt}((a*x + b)/x)]/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), 1/24*(24*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*\text{sqrt}(-a)*\text{arctan}(\text{sqrt}(-a)*\text{sqrt}((a*x + b)/x)/a) + 3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*\text{sqrt}(-d/(b*c - a*d))*\text{log}(-(2*(b*c - a*d)*x*\text{sqrt}(-d/(b*c - a*d))*\text{sqrt}((a*x + b)/x) - b*d + (b*c - 2*a*d)*x)/(c*x + d)) + 2*(12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5$

$$\begin{aligned}
& ^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 \\
& + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - \\
& 56*a^2*b^5*c^4*d^3 + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*\sqrt{(a*x + b)/x)/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 \\
& - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d \\
& + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 \\
& + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 \\
& + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 \\
& + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*\sqrt{d/(b*c - a*d)} \\
& *\arctan(-(b*c - a*d)*x*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x)/(a*d*x + b*d)) - 6*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 \\
& + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 \\
& + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 \\
& + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 + 6*a^6*b*d^7)*x)*\sqrt{a} \\
& *\log(2*a*x - 2*\sqrt{a})*x*\sqrt{(a*x + b)/x) + b) - (12*(a^3*b^4*c^7 - 4*a^4*b^3*c^6*d + 6*a^5*b^2*c^5*d^2 - 4*a^6*b*c^4*d^3 + a^7*c^3*d^4)*x^5 + (80*a^2*b^5*c^7 - 200*a^3*b^4*c^6*d \\
& + 48*a^4*b^3*c^5*d^2 + 48*a^5*b^2*c^4*d^3 - 135*a^6*b*c^3*d^4 + 54*a^7*c^2*d^5)*x^4 + (60*a*b^6*c^7 - 8*a^2*b^5*c^6*d - 364*a^3*b^4*c^5*d^2 + 192*a^4*b^3*c^4*d^3 - 234*a^5*b^2*c^3*d^4 \\
& + 3*a^6*b*c^2*d^5 + 36*a^7*c*d^6)*x^3 + (120*a*b^6*c^6*d - 256*a^2*b^5*c^5*d^2 - 80*a^3*b^4*c^4*d^3 - 15*a^4*b^3*c^3*d^4 - 156*a^5*b^2*c^2*d^5 + 72*a^6*b*c*d^6)*x^2 + 3*(20*a*b^6*c^5*d^2 - 56*a^2*b^5*c^4*d^3 \\
& + 24*a^3*b^4*c^3*d^4 - 35*a^4*b^3*c^2*d^5 + 12*a^5*b^2*c*d^6)*x)*\sqrt{(a*x + b)/x)/(a^4*b^6*c^8*d^2 - 4*a^5*b^5*c^7*d^3 + 6*a^6*b^4*c^6*d^4 - 4*a^7*b^3*c^5*d^5 + a^8*b^2*c^4*d^6 \\
& + (a^6*b^4*c^10 - 4*a^7*b^3*c^9*d + 6*a^8*b^2*c^8*d^2 - 4*a^9*b*c^7*d^3 + a^10*c^6*d^4)*x^4 + 2*(a^5*b^5*c^10 - 3*a^6*b^4*c^9*d + 2*a^7*b^3*c^8*d^2 + 2*a^8*b^2*c^7*d^3 - 3*a^9*b*c^6*d^4 \\
& + a^10*c^5*d^5)*x^3 + (a^4*b^6*c^10 - 9*a^6*b^4*c^8*d^2 + 16*a^7*b^3*c^7*d^3 - 9*a^8*b^2*c^6*d^4 + a^10*c^4*d^6)*x^2 + 2*(a^4*b^6*c^9*d - 3*a^5*b^5*c^8*d^2 + 2*a^6*b^4*c^7*d^3 + 2*a^7*b^3*c^6*d^4 \\
& - 3*a^8*b^2*c^5*d^5 + a^9*b*c^4*d^6)*x), -1/12*(3*(99*a^4*b^4*c^2*d^5 - 88*a^5*b^3*c*d^6 + 24*a^6*b^2*d^7 + (99*a^6*b^2*c^4*d^3 - 88*a^7*b*c^3*d^4 + 24*a^8*c^2*d^5)*x^4 \\
& + 2*(99*a^5*b^3*c^4*d^3 + 11*a^6*b^2*c^3*d^4 - 64*a^7*b*c^2*d^5 + 24*a^8*c*d^6)*x^3 + (99*a^4*b^4*c^4*d^3 + 308*a^5*b^3*c^3*d^4 - 229*a^6*b^2*c^2*d^5 + 8*a^7*b*c*d^6 + 24*a^8*d^7)*x^2 \\
& + 2*(99*a^4*b^4*c^3*d^4 + 11*a^5*b^3*c^2*d^5 - 64*a^6*b^2*c*d^6 + 24*a^7*b*d^7)*x)*\sqrt{d/(b*c - a*d)}*\arctan(-(b*c - a*d)*x*\sqrt{d/(b*c - a*d)}*\sqrt{(a*x + b)/x)/(a*d*x + b*d)) \\
& - 12*(5*b^7*c^5*d^2 - 14*a*b^6*c^4*d^3 + 6*a^2*b^5*c^3*d^4 + 16*a^3*b^4*c^2*d^5 - 19*a^4*b^3*c*d^6 + 6*a^5*b^2*d^7 + (5*a^2*b^5*c^7 - 14*a^3*b^4*c^6*d + 6*a^4*b^3*c^5*d^2 + 16*a^5*b^2*c^4*d^3 \\
& - 19*a^6*b*c^3*d^4 + 6*a^7*c^2*d^5)*x^4 + 2*(5*a*b^6*c^7 - 9*a^2*b^5*c^6*d - 8*a^3*b^4*c^5*d^2 + 22*a^4*b^3*c^4*d^3 - 3*a^5*b^2*c^3*d^4 - 13*a^6*b*c^2*d^5 + 6*a^7*c*d^6)*x^3 + (5*b^7*c^7 + 6*a*b^6*c^6*d - 45*a^2*b^5*c^5*d^2 \\
& + 26*a^3*b^4*c^4*d^3 + 51*a^4*b^3*c^3*d^4 - 54*a^5*b^2*c^2*d^5 + 5*a^6*b*c*d^6 + 6*a^7*d^7)*x^2 + 2*(5*b^7*c^6*d - 9*a*b^6*c^5*d^2 - 8*a^2*b^5*c^4*d^3 + 22*a^3*b^4*c^3*d^4 - 3*a^4*b^3*c^2*d^5 - 13*a^5*b^2*c*d^6 \\
& + 6*a^6*b*d^7)*x)*\sqrt{-a}*\arctan(\sqrt{-a}*\sqrt{(a*x + b)/x)/a) - (12*(
\end{aligned}$$

$$a^3b^4c^7 - 4a^4b^3c^6d + 6a^5b^2c^5d^2 - 4a^6b^1c^4d^3 + a^7c^3d^4)x^5 + (80a^2b^5c^7 - 200a^3b^4c^6d + 48a^4b^3c^5d^2 + 48a^5b^2c^4d^3 - 135a^6b^1c^3d^4 + 54a^7c^2d^5)x^4 + (60a^1b^6c^7 - 8a^2b^5c^6d - 364a^3b^4c^5d^2 + 192a^4b^3c^4d^3 - 234a^5b^2c^3d^4 + 3a^6b^1c^2d^5 + 36a^7c^1d^6)x^3 + (120a^1b^6c^6d - 256a^2b^5c^5d^2 - 80a^3b^4c^4d^3 - 15a^4b^3c^3d^4 - 156a^5b^2c^2d^5 + 72a^6b^1c^1d^6)x^2 + 3(20a^1b^6c^5d^2 - 56a^2b^5c^4d^3 + 24a^3b^4c^3d^4 - 35a^4b^3c^2d^5 + 12a^5b^2c^1d^6)x) \sqrt{(ax+b)/x} / (a^4b^6c^8d^2 - 4a^5b^5c^7d^3 + 6a^6b^4c^6d^4 - 4a^7b^3c^5d^5 + a^8b^2c^4d^6 + (a^6b^4c^{10} - 4a^7b^3c^9d + 6a^8b^2c^8d^2 - 4a^9b^1c^7d^3 + a^{10}c^6d^4)x^4 + 2(a^5b^5c^{10} - 3a^6b^4c^9d + 2a^7b^3c^8d^2 + 2a^8b^2c^7d^3 - 3a^9b^1c^6d^4 + a^{10}c^5d^5)x^3 + (a^4b^6c^{10} - 9a^6b^4c^8d^2 + 16a^7b^3c^7d^3 - 9a^8b^2c^6d^4 + a^{10}c^4d^6)x^2 + 2(a^4b^6c^9d - 3a^5b^5c^8d^2 + 2a^6b^4c^7d^3 + 2a^7b^3c^6d^4 - 3a^8b^2c^5d^5 + a^9b^1c^4d^6)x]$$

**giac** [A] time = 0.29, size = 523, normalized size = 1.28

$$-\frac{1}{12} b^4 \left( \frac{3(99b^2c^2d^4 - 88abcd^5 + 24a^2d^6) \arctan\left(\frac{d\sqrt{\frac{ax+b}{x}}}{\sqrt{bcd-ad^2}}\right)}{(b^8c^8 - 4ab^7c^7d + 6a^2b^6c^6d^2 - 4a^3b^5c^5d^3 + a^4b^4c^4d^4)\sqrt{bcd-ad^2}} - \frac{8\left(abc - a^2d + \frac{6(ax+b)}{x}\right)}{(a^3b^4c^4 - 4a^4b^3c^3d + 6a^5b^2c^2d^2 - \dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="giac")

[Out]  $-1/12*b^4*(3*(99*b^2*c^2*d^4 - 88*a*b*c*d^5 + 24*a^2*d^6)*\arctan(d*\sqrt{(a*x + b)/x}/\sqrt{b*c*d - a*d^2}))/((b^8*c^8 - 4*a*b^7*c^7*d + 6*a^2*b^6*c^6*d^2 - 4*a^3*b^5*c^5*d^3 + a^4*b^4*c^4*d^4)*\sqrt{b*c*d - a*d^2}) - 8*(a*b*c - a^2*d + 6*(a*x + b)*b*c/x - 15*(a*x + b)*a*d/x)*x/((a^3*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b^1*c^1*d^3 + a^7*d^4)*(a*x + b)*\sqrt{(a*x + b)/x}) + 3*(21*b^2*c^2*d^4*\sqrt{(a*x + b)/x} - 29*a*b*c*d^5*\sqrt{(a*x + b)/x} + 8*a^2*d^6*\sqrt{(a*x + b)/x} + 19*(a*x + b)*b*c*d^5*\sqrt{(a*x + b)/x}/x - 8*(a*x + b)*a*d^6*\sqrt{(a*x + b)/x}/x)/((b^7*c^7 - 4*a*b^6*c^6*d + 6*a^2*b^5*c^5*d^2 - 4*a^3*b^4*c^4*d^3 + a^4*b^3*c^3*d^4)*(b*c - a*d + (a*x + b)*d/x)^2) + 12*\sqrt{(a*x + b)/x}/((a - (a*x + b)/x)*a^3*b^3*c^3) - 12*(5*b*c + 6*a*d)*\arctan(\sqrt{(a*x + b)/x}/\sqrt{-a})/(\sqrt{-a}*a^3*b^4*c^4)$

**maple** [B] time = 0.08, size = 7300, normalized size = 17.85

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x)^(5/2)/(c+d/x)^3,x)

[Out] result too large to display

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + \frac{b}{x}\right)^{\frac{5}{2}} \left(c + \frac{d}{x}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)^(5/2)/(c+d/x)^3,x, algorithm="maxima")

[Out] integrate(1/((a + b/x)^(5/2)\*(c + d/x)^3), x)

mupad [B] time = 8.23, size = 4284, normalized size = 10.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\int \frac{1}{(a + b/x)^{5/2} (c + d/x)^3} dx$

[Out] 
$$\frac{(2b^4)/(3(a^2d - abc)) + (2b^4(a + b/x)(12ad - 5bc))/(3(a^2d - abc)^2) + (b(a + b/x)^2(36a^5d^5 - 60b^5c^5 - 456a^2b^3c^3d^2 + 120a^3b^2c^2d^3 + 308a^4b^4c^4d - 123a^4b^3c^4d^4))/(12a^2c^3(a^2d - abc)(ad - bc)^2) + (b(a + b/x)^4(12a^4d^6 + 20b^4c^4d^2 - 56a^2b^3c^3d^3 + 24a^2b^2c^2d^4 - 35a^3b^3c^4d^5))/(4a^2c^3(a^2d - abc)(ad - bc)^3) - (b(a + b/x)^3(72a^5d^6 - 120b^5c^5d + 496a^4b^4c^4d^2 - 592a^2b^3c^3d^3 + 303a^3b^2c^2d^4 - 264a^4b^3c^4d^5))/(12a^2c^3(a^2d - abc)(ad - bc)^3)}{(a + b/x)^{5/2} (3a^2d^2 + b^2c^2 - 4abc) - (a + b/x)^{7/2} (3ad^2 - 2bcd) + d^2(a + b/x)^{9/2} - (a + b/x)^{3/2} (a^3d^2 + ab^2c^2 - 2a^2bcd)} + \text{atan}\left(\frac{a^{15}b^{24}c^{24}(a + b/x)^{1/2} \cdot 2000i + a^{17}b^{22}c^{22}d^2(a + b/x)^{1/2} \cdot 77440i - a^{18}b^{21}c^{21}d^3(a + b/x)^{1/2} \cdot 1325984i + a^{19}b^{20}c^{20}d^4(a + b/x)^{1/2} \cdot 4135824i - a^{20}b^{19}c^{19}d^5(a + b/x)^{1/2} \cdot 8371440i + a^{21}b^{18}c^{18}d^6(a + b/x)^{1/2} \cdot 9129120i + a^{22}b^{17}c^{17}d^7(a + b/x)^{1/2} \cdot 3058605i - a^{23}b^{16}c^{16}d^8(a + b/x)^{1/2} \cdot 32337558i + a^{24}b^{15}c^{15}d^9(a + b/x)^{1/2} \cdot 63677218i - a^{25}b^{14}c^{14}d^{10}(a + b/x)^{1/2} \cdot 66665280i + a^{26}b^{13}c^{13}d^{11}(a + b/x)^{1/2} \cdot 24871035i + a^{27}b^{12}c^{12}d^{12}(a + b/x)^{1/2} \cdot 40203170i - a^{28}b^{11}c^{11}d^{13}(a + b/x)^{1/2} \cdot 85652532i + a^{29}b^{10}c^{10}d^{14}(a + b/x)^{1/2} \cdot 88170192i - a^{30}b^9c^9d^{15}(a + b/x)^{1/2} \cdot 60362445i + a^{31}b^8c^8d^{16}(a + b/x)^{1/2} \cdot 29178270i - a^{32}b^7c^7d^{17}(a + b/x)^{1/2} \cdot 9940590i + a^{33}b^6c^6d^{18}(a + b/x)^{1/2} \cdot 2287824i - a^{34}b^5c^5d^{19}(a + b/x)^{1/2} \cdot 320859i + a^{35}b^4c^4d^{20}(a + b/x)^{1/2} \cdot 20790i - a^{16}b^{23}c^{23}d(a + b/x)^{1/2} \cdot 34800i}{(a^7(a^7(a^7(29178270b^8c^8d^{16} - 9940590ab^7c^7d^{17} + 2287824a^2b^6c^6d^{18} - 320859a^3b^5c^5d^{19} + 20790a^4b^4c^4d^{20}) + 63677218b^{15}c^{15}d^9 - 66665280ab^{14}c^{14}d^{10} + 24871035a^2b^{13}c^{13}d^{11} + 40203170a^3b^{12}c^{12}d^{12} - 85652532a^4b^{11}c^{11}d^{13} + 88170192a^5b^{10}c^{10}d^{14} - 60362445a^6b^9c^9d^{15}) + 277440b^{22}c^{22}d^2 - 1325984ab^{21}c^{21}d^3 + 4135824a^2b^{20}c^{20}d^4 - 8371440a^3b^{19}c^{19}d^5 + 9129120a^4b^{18}c^{18}d^6 + 3058605a^5b^{17}c^{17}d^7 - 32337558a^6b^{16}c^{16}d^8) + 2000a^5b^{24}c^{24} - 34800a^6b^{23}c^{23}d)} \cdot (6ad + 5bc) \cdot \frac{1}{c^4(a^7)^{1/2}} + \log(400b^{25}c^{25}d^4 - 8240ab^{24}c^{24}d^5 - 1152a^{11}d^5(d^7(ad - bc)^9)^{3/2}(a + b/x)^{1/2} + 1152a^{20}d^{21}(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 79696a^2b^{23}c^{23}d^6 - 478768a^3b^{22}c^{22}d^7 + 1987568a^4b^{21}c^{21}d^8 - 5978896a^5b^{20}c^{20}d^9 + 13176240a^6b^{19}c^{19}d^{10} - 20525703a^7b^{18}c^{18}d^{11} + 18765714a^8b^{17}c^{17}d^{12} + 3763331a^9b^{16}c^{16}d^{13} - 49787452a^{10}b^{15}c^{15}d^{14} + 104120705a^{11}b^{14}c^{14}d^{15} - 140185682a^{12}b^{13}c^{13}d^{16} + 139985251a^{13}b^{12}c^{12}d^{17} - 108046616a^{14}b^{11}c^{11}d^{18} + 65184867a^{15}b^{10}c^{10}d^{19} - 30607170a^{16}b^9c^9d^{20} + 10996689a^{17}b^8c^8d^{21} - 2926572a^{18}b^7c^7d^{22} + 544467a^{19}b^6c^6d^{23} - 63294a^{20}b^5c^5d^{24} + 3465a^{21}b^4c^4d^{25} + 400b^{20}c^{20}d(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 9801a^6b^5c^5(d^7(ad - bc)^9)^{3/2}(a + b/x)^{1/2} - 37026a^7b^4c^4d(d^7(ad - bc)^9)^{3/2}(a + b/x)^{1/2} - 6240ab^{19}c^{19}d^2(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 47344a^8b^3c^3d^2(d^7(ad - bc)^9)^{3/2}(a + b/x)^{1/2} - 29216a^9b^2c^2d^3(d^7(ad - bc)^9)^{3/2}(a + b/x)^{1/2} + 44496a^2b^{18}c^{18}d^3(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} - 189888a^3b^{17}c^{17}d^4(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 528768a^4b^{16}c^{16}d^5(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} - 959616a^5b^{15}c^{15}d^6(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 972681a^6b^{14}c^{14}d^7(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} + 41238a^7b^{13}c^{13}d^8(d^7(ad - bc)^9)^{1/2}(a + b/x)^{1/2} - 1727195a^8$$

```

*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^(
1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(3*a^2*d^2 + (99*b^2*c^2)/8 - 11*a*b*c*d))/(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d) - (log(8240*a*b^24*c^24*d^5 - 400*b^25*c^25*d^4 - 1152
*a^11*d^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) + 1152*a^20*d^21*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 79696*a^2*b^23*c^23*d^6 + 478768*a^3*
b^22*c^22*d^7 - 1987568*a^4*b^21*c^21*d^8 + 5978896*a^5*b^20*c^20*d^9 - 131
76240*a^6*b^19*c^19*d^10 + 20525703*a^7*b^18*c^18*d^11 - 18765714*a^8*b^17*
c^17*d^12 - 3763331*a^9*b^16*c^16*d^13 + 49787452*a^10*b^15*c^15*d^14 - 104
120705*a^11*b^14*c^14*d^15 + 140185682*a^12*b^13*c^13*d^16 - 139985251*a^13
*b^12*c^12*d^17 + 108046616*a^14*b^11*c^11*d^18 - 65184867*a^15*b^10*c^10*d
^19 + 30607170*a^16*b^9*c^9*d^20 - 10996689*a^17*b^8*c^8*d^21 + 2926572*a^1
8*b^7*c^7*d^22 - 544467*a^19*b^6*c^6*d^23 + 63294*a^20*b^5*c^5*d^24 - 3465*
a^21*b^4*c^4*d^25 + 400*b^20*c^20*d*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/
2) + 9801*a^6*b^5*c^5*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 37026*a^7
*b^4*c^4*d*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 6240*a*b^19*c^19*d^2
*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 47344*a^8*b^3*c^3*d^2*(d^7*(a*
d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 29216*a^9*b^2*c^2*d^3*(d^7*(a*d - b*c)^
9)^(3/2)*(a + b/x)^(1/2) + 44496*a^2*b^18*c^18*d^3*(d^7*(a*d - b*c)^9)^(1/2
)*(a + b/x)^(1/2) - 189888*a^3*b^17*c^17*d^4*(d^7*(a*d - b*c)^9)^(1/2)*(a +
b/x)^(1/2) + 528768*a^4*b^16*c^16*d^5*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(
1/2) - 959616*a^5*b^15*c^15*d^6*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2)
+ 972681*a^6*b^14*c^14*d^7*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 4123
8*a^7*b^13*c^13*d^8*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 1727195*a^8
*b^12*c^12*d^9*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 2139672*a^9*b^11
*c^11*d^10*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 786834*a^10*b^10*c^1
0*d^11*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 6551292*a^11*b^9*c^9*d^1
2*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 11685186*a^12*b^8*c^8*d^13*(d
^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) - 12876696*a^13*b^7*c^7*d^14*(d^7*(
a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2) + 10033077*a^14*b^6*c^6*d^15*(d^7*(a*d
- b*c)^9)^(1/2)*(a + b/x)^(1/2) - 5737770*a^15*b^5*c^5*d^16*(d^7*(a*d - b*c
)^9)^(1/2)*(a + b/x)^(1/2) + 2414601*a^16*b^4*c^4*d^17*(d^7*(a*d - b*c)^9)^(
1/2)*(a + b/x)^(1/2) - 731920*a^17*b^3*c^3*d^18*(d^7*(a*d - b*c)^9)^(1/2)*
(a + b/x)^(1/2) + 151904*a^18*b^2*c^2*d^19*(d^7*(a*d - b*c)^9)^(1/2)*(a + b
/x)^(1/2) + 9024*a^10*b*c*d^4*(d^7*(a*d - b*c)^9)^(3/2)*(a + b/x)^(1/2) - 1
9392*a^19*b*c*d^20*(d^7*(a*d - b*c)^9)^(1/2)*(a + b/x)^(1/2))*(d^7*(a*d - b
*c)^9)^(1/2)*(24*a^2*d^2 + 99*b^2*c^2 - 88*a*b*c*d))/(8*(b^9*c^13 - a^9*c^4
*d^9 + 9*a^8*b*c^5*d^8 + 36*a^2*b^7*c^11*d^2 - 84*a^3*b^6*c^10*d^3 + 126*a^
4*b^5*c^9*d^4 - 126*a^5*b^4*c^8*d^5 + 84*a^6*b^3*c^7*d^6 - 36*a^7*b^2*c^6*d
^7 - 9*a*b^8*c^12*d))

```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x)\*\*(5/2)/(c+d/x)\*\*3,x)



[Out] Timed out

$$3.266 \quad \int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Optimal. Leaf size=123

$$x\sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} + \frac{(ad + bc) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{b}\sqrt{c + \frac{d}{x}}}\right)$$

[Out] (a\*d+b\*c)\*arctanh(c^(1/2)\*(a+b/x)^(1/2)/a^(1/2)/(c+d/x)^(1/2))/a^(1/2)/c^(1/2)-2\*arctanh(d^(1/2)\*(a+b/x)^(1/2)/b^(1/2)/(c+d/x)^(1/2))\*b^(1/2)\*d^(1/2)+x\*(a+b/x)^(1/2)\*(c+d/x)^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$ , Rules used = {375, 97, 157, 63, 217, 206, 93, 208}

$$x\sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} + \frac{(ad + bc) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{x}}}{\sqrt{a}\sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a}\sqrt{c}} - 2\sqrt{b}\sqrt{d} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{a + \frac{b}{x}}}{\sqrt{b}\sqrt{c + \frac{d}{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]\*Sqrt[c + d/x], x]

[Out] Sqrt[a + b/x]\*Sqrt[c + d/x]\*x + ((b\*c + a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*Sqrt[c]) - 2\*Sqrt[b]\*Sqrt[d]\*ArcTanh[(Sqrt[d]\*Sqrt[a + b/x])/(Sqrt[b]\*Sqrt[c + d/x])]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 97

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p)/(b\*(m + 1)), x] - Dist[1/(b\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p - 1)\*Simp[d\*e\*n + c\*f\*p + d\*f\*(n + p)\*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

### Rule 157

Int[(((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_)\*((g\_.) + (h\_.)\*(x\_)))/((a\_.) + (b\_.)\*(x\_)), x\_Symbol] :> Dist[h/b, Int[(c + d\*x)^n\*(e + f\*x)^

$p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, h, n, p\}, x]$

### Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b] \&\& (\text{GtQ}[a, 0] \parallel \text{LtQ}[b, 0])$

### Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x\_Symbol] :> \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{NegQ}[a/b]$

### Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2)], x\_Symbol] :> \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}\{a, b\}, x] \&\& !\text{GtQ}[a, 0]$

### Rule 375

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol] :> -\text{Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[n, 0]$

### Rubi steps

$$\begin{aligned}
 \int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx} \sqrt{c + dx}}{x^2} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} x - \text{Subst} \left( \int \frac{\frac{1}{2}(bc + ad) + bdx}{x\sqrt{a + bx} \sqrt{c + dx}} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} x - (bd) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} dx, x, \frac{1}{x} \right) - \frac{1}{2}(bc + ad) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} x - (2d) \text{Subst} \left( \int \frac{1}{\sqrt{c - \frac{ad}{b} + \frac{dx^2}{b}}} dx, x, \sqrt{a + \frac{b}{x}} \right) - (bc + ad) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx} \sqrt{c + dx}} dx, x, \frac{1}{x} \right) \\
 &= \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} x + \frac{(bc + ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a} \sqrt{c}} - (2d) \text{Subst} \left( \int \frac{1}{1 - \frac{dx^2}{b}} dx, x, \frac{\sqrt{a}}{\sqrt{c}} \right) \\
 &= \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} x + \frac{(bc + ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a} \sqrt{c}} - 2\sqrt{b} \sqrt{d} \tanh^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{b} \sqrt{c + \frac{d}{x}}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 1.40, size = 167, normalized size = 1.36

$$\frac{\sqrt{a + \frac{b}{x}} (cx + d) - 2\sqrt{d} \sqrt{bc - ad} \sqrt{\frac{bcx + bd}{bcx - ad}} \sinh^{-1} \left( \frac{\sqrt{d} \sqrt{a + \frac{b}{x}}}{\sqrt{bc - ad}} \right) + \frac{\sqrt{c + \frac{d}{x}} (ad + bc) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}} \right)}{\sqrt{a} \sqrt{c}}}{\sqrt{c + \frac{d}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]\*Sqrt[c + d/x],x]

[Out] (Sqrt[a + b/x]\*(d + c\*x) - 2\*Sqrt[d]\*Sqrt[b\*c - a\*d]\*Sqrt[(b\*d + b\*c\*x)/(b\*c\*x - a\*d\*x)]\*ArcSinh[(Sqrt[d]\*Sqrt[a + b/x])/Sqrt[b\*c - a\*d]] + ((b\*c + a\*d)\*Sqrt[c + d/x]\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*Sqrt[c])/Sqrt[c + d/x]

**fricas** [A] time = 2.16, size = 890, normalized size = 7.24

$$\frac{4acx\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}} + 2\sqrt{bd}ac\log\left(-\frac{8b^2d^2+(b^2c^2+6abcd+a^2d^2)x^2-4(2bdx+(bc+ad)x^2)\sqrt{bd}\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}+8(b^2cd+abd^2)x}{x^2}\right) + \sqrt{ac}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)\*(a+b/x)^(1/2),x, algorithm="fricas")

[Out] [1/4\*(4\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 2\*sqrt(b\*d)\*a\*c\*log(-(8\*b^2\*d^2 + (b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^2 - 4\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 8\*(b^2\*c\*d + a\*b\*d^2)\*x)/x^2) + sqrt(a\*c)\*(b\*c + a\*d)\*log(-8\*a^2\*c^2\*x^2 - b^2\*c^2 - 6\*a\*b\*c\*d - a^2\*d^2 - 4\*(2\*a\*c\*x^2 + (b\*c + a\*d)\*x)\*sqrt(a\*c)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)/(a\*c), 1/4\*(4\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 4\*sqrt(-b\*d)\*a\*c\*arctan(1/2\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(-b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(a\*b\*c\*d\*x^2 + b^2\*d^2 + (b^2\*c\*d + a\*b\*d^2)\*x)) + sqrt(a\*c)\*(b\*c + a\*d)\*log(-8\*a^2\*c^2\*x^2 - b^2\*c^2 - 6\*a\*b\*c\*d - a^2\*d^2 - 4\*(2\*a\*c\*x^2 + (b\*c + a\*d)\*x)\*sqrt(a\*c)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)/(a\*c), 1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + sqrt(b\*d)\*a\*c\*log(-(8\*b^2\*d^2 + (b^2\*c^2 + 6\*a\*b\*c\*d + a^2\*d^2)\*x^2 - 4\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 8\*(b^2\*c\*d + a\*b\*d^2)\*x)/x^2) - sqrt(-a\*c)\*(b\*c + a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d))/(a\*c), 1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) + 2\*sqrt(-b\*d)\*a\*c\*arctan(1/2\*(2\*b\*d\*x + (b\*c + a\*d)\*x^2)\*sqrt(-b\*d)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(a\*b\*c\*d\*x^2 + b^2\*d^2 + (b^2\*c\*d + a\*b\*d^2)\*x)) - sqrt(-a\*c)\*(b\*c + a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d))/(a\*c)]

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)^(1/2)\*(a+b/x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x)\*sqrt(c + d/x), x)

**maple** [B] time = 0.10, size = 253, normalized size = 2.06

$$\frac{\sqrt{\frac{ax+b}{x}}\sqrt{\frac{cx+d}{x}}\left(\sqrt{bd}ad\ln\left(\frac{2acx+ad+bc+2\sqrt{acx^2+adx+bcx+bd}\sqrt{ac}}{2\sqrt{ac}}\right)+\sqrt{bd}bc\ln\left(\frac{2acx+ad+bc+2\sqrt{acx^2+adx+bcx+bd}\sqrt{ac}}{2\sqrt{ac}}\right)-2\sqrt{acx^2+adx+bcx+bd}\sqrt{ac}\right)}{2\sqrt{acx^2+adx+bcx+bd}\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x)^(1/2)\*(a+b/x)^(1/2),x)

```
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*((b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*a*d+(b*d)^(1/2)*ln(1/2*(2*a*c*x+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)+a*d+b*c)/(a*c)^(1/2))*b*c-2*b*d*ln((a*d*x+b*c*x+2*(b*d)^(1/2)*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)+2*b*d)/x)*(a*c)^(1/2)+2*(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)*(a*c)^(1/2)*(b*d)^(1/2))/(a*c*x^2+a*d*x+b*c*x+b*d)^(1/2)/(a*c)^(1/2)/(b*d)^(1/2)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x)^(1/2)*(a+b/x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + b/x)*sqrt(c + d/x), x)
```

**mupad** [B] time = 22.22, size = 4674, normalized size = 38.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/x)^(1/2)*(c + d/x)^(1/2),x)
```

```
[Out] atan(((b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5)))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2)))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6)))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6 + 16*a^(21/2)*b^3*c^(7/2)*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) - (2*(a^(7/2)*b^11*c^(21/2) + 16*a^(9/2)*b^10*c^(19/2)*d - 42*a^(11/2)*b^9*c^(17/2)*d^2 + 25*a^(13/2)*b^8*c^(15/2)*d^3 + 25*a^(15/2)*b^7*c^(13/2)*d^4 - 42*a^(17/2)*b^6*c^(11/2)*d^5 + 16*a^(19/2)*b^5*c^(9/2)*d^6 + a^(21/2)*b^4*c^(7/2)*d^7))/(a^7*c^7*d^9) + (((a + b/x)^(1/2) - a^(1/2))*(146*a^4*b^10*c^10*d - 556*a^5*b^9*c^9*d^2 + 1006*a^6*b^8*c^8*d^3 - 1192*a^7*b^7*c^7*d^4 + 1006*a^8*b^6*c^6*d^5 - 556*a^9*b^5*c^5*d^6 + 146*a^10*b^4*c^4*d^7))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(2*a^4*b^11*c^10*d + 8*a^5*b^10*c^9*d^2 - 2*a^6*b^9*c^8*d^3 - 16*a^7*b^8*c^7*d^4 - 2*a^8*b^7*c^6*d^5 + 8*a^9*b^6*c^5*d^6 + 2*a^10*b^5*c^4*d^7))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(65*a^(7/2)*b^11*c^(21/2)*d - 297*a^(9/2)*b^10*c^(19/2)*d^2 + 597*a^(11/2)*b^9*c^(17/2)*d^3 - 365*a^(13/2)*b^8*c^(15/2)*d^4 - 365*a^(15/2)*b^7*c^(13/2)*d^5 + 597*a^(17/2)*b^6*c^(11/2)*d^6 - 297*a^(19/2)*b^5*c^(9/2)*d^7 + 65*a^(21/2)*b^4*c^(7/2)*d^8))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2))))*1i - (b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*(2*(b*d)^(1/2)*((2*(4*a^(9/2)*b^9*c^(19/2) - 4*a^(13/2)*b^7*c^(15/2)*d^2 - 4*a^(15/2)*b^6*c^(13/2)*d^3 + 4*a^(19/2)*b^4*c^(9/2)*d^5)))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(32*a^4*b^9*c^10 - 120*a^5*b^8*c^9*d + 288*a^6*b^7*c^8*d^2 - 400*a^7*b^6*c^7*d^3 + 288*a^8*b^5*c^6*d^4 - 120*a^9*b^4*c^5*d^5 + 32*a^10*b^3*c^4*d^6)))/(2*a^7*c^7*d^9*((c + d/x)^(1/2) - c^(1/2)))) + (2*(8*a^5*b^9*c^9*d + 16*a^6*b^8*c^8*d^2 - 48*a^7*b^7*c^7*d^3 + 16*a^8*b^6*c^6*d^4 + 8*a^9*b^5*c^5*d^5))/(a^7*c^7*d^9) - (((a + b/x)^(1/2) - a^(1/2))*(16*a^(7/2)*b^10*c^(21/2) - 76*a^(9/2)*b^9*c^(19/2)*d + 228*a^(11/2)*b^8*c^(17/2)*d^2 - 168*a^(13/2)*b^7*c^(15/2)*d^3 - 168*a^(15/2)*b^6*c^(13/2)*d^4 + 228*a^(17/2)*b^5*c^(11/2)*d^5 - 76*a^(19/2)*b^4*c^(9/2)*d^6
```



$$\begin{aligned}
& - a^{(1/2)}(65a^{(7/2)}b^{11}c^{(21/2)}d - 297a^{(9/2)}b^{10}c^{(19/2)}d^2 + 597a^{(11/2)}b^9c^{(17/2)}d^3 - 365a^{(13/2)}b^8c^{(15/2)}d^4 - 365a^{(15/2)} \\
& *b^7c^{(13/2)}d^5 + 597a^{(17/2)}b^6c^{(11/2)}d^6 - 297a^{(19/2)}b^5c^{(9/2)}d^7 + 65a^{(21/2)}b^4c^{(7/2)}d^8) / (2a^7c^7d^9((c + d/x)^{(1/2)} - c^{(1/2)})) \\
& + (7a^{(7/2)}b^{12}c^{(21/2)}d - 7a^{(9/2)}b^{11}c^{(19/2)}d^2 - 21a^{(11/2)}b^{10}c^{(17/2)}d^3 + 21a^{(13/2)}b^9c^{(15/2)}d^4 + 21a^{(15/2)}b^8c^{(13/2)}d^5 \\
& - 21a^{(17/2)}b^7c^{(11/2)}d^6 - 7a^{(19/2)}b^6c^{(9/2)}d^7 + 7a^{(21/2)}b^5c^{(7/2)}d^8) / (a^7c^7d^9 + ((a + b/x)^{(1/2)} - a^{(1/2)})(112a^5b^{10}c^9d^3 \\
& - 56a^4b^{11}c^{10}d^2 + 56a^6b^9c^8d^4 - 224a^7b^8c^7d^5 + 56a^8b^7c^6d^6 + 112a^9b^6c^5d^7 - 56a^{10}b^5c^4d^8)) / (2a^7c^7d^9((c + d/x)^{(1/2)} - c^{(1/2)})) \\
& * (b*d)^{(1/2)} * 4i - (((a + b/x)^{(1/2)} - a^{(1/2)}) * ((b^2*c)/4 + (a*b*d)/4)) / (a^{(1/2)} * c^{(1/2)} * d * ((c + d/x)^{(1/2)} - c^{(1/2)})) \\
& - b^2/(4*d) + (((a + b/x)^{(1/2)} - a^{(1/2)})^2 * ((a^2*d^2)/4 + (b^2*c^2)/4 - (3*a*b*c*d)/4)) / (a*c*d * ((c + d/x)^{(1/2)} - c^{(1/2)})^2) / (((a + b/x)^{(1/2)} - a^{(1/2)})^3 / ((c + d/x)^{(1/2)} - c^{(1/2)})^3 \\
& + (b * ((a + b/x)^{(1/2)} - a^{(1/2)})) / (d * ((c + d/x)^{(1/2)} - c^{(1/2)})) - (((a + b/x)^{(1/2)} - a^{(1/2)})^2 * (a*d + b*c)) / (a^{(1/2)} * c^{(1/2)} * d * ((c + d/x)^{(1/2)} - c^{(1/2)})^2) \\
& + (d * ((a + b/x)^{(1/2)} - a^{(1/2)})) / (4 * ((c + d/x)^{(1/2)} - c^{(1/2)})) + (\log(((a + b/x)^{(1/2)} - a^{(1/2)}) / ((c + d/x)^{(1/2)} - c^{(1/2)})) * (a*d + b*c)) / (2*a^{(1/2)} * c^{(1/2)}) \\
& - (\log(((c^{(1/2)} * (a + b/x)^{(1/2)} - a^{(1/2)} * (c + d/x)^{(1/2)}) * (b*c^{(1/2)} - (a^{(1/2)} * d * ((a + b/x)^{(1/2)} - a^{(1/2)})) / ((c + d/x)^{(1/2)} - c^{(1/2)}))) / ((c + d/x)^{(1/2)} - c^{(1/2)})) * (a^{(1/2)} * b * c^{(3/2)} + a^{(3/2)} * c^{(1/2)} * d)) / (2*a * c)
\end{aligned}$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x)\*\*(1/2)\*(a+b/x)\*\*(1/2),x)

[Out] Integral(sqrt(a + b/x)\*sqrt(c + d/x), x)

$$3.267 \quad \int \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} dx$$

**Optimal.** Leaf size=81

$$\frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a+\frac{b}{x}}}{\sqrt{a} \sqrt{c+\frac{d}{x}}} \right)}{\sqrt{a} c^{3/2}} + \frac{x \sqrt{a+\frac{b}{x}} \sqrt{c+\frac{d}{x}}}{c}$$

[Out]  $(-a*d+b*c)*\operatorname{arctanh}(c^{(1/2)}*(a+b/x)^{(1/2)}/a^{(1/2)/(c+d/x)^{(1/2)})/c^{(3/2)}/a^{(1/2)+x*(a+b/x)^{(1/2)}*(c+d/x)^{(1/2)}/c$

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$ , Rules used = {375, 94, 93, 208}

$$\frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a+\frac{b}{x}}}{\sqrt{a} \sqrt{c+\frac{d}{x}}} \right)}{\sqrt{a} c^{3/2}} + \frac{x \sqrt{a+\frac{b}{x}} \sqrt{c+\frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/Sqrt[c + d/x], x]

[Out]  $(\operatorname{Sqrt}[a + b/x]*\operatorname{Sqrt}[c + d/x]*x)/c + ((b*c - a*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b/x])/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c + d/x])])/(\operatorname{Sqrt}[a]*c^{(3/2)})$

#### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

#### Rule 94

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*e - a\*f)), x] - Dist[(n\*(d\*e - c\*f))/((m + 1)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

#### Rule 208

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

#### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps



$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx}}{x^2 \sqrt{c + dx}} dx, x, \frac{1}{x} \right) \\
&= \frac{\sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}}}{c} - \frac{(bc - ad) \text{Subst} \left( \int \frac{1}{x \sqrt{a+bx} \sqrt{c+dx}} dx, x, \frac{1}{x} \right)}{2c} \\
&= \frac{\sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}}}{c} - \frac{(bc - ad) \text{Subst} \left( \int \frac{1}{-a+cx^2} dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}} \right)}{c} \\
&= \frac{\sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}}}{c} + \frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a+\frac{b}{x}}}{\sqrt{a} \sqrt{c+\frac{d}{x}}} \right)}{\sqrt{a} c^{3/2}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 81, normalized size = 1.00

$$\frac{(bc - ad) \tanh^{-1} \left( \frac{\sqrt{c} \sqrt{a+\frac{b}{x}}}{\sqrt{a} \sqrt{c+\frac{d}{x}}} \right)}{\sqrt{a} c^{3/2}} + \frac{x \sqrt{a + \frac{b}{x}} \sqrt{c + \frac{d}{x}}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/Sqrt[c + d/x], x]

[Out] (Sqrt[a + b/x]\*Sqrt[c + d/x]\*x)/c + ((b\*c - a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(3/2))

**fricas [A]** time = 0.93, size = 247, normalized size = 3.05

$$\left[ \frac{4 acx \sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} - \sqrt{ac} (bc - ad) \log \left( -8 a^2 c^2 x^2 - b^2 c^2 - 6 abcd - a^2 d^2 + 4 (2 acx^2 + (bc + ad)x) \sqrt{ac} \sqrt{\frac{ax+b}{x}} \right)}{4 ac^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2), x, algorithm="fricas")

[Out] [1/4\*(4\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - sqrt(a\*c)\*(b\*c - a\*d)\*log(-8\*a^2\*c^2\*x^2 - b^2\*c^2 - 6\*a\*b\*c\*d - a^2\*d^2 + 4\*(2\*a\*c\*x^2 + (b\*c + a\*d)\*x)\*sqrt(a\*c)\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - 8\*(a\*b\*c^2 + a^2\*c\*d)\*x)/(a\*c^2), 1/2\*(2\*a\*c\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x) - sqrt(-a\*c)\*(b\*c - a\*d)\*arctan(2\*sqrt(-a\*c)\*x\*sqrt((a\*x + b)/x)\*sqrt((c\*x + d)/x)/(2\*a\*c\*x + b\*c + a\*d)))/(a\*c^2)]

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)

**maple [B]** time = 0.10, size = 155, normalized size = 1.91

$$\frac{\sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} \left( -ad \ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)} \sqrt{ac}}{2\sqrt{ac}}\right) + bc \ln\left(\frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)} \sqrt{ac}}{2\sqrt{ac}}\right) + 2\sqrt{(ax+b)(cx+d)} \sqrt{ac} \right)}{2\sqrt{(ax+b)(cx+d)} \sqrt{ac} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^(1/2)/(c+d/x)^(1/2), x)

[Out] 1/2\*((a\*x+b)/x)^(1/2)\*x\*((c\*x+d)/x)^(1/2)\*(-ln(1/2\*(2\*a\*c\*x+2\*((a\*x+b)\*(c\*x+d))^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*a\*d+ln(1/2\*(2\*a\*c\*x+2\*((a\*x+b)\*(c\*x+d))^(1/2)\*(a\*c)^(1/2)+a\*d+b\*c)/(a\*c)^(1/2))\*b\*c+2\*((a\*x+b)\*(c\*x+d))^(1/2)\*(a\*c)^(1/2))/((a\*x+b)\*(c\*x+d))^(1/2)/c/(a\*c)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^(1/2)/(c+d/x)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x)/sqrt(c + d/x), x)

**mupad [B]** time = 6.58, size = 478, normalized size = 5.90

$$\frac{d \left( \sqrt{a + \frac{b}{x}} - \sqrt{a} \right)}{4c \left( \sqrt{c + \frac{d}{x}} - \sqrt{c} \right)} - \frac{\left( \sqrt{a + \frac{b}{x}} - \sqrt{a} \right) \left( \frac{cb^2}{4} + \frac{adb}{4} \right)}{\sqrt{a} c^{3/2} d \left( \sqrt{c + \frac{d}{x}} - \sqrt{c} \right)} - \frac{b^2}{4cd} + \frac{\left( \sqrt{a + \frac{b}{x}} - \sqrt{a} \right)^2 \left( \frac{a^2 d^2}{4} - \frac{3abcd}{4} + \frac{b^2 c^2}{4} \right)}{a c^2 d \left( \sqrt{c + \frac{d}{x}} - \sqrt{c} \right)^2} + \frac{\ln \left( \frac{\sqrt{a + \frac{b}{x}} - \sqrt{a}}{\sqrt{c + \frac{d}{x}} - \sqrt{c}} \right) \left( \sqrt{a} b c^{3/2} - a^{3/2} \sqrt{c} \right)}{2 a c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)/(c + d/x)^(1/2), x)

[Out] (d\*((a + b/x)^(1/2) - a^(1/2)))/(4\*c\*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2) - a^(1/2))\*((b^2\*c)/4 + (a\*b\*d)/4))/(a^(1/2)\*c^(3/2)\*d\*((c + d/x)^(1/2) - c^(1/2))) - b^2/(4\*c\*d) + (((a + b/x)^(1/2) - a^(1/2))^2\*((a^2\*d^2)/4 + (b^2\*c^2)/4 - (3\*a\*b\*c\*d)/4))/(a\*c^2\*d\*((c + d/x)^(1/2) - c^(1/2))^2) + (((a + b/x)^(1/2) - a^(1/2))^3)/((c + d/x)^(1/2) - c^(1/2))^3 + (b\*((a + b/x)^(1/2) - a^(1/2)))/(d\*((c + d/x)^(1/2) - c^(1/2))) - (((a + b/x)^(1/2) - a^(1/2))^2\*(a\*d + b\*c))/(a^(1/2)\*c^(1/2)\*d\*((c + d/x)^(1/2) - c^(1/2))^2) + (log(((a + b/x)^(1/2) - a^(1/2)))/((c + d/x)^(1/2) - c^(1/2)))\*(a^(1/2)\*b\*c^(3/2) - a^(3/2)\*c^(1/2)\*d)/(2\*a\*c^2) - (log(((c^(1/2)\*a + b/x)^(1/2) - a^(1/2)\*c + d/x)^(1/2)))/((c + d/x)^(1/2) - c^(1/2)) + (log(((c^(1/2)\*a + b/x)^(1/2) - a^(1/2)\*c + d/x)^(1/2)))/((c + d/x)^(1/2) - c^(1/2)))/(a^(1/2)\*b\*c^(3/2) - a^(3/2)\*c^(1/2)\*d)/(2\*a\*c^2)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\sqrt{c + \frac{d}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)**(1/2)/(c+d/x)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b/x)/sqrt(c + d/x), x)
```

$$3.268 \quad \int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx$$

**Optimal.** Leaf size=122

$$\frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a} c^{5/2}} - \frac{\sqrt{a + \frac{b}{x}} (bc - 3ad)}{ac^2 \sqrt{c + \frac{d}{x}}} + \frac{x \left(a + \frac{b}{x}\right)^{3/2}}{ac \sqrt{c + \frac{d}{x}}}$$

[Out]  $(-3*a*d+b*c)*\operatorname{arctanh}(c^{1/2}*(a+b/x)^{1/2}/a^{1/2}/(c+d/x)^{1/2})/c^{5/2}/a^{1/2}+(a+b/x)^{3/2}*x/a/c/(c+d/x)^{1/2}-(-3*a*d+b*c)*(a+b/x)^{1/2}/a/c^2/(c+d/x)^{1/2}$

**Rubi [A]** time = 0.08, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$ , Rules used = {375, 96, 94, 93, 208}

$$-\frac{\sqrt{a + \frac{b}{x}} (bc - 3ad)}{ac^2 \sqrt{c + \frac{d}{x}}} + \frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{a + \frac{b}{x}}}{\sqrt{a} \sqrt{c + \frac{d}{x}}}\right)}{\sqrt{a} c^{5/2}} + \frac{x \left(a + \frac{b}{x}\right)^{3/2}}{ac \sqrt{c + \frac{d}{x}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x]/(c + d/x)^(3/2), x]

[Out]  $-(((b*c - 3*a*d)*\operatorname{Sqrt}[a + b/x])/(a*c^2*\operatorname{Sqrt}[c + d/x])) + ((a + b/x)^{3/2}*x)/(a*c*\operatorname{Sqrt}[c + d/x]) + ((b*c - 3*a*d)*\operatorname{ArcTanh}[(\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + b/x])]/(\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c + d/x]))/(\operatorname{Sqrt}[a]*c^{5/2})$

### Rule 93

Int[(((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_))/((e\_.) + (f\_.)\*(x\_)), x\_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q\*(m + 1) - 1)/(b\*e - a\*f - (d\*e - c\*f)\*x^q), x], x, (a + b\*x)^(1/q)/(c + d\*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b\*x, c + d\*x]

### Rule 94

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/(m + 1)\*(b\*e - a\*f), x] - Dist[(n\*(d\*e - c\*f))/(m + 1)\*(b\*e - a\*f), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

### Rule 96

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[(a\*d\*f\*(m + 1) + b\*c\*f\*(n + 1) + b\*d\*e\*(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[-(a/b), 2]\*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 375

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx &= -\text{Subst}\left(\int \frac{\sqrt{a + bx}}{x^2(c + dx)^{3/2}} dx, x, \frac{1}{x}\right) \\ &= \frac{\left(a + \frac{b}{x}\right)^{3/2} x}{ac\sqrt{c + \frac{d}{x}}} + \frac{\left(-\frac{bc}{2} + \frac{3ad}{2}\right) \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x(c+dx)^{3/2}} dx, x, \frac{1}{x}\right)}{ac} \\ &= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{ac^2\sqrt{c + \frac{d}{x}}} + \frac{\left(a + \frac{b}{x}\right)^{3/2} x}{ac\sqrt{c + \frac{d}{x}}} - \frac{(bc - 3ad) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}\sqrt{c+dx}} dx, x, \frac{1}{x}\right)}{2c^2} \\ &= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{ac^2\sqrt{c + \frac{d}{x}}} + \frac{\left(a + \frac{b}{x}\right)^{3/2} x}{ac\sqrt{c + \frac{d}{x}}} - \frac{(bc - 3ad) \text{Subst}\left(\int \frac{1}{-a+cx^2} dx, x, \frac{\sqrt{a+\frac{b}{x}}}{\sqrt{c+\frac{d}{x}}}\right)}{c^2} \\ &= -\frac{(bc - 3ad)\sqrt{a + \frac{b}{x}}}{ac^2\sqrt{c + \frac{d}{x}}} + \frac{\left(a + \frac{b}{x}\right)^{3/2} x}{ac\sqrt{c + \frac{d}{x}}} + \frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 87, normalized size = 0.71

$$\frac{(bc - 3ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+\frac{b}{x}}}{\sqrt{a}\sqrt{c+\frac{d}{x}}}\right)}{\sqrt{a}c^{5/2}} + \frac{\sqrt{a + \frac{b}{x}}(cx + 3d)}{c^2\sqrt{c + \frac{d}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x]/(c + d/x)^(3/2), x]

[Out] (Sqrt[a + b/x]\*(3\*d + c\*x))/(c^2\*Sqrt[c + d/x]) + ((b\*c - 3\*a\*d)\*ArcTanh[(Sqrt[c]\*Sqrt[a + b/x])/(Sqrt[a]\*Sqrt[c + d/x])])/(Sqrt[a]\*c^(5/2))

**fricas [A]** time = 1.30, size = 319, normalized size = 2.61

$$\left[ \frac{(bcd - 3ad^2 + (bc^2 - 3acd)x)\sqrt{ac} \log\left(-8a^2c^2x^2 - b^2c^2 - 6abcd - a^2d^2 + 4(2acx^2 + (bc + ad)x)\sqrt{ac}\sqrt{\frac{ax-b}{x}}\right)}{4(ac^4x + ac^3d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="fricas")
[Out] [-1/4*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(a*c)*log(-8*a^2*c^2*x^2 - b^2*c^2 - 6*a*b*c*d - a^2*d^2 + 4*(2*a*c*x^2 + (b*c + a*d)*x)*sqrt(a*c)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x) - 8*(a*b*c^2 + a^2*c*d)*x) - 4*(a*c^2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d), - 1/2*((b*c*d - 3*a*d^2 + (b*c^2 - 3*a*c*d)*x)*sqrt(-a*c)*arctan(2*sqrt(-a*c)*x*sqrt((a*x + b)/x)*sqrt((c*x + d)/x)/(2*a*c*x + b*c + a*d)) - 2*(a*c^2*x^2 + 3*a*c*d*x)*sqrt((a*x + b)/x)*sqrt((c*x + d)/x))/(a*c^4*x + a*c^3*d)]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Warning, integration of abs or sign assumes constant sign by intervals (cor
rect if the argument is real):Check [abs(t_nostep)]Unable to divide, perhap
s due to rounding error%%{%%{1, [1]%%}, [2, 1, 2]%%}+%%{%%{[-2, 0] : [1, 0, %%
{-1, [1]%%}]%%}, [1, 1, 3]%%}+%%{1, [0, 1, 4]%%} / %%{%%{1, [2]%%}, [2, 0, 0]%%
}+%%{%%{[%%{-2, [1]%%}, 0] : [1, 0, %%{-1, [1]%%}]%%}, [1, 0, 1]%%}+%%{%%{1,
[1]%%}, [0, 0, 2]%%} Error: Bad Argument Value
maple [B] time = 0.08, size = 280, normalized size = 2.30
```

$$\frac{\sqrt{\frac{ax+b}{x}} \sqrt{\frac{cx+d}{x}} \left( -3acd x \ln \left( \frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)} \sqrt{ac}}{2\sqrt{ac}} \right) + b c^2 x \ln \left( \frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)} \sqrt{ac}}{2\sqrt{ac}} \right) - 3a d^2 \ln \left( \frac{2acx+ad+bc+2\sqrt{(ax+b)(cx+d)} \sqrt{ac}}{2\sqrt{ac}} \right) \right)}{(cx+d)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/x)^(1/2)/(c+d/x)^(3/2),x)
[Out] 1/2*((a*x+b)/x)^(1/2)*x*((c*x+d)/x)^(1/2)*(-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)))/(a*c)^(1/2))*x*a*c*d+ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)))/(a*c)^(1/2))*x*b*c^2+2*x*c*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)-3*ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)))/(a*c)^(1/2))*a*d^2+ln(1/2*(2*a*c*x+a*d+b*c+2*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2)))/(a*c)^(1/2))*b*c*d+6*d*((a*x+b)*(c*x+d))^(1/2)*(a*c)^(1/2))/(a*c)^(1/2)/(c*x+d)/((a*x+b)*(c*x+d))^(1/2)/c^2
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/x)^(1/2)/(c+d/x)^(3/2),x, algorithm="maxima")
[Out] integrate(sqrt(a + b/x)/(c + d/x)^(3/2), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)

[Out] int((a + b/x)^(1/2)/(c + d/x)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x}}}{\left(c + \frac{d}{x}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)\*\*(1/2)/(c+d/x)\*\*(3/2), x)

[Out] Integral(sqrt(a + b/x)/(c + d/x)\*\*(3/2), x)

$$3.269 \quad \int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

**Optimal.** Leaf size=96

$$\frac{b \left(a + \frac{b}{x}\right)^{p+1} \left(c + \frac{d}{x}\right)^q \left(\frac{b\left(c + \frac{d}{x}\right)}{bc-ad}\right)^{-q} F_1\left(p+1; -q, 2; p+2; -\frac{d\left(a + \frac{b}{x}\right)}{bc-ad}, \frac{a + \frac{b}{x}}{a}\right)}{a^2(p+1)}$$

[Out]  $-b*(a+b/x)^{(1+p)}*(c+d/x)^q*AppellF1(1+p, 2, -q, 2+p, (a+b/x)/a, -d*(a+b/x)/(-a*d+b*c))/a^2/(1+p)/((b*(c+d/x)/(-a*d+b*c))^q)$

**Rubi [A]** time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {375, 137, 136}

$$\frac{b \left(a + \frac{b}{x}\right)^{p+1} \left(c + \frac{d}{x}\right)^q \left(\frac{b\left(c + \frac{d}{x}\right)}{bc-ad}\right)^{-q} F_1\left(p+1; -q, 2; p+2; -\frac{d\left(a + \frac{b}{x}\right)}{bc-ad}, \frac{a + \frac{b}{x}}{a}\right)}{a^2(p+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b/x)^p*(c + d/x)^q, x]$

[Out]  $-((b*(a + b/x)^{(1 + p)}*(c + d/x)^q*AppellF1[1 + p, -q, 2, 2 + p, -((d*(a + b/x))/(b*c - a*d)), (a + b/x)/a])/a^2*(1 + p)*((b*(c + d/x))/(b*c - a*d))^{-q})$

#### Rule 136

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x\_Symbol] :> \text{Simp}[(b*e - a*f)^p*(a + b*x)^{(m + 1)}*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^{(p + 1)}*(m + 1)*(b/(b*c - a*d))^n), x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& !(\text{GtQ}[d/(d*a - c*b), 0]) \&\& \text{SimplerQ}[c + d*x, a + b*x]$

#### Rule 137

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}), x\_Symbol] :> \text{Dist}[(c + d*x)^{\text{FracPart}[n]}/((b/(b*c - a*d))^{\text{IntPart}[n]}*((b*(c + d*x))/(b*c - a*d))^{\text{FracPart}[n]}), \text{Int}[(a + b*x)^m*((b*c)/(b*c - a*d) + (b*d*x)/(b*c - a*d))^n*(e + f*x)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n\}, x] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{IntegerQ}[p] \&\& !\text{GtQ}[b/(b*c - a*d), 0] \&\& !\text{SimplerQ}[c + d*x, a + b*x]$

#### Rule 375

$\text{Int}[(a_ + (b_)*(x_)^{(n_}))^{(p_)}*((c_ + (d_)*(x_)^{(n_}))^{(q_)}), x\_Symbol] :> -\text{Subst}[\text{Int}[(a + b/x^n)^p*(c + d/x^n)^q/x^2, x], x, 1/x] /; \text{FreeQ}\{a, b, c, d, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{ILtQ}[n, 0]$

#### Rubi steps



$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx = -\text{Subst} \left( \int \frac{(a + bx)^p (c + dx)^q}{x^2} dx, x, \frac{1}{x} \right)$$

$$= - \left( \left( c + \frac{d}{x} \right)^q \left( \frac{b \left( c + \frac{d}{x} \right)}{bc - ad} \right)^{-q} \right) \text{Subst} \left( \int \frac{(a + bx)^p \left( \frac{bc}{bc - ad} + \frac{bdx}{bc - ad} \right)^q}{x^2} dx, x, \frac{1}{x} \right)$$

$$= - \frac{b \left( a + \frac{b}{x} \right)^{1+p} \left( c + \frac{d}{x} \right)^q \left( \frac{b \left( c + \frac{d}{x} \right)}{bc - ad} \right)^{-q} F_1 \left( 1 + p; -q, 2; 2 + p; -\frac{d \left( a + \frac{b}{x} \right)}{bc - ad}, \frac{a + \frac{b}{x}}{a} \right)}{a^2(1 + p)}$$

**Mathematica** [B] time = 0.37, size = 206, normalized size = 2.15

$$\frac{bdx(p+q-2) \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q F_1(-p-q+1; -p, -q; -p-q+2, -p-q+1; -\frac{ax}{b}, -\frac{cx}{d}) + bcqF_1(-p-q+2; -p, 1-q; -p-q+3; -p-q+2, -p-q+1; -\frac{ax}{b}, -\frac{cx}{d})}{(p+q-1) \left(x \left(adpF_1(-p-q+2; 1-p, -q; -p-q+3; -\frac{ax}{b}, -\frac{cx}{d}) + bcqF_1(-p-q+2; -p, 1-q; -p-q+3; -p-q+2, -p-q+1; -\frac{ax}{b}, -\frac{cx}{d})\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b/x)^p\*(c + d/x)^q,x]

[Out] (b\*d\*(-2 + p + q)\*(a + b/x)^p\*(c + d/x)^q\*x\*AppellF1[1 - p - q, -p, -q, 2 - p - q, -((a\*x)/b), -((c\*x)/d)]/((-1 + p + q)\*(-b\*d\*(-2 + p + q)\*AppellF1[1 - p - q, -p, -q, 2 - p - q, -((a\*x)/b), -((c\*x)/d)] + x\*(a\*d\*p\*AppellF1[2 - p - q, 1 - p, -q, 3 - p - q, -((a\*x)/b), -((c\*x)/d)] + b\*c\*q\*AppellF1[2 - p - q, -p, 1 - q, 3 - p - q, -((a\*x)/b), -((c\*x)/d)]))

**fricas** [F] time = 1.04, size = 0, normalized size = 0.00

$$\text{integral} \left( \left( \frac{ax + b}{x} \right)^p \left( \frac{cx + d}{x} \right)^q, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^p\*(c+d/x)^q,x, algorithm="fricas")

[Out] integral(((a\*x + b)/x)^p\*((c\*x + d)/x)^q, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x)^p\*(c+d/x)^q,x, algorithm="giac")

[Out] integrate((a + b/x)^p\*(c + d/x)^q, x)

**maple** [F] time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x)^p\*(c+d/x)^q,x)

[Out] `int((a+b/x)^p*(c+d/x)^q,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)^p*(c+d/x)^q,x, algorithm="maxima")`

[Out] `integrate((a + b/x)^p*(c + d/x)^q, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x)^p*(c + d/x)^q,x)`

[Out] `int((a + b/x)^p*(c + d/x)^q, x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x}\right)^p \left(c + \frac{d}{x}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x)**p*(c+d/x)**q,x)`

[Out] `Integral((a + b/x)**p*(c + d/x)**q, x)`

$$3.270 \quad \int \frac{a + \frac{b}{x^2}}{c + \frac{d}{x^2}} dx$$

**Optimal.** Leaf size=39

$$\frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right) + \frac{ax}{c}}{c^{3/2} \sqrt{d}}$$

[Out] a\*x/c+(-a\*d+b\*c)\*arctan(x\*c^(1/2)/d^(1/2))/c^(3/2)/d^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {374, 388, 205}

$$\frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right) + \frac{ax}{c}}{c^{3/2} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)/(c + d/x^2), x]

[Out] (a\*x)/c + ((b\*c - a\*d)\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]])/(c^(3/2)\*Sqrt[d])

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 374

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Int[x^(n\*(p+q))\*(b + a/x^n)^p\*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IntegersQ[p, q] && NegQ[n]

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(b\*(n\*(p+1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{a + \frac{b}{x^2}}{c + \frac{d}{x^2}} dx &= \int \frac{b + ax^2}{d + cx^2} dx \\ &= \frac{ax}{c} - \frac{(-bc + ad) \int \frac{1}{d+cx^2} dx}{c} \\ &= \frac{ax}{c} + \frac{(bc - ad) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 40, normalized size = 1.03

$$\frac{ax}{c} - \frac{(ad - bc) \tan^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right)}{c^{3/2} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^2)/(c + d/x^2), x]

[Out] (a\*x)/c - ((-b\*c) + a\*d)\*ArcTan[(Sqrt[c]\*x)/Sqrt[d]]/(c^(3/2)\*Sqrt[d])

**fricas** [A] time = 0.88, size = 98, normalized size = 2.51

$$\left[ \frac{2acdx + (bc - ad)\sqrt{-cd} \log\left(\frac{cx^2 + 2\sqrt{-cd}x - d}{cx^2 + d}\right)}{2c^2d}, \frac{acdx + (bc - ad)\sqrt{cd} \arctan\left(\frac{\sqrt{cd}x}{d}\right)}{c^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2), x, algorithm="fricas")

[Out] [1/2\*(2\*a\*c\*d\*x + (b\*c - a\*d)\*sqrt(-c\*d)\*log((c\*x^2 + 2\*sqrt(-c\*d)\*x - d)/(c\*x^2 + d)))/(c^2\*d), (a\*c\*d\*x + (b\*c - a\*d)\*sqrt(c\*d)\*arctan(sqrt(c\*d)\*x/d))/(c^2\*d)]

**giac** [A] time = 0.15, size = 33, normalized size = 0.85

$$\frac{ax}{c} + \frac{(bc - ad) \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2), x, algorithm="giac")

[Out] a\*x/c + (b\*c - a\*d)\*arctan(c\*x/sqrt(c\*d))/(sqrt(c\*d)\*c)

**maple** [A] time = 0.05, size = 45, normalized size = 1.15

$$-\frac{ad \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c} + \frac{b \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}} + \frac{ax}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)/(c+d/x^2), x)

[Out] a\*x/c - 1/c/(c\*d)^(1/2)\*arctan(c\*x/(c\*d)^(1/2)) + a\*d + 1/(c\*d)^(1/2)\*arctan(c\*x/(c\*d)^(1/2))\*b

**maxima** [A] time = 1.30, size = 33, normalized size = 0.85

$$\frac{ax}{c} + \frac{(bc - ad) \arctan\left(\frac{cx}{\sqrt{cd}}\right)}{\sqrt{cd}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)/(c+d/x^2), x, algorithm="maxima")

[Out] a\*x/c + (b\*c - a\*d)\*arctan(c\*x/sqrt(c\*d))/(sqrt(c\*d)\*c)

**mupad** [B] time = 0.07, size = 32, normalized size = 0.82

$$\frac{ax}{c} - \frac{\operatorname{atan}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)(ad - bc)}{c^{3/2}\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)/(c + d/x^2),x)`

[Out] `(a*x)/c - (atan((c^(1/2)*x)/d^(1/2))*(a*d - b*c))/(c^(3/2)*d^(1/2))`

**sympy [B]** time = 0.33, size = 82, normalized size = 2.10

$$\frac{ax}{c} + \frac{\sqrt{-\frac{1}{c^3d}} (ad - bc) \log\left(-cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2} - \frac{\sqrt{-\frac{1}{c^3d}} (ad - bc) \log\left(cd\sqrt{-\frac{1}{c^3d}} + x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)/(c+d/x**2),x)`

[Out] `a*x/c + sqrt(-1/(c**3*d))*(a*d - b*c)*log(-c*d*sqrt(-1/(c**3*d)) + x)/2 - s  
qrt(-1/(c**3*d))*(a*d - b*c)*log(c*d*sqrt(-1/(c**3*d)) + x)/2`

$$3.271 \quad \int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

**Optimal.** Leaf size=233

$$-\frac{2d\sqrt{a + \frac{b}{x^2}}}{x\sqrt{c + \frac{d}{x^2}}} + x\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} - \frac{\sqrt{c}\sqrt{a + \frac{b}{x^2}}(ad + bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| \frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}\right)}{\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

[Out]  $-2*d*(a+b/x^2)^{(1/2)}/x/(c+d/x^2)^{(1/2)}-(a*d+b*c)*(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x*(1+x^2*c/d)^{(1/2)}*EllipticF(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/a/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+2*(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x*d*(1+x^2*c/d)^{(1/2)}*EllipticE(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+x*(a+b/x^2)^{(1/2)}*(c+d/x^2)^{(1/2)}$

**Rubi [A]** time = 0.22, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, number of rules / integrand size = 0.261, Rules used = {375, 473, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a + \frac{b}{x^2}}}{x\sqrt{c + \frac{d}{x^2}}} + x\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} - \frac{\sqrt{c}\sqrt{a + \frac{b}{x^2}}(ad + bc)F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| \frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}\right)}{\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x^2]\*Sqrt[c + d/x^2],x]

[Out]  $(-2*d*Sqrt[a + b/x^2])/(Sqrt[c + d/x^2]*x) + Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x + (2*Sqrt[c]*Sqrt[d]*Sqrt[a + b/x^2]*EllipticE[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)]/(Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2]) - (Sqrt[c]*(b*c + a*d)*Sqrt[a + b/x^2]*EllipticF[ArcCot[(Sqrt[c]*x)/Sqrt[d]], 1 - (b*c)/(a*d)]/(a*Sqrt[d]*Sqrt[(c*(a + b/x^2))/(a*(c + d/x^2))]*Sqrt[c + d/x^2])$

#### Rule 375

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

#### Rule 411

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/((c\_) + (d\_)\*(x\_)^2)^(3/2), x\_Symbol] := Simp[(Sqrt[a + b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(c\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

#### Rule 418

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)^2]\*Sqrt[(c\_) + (d\_)\*(x\_)^2]), x\_Symbol] := Simp[(Sqrt[a + b\*x^2]\*EllipticF[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(a\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]

#### Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] := Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, f, n, p, q}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx^2} \sqrt{c + dx^2}}{x^2} dx, x, \frac{1}{x} \right) \\ &= \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} x - 2 \text{Subst} \left( \int \frac{\frac{1}{2}(bc + ad) + bdx^2}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\ &= \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} x - (2bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) - (bc + ad) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\ &= -\frac{2d\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}} x} + \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} x - \frac{\sqrt{c}(bc + ad)\sqrt{a + \frac{b}{x^2}} F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{a\sqrt{d} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}} \sqrt{c + \frac{d}{x^2}}} \\ &= -\frac{2d\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}} x} + \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} x + \frac{2\sqrt{c}\sqrt{d}\sqrt{a + \frac{b}{x^2}} E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right) \middle| 1 - \frac{bc}{ad}\right)}{\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}} \sqrt{c + \frac{d}{x^2}}} \end{aligned}$$

**Mathematica [C]** time = 0.38, size = 205, normalized size = 0.88

$$\frac{x\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} \left( \sqrt{\frac{a}{b}} (ax^2 + b)(cx^2 + d) + ix\sqrt{\frac{ax^2}{b} + 1} \sqrt{\frac{cx^2}{d} + 1} (bc - ad) F\left(i \sinh^{-1}\left(\sqrt{\frac{a}{b}} x\right) \middle| \frac{bc}{ad}\right) + 2iadx \right)}{\sqrt{\frac{a}{b}} (ax^2 + b)(cx^2 + d)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/x^2]*Sqrt[c + d/x^2], x]
[Out] -((Sqrt[a + b/x^2]*Sqrt[c + d/x^2]*x*(Sqrt[a/b]*(b + a*x^2)*(d + c*x^2) + (2*I)*a*d*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d]*EllipticE[I*ArcSinh[Sqrt[a/b]*x], (b*c)/(a*d)] + I*(b*c - a*d)*x*Sqrt[1 + (a*x^2)/b]*Sqrt[1 + (c*x^2)/d])
```

2)/d)\*EllipticF[I\*ArcSinh[Sqrt[a/b]\*x], (b\*c)/(a\*d)])/(Sqrt[a/b]\*(b + a\*x^2)\*(d + c\*x^2)))

**fricas** [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{\frac{ax^2+b}{x^2}}\sqrt{\frac{cx^2+d}{x^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x^2)^(1/2)\*(a+b/x^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt((a\*x^2 + b)/x^2)\*sqrt((c\*x^2 + d)/x^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x^2)^(1/2)\*(a+b/x^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x^2)\*sqrt(c + d/x^2), x)

**maple** [A] time = 0.14, size = 277, normalized size = 1.19

$$\frac{\sqrt{\frac{ax^2+b}{x^2}} \sqrt{\frac{cx^2+d}{x^2}} \left( -\sqrt{-\frac{c}{d}} acx^4 - \sqrt{-\frac{c}{d}} adx^2 + \sqrt{\frac{cx^2+d}{d}} \sqrt{\frac{ax^2+b}{b}} adx \text{EllipticF}\left(\sqrt{-\frac{c}{d}} x, \sqrt{\frac{ad}{bc}}\right) - \sqrt{-\frac{c}{d}} bcx^2 + 2\sqrt{\frac{ad}{bc}} \right)}{(acx^4 + adx^2 + bcx^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c+d/x^2)^(1/2)\*(a+b/x^2)^(1/2),x)

[Out] ((a\*x^2+b)/x^2)^(1/2)\*x\*((c\*x^2+d)/x^2)^(1/2)\*(-(-c/d)^(1/2)\*x^4\*a\*c+((c\*x^2+d)/d)^(1/2)\*((a\*x^2+b)/b)^(1/2)\*EllipticF(x\*(-c/d)^(1/2), (a/b/c\*d)^(1/2)))\*x\*a\*d-c\*b\*((c\*x^2+d)/d)^(1/2)\*((a\*x^2+b)/b)^(1/2)\*x\*EllipticF(x\*(-c/d)^(1/2), (a/b/c\*d)^(1/2))+2\*c\*b\*((c\*x^2+d)/d)^(1/2)\*((a\*x^2+b)/b)^(1/2)\*x\*EllipticE(x\*(-c/d)^(1/2), (a/b/c\*d)^(1/2))-(-c/d)^(1/2)\*x^2\*a\*d-(-c/d)^(1/2)\*x^2\*b\*c-(-c/d)^(1/2)\*b\*d)/(a\*c\*x^4+a\*d\*x^2+b\*c\*x^2+b\*d)/(-c/d)^(1/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d/x^2)^(1/2)\*(a+b/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x^2)\*sqrt(c + d/x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)^(1/2)\*(c + d/x^2)^(1/2),x)



```
[Out] int((a + b/x^2)^(1/2)*(c + d/x^2)^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d/x**2)**(1/2)*(a+b/x**2)**(1/2), x)
```

```
[Out] Integral(sqrt(a + b/x**2)*sqrt(c + d/x**2), x)
```

$$3.272 \quad \int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

**Optimal.** Leaf size=232

$$\frac{d\sqrt{a + \frac{b}{x^2}}}{cx\sqrt{c + \frac{d}{x^2}}} + \frac{x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left|1 - \frac{bc}{ad}\right.\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} + \frac{\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left|1 - \frac{bc}{ad}\right.\right)}{\sqrt{c}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

[Out]  $-d*(a+b/x^2)^{(1/2)}/c/x/(c+d/x^2)^{(1/2)}-b*(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x*(1+x^2*c/d)^{(1/2)}*EllipticF(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/a/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x/c*d*(1+x^2*c/d)^{(1/2)}*EllipticE(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+x*(a+b/x^2)^{(1/2)}*(c+d/x^2)^{(1/2)}/c$

**Rubi [A]** time = 0.20, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, number of rules / integrand size = 0.304, Rules used = {375, 475, 21, 422, 418, 492, 411}

$$\frac{d\sqrt{a + \frac{b}{x^2}}}{cx\sqrt{c + \frac{d}{x^2}}} + \frac{x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c} - \frac{b\sqrt{c}\sqrt{a + \frac{b}{x^2}}F\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left|1 - \frac{bc}{ad}\right.\right)}{a\sqrt{d}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} + \frac{\sqrt{d}\sqrt{a + \frac{b}{x^2}}E\left(\cot^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d}}\right)\left|1 - \frac{bc}{ad}\right.\right)}{\sqrt{c}\sqrt{c + \frac{d}{x^2}}\sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x^2]/Sqrt[c + d/x^2], x]

[Out]  $-((d*\text{Sqrt}[a + b/x^2])/c*\text{Sqrt}[c + d/x^2]*x) + (\text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x)/c + (\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]/(\text{Sqrt}[c]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])* \text{Sqrt}[c + d/x^2]) - (b*\text{Sqrt}[c]*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]/a*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))])* \text{Sqrt}[c + d/x^2])$

### Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] :> Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

### Rule 375

Int[((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q]/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

### Rule 411

Int[Sqrt[(a\_) + (b\_)\*(x\_)^2]/((c\_) + (d\_)\*(x\_)^2)^(3/2), x\_Symbol] :> Simp[(Sqrt[a + b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(c\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] := Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 475

```
Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
:= Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
+ b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx^2}}{x^2 \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) \\
&= \frac{\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c} - \frac{\text{Subst} \left( \int \frac{bc + bdx^2}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c} - \frac{b \text{Subst} \left( \int \frac{\sqrt{c + dx^2}}{\sqrt{a + bx^2}} dx, x, \frac{1}{x} \right)}{c} \\
&= \frac{\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c} - b \text{Subst} \left( \int \frac{1}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right) - \frac{(bd) \text{Subst} \left( \int \frac{x^2}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{c} \\
&= -\frac{d \sqrt{a + \frac{b}{x^2}}}{c \sqrt{c + \frac{d}{x^2}}} + \frac{\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c} - \frac{b \sqrt{c} \sqrt{a + \frac{b}{x^2}} F \left( \cot^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right) \middle| 1 - \frac{bc}{ad} \right)}{a \sqrt{d} \sqrt{\frac{c \left( a + \frac{b}{x^2} \right)}{a \left( c + \frac{d}{x^2} \right)}} \sqrt{c + \frac{d}{x^2}}} + d \text{Subst} \left( \int \frac{\sqrt{c + dx^2}}{\sqrt{a + bx^2}} dx, x, \frac{1}{x} \right) \\
&= -\frac{d \sqrt{a + \frac{b}{x^2}}}{c \sqrt{c + \frac{d}{x^2}}} + \frac{\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c} + \frac{\sqrt{d} \sqrt{a + \frac{b}{x^2}} E \left( \cot^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right) \middle| 1 - \frac{bc}{ad} \right)}{\sqrt{c} \sqrt{\frac{c \left( a + \frac{b}{x^2} \right)}{a \left( c + \frac{d}{x^2} \right)}} \sqrt{c + \frac{d}{x^2}}} - \frac{b \sqrt{c} \sqrt{a + \frac{b}{x^2}} F \left( \cot^{-1} \left( \frac{\sqrt{c} x}{\sqrt{d}} \right) \middle| 1 - \frac{bc}{ad} \right)}{a \sqrt{d} \sqrt{\frac{c \left( a + \frac{b}{x^2} \right)}{a \left( c + \frac{d}{x^2} \right)}} \sqrt{c + \frac{d}{x^2}}}
\end{aligned}$$

**Mathematica [A]** time = 0.07, size = 86, normalized size = 0.37

$$\frac{\sqrt{a + \frac{b}{x^2}} \sqrt{\frac{cx^2+d}{d}} E\left(\sin^{-1}\left(\sqrt{\frac{-c}{d}} x\right) \middle| \frac{ad}{bc}\right)}{\sqrt{\frac{-c}{d}} \sqrt{\frac{ax^2+b}{b}} \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x^2]/Sqrt[c + d/x^2], x]

[Out] (Sqrt[a + b/x^2]\*Sqrt[(d + c\*x^2)/d]\*EllipticE[ArcSin[Sqrt[-(c/d)]\*x], (a\*d)/(b\*c)))/(Sqrt[-(c/d)]\*Sqrt[c + d/x^2]\*Sqrt[(b + a\*x^2)/b])

**fricas [F]** time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{x^2 \sqrt{\frac{ax^2+b}{x^2}} \sqrt{\frac{cx^2+d}{x^2}}}{cx^2 + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2), x, algorithm="fricas")

[Out] integral(x^2\*sqrt((a\*x^2 + b)/x^2)\*sqrt((c\*x^2 + d)/x^2)/(c\*x^2 + d), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)

**maple [A]** time = 0.07, size = 94, normalized size = 0.41

$$\frac{\sqrt{\frac{ax^2+b}{x^2}} \sqrt{\frac{ax^2+b}{b}} \sqrt{\frac{cx^2+d}{d}} b \text{EllipticE}\left(\sqrt{\frac{-c}{d}} x, \sqrt{\frac{ad}{bc}}\right)}{(ax^2 + b) \sqrt{\frac{-c}{d}} \sqrt{\frac{cx^2+d}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2), x)

[Out] ((a\*x^2+b)/x^2)^(1/2)/(a\*x^2+b)\*EllipticE((-c/d)^(1/2)\*x, (a/b/c\*d)^(1/2))\*((a\*x^2+b)/b)^(1/2)\*((c\*x^2+d)/d)^(1/2)\*b/(-c/d)^(1/2)/((c\*x^2+d)/x^2)^(1/2)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x^2)/sqrt(c + d/x^2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)

[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(1/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\sqrt{c + \frac{d}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x\*\*2)\*\*(1/2)/(c+d/x\*\*2)\*\*(1/2), x)

[Out] Integral(sqrt(a + b/x\*\*2)/sqrt(c + d/x\*\*2), x)

$$3.273 \quad \int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx$$

**Optimal.** Leaf size=262

$$\frac{2\sqrt{d}\sqrt{a + \frac{b}{x^2}} E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \left|1 - \frac{bc}{ad}\right.\right)}{c^{3/2}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} - \frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2 x \sqrt{c + \frac{d}{x^2}}} + \frac{2x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c^2} - \frac{x\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} - \frac{b\sqrt{a + \frac{b}{x^2}} F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \left|1 - \frac{bc}{ad}\right.\right)}{a\sqrt{c}\sqrt{d}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

[Out]  $-2*d*(a+b/x^2)^{(1/2)}/c^2/x/(c+d/x^2)^{(1/2)}-x*(a+b/x^2)^{(1/2)}/c/(c+d/x^2)^{(1/2)}-b*(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x/c*(1+x^2*c/d)^{(1/2)}*EllipticF(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/a/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+2*(x^2*c/d/(1+x^2*c/d))^{(1/2)}/x/c^2*d*(1+x^2*c/d)^{(1/2)}*EllipticE(1/(1+x^2*c/d)^{(1/2)},(1-b*c/a/d)^{(1/2)})*(a+b/x^2)^{(1/2)}/(c*(a+b/x^2)/a/(c+d/x^2))^{(1/2)}/(c+d/x^2)^{(1/2)}+2*x*(a+b/x^2)^{(1/2)}*(c+d/x^2)^{(1/2)}/c^2$

**Rubi [A]** time = 0.28, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$ , Rules used = {375, 469, 583, 531, 418, 492, 411}

$$-\frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2 x \sqrt{c + \frac{d}{x^2}}} + \frac{2x\sqrt{a + \frac{b}{x^2}}\sqrt{c + \frac{d}{x^2}}}{c^2} + \frac{2\sqrt{d}\sqrt{a + \frac{b}{x^2}} E\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \left|1 - \frac{bc}{ad}\right.\right)}{c^{3/2}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}} - \frac{x\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} - \frac{b\sqrt{a + \frac{b}{x^2}} F\left(\cot^{-1}\left(\frac{\sqrt{cx}}{\sqrt{d}}\right) \left|1 - \frac{bc}{ad}\right.\right)}{a\sqrt{c}\sqrt{d}\sqrt{c + \frac{d}{x^2}} \sqrt{\frac{c\left(a + \frac{b}{x^2}\right)}{a\left(c + \frac{d}{x^2}\right)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2), x]

[Out]  $(-2*d*\text{Sqrt}[a + b/x^2])/(c^2*\text{Sqrt}[c + d/x^2]*x) - (\text{Sqrt}[a + b/x^2]*x)/(c*\text{Sqrt}[c + d/x^2]) + (2*\text{Sqrt}[a + b/x^2]*\text{Sqrt}[c + d/x^2]*x)/c^2 + (2*\text{Sqrt}[d]*\text{Sqrt}[a + b/x^2]*\text{EllipticE}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)])/(c^(3/2)*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))]*\text{Sqrt}[c + d/x^2]) - (b*\text{Sqrt}[a + b/x^2]*\text{EllipticF}[\text{ArcCot}[(\text{Sqrt}[c]*x)/\text{Sqrt}[d]], 1 - (b*c)/(a*d)]/(a*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[(c*(a + b/x^2))/(a*(c + d/x^2))]*\text{Sqrt}[c + d/x^2])$

#### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

#### Rule 411

Int[Sqrt[(a\_) + (b\_.)\*(x\_)^2]/((c\_) + (d\_.)\*(x\_)^2)^(3/2), x\_Symbol] :> Simp[(Sqrt[a + b\*x^2]\*EllipticE[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(c\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]

#### Rule 418

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)^2]\*Sqrt[(c\_) + (d\_.)\*(x\_)^2]), x\_Symbol] :> Simp[(Sqrt[a + b\*x^2]\*EllipticF[ArcTan[Rt[d/c, 2]\*x], 1 - (b\*c)/(a\*d)]/(a\*Rt[d/c, 2]\*Sqrt[c + d\*x^2]\*Sqrt[(c\*(a + b\*x^2))/(a\*(c + d\*x^2))]), x] /; Fre

$eQ[\{a, b, c, d\}, x] \&\& \text{PosQ}[d/c] \&\& \text{PosQ}[b/a] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

#### Rule 469

$\text{Int}[(e_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}((c_*) + (d_*)(x_*)^{(n_*)})^{(q_*)}, x\_Symbol] \rightarrow -\text{Simp}[(e*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^q / (a*e*n*(p+1)), x] + \text{Dist}[1/(a*n*(p+1)), \text{Int}[(e*x)^m*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q-1)}*\text{Simp}[c*(m+n*(p+1)+1) + d*(m+n*(p+q+1)+1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, m\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[p, -1] \&\& \text{LtQ}[0, q, 1] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

#### Rule 492

$\text{Int}[(x_*)^2/(\text{Sqrt}[(a_*) + (b_*)(x_*)^2]*\text{Sqrt}[(c_*) + (d_*)(x_*)^2]), x\_Symbol] \rightarrow \text{Simp}[(x*\text{Sqrt}[a + b*x^2])/(b*\text{Sqrt}[c + d*x^2]), x] - \text{Dist}[c/b, \text{Int}[\text{Sqrt}[a + b*x^2]/(c + d*x^2)^{(3/2)}, x], x] /; \text{FreeQ}[\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{PosQ}[b/a] \&\& \text{PosQ}[d/c] \&\& \text{!SimplerSqrtQ}[b/a, d/c]$

#### Rule 531

$\text{Int}[(a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}((c_*) + (d_*)(x_*)^{(n_*)})^{(q_*)}((e_*) + (f_*)(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Dist}[e, \text{Int}[(a + b*x^n)^p*(c + d*x^n)^q, x], x] + \text{Dist}[f, \text{Int}[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p, q\}, x]$

#### Rule 583

$\text{Int}[(g_*)(x_*)^{(m_*)}((a_*) + (b_*)(x_*)^{(n_*)})^{(p_*)}((c_*) + (d_*)(x_*)^{(n_*)})^{(q_*)}((e_*) + (f_*)(x_*)^{(n_*)}), x\_Symbol] \rightarrow \text{Simp}[(e*(g*x)^{(m+1)}*(a + b*x^n)^{(p+1)}*(c + d*x^n)^{(q+1)}) / (a*c*g*(m+1)), x] + \text{Dist}[1/(a*c*g^n*(m+1)), \text{Int}[(g*x)^{(m+n)}*(a + b*x^n)^p*(c + d*x^n)^q*\text{Simp}[a*f*c*(m+1) - e*(b*c + a*d)*(m+n+1) - e*n*(b*c*p + a*d*q) - b*e*d*(m+n*(p+q+2)+1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, g, p, q\}, x] \&\& \text{IGtQ}[n, 0] \&\& \text{LtQ}[m, -1]$

#### Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{3/2}} dx &= -\text{Subst} \left( \int \frac{\sqrt{a + bx^2}}{x^2 (c + dx^2)^{3/2}} dx, x, \frac{1}{x} \right) \\
&= -\frac{\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} + \frac{\text{Subst} \left( \int \frac{-2a - bx^2}{x^2 \sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{c} \\
&= -\frac{\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} + \frac{2\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c^2} - \frac{\text{Subst} \left( \int \frac{abc + 2abdx^2}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{ac^2} \\
&= -\frac{\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} + \frac{2\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c^2} - \frac{b \text{Subst} \left( \int \frac{1}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{c} - \frac{(2bd) \text{Subst} \left( \int \frac{1}{\sqrt{a + bx^2} \sqrt{c + dx^2}} dx, x, \frac{1}{x} \right)}{c} \\
&= -\frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2\sqrt{c + \frac{d}{x^2}}} - \frac{\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} + \frac{2\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c^2} - \frac{b\sqrt{a + \frac{b}{x^2}} F \left( \cot^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right) \middle| 1 - \frac{bc}{ad} \right)}{a\sqrt{c}\sqrt{d} \sqrt{\frac{c(a + \frac{b}{x^2})}{a(c + \frac{d}{x^2})}} \sqrt{c + \frac{d}{x^2}}} + \frac{(2d)}{c^2} \\
&= -\frac{2d\sqrt{a + \frac{b}{x^2}}}{c^2\sqrt{c + \frac{d}{x^2}}} - \frac{\sqrt{a + \frac{b}{x^2}}}{c\sqrt{c + \frac{d}{x^2}}} + \frac{2\sqrt{a + \frac{b}{x^2}} \sqrt{c + \frac{d}{x^2}}}{c^2} + \frac{2\sqrt{d} \sqrt{a + \frac{b}{x^2}} E \left( \cot^{-1} \left( \frac{\sqrt{c}x}{\sqrt{d}} \right) \middle| 1 - \frac{bc}{ad} \right)}{c^{3/2} \sqrt{\frac{c(a + \frac{b}{x^2})}{a(c + \frac{d}{x^2})}} \sqrt{c + \frac{d}{x^2}}} - \frac{(2d)}{c^2}
\end{aligned}$$

**Mathematica [C]** time = 0.33, size = 191, normalized size = 0.73

$$\frac{\sqrt{a + \frac{b}{x^2}} \left( i\sqrt{\frac{ax^2}{b}} + 1 \sqrt{\frac{cx^2}{d}} + 1 (bc - 2ad) F \left( i \sinh^{-1} \left( \sqrt{\frac{a}{b}} x \right) \middle| \frac{bc}{ad} \right) + 2iad \sqrt{\frac{ax^2}{b}} + 1 \sqrt{\frac{cx^2}{d}} + 1 E \left( i \sinh^{-1} \left( \sqrt{\frac{a}{b}} x \right) \middle| \frac{bc}{ad} \right) \right)}{c^2 \sqrt{\frac{a}{b}} (ax^2 + b) \sqrt{c + \frac{d}{x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/x^2]/(c + d/x^2)^(3/2), x]

[Out] -((Sqrt[a + b/x^2]\*(Sqrt[a/b]\*c\*x\*(b + a\*x^2) + (2\*I)\*a\*d\*Sqrt[1 + (a\*x^2)/b]\*Sqrt[1 + (c\*x^2)/d]\*EllipticE[I\*ArcSinh[Sqrt[a/b]\*x], (b\*c)/(a\*d)] + I\*(b\*c - 2\*a\*d)\*Sqrt[1 + (a\*x^2)/b]\*Sqrt[1 + (c\*x^2)/d]\*EllipticF[I\*ArcSinh[Sqrt[a/b]\*x], (b\*c)/(a\*d)]))/(Sqrt[a/b]\*c^2\*Sqrt[c + d/x^2]\*(b + a\*x^2))

**fricas [F]** time = 0.80, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{x^4 \sqrt{\frac{ax^2 + b}{x^2}} \sqrt{\frac{cx^2 + d}{x^2}}}{c^2 x^4 + 2cdx^2 + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2), x, algorithm="fricas")

[Out] integral(x^4\*sqrt((a\*x^2 + b)/x^2)\*sqrt((c\*x^2 + d)/x^2)/(c^2\*x^4 + 2\*c\*d\*x^2 + d^2), x)



**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="giac")

[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)

**maple** [A] time = 0.17, size = 185, normalized size = 0.71

$$\frac{\sqrt{\frac{ax^2+b}{x^2}} \left( \sqrt{-\frac{c}{d}} ax^3 + \sqrt{-\frac{c}{d}} bx - 2\sqrt{\frac{cx^2+d}{d}} \sqrt{\frac{ax^2+b}{b}} b \operatorname{EllipticE} \left( \sqrt{-\frac{c}{d}} x, \sqrt{\frac{ad}{bc}} \right) + \sqrt{\frac{cx^2+d}{d}} \sqrt{\frac{ax^2+b}{b}} b \operatorname{EllipticF} \left( \sqrt{-\frac{c}{d}} x, \sqrt{\frac{ad}{bc}} \right) \right)}{(ax^2 + b) \sqrt{-\frac{c}{d}} \left( \frac{cx^2+d}{x^2} \right)^{\frac{3}{2}}} cx^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x)

[Out] -((a\*x^2+b)/x^2)^(1/2)/x^2/(a\*x^2+b)\*(x^3\*a\*(-c/d)^(1/2)+EllipticF((-c/d)^(1/2)\*x, (a/b/c\*d)^(1/2))\*b\*((c\*x^2+d)/d)^(1/2)\*((a\*x^2+b)/b)^(1/2)-2\*EllipticE((-c/d)^(1/2)\*x, (a/b/c\*d)^(1/2))\*b\*((c\*x^2+d)/d)^(1/2)\*((a\*x^2+b)/b)^(1/2)+x\*b\*(-c/d)^(1/2)\*(c\*x^2+d)/(-c/d)^(1/2)/c/((c\*x^2+d)/x^2)^(3/2)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^(1/2)/(c+d/x^2)^(3/2),x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x^2)/(c + d/x^2)^(3/2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2),x)

[Out] int((a + b/x^2)^(1/2)/(c + d/x^2)^(3/2), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x^2}}}{\left(c + \frac{d}{x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x\*\*2)\*\*(1/2)/(c+d/x\*\*2)\*\*(3/2),x)

[Out] Integral(sqrt(a + b/x\*\*2)/(c + d/x\*\*2)\*\*(3/2), x)

$$3.274 \quad \int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

**Optimal.** Leaf size=79

$$x \left(a + \frac{b}{x^2}\right)^p \left(\frac{b}{ax^2} + 1\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(\frac{d}{cx^2} + 1\right)^{-q} F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

[Out]  $(a+b/x^2)^p(c+d/x^2)^q*x*AppellF1(-1/2,-p,-q,1/2,-b/a/x^2,-d/c/x^2)/((1+b/a/x^2)^p)/((1+d/c/x^2)^q)$

**Rubi [A]** time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {375, 511, 510}

$$x \left(a + \frac{b}{x^2}\right)^p \left(\frac{b}{ax^2} + 1\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(\frac{d}{cx^2} + 1\right)^{-q} F_1\left(-\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^2)^p\*(c + d/x^2)^q,x]

[Out]  $((a + b/x^2)^p(c + d/x^2)^q*x*AppellF1[-1/2, -p, -q, 1/2, -(b/(a*x^2)), -(d/(c*x^2))])/((1 + b/(a*x^2))^p*(1 + d/(c*x^2))^q)$

#### Rule 375

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> -Subst[Int[((a + b/x^n)^p\*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && ILtQ[n, 0]

#### Rule 510

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Simp[(a^p\*c^q\*(e\*x)^(m+1)\*AppellF1[(m+1)/n, -p, -q, 1 + (m+1)/n, -(b\*x^n)/a, -(d\*x^n)/c])/(e\*(m+1)), x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 511

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(e\*x)^m\*(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m, -1] && NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned}
\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx &= -\text{Subst} \left( \int \frac{(a + bx^2)^p (c + dx^2)^q}{x^2} dx, x, \frac{1}{x} \right) \\
&= - \left( \left( \left(a + \frac{b}{x^2}\right)^p \left(1 + \frac{b}{ax^2}\right)^{-p} \right) \text{Subst} \left( \int \frac{\left(1 + \frac{bx^2}{a}\right)^p (c + dx^2)^q}{x^2} dx, x, \frac{1}{x} \right) \right) \\
&= - \left( \left( \left(a + \frac{b}{x^2}\right)^p \left(1 + \frac{b}{ax^2}\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(1 + \frac{d}{cx^2}\right)^{-q} \right) \text{Subst} \left( \int \frac{\left(1 + \frac{bx^2}{a}\right)^p \left(1 + \frac{dx^2}{c}\right)^{-q}}{x^2} dx, x, \frac{1}{x} \right) \right) \\
&= \left(a + \frac{b}{x^2}\right)^p \left(1 + \frac{b}{ax^2}\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(1 + \frac{d}{cx^2}\right)^{-q} x F_1 \left( -\frac{1}{2}; -p, -q; \frac{1}{2}; -\frac{b}{ax^2}, -\frac{d}{cx^2} \right)
\end{aligned}$$

**Mathematica [A]** time = 0.15, size = 104, normalized size = 1.32

$$\frac{x \left(a + \frac{b}{x^2}\right)^p \left(\frac{ax^2}{b} + 1\right)^{-p} \left(c + \frac{d}{x^2}\right)^q \left(\frac{cx^2}{d} + 1\right)^{-q} F_1 \left( -p - q + \frac{1}{2}; -p, -q; -p - q + \frac{3}{2}; -\frac{ax^2}{b}, -\frac{cx^2}{d} \right)}{2p + 2q - 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b/x^2)^p\*(c + d/x^2)^q,x]

[Out] -(((a + b/x^2)^p\*(c + d/x^2)^q\*x\*AppellF1[1/2 - p - q, -p, -q, 3/2 - p - q, -(a\*x^2)/b, -(c\*x^2)/d])/((-1 + 2\*p + 2\*q)\*(1 + (a\*x^2)/b)^p\*(1 + (c\*x^2)/d)^q))

**fricas [F]** time = 1.33, size = 0, normalized size = 0.00

$$\text{integral} \left( \left( \frac{ax^2 + b}{x^2} \right)^p \left( \frac{cx^2 + d}{x^2} \right)^q, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^p\*(c+d/x^2)^q,x, algorithm="fricas")

[Out] integral(((a\*x^2 + b)/x^2)^p\*((c\*x^2 + d)/x^2)^q, x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^2)^p\*(c+d/x^2)^q,x, algorithm="giac")

[Out] integrate((a + b/x^2)^p\*(c + d/x^2)^q, x)

**maple [F]** time = 0.22, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^2)^p\*(c+d/x^2)^q,x)

[Out] `int((a+b/x^2)^p*(c+d/x^2)^q,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x^2)^p*(c+d/x^2)^q,x, algorithm="maxima")`

[Out] `integrate((a + b/x^2)^p*(c + d/x^2)^q, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{b}{x^2}\right)^p \left(c + \frac{d}{x^2}\right)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/x^2)^p*(c + d/x^2)^q,x)`

[Out] `int((a + b/x^2)^p*(c + d/x^2)^q, x)`

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/x**2)**p*(c+d/x**2)**q,x)`

[Out] Timed out

$$3.275 \quad \int \frac{a + \frac{b}{x^3}}{c + \frac{d}{x^3}} dx$$

**Optimal.** Leaf size=145

$$\frac{(bc - ad) \log\left(c^{2/3}x^2 - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3}\right)}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \log\left(\sqrt[3]{c}x + \sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{c}x}{\sqrt{3} \sqrt[3]{d}}\right)}{\sqrt{3} c^{4/3}d^{2/3}} + \frac{ax}{c}$$

[Out] a\*x/c+1/3\*(-a\*d+b\*c)\*ln(d^(1/3)+c^(1/3)\*x)/c^(4/3)/d^(2/3)-1/6\*(-a\*d+b\*c)\*ln(d^(2/3)-c^(1/3)\*d^(1/3)\*x+c^(2/3)\*x^2)/c^(4/3)/d^(2/3)-1/3\*(-a\*d+b\*c)\*arctan(1/3\*(d^(1/3)-2\*c^(1/3)\*x)/d^(1/3)\*3^(1/2))/c^(4/3)/d^(2/3)\*3^(1/2)

**Rubi [A]** time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$ , Rules used = {374, 388, 200, 31, 634, 617, 204, 628}

$$\frac{(bc - ad) \log\left(c^{2/3}x^2 - \sqrt[3]{c} \sqrt[3]{d} x + d^{2/3}\right)}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \log\left(\sqrt[3]{c}x + \sqrt[3]{d}\right)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{c}x}{\sqrt{3} \sqrt[3]{d}}\right)}{\sqrt{3} c^{4/3}d^{2/3}} + \frac{ax}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/x^3)/(c + d/x^3), x]

[Out] (a\*x)/c - ((b\*c - a\*d)\*ArcTan[(d^(1/3) - 2\*c^(1/3)\*x)/(Sqrt[3]\*d^(1/3))]/(Sqrt[3]\*c^(4/3)\*d^(2/3)) + ((b\*c - a\*d)\*Log[d^(1/3) + c^(1/3)\*x]/(3\*c^(4/3)\*d^(2/3)) - ((b\*c - a\*d)\*Log[d^(2/3) - c^(1/3)\*d^(1/3)\*x + c^(2/3)\*x^2]/(6\*c^(4/3)\*d^(2/3)))

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 374

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Int[x^(n\*(p+q))\*(b + a/x^n)^p\*(d + c/x^n)^q, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IntegersQ[p, q] && NegQ[n]

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(b\*(n\*(p+1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b,

c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simplify[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned} \int \frac{a + \frac{b}{x^3}}{c + \frac{d}{x^3}} dx &= \int \frac{b + ax^3}{d + cx^3} dx \\ &= \frac{ax}{c} - \frac{(-bc + ad) \int \frac{1}{d+cx^3} dx}{c} \\ &= \frac{ax}{c} + \frac{(bc - ad) \int \frac{1}{\sqrt[3]{d} + \sqrt[3]{c}x} dx}{3cd^{2/3}} + \frac{(bc - ad) \int \frac{2\sqrt[3]{d} - \sqrt[3]{c}x}{d^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + c^{2/3}x^2} dx}{3cd^{2/3}} \\ &= \frac{ax}{c} + \frac{(bc - ad) \log(\sqrt[3]{d} + \sqrt[3]{c}x)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \int \frac{-\sqrt[3]{c}\sqrt[3]{d} + 2c^{2/3}x}{d^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + c^{2/3}x^2} dx}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \int \frac{1}{d^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + c^{2/3}x^2} dx}{2c\sqrt[3]{d}} \\ &= \frac{ax}{c} + \frac{(bc - ad) \log(\sqrt[3]{d} + \sqrt[3]{c}x)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \log(d^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + c^{2/3}x^2)}{6c^{4/3}d^{2/3}} + \frac{(bc - ad) \operatorname{Subst}\left(\int \frac{1}{u^2 - \sqrt[3]{c}u + c^{2/3}} du\right)}{c^{4/3}d^{2/3}} \\ &= \frac{ax}{c} - \frac{(bc - ad) \tan^{-1}\left(\frac{\sqrt[3]{d} - 2\sqrt[3]{c}x}{\sqrt{3}\sqrt[3]{d}}\right)}{\sqrt{3}c^{4/3}d^{2/3}} + \frac{(bc - ad) \log(\sqrt[3]{d} + \sqrt[3]{c}x)}{3c^{4/3}d^{2/3}} - \frac{(bc - ad) \log(d^{2/3} - \sqrt[3]{c}\sqrt[3]{d}x + c^{2/3}x^2)}{6c^{4/3}d^{2/3}} \end{aligned}$$

**Mathematica** [A] time = 0.10, size = 129, normalized size = 0.89

$$\frac{-(bc - ad) \log(c^{2/3}x^2 - \sqrt[3]{c}\sqrt[3]{d}x + d^{2/3}) + 2(bc - ad) \log(\sqrt[3]{c}x + \sqrt[3]{d}) - 2\sqrt{3}(bc - ad) \tan^{-1}\left(\frac{1 - \frac{2\sqrt[3]{c}x}{\sqrt[3]{d}}}{\sqrt{3}}\right) + 6a\sqrt[3]{c}d^{2/3}}{6c^{4/3}d^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/x^3)/(c + d/x^3), x]

[Out] (6\*a\*c^(1/3)\*d^(2/3)\*x - 2\*Sqrt[3]\*(b\*c - a\*d)\*ArcTan[(1 - (2\*c^(1/3)\*x)/d^(1/3))/Sqrt[3]] + 2\*(b\*c - a\*d)\*Log[d^(1/3) + c^(1/3)\*x] - (b\*c - a\*d)\*Log[d^(2/3) - c^(1/3)\*d^(1/3)\*x + c^(2/3)\*x^2]/(6\*c^(4/3)\*d^(2/3))

**fricas** [A] time = 0.66, size = 390, normalized size = 2.69

$$\frac{6acd^2x - 3\sqrt{\frac{1}{3}}(bc^2d - acd^2)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}} \log\left(\frac{2cdx^3 + 3(-cd^2)^{\frac{1}{3}}dx - d^2 - 3\sqrt{\frac{1}{3}}\left(2cdx^2 + (-cd^2)^{\frac{2}{3}}x + (-cd^2)^{\frac{1}{3}}d\right)\sqrt{\frac{(-cd^2)^{\frac{1}{3}}}{c}}}{cx^3 + d}\right) - (-cd^2)^{\frac{2}{3}}}{6c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="fricas")

[Out] [1/6\*(6\*a\*c\*d^2\*x - 3\*sqrt(1/3)\*(b\*c^2\*d - a\*c\*d^2)\*sqrt((-c\*d^2)^(1/3)/c)\*log((2\*c\*d\*x^3 + 3\*(-c\*d^2)^(1/3)\*d\*x - d^2 - 3\*sqrt(1/3)\*(2\*c\*d\*x^2 + (-c\*d^2)^(2/3)\*x + (-c\*d^2)^(1/3)\*d)\*sqrt((-c\*d^2)^(1/3)/c))/(c\*x^3 + d)) - (-c\*d^2)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x^2 - (-c\*d^2)^(2/3)\*x - (-c\*d^2)^(1/3)\*d) + 2\*(-c\*d^2)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x + (-c\*d^2)^(2/3)))/(c^2\*d^2), 1/6\*(6\*a\*c\*d^2\*x + 6\*sqrt(1/3)\*(b\*c^2\*d - a\*c\*d^2)\*sqrt(-(-c\*d^2)^(1/3)/c)\*arctan(sqrt(1/3)\*(2\*(-c\*d^2)^(2/3)\*x + (-c\*d^2)^(1/3)\*d)\*sqrt(-(-c\*d^2)^(1/3)/c)/d^2) - (-c\*d^2)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x^2 - (-c\*d^2)^(2/3)\*x - (-c\*d^2)^(1/3)\*d) + 2\*(-c\*d^2)^(2/3)\*(b\*c - a\*d)\*log(c\*d\*x + (-c\*d^2)^(2/3)))/(c^2\*d^2)]

**giac** [A] time = 0.22, size = 133, normalized size = 0.92

$$\frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x + \left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(-\frac{d}{c}\right)^{\frac{1}{3}}}\right) + (bc - ad) \log\left(x^2 + x\left(-\frac{d}{c}\right)^{\frac{1}{3}} + \left(-\frac{d}{c}\right)^{\frac{2}{3}}\right) + \frac{ax}{c} - (bc - ad)\left(-\frac{d}{c}\right)^{\frac{1}{3}} \log\left(x - \left(-\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3(-c^2d)^{\frac{2}{3}} - 6(-c^2d)^{\frac{2}{3}} + 3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="giac")

[Out] -1/3\*sqrt(3)\*(b\*c - a\*d)\*arctan(1/3\*sqrt(3)\*(2\*x + (-d/c)^(1/3))/(-d/c)^(1/3))/(-c^2\*d)^(2/3) - 1/6\*(b\*c - a\*d)\*log(x^2 + x\*(-d/c)^(1/3) + (-d/c)^(2/3))/(-c^2\*d)^(2/3) + a\*x/c - 1/3\*(b\*c - a\*d)\*(-d/c)^(1/3)\*log(abs(x - (-d/c)^(1/3)))/(c\*d)

**maple** [A] time = 0.05, size = 195, normalized size = 1.34

$$\frac{ax}{c} - \frac{\sqrt{3} ad \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2} - \frac{ad \ln\left(x + \left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2} + \frac{ad \ln\left(x^2 - \left(\frac{d}{c}\right)^{\frac{1}{3}}x + \left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6\left(\frac{d}{c}\right)^{\frac{2}{3}}c^2} + \frac{\sqrt{3} b \arctan\left(\frac{\sqrt{3}\left(\frac{2x}{\left(\frac{d}{c}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3\left(\frac{d}{c}\right)^{\frac{2}{3}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/x^3)/(c+d/x^3),x)

[Out]  $a/c*x-1/3/c^2/(1/c*d)^{(2/3)}*\ln(x+(1/c*d)^{(1/3)})*a*d+1/3/c/(1/c*d)^{(2/3)}*\ln(x+(1/c*d)^{(1/3)})*b+1/6/c^2/(1/c*d)^{(2/3)}*\ln(x^2-(1/c*d)^{(1/3)}*x+(1/c*d)^{(2/3)})*a*d-1/6/c/(1/c*d)^{(2/3)}*\ln(x^2-(1/c*d)^{(1/3)}*x+(1/c*d)^{(2/3)})*b-1/3/c^2/(1/c*d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/c*d)^{(1/3)}*x-1))*a*d+1/3/c/(1/c*d)^{(2/3)}*3^{(1/2)}*\arctan(1/3*3^{(1/2)}*(2/(1/c*d)^{(1/3)}*x-1))*b$

**maxima** [A] time = 1.29, size = 128, normalized size = 0.88

$$\frac{ax}{c} + \frac{\sqrt{3}(bc - ad) \arctan\left(\frac{\sqrt{3}\left(2x - \left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3\left(\frac{d}{c}\right)^{\frac{1}{3}}}\right)}{3c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} - \frac{(bc - ad) \log\left(x^2 - x\left(\frac{d}{c}\right)^{\frac{1}{3}} + \left(\frac{d}{c}\right)^{\frac{2}{3}}\right)}{6c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}} + \frac{(bc - ad) \log\left(x + \left(\frac{d}{c}\right)^{\frac{1}{3}}\right)}{3c^2\left(\frac{d}{c}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x^3)/(c+d/x^3),x, algorithm="maxima")

[Out]  $a*x/c + 1/3*\sqrt{3}*(b*c - a*d)*\arctan(1/3*\sqrt{3}*(2*x - (d/c)^{(1/3)})/(d/c)^{(1/3)})/(c^2*(d/c)^{(2/3)}) - 1/6*(b*c - a*d)*\log(x^2 - x*(d/c)^{(1/3)} + (d/c)^{(2/3)})/(c^2*(d/c)^{(2/3)}) + 1/3*(b*c - a*d)*\log(x + (d/c)^{(1/3)})/(c^2*(d/c)^{(2/3)})$

**mupad** [B] time = 0.27, size = 123, normalized size = 0.85

$$\frac{ax}{c} - \frac{\ln(c^{1/3}x + d^{1/3})(ad - bc)}{3c^{4/3}d^{2/3}} + \frac{\ln(d^{1/3} - 2c^{1/3}x + \sqrt{3}d^{1/3}1i)\left(\frac{1}{2} + \frac{\sqrt{3}1i}{2}\right)(ad - bc)}{3c^{4/3}d^{2/3}} - \frac{\ln(2c^{1/3}x - d^{1/3} + \sqrt{3}d^{1/3}1i)\left(\frac{1}{2} - \frac{\sqrt{3}1i}{2}\right)(ad - bc)}{3c^{4/3}d^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x^3)/(c + d/x^3),x)

[Out]  $(a*x)/c - (\log(c^{(1/3)}*x + d^{(1/3)})*(a*d - b*c))/(3*c^{(4/3)}*d^{(2/3)}) + (\log(3^{(1/2)}*d^{(1/3)}*1i - 2*c^{(1/3)}*x + d^{(1/3)})*((3^{(1/2)}*1i)/2 + 1/2)*(a*d - b*c))/(3*c^{(4/3)}*d^{(2/3)}) - (\log(3^{(1/2)}*d^{(1/3)}*1i + 2*c^{(1/3)}*x - d^{(1/3)})*((3^{(1/2)}*1i)/2 - 1/2)*(a*d - b*c))/(3*c^{(4/3)}*d^{(2/3)})$

**sympy** [A] time = 0.45, size = 71, normalized size = 0.49

$$\frac{ax}{c} + \text{RootSum}\left(27t^3c^4d^2 + a^3d^3 - 3a^2bcd^2 + 3ab^2c^2d - b^3c^3, \left(t \mapsto t \log\left(-\frac{3tcd}{ad - bc} + x\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/x\*\*3)/(c+d/x\*\*3),x)

[Out]  $a*x/c + \text{RootSum}(27*_t**3*c**4*d**2 + a**3*d**3 - 3*a**2*b*c*d**2 + 3*a*b**2*c**2*d - b**3*c**3, \text{Lambda}(_t, _t*\log(-3*_t*c*d/(a*d - b*c) + x))$



$$3.276 \quad \int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx$$

Optimal. Leaf size=49

$$\frac{2c(bc-ad)\log(c+d\sqrt{x})}{d^3} - \frac{2\sqrt{x}(bc-ad)}{d^2} + \frac{bx}{d}$$

[Out]  $b*x/d+2*c*(-a*d+b*c)*\ln(c+d*x^{(1/2)})/d^3-2*(-a*d+b*c)*x^{(1/2)}/d^2$

**Rubi [A]** time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$ , Rules used = {376, 77}

$$-\frac{2\sqrt{x}(bc-ad)}{d^2} + \frac{2c(bc-ad)\log(c+d\sqrt{x})}{d^3} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*Sqrt[x])/(c + d\*Sqrt[x]), x]

[Out]  $(-2*(b*c - a*d)*\text{Sqrt}[x])/d^2 + (b*x)/d + (2*c*(b*c - a*d)*\text{Log}[c + d*\text{Sqrt}[x]])/d^3$

Rule 77

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 376

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g-1)\*(a + b\*x^(g\*n))^p\*(c + d\*x^(g\*n))^q, x], x, x^(1/g)], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{a+b\sqrt{x}}{c+d\sqrt{x}} dx &= 2 \text{Subst} \left( \int \frac{x(a+bx)}{c+dx} dx, x, \sqrt{x} \right) \\ &= 2 \text{Subst} \left( \int \left( \frac{-bc+ad}{d^2} + \frac{bx}{d} + \frac{c(bc-ad)}{d^2(c+dx)} \right) dx, x, \sqrt{x} \right) \\ &= -\frac{2(bc-ad)\sqrt{x}}{d^2} + \frac{bx}{d} + \frac{2c(bc-ad)\log(c+d\sqrt{x})}{d^3} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 41, normalized size = 0.84

$$\frac{2(ad-bc)(d\sqrt{x}-c\log(c+d\sqrt{x}))}{d^3} + \frac{bx}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*Sqrt[x])/(c + d\*Sqrt[x]), x]

[Out]  $(b*x)/d + (2*(-(b*c) + a*d)*(d*\text{Sqrt}[x] - c*\text{Log}[c + d*\text{Sqrt}[x]]))/d^3$

**fricas** [A] time = 0.97, size = 48, normalized size = 0.98

$$\frac{bd^2x + 2(bc^2 - acd)\log(d\sqrt{x} + c) - 2(bcd - ad^2)\sqrt{x}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="fricas")`

[Out]  $(b*d^2*x + 2*(b*c^2 - a*c*d)*\log(d*\text{sqrt}(x) + c) - 2*(b*c*d - a*d^2)*\text{sqrt}(x))/d^3$

**giac** [A] time = 0.19, size = 49, normalized size = 1.00

$$\frac{bdx - 2bc\sqrt{x} + 2ad\sqrt{x}}{d^2} + \frac{2(bc^2 - acd)\log(|d\sqrt{x} + c|)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="giac")`

[Out]  $(b*d*x - 2*b*c*\text{sqrt}(x) + 2*a*d*\text{sqrt}(x))/d^2 + 2*(b*c^2 - a*c*d)*\log(\text{abs}(d*\text{sqrt}(x) + c))/d^3$

**maple** [A] time = 0.05, size = 59, normalized size = 1.20

$$-\frac{2ac \ln(d\sqrt{x} + c)}{d^2} + \frac{2bc^2 \ln(d\sqrt{x} + c)}{d^3} + \frac{bx}{d} + \frac{2a\sqrt{x}}{d} - \frac{2bc\sqrt{x}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^(1/2)+a)/(c+d*x^(1/2)),x)`

[Out]  $b/d*x + 2/d*a*x^(1/2) - 2/d^2*b*c*x^(1/2) - 2*c/d^2*\ln(c+d*x^(1/2))*a + 2*c^2/d^3*\ln(c+d*x^(1/2))*b$

**maxima** [A] time = 0.64, size = 47, normalized size = 0.96

$$\frac{bdx - 2(bc - ad)\sqrt{x}}{d^2} + \frac{2(bc^2 - acd)\log(d\sqrt{x} + c)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^(1/2))/(c+d*x^(1/2)),x, algorithm="maxima")`

[Out]  $(b*d*x - 2*(b*c - a*d)*\text{sqrt}(x))/d^2 + 2*(b*c^2 - a*c*d)*\log(d*\text{sqrt}(x) + c)/d^3$

**mupad** [B] time = 0.07, size = 49, normalized size = 1.00

$$\sqrt{x} \left( \frac{2a}{d} - \frac{2bc}{d^2} \right) + \frac{\ln(c + d\sqrt{x})(2bc^2 - 2acd)}{d^3} + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^(1/2))/(c + d*x^(1/2)),x)`

[Out]  $x^(1/2)*((2*a)/d - (2*b*c)/d^2) + (\log(c + d*x^(1/2))*(2*b*c^2 - 2*a*c*d))/d^3 + (b*x)/d$

sympy [A] time = 0.30, size = 82, normalized size = 1.67

$$\begin{cases} -\frac{2ac \log\left(\frac{c}{d} + \sqrt{x}\right)}{d^2} + \frac{2a\sqrt{x}}{d} + \frac{2bc^2 \log\left(\frac{c}{d} + \sqrt{x}\right)}{d^3} - \frac{2bc\sqrt{x}}{d^2} + \frac{bx}{d} & \text{for } d \neq 0 \\ \frac{ax + \frac{2bx^{\frac{3}{2}}}{3}}{c} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*(1/2))/(c+d\*x\*\*(1/2)),x)

[Out] Piecewise((-2\*a\*c\*log(c/d + sqrt(x))/d\*\*2 + 2\*a\*sqrt(x)/d + 2\*b\*c\*\*2\*log(c/d + sqrt(x))/d\*\*3 - 2\*b\*c\*sqrt(x)/d\*\*2 + b\*x/d, Ne(d, 0)), ((a\*x + 2\*b\*x\*\*(3/2))/3)/c, True)

$$3.277 \quad \int \frac{-1 + \sqrt[3]{x}}{1 + \sqrt[3]{x}} dx$$

**Optimal.** Leaf size=26

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log(\sqrt[3]{x} + 1)$$

[Out]  $6x^{1/3} - 3x^{2/3} + x - 6\ln(x^{1/3} + 1)$

**Rubi [A]** time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {376, 77}

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log(\sqrt[3]{x} + 1)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^(1/3))/(1 + x^(1/3)), x]

[Out]  $6x^{1/3} - 3x^{2/3} + x - 6\text{Log}[1 + x^{1/3}]$

**Rule 77**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_) + (d\_.)\*(x\_)^(n\_.))\*((e\_.) + (f\_.)\*(x\_)^(p\_.)), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x)\*(c + d\*x)^n\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b\*c - a\*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9\*p + 5\*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

**Rule 376**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)\*(a + b\*x^(g\*n))^p\*(c + d\*x^(g\*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[n]

**Rubi steps**

$$\begin{aligned} \int \frac{-1 + \sqrt[3]{x}}{1 + \sqrt[3]{x}} dx &= 3 \text{Subst} \left( \int \frac{(-1 + x)x^2}{1 + x} dx, x, \sqrt[3]{x} \right) \\ &= 3 \text{Subst} \left( \int \left( 2 - 2x + x^2 - \frac{2}{1 + x} \right) dx, x, \sqrt[3]{x} \right) \\ &= 6\sqrt[3]{x} - 3x^{2/3} + x - 6\log(1 + \sqrt[3]{x}) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 26, normalized size = 1.00

$$-3x^{2/3} + x + 6\sqrt[3]{x} - 6\log(\sqrt[3]{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^(1/3))/(1 + x^(1/3)), x]

[Out]  $6x^{1/3} - 3x^{2/3} + x - 6\text{Log}[1 + x^{1/3}]$

**fricas [A]** time = 0.91, size = 20, normalized size = 0.77

$$x - 3x^{2/3} + 6x^{1/3} - 6\log(x^{1/3} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="fricas")

[Out]  $x - 3x^{2/3} + 6x^{1/3} - 6\log(x^{1/3} + 1)$

**giac** [A] time = 0.20, size = 20, normalized size = 0.77

$$x - 3x^{2/3} + 6x^{1/3} - 6\log\left(x^{1/3} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="giac")

[Out]  $x - 3x^{2/3} + 6x^{1/3} - 6\log(x^{1/3} + 1)$

**maple** [A] time = 0.04, size = 21, normalized size = 0.81

$$x - 6\ln\left(x^{1/3} + 1\right) - 3x^{2/3} + 6x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3)-1)/(x^(1/3)+1),x)

[Out]  $6x^{1/3} - 3x^{2/3} + x - 6\ln(x^{1/3} + 1)$

**maxima** [A] time = 0.55, size = 20, normalized size = 0.77

$$x - 3x^{2/3} + 6x^{1/3} - 6\log\left(x^{1/3} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x^(1/3))/(1+x^(1/3)),x, algorithm="maxima")

[Out]  $x - 3x^{2/3} + 6x^{1/3} - 6\log(x^{1/3} + 1)$

**mupad** [B] time = 0.03, size = 20, normalized size = 0.77

$$x - 6\ln\left(x^{1/3} + 1\right) + 6x^{1/3} - 3x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3) - 1)/(x^(1/3) + 1),x)

[Out]  $x - 6\log(x^{1/3} + 1) + 6x^{1/3} - 3x^{2/3}$

**sympy** [A] time = 0.20, size = 24, normalized size = 0.92

$$-3x^{2/3} + 6\sqrt[3]{x} + x - 6\log\left(\sqrt[3]{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x\*\*(1/3))/(1+x\*\*(1/3)),x)

[Out]  $-3x^{2/3} + 6x^{1/3} + x - 6\log(x^{1/3} + 1)$

$$3.278 \quad \int \frac{1+x^{2/3}}{-1+x^{2/3}} dx$$

Optimal. Leaf size=17

$$x + 6\sqrt[3]{x} - 6 \tanh^{-1}\left(\sqrt[3]{x}\right)$$

[Out] 6\*x^(1/3)+x-6\*arctanh(x^(1/3))

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$ , Rules used = {376, 459, 321, 207}

$$x + 6\sqrt[3]{x} - 6 \tanh^{-1}\left(\sqrt[3]{x}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(2/3))/(-1 + x^(2/3)),x]

[Out] 6\*x^(1/3) + x - 6\*ArcTanh[x^(1/3)]

#### Rule 207

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := -Simp[ArcTanh[(Rt[b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

#### Rule 321

Int[((c\_.)\*(x\_)^(m\_))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n - 1)\*(c\*x)^(m - n + 1)\*(a + b\*x^n)^(p + 1))/(b\*(m + n\*p + 1)), x] - Dist[(a\*c^n\*(m - n + 1))/(b\*(m + n\*p + 1)), Int[(c\*x)^(m - n)\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n\*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 376

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)\*(a + b\*x^(g\*n))^p\*(c + d\*x^(g\*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b\*c - a\*d, 0] && FractionQ[n]

#### Rule 459

Int[((e\_.)\*(x\_)^(m\_.))\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{1+x^{2/3}}{-1+x^{2/3}} dx &= 3 \operatorname{Subst} \left( \int \frac{x^2(1+x^2)}{-1+x^2} dx, x, \sqrt[3]{x} \right) \\
&= x + 6 \operatorname{Subst} \left( \int \frac{x^2}{-1+x^2} dx, x, \sqrt[3]{x} \right) \\
&= 6\sqrt[3]{x} + x + 6 \operatorname{Subst} \left( \int \frac{1}{-1+x^2} dx, x, \sqrt[3]{x} \right) \\
&= 6\sqrt[3]{x} + x - 6 \tanh^{-1}(\sqrt[3]{x})
\end{aligned}$$

**Mathematica** [A] time = 0.01, size = 17, normalized size = 1.00

$$x + 6\sqrt[3]{x} - 6 \tanh^{-1}(\sqrt[3]{x})$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^(2/3))/(-1 + x^(2/3)), x]

[Out] 6\*x^(1/3) + x - 6\*ArcTanh[x^(1/3)]

**fricas** [A] time = 0.69, size = 23, normalized size = 1.35

$$x + 6x^{\frac{1}{3}} - 3 \log(x^{\frac{1}{3}} + 1) + 3 \log(x^{\frac{1}{3}} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(2/3))/(-1+x^(2/3)), x, algorithm="fricas")

[Out] x + 6\*x^(1/3) - 3\*log(x^(1/3) + 1) + 3\*log(x^(1/3) - 1)

**giac** [A] time = 0.18, size = 24, normalized size = 1.41

$$x + 6x^{\frac{1}{3}} - 3 \log(x^{\frac{1}{3}} + 1) + 3 \log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(2/3))/(-1+x^(2/3)), x, algorithm="giac")

[Out] x + 6\*x^(1/3) - 3\*log(x^(1/3) + 1) + 3\*log(abs(x^(1/3) - 1))

**maple** [A] time = 0.05, size = 24, normalized size = 1.41

$$x - 3 \ln(x^{\frac{1}{3}} + 1) + 3 \ln(x^{\frac{1}{3}} - 1) + 6x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(2/3)+1)/(x^(2/3)-1), x)

[Out] x+6\*x^(1/3)+3\*ln(x^(1/3)-1)-3\*ln(x^(1/3)+1)

**maxima** [A] time = 0.56, size = 23, normalized size = 1.35

$$x + 6x^{\frac{1}{3}} - 3 \log(x^{\frac{1}{3}} + 1) + 3 \log(x^{\frac{1}{3}} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(2/3))/(-1+x^(2/3)), x, algorithm="maxima")

[Out]  $x + 6x^{1/3} - 3\log(x^{1/3} + 1) + 3\log(x^{1/3} - 1)$

**mupad** [B] time = 1.46, size = 13, normalized size = 0.76

$$x - 6 \operatorname{atanh}(x^{1/3}) + 6x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(2/3) + 1)/(x^(2/3) - 1),x)`

[Out]  $x - 6\operatorname{atanh}(x^{1/3}) + 6x^{1/3}$

**sympy** [A] time = 0.27, size = 27, normalized size = 1.59

$$6\sqrt[3]{x} + x + 3\log(\sqrt[3]{x} - 1) - 3\log(\sqrt[3]{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x**(2/3))/(-1+x**(2/3)),x)`

[Out]  $6x^{1/3} + x + 3\log(x^{1/3} - 1) - 3\log(x^{1/3} + 1)$



$$3.279 \quad \int \frac{-16+x^{3/4}}{16+x^{3/4}} dx$$

**Optimal.** Leaf size=104

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\cdot 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

[Out] -128\*x^(1/4)+x+256/3\*2^(1/3)\*ln(2\*2^(1/3)+x^(1/4))-128/3\*2^(1/3)\*ln(4\*2^(2/3)-2\*2^(1/3)\*x^(1/4)+x^(1/2))-256/3\*2^(1/3)\*arctan(1/6\*(2^(1/3)-x^(1/4))\*2^(2/3)\*3^(1/2))\*3^(1/2)

**Rubi [A]** time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$ , Rules used = {376, 459, 321, 200, 31, 634, 617, 204, 628}

$$x-128\sqrt[4]{x}+\frac{256}{3}\sqrt[3]{2}\log\left(\sqrt[4]{x}+2\sqrt[3]{2}\right)-\frac{128}{3}\sqrt[3]{2}\log\left(\sqrt{x}-2\sqrt[3]{2}\sqrt[4]{x}+4\cdot 2^{2/3}\right)-\frac{256\sqrt[3]{2}\tan^{-1}\left(\frac{\sqrt[3]{2}-\sqrt[4]{x}}{\sqrt[3]{2}\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(-16 + x^(3/4))/(16 + x^(3/4)), x]

[Out] -128\*x^(1/4) + x - (256\*2^(1/3)\*ArcTan[(2^(1/3) - x^(1/4))/(2^(1/3)\*Sqrt[3]])/Sqrt[3] + (256\*2^(1/3)\*Log[2\*2^(1/3) + x^(1/4)]/3 - (128\*2^(1/3)\*Log[4\*2^(2/3) - 2\*2^(1/3)\*x^(1/4) + Sqrt[x]]/3

#### Rule 31

Int[((a\_) + (b\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[Log[RemoveContent[a + b\*x, x]]/b, x] /; FreeQ[{a, b}, x]

#### Rule 200

Int[((a\_) + (b\_.)\*(x\_)^3)^(n\_), x\_Symbol] := Dist[1/(3\*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]\*x), x], x] + Dist[1/(3\*Rt[a, 3]^2), Int[(2\*Rt[a, 3] - Rt[b, 3]\*x)/(Rt[a, 3]^2 - Rt[a, 3]\*Rt[b, 3]\*x + Rt[b, 3]^2\*x^2), x], x] /; FreeQ[{a, b}, x]

#### Rule 204

Int[((a\_) + (b\_.)\*(x\_)^2)^(n\_), x\_Symbol] := -Simp[ArcTan[(Rt[-b, 2]\*x)/Rt[-a, 2]]/(Rt[-a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

#### Rule 321

Int[((c\_.)\*(x\_))^(m\_)\*((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(c^(n-1)\*(c\*x)^(m-n+1)\*(a+b\*x^n)^(p+1))/(b\*(m+n\*p+1)), x] - Dist[(a\*c^n\*(m-n+1))/(b\*(m+n\*p+1)), Int[(c\*x)^(m-n)\*(a+b\*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n\*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

#### Rule 376

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g-1)\*(a+b\*x^(g\*n))^p\*(c+d\*x^(g\*n))^q, x], x, x^(1/g)], x] /; FreeQ[{a, b, c, d, p, q}, x]

&& NeQ[b\*c - a\*d, 0] && FractionQ[n]

### Rule 459

Int[((e\_)\*(x\_))^(m\_)\*((a\_) + (b\_)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a + b\*x^n)^(p + 1))/(b\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a\*d\*(m + 1) - b\*c\*(m + n\*(p + 1) + 1))/(b\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b\*c - a\*d, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rule 617

Int[((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2)^(-1), x\_Symbol] := With[{q = 1 - 4\*Simplify[(a\*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2\*c\*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4\*a\*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4\*a\*c, 0]

### Rule 628

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Simp[(d\*Log[RemoveContent[a + b\*x + c\*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2\*c\*d - b\*e, 0]

### Rule 634

Int[((d\_) + (e\_)\*(x\_))/((a\_) + (b\_)\*(x\_) + (c\_)\*(x\_)^2), x\_Symbol] := Dist[(2\*c\*d - b\*e)/(2\*c), Int[1/(a + b\*x + c\*x^2), x], x] + Dist[e/(2\*c), Int[(b + 2\*c\*x)/(a + b\*x + c\*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2\*c\*d - b\*e, 0] && NeQ[b^2 - 4\*a\*c, 0] && !NiceSqrtQ[b^2 - 4\*a\*c]

### Rubi steps

$$\begin{aligned}
 \int \frac{-16 + x^{3/4}}{16 + x^{3/4}} dx &= 4 \operatorname{Subst} \left( \int \frac{x^3 (-16 + x^3)}{16 + x^3} dx, x, \sqrt[4]{x} \right) \\
 &= x - 128 \operatorname{Subst} \left( \int \frac{x^3}{16 + x^3} dx, x, \sqrt[4]{x} \right) \\
 &= -128 \sqrt[4]{x} + x + 2048 \operatorname{Subst} \left( \int \frac{1}{16 + x^3} dx, x, \sqrt[4]{x} \right) \\
 &= -128 \sqrt[4]{x} + x + \frac{1}{3} (256 \sqrt[3]{2}) \operatorname{Subst} \left( \int \frac{1}{2 \sqrt[3]{2} + x} dx, x, \sqrt[4]{x} \right) + \frac{1}{3} (256 \sqrt[3]{2}) \operatorname{Subst} \left( \int \frac{4 \sqrt[3]{2}}{4 \cdot 2^{2/3} - 2} dx, x, \sqrt[4]{x} \right) \\
 &= -128 \sqrt[4]{x} + x + \frac{256}{3} \sqrt[3]{2} \log(2 \sqrt[3]{2} + \sqrt[4]{x}) - \frac{1}{3} (128 \sqrt[3]{2}) \operatorname{Subst} \left( \int \frac{-2 \sqrt[3]{2} + 2x}{4 \cdot 2^{2/3} - 2 \sqrt[3]{2} x + x^2} dx, x, \sqrt[4]{x} \right) \\
 &= -128 \sqrt[4]{x} + x + \frac{256}{3} \sqrt[3]{2} \log(2 \sqrt[3]{2} + \sqrt[4]{x}) - \frac{128}{3} \sqrt[3]{2} \log(4 \cdot 2^{2/3} - 2 \sqrt[3]{2} \sqrt[4]{x} + \sqrt{x}) + (256 \sqrt[3]{2}) \operatorname{Subst} \left( \int \frac{1}{4 \cdot 2^{2/3} - 2} dx, x, \sqrt[4]{x} \right) \\
 &= -128 \sqrt[4]{x} + x - \frac{256 \sqrt[3]{2} \tan^{-1} \left( \frac{2 - 2^{2/3} \sqrt[4]{x}}{2 \sqrt{3}} \right)}{\sqrt{3}} + \frac{256}{3} \sqrt[3]{2} \log(2 \sqrt[3]{2} + \sqrt[4]{x}) - \frac{128}{3} \sqrt[3]{2} \log(4 \cdot 2^{2/3} - 2 \sqrt[3]{2} \sqrt[4]{x} + \sqrt{x})
 \end{aligned}$$

**Mathematica** [C] time = 0.01, size = 22, normalized size = 0.21

$$x - 2x {}_2F_1 \left( 1, \frac{4}{3}; \frac{7}{3}; -\frac{x^{3/4}}{16} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-16 + x^(3/4))/(16 + x^(3/4)), x]

[Out] x - 2\*x\*Hypergeometric2F1[1, 4/3, 7/3, -1/16\*x^(3/4)]

**fricas** [A] time = 0.82, size = 71, normalized size = 0.68

$$\frac{256}{3} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} x^{\frac{1}{4}} - \frac{1}{3} \sqrt{3}\right) - \frac{128}{3} \cdot 2^{\frac{1}{3}} \log\left(4 \cdot 2^{\frac{2}{3}} - 2 \cdot 2^{\frac{1}{3}} x^{\frac{1}{4}} + \sqrt{x}\right) + \frac{256}{3} \cdot 2^{\frac{1}{3}} \log\left(2 \cdot 2^{\frac{1}{3}} + x^{\frac{1}{4}}\right) + x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16+x^(3/4))/(16+x^(3/4)), x, algorithm="fricas")

[Out] 256/3\*sqrt(3)\*2^(1/3)\*arctan(1/6\*sqrt(3)\*2^(2/3)\*x^(1/4) - 1/3\*sqrt(3)) - 128/3\*2^(1/3)\*log(4\*2^(2/3) - 2\*2^(1/3)\*x^(1/4) + sqrt(x)) + 256/3\*2^(1/3)\*log(2\*2^(1/3) + x^(1/4)) + x - 128\*x^(1/4)

**giac** [A] time = 0.17, size = 71, normalized size = 0.68

$$\frac{256}{3} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(-\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} - x^{\frac{1}{4}}\right)\right) - \frac{128}{3} \cdot 2^{\frac{1}{3}} \log\left(4 \cdot 2^{\frac{2}{3}} - 2 \cdot 2^{\frac{1}{3}} x^{\frac{1}{4}} + \sqrt{x}\right) + \frac{256}{3} \cdot 2^{\frac{1}{3}} \log\left(2 \cdot 2^{\frac{1}{3}} + x^{\frac{1}{4}}\right) + x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16+x^(3/4))/(16+x^(3/4)), x, algorithm="giac")

[Out] 256/3\*sqrt(3)\*2^(1/3)\*arctan(-1/6\*sqrt(3)\*2^(2/3)\*(2^(1/3) - x^(1/4))) - 128/3\*2^(1/3)\*log(4\*2^(2/3) - 2\*2^(1/3)\*x^(1/4) + sqrt(x)) + 256/3\*2^(1/3)\*log(2\*2^(1/3) + x^(1/4)) + x - 128\*x^(1/4)

**maple** [A] time = 0.04, size = 66, normalized size = 0.63

$$x + \frac{128 16^{\frac{1}{3}} \sqrt{3} \arctan\left(\frac{\sqrt{3} \left(\frac{16^{\frac{2}{3}} x^{\frac{1}{4}}}{8} - 1\right)}{3}\right)}{3} + \frac{128 16^{\frac{1}{3}} \ln\left(x^{\frac{1}{4}} + 16^{\frac{1}{3}}\right)}{3} - \frac{64 16^{\frac{1}{3}} \ln\left(\sqrt{x} - 16^{\frac{1}{3}} x^{\frac{1}{4}} + 16^{\frac{2}{3}}\right)}{3} - 128 x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-16+x^(3/4))/(16+x^(3/4)), x)

[Out] x-128\*x^(1/4)+128/3\*16^(1/3)\*ln(x^(1/4)+16^(1/3))-64/3\*16^(1/3)\*ln(x^(1/2)-16^(1/3)\*x^(1/4)+16^(2/3))+128/3\*16^(1/3)\*3^(1/2)\*arctan(1/3\*3^(1/2)\*(1/8\*16^(2/3)\*x^(1/4)-1))

**maxima** [A] time = 1.13, size = 71, normalized size = 0.68

$$\frac{256}{3} \sqrt{3} 2^{\frac{1}{3}} \arctan\left(-\frac{1}{6} \sqrt{3} 2^{\frac{2}{3}} \left(2^{\frac{1}{3}} - x^{\frac{1}{4}}\right)\right) - \frac{128}{3} \cdot 2^{\frac{1}{3}} \log\left(4 \cdot 2^{\frac{2}{3}} - 2 \cdot 2^{\frac{1}{3}} x^{\frac{1}{4}} + \sqrt{x}\right) + \frac{256}{3} \cdot 2^{\frac{1}{3}} \log\left(2 \cdot 2^{\frac{1}{3}} + x^{\frac{1}{4}}\right) + x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-16+x^(3/4))/(16+x^(3/4)), x, algorithm="maxima")

[Out] 256/3\*sqrt(3)\*2^(1/3)\*arctan(-1/6\*sqrt(3)\*2^(2/3)\*(2^(1/3) - x^(1/4))) - 128/3\*2^(1/3)\*log(4\*2^(2/3) - 2\*2^(1/3)\*x^(1/4) + sqrt(x)) + 256/3\*2^(1/3)\*log(2\*2^(1/3) + x^(1/4)) + x - 128\*x^(1/4)

**mupad** [B] time = 1.50, size = 90, normalized size = 0.87

$$x + \frac{256 2^{1/3} \ln\left(12288 2^{1/3} + 6144 x^{1/4}\right)}{3} - 128 x^{1/4} + \frac{128 2^{1/3} \ln\left(6144 x^{1/4} + 6144 2^{1/3} (-1 + \sqrt{3} i)\right) (-1 + \sqrt{3} i)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(3/4) - 16)/(x^(3/4) + 16),x)`

[Out]  $x + \frac{256 \cdot 2^{1/3} \log(12288 \cdot 2^{1/3} + 6144 \cdot x^{1/4})}{3} - 128 \cdot x^{1/4} + \frac{128 \cdot 2^{1/3} \log(6144 \cdot x^{1/4} + 6144 \cdot 2^{1/3} \cdot (3^{1/2} \cdot 1i - 1)) \cdot (3^{1/2} \cdot 1i - 1)}{3} - \frac{128 \cdot 2^{1/3} \log(6144 \cdot x^{1/4} - 6144 \cdot 2^{1/3} \cdot (3^{1/2} \cdot 1i + 1)) \cdot (3^{1/2} \cdot 1i + 1)}{3}$

**sympy** [A] time = 5.72, size = 102, normalized size = 0.98

$$-128 \sqrt[4]{x} + x + \frac{256 \sqrt[3]{2} \log(\sqrt[4]{x} + 2 \sqrt[3]{2})}{3} - \frac{128 \sqrt[3]{2} \log(-2 \sqrt[3]{2} \sqrt[4]{x} + \sqrt{x} + 4 \cdot 2^{2/3})}{3} + \frac{256 \sqrt[3]{2} \sqrt{3} \operatorname{atan}\left(\frac{2^{2/3} \sqrt{3} \sqrt[4]{x}}{6} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-16+x**(3/4))/(16+x**(3/4)),x)`

[Out]  $-128 \cdot x^{1/4} + x + \frac{256 \cdot 2^{1/3} \log(x^{1/4} + 2 \cdot 2^{1/3})}{3} - \frac{128 \cdot 2^{1/3} \log(-2 \cdot 2^{1/3} \cdot x^{1/4} + \sqrt{x} + 4 \cdot 2^{2/3})}{3} + \frac{256 \cdot 2^{1/3} \sqrt{3} \operatorname{atan}(2^{2/3} \sqrt{3} \cdot x^{1/4} / 6 - \sqrt{3} / 3)}{3}$

$$3.280 \quad \int \frac{1 + \frac{1}{\sqrt[3]{x}}}{-1 + \frac{1}{\sqrt[3]{x}}} dx$$

Optimal. Leaf size=30

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6 \log(1 - \sqrt[3]{x})$$

[Out]  $-6*x^{(1/3)}-3*x^{(2/3)}-x-6*\ln(1-x^{(1/3)})$

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {374, 376, 77}

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6 \log(1 - \sqrt[3]{x})$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + x^{(-1/3)})/(-1 + x^{(-1/3)}), x]$

[Out]  $-6*x^{(1/3)} - 3*x^{(2/3)} - x - 6*\text{Log}[1 - x^{(1/3)}]$

Rule 77

$\text{Int}[(a_. + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^{(n_.)}*((e_.) + (f_.)*(x_.))^{(p_.)}, x\_Symbol] :> \text{Int}[\text{ExpandIntegrand}[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& ((\text{ILtQ}[n, 0] \&\& \text{ILtQ}[p, 0]) || \text{EqQ}[p, 1] || (\text{IGtQ}[p, 0] \&\& (!\text{IntegerQ}[n] || \text{LeQ}[9*p + 5*(n + 2), 0] || \text{GeQ}[n + p + 1, 0] || (\text{GeQ}[n + p + 2, 0] \&\& \text{RationalQ}[a, b, c, d, e, f])))$

Rule 374

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x\_Symbol] :> \text{Int}[x^{(n*(p + q))}*(b + a/x^n)^p*(d + c/x^n)^q, x] /; \text{FreeQ}[\{a, b, c, d, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IntegersQ}[p, q] \&\& \text{NegQ}[n]$

Rule 376

$\text{Int}[(a_. + (b_.)*(x_.)^{(n_.)})^{(p_.)}*((c_.) + (d_.)*(x_.)^{(n_.)})^{(q_.)}, x\_Symbol] :> \text{With}[\{g = \text{Denominator}[n]\}, \text{Dist}[g, \text{Subst}[\text{Int}[x^{(g - 1)}*(a + b*x^{(g*n)})^p*(c + d*x^{(g*n)})^q, x], x, x^{(1/g)}], x]] /; \text{FreeQ}[\{a, b, c, d, p, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{FractionQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{1 + \frac{1}{\sqrt[3]{x}}}{-1 + \frac{1}{\sqrt[3]{x}}} dx &= \int \frac{1 + \sqrt[3]{x}}{1 - \sqrt[3]{x}} dx \\ &= 3 \text{Subst} \left( \int \frac{x^2(1+x)}{1-x} dx, x, \sqrt[3]{x} \right) \\ &= 3 \text{Subst} \left( \int \left( -2 - \frac{2}{-1+x} - 2x - x^2 \right) dx, x, \sqrt[3]{x} \right) \\ &= -6\sqrt[3]{x} - 3x^{2/3} - x - 6 \log(1 - \sqrt[3]{x}) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 30, normalized size = 1.00

$$-3x^{2/3} - x - 6\sqrt[3]{x} - 6\log(1 - \sqrt[3]{x})$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^(-1/3))/(-1 + x^(-1/3)),x]

[Out] -6\*x^(1/3) - 3\*x^(2/3) - x - 6\*Log[1 - x^(1/3)]

**fricas [A]** time = 0.59, size = 22, normalized size = 0.73

$$-x - 3x^{2/3} - 6x^{1/3} - 6\log(x^{1/3} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="fricas")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*log(x^(1/3) - 1)

**giac [A]** time = 0.15, size = 23, normalized size = 0.77

$$-x - 3x^{2/3} - 6x^{1/3} - 6\log\left(\left|x^{1/3} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="giac")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*log(abs(x^(1/3) - 1))

**maple [A]** time = 0.04, size = 23, normalized size = 0.77

$$-x - 6\ln(x^{1/3} - 1) - 3x^{2/3} - 6x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/x^(1/3))/(-1+1/x^(1/3)),x)

[Out] -x-3\*x^(2/3)-6\*x^(1/3)-6\*ln(x^(1/3)-1)

**maxima [A]** time = 0.56, size = 22, normalized size = 0.73

$$-x - 3x^{2/3} - 6x^{1/3} - 6\log(x^{1/3} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x^(1/3))/(-1+1/x^(1/3)),x, algorithm="maxima")

[Out] -x - 3\*x^(2/3) - 6\*x^(1/3) - 6\*log(x^(1/3) - 1)

**mupad [B]** time = 0.04, size = 22, normalized size = 0.73

$$-x - 6\ln(x^{1/3} - 1) - 6x^{1/3} - 3x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^(1/3) + 1)/(1/x^(1/3) - 1),x)

[Out] - x - 6\*log(x^(1/3) - 1) - 6\*x^(1/3) - 3\*x^(2/3)

sympy [A] time = 0.18, size = 26, normalized size = 0.87

$$-3x^{\frac{2}{3}} - 6\sqrt[3]{x} - x - 6\log(\sqrt[3]{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x**(1/3))/(-1+1/x**(1/3)),x)
```

```
[Out] -3*x**(2/3) - 6*x**(1/3) - x - 6*log(x**(1/3) - 1)
```

$$3.281 \quad \int (a - bx^n)^{3/2} (a + bx^n)^{3/2} dx$$

**Optimal.** Leaf size=79

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

[Out]  $a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right) / \sqrt{1 - \frac{b^2 x^{2n}}{a^2}}$

**Rubi [A]** time = 0.03, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {253, 246, 245}

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^(3/2)\*(a + b\*x^n)^(3/2),x]

[Out]  $(a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} \text{Hypergeometric2F1}[-3/2, 1/(2n), (2 + n(-1))/2, (b^2 x^{2n})/a^2]) / \sqrt{1 - (b^2 x^{2n})/a^2}$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p]) / (1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 253

Int[((a1\_.) + (b1\_.)\*(x\_)^(n\_))^(p\_)\*((a2\_.) + (b2\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[((a1 + b1\*x^n)^FracPart[p]\*(a2 + b2\*x^n)^FracPart[p]) / (a1\*a2 + b1\*b2\*x^(2\*n))^FracPart[p], Int[(a1\*a2 + b1\*b2\*x^(2\*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2\*b1 + a1\*b2, 0] && !IntegerQ[p]

#### Rubi steps



$$\begin{aligned} \int (a - bx^n)^{3/2} (a + bx^n)^{3/2} dx &= \frac{(\sqrt{a - bx^n} \sqrt{a + bx^n}) \int (a^2 - b^2 x^{2n})^{3/2} dx}{\sqrt{a^2 - b^2 x^{2n}}} \\ &= \frac{(a^2 \sqrt{a - bx^n} \sqrt{a + bx^n}) \int \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{3/2} dx}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}} \\ &= \frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; \frac{1}{2} \left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}} \end{aligned}$$

**Mathematica** [A] time = 0.04, size = 79, normalized size = 1.00

$$\frac{a^2 x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{3}{2}, \frac{1}{2n}; 1 + \frac{1}{2n}; \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^n)^(3/2)\*(a + b\*x^n)^(3/2), x]

[Out] (a^2\*x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-3/2, 1/(2\*n), 1 + 1/(2\*n), (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(3/2)\*(a+b\*x^n)^(3/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} (-bx^n + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(3/2)\*(a+b\*x^n)^(3/2), x, algorithm="giac")

[Out] integrate((b\*x^n + a)^(3/2)\*(-b\*x^n + a)^(3/2), x)

**maple** [F] time = 0.77, size = 0, normalized size = 0.00

$$\int (-b x^n + a)^{\frac{3}{2}} (b x^n + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-b\*x^n)^(3/2)\*(b\*x^n+a)^(3/2), x)

[Out] int((a-b\*x^n)^(3/2)\*(b\*x^n+a)^(3/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{3}{2}} (-bx^n + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(3/2)\*(a+b\*x^n)^(3/2),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^(3/2)\*(-b\*x^n + a)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b x^n)^{3/2} (a - b x^n)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^(3/2)\*(a - b\*x^n)^(3/2),x)

[Out] int((a + b\*x^n)^(3/2)\*(a - b\*x^n)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a - b x^n)^{\frac{3}{2}} (a + b x^n)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*n)\*\*(3/2)\*(a+b\*x\*\*n)\*\*(3/2),x)

[Out] Integral((a - b\*x\*\*n)\*\*(3/2)\*(a + b\*x\*\*n)\*\*(3/2), x)

### 3.282 $\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$

**Optimal.** Leaf size=76

$$\frac{x\sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2x^{2n}}{a^2}}}$$

[Out] x\*hypergeom([-1/2, 1/2/n], [1+1/2/n], b^2\*x^(2\*n)/a^2)\*(a-b\*x^n)^(1/2)\*(a+b\*x^n)^(1/2)/(1-b^2\*x^(2\*n)/a^2)^(1/2)

**Rubi [A]** time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$ , Rules used = {253, 246, 245}

$$\frac{x\sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n], x]

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-1/2, 1/(2\*n), (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 253

Int[((a1\_.) + (b1\_.)\*(x\_)^(n\_))^(p\_)\*((a2\_.) + (b2\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[((a1 + b1\*x^n)^FracPart[p]\*(a2 + b2\*x^n)^FracPart[p])/(a1\*a2 + b1\*b2\*x^(2\*n))^FracPart[p], Int[(a1\*a2 + b1\*b2\*x^(2\*n))^p, x], x] /; FreeQ[{a1, b1, a2, b2, n, p}, x] && EqQ[a2\*b1 + a1\*b2, 0] && !IntegerQ[p]

#### Rubi steps

$$\begin{aligned} \int \sqrt{a - bx^n} \sqrt{a + bx^n} dx &= \frac{(\sqrt{a - bx^n} \sqrt{a + bx^n}) \int \sqrt{a^2 - b^2 x^{2n}} dx}{\sqrt{a^2 - b^2 x^{2n}}} \\ &= \frac{(\sqrt{a - bx^n} \sqrt{a + bx^n}) \int \sqrt{1 - \frac{b^2 x^{2n}}{a^2}} dx}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}} \\ &= \frac{x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; \frac{1}{2} \left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}} \end{aligned}$$

**Mathematica** [A] time = 0.02, size = 76, normalized size = 1.00

$$\frac{x \sqrt{a - bx^n} \sqrt{a + bx^n} {}_2F_1\left(-\frac{1}{2}, \frac{1}{2n}; 1 + \frac{1}{2n}; \frac{b^2 x^{2n}}{a^2}\right)}{\sqrt{1 - \frac{b^2 x^{2n}}{a^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n], x]

[Out] (x\*Sqrt[a - b\*x^n]\*Sqrt[a + b\*x^n]\*Hypergeometric2F1[-1/2, 1/(2\*n), 1 + 1/(2\*n), (b^2\*x^(2\*n))/a^2])/Sqrt[1 - (b^2\*x^(2\*n))/a^2]

**fricas** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(1/2)\*(a+b\*x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a} \sqrt{-bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(1/2)\*(a+b\*x^n)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b\*x^n + a)\*sqrt(-b\*x^n + a), x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \sqrt{-b x^n + a} \sqrt{b x^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^n+a)^(1/2)\*(b\*x^n+a)^(1/2), x)

[Out] int((-b\*x^n+a)^(1/2)\*(b\*x^n+a)^(1/2), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{bx^n + a} \sqrt{-bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^(1/2)\*(a+b\*x^n)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b\*x^n + a)\*sqrt(-b\*x^n + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a + bx^n} \sqrt{a - bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^(1/2)\*(a - b\*x^n)^(1/2),x)

[Out] int((a + b\*x^n)^(1/2)\*(a - b\*x^n)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a - bx^n} \sqrt{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*n)\*\*(1/2)\*(a+b\*x\*\*n)\*\*(1/2),x)

[Out] Integral(sqrt(a - b\*x\*\*n)\*sqrt(a + b\*x\*\*n), x)

### 3.283 $\int (a - bx^n)^p (a + bx^n)^p dx$

**Optimal.** Leaf size=72

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)$$

[Out]  $x*(a-b*x^n)^p*(a+b*x^n)^p*\text{hypergeom}([-p, 1/2/n], [1+1/2/n], b^2*x^(2*n)/a^2)/($   
 $((1-b^2*x^(2*n))/a^2)^p)$

**Rubi [A]** time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$ , Rules used = {253, 246, 245}

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^n)^p*(a + b*x^n)^p, x]$

[Out]  $(x*(a - b*x^n)^p*(a + b*x^n)^p*\text{Hypergeometric2F1}[1/(2*n), -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2])/(1 - (b^2*x^(2*n))/a^2)^p$

#### Rule 245

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_), x\_Symbol] := \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /;$   $\text{FreeQ}\{a, b, n, p\}, x \&\& \text{!IGtQ}[p, 0] \&\& \text{!IntegerQ}[1/n] \&\& \text{!ILtQ}[\text{Simplify}[1/n + p], 0] \&\& (\text{IntegerQ}[p] \parallel \text{GtQ}[a, 0])$

#### Rule 246

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_), x\_Symbol] := \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p, x], x] /;$   $\text{FreeQ}\{a, b, n, p\}, x \&\& \text{!IGtQ}[p, 0] \&\& \text{!IntegerQ}[1/n] \&\& \text{!ILtQ}[\text{Simplify}[1/n + p], 0] \&\& \text{!(IntegerQ}[p] \parallel \text{GtQ}[a, 0])$

#### Rule 253

$\text{Int}[(a1_ + (b1_)*(x_)^(n_))^(p_)*((a2_ + (b2_)*(x_)^(n_))^(p_), x\_Symbol] := \text{Dist}[(a1 + b1*x^n)^{\text{FracPart}[p]}*(a2 + b2*x^n)^{\text{FracPart}[p]})/(a1*a2 + b1*b2*x^{(2*n)})^{\text{FracPart}[p]}, \text{Int}[(a1*a2 + b1*b2*x^{(2*n)})^p, x], x] /;$   $\text{FreeQ}\{a1, b1, a2, b2, n, p\}, x \&\& \text{EqQ}[a2*b1 + a1*b2, 0] \&\& \text{!IntegerQ}[p]$

#### Rubi steps

$$\begin{aligned} \int (a - bx^n)^p (a + bx^n)^p dx &= \left( (a - bx^n)^p (a + bx^n)^p (a^2 - b^2 x^{2n})^{-p} \right) \int (a^2 - b^2 x^{2n})^p dx \\ &= \left( (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} \right) \int \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^p dx \\ &= x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}\right) \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 72, normalized size = 1.00

$$x (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} {}_2F_1\left(\frac{1}{2n}, -p; 1 + \frac{1}{2n}; \frac{b^2 x^{2n}}{a^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p,x]

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*Hypergeometric2F1[1/(2\*n), -p, 1 + 1/(2\*n), (b^2\*x^(2\*n))/a^2])/(1 - (b^2\*x^(2\*n))/a^2)^p

**fricas** [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}((bx^n + a)^p(-bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**maple** [F] time = 0.93, size = 0, normalized size = 0.00

$$\int (-bx^n + a)^p (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^n+a)^p\*(b\*x^n+a)^p,x)

[Out] int((-b\*x^n+a)^p\*(b\*x^n+a)^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + bx^n)^p (a - bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p\*(a - b\*x^n)^p,x)

[Out] int((a + b\*x^n)^p\*(a - b\*x^n)^p, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a - bx^n)^p (a + bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p,x)

[Out] Integral((a - b\*x\*\*n)\*\*p\*(a + b\*x\*\*n)\*\*p, x)

### 3.284 $\int (a + bx^n)(c + dx^n)^4 dx$

**Optimal.** Leaf size=132

$$\frac{c^3 x^{n+1}(4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n+1}(3ad + 2bc)}{2n + 1} + \frac{d^3 x^{4n+1}(ad + 4bc)}{4n + 1} + \frac{2cd^2 x^{3n+1}(2ad + 3bc)}{3n + 1} + ac^4 x + \frac{bd^4 x^{5n+1}}{5n + 1}$$

[Out]  $a*c^4*x+c^3*(4*a*d+b*c)*x^{(1+n)/(1+n)+2*c^2*d*(3*a*d+2*b*c)*x^{(1+2*n)/(1+2*n)}+2*c*d^2*(2*a*d+3*b*c)*x^{(1+3*n)/(1+3*n)}+d^3*(a*d+4*b*c)*x^{(1+4*n)/(1+4*n)}+b*d^4*x^{(1+5*n)/(1+5*n)}$

**Rubi [A]** time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{c^3 x^{n+1}(4ad + bc)}{n + 1} + \frac{2c^2 dx^{2n+1}(3ad + 2bc)}{2n + 1} + \frac{2cd^2 x^{3n+1}(2ad + 3bc)}{3n + 1} + \frac{d^3 x^{4n+1}(ad + 4bc)}{4n + 1} + ac^4 x + \frac{bd^4 x^{5n+1}}{5n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^4, x]

[Out]  $a*c^4*x + (c^3*(b*c + 4*a*d)*x^{(1 + n)/(1 + n)} + (2*c^2*d*(2*b*c + 3*a*d)*x^{(1 + 2*n)/(1 + 2*n)} + (2*c*d^2*(3*b*c + 2*a*d)*x^{(1 + 3*n)/(1 + 3*n)} + (d^3*(4*b*c + a*d)*x^{(1 + 4*n)/(1 + 4*n)} + (b*d^4*x^{(1 + 5*n)/(1 + 5*n)})$

#### Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rubi steps

$$\int (a + bx^n)(c + dx^n)^4 dx = \int (ac^4 + c^3(bc + 4ad)x^n + 2c^2d(2bc + 3ad)x^{2n} + 2cd^2(3bc + 2ad)x^{3n} + d^3(4bc + ad)x^{4n} + bd^4x^{5n}) dx$$

$$= ac^4 x + \frac{c^3(bc + 4ad)x^{1+n}}{1+n} + \frac{2c^2d(2bc + 3ad)x^{1+2n}}{1+2n} + \frac{2cd^2(3bc + 2ad)x^{1+3n}}{1+3n} + \frac{d^3(4bc + ad)x^{1+4n}}{1+4n} + \frac{bd^4x^{1+5n}}{1+5n}$$

**Mathematica [A]** time = 0.28, size = 110, normalized size = 0.83

$$\frac{bx(c + dx^n)^5 - x \left( c^4 + \frac{4c^3 dx^n}{n+1} + \frac{6c^2 d^2 x^{2n}}{2n+1} + \frac{4cd^3 x^{3n}}{3n+1} + \frac{d^4 x^{4n}}{4n+1} \right) (bc - ad(5n + 1))}{5dn + d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^4, x]

[Out]  $(b*x*(c + d*x^n)^5 - (b*c - a*d*(1 + 5*n))*x*(c^4 + (4*c^3*d*x^n)/(1 + n) + (6*c^2*d^2*x^(2*n))/(1 + 2*n) + (4*c*d^3*x^(3*n))/(1 + 3*n) + (d^4*x^(4*n))/(1 + 4*n)))/(d + 5*d*n)$

**fricas [B]** time = 0.68, size = 527, normalized size = 3.99

$$\frac{(24bd^4n^4 + 50bd^4n^3 + 35bd^4n^2 + 10bd^4n + bd^4)xx^{5n} + (4bcd^3 + ad^4 + 30(4bcd^3 + ad^4)n^4 + 61(4bcd^3 + ad^4))x^{5n}}{(5dn + d)}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*x^n)\*(c+d\*x^n)^4,x, algorithm="fricas")

[Out] ((24\*b\*d^4\*n^4 + 50\*b\*d^4\*n^3 + 35\*b\*d^4\*n^2 + 10\*b\*d^4\*n + b\*d^4)\*x\*x^(5\*n) + (4\*b\*c\*d^3 + a\*d^4 + 30\*(4\*b\*c\*d^3 + a\*d^4)\*n^4 + 61\*(4\*b\*c\*d^3 + a\*d^4)\*n^3 + 41\*(4\*b\*c\*d^3 + a\*d^4)\*n^2 + 11\*(4\*b\*c\*d^3 + a\*d^4)\*n)\*x\*x^(4\*n) + 2\*(3\*b\*c^2\*d^2 + 2\*a\*c\*d^3 + 40\*(3\*b\*c^2\*d^2 + 2\*a\*c\*d^3)\*n^4 + 78\*(3\*b\*c^2\*d^2 + 2\*a\*c\*d^3)\*n^3 + 49\*(3\*b\*c^2\*d^2 + 2\*a\*c\*d^3)\*n^2 + 12\*(3\*b\*c^2\*d^2 + 2\*a\*c\*d^3)\*n)\*x\*x^(3\*n) + 2\*(2\*b\*c^3\*d + 3\*a\*c^2\*d^2 + 60\*(2\*b\*c^3\*d + 3\*a\*c^2\*d^2)\*n^4 + 107\*(2\*b\*c^3\*d + 3\*a\*c^2\*d^2)\*n^3 + 59\*(2\*b\*c^3\*d + 3\*a\*c^2\*d^2)\*n^2 + 13\*(2\*b\*c^3\*d + 3\*a\*c^2\*d^2)\*n)\*x\*x^(2\*n) + (b\*c^4 + 4\*a\*c^3\*d + 120\*(b\*c^4 + 4\*a\*c^3\*d)\*n^4 + 154\*(b\*c^4 + 4\*a\*c^3\*d)\*n^3 + 71\*(b\*c^4 + 4\*a\*c^3\*d)\*n^2 + 14\*(b\*c^4 + 4\*a\*c^3\*d)\*n)\*x\*x^n + (120\*a\*c^4\*n^5 + 274\*a\*c^4\*n^4 + 225\*a\*c^4\*n^3 + 85\*a\*c^4\*n^2 + 15\*a\*c^4\*n + a\*c^4)\*x)/(120\*n^5 + 274\*n^4 + 225\*n^3 + 85\*n^2 + 15\*n + 1)

**giac** [B] time = 0.22, size = 740, normalized size = 5.61

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$$120 ac^4 n^5 x + 24 bd^4 n^4 x x^{5n} + 120 bcd^3 n^4 x x^{4n} + 30 ad^4 n^4 x x^{4n} + 240 bc^2 d^2 n^4 x x^{3n} + 160 acd^3 n^4 x x^{3n} + 240 bc^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^4,x, algorithm="giac")

[Out] (120\*a\*c^4\*n^5\*x + 24\*b\*d^4\*n^4\*x\*x^(5\*n) + 120\*b\*c\*d^3\*n^4\*x\*x^(4\*n) + 30\*a\*d^4\*n^4\*x\*x^(4\*n) + 240\*b\*c^2\*d^2\*n^4\*x\*x^(3\*n) + 160\*a\*c\*d^3\*n^4\*x\*x^(3\*n) + 240\*b\*c^3\*d\*n^4\*x\*x^(2\*n) + 360\*a\*c^2\*d^2\*n^4\*x\*x^(2\*n) + 120\*b\*c^4\*n^4\*x\*x^n + 480\*a\*c^3\*d\*n^4\*x\*x^n + 274\*a\*c^4\*n^4\*x + 50\*b\*d^4\*n^3\*x\*x^(5\*n) + 244\*b\*c\*d^3\*n^3\*x\*x^(4\*n) + 61\*a\*d^4\*n^3\*x\*x^(4\*n) + 468\*b\*c^2\*d^2\*n^3\*x\*x^(3\*n) + 312\*a\*c\*d^3\*n^3\*x\*x^(3\*n) + 428\*b\*c^3\*d\*n^3\*x\*x^(2\*n) + 642\*a\*c^2\*d^2\*n^3\*x\*x^(2\*n) + 154\*b\*c^4\*n^3\*x\*x^n + 616\*a\*c^3\*d\*n^3\*x\*x^n + 225\*a\*c^4\*n^3\*x + 35\*b\*d^4\*n^2\*x\*x^(5\*n) + 164\*b\*c\*d^3\*n^2\*x\*x^(4\*n) + 41\*a\*d^4\*n^2\*x\*x^(4\*n) + 294\*b\*c^2\*d^2\*n^2\*x\*x^(3\*n) + 196\*a\*c\*d^3\*n^2\*x\*x^(3\*n) + 236\*b\*c^3\*d\*n^2\*x\*x^(2\*n) + 354\*a\*c^2\*d^2\*n^2\*x\*x^(2\*n) + 71\*b\*c^4\*n^2\*x\*x^n + 284\*a\*c^3\*d\*n^2\*x\*x^n + 85\*a\*c^4\*n^2\*x + 10\*b\*d^4\*n\*x\*x^(5\*n) + 44\*b\*c\*d^3\*n\*x\*x^(4\*n) + 11\*a\*d^4\*n\*x\*x^(4\*n) + 72\*b\*c^2\*d^2\*n\*x\*x^(3\*n) + 48\*a\*c\*d^3\*n\*x\*x^(3\*n) + 52\*b\*c^3\*d\*n\*x\*x^(2\*n) + 78\*a\*c^2\*d^2\*n\*x\*x^(2\*n) + 14\*b\*c^4\*n\*x\*x^n + 56\*a\*c^3\*d\*n\*x\*x^n + 15\*a\*c^4\*n\*x + b\*d^4\*x\*x^(5\*n) + 4\*b\*c\*d^3\*x\*x^(4\*n) + a\*d^4\*x\*x^(4\*n) + 6\*b\*c^2\*d^2\*x\*x^(3\*n) + 4\*a\*c\*d^3\*x\*x^(3\*n) + 4\*b\*c^3\*d\*x\*x^(2\*n) + 6\*a\*c^2\*d^2\*x\*x^(2\*n) + b\*c^4\*x\*x^n + 4\*a\*c^3\*d\*x\*x^n + a\*c^4\*x)/(120\*n^5 + 274\*n^4 + 225\*n^3 + 85\*n^2 + 15\*n + 1)

**maple** [A] time = 0.05, size = 138, normalized size = 1.05

$$\frac{bd^4 x e^{5n \ln(x)}}{5n+1} + a c^4 x + \frac{(4ad+bc)c^3 x e^{n \ln(x)}}{n+1} + \frac{2(3ad+2bc)c^2 dx e^{2n \ln(x)}}{2n+1} + \frac{2(2ad+3bc)c d^2 x e^{3n \ln(x)}}{3n+1} + \frac{(ad+4bc)c^4 x e^{4n \ln(x)}}{4n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(c+d\*x^n)^4,x)

[Out] a\*c^4\*x+b\*d^4/(1+5\*n)\*x\*exp(n\*ln(x))^5+c^3\*(4\*a\*d+b\*c)/(n+1)\*x\*exp(n\*ln(x))+d^3\*(a\*d+4\*b\*c)/(4\*n+1)\*x\*exp(n\*ln(x))^4+2\*c\*d^2\*(2\*a\*d+3\*b\*c)/(3\*n+1)\*x\*exp(n\*ln(x))^3+2\*c^2\*d\*(3\*a\*d+2\*b\*c)/(2\*n+1)\*x\*exp(n\*ln(x))^2

**maxima** [A] time = 0.61, size = 186, normalized size = 1.41

$$ac^4 x + \frac{bd^4 x^{5n+1}}{5n+1} + \frac{4bcd^3 x^{4n+1}}{4n+1} + \frac{ad^4 x^{4n+1}}{4n+1} + \frac{6bc^2 d^2 x^{3n+1}}{3n+1} + \frac{4acd^3 x^{3n+1}}{3n+1} + \frac{4bc^3 dx^{2n+1}}{2n+1} + \frac{6ac^2 d^2 x^{2n+1}}{2n+1} + \frac{bc^4 x^{n+1}}{n+1} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^4,x, algorithm="maxima")

```
[Out] a*c^4*x + b*d^4*x^(5*n + 1)/(5*n + 1) + 4*b*c*d^3*x^(4*n + 1)/(4*n + 1) + a
*d^4*x^(4*n + 1)/(4*n + 1) + 6*b*c^2*d^2*x^(3*n + 1)/(3*n + 1) + 4*a*c*d^3*
x^(3*n + 1)/(3*n + 1) + 4*b*c^3*d*x^(2*n + 1)/(2*n + 1) + 6*a*c^2*d^2*x^(2*
n + 1)/(2*n + 1) + b*c^4*x^(n + 1)/(n + 1) + 4*a*c^3*d*x^(n + 1)/(n + 1)
```

**mupad [B]** time = 1.64, size = 131, normalized size = 0.99

$$a c^4 x + \frac{x x^n (b c^4 + 4 a d c^3)}{n + 1} + \frac{x x^{4 n} (a d^4 + 4 b c d^3)}{4 n + 1} + \frac{b d^4 x x^{5 n}}{5 n + 1} + \frac{2 c^2 d x x^{2 n} (3 a d + 2 b c)}{2 n + 1} + \frac{2 c d^2 x x^{3 n} (2 a d + 3 b c)}{3 n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)*(c + d*x^n)^4,x)
```

```
[Out] a*c^4*x + (x*x^n*(b*c^4 + 4*a*c^3*d))/(n + 1) + (x*x^(4*n)*(a*d^4 + 4*b*c*d
^3))/(4*n + 1) + (b*d^4*x*x^(5*n))/(5*n + 1) + (2*c^2*d*x*x^(2*n))*(3*a*d +
2*b*c))/(2*n + 1) + (2*c*d^2*x*x^(3*n))*(2*a*d + 3*b*c))/(3*n + 1)
```

**sympy [A]** time = 3.67, size = 2744, normalized size = 20.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**4,x)
```

```
[Out] Piecewise((a*c**4*x + 4*a*c**3*d*log(x) - 6*a*c**2*d**2/x - 2*a*c*d**3/x**2
- a*d**4/(3*x**3) + b*c**4*log(x) - 4*b*c**3*d/x - 3*b*c**2*d**2/x**2 - 4*
b*c*d**3/(3*x**3) - b*d**4/(4*x**4), Eq(n, -1)), (a*c**4*x + 8*a*c**3*d*sqrt
t(x) + 6*a*c**2*d**2*log(x) - 8*a*c*d**3/sqrt(x) - a*d**4/x + 2*b*c**4*sqrt
(x) + 4*b*c**3*d*log(x) - 12*b*c**2*d**2/sqrt(x) - 4*b*c*d**3/x - 2*b*d**4/
(3*x**(3/2)), Eq(n, -1/2)), (a*c**4*x + 6*a*c**3*d*x**(2/3) + 18*a*c**2*d**
2*x**(1/3) + 4*a*c*d**3*log(x) - 3*a*d**4/x**(1/3) + 3*b*c**4*x**(2/3)/2 +
12*b*c**3*d*x**(1/3) + 6*b*c**2*d**2*log(x) - 12*b*c*d**3/x**(1/3) - 3*b*d*
**4/(2*x**(2/3)), Eq(n, -1/3)), (a*c**4*x + 16*a*c**3*d*x**(3/4)/3 + 12*a*c*
**2*d**2*sqrt(x) + 16*a*c*d**3*x**(1/4) + a*d**4*log(x) + 4*b*c**4*x**(3/4)/
3 + 8*b*c**3*d*sqrt(x) + 24*b*c**2*d**2*x**(1/4) + 4*b*c*d**3*log(x) - 4*b*
d**4/x**(1/4), Eq(n, -1/4)), (a*c**4*x + 5*a*c**3*d*x**(4/5) + 10*a*c**2*d*
**2*x**(3/5) + 10*a*c*d**3*x**(2/5) + 5*a*d**4*x**(1/5) + 5*b*c**4*x**(4/5)/
4 + 20*b*c**3*d*x**(3/5)/3 + 15*b*c**2*d**2*x**(2/5) + 20*b*c*d**3*x**(1/5)
+ b*d**4*log(x), Eq(n, -1/5)), (120*a*c**4*n**5*x/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 274*a*c**4*n**4*x/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 225*a*c**4*n**3*x/(120*n**5 + 274*n**4 + 225
*n**3 + 85*n**2 + 15*n + 1) + 85*a*c**4*n**2*x/(120*n**5 + 274*n**4 + 225*n
**3 + 85*n**2 + 15*n + 1) + 15*a*c**4*n*x/(120*n**5 + 274*n**4 + 225*n**3 +
85*n**2 + 15*n + 1) + a*c**4*x/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 +
15*n + 1) + 480*a*c**3*d*n**4*x*x**n/(120*n**5 + 274*n**4 + 225*n**3 + 85*
n**2 + 15*n + 1) + 616*a*c**3*d*n**3*x*x**n/(120*n**5 + 274*n**4 + 225*n**3
+ 85*n**2 + 15*n + 1) + 284*a*c**3*d*n**2*x*x**n/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 56*a*c**3*d*n*x*x**n/(120*n**5 + 274*n**4 +
225*n**3 + 85*n**2 + 15*n + 1) + 4*a*c**3*d*x*x**n/(120*n**5 + 274*n**4 + 2
25*n**3 + 85*n**2 + 15*n + 1) + 360*a*c**2*d**2*n**4*x*x**(2*n)/(120*n**5 +
274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 642*a*c**2*d**2*n**3*x*x**(2*n
)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 354*a*c**2*d**2*n
**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 78*a
*c**2*d**2*n*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 15*n +
1) + 6*a*c**2*d**2*x*x**(2*n)/(120*n**5 + 274*n**4 + 225*n**3 + 85*n**2 + 1
5*n + 1) + 160*a*c*d**3*n**4*x*x**(3*n)/(120*n**5 + 274*n**4 + 225*n**3 + 8
5*n**2 + 15*n + 1) + 312*a*c*d**3*n**3*x*x**(3*n)/(120*n**5 + 274*n**4 + 22
5*n**3 + 85*n**2 + 15*n + 1) + 196*a*c*d**3*n**2*x*x**(3*n)/(120*n**5 + 274
*n**4 + 225*n**3 + 85*n**2 + 15*n + 1) + 48*a*c*d**3*n*x*x**(3*n)/(120*n**5
```

$$\begin{aligned}
& + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 4ac^3d^3x^3x^3(3n)/(120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 30ad^4n^4x^3x^3(4n) \\
& / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 61ad^4n^3x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 41ad^4n^2x^3x^3(4n) \\
& / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 11ad^4n^1x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + \\
& ad^4x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 120b^4c^4n^4x^3x^3n / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 154b^4c^4n^3x^3x^3n / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 71b^4c^4n^2x^3x^3n / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 14b^4c^4n^1x^3x^3n / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + b^4c^4x^3x^3n / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 240b^3c^3d^4n^4x^3x^3(2n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 428b^3c^3d^3n^3x^3x^3(2n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 236b^3c^3d^2n^2x^3x^3(2n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 52b^3c^3d^1n^1x^3x^3(2n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 4b^3c^3d^0x^3x^3(2n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 240b^2c^2d^2n^4x^3x^3(3n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 468b^2c^2d^2n^3x^3x^3(3n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 294b^2c^2d^2n^2x^3x^3(3n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 72b^2c^2d^2n^1x^3x^3(3n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 6b^2c^2d^2x^3x^3(3n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 120b^2c^2d^1n^4x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 244b^2c^2d^1n^3x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 164b^2c^2d^1n^2x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 44b^2c^2d^1n^1x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 4b^2c^2d^1x^3x^3(4n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 24b^1d^4n^4x^3x^3(5n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 50b^1d^4n^3x^3x^3(5n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + 35b^1d^4n^2x^3x^3(5n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) \\
& + 10b^1d^4n^1x^3x^3(5n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1) + b^1d^4x^3x^3(5n) / (120n^5 + 274n^4 + 225n^3 + 85n^2 + 15n + 1), True))
\end{aligned}$$

### 3.285 $\int (a + bx^n)(c + dx^n)^3 dx$

**Optimal.** Leaf size=99

$$\frac{c^2x^{n+1}(3ad + bc)}{n + 1} + \frac{d^2x^{3n+1}(ad + 3bc)}{3n + 1} + \frac{3cdx^{2n+1}(ad + bc)}{2n + 1} + ac^3x + \frac{bd^3x^{4n+1}}{4n + 1}$$

[Out]  $a*c^3*x + c^2*(3*a*d + b*c)*x^{(1+n)}/(1+n) + 3*c*d*(a*d + b*c)*x^{(1+2*n)}/(1+2*n) + d^2*(a*d + 3*b*c)*x^{(1+3*n)}/(1+3*n) + b*d^3*x^{(1+4*n)}/(1+4*n)$

**Rubi [A]** time = 0.07, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{c^2x^{n+1}(3ad + bc)}{n + 1} + \frac{d^2x^{3n+1}(ad + 3bc)}{3n + 1} + \frac{3cdx^{2n+1}(ad + bc)}{2n + 1} + ac^3x + \frac{bd^3x^{4n+1}}{4n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^3, x]

[Out]  $a*c^3*x + (c^2*(b*c + 3*a*d)*x^{(1 + n)})/(1 + n) + (3*c*d*(b*c + a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (d^2*(3*b*c + a*d)*x^{(1 + 3*n)})/(1 + 3*n) + (b*d^3*x^{(1 + 4*n)})/(1 + 4*n)$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^n)(c + dx^n)^3 dx &= \int (ac^3 + c^2(bc + 3ad)x^n + 3cd(bc + ad)x^{2n} + d^2(3bc + ad)x^{3n} + bd^3x^{4n}) dx \\ &= ac^3x + \frac{c^2(bc + 3ad)x^{1+n}}{1 + n} + \frac{3cd(bc + ad)x^{1+2n}}{1 + 2n} + \frac{d^2(3bc + ad)x^{1+3n}}{1 + 3n} + \frac{bd^3x^{1+4n}}{1 + 4n} \end{aligned}$$

**Mathematica [A]** time = 0.13, size = 90, normalized size = 0.91

$$\frac{bx(c + dx^n)^4 - x\left(c^3 + \frac{3c^2dx^n}{n+1} + \frac{3cd^2x^{2n}}{2n+1} + \frac{d^3x^{3n}}{3n+1}\right)(bc - ad(4n + 1))}{4dn + d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^3, x]

[Out]  $(b*x*(c + d*x^n)^4 - (b*c - a*d*(1 + 4*n))*x*(c^3 + (3*c^2*d*x^n)/(1 + n) + (3*c*d^2*x^{(2*n)})/(1 + 2*n) + (d^3*x^{(3*n)})/(1 + 3*n)))/(d + 4*d*n)$

**fricas [B]** time = 1.01, size = 319, normalized size = 3.22

$$\frac{(6bd^3n^3 + 11bd^3n^2 + 6bd^3n + bd^3)xx^{4n} + (3bcd^2 + ad^3 + 8(3bcd^2 + ad^3)n^3 + 14(3bcd^2 + ad^3)n^2 + 7(3bcd^2 + ad^3)n + 7ad^3)x^{4n+1}}{4dn + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^3,x, algorithm="fricas")

[Out]  $((6*b*d^3*n^3 + 11*b*d^3*n^2 + 6*b*d^3*n + b*d^3)*x*x^(4*n) + (3*b*c*d^2 + a*d^3 + 8*(3*b*c*d^2 + a*d^3)*n^3 + 14*(3*b*c*d^2 + a*d^3)*n^2 + 7*(3*b*c*d^2 + a*d^3)*n)*x*x^(3*n) + 3*(b*c^2*d + a*c*d^2 + 12*(b*c^2*d + a*c*d^2)*n^3 + 19*(b*c^2*d + a*c*d^2)*n^2 + 8*(b*c^2*d + a*c*d^2)*n)*x*x^(2*n) + (b*c^3 + 3*a*c^2*d + 24*(b*c^3 + 3*a*c^2*d)*n^3 + 26*(b*c^3 + 3*a*c^2*d)*n^2 + 9*(b*c^3 + 3*a*c^2*d)*n)*x*x^n + (24*a*c^3*n^4 + 50*a*c^3*n^3 + 35*a*c^3*n^2 + 10*a*c^3*n + a*c^3)*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)$

**giac** [B] time = 0.19, size = 450, normalized size = 4.55

$$\frac{24ac^3n^4x + 6bd^3n^3xx^{4n} + 24bcd^2n^3xx^{3n} + 8ad^3n^3xx^{3n} + 36bc^2dn^3xx^{2n} + 36acd^2n^3xx^{2n} + 24bc^3n^3xx^n + \dots}{24n^4 + 50n^3 + 35n^2 + 10n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^3,x, algorithm="giac")

[Out]  $(24*a*c^3*n^4*x + 6*b*d^3*n^3*x*x^(4*n) + 24*b*c*d^2*n^3*x*x^(3*n) + 8*a*d^3*n^3*x*x^(3*n) + 36*b*c^2*d*n^3*x*x^(2*n) + 36*a*c*d^2*n^3*x*x^(2*n) + 24*b*c^3*n^3*x*x^n + 72*a*c^2*d*n^3*x*x^n + 50*a*c^3*n^3*x + 11*b*d^3*n^2*x*x^(4*n) + 42*b*c*d^2*n^2*x*x^(3*n) + 14*a*d^3*n^2*x*x^(3*n) + 57*b*c^2*d*n^2*x*x^(2*n) + 57*a*c*d^2*n^2*x*x^(2*n) + 26*b*c^3*n^2*x*x^n + 78*a*c^2*d*n^2*x*x^n + 35*a*c^3*n^2*x + 6*b*d^3*n*x*x^(4*n) + 21*b*c*d^2*n*x*x^(3*n) + 7*a*d^3*n*x*x^(3*n) + 24*b*c^2*d*n*x*x^(2*n) + 24*a*c*d^2*n*x*x^(2*n) + 9*b*c^3*n*x*x^n + 27*a*c^2*d*n*x*x^n + 10*a*c^3*n*x + b*d^3*x*x^(4*n) + 3*b*c*d^2*x*x^(3*n) + a*d^3*x*x^(3*n) + 3*b*c^2*d*x*x^(2*n) + 3*a*c*d^2*x*x^(2*n) + b*c^3*x*x^n + 3*a*c^2*d*x*x^n + a*c^3*x)/(24*n^4 + 50*n^3 + 35*n^2 + 10*n + 1)$

**maple** [A] time = 0.06, size = 104, normalized size = 1.05

$$\frac{bd^3xe^{4n\ln(x)}}{4n+1} + ac^3x + \frac{(3ad+bc)c^2xe^{n\ln(x)}}{n+1} + \frac{3(ad+bc)cdxe^{2n\ln(x)}}{2n+1} + \frac{(ad+3bc)d^2xe^{3n\ln(x)}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(c+d\*x^n)^3,x)

[Out]  $a*c^3*x + b*d^3/(4*n+1)*x*\exp(n*\ln(x))^4 + c^2*(3*a*d + b*c)/(n+1)*x*\exp(n*\ln(x)) + d^2*(a*d + 3*b*c)/(3*n+1)*x*\exp(n*\ln(x))^3 + 3*c*d*(a*d + b*c)/(2*n+1)*x*\exp(n*\ln(x))^2$

**maxima** [A] time = 0.50, size = 140, normalized size = 1.41

$$ac^3x + \frac{bd^3x^{4n+1}}{4n+1} + \frac{3bcd^2x^{3n+1}}{3n+1} + \frac{ad^3x^{3n+1}}{3n+1} + \frac{3bc^2dx^{2n+1}}{2n+1} + \frac{3acd^2x^{2n+1}}{2n+1} + \frac{bc^3x^{n+1}}{n+1} + \frac{3ac^2dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^3,x, algorithm="maxima")

[Out]  $a*c^3*x + b*d^3*x^(4*n + 1)/(4*n + 1) + 3*b*c*d^2*x^(3*n + 1)/(3*n + 1) + a*d^3*x^(3*n + 1)/(3*n + 1) + 3*b*c^2*d*x^(2*n + 1)/(2*n + 1) + 3*a*c*d^2*x^(2*n + 1)/(2*n + 1) + b*c^3*x^(n + 1)/(n + 1) + 3*a*c^2*d*x^(n + 1)/(n + 1)$

**mupad** [B] time = 1.56, size = 99, normalized size = 1.00

$$ac^3x + \frac{xx^n(bc^3 + 3ad^2c^2)}{n+1} + \frac{xx^{3n}(ad^3 + 3bc^2d^2)}{3n+1} + \frac{bd^3xx^{4n}}{4n+1} + \frac{3cdxx^{2n}(ad+bc)}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)\*(c + d\*x^n)^3,x)

```
[Out] a*c^3*x + (x*x^n*(b*c^3 + 3*a*c^2*d))/(n + 1) + (x*x^(3*n)*(a*d^3 + 3*b*c*d^2))/(3*n + 1) + (b*d^3*x*x^(4*n))/(4*n + 1) + (3*c*d*x*x^(2*n)*(a*d + b*c))/(2*n + 1)
```

```
sympy [A] time = 3.35, size = 1540, normalized size = 15.56
```

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**3,x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*log(x) - 3*a*c*d**2/x - a*d**3/(2*x**2) + b*c**3*log(x) - 3*b*c**2*d/x - 3*b*c*d**2/(2*x**2) - b*d**3/(3*x**3), Eq(n, -1)), (a*c**3*x + 6*a*c**2*d*sqrt(x) + 3*a*c*d**2*log(x) - 2*a*d**3/sqrt(x) + 2*b*c**3*sqrt(x) + 3*b*c**2*d*log(x) - 6*b*c*d**2/sqrt(x) - b*d**3/x, Eq(n, -1/2)), (a*c**3*x + 9*a*c**2*d*x**(2/3)/2 + 9*a*c*d**2*x**(1/3) + a*d**3*log(x) + 3*b*c**3*x**(2/3)/2 + 9*b*c**2*d*x**(1/3) + 3*b*c*d**2*log(x) - 3*b*d**3/x**(1/3), Eq(n, -1/3)), (a*c**3*x + 4*a*c**2*d*x**(3/4) + 6*a*c*d**2*sqrt(x) + 4*a*d**3*x**(1/4) + 4*b*c**3*x**(3/4)/3 + 6*b*c**2*d*sqrt(x) + 12*b*c*d**2*x**(1/4) + b*d**3*log(x), Eq(n, -1/4)), (24*a*c**3*n**4*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a*c**3*n**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a*c**3*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10*a*c**3*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*c**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 72*a*c**2*d*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 78*a*c**2*d*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 27*a*c**2*d*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c**2*d*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*a*c*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*a*c*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*a*c*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*a*c*d**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a*d**3*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*d**3*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 7*a*d**3*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a*d**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**3*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 26*b*c**3*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 9*b*c**3*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*c**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 36*b*c**2*d*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 57*b*c**2*d*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c**2*d*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*b*c**2*d*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 24*b*c*d**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 42*b*c*d**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 21*b*c*d**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 3*b*c*d**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b*d**3*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b*d**3*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b*d**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

### 3.286 $\int (a + bx^n)(c + dx^n)^2 dx$

**Optimal.** Leaf size=70

$$\frac{cx^{n+1}(2ad + bc)}{n + 1} + \frac{dx^{2n+1}(ad + 2bc)}{2n + 1} + ac^2x + \frac{bd^2x^{3n+1}}{3n + 1}$$

[Out]  $a*c^2*x+c*(2*a*d+b*c)*x^{(1+n)/(1+n)}+d*(a*d+2*b*c)*x^{(1+2*n)/(1+2*n)}+b*d^2*x^{(1+3*n)/(1+3*n)}$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$\frac{cx^{n+1}(2ad + bc)}{n + 1} + \frac{dx^{2n+1}(ad + 2bc)}{2n + 1} + ac^2x + \frac{bd^2x^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^2,x]

[Out]  $a*c^2*x + (c*(b*c + 2*a*d)*x^{(1 + n)/(1 + n)} + (d*(2*b*c + a*d)*x^{(1 + 2*n)/(1 + 2*n)} + (b*d^2*x^{(1 + 3*n)/(1 + 3*n)}))$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^n)(c + dx^n)^2 dx &= \int (ac^2 + c(bc + 2ad)x^n + d(2bc + ad)x^{2n} + bd^2x^{3n}) dx \\ &= ac^2x + \frac{c(bc + 2ad)x^{1+n}}{1 + n} + \frac{d(2bc + ad)x^{1+2n}}{1 + 2n} + \frac{bd^2x^{1+3n}}{1 + 3n} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 70, normalized size = 1.00

$$\frac{bx(c + dx^n)^3 - x\left(c^2 + \frac{2cdx^n}{n+1} + \frac{d^2x^{2n}}{2n+1}\right)(bc - ad(3n + 1))}{3dn + d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^2,x]

[Out]  $(b*x*(c + d*x^n)^3 - (b*c - a*d*(1 + 3*n))*x*(c^2 + (2*c*d*x^n)/(1 + n) + (d^2*x^{(2*n)})/(1 + 2*n)))/(d + 3*d*n)$

**fricas [B]** time = 0.98, size = 175, normalized size = 2.50

$$\frac{(2bd^2n^2 + 3bd^2n + bd^2)xx^{3n} + (2bcd + ad^2 + 3(2bcd + ad^2)n^2 + 4(2bcd + ad^2)n)xx^{2n} + (bc^2 + 2acd + 6cd^2n)}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^2,x, algorithm="fricas")

[Out]  $((2*b*d^2*n^2 + 3*b*d^2*n + b*d^2)*x*x^(3*n) + (2*b*c*d + a*d^2 + 3*(2*b*c*d + a*d^2)*n^2 + 4*(2*b*c*d + a*d^2)*n)*x*x^(2*n) + (b*c^2 + 2*a*c*d + 6*(b*c^2 + 2*a*c*d)*n^2 + 5*(b*c^2 + 2*a*c*d)*n)*x*x^n + (6*a*c^2*n^3 + 11*a*c^2*n^2 + 6*a*c^2*n + a*c^2)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**giac** [B] time = 0.19, size = 232, normalized size = 3.31

$$\frac{6ac^2n^3x + 2bd^2n^2xx^{3n} + 6bcdn^2xx^{2n} + 3ad^2n^2xx^{2n} + 6bc^2n^2xx^n + 12acdn^2xx^n + 11ac^2n^2x + 3bd^2nxx^{3n} + 8bd^2nxx^{2n}}{6n^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^2,x, algorithm="giac")

[Out]  $(6*a*c^2*n^3*x + 2*b*d^2*n^2*x*x^(3*n) + 6*b*c*d*n^2*x*x^(2*n) + 3*a*d^2*n^2*x*x^(2*n) + 6*b*c^2*n^2*x*x^n + 12*a*c*d*n^2*x*x^n + 11*a*c^2*n^2*x + 3*b*d^2*n*x*x^(3*n) + 8*b*c*d*n*x*x^(2*n) + 4*a*d^2*n*x*x^(2*n) + 5*b*c^2*n*x*x^n + 10*a*c*d*n*x*x^n + 6*a*c^2*n*x + b*d^2*x*x^(3*n) + 2*b*c*d*x*x^(2*n) + a*d^2*x*x^(2*n) + b*c^2*x*x^n + 2*a*c*d*x*x^n + a*c^2*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**maple** [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{bd^2xe^{3n\ln(x)}}{3n+1} + ac^2x + \frac{(2ad+bc)cx e^{n\ln(x)}}{n+1} + \frac{(ad+2bc)dx e^{2n\ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(c+d\*x^n)^2,x)

[Out]  $a*c^2*x + b*d^2/(3*n+1)*x*\exp(n*\ln(x))^3 + c*(2*a*d+b*c)/(n+1)*x*\exp(n*\ln(x)) + d*(a*d+2*b*c)/(2*n+1)*x*\exp(n*\ln(x))^2$

**maxima** [A] time = 0.44, size = 94, normalized size = 1.34

$$ac^2x + \frac{bd^2x^{3n+1}}{3n+1} + \frac{2bcdx^{2n+1}}{2n+1} + \frac{ad^2x^{2n+1}}{2n+1} + \frac{bc^2x^{n+1}}{n+1} + \frac{2acdx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^2,x, algorithm="maxima")

[Out]  $a*c^2*x + b*d^2*x^(3*n + 1)/(3*n + 1) + 2*b*c*d*x^(2*n + 1)/(2*n + 1) + a*d^2*x^(2*n + 1)/(2*n + 1) + b*c^2*x^(n + 1)/(n + 1) + 2*a*c*d*x^(n + 1)/(n + 1)$

**mupad** [B] time = 1.53, size = 71, normalized size = 1.01

$$ac^2x + \frac{xx^{2n}(ad^2 + 2bcd)}{2n+1} + \frac{xx^n(bc^2 + 2adc)}{n+1} + \frac{bd^2xx^{3n}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)\*(c + d\*x^n)^2,x)

[Out]  $a*c^2*x + (x*x^(2*n)*(a*d^2 + 2*b*c*d))/(2*n + 1) + (x*x^n*(b*c^2 + 2*a*c*d))/(n + 1) + (b*d^2*x*x^(3*n))/(3*n + 1)$



sympy [A] time = 1.96, size = 726, normalized size = 10.37

$$\left\{ \begin{array}{l} ac^2x + 2acd \log(x) - \frac{ad^2}{x} + bc^2 \log(x) - \frac{2bcd}{x} - \frac{bd^2}{2x^2} \\ ac^2x + 4acd\sqrt{x} + ad^2 \log(x) + 2bc^2\sqrt{x} + 2bcd \log(x) - \frac{2bd^2}{\sqrt{x}} \\ ac^2x + 3acdx^{\frac{2}{3}} + 3ad^2\sqrt[3]{x} + \frac{3bc^2x^{\frac{2}{3}}}{2} + 6bcd\sqrt[3]{x} + bd^2 \log(x) \\ \frac{6ac^2n^3x}{6n^3+11n^2+6n+1} + \frac{11ac^2n^2x}{6n^3+11n^2+6n+1} + \frac{6ac^2nx}{6n^3+11n^2+6n+1} + \frac{ac^2x}{6n^3+11n^2+6n+1} + \frac{12acd n^2 x x^n}{6n^3+11n^2+6n+1} + \frac{10acd n x x^n}{6n^3+11n^2+6n+1} + \frac{2acdx x^n}{6n^3+11n^2+6n+1} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**2,x)
```

```
[Out] Piecewise((a*c**2*x + 2*a*c*d*log(x) - a*d**2/x + b*c**2*log(x) - 2*b*c*d/x - b*d**2/(2*x**2), Eq(n, -1)), (a*c**2*x + 4*a*c*d*sqrt(x) + a*d**2*log(x) + 2*b*c**2*sqrt(x) + 2*b*c*d*log(x) - 2*b*d**2/sqrt(x), Eq(n, -1/2)), (a*c**2*x + 3*a*c*d*x**(2/3) + 3*a*d**2*x**(1/3) + 3*b*c**2*x**(2/3)/2 + 6*b*c*d*x**(1/3) + b*d**2*log(x), Eq(n, -1/3)), (6*a*c**2*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a*c**2*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*c**2*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a*c**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*c*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*c*d*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*c*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 3*a*d**2*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*a*d**2*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + a*d**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*c**2*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*b*c**2*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + b*c**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*b*c*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*b*c*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*c*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b*d**2*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b*d**2*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b*d**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```

### 3.287 $\int (a + bx^n)(c + dx^n) dx$

**Optimal.** Leaf size=40

$$\frac{x^{n+1}(ad + bc)}{n + 1} + acx + \frac{bdx^{2n+1}}{2n + 1}$$

[Out]  $a*c*x + (a*d+b*c)*x^{(1+n)}/(1+n) + b*d*x^{(1+2*n)}/(1+2*n)$

**Rubi [A]** time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {373}

$$\frac{x^{n+1}(ad + bc)}{n + 1} + acx + \frac{bdx^{2n+1}}{2n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n), x]

[Out]  $a*c*x + ((b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*d*x^{(1 + 2*n)})/(1 + 2*n)$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\begin{aligned} \int (a + bx^n)(c + dx^n) dx &= \int (ac + (bc + ad)x^n + bdx^{2n}) dx \\ &= acx + \frac{(bc + ad)x^{1+n}}{1 + n} + \frac{bdx^{1+2n}}{1 + 2n} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 37, normalized size = 0.92

$$x \left( \frac{x^n(ad + bc)}{n + 1} + ac + \frac{bdx^{2n}}{2n + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n), x]

[Out]  $x*(a*c + ((b*c + a*d)*x^n)/(1 + n) + (b*d*x^{(2*n)})/(1 + 2*n))$

**fricas [A]** time = 0.98, size = 69, normalized size = 1.72

$$\frac{(bdn + bd)xx^{2n} + (bc + ad + 2(bc + ad)n)xx^n + (2acn^2 + 3acn + ac)x}{2n^2 + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n), x, algorithm="fricas")

[Out]  $((b*d*n + b*d)*x*x^{(2*n)} + (b*c + a*d + 2*(b*c + a*d)*n)*x*x^n + (2*a*c*n^2 + 3*a*c*n + a*c)*x)/(2*n^2 + 3*n + 1)$

**giac [B]** time = 0.17, size = 83, normalized size = 2.08

$$\frac{2acn^2x + bdnxx^{2n} + 2bcnxx^n + 2adnxx^n + 3acnx + bdx^{2n} + bcxx^n + adxx^n + acx}{2n^2 + 3n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n),x, algorithm="giac")

[Out]  $(2*a*c*n^2*x + b*d*n*x*x^(2*n) + 2*b*c*n*x*x^n + 2*a*d*n*x*x^n + 3*a*c*n*x + b*d*x*x^(2*n) + b*c*x*x^n + a*d*x*x^n + a*c*x)/(2*n^2 + 3*n + 1)$

**maple** [A] time = 0.04, size = 43, normalized size = 1.08

$$\frac{bdx e^{2n \ln(x)}}{2n+1} + acx + \frac{(ad+bc)x e^{n \ln(x)}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(c+d\*x^n),x)

[Out]  $a*c*x+(a*d+b*c)/(n+1)*x*\exp(n*\ln(x))+b*d/(2*n+1)*x*\exp(n*\ln(x))^2$

**maxima** [A] time = 0.51, size = 48, normalized size = 1.20

$$acx + \frac{bdx^{2n+1}}{2n+1} + \frac{bcx^{n+1}}{n+1} + \frac{adx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n),x, algorithm="maxima")

[Out]  $a*c*x + b*d*x^(2*n + 1)/(2*n + 1) + b*c*x^(n + 1)/(n + 1) + a*d*x^(n + 1)/(n + 1)$

**mupad** [B] time = 1.48, size = 38, normalized size = 0.95

$$acx + \frac{xx^n(ad+bc)}{n+1} + \frac{bdxx^{2n}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)\*(c + d\*x^n),x)

[Out]  $a*c*x + (x*x^n*(a*d + b*c))/(n + 1) + (b*d*x*x^(2*n))/(2*n + 1)$

**sympy** [A] time = 0.65, size = 236, normalized size = 5.90

$$\begin{cases} acx + ad \log(x) + bc \log(x) - \frac{bd}{x} & \text{for } n = \\ acx + 2ad\sqrt{x} + 2bc\sqrt{x} + bd \log(x) & \text{for } n = \\ \frac{2acn^2x}{2n^2+3n+1} + \frac{3acnx}{2n^2+3n+1} + \frac{acx}{2n^2+3n+1} + \frac{2adnxx^n}{2n^2+3n+1} + \frac{adxx^n}{2n^2+3n+1} + \frac{2bcnxx^n}{2n^2+3n+1} + \frac{bcxx^n}{2n^2+3n+1} + \frac{bdnxx^{2n}}{2n^2+3n+1} + \frac{bdxx^{2n}}{2n^2+3n+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n),x)

[Out] Piecewise((a\*c\*x + a\*d\*log(x) + b\*c\*log(x) - b\*d/x, Eq(n, -1)), (a\*c\*x + 2\*a\*d\*sqrt(x) + 2\*b\*c\*sqrt(x) + b\*d\*log(x), Eq(n, -1/2)), (2\*a\*c\*n\*\*2\*x/(2\*n\*\*2 + 3\*n + 1) + 3\*a\*c\*n\*x/(2\*n\*\*2 + 3\*n + 1) + a\*c\*x/(2\*n\*\*2 + 3\*n + 1) + 2\*a\*d\*n\*x\*x\*\*n/(2\*n\*\*2 + 3\*n + 1) + a\*d\*x\*x\*\*n/(2\*n\*\*2 + 3\*n + 1) + 2\*b\*c\*n\*x\*x\*\*n/(2\*n\*\*2 + 3\*n + 1) + b\*c\*x\*x\*\*n/(2\*n\*\*2 + 3\*n + 1) + b\*d\*n\*x\*x\*\*(2\*n)/(2\*n\*\*2 + 3\*n + 1) + b\*d\*x\*x\*\*(2\*n)/(2\*n\*\*2 + 3\*n + 1), True))

$$3.288 \quad \int \frac{a+bx^n}{c+dx^n} dx$$

Optimal. Leaf size=43

$$\frac{bx}{d} - \frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd}$$

[Out] b\*x/d-(-a\*d+b\*c)\*x\*hypergeom([1, 1/n], [1+1/n], -d\*x^n/c)/c/d

**Rubi [A]** time = 0.02, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {388, 245}

$$\frac{bx}{d} - \frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n), x]

[Out] (b\*x)/d - ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*d)

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^n}{c+dx^n} dx &= \frac{bx}{d} - \frac{(bc-ad) \int \frac{1}{c+dx^n} dx}{d} \\ &= \frac{bx}{d} - \frac{(bc-ad)x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.93

$$\frac{x\left((ad-bc) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + bc\right)}{cd}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n), x]

[Out] (x\*(b\*c + (-b\*c) + a\*d)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)]))/(c\*d)

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n + a}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n),x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d\*x^n + c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n),x, algorithm="giac")

[Out] integrate((b\*x^n + a)/(d\*x^n + c), x)

**maple** [F] time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)/(c+d\*x^n),x)

[Out] int((b\*x^n+a)/(c+d\*x^n),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-(bc - ad) \int \frac{1}{d^2x^n + cd} dx + \frac{bx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n),x, algorithm="maxima")

[Out] -(b\*c - a\*d)\*integrate(1/(d^2\*x^n + c\*d), x) + b\*x/d

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + bx^n}{c + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n),x)

[Out] int((a + b\*x^n)/(c + d\*x^n), x)

**sympy** [C] time = 3.33, size = 73, normalized size = 1.70

$$\frac{ax\Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{cn^2\Gamma\left(1 + \frac{1}{n}\right)} - \frac{bx\Phi\left(\frac{cx^{-n} e^{i\pi}}{d}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{dn^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)/(c+d\*x\*\*n),x)

[Out] a\*x\*lerchphi(d\*x\*\*n\*exp\_polar(I\*pi)/c, 1, 1/n)\*gamma(1/n)/(c\*n\*\*2\*gamma(1 + 1/n)) - b\*x\*lerchphi(c\*x\*\*(-n)\*exp\_polar(I\*pi)/d, 1, exp\_polar(I\*pi)/n)\*gamma(1/n)/(d\*n\*\*2\*gamma(1 + 1/n))

$$3.289 \quad \int \frac{a+bx^n}{(c+dx^n)^2} dx$$

Optimal. Leaf size=73

$$\frac{x(bc - ad(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn} - \frac{x(bc - ad)}{cdn(c + dx^n)}$$

[Out]  $-(a*d+b*c)*x/c/d/n/(c+d*x^n)+(b*c-a*d*(1-n))*x*\text{hypergeom}([1, 1/n], [1+1/n], -d*x^n/c)/c^2/d/n$

**Rubi [A]** time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {385, 245}

$$\frac{x(bc - ad(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn} - \frac{x(bc - ad)}{cdn(c + dx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^2, x]

[Out]  $-(((b*c - a*d)*x)/(c*d*n*(c + d*x^n))) + ((b*c - a*d*(1 - n))*x*\text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)])/(c^2*d*n)$

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \frac{a + bx^n}{(c + dx^n)^2} dx &= -\frac{(bc - ad)x}{cdn(c + dx^n)} + \frac{(bc - ad(1 - n)) \int \frac{1}{c + dx^n} dx}{cdn} \\ &= -\frac{(bc - ad)x}{cdn(c + dx^n)} + \frac{(bc - ad(1 - n))x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 dn} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 56, normalized size = 0.77

$$\frac{x\left(\frac{b}{c+dx^n} - \frac{(ad(n-1)+bc) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2}\right)}{d - dn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^2, x]

[Out] (x\*(b/(c + d\*x^n) - ((b\*c + a\*d\*(-1 + n))\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/c^2))/(d - d\*n)

**fricas** [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n + a}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^2, x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^2, x, algorithm="giac")

[Out] integrate((b\*x^n + a)/(d\*x^n + c)^2, x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)/(d\*x^n+c)^2, x)

[Out] int((b\*x^n+a)/(d\*x^n+c)^2, x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(ad(n-1) + bc) \int \frac{1}{cd^2nx^n + c^2dn} dx - \frac{(bc - ad)x}{cd^2nx^n + c^2dn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^2, x, algorithm="maxima")

[Out] (a\*d\*(n - 1) + b\*c)\*integrate(1/(c\*d^2\*n\*x^n + c^2\*d\*n), x) - (b\*c - a\*d)\*x/(c\*d^2\*n\*x^n + c^2\*d\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + bx^n}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n)^2, x)

[Out] int((a + b\*x^n)/(c + d\*x^n)^2, x)

**sympy** [C] time = 8.69, size = 592, normalized size = 8.11

$$a \left( \frac{nx\Phi\left(\frac{dx^ne^{i\pi}}{c}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1 + \frac{1}{n}\right) + dn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} + \frac{nx\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1 + \frac{1}{n}\right) + dn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} - \frac{x\Phi\left(\frac{dx^ne^{i\pi}}{c}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{c\left(cn^3\Gamma\left(1 + \frac{1}{n}\right) + dn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*2,x)

[Out]  $a*(n*x*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1/n)*\gamma(1/n)/(c*(c*n**3*\gamma(1 + 1/n) + d*n**3*x**n*\gamma(1 + 1/n))) + n*x*\gamma(1/n)/(c*(c*n**3*\gamma(1 + 1/n) + d*n**3*x**n*\gamma(1 + 1/n))) - x*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1/n)*\gamma(1/n)/(c*(c*n**3*\gamma(1 + 1/n) + d*n**3*x**n*\gamma(1 + 1/n))) + d*n*x*x**n*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1/n)*\gamma(1/n)/(c**2*(c*n**3*\gamma(1 + 1/n) + d*n**3*x**n*\gamma(1 + 1/n))) - d*x*x**n*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1/n)*\gamma(1/n)/(c**2*(c*n**3*\gamma(1 + 1/n) + d*n**3*x**n*\gamma(1 + 1/n))) + b*(n**2*x*x**n*\gamma(1 + 1/n)/(c*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n))) - n*x*x**n*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1 + 1/n)*\gamma(1 + 1/n)/(c*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n))) + n*x*x**n*\gamma(1 + 1/n)/(c*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n))) - x*x**n*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1 + 1/n)*\gamma(1 + 1/n)/(c*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n))) - d*n*x*x**(2*n)*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1 + 1/n)*\gamma(1 + 1/n)/(c**2*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n))) - d*x*x**(2*n)*\operatorname{lerchphi}(d*x**n*\exp\_polar(I*\pi)/c, 1, 1 + 1/n)*\gamma(1 + 1/n)/(c**2*(c*n**3*\gamma(2 + 1/n) + d*n**3*x**n*\gamma(2 + 1/n)))$



$$3.290 \quad \int \frac{a+bx^n}{(c+dx^n)^3} dx$$

Optimal. Leaf size=78

$$\frac{x(bc - ad(1 - 2n)) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc - ad)}{2cdn(c + dx^n)^2}$$

[Out] -1/2\*(-a\*d+b\*c)\*x/c/d/n/(c+d\*x^n)^2+1/2\*(b\*c-a\*d\*(1-2\*n))\*x\*hypergeom([2, 1/n], [1+1/n], -d\*x^n/c)/c^3/d/n

**Rubi [A]** time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {385, 245}

$$\frac{x(bc - ad(1 - 2n)) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn} - \frac{x(bc - ad)}{2cdn(c + dx^n)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^3, x]

[Out] -((b\*c - a\*d)\*x)/(2\*c\*d\*n\*(c + d\*x^n)^2) + ((b\*c - a\*d\*(1 - 2\*n))\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(2\*c^3\*d\*n)

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \frac{a + bx^n}{(c + dx^n)^3} dx &= -\frac{(bc - ad)x}{2cdn(c + dx^n)^2} + \frac{(bc - ad(1 - 2n)) \int \frac{1}{(c + dx^n)^2} dx}{2cdn} \\ &= -\frac{(bc - ad)x}{2cdn(c + dx^n)^2} + \frac{(bc - ad(1 - 2n))x {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3dn} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 58, normalized size = 0.74

$$\frac{x \left( \frac{b}{(c+dx^n)^2} - \frac{(ad(2n-1)+bc) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^3} \right)}{d - 2dn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^3,x]

[Out] (x\*(b/(c + d\*x^n)^2 - ((b\*c + a\*d\*(-1 + 2\*n))\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/c^3))/(d - 2\*d\*n)

**fricas** [F] time = 0.96, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n + a}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="giac")

[Out] integrate((b\*x^n + a)/(d\*x^n + c)^3, x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)/(d\*x^n+c)^3,x)

[Out] int((b\*x^n+a)/(d\*x^n+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left((2n^2 - 3n + 1)ad + bc(n - 1)\right) \int \frac{1}{2(c^2d^2n^2x^n + c^3dn^2)} dx + \frac{(ad^2(2n - 1) + bcd)xx^n + (acd(3n - 1) - bc^2(n - 1))}{2(c^2d^3n^2x^{2n} + 2c^3d^2n^2x^n + c^4dn^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*a\*d + b\*c\*(n - 1))\*integrate(1/2/(c^2\*d^2\*n^2\*x^n + c^3\*d\*n^2), x) + 1/2\*((a\*d^2\*(2\*n - 1) + b\*c\*d)\*x\*x^n + (a\*c\*d\*(3\*n - 1) - b\*c^2\*(n - 1))\*x)/(c^2\*d^3\*n^2\*x^(2\*n) + 2\*c^3\*d^2\*n^2\*x^n + c^4\*d\*n^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + bx^n}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n)^3,x)

[Out] int((a + b\*x^n)/(c + d\*x^n)^3, x)

sympy [C] time = 64.99, size = 3706, normalized size = 47.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*3,x)

[Out] 
$$\begin{aligned} & a(2c^n x^2 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + 3c^n x^2 \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) - 3c^n x \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) - c^n x \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + c x \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + 6d^n x^2 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + 5d^n x^2 \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) - 9d^n x \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) - 2d^n x \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + 3d^n x \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n)) + 6d^{n+2} x^2 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) + 2d^{n+2} x^2 \Gamma(1/n) / (c(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) - 9d^{n+2} x^2 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) - d^{n+2} x^2 \Gamma(1/n) / (c(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) + 3d^{n+2} x^2 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) + 2d^{n+3} x^3 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c^2(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) - 3d^{n+3} x^3 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c^2(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) + d^{n+3} x^3 \operatorname{lerchphi}(d x^n \exp_{\text{polar}}(I\pi)/c, 1, 1/n) \Gamma(1/n) / (c^2(2c^{n+4} \Gamma(1 + 1/n) + 6c^{n+3} d^n x^n \Gamma(1 + 1/n) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(1 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(1 + 1/n))) + b(2c^n x^3 \Gamma(1 + 1/n) / (2c^{n+4} \Gamma(2 + 1/n) + 6c^{n+3} d^n x^n \Gamma(2 + 1/n)) + 6c^{n+2} d^{n+2} x^{2n} \Gamma(2n) \Gamma(2 + 1/n) + 2c^d d^{n+3} x^{3n} \Gamma(3n) \Gamma(2 + 1/n)) \end{aligned}$$



$$3.291 \quad \int \frac{a+bx^n}{(c+dx^n)^4} dx$$

Optimal. Leaf size=78

$$\frac{x(bc - ad(1 - 3n)) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc - ad)}{3cdn(c + dx^n)^3}$$

[Out] -1/3\*(-a\*d+b\*c)\*x/c/d/n/(c+d\*x^n)^3+1/3\*(b\*c-a\*d\*(1-3\*n))\*x\*hypergeom([3, 1/n], [1+1/n], -d\*x^n/c)/c^4/d/n

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {385, 245}

$$\frac{x(bc - ad(1 - 3n)) {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn} - \frac{x(bc - ad)}{3cdn(c + dx^n)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)/(c + d\*x^n)^4, x]

[Out] -((b\*c - a\*d)\*x)/(3\*c\*d\*n\*(c + d\*x^n)^3) + ((b\*c - a\*d\*(1 - 3\*n))\*x\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(3\*c^4\*d\*n)

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \frac{a + bx^n}{(c + dx^n)^4} dx &= -\frac{(bc - ad)x}{3cdn(c + dx^n)^3} + \frac{(bc - ad(1 - 3n)) \int \frac{1}{(c + dx^n)^3} dx}{3cdn} \\ &= -\frac{(bc - ad)x}{3cdn(c + dx^n)^3} + \frac{(bc - ad(1 - 3n))x {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{3c^4dn} \end{aligned}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.74

$$\frac{x \left( \frac{b}{(c+dx^n)^3} - \frac{(ad(3n-1)+bc) {}_2F_1\left(4, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^4} \right)}{d - 3dn}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)/(c + d\*x^n)^4,x]

[Out] (x\*(b/(c + d\*x^n)^3 - ((b\*c + a\*d\*(-1 + 3\*n))\*Hypergeometric2F1[4, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/c^4))/(d - 3\*d\*n)

**fricas** [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{bx^n + a}{d^4x^{4n} + 4cd^3x^{3n} + 6c^2d^2x^{2n} + 4c^3dx^n + c^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^4,x, algorithm="fricas")

[Out] integral((b\*x^n + a)/(d^4\*x^(4\*n) + 4\*c\*d^3\*x^(3\*n) + 6\*c^2\*d^2\*x^(2\*n) + 4\*c^3\*d\*x^n + c^4), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^4,x, algorithm="giac")

[Out] integrate((b\*x^n + a)/(d\*x^n + c)^4, x)

**maple** [F] time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{bx^n + a}{(dx^n + c)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)/(d\*x^n+c)^4,x)

[Out] int((b\*x^n+a)/(d\*x^n+c)^4,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left((2n^2 - 3n + 1)bc + (6n^3 - 11n^2 + 6n - 1)ad\right) \int \frac{1}{6(c^3d^2n^3x^n + c^4dn^3)} dx + \frac{((6n^2 - 5n + 1)ad^3 + bcd^2(2n - 1))}{6(c^3d^2n^3x^n + c^4dn^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)/(c+d\*x^n)^4,x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*b\*c + (6\*n^3 - 11\*n^2 + 6\*n - 1)\*a\*d)\*integrate(1/6/(c^3\*d^2\*n^3\*x^n + c^4\*d\*n^3), x) + 1/6\*(((6\*n^2 - 5\*n + 1)\*a\*d^3 + b\*c\*d^2\*(2\*n - 1))\*x\*x^(2\*n) + ((15\*n^2 - 11\*n + 2)\*a\*c\*d^2 + b\*c^2\*d\*(5\*n - 2))\*x\*x^n - ((2\*n^2 - 3\*n + 1)\*b\*c^3 - (11\*n^2 - 6\*n + 1)\*a\*c^2\*d)\*x)/(c^3\*d^4\*n^3\*x^(3\*n) + 3\*c^4\*d^3\*n^3\*x^(2\*n) + 3\*c^5\*d^2\*n^3\*x^n + c^6\*d\*n^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + bx^n}{(c + dx^n)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n)^4,x)

```
[Out] int((a + b*x^n)/(c + d*x^n)^4, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)/(c+d*x**n)**4,x)
```

```
[Out] Timed out
```

### 3.292 $\int (a + bx^n)^2 (d + ex^n)^3 dx$

**Optimal.** Leaf size=158

$$\frac{dx^{2n+1} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n+1} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3x + \frac{ad^2x^{n+1}(3ae + 2bd)}{n+1} + \frac{be^2x^{4n+1}(2ae + 3bd)}{4n+1}$$

[Out]  $a^2d^3x + a^2d^2(3ae + 2bd)x^{n+1}/(n+1) + d(3a^2e^2 + 6abde + b^2d^2)x^{2n+1}/(2n+1) + e(a^2e^2 + 6abde + 3b^2d^2)x^{3n+1}/(3n+1) + b^2e^2x^{4n+1}/(4n+1)$

**Rubi [A]** time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{dx^{2n+1} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n+1} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3x + \frac{ad^2x^{n+1}(3ae + 2bd)}{n+1} + \frac{be^2x^{4n+1}(2ae + 3bd)}{4n+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(d + e\*x^n)^3,x]

[Out]  $a^2d^3x + (a^2d^2(2bd + 3ae)x^{n+1}/(n+1) + d(b^2d^2 + 6abde + 3a^2e^2)x^{2n+1}/(2n+1) + e(3b^2d^2 + 6abde + a^2e^2)x^{3n+1}/(3n+1) + (b^2e^2(3bd + 2ae)x^{4n+1}/(4n+1) + b^2e^3x^{5n+1}/(5n+1))$

#### Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

#### Rubi steps

$$\int (a + bx^n)^2 (d + ex^n)^3 dx = \int (a^2d^3 + ad^2(2bd + 3ae)x^n + d(b^2d^2 + 6abde + 3a^2e^2)x^{2n} + e(3b^2d^2 + 6abde + a^2e^2)x^{3n}) dx$$

$$= a^2d^3x + \frac{ad^2(2bd + 3ae)x^{1+n}}{1+n} + \frac{d(b^2d^2 + 6abde + 3a^2e^2)x^{1+2n}}{1+2n} + \frac{e(3b^2d^2 + 6abde + a^2e^2)x^{1+3n}}{1+3n} + \frac{b^2e^2(3bd + 2ae)x^{1+4n}}{1+4n} + \frac{b^2e^3x^{1+5n}}{1+5n}$$

**Mathematica [A]** time = 0.21, size = 149, normalized size = 0.94

$$x \left( \frac{dx^{2n} (3a^2e^2 + 6abde + b^2d^2)}{2n+1} + \frac{ex^{3n} (a^2e^2 + 6abde + 3b^2d^2)}{3n+1} + a^2d^3 + \frac{ad^2x^n(3ae + 2bd)}{n+1} + \frac{be^2x^{4n}(2ae + 3bd)}{4n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(d + e\*x^n)^3,x]

[Out]  $x(a^2d^3 + a^2d^2(2bd + 3ae)x^n/(n+1) + d(b^2d^2 + 6abde + 3a^2e^2)x^{2n}/(2n+1) + e(3b^2d^2 + 6abde + a^2e^2)x^{3n}/(3n+1) + (b^2e^2(3bd + 2ae)x^{4n}/(4n+1) + b^2e^3x^{5n}/(5n+1)))$

**fricas [B]** time = 1.00, size = 667, normalized size = 4.22

$$(24b^2e^3n^4 + 50b^2e^3n^3 + 35b^2e^3n^2 + 10b^2e^3n + b^2e^3)xx^{5n} + (3b^2de^2 + 2abe^3 + 30(3b^2de^2 + 2abe^3)n^4 + 61(3$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="fricas")
```

```
[Out] ((24*b^2*e^3*n^4 + 50*b^2*e^3*n^3 + 35*b^2*e^3*n^2 + 10*b^2*e^3*n + b^2*e^3)
)*x*x^(5*n) + (3*b^2*d*e^2 + 2*a*b*e^3 + 30*(3*b^2*d*e^2 + 2*a*b*e^3)*n^4 +
61*(3*b^2*d*e^2 + 2*a*b*e^3)*n^3 + 41*(3*b^2*d*e^2 + 2*a*b*e^3)*n^2 + 11*(
3*b^2*d*e^2 + 2*a*b*e^3)*n)*x*x^(4*n) + (3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^
3 + 40*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^4 + 78*(3*b^2*d^2*e + 6*a*b*
d*e^2 + a^2*e^3)*n^3 + 49*(3*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n^2 + 12*(3
*b^2*d^2*e + 6*a*b*d*e^2 + a^2*e^3)*n)*x*x^(3*n) + (b^2*d^3 + 6*a*b*d^2*e +
3*a^2*d*e^2 + 60*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n^4 + 107*(b^2*d^3
+ 6*a*b*d^2*e + 3*a^2*d*e^2)*n^3 + 59*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)
)*n^2 + 13*(b^2*d^3 + 6*a*b*d^2*e + 3*a^2*d*e^2)*n)*x*x^(2*n) + (2*a*b*d^3 +
3*a^2*d^2*e + 120*(2*a*b*d^3 + 3*a^2*d^2*e)*n^4 + 154*(2*a*b*d^3 + 3*a^2*d
^2*e)*n^3 + 71*(2*a*b*d^3 + 3*a^2*d^2*e)*n^2 + 14*(2*a*b*d^3 + 3*a^2*d^2*e)
)*n)*x*x^n + (120*a^2*d^3*n^5 + 274*a^2*d^3*n^4 + 225*a^2*d^3*n^3 + 85*a^2*d
^3*n^2 + 15*a^2*d^3*n + a^2*d^3)*x)/(120*n^5 + 274*n^4 + 225*n^3 + 85*n^2 +
15*n + 1)
```

**giac [B]** time = 0.24, size = 947, normalized size = 5.99

$$\frac{120 a^2 d^3 n^5 x + 60 b^2 d^3 n^4 x x^{2n} + 240 a b d^3 n^4 x x^n + 120 b^2 d^2 n^4 x x^{3n} e + 360 a b d^2 n^4 x x^{2n} e + 360 a^2 d^2 n^4 x x^n e + 274 a^2 d^3 n^4 x + 107 b^2 d^3 n^3 x x^{2n} + 308 a^2 b d^3 n^3 x x^n + 90 b^2 d^2 n^4 x x^{4n} e^2 + 240 a^2 b d^2 n^4 x x^{3n} e^2 + 180 a^2 d^2 n^4 x x^{2n} e^2 + 234 b^2 d^2 n^3 x x^{3n} e + 642 a^2 b d^2 n^3 x x^{2n} e + 462 a^2 d^2 n^3 x x^n e + 225 a^2 d^3 n^3 x + 59 b^2 d^3 n^2 x x^{2n} + 142 a^2 b d^3 n^2 x x^n + 24 b^2 n^4 x x^{5n} e^3 + 60 a^2 b n^4 x x^{4n} e^3 + 40 a^2 n^4 x x^{3n} e^3 + 183 b^2 d^2 n^3 x x^{4n} e^2 + 468 a^2 b d^2 n^3 x x^{3n} e^2 + 321 a^2 d^2 n^3 x x^{2n} e^2 + 147 b^2 d^2 n^2 x x^{3n} e + 354 a^2 b d^2 n^2 x x^{2n} e + 213 a^2 d^2 n^2 x x^n e + 85 a^2 d^3 n^2 x + 13 b^2 d^3 n^2 x x^{2n} + 28 a^2 b d^3 n^2 x x^n + 50 b^2 n^3 x x^{5n} e^3 + 122 a^2 b n^3 x x^{4n} e^3 + 78 a^2 n^3 x x^{3n} e^3 + 123 b^2 d^2 n^2 x x^{4n} e^2 + 294 a^2 b d^2 n^2 x x^{3n} e^2 + 177 a^2 d^2 n^2 x x^{2n} e^2 + 36 b^2 d^2 n^2 x x^{3n} e + 78 a^2 b d^2 n^2 x x^{2n} e + 42 a^2 d^2 n^2 x x^n e + 15 a^2 d^3 n^2 x + b^2 d^3 x x^{2n} + 2 a^2 b d^3 x x^n + 35 b^2 n^2 x x^{5n} e^3 + 82 a^2 b n^2 x x^{4n} e^3 + 49 a^2 n^2 x x^{3n} e^3 + 33 b^2 d^2 n^2 x x^{4n} e^2 + 72 a^2 b d^2 n^2 x x^{3n} e^2 + 39 a^2 d^2 n^2 x x^{2n} e^2 + 3 b^2 d^2 n^2 x x^{3n} e + 6 a^2 b d^2 n^2 x x^{2n} e + 3 a^2 d^2 n^2 x x^n e + a^2 d^3 n^2 x + 10 b^2 n^2 x x^{5n} e^3 + 22 a^2 b n^2 x x^{4n} e^3 + 12 a^2 n^2 x x^{3n} e^3 + 3 b^2 d^2 n^2 x x^{4n} e^2 + 6 a^2 b d^2 n^2 x x^{3n} e^2 + 3 a^2 d^2 n^2 x x^{2n} e^2 + b^2 x x^{5n} e^3 + 2 a^2 b x x^{4n} e^3 + a^2 x x^{3n} e^3)/(120 n^5 + 274 n^4 + 225 n^3 + 85 n^2 + 15 n + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^2*(d+e*x^n)^3,x, algorithm="giac")
```

```
[Out] (120*a^2*d^3*n^5*x + 60*b^2*d^3*n^4*x*x^(2*n) + 240*a*b*d^3*n^4*x*x^n + 120
)*b^2*d^2*n^4*x*x^(3*n)*e + 360*a*b*d^2*n^4*x*x^(2*n)*e + 360*a^2*d^2*n^4*x*
x^n*e + 274*a^2*d^3*n^4*x + 107*b^2*d^3*n^3*x*x^(2*n) + 308*a*b*d^3*n^3*x*x
^n + 90*b^2*d^2*n^4*x*x^(4*n)*e^2 + 240*a^2*b*d^2*n^4*x*x^(3*n)*e^2 + 180*a^2*d^2*n
^4*x*x^(2*n)*e^2 + 234*b^2*d^2*n^3*x*x^(3*n)*e + 642*a^2*b*d^2*n^3*x*x^(2*n)*
e + 462*a^2*d^2*n^3*x*x^n*e + 225*a^2*d^3*n^3*x + 59*b^2*d^3*n^2*x*x^(2*n)
+ 142*a^2*b*d^3*n^2*x*x^n + 24*b^2*n^4*x*x^(5*n)*e^3 + 60*a^2*b*n^4*x*x^(4*n)*e
^3 + 40*a^2*n^4*x*x^(3*n)*e^3 + 183*b^2*d^2*n^3*x*x^(4*n)*e^2 + 468*a^2*b*d^2*n^3
*x*x^(3*n)*e^2 + 321*a^2*d^2*n^3*x*x^(2*n)*e^2 + 147*b^2*d^2*n^2*x*x^(3*n)*e
+ 354*a^2*b*d^2*n^2*x*x^(2*n)*e + 213*a^2*d^2*n^2*x*x^n*e + 85*a^2*d^3*n^2*x
+ 13*b^2*d^3*n^2*x*x^(2*n) + 28*a^2*b*d^3*n^2*x*x^n + 50*b^2*n^3*x*x^(5*n)*e^3 +
122*a^2*b*n^3*x*x^(4*n)*e^3 + 78*a^2*n^3*x*x^(3*n)*e^3 + 123*b^2*d^2*n^2*x*x^(4
*n)*e^2 + 294*a^2*b*d^2*n^2*x*x^(3*n)*e^2 + 177*a^2*d^2*n^2*x*x^(2*n)*e^2 + 36*b^
2*d^2*n^2*x*x^(3*n)*e + 78*a^2*b*d^2*n^2*x*x^(2*n)*e + 42*a^2*d^2*n^2*x*x^n*e + 15*
a^2*d^3*n^2*x + b^2*d^3*x*x^(2*n) + 2*a^2*b*d^3*x*x^n + 35*b^2*n^2*x*x^(5*n)*e^
3 + 82*a^2*b*n^2*x*x^(4*n)*e^3 + 49*a^2*n^2*x*x^(3*n)*e^3 + 33*b^2*d^2*n^2*x*x^(4
*n)*e^2 + 72*a^2*b*d^2*n^2*x*x^(3*n)*e^2 + 39*a^2*d^2*n^2*x*x^(2*n)*e^2 + 3*b^2*d^2*n
^2*x*x^(3*n)*e + 6*a^2*b*d^2*n^2*x*x^(2*n)*e + 3*a^2*d^2*n^2*x*x^n*e + a^2*d^3*n^2*x
+ 10*b^2*n^2*x*x^(5*n)*e^3 + 22*a^2*b*n^2*x*x^(4*n)*e^3 + 12*a^2*n^2*x*x^(3*n)*e^3 + 3*b^2*d
^2*n^2*x*x^(4*n)*e^2 + 6*a^2*b*d^2*n^2*x*x^(3*n)*e^2 + 3*a^2*d^2*n^2*x*x^(2*n)*e^2 + b^2*x*x^(
5*n)*e^3 + 2*a^2*b*x*x^(4*n)*e^3 + a^2*x*x^(3*n)*e^3)/(120*n^5 + 274*n^4 + 22
5*n^3 + 85*n^2 + 15*n + 1)
```

**maple [A]** time = 0.06, size = 164, normalized size = 1.04

$$\frac{b^2 e^3 x e^{5n \ln(x)}}{5n + 1} + a^2 d^3 x + \frac{(3ae + 2bd) a d^2 x e^{n \ln(x)}}{n + 1} + \frac{(2ae + 3bd) b e^2 x e^{4n \ln(x)}}{4n + 1} + \frac{(3a^2 e^2 + 6abde + b^2 d^2) dx e^{2n \ln(x)}}{2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^2*(d+e*x^n)^3,x)
```

[Out]  $a^2 d^3 x + b^2 e^3 / (5n+1) * x * \exp(n * \ln(x))^{5+d} * (3a^2 e^2 + 6a * b * d * e + b^2 d^2) / (2n+1) * x * \exp(n * \ln(x))^{2+e} * (a^2 e^2 + 6a * b * d * e + 3b^2 d^2) / (3n+1) * x * \exp(n * \ln(x))^{3+a} * d^2 * (3a * e + 2 * b * d) / (n+1) * x * \exp(n * \ln(x)) + b * e^2 * (2a * e + 3 * b * d) / (4n+1) * x * \exp(n * \ln(x))^{4}$

**maxima** [A] time = 0.62, size = 242, normalized size = 1.53

$$a^2 d^3 x + \frac{b^2 e^3 x^{5n+1}}{5n+1} + \frac{3b^2 d e^2 x^{4n+1}}{4n+1} + \frac{2 a b e^3 x^{4n+1}}{4n+1} + \frac{3 b^2 d^2 e x^{3n+1}}{3n+1} + \frac{6 a b d e^2 x^{3n+1}}{3n+1} + \frac{a^2 e^3 x^{3n+1}}{3n+1} + \frac{b^2 d^3 x^{2n+1}}{2n+1} + \frac{6 a b d^2 e x^{2n+1}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(d+e\*x^n)^3,x, algorithm="maxima")

[Out]  $a^2 d^3 x + b^2 e^3 x^{5n+1} / (5n+1) + 3 * b^2 * d * e^2 * x^{4n+1} / (4n+1) + 2 * a * b * e^3 * x^{4n+1} / (4n+1) + 3 * b^2 * d^2 * e * x^{3n+1} / (3n+1) + 6 * a * b * d * e^2 * x^{3n+1} / (3n+1) + a^2 * e^3 * x^{3n+1} / (3n+1) + b^2 * d^3 * x^{2n+1} / (2n+1) + 6 * a * b * d^2 * e * x^{2n+1} / (2n+1) + 3 * a^2 * d * e^2 * x^{2n+1} / (2n+1) + 2 * a * b * d^3 * x^{n+1} / (n+1) + 3 * a^2 * d^2 * e * x^{n+1} / (n+1)$

**mupad** [B] time = 1.71, size = 157, normalized size = 0.99

$$a^2 d^3 x + \frac{x x^{2n} (3 a^2 d e^2 + 6 a b d^2 e + b^2 d^3)}{2n+1} + \frac{x x^{3n} (a^2 e^3 + 6 a b d e^2 + 3 b^2 d^2 e)}{3n+1} + \frac{b^2 e^3 x x^{5n}}{5n+1} + \frac{a d^2 x x^n (3 a e + 2 b d)}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2\*(d + e\*x^n)^3,x)

[Out]  $a^2 d^3 x + (x * x^{2n} * (b^2 d^3 + 3a^2 d * e^2 + 6a * b * d^2 * e)) / (2n+1) + (x * x^{3n} * (a^2 e^3 + 3b^2 d^2 * e + 6a * b * d * e^2)) / (3n+1) + (b^2 e^3 * x * x^{5n}) / (5n+1) + (a * d^2 * x * x^n * (3a * e + 2 * b * d)) / (n+1) + (b * e^2 * x * x^{4n} * (2a * e + 3 * b * d)) / (4n+1)$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2\*(d+e\*x\*\*n)\*\*3,x)

[Out] Timed out

### 3.293 $\int (a + bx^n)^2 (d + ex^n)^2 dx$

**Optimal.** Leaf size=112

$$\frac{x^{2n+1}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2x + \frac{2adx^{n+1}(ae + bd)}{n+1} + \frac{2bex^{3n+1}(ae + bd)}{3n+1} + \frac{b^2e^2x^{4n+1}}{4n+1}$$

[Out]  $a^2d^2x + 2a^2d^2x^{2n+1}/(2n+1) + a^2d^2x + 2a^2d^2x^{2n+1}/(2n+1) + (a^2e^2 + 4abde + b^2d^2)x^{2n+1}/(2n+1) + 2bex^{3n+1}(ae + bd)/(3n+1) + b^2e^2x^{4n+1}/(4n+1)$

**Rubi [A]** time = 0.08, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$ , Rules used = {373}

$$\frac{x^{2n+1}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2x + \frac{2adx^{n+1}(ae + bd)}{n+1} + \frac{2bex^{3n+1}(ae + bd)}{3n+1} + \frac{b^2e^2x^{4n+1}}{4n+1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(d + e\*x^n)^2,x]

[Out]  $a^2d^2x + (2a^2d^2(bd + ae)x^{1+n})/(1+n) + ((b^2d^2 + 4a^2bd + a^2e^2)x^{1+2n})/(1+2n) + (2b^2e^2(bd + ae)x^{1+3n})/(1+3n) + (b^2e^2e^2x^{1+4n})/(1+4n)$

**Rule 373**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] :> Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

**Rubi steps**

$$\int (a + bx^n)^2 (d + ex^n)^2 dx = \int (a^2d^2 + 2ad(bd + ae)x^n + (b^2d^2 + 4abde + a^2e^2)x^{2n} + 2be(bd + ae)x^{3n} + b^2e^2x^{4n}) dx$$

$$= a^2d^2x + \frac{2ad(bd + ae)x^{1+n}}{1+n} + \frac{(b^2d^2 + 4abde + a^2e^2)x^{1+2n}}{1+2n} + \frac{2be(bd + ae)x^{1+3n}}{1+3n} + \frac{b^2e^2x^{1+4n}}{1+4n}$$

**Mathematica [A]** time = 0.20, size = 105, normalized size = 0.94

$$x \left( \frac{x^{2n}(a^2e^2 + 4abde + b^2d^2)}{2n+1} + a^2d^2 + \frac{2bex^{3n}(ae + bd)}{3n+1} + \frac{2adx^n(ae + bd)}{n+1} + \frac{b^2e^2x^{4n}}{4n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(d + e\*x^n)^2,x]

[Out]  $x(a^2d^2 + 2a^2d^2x^{2n}/(2n+1) + (b^2d^2 + 4a^2bd + a^2e^2)x^{2n+1}/(2n+1) + 2b^2e^2(bd + ae)x^{3n+1}/(3n+1) + b^2e^2e^2x^{4n+1}/(4n+1))$

**fricas [B]** time = 0.86, size = 370, normalized size = 3.30

$$\frac{(6b^2e^2n^3 + 11b^2e^2n^2 + 6b^2e^2n + b^2e^2)xx^{4n} + 2(b^2de + abe^2 + 8(b^2de + abe^2)n^3 + 14(b^2de + abe^2)n^2 + 7(b^2de + abe^2)n + b^2de + abe^2)x^{3n+1} + (b^2de + abe^2)x^{2n+1} + a^2d^2x}{(2n+1)(1+n)(1+2n)(1+3n)(1+4n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(d+e\*x^n)^2,x, algorithm="fricas")

[Out] ((6\*b^2\*e^2\*n^3 + 11\*b^2\*e^2\*n^2 + 6\*b^2\*e^2\*n + b^2\*e^2)\*x\*x^(4\*n) + 2\*(b^2\*d\*e + a\*b\*e^2 + 8\*(b^2\*d\*e + a\*b\*e^2)\*n^3 + 14\*(b^2\*d\*e + a\*b\*e^2)\*n^2 + 7\*(b^2\*d\*e + a\*b\*e^2)\*n)\*x\*x^(3\*n) + (b^2\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2 + 12\*(b^2\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2)\*n^3 + 19\*(b^2\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2)\*n^2 + 8\*(b^2\*d^2 + 4\*a\*b\*d\*e + a^2\*e^2)\*n)\*x\*x^(2\*n) + 2\*(a\*b\*d^2 + a^2\*d\*e + 2\*4\*(a\*b\*d^2 + a^2\*d\*e)\*n^3 + 26\*(a\*b\*d^2 + a^2\*d\*e)\*n^2 + 9\*(a\*b\*d^2 + a^2\*d\*e)\*n)\*x\*x^n + (24\*a^2\*d^2\*n^4 + 50\*a^2\*d^2\*n^3 + 35\*a^2\*d^2\*n^2 + 10\*a^2\*d^2\*n + a^2\*d^2)\*x)/(24\*n^4 + 50\*n^3 + 35\*n^2 + 10\*n + 1)

**giac [B]** time = 0.22, size = 539, normalized size = 4.81

$$24 a^2 d^2 n^4 x + 12 b^2 d^2 n^3 x x^{2n} + 48 a b d^2 n^3 x x^n + 16 b^2 d n^3 x x^{3n} e + 48 a b d n^3 x x^{2n} e + 48 a^2 d n^3 x x^n e + 50 a^2 d^2 n^3 x + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(d+e\*x^n)^2,x, algorithm="giac")

[Out] (24\*a^2\*d^2\*n^4\*x + 12\*b^2\*d^2\*n^3\*x\*x^(2\*n) + 48\*a\*b\*d^2\*n^3\*x\*x^n + 16\*b^2\*d\*n^3\*x\*x^(3\*n)\*e + 48\*a\*b\*d\*n^3\*x\*x^(2\*n)\*e + 48\*a^2\*d\*n^3\*x\*x^n\*e + 50\*a^2\*d^2\*n^3\*x + 19\*b^2\*d^2\*n^2\*x\*x^(2\*n) + 52\*a\*b\*d^2\*n^2\*x\*x^n + 6\*b^2\*n^3\*x\*x^(4\*n)\*e^2 + 16\*a\*b\*n^3\*x\*x^(3\*n)\*e^2 + 12\*a^2\*n^3\*x\*x^(2\*n)\*e^2 + 28\*b^2\*d\*n^2\*x\*x^(3\*n)\*e + 76\*a\*b\*d\*n^2\*x\*x^(2\*n)\*e + 52\*a^2\*d\*n^2\*x\*x^n\*e + 35\*a^2\*d^2\*n^2\*x + 8\*b^2\*d^2\*n\*x\*x^(2\*n) + 18\*a\*b\*d^2\*n\*x\*x^n + 11\*b^2\*n^2\*x\*x^(4\*n)\*e^2 + 28\*a\*b\*n^2\*x\*x^(3\*n)\*e^2 + 19\*a^2\*n^2\*x\*x^(2\*n)\*e^2 + 14\*b^2\*d\*n\*x\*x^(3\*n)\*e + 32\*a\*b\*d\*n\*x\*x^(2\*n)\*e + 18\*a^2\*d\*n\*x\*x^n\*e + 10\*a^2\*d^2\*n\*x + b^2\*d^2\*x\*x^(2\*n) + 2\*a\*b\*d^2\*x\*x^n + 6\*b^2\*n\*x\*x^(4\*n)\*e^2 + 14\*a\*b\*n\*x\*x^(3\*n)\*e^2 + 8\*a^2\*n\*x\*x^(2\*n)\*e^2 + 2\*b^2\*d\*x\*x^(3\*n)\*e + 4\*a\*b\*d\*x\*x^(2\*n)\*e + 2\*a^2\*d\*x\*x^n\*e + a^2\*d^2\*x + b^2\*x\*x^(4\*n)\*e^2 + 2\*a\*b\*x\*x^(3\*n)\*e^2 + a^2\*x\*x^(2\*n)\*e^2)/(24\*n^4 + 50\*n^3 + 35\*n^2 + 10\*n + 1)

**maple [A]** time = 0.06, size = 117, normalized size = 1.04

$$\frac{b^2 e^2 x e^{4n \ln(x)}}{4n + 1} + a^2 d^2 x + \frac{2(ae + bd) adx e^{n \ln(x)}}{n + 1} + \frac{2(ae + bd) bex e^{3n \ln(x)}}{3n + 1} + \frac{(a^2 e^2 + 4abde + b^2 d^2) x e^{2n \ln(x)}}{2n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2\*(d+e\*x^n)^2,x)

[Out] a^2\*d^2\*x+(a^2\*e^2+4\*a\*b\*d\*e+b^2\*d^2)/(2\*n+1)\*x\*exp(n\*ln(x))^2+b^2\*e^2/(4\*n+1)\*x\*exp(n\*ln(x))^4+2\*a\*d\*(a\*e+b\*d)/(n+1)\*x\*exp(n\*ln(x))+2\*b\*e\*(a\*e+b\*d)/(3\*n+1)\*x\*exp(n\*ln(x))^3

**maxima [A]** time = 0.60, size = 168, normalized size = 1.50

$$a^2 d^2 x + \frac{b^2 e^2 x^{4n+1}}{4n + 1} + \frac{2 b^2 d e x^{3n+1}}{3n + 1} + \frac{2 a b e^2 x^{3n+1}}{3n + 1} + \frac{b^2 d^2 x^{2n+1}}{2n + 1} + \frac{4 a b d e x^{2n+1}}{2n + 1} + \frac{a^2 e^2 x^{2n+1}}{2n + 1} + \frac{2 a b d^2 x^{n+1}}{n + 1} + \frac{2 a^2 d e x^{n+1}}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(d+e\*x^n)^2,x, algorithm="maxima")

[Out] a^2\*d^2\*x + b^2\*e^2\*x^(4\*n + 1)/(4\*n + 1) + 2\*b^2\*d\*e\*x^(3\*n + 1)/(3\*n + 1) + 2\*a\*b\*e^2\*x^(3\*n + 1)/(3\*n + 1) + b^2\*d^2\*x^(2\*n + 1)/(2\*n + 1) + 4\*a\*b\*d\*e\*x^(2\*n + 1)/(2\*n + 1) + a^2\*e^2\*x^(2\*n + 1)/(2\*n + 1) + 2\*a\*b\*d^2\*x^(n + 1)/(n + 1) + 2\*a^2\*d\*e\*x^(n + 1)/(n + 1)

**mupad [B]** time = 1.57, size = 108, normalized size = 0.96

$$a^2 d^2 x + \frac{x x^{2n} (a^2 e^2 + 4 a b d e + b^2 d^2)}{2n + 1} + \frac{b^2 e^2 x x^{4n}}{4n + 1} + \frac{2 b e x x^{3n} (a e + b d)}{3n + 1} + \frac{2 a d x x^n (a e + b d)}{n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2*(d + e*x^n)^2,x)
```

```
[Out] a^2*d^2*x + (x*x^(2*n))*(a^2*e^2 + b^2*d^2 + 4*a*b*d*e)/(2*n + 1) + (b^2*e^2*x*x^(4*n))/(4*n + 1) + (2*b*e*x*x^(3*n)*(a*e + b*d))/(3*n + 1) + (2*a*d*x*x^n*(a*e + b*d))/(n + 1)
```

**sympy** [A] time = 77.47, size = 1765, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(d+e*x**n)**2,x)
```

```
[Out] Piecewise((a**2*d**2*x + 2*a**2*d*e*log(x) - a**2*e**2/x + 2*a*b*d**2*log(x) - 4*a*b*d*e/x - a*b*e**2/x**2 - b**2*d**2/x - b**2*d*e/x**2 - b**2*e**2/(3*x**3), Eq(n, -1)), (a**2*d**2*x + 4*a**2*d*e*sqrt(x) + a**2*e**2*log(x) + 4*a*b*d**2*sqrt(x) + 4*a*b*d*e*log(x) - 4*a*b*e**2/sqrt(x) + b**2*d**2*log(x) - 4*b**2*d*e/sqrt(x) - b**2*e**2/x, Eq(n, -1/2)), (a**2*d**2*x + 3*a**2*d*e*x**(2/3) + 3*a**2*e**2*x**(1/3) + 3*a*b*d**2*x**(2/3) + 12*a*b*d*e*x**(1/3) + 2*a*b*e**2*log(x) + 3*b**2*d**2*x**(1/3) + 2*b**2*d*e*log(x) - 3*b**2*e**2/x**(1/3), Eq(n, -1/3)), (a**2*d**2*x + 8*a*d*x**(3/4)*(a*e + b*d)/3 - 4*b**2*e**2*log(x**(-1/4)) + 8*b*e*x**(1/4)*(a*e + b*d) - sqrt(x)*(-4*a**2*e**2 - 16*a*b*d*e - 4*b**2*d**2)/2, Eq(n, -1/4)), (24*a**2*d**2*n**4*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 50*a**2*d**2*n**3*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 35*a**2*d**2*n**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 10*a**2*d**2*n*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a**2*d**2*x/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a**2*d*e*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a**2*d*e*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a**2*d*e*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a**2*d*e*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*a**2*e**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*a**2*e**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*a**2*e**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + a**2*e**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*d**2*n**3*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 52*a*b*d**2*n**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 18*a*b*d**2*n*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*d**2*x*x**n/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 48*a*b*d*e*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 76*a*b*d*e*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 32*a*b*d*e*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 4*a*b*d*e*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16*a*b*e**2*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*a*b*e**2*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*a*b*e**2*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*a*b*e**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 12*b**2*d**2*n**3*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 19*b**2*d**2*n**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 8*b**2*d**2*n*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*d**2*x*x**(2*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 16*b**2*d*e*n**3*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 28*b**2*d*e*n**2*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 14*b**2*d*e*n*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 2*b**2*d*e*x*x**(3*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e**2*n**3*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 11*b**2*e**2*n**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + 6*b**2*e**2*n*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1) + b**2*e**2*x*x**(4*n)/(24*n**4 + 50*n**3 + 35*n**2 + 10*n + 1), True))
```

### 3.294 $\int (a + bx^n)^2 (c + dx^n) dx$

Optimal. Leaf size=70

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

[Out]  $a^2c*x + a*(a*d + 2*b*c)*x^{(1+n)}/(1+n) + b*(2*a*d + b*c)*x^{(1+2*n)}/(1+2*n) + b^2*d*x^{(1+3*n)}/(1+3*n)$

**Rubi [A]** time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$ , Rules used = {373}

$$a^2cx + \frac{ax^{n+1}(ad + 2bc)}{n + 1} + \frac{bx^{2n+1}(2ad + bc)}{2n + 1} + \frac{b^2dx^{3n+1}}{3n + 1}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(c + d\*x^n), x]

[Out]  $a^2c*x + (a*(2*b*c + a*d)*x^{(1 + n)})/(1 + n) + (b*(b*c + 2*a*d)*x^{(1 + 2*n)})/(1 + 2*n) + (b^2*d*x^{(1 + 3*n)})/(1 + 3*n)$

Rule 373

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := Int[ExpandIntegrand[(a + b\*x^n)^p\*(c + d\*x^n)^q, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int (a + bx^n)^2 (c + dx^n) dx &= \int (a^2c + a(2bc + ad)x^n + b(bc + 2ad)x^{2n} + b^2dx^{3n}) dx \\ &= a^2cx + \frac{a(2bc + ad)x^{1+n}}{1 + n} + \frac{b(bc + 2ad)x^{1+2n}}{1 + 2n} + \frac{b^2dx^{1+3n}}{1 + 3n} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 70, normalized size = 1.00

$$\frac{dx (a + bx^n)^3 - x \left( a^2 + \frac{2abx^n}{n+1} + \frac{b^2x^{2n}}{2n+1} \right) (ad - b(3cn + c))}{3bn + b}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(c + d\*x^n), x]

[Out]  $(d*x*(a + b*x^n)^3 - (a*d - b*(c + 3*c*n))*x*(a^2 + (2*a*b*x^n)/(1 + n) + (b^2*x^{(2*n)})/(1 + 2*n)))/(b + 3*b*n)$

**fricas [B]** time = 1.08, size = 175, normalized size = 2.50

$$\frac{(2b^2dn^2 + 3b^2dn + b^2d)xx^{3n} + (b^2c + 2abd + 3(b^2c + 2abd)n^2 + 4(b^2c + 2abd)n)xx^{2n} + (2abc + a^2d + 6(2a^2b + b^2c)n)x^{1+n}}{6n^3 + 11n^2 + 6n + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n), x, algorithm="fricas")

[Out]  $((2*b^2*d*n^2 + 3*b^2*d*n + b^2*d)*x*x^{(3*n)} + (b^2*c + 2*a*b*d + 3*(b^2*c + 2*a*b*d)*n^2 + 4*(b^2*c + 2*a*b*d)*n)*x*x^{(2*n)} + (2*a*b*c + a^2*d + 6*(2*a*b*c + a^2*d)*n^2 + 5*(2*a*b*c + a^2*d)*n)*x*x^n + (6*a^2*c*n^3 + 11*a^2*c*n^2 + 6*a^2*c*n + a^2*c)*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**giac** [B] time = 0.27, size = 232, normalized size = 3.31

$$\frac{6 a^2 c n^3 x + 2 b^2 d n^2 x x^{3 n} + 3 b^2 c n^2 x x^{2 n} + 6 a b d n^2 x x^{2 n} + 12 a b c n^2 x x^n + 6 a^2 d n^2 x x^n + 11 a^2 c n^2 x + 3 b^2 d n x x^{3 n}}{6 n^3 + 11 n^2 + 6 n + 1}$$

6

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n),x, algorithm="giac")

[Out]  $(6*a^2*c*n^3*x + 2*b^2*d*n^2*x*x^{(3*n)} + 3*b^2*c*n^2*x*x^{(2*n)} + 6*a*b*d*n^2*x*x^{(2*n)} + 12*a*b*c*n^2*x*x^n + 6*a^2*d*n^2*x*x^n + 11*a^2*c*n^2*x + 3*b^2*d*n*x*x^{(3*n)} + 4*b^2*c*n*x*x^{(2*n)} + 8*a*b*d*n*x*x^{(2*n)} + 10*a*b*c*n*x*x^n + 5*a^2*d*n*x*x^n + 6*a^2*c*n*x + b^2*d*x*x^{(3*n)} + b^2*c*x*x^{(2*n)} + 2*a*b*d*x*x^{(2*n)} + 2*a*b*c*x*x^n + a^2*d*x*x^n + a^2*c*x)/(6*n^3 + 11*n^2 + 6*n + 1)$

**maple** [A] time = 0.05, size = 74, normalized size = 1.06

$$\frac{b^2 dx e^{3n \ln(x)}}{3n+1} + a^2 cx + \frac{(ad+2bc) ax e^{n \ln(x)}}{n+1} + \frac{(2ad+bc) bx e^{2n \ln(x)}}{2n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2\*(d\*x^n+c),x)

[Out]  $a^2*c*x+a*(a*d+2*b*c)/(n+1)*x*\exp(n*\ln(x))+b*(2*a*d+b*c)/(2*n+1)*x*\exp(n*\ln(x))^2+b^2*d/(3*n+1)*x*\exp(n*\ln(x))^3$

**maxima** [A] time = 0.63, size = 94, normalized size = 1.34

$$a^2 cx + \frac{b^2 dx^{3n+1}}{3n+1} + \frac{b^2 cx^{2n+1}}{2n+1} + \frac{2 ab dx^{2n+1}}{2n+1} + \frac{2 ab cx^{n+1}}{n+1} + \frac{a^2 dx^{n+1}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n),x, algorithm="maxima")

[Out]  $a^2*c*x + b^2*d*x^{(3*n+1)}/(3*n+1) + b^2*c*x^{(2*n+1)}/(2*n+1) + 2*a*b*d*x^{(2*n+1)}/(2*n+1) + 2*a*b*c*x^{(n+1)}/(n+1) + a^2*d*x^{(n+1)}/(n+1)$

**mupad** [B] time = 1.53, size = 71, normalized size = 1.01

$$a^2 cx + \frac{xx^{2n}(cb^2+2adb)}{2n+1} + \frac{xx^n(da^2+2bca)}{n+1} + \frac{b^2 dx x^{3n}}{3n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2\*(c + d\*x^n),x)

[Out]  $a^2*c*x + (x*x^{(2*n)}*(b^2*c + 2*a*b*d))/(2*n+1) + (x*x^n*(a^2*d + 2*a*b*c))/(n+1) + (b^2*d*x*x^{(3*n)})/(3*n+1)$

sympy [A] time = 2.74, size = 726, normalized size = 10.37

$$\left\{ \begin{array}{l} a^2cx + a^2d \log(x) + 2abc \log(x) - \frac{2abd}{x} - \frac{b^2c}{x} - \frac{b^2d}{2x^2} \\ a^2cx + 2a^2d\sqrt{x} + 4abc\sqrt{x} + 2abd \log(x) + b^2c \log(x) - \frac{2b^2d}{\sqrt{x}} \\ a^2cx + \frac{3a^2dx^{\frac{2}{3}}}{2} + 3abcx^{\frac{2}{3}} + 6abd\sqrt[3]{x} + 3b^2c\sqrt[3]{x} + b^2d \log(x) \\ \frac{6a^2cn^3x}{6n^3+11n^2+6n+1} + \frac{11a^2cn^2x}{6n^3+11n^2+6n+1} + \frac{6a^2cnx}{6n^3+11n^2+6n+1} + \frac{a^2cx}{6n^3+11n^2+6n+1} + \frac{6a^2dn^2xx^n}{6n^3+11n^2+6n+1} + \frac{5a^2dnxx^n}{6n^3+11n^2+6n+1} + \frac{a^2dxx^n}{6n^3+11n^2+6n+1} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n),x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*log(x) + 2*a*b*c*log(x) - 2*a*b*d/x - b**2*c/x - b**2*d/(2*x**2), Eq(n, -1)), (a**2*c*x + 2*a**2*d*sqrt(x) + 4*a*b*c*sqrt(x) + 2*a*b*d*log(x) + b**2*c*log(x) - 2*b**2*d/sqrt(x), Eq(n, -1/2)), (a**2*c*x + 3*a**2*d*x**(2/3)/2 + 3*a*b*c*x**(2/3) + 6*a*b*d*x**(1/3) + 3*b**2*c*x**(1/3) + b**2*d*log(x), Eq(n, -1/3)), (6*a**2*c*n**3*x/(6*n**3 + 11*n**2 + 6*n + 1) + 11*a**2*c*n**2*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*c*n*x/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*c*x/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a**2*d*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 5*a**2*d*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + a**2*d*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 12*a*b*c*n**2*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 10*a*b*c*n*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*c*x*x**n/(6*n**3 + 11*n**2 + 6*n + 1) + 6*a*b*d*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 8*a*b*d*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*a*b*d*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*c*n**2*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 4*b**2*c*n*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*c*x*x**(2*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 2*b**2*d*n**2*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + 3*b**2*d*n*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1) + b**2*d*x*x**(3*n)/(6*n**3 + 11*n**2 + 6*n + 1), True))
```



$$3.295 \quad \int \frac{(a+bx^n)^2}{c+dx^n} dx$$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{bx(bc(n+1) - ad(2n+1))}{d^2(n+1)} + \frac{bx(a+bx^n)}{d(n+1)}$$

[Out]  $-b*(b*c*(1+n)-a*d*(1+2*n))*x/d^2/(1+n)+b*x*(a+b*x^n)/d/(1+n)+(-a*d+b*c)^2*x*$   
 $*hypergeom([1, 1/n], [1+1/n], -d*x^n/c)/c/d^2$

**Rubi [A]** time = 0.09, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{bx(bc(n+1) - ad(2n+1))}{d^2(n+1)} + \frac{bx(a+bx^n)}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n), x]

[Out]  $-((b*(b*c*(1+n) - a*d*(1+2*n))*x)/(d^2*(1+n))) + (b*x*(a + b*x^n))/(d*(1+n)) + ((b*c - a*d)^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d*x^n/c)])/(c*d^2)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(b\*(n\*(p+1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q) + 1)), x] + Dist[1/(b\*(n\*(p+q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q) + 1) - a\*d) + d\*(b\*c\*(n\*(p+2\*q-1) + 1) - a\*d\*(n\*(q-1) + 1))\*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^n)^2}{c + dx^n} dx &= \frac{bx(a + bx^n)}{d(1+n)} + \frac{\int \frac{-a(bc-ad(1+n))-b(bc(1+n)-ad(1+2n))x^n}{c+dx^n} dx}{d(1+n)} \\ &= -\frac{b(bc(1+n) - ad(1+2n))x}{d^2(1+n)} + \frac{bx(a + bx^n)}{d(1+n)} + \frac{(bc-ad)^2 \int \frac{1}{c+dx^n} dx}{d^2} \\ &= -\frac{b(bc(1+n) - ad(1+2n))x}{d^2(1+n)} + \frac{bx(a + bx^n)}{d(1+n)} + \frac{(bc-ad)^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} \end{aligned}$$

**Mathematica** [A] time = 0.05, size = 82, normalized size = 0.98

$$\frac{a^2x}{c} + \frac{x(ad - bc)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{cd^2} - \frac{x(bc - ad)^2}{cd^2} + \frac{b^2x^{n+1}}{d(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n), x]

[Out] (a^2\*x)/c - ((b\*c - a\*d)^2\*x)/(c\*d^2) + (b^2\*x^(1 + n))/(d\*(1 + n)) + ((-(b\*c) + a\*d)^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*d^2)

**fricas** [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^{2n} + 2abx^n + a^2}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n), x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d\*x^n + c), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n), x, algorithm="giac")

[Out] integrate((b\*x^n + a)^2/(d\*x^n + c), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2/(d\*x^n+c), x)

[Out] int((b\*x^n+a)^2/(d\*x^n+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(b^2c^2 - 2abcd + a^2d^2) \int \frac{1}{d^3x^n + cd^2} dx + \frac{b^2dxx^n - (b^2c(n+1) - 2abd(n+1))x}{d^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n),x, algorithm="maxima")

[Out] (b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*integrate(1/(d^3\*x^n + c\*d^2), x) + (b^2\*d\*x\*x^n - (b^2\*c\*(n + 1) - 2\*a\*b\*d\*(n + 1))\*x)/(d^2\*(n + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b x^n)^2}{c + d x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2/(c + d\*x^n), x)

[Out] int((a + b\*x^n)^2/(c + d\*x^n), x)

sympy [C] time = 6.83, size = 170, normalized size = 2.02

$$\frac{a^2 x \Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, \frac{1}{n}\right) \Gamma\left(\frac{1}{n}\right)}{cn^2 \Gamma\left(1 + \frac{1}{n}\right)} - \frac{2abx \Phi\left(\frac{cx^{-n} e^{i\pi}}{d}, 1, \frac{e^{i\pi}}{n}\right) \Gamma\left(\frac{1}{n}\right)}{dn^2 \Gamma\left(1 + \frac{1}{n}\right)} + \frac{2b^2 x x^{2n} \Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, 2 + \frac{1}{n}\right) \Gamma\left(2 + \frac{1}{n}\right)}{cn \Gamma\left(3 + \frac{1}{n}\right)} + \frac{b^2 x x^{2n} \Phi\left(\frac{dx^n e^{i\pi}}{c}, 1, 2 + \frac{1}{n}\right) \Gamma\left(2 + \frac{1}{n}\right)}{cn^2 \Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n),x)

[Out] a\*\*2\*x\*lerchphi(d\*x\*\*n\*exp\_polar(I\*pi)/c, 1, 1/n)\*gamma(1/n)/(c\*n\*\*2\*gamma(1 + 1/n)) - 2\*a\*b\*x\*lerchphi(c\*x\*\*(-n)\*exp\_polar(I\*pi)/d, 1, exp\_polar(I\*pi)/n)\*gamma(1/n)/(d\*n\*\*2\*gamma(1 + 1/n)) + 2\*b\*\*2\*x\*x\*\*(2\*n)\*lerchphi(d\*x\*\*n\*exp\_polar(I\*pi)/c, 1, 2 + 1/n)\*gamma(2 + 1/n)/(c\*n\*gamma(3 + 1/n)) + b\*\*2\*x\*x\*\*(2\*n)\*lerchphi(d\*x\*\*n\*exp\_polar(I\*pi)/c, 1, 2 + 1/n)\*gamma(2 + 1/n)/(c\*n\*\*2\*gamma(3 + 1/n))

$$3.296 \quad \int \frac{(a+bx^n)^2}{(c+dx^n)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 d^2 n} - \frac{bx(ad-bc(n+1))}{cd^2 n} - \frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

[Out] -b\*(a\*d-b\*c\*(1+n))\*x/c/d^2/n-(-a\*d+b\*c)\*x\*(a+b\*x^n)/c/d/n/(c+d\*x^n)+(-a\*d+b\*c)\*(a\*d\*(1-n)-b\*c\*(1+n))\*x\*hypergeom([1, 1/n],[1+1/n],-d\*x^n/c)/c^2/d^2/n

**Rubi [A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {413, 388, 245}

$$\frac{x(bc-ad)(ad(1-n)-bc(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 d^2 n} - \frac{bx(ad-bc(n+1))}{cd^2 n} - \frac{x(bc-ad)(a+bx^n)}{cdn(c+dx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n)^2, x]

[Out] -((b\*(a\*d - b\*c\*(1 + n))\*x)/(c\*d^2\*n)) - ((b\*c - a\*d)\*x\*(a + b\*x^n))/(c\*d\*n\*(c + d\*x^n)) + ((b\*c - a\*d)\*(a\*d\*(1 - n) - b\*c\*(1 + n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c^2\*d^2\*n)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned} \int \frac{(a + bx^n)^2}{(c + dx^n)^2} dx &= -\frac{(bc - ad)x(a + bx^n)}{cdn(c + dx^n)} + \frac{\int \frac{a(bc - ad(1-n)) - b(ad - bc(1+n))x^n}{c + dx^n} dx}{cdn} \\ &= -\frac{b(ad - bc(1+n))x}{cd^2n} - \frac{(bc - ad)x(a + bx^n)}{cdn(c + dx^n)} + \frac{((bc - ad)(ad(1-n) - bc(1+n))) \int \frac{1}{c + dx^n} dx}{cd^2n} \\ &= -\frac{b(ad - bc(1+n))x}{cd^2n} - \frac{(bc - ad)x(a + bx^n)}{cdn(c + dx^n)} + \frac{(bc - ad)(ad(1-n) - bc(1+n))x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2d^2n} \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 95, normalized size = 0.83

$$\frac{x \left( \frac{c(a^2d^2 - 2abcd + b^2c(cn + c + dnx^n))}{c + dx^n} - (bc - ad)(ad(n - 1) + bc(n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) \right)}{c^2d^2n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n)^2,x]

[Out] (x\*((c\*(-2\*a\*b\*c\*d + a^2\*d^2 + b^2\*c\*(c + c\*n + d\*n\*x^n)))/(c + d\*x^n) - (b\*c - a\*d)\*(a\*d\*(-1 + n) + b\*c\*(1 + n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c]))/(c^2\*d^2\*n)

**fricas [F]** time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{b^2x^{2n} + 2abx^n + a^2}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^2/(d\*x^n + c)^2, x)

**maple [F]** time = 0.58, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2/(d\*x^n+c)^2,x)

[Out] int((b\*x^n+a)^2/(d\*x^n+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-(b^2c^2(n+1) - a^2d^2(n-1) - 2abcd) \int \frac{1}{cd^3nx^n + c^2d^2n} dx + \frac{b^2cdnxx^n + (b^2c^2(n+1) - 2abcd + a^2d^2)x}{cd^3nx^n + c^2d^2n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="maxima")

[Out] -(b^2\*c^2\*(n + 1) - a^2\*d^2\*(n - 1) - 2\*a\*b\*c\*d)\*integrate(1/(c\*d^3\*n\*x^n + c^2\*d^2\*n), x) + (b^2\*c\*d\*n\*x\*x^n + (b^2\*c^2\*(n + 1) - 2\*a\*b\*c\*d + a^2\*d^2)\*x)/(c\*d^3\*n\*x^n + c^2\*d^2\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2/(c + d\*x^n)^2,x)

[Out] int((a + b\*x^n)^2/(c + d\*x^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2,x)

[Out] Integral((a + b\*x\*\*n)\*\*2/(c + d\*x\*\*n)\*\*2, x)

$$3.297 \quad \int \frac{(a+bx^n)^2}{(c+dx^n)^3} dx$$

**Optimal.** Leaf size=160

$$\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3d^2n^2} + \frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2(c+dx^n)}$$

[Out]  $-1/2*(-a*d+b*c)*x*(a+b*x^n)/c/d/n/(c+d*x^n)^2+1/2*(-a*d+b*c)*(a*d*(1-2*n)-b*c*(1+n))*x/c^2/d^2/n^2/(c+d*x^n)-1/2*(2*a*b*c*d*(1-n)-b^2*c^2*(1+n)-a^2*d^2*(2*n^2-3*n+1))*x*hypergeom([1, 1/n], [1+1/n], -d*x^n/c)/c^3/d^2/n^2$

**Rubi [A]** time = 0.16, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {413, 385, 245}

$$\frac{x(-a^2d^2(2n^2-3n+1)+2abcd(1-n)-b^2c^2(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3d^2n^2} + \frac{x(bc-ad)(ad(1-2n)-bc(n+1))}{2c^2d^2n^2(c+dx^n)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2/(c + d\*x^n)^3, x]

[Out]  $-((b*c - a*d)*x*(a + b*x^n))/(2*c*d*n*(c + d*x^n)^2) + ((b*c - a*d)*(a*d*(1 - 2*n) - b*c*(1 + n))*x)/(2*c^2*d^2*n^2*(c + d*x^n)) - ((2*a*b*c*d*(1 - n) - b^2*c^2*(1 + n) - a^2*d^2*(1 - 3*n + 2*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(2*c^3*d^2*n^2)$

**Rule 245**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

**Rule 385**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

**Rule 413**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

**Rubi steps**

$$\begin{aligned} \int \frac{(a + bx^n)^2}{(c + dx^n)^3} dx &= -\frac{(bc - ad)x(a + bx^n)}{2cdn(c + dx^n)^2} + \frac{\int \frac{a(bc - ad(1 - 2n)) - b(ad(1 - n) - bc(1 + n))x^n}{(c + dx^n)^2} dx}{2cdn} \\ &= -\frac{(bc - ad)x(a + bx^n)}{2cdn(c + dx^n)^2} + \frac{(bc - ad)(ad(1 - 2n) - bc(1 + n))x}{2c^2d^2n^2(c + dx^n)} - \frac{(2abcd(1 - n) - b^2c^2(1 + n) - a^2d^2)}{2c^2d^2n^2} \\ &= -\frac{(bc - ad)x(a + bx^n)}{2cdn(c + dx^n)^2} + \frac{(bc - ad)(ad(1 - 2n) - bc(1 + n))x}{2c^2d^2n^2(c + dx^n)} - \frac{(2abcd(1 - n) - b^2c^2(1 + n) - a^2d^2)}{2c^2d^2n^2} \end{aligned}$$

**Mathematica [A]** time = 0.19, size = 133, normalized size = 0.83

$$\frac{x \left( (a^2d^2(2n^2 - 3n + 1) + 2abcd(n - 1) + b^2c^2(n + 1)) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + \frac{c^2n(bc - ad)^2}{(c + dx^n)^2} - \frac{c(bc - ad)(ad(2n - 1) + b(2cn + a^2))}{c + dx^n} \right)}{2c^3d^2n^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2/(c + d\*x^n)^3,x]

[Out] (x\*((c^2\*(b\*c - a\*d)^2\*n)/(c + d\*x^n)^2 - (c\*(b\*c - a\*d)\*(a\*d\*(-1 + 2\*n) + b\*(c + 2\*c\*n)))/(c + d\*x^n) + (2\*a\*b\*c\*d\*(-1 + n) + b^2\*c^2\*(1 + n) + a^2\*d^2\*(1 - 3\*n + 2\*n^2))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(2\*c^3\*d^2\*n^2)

**fricas [F]** time = 0.94, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{b^2x^{2n} + 2abx^n + a^2}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^2/(d\*x^n + c)^3, x)

**maple [F]** time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^2}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2/(d\*x^n+c)^3,x)

[Out] int((b\*x^n+a)^2/(d\*x^n+c)^3,x)



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( (2n^2 - 3n + 1)a^2d^2 + b^2c^2(n+1) + 2abcd(n-1) \right) \int \frac{1}{2(c^2d^3n^2x^n + c^3d^2n^2)} dx - \frac{(b^2c^2d(2n+1) - a^2d^3(2n -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="maxima")

[Out] ((2\*n^2 - 3\*n + 1)\*a^2\*d^2 + b^2\*c^2\*(n + 1) + 2\*a\*b\*c\*d\*(n - 1))\*integrate  
 (1/2/(c^2\*d^3\*n^2\*x^n + c^3\*d^2\*n^2), x) - 1/2\*((b^2\*c^2\*d\*(2\*n + 1) - a^2\*d  
 ^3\*(2\*n - 1) - 2\*a\*b\*c\*d^2)\*x\*x^n - (a^2\*c\*d^2\*(3\*n - 1) - b^2\*c^3\*(n + 1)  
 - 2\*a\*b\*c^2\*d\*(n - 1))\*x)/(c^2\*d^4\*n^2\*x^(2\*n) + 2\*c^3\*d^3\*n^2\*x^n + c^4\*d  
 ^2\*n^2)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b x^n)^2}{(c + d x^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2/(c + d\*x^n)^3,x)

[Out] int((a + b\*x^n)^2/(c + d\*x^n)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*3,x)

[Out] Timed out

$$3.298 \quad \int \frac{(c+dx^n)^4}{a+bx^n} dx$$

**Optimal.** Leaf size=310

$$\frac{dx(c+dx^n)(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1))}{b^3(n+1)(2n+1)(3n+1)} - \frac{dx(a^3d^3(6n^3+11n^2+6n+1))}{b^3(n+1)(2n+1)(3n+1)}$$

[Out] -d\*(a^3\*d^3\*(6\*n^3+11\*n^2+6\*n+1)-b^3\*c^3\*(24\*n^3+18\*n^2+7\*n+1)-a^2\*b\*c\*d^2\*(24\*n^3+38\*n^2+19\*n+3)+a\*b^2\*c^2\*d\*(36\*n^3+45\*n^2+20\*n+3))\*x/b^4/(6\*n^3+11\*n^2+6\*n+1)-d\*(2\*a\*b\*c\*d\*(1+3\*n)^2-a^2\*d^2\*(6\*n^2+5\*n+1)-b^2\*c^2\*(18\*n^2+7\*n+1))\*x\*(c+d\*x^n)/b^3/(6\*n^3+11\*n^2+6\*n+1)-d\*(a\*d\*(1+3\*n)-b\*(6\*c\*n+c))\*x\*(c+d\*x^n)^2/b^2/(6\*n^2+5\*n+1)+d\*x\*(c+d\*x^n)^3/b/(1+3\*n)+(-a\*d+b\*c)^4\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a/b^4

**Rubi [A]** time = 0.50, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 528, 388, 245}

$$\frac{dx(c+dx^n)(-a^2d^2(6n^2+5n+1)+2abcd(3n+1)^2-b^2c^2(18n^2+7n+1))}{b^3(n+1)(2n+1)(3n+1)} - \frac{dx(-a^2bcd^2(24n^3+38n^2+19n+3))}{b^3(n+1)(2n+1)(3n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^4/(a + b\*x^n), x]

[Out] -((d\*(a^3\*d^3\*(1+6\*n+11\*n^2+6\*n^3)-b^3\*c^3\*(1+7\*n+18\*n^2+24\*n^3)-a^2\*b\*c\*d^2\*(3+19\*n+38\*n^2+24\*n^3)+a\*b^2\*c^2\*d\*(3+20\*n+45\*n^2+36\*n^3))\*x)/(b^4\*(1+n)\*(1+2\*n)\*(1+3\*n))-((d\*(2\*a\*b\*c\*d\*(1+3\*n)^2-a^2\*d^2\*(1+5\*n+6\*n^2)-b^2\*c^2\*(1+7\*n+18\*n^2))\*x\*(c+d\*x^n))/(b^3\*(1+n)\*(1+2\*n)\*(1+3\*n))-((d\*(a\*d\*(1+3\*n)-b\*(c+6\*c\*n))\*x\*(c+d\*x^n)^2)/(b^2\*(1+5\*n+6\*n^2))+((d\*x\*(c+d\*x^n)^3)/(b\*(1+3\*n)))+(b\*c-a\*d)^4\*x\*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -(b\*x^n)/a])/a/b^4

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(b\*(n\*(p + q) + 1)), x] + Dist[1/(b\*(n\*(p + q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 2)\*Simp[c\*(b\*c\*(n\*(p + q) + 1) - a\*d] + d\*(b\*c\*(n\*(p + 2\*q - 1) + 1) - a\*d\*(n\*(q - 1) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 528

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(f\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(b\*(n\*(p + q + 1) + 1)), x] + Dist[1/(b\*(n\*(p + q + 1) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q - 1)\*Simp[c\*(b\*e - a\*f + b\*e\*n\*(p + q + 1)) + (d\*(b\*e - a\*f) + f\*n\*q\*(b\*c - a\*d) + b\*d\*e\*n\*(p + q + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n\*(p + q + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{(c + dx^n)^4}{a + bx^n} dx &= \frac{dx (c + dx^n)^3}{b(1 + 3n)} + \frac{\int \frac{(c+dx^n)^2(-c(ad-b(c+3cn))-d(ad(1+3n)-b(c+6cn))x^n)}{a+bx^n} dx}{b(1 + 3n)} \\ &= -\frac{d(ad(1 + 3n) - b(c + 6cn))x (c + dx^n)^2}{b^2(1 + 5n + 6n^2)} + \frac{dx (c + dx^n)^3}{b(1 + 3n)} + \frac{\int \frac{(c+dx^n)(c(a^2d^2(1+3n)-2abcd(1+4n)+d^2c^2(1+7n+18n^2))x^n)}{a+bx^n} dx}{b^2(1 + 5n + 6n^2)} \\ &= -\frac{d(2abcd(1 + 3n)^2 - a^2d^2(1 + 5n + 6n^2) - b^2c^2(1 + 7n + 18n^2))x (c + dx^n)}{b^3(1 + n)(1 + 5n + 6n^2)} - \frac{d(ad(1 + 3n) - b(c + 6cn))x (c + dx^n)^2}{b^2(1 + 5n + 6n^2)} + \frac{dx (c + dx^n)^3}{b(1 + 3n)} \\ &= -\frac{d(a^3d^3(1 + 6n + 11n^2 + 6n^3) - b^3c^3(1 + 7n + 18n^2 + 24n^3) - a^2bcd^2(3 + 19n + 38n^2 + 24n^3))x (c + dx^n)}{b^4(1 + n)(1 + 5n + 6n^2)} - \frac{d(ad(1 + 3n) - b(c + 6cn))x (c + dx^n)^2}{b^2(1 + 5n + 6n^2)} + \frac{dx (c + dx^n)^3}{b(1 + 3n)} \\ &= -\frac{d(a^3d^3(1 + 6n + 11n^2 + 6n^3) - b^3c^3(1 + 7n + 18n^2 + 24n^3) - a^2bcd^2(3 + 19n + 38n^2 + 24n^3))x (c + dx^n)}{b^4(1 + n)(1 + 5n + 6n^2)} - \frac{d(ad(1 + 3n) - b(c + 6cn))x (c + dx^n)^2}{b^2(1 + 5n + 6n^2)} + \frac{dx (c + dx^n)^3}{b(1 + 3n)} \end{aligned}$$

**Mathematica [C]** time = 4.12, size = 133, normalized size = 0.43

$$\frac{x \left( c^4 \Phi \left( -\frac{bx^n}{a}, 1, \frac{1}{n} \right) + 4c^3 dx^n \Phi \left( -\frac{bx^n}{a}, 1, 1 + \frac{1}{n} \right) + 6c^2 d^2 x^{2n} \Phi \left( -\frac{bx^n}{a}, 1, 2 + \frac{1}{n} \right) + 4cd^3 x^{3n} \Phi \left( -\frac{bx^n}{a}, 1, 3 + \frac{1}{n} \right) + d^4 x^{4n} \Phi \left( -\frac{bx^n}{a}, 1, 4 + \frac{1}{n} \right) \right)}{an}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^4/(a + b\*x^n), x]

[Out] (x\*(4\*c^3\*d\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 1 + n^(-1)] + 6\*c^2\*d^2\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 2 + n^(-1)] + 4\*c\*d^3\*x^(3\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 3 + n^(-1)] + d^4\*x^(4\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 4 + n^(-1)] + c^4\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)]))/(a\*n)

**fricas [F]** time = 0.93, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{d^4 x^{4n} + 4cd^3 x^{3n} + 6c^2 d^2 x^{2n} + 4c^3 dx^n + c^4}{bx^n + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^4/(a+b\*x^n), x, algorithm="fricas")

[Out] integral((d^4\*x^(4\*n) + 4\*c\*d^3\*x^(3\*n) + 6\*c^2\*d^2\*x^(2\*n) + 4\*c^3\*d\*x^n + c^4)/(b\*x^n + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^4}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="giac")
```

```
[Out] integrate((d*x^n + c)^4/(b*x^n + a), x)
```

```
maple [F] time = 0.65, size = 0, normalized size = 0.00
```

$$\int \frac{(dx^n + c)^4}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^n+c)^4/(b*x^n+a),x)
```

```
[Out] int((d*x^n+c)^4/(b*x^n+a),x)
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$(b^4c^4 - 4ab^3c^3d + 6a^2b^2c^2d^2 - 4a^3bcd^3 + a^4d^4) \int \frac{1}{b^5x^n + ab^4} dx + \frac{(2n^2 + 3n + 1)b^3d^4xx^{3n} + (4(3n^2 + 4n + 1)ab^2d^3x^{2n} + (6n^2 + 5n + 1)a^2bd^2c^2x^{2n} - 4(6n^2 + 5n + 1)ab^2cd^3x^{2n} + (6n^2 + 5n + 1)a^2b^2d^4)x^{2n} + (4(6n^3 + 11n^2 + 6n + 1)b^3c^3d - 6(6n^3 + 11n^2 + 6n + 1)ab^2c^2d^2 + 4(6n^3 + 11n^2 + 6n + 1)a^2b^2cd^3 - (6n^3 + 11n^2 + 6n + 1)a^3d^4)x^{2n}}{(6n^3 + 11n^2 + 6n + 1)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x^n)^4/(a+b*x^n),x, algorithm="maxima")
```

```
[Out] (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*integrate(1/(b^5*x^n + a*b^4), x) + ((2*n^2 + 3*n + 1)*b^3*d^4*x*x^(3*n) + (4*(3*n^2 + 4*n + 1)*b^3*c*d^3 - (3*n^2 + 4*n + 1)*a*b^2*d^4)*x*x^(2*n) + (6*(6*n^2 + 5*n + 1)*b^3*c^2*d^2 - 4*(6*n^2 + 5*n + 1)*a*b^2*c*d^3 + (6*n^2 + 5*n + 1)*a^2*b*d^4)*x*x^n + (4*(6*n^3 + 11*n^2 + 6*n + 1)*b^3*c^3*d - 6*(6*n^3 + 11*n^2 + 6*n + 1)*a*b^2*c^2*d^2 + 4*(6*n^3 + 11*n^2 + 6*n + 1)*a^2*b*c*d^3 - (6*n^3 + 11*n^2 + 6*n + 1)*a^3*d^4)*x)/((6*n^3 + 11*n^2 + 6*n + 1)*b^4)
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{(c + dx^n)^4}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^n)^4/(a + b*x^n),x)
```

```
[Out] int((c + d*x^n)^4/(a + b*x^n), x)
```

```
sympy [C] time = 9.73, size = 369, normalized size = 1.19
```

$$-\frac{4c^3dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{c^4x\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{12c^2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an\Gamma\left(3 + \frac{1}{n}\right)} + \frac{6c^2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an\Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**4/(a+b*x**n),x)
```

```
[Out] -4*c**3*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**4*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 12*c**2*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) + 6*c**2*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 12*c*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n**3*gamma(4 + 1/n))
```

```

*n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + 4*c
*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1
/n)/(a*n**2*gamma(4 + 1/n)) + 4*d**4*x*x**(4*n)*lerchphi(b*x**n*exp_polar(I
*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n*gamma(5 + 1/n)) + d**4*x*x**(4*n)*l
erchphi(b*x**n*exp_polar(I*pi)/a, 1, 4 + 1/n)*gamma(4 + 1/n)/(a*n**2*gamma(
5 + 1/n))

```

$$3.299 \quad \int \frac{(c+dx^n)^3}{a+bx^n} dx$$

**Optimal.** Leaf size=173

$$\frac{dx \left( a^2 d^2 (2n^2 + 3n + 1) - abcd (6n^2 + 7n + 2) + b^2 c^2 (6n^2 + 4n + 1) \right)}{b^3 (n+1)(2n+1)} + \frac{x(bc-ad)^3 {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{ab^3} dx$$

[Out] d\*(a^2\*d^2\*(2\*n^2+3\*n+1)+b^2\*c^2\*(6\*n^2+4\*n+1)-a\*b\*c\*d\*(6\*n^2+7\*n+2))\*x/b^3/(2\*n^2+3\*n+1)-d\*(a\*d\*(1+2\*n)-b\*(4\*c\*n+c))\*x\*(c+d\*x^n)/b^2/(2\*n^2+3\*n+1)+d\*x\*(c+d\*x^n)^2/b/(1+2\*n)+(-a\*d+b\*c)^3\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a/b^3

**Rubi [A]** time = 0.27, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 528, 388, 245}

$$\frac{dx \left( a^2 d^2 (2n^2 + 3n + 1) - abcd (6n^2 + 7n + 2) + b^2 c^2 (6n^2 + 4n + 1) \right)}{b^3 (n+1)(2n+1)} + \frac{x(bc-ad)^3 {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a} \right)}{ab^3} dx$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^3/(a + b\*x^n), x]

[Out] (d\*(a^2\*d^2\*(1 + 3\*n + 2\*n^2) + b^2\*c^2\*(1 + 4\*n + 6\*n^2) - a\*b\*c\*d\*(2 + 7\*n + 6\*n^2))\*x)/(b^3\*(1 + n)\*(1 + 2\*n)) - (d\*(a\*d\*(1 + 2\*n) - b\*(c + 4\*c\*n))\*x\*(c + d\*x^n))/(b^2\*(1 + n)\*(1 + 2\*n)) + (d\*x\*(c + d\*x^n)^2)/(b\*(1 + 2\*n)) + ((b\*c - a\*d)^3\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a\*b^3)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q)+1)), x] + Dist[1/(b\*(n\*(p+q)+1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q)+1) - a\*d] + d\*(b\*c\*(n\*(p+2\*q-1)+1) - a\*d\*(n\*(q-1)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 528

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(f\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(b\*(n\*(p+q)+1)), x] + Dist[1/(b\*(n\*(p+q)+1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^q]

$n)^p(c + dx^n)^{q-1} \text{Simp}[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e - a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, n, p\}, x] \&\& \text{GtQ}[q, 0] \&\& \text{NeQ}[n*(p + q + 1) + 1, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^n)^3}{a + bx^n} dx &= \frac{dx(c + dx^n)^2}{b(1 + 2n)} + \int \frac{(c+dx^n)(-c(ad-b(c+2cn))-d(ad(1+2n)-b(c+4cn))x^n)}{a+bx^n} dx \\ &= -\frac{d(ad(1 + 2n) - b(c + 4cn))x(c + dx^n)}{b^2(1 + n)(1 + 2n)} + \frac{dx(c + dx^n)^2}{b(1 + 2n)} + \int \frac{c(a^2d^2(1+2n)-abcd(2+5n)+b^2c^2(1+3n))}{b^3(1+n)(1+2n)} dx \\ &= \frac{d(a^2d^2(1 + 3n + 2n^2) + b^2c^2(1 + 4n + 6n^2) - abcd(2 + 7n + 6n^2))x}{b^3(1 + n)(1 + 2n)} - \frac{d(ad(1 + 2n) - b(c + 4cn))x}{b^2(1 + n)(1 + 2n)} \\ &= \frac{d(a^2d^2(1 + 3n + 2n^2) + b^2c^2(1 + 4n + 6n^2) - abcd(2 + 7n + 6n^2))x}{b^3(1 + n)(1 + 2n)} - \frac{d(ad(1 + 2n) - b(c + 4cn))x}{b^2(1 + n)(1 + 2n)} \end{aligned}$$

**Mathematica [C]** time = 1.59, size = 104, normalized size = 0.60

$$\frac{x \left( c^3 \Phi \left( -\frac{bx^n}{a}, 1, \frac{1}{n} \right) + 3c^2 dx^n \Phi \left( -\frac{bx^n}{a}, 1, 1 + \frac{1}{n} \right) + 3cd^2 x^{2n} \Phi \left( -\frac{bx^n}{a}, 1, 2 + \frac{1}{n} \right) + d^3 x^{3n} \Phi \left( -\frac{bx^n}{a}, 1, 3 + \frac{1}{n} \right) \right)}{an}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^3/(a + b\*x^n), x]

[Out] (x\*(3\*c^2\*d\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 1 + n^(-1)] + 3\*c\*d^2\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 2 + n^(-1)] + d^3\*x^(3\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 3 + n^(-1)] + c^3\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)]))/(a\*n)

**fricas [F]** time = 1.02, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{d^3 x^{3n} + 3cd^2 x^{2n} + 3c^2 dx^n + c^3}{bx^n + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^3/(a+b\*x^n), x, algorithm="fricas")

[Out] integral((d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3)/(b\*x^n + a), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^3}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^3/(a+b\*x^n), x, algorithm="giac")

[Out] integrate((d\*x^n + c)^3/(b\*x^n + a), x)

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^3}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d*x^n+c)^3/(b*x^n+a),x)`

[Out] `int((d*x^n+c)^3/(b*x^n+a),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(b^3c^3 - 3ab^2c^2d + 3a^2bcd^2 - a^3d^3) \int \frac{1}{b^4x^n + ab^3} dx + \frac{b^2d^3(n+1)xx^{2n} + (3b^2cd^2(2n+1) - abd^3(2n+1))xx^n + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x^n)^3/(a+b*x^n),x, algorithm="maxima")`

[Out] `(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*integrate(1/(b^4*x^n + a*b^3), x) + (b^2*d^3*(n + 1)*x*x^(2*n) + (3*b^2*c*d^2*(2*n + 1) - a*b*d^3*(2*n + 1))*x*x^n + (3*(2*n^2 + 3*n + 1)*b^2*c^2*d - 3*(2*n^2 + 3*n + 1)*a*b*c*d^2 + (2*n^2 + 3*n + 1)*a^2*d^3)*x)/((2*n^2 + 3*n + 1)*b^3)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx^n)^3}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^n)^3/(a + b*x^n),x)`

[Out] `int((c + d*x^n)^3/(a + b*x^n), x)`

**sympy** [C] time = 13.01, size = 269, normalized size = 1.55

$$-\frac{3c^2dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{c^3x\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{6cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an\Gamma\left(3 + \frac{1}{n}\right)} + \frac{3cd^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an^2\Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c+d*x**n)**3/(a+b*x**n),x)`

[Out] `-3*c**2*d*x*lerchphi(a*x**(-n)*exp_polar(I*pi)/b, 1, exp_polar(I*pi)/n)*gamma(1/n)/(b*n**2*gamma(1 + 1/n)) + c**3*x*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 1/n)*gamma(1/n)/(a*n**2*gamma(1 + 1/n)) + 6*c*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n*gamma(3 + 1/n)) + 3*c*d**2*x*x**(2*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 2 + 1/n)*gamma(2 + 1/n)/(a*n**2*gamma(3 + 1/n)) + 3*d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n*gamma(4 + 1/n)) + d**3*x*x**(3*n)*lerchphi(b*x**n*exp_polar(I*pi)/a, 1, 3 + 1/n)*gamma(3 + 1/n)/(a*n**2*gamma(4 + 1/n))`



$$3.300 \quad \int \frac{(c+dx^n)^2}{a+bx^n} dx$$

Optimal. Leaf size=84

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} - \frac{dx(ad(n+1) - b(2cn+c))}{b^2(n+1)} + \frac{dx(c+dx^n)}{b(n+1)}$$

[Out] -d\*(a\*d\*(1+n)-b\*(2\*c\*n+c))\*x/b^2/(1+n)+d\*x\*(c+d\*x^n)/b/(1+n)+(-a\*d+b\*c)^2\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a/b^2

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {416, 388, 245}

$$\frac{x(bc-ad)^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} - \frac{dx(ad(n+1) - b(2cn+c))}{b^2(n+1)} + \frac{dx(c+dx^n)}{b(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^2/(a + b\*x^n), x]

[Out] -((d\*(a\*d\*(1+n) - b\*(c + 2\*c\*n))\*x)/(b^2\*(1+n))) + (d\*x\*(c + d\*x^n))/(b\*(1+n)) + ((b\*c - a\*d)^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n/a)])/(a\*b^2)

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1) + 1))/(b\*(n\*(p+1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1) + 1, 0]

Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q) + 1)), x] + Dist[1/(b\*(n\*(p+q) + 1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q) + 1) - a\*d) + d\*(b\*c\*(n\*(p+2\*q-1) + 1) - a\*d\*(n\*(q-1) + 1))\*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rubi steps

$$\begin{aligned} \int \frac{(c + dx^n)^2}{a + bx^n} dx &= \frac{dx(c + dx^n)}{b(1+n)} + \frac{\int \frac{-c(ad-bc(1+n))-d(ad(1+n)-b(c+2cn))x^n}{a+bx^n} dx}{b(1+n)} \\ &= -\frac{d(ad(1+n) - b(c + 2cn))x}{b^2(1+n)} + \frac{dx(c + dx^n)}{b(1+n)} + \frac{(bc - ad)^2 \int \frac{1}{a+bx^n} dx}{b^2} \\ &= -\frac{d(ad(1+n) - b(c + 2cn))x}{b^2(1+n)} + \frac{dx(c + dx^n)}{b(1+n)} + \frac{(bc - ad)^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab^2} \end{aligned}$$

**Mathematica** [C] time = 0.36, size = 75, normalized size = 0.89

$$\frac{x \left( c^2 \Phi\left(-\frac{bx^n}{a}, 1, \frac{1}{n}\right) + 2cdx^n \Phi\left(-\frac{bx^n}{a}, 1, 1 + \frac{1}{n}\right) + d^2 x^{2n} \Phi\left(-\frac{bx^n}{a}, 1, 2 + \frac{1}{n}\right) \right)}{an}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^2/(a + b\*x^n), x]

[Out] (x\*(2\*c\*d\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 1 + n^(-1)] + d^2\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 2 + n^(-1)] + c^2\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)]))/(a\*n)

**fricas** [F] time = 0.67, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2 x^{2n} + 2cdx^n + c^2}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n), x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)/(b\*x^n + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^2}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n), x, algorithm="giac")

[Out] integrate((d\*x^n + c)^2/(b\*x^n + a), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^2}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^2/(b\*x^n+a), x)

[Out] int((d\*x^n+c)^2/(b\*x^n+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(b^2c^2 - 2abcd + a^2d^2) \int \frac{1}{b^3x^n + ab^2} dx + \frac{bd^2xx^n + (2bcd(n+1) - ad^2(n+1))x}{b^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n),x, algorithm="maxima")

[Out] (b^2\*c^2 - 2\*a\*b\*c\*d + a^2\*d^2)\*integrate(1/(b^3\*x^n + a\*b^2), x) + (b\*d^2\*x\*x^n + (2\*b\*c\*d\*(n + 1) - a\*d^2\*(n + 1))\*x)/(b^2\*(n + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx^n)^2}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^2/(a + b\*x^n), x)

[Out] int((c + d\*x^n)^2/(a + b\*x^n), x)

sympy [C] time = 12.79, size = 170, normalized size = 2.02

$$\frac{2cdx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{c^2x\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{2d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an\Gamma\left(3 + \frac{1}{n}\right)} + \frac{d^2xx^{2n}\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, 2 + \frac{1}{n}\right)\Gamma\left(2 + \frac{1}{n}\right)}{an^2\Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n),x)

[Out] -2\*c\*d\*x\*lerchphi(a\*x\*\*(-n)\*exp\_polar(I\*pi)/b, 1, exp\_polar(I\*pi)/n)\*gamma(1/n)/(b\*n\*\*2\*gamma(1 + 1/n)) + c\*\*2\*x\*lerchphi(b\*x\*\*n\*exp\_polar(I\*pi)/a, 1, 1/n)\*gamma(1/n)/(a\*n\*\*2\*gamma(1 + 1/n)) + 2\*d\*\*2\*x\*x\*\*(2\*n)\*lerchphi(b\*x\*\*n\*exp\_polar(I\*pi)/a, 1, 2 + 1/n)\*gamma(2 + 1/n)/(a\*n\*gamma(3 + 1/n)) + d\*\*2\*x\*x\*\*(2\*n)\*lerchphi(b\*x\*\*n\*exp\_polar(I\*pi)/a, 1, 2 + 1/n)\*gamma(2 + 1/n)/(a\*n\*\*2\*gamma(3 + 1/n))

### 3.301 $\int \frac{c+dx^n}{a+bx^n} dx$

**Optimal.** Leaf size=42

$$\frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab} + \frac{dx}{b}$$

[Out] d\*x/b+(-a\*d+b\*c)\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a/b

**Rubi [A]** time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {388, 245}

$$\frac{x(bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab} + \frac{dx}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)/(a + b\*x^n), x]

[Out] (d\*x)/b + ((b\*c - a\*d)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a\*b)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{c+dx^n}{a+bx^n} dx &= \frac{dx}{b} - \frac{(-bc+ad) \int \frac{1}{a+bx^n} dx}{b} \\ &= \frac{dx}{b} + \frac{(bc-ad)x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{ab} \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 40, normalized size = 0.95

$$\frac{x\left((bc-ad) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + ad\right)}{ab}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)/(a + b\*x^n), x]

[Out] (x\*(a\*d + (b\*c - a\*d)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)]))/(a\*b)

**fricas** [F] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^n + c}{bx^n + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n),x, algorithm="fricas")

[Out] integral((d\*x^n + c)/(b\*x^n + a), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^n + c}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n),x, algorithm="giac")

[Out] integrate((d\*x^n + c)/(b\*x^n + a), x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{dx^n + c}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)/(b\*x^n+a),x)

[Out] int((d\*x^n+c)/(b\*x^n+a),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(bc - ad) \int \frac{1}{b^2x^n + ab} dx + \frac{dx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n),x, algorithm="maxima")

[Out] (b\*c - a\*d)\*integrate(1/(b^2\*x^n + a\*b), x) + d\*x/b

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c + dx^n}{a + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)/(a + b\*x^n),x)

[Out] int((c + d\*x^n)/(a + b\*x^n), x)

**sympy** [C] time = 4.11, size = 73, normalized size = 1.74

$$-\frac{dx\Phi\left(\frac{ax^{-n}e^{i\pi}}{b}, 1, \frac{e^{i\pi}}{n}\right)\Gamma\left(\frac{1}{n}\right)}{bn^2\Gamma\left(1 + \frac{1}{n}\right)} + \frac{cx\Phi\left(\frac{bx^n e^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{an^2\Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)/(a+b\*x\*\*n),x)

[Out] -d\*x\*lerchphi(a\*x\*\*(-n)\*exp\_polar(I\*pi)/b, 1, exp\_polar(I\*pi)/n)\*gamma(1/n)/(b\*n\*\*2\*gamma(1 + 1/n)) + c\*x\*lerchphi(b\*x\*\*n\*exp\_polar(I\*pi)/a, 1, 1/n)\*gamma(1/n)/(a\*n\*\*2\*gamma(1 + 1/n))

$$3.302 \quad \int \frac{1}{(a+bx^n)(c+dx^n)} dx$$

**Optimal.** Leaf size=72

$$\frac{bx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)}$$

[Out] b\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a/(-a\*d+b\*c)-d\*x\*hypergeom([1, 1/n], [1+1/n], -d\*x^n/c)/c/(-a\*d+b\*c)

**Rubi [A]** time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {391, 245}

$$\frac{bx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)),x]

[Out] (b\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a\*(b\*c - a\*d)) - (d\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c\*(b\*c - a\*d))

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 391

Int[1/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Dist[b/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[d/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a+bx^n)(c+dx^n)} dx &= \frac{b \int \frac{1}{a+bx^n} dx}{bc-ad} - \frac{d \int \frac{1}{c+dx^n} dx}{bc-ad} \\ &= \frac{bx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)} - \frac{dx {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)} \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 64, normalized size = 0.89

$$\frac{x \left( ad {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) - bc {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) \right)}{ac(ad-bc)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)),x]

[Out]  $(x * (- (b * c * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((b * x^n)/a)]) + a * d * \text{Hypergeometric2F1}[1, n^{(-1)}, 1 + n^{(-1)}, -((d * x^n)/c)])) / (a * c * (- (b * c) + a * d))$

**fricas** [F] time = 1.01, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bdx^{2n} + ac + (bc + ad)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x^n)/(c+d*x^n), x, algorithm="fricas")`

[Out] `integral(1/(b*d*x^(2*n) + a*c + (b*c + a*d)*x^n), x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x^n)/(c+d*x^n), x, algorithm="giac")`

[Out] `integrate(1/((b*x^n + a)*(d*x^n + c)), x)`

**maple** [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^n+a)/(d*x^n+c), x)`

[Out] `int(1/(b*x^n+a)/(d*x^n+c), x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x^n)/(c+d*x^n), x, algorithm="maxima")`

[Out] `integrate(1/((b*x^n + a)*(d*x^n + c)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + bx^n)(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^n)*(c + d*x^n)), x)`

[Out] `int(1/((a + b*x^n)*(c + d*x^n)), x)`

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^n)(c + dx^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)/(c+d*x**n), x)`

[Out] `Integral(1/((a + b*x**n)*(c + d*x**n)), x)`

$$3.303 \quad \int \frac{1}{(a+bx^n)(c+dx^n)^2} dx$$

**Optimal.** Leaf size=123

$$\frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{dx(bc(1-2n) - ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 n(bc-ad)^2} - \frac{dx}{cn(bc-ad)(c+dx^n)}$$

[Out]  $-d*x/c/(-a*d+b*c)/n/(c+d*x^n)+b^2*x*hypergeom([1, 1/n], [1+1/n], -b*x^n/a)/a/(-a*d+b*c)^2+d*(b*c*(1-2*n)-a*d*(1-n))*x*hypergeom([1, 1/n], [1+1/n], -d*x^n/c)/c^2/(-a*d+b*c)^2/n$

**Rubi [A]** time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {414, 522, 245}

$$\frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc-ad)^2} + \frac{dx(bc(1-2n) - ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2 n(bc-ad)^2} - \frac{dx}{cn(bc-ad)(c+dx^n)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^2), x]

[Out]  $-((d*x)/(c*(b*c - a*d)*n*(c + d*x^n))) + (b^2*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)])/(a*(b*c - a*d)^2) + (d*(b*c*(1 - 2*n) - a*d*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)])/(c^2*(b*c - a*d)^2*n)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rubi steps



$$\begin{aligned} \int \frac{1}{(a + bx^n)(c + dx^n)^2} dx &= -\frac{dx}{c(bc - ad)n(c + dx^n)} + \frac{\int \frac{bcn + a(d - dn) + bd(1 - n)x^n}{(a + bx^n)(c + dx^n)} dx}{c(bc - ad)n} \\ &= -\frac{dx}{c(bc - ad)n(c + dx^n)} + \frac{b^2 \int \frac{1}{a + bx^n} dx}{(bc - ad)^2} - \frac{(d(ad(1 - n) - b(c - 2cn))) \int \frac{1}{c + dx^n} dx}{c(bc - ad)^2 n} \\ &= -\frac{dx}{c(bc - ad)n(c + dx^n)} + \frac{b^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)^2} + \frac{d(bc(1 - 2n) - ad(1 - n))x}{c^2(bc - ad)^2} \end{aligned}$$

**Mathematica [A]** time = 0.22, size = 121, normalized size = 0.98

$$\frac{x \left( b^2 c^2 n (c + dx^n) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + ad \left( (c + dx^n) (ad(n - 1) + b(c - 2cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + c(ad(n - 1) + b(c - 2cn)) \right) \right)}{ac^2 n (bc - ad)^2 (c + dx^n)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^2), x]

[Out] (x\*(b^2\*c^2\*n\*(c + d\*x^n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)] + a\*d\*(c\*(-b\*c) + a\*d) + (a\*d\*(-1 + n) + b\*(c - 2\*c\*n))\*(c + d\*x^n)\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)]))/(a\*c^2\*(b\*c - a\*d)^2\*n\*(c + d\*x^n))

**fricas [F]** time = 1.23, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bd^2x^{3n} + ac^2 + (2bcd + ad^2)x^{2n} + (bc^2 + 2acd)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^2,x, algorithm="fricas")

[Out] integral(1/(b\*d^2\*x^(3\*n) + a\*c^2 + (2\*b\*c\*d + a\*d^2)\*x^(2\*n) + (b\*c^2 + 2\*a\*c\*d)\*x^n), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^2), x)

**maple [F]** time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)/(d\*x^n+c)^2,x)

[Out] int(1/(b\*x^n+a)/(d\*x^n+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$b^2 \int \frac{1}{ab^2c^2 - 2a^2bcd + a^3d^2 + (b^3c^2 - 2ab^2cd + a^2bd^2)x^n} dx - (bcd(2n-1) - ad^2(n-1)) \int \frac{1}{b^2c^4n - 2abc^3dn + a^2bd^4n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^2,x, algorithm="maxima")

[Out] b^2\*integrate(1/(a\*b^2\*c^2 - 2\*a^2\*b\*c\*d + a^3\*d^2 + (b^3\*c^2 - 2\*a\*b^2\*c\*d + a^2\*b\*d^2)\*x^n), x) - (b\*c\*d\*(2\*n - 1) - a\*d^2\*(n - 1))\*integrate(1/(b^2\*c^4\*n - 2\*a\*b\*c^3\*d\*n + a^2\*c^2\*d^2\*n + (b^2\*c^3\*d\*n - 2\*a\*b\*c^2\*d^2\*n + a^2\*c\*d^3\*n)\*x^n), x) - d\*x/(b\*c^3\*n - a\*c^2\*d\*n + (b\*c^2\*d\*n - a\*c\*d^2\*n)\*x^n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b x^n) (c + d x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)\*(c + d\*x^n)^2),x)

[Out] int(1/((a + b\*x^n)\*(c + d\*x^n)^2), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*2,x)

[Out] Exception raised: HeuristicGCDFailed

$$3.304 \quad \int \frac{1}{(a+bx^n)(c+dx^n)^3} dx$$

**Optimal.** Leaf size=210

$$\frac{dx \left( a^2 d^2 (2n^2 - 3n + 1) - 2abcd (3n^2 - 4n + 1) + b^2 c^2 (6n^2 - 5n + 1) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + \frac{b^3 x {}_2F_1 \left( 1, \frac{1}{n}; \dots \right)}{2c^3 n^2 (bc - ad)^3} + \frac{\dots}{a(bc - \dots)}$$

[Out]  $-1/2*d*x/c/(-a*d+b*c)/n/(c+d*x^n)^2-1/2*d*(a*d*(1-2*n)-b*(-4*c*n+c))*x/c^2/(-a*d+b*c)^2/n^2/(c+d*x^n)+b^3*x*hypergeom([1, 1/n], [1+1/n], -b*x^n/a)/a/(-a*d+b*c)^3-1/2*d*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(3*n^2-4*n+1)+b^2*c^2*(6*n^2-5*n+1))*x*hypergeom([1, 1/n], [1+1/n], -d*x^n/c)/c^3/(-a*d+b*c)^3/n^2$

**Rubi [A]** time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {414, 527, 522, 245}

$$\frac{dx \left( a^2 d^2 (2n^2 - 3n + 1) - 2abcd (3n^2 - 4n + 1) + b^2 c^2 (6n^2 - 5n + 1) \right) {}_2F_1 \left( 1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + \frac{b^3 x {}_2F_1 \left( 1, \frac{1}{n}; \dots \right)}{2c^3 n^2 (bc - ad)^3} + \frac{\dots}{a(bc - \dots)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^3), x]

[Out]  $-(d*x)/(2*c*(b*c - a*d)*n*(c + d*x^n)^2) - (d*(a*d*(1 - 2*n) - b*(c - 4*c*n)) * x)/(2*c^2*(b*c - a*d)^2*n^2*(c + d*x^n)) + (b^3*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]/(a*(b*c - a*d)^3) - (d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]/(2*c^3*(b*c - a*d)^3*n^2)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p + 1)\*(b\*c - a\*d) + d\*b\*(n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p + 1)\*(c +

$d*x^n)^{(q+1)}/(a*n*(b*c - a*d)*(p+1)), x] + \text{Dist}[1/(a*n*(b*c - a*d)*(p+1)), \text{Int}[(a + b*x^n)^{(p+1)}*(c + d*x^n)^q*\text{Simp}[c*(b*e - a*f) + e*n*(b*c - a*d)*(p+1) + d*(b*e - a*f)*(n*(p+q+2) + 1)*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, q\}, x] \&\& \text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^n)(c + dx^n)^3} dx &= -\frac{dx}{2c(bc - ad)n(c + dx^n)^2} + \frac{\int \frac{2bcn+a(d-2dn)+bd(1-2n)x^n}{(a+bx^n)(c+dx^n)^2} dx}{2c(bc - ad)n} \\ &= -\frac{dx}{2c(bc - ad)n(c + dx^n)^2} + \frac{d(bc(1 - 4n) - ad(1 - 2n))x}{2c^2(bc - ad)^2n^2(c + dx^n)} + \frac{\int \frac{2b^2c^2n^2+a^2d^2(1-3n+2n^2)-ab}{2} dx}{2c^2(bc - ad)^2n^2(c + dx^n)} \\ &= -\frac{dx}{2c(bc - ad)n(c + dx^n)^2} + \frac{d(bc(1 - 4n) - ad(1 - 2n))x}{2c^2(bc - ad)^2n^2(c + dx^n)} + \frac{b^3 \int \frac{1}{a+bx^n} dx}{(bc - ad)^3} - \frac{d(a^2d^2(1-3n+2n^2)-ab)}{2c^2(bc - ad)^2n^2(c + dx^n)} \\ &= -\frac{dx}{2c(bc - ad)n(c + dx^n)^2} + \frac{d(bc(1 - 4n) - ad(1 - 2n))x}{2c^2(bc - ad)^2n^2(c + dx^n)} + \frac{b^3 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a(bc - ad)^3} \end{aligned}$$

**Mathematica [A]** time = 0.26, size = 210, normalized size = 1.00

$$\frac{x \left( -ad(c + dx^n)^2 (a^2d^2(2n^2 - 3n + 1) - 2abcd(3n^2 - 4n + 1) + b^2c^2(6n^2 - 5n + 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right) + 2b^3x \right)}{2ac^3n^2(bc - ad)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^3), x]

[Out]  $(x*(-(a*c^2*d*(b*c - a*d)^2*n) + a*c*d*(b*c - a*d)*(a*d*(-1 + 2*n) + b*(c - 4*c*n))*(c + d*x^n) + 2*b^3*c^3*n^2*(c + d*x^n)^2*\text{Hypergeometric2F1}[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)] - a*d*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 4*n + 3*n^2) + b^2*c^2*(1 - 5*n + 6*n^2))*(c + d*x^n)^2*\text{Hypergeometric2F1}[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]))/(2*a*c^3*(b*c - a*d)^3*n^2*(c + d*x^n)^2)$

**fricas [F]** time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{bd^3x^{4n} + ac^3 + (3bcd^2 + ad^3)x^{3n} + 3(bc^2d + acd^2)x^{2n} + (bc^3 + 3ac^2d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="fricas")

[Out]  $\text{integral}(1/(b*d^3*x^{(4*n)} + a*c^3 + (3*b*c*d^2 + a*d^3)*x^{(3*n)} + 3*(b*c^2*d + a*c*d^2)*x^{(2*n)} + (b*c^3 + 3*a*c^2*d)*x^n), x)$

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^3), x)

**maple** [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)/(d\*x^n+c)^3,x)

[Out] int(1/(b\*x^n+a)/(d\*x^n+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-b^3 \int \frac{1}{ab^3c^3 - 3a^2b^2c^2d + 3a^3bcd^2 - a^4d^3 + (b^4c^3 - 3ab^3c^2d + 3a^2b^2cd^2 - a^3bd^3)x^n} dx + ((6n^2 - 5n + 1)b^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/(c+d\*x^n)^3,x, algorithm="maxima")

[Out] -b^3\*integrate(-1/(a\*b^3\*c^3 - 3\*a^2\*b^2\*c^2\*d + 3\*a^3\*b\*c\*d^2 - a^4\*d^3 + (b^4\*c^3 - 3\*a\*b^3\*c^2\*d + 3\*a^2\*b^2\*c\*d^2 - a^3\*b\*d^3)\*x^n), x) + ((6\*n^2 - 5\*n + 1)\*b^2\*c^2\*d - 2\*(3\*n^2 - 4\*n + 1)\*a\*b\*c\*d^2 + (2\*n^2 - 3\*n + 1)\*a^2\*d^3)\*integrate(-1/2/(b^3\*c^6\*n^2 - 3\*a\*b^2\*c^5\*d\*n^2 + 3\*a^2\*b\*c^4\*d^2\*n^2 - a^3\*c^3\*d^3\*n^2 + (b^3\*c^5\*d\*n^2 - 3\*a\*b^2\*c^4\*d^2\*n^2 + 3\*a^2\*b\*c^3\*d^3\*n^2 - a^3\*c^2\*d^4\*n^2)\*x^n), x) - 1/2\*((b\*c\*d^2\*(4\*n - 1) - a\*d^3\*(2\*n - 1))\*x\*x^n + (b\*c^2\*d\*(5\*n - 1) - a\*c\*d^2\*(3\*n - 1))\*x)/(b^2\*c^6\*n^2 - 2\*a\*b\*c^5\*d\*n^2 + a^2\*c^4\*d^2\*n^2 + (b^2\*c^4\*d^2\*n^2 - 2\*a\*b\*c^3\*d^3\*n^2 + a^2\*c^2\*d^4\*n^2)\*x^(2\*n) + 2\*(b^2\*c^5\*d\*n^2 - 2\*a\*b\*c^4\*d^2\*n^2 + a^2\*c^3\*d^3\*n^2)\*x^n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + bx^n)(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)\*(c + d\*x^n)^3), x)

[Out] int(1/((a + b\*x^n)\*(c + d\*x^n)^3), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)/(c+d\*x\*\*n)\*\*3,x)

[Out] Exception raised: HeuristicGCDFailed

### 3.305 $\int \frac{(c+dx^n)^4}{(a+bx^n)^2} dx$

**Optimal.** Leaf size=341

$$\frac{x(bc-ad)^3(bc(1-n)-ad(3n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) dx (c+dx^n) \left(a^2d^2(6n^2+5n+1) - 2abcd(5n^2+4n+1)\right)}{a^2b^4n ab^3n(n+1)(2n+1)}$$

[Out]  $-d*(b^3*c^3*(2*n^2+3*n+1)-a^3*d^3*(6*n^3+11*n^2+6*n+1)-a*b^2*c^2*d*(12*n^3+17*n^2+12*n+3)+a^2*b*c*d^2*(16*n^3+26*n^2+15*n+3))*x/a/b^4/n/(2*n^2+3*n+1)-d*(b^2*c^2*(2*n^2+3*n+1)-2*a*b*c*d*(5*n^2+4*n+1)+a^2*d^2*(6*n^2+5*n+1))*x*(c+d*x^n)/a/b^3/n/(2*n^2+3*n+1)-d*(-3*a*d*n+2*b*c*n-a*d+b*c)*x*(c+d*x^n)^2/a/b^2/n/(1+2*n)+(-a*d+b*c)*x*(c+d*x^n)^3/a/b/n/(a+b*x^n)-(-a*d+b*c)^3*(b*c*(1-n)-a*d*(1+3*n))*x*hypergeom([1, 1/n], [1+1/n], -b*x^n/a)/a^2/b^4/n$

**Rubi [A]** time = 0.55, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {413, 528, 388, 245}

$$\frac{dx (c+dx^n) \left(a^2d^2(6n^2+5n+1) - 2abcd(5n^2+4n+1) + b^2c^2(2n^2+3n+1)\right) dx \left(a^2bcd^2(16n^3+26n^2+15n+3)\right)}{ab^3n(n+1)(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^4/(a + b\*x^n)^2, x]

[Out]  $-((d*(b^3*c^3*(1+3*n+2*n^2) - a^3*d^3*(1+6*n+11*n^2+6*n^3) - a*b^2*c^2*d*(3+12*n+17*n^2+12*n^3) + a^2*b*c*d^2*(3+15*n+26*n^2+16*n^3))*x)/(a*b^4*n*(1+n)*(1+2*n)) - (d*(b^2*c^2*(1+3*n+2*n^2) - 2*a*b*c*d*(1+4*n+5*n^2) + a^2*d^2*(1+5*n+6*n^2))*x*(c+d*x^n))/(a*b^3*n*(1+n)*(1+2*n)) + (d*(a*d*(1+3*n) - b*(c+2*c*n))*x*(c+d*x^n)^2)/(a*b^2*n*(1+2*n)) + ((b*c - a*d)*x*(c+d*x^n)^3)/(a*b*n*(a+b*x^n)) - ((b*c - a*d)^3*(b*c*(1-n) - a*d*(1+3*n))*x*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -(b*x^n/a)])/(a^2*b^4*n)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1)+1)) + d\*(a\*d\*(n\*(q-1)+1) - b\*c\*(n\*(p+q)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 528

```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (
f_)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^n)^4}{(a + bx^n)^2} dx &= \frac{(bc - ad)x(c + dx^n)^3}{abn(a + bx^n)} + \frac{\int \frac{(c+dx^n)^2(c(ad-bc(1-n))+d(ad(1+3n)-b(c+2cn))x^n)}{a+bx^n} dx}{abn} \\ &= \frac{d(ad(1 + 3n) - b(c + 2cn))x(c + dx^n)^2}{ab^2n(1 + 2n)} + \frac{(bc - ad)x(c + dx^n)^3}{abn(a + bx^n)} + \frac{\int \frac{(c+dx^n)(c(2abcd(1+2n)-a^2d^2)}{a+bx^n} dx}{abn} \\ &= -\frac{d(b^2c^2(1 + 3n + 2n^2) - 2abcd(1 + 4n + 5n^2) + a^2d^2(1 + 5n + 6n^2))x(c + dx^n)}{ab^3n(1 + n)(1 + 2n)} + \frac{d(ad(1 + 3n) - b(c + 2cn))x(c + dx^n)^2}{abn(a + bx^n)} \\ &= -\frac{d(b^3c^3(1 + 3n + 2n^2) - a^3d^3(1 + 6n + 11n^2 + 6n^3) - ab^2c^2d(3 + 12n + 17n^2 + 12n^3) + a^2d^2(1 + 5n + 6n^2))x(c + dx^n)}{ab^4n(1 + n)(1 + 2n)} + \frac{d(ad(1 + 3n) - b(c + 2cn))x(c + dx^n)^2}{abn(a + bx^n)} \\ &= -\frac{d(b^3c^3(1 + 3n + 2n^2) - a^3d^3(1 + 6n + 11n^2 + 6n^3) - ab^2c^2d(3 + 12n + 17n^2 + 12n^3) + a^2d^2(1 + 5n + 6n^2))x(c + dx^n)}{ab^4n(1 + n)(1 + 2n)} + \frac{d(ad(1 + 3n) - b(c + 2cn))x(c + dx^n)^2}{abn(a + bx^n)} \end{aligned}$$

**Mathematica [A]** time = 5.26, size = 217, normalized size = 0.64

$$x \left( \frac{(bc-ad)^3(ad(3n+1)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n} + \frac{(ad-bc)^3(ad(3n+1)+bc(n-1))}{a^2n} + \frac{-a^4d^4+4a^3bcd^3-6a^2b^2c^2d^2+4ab^3c^3d+b^4c^4(n-1)}{a^2n} + \frac{2b^4}{a^2n} \right) / b^4$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^4/(a + b\*x^n)^2, x]

[Out] (x\*((4\*a\*b^3\*c^3\*d - 6\*a^2\*b^2\*c^2\*d^2 + 4\*a^3\*b\*c\*d^3 - a^4\*d^4 + b^4\*c^4\*(-1 + n))/(a^2\*n) + ((-(b\*c) + a\*d)^3\*(b\*c\*(-1 + n) + a\*d\*(1 + 3\*n)))/(a^2\*n) + (2\*b\*d^3\*(2\*b\*c - a\*d)\*x^n)/(1 + n) + (b^2\*d^4\*x^(2\*n))/(1 + 2\*n) + (b\*c - a\*d)^4/(a\*n\*(a + b\*x^n)) + ((b\*c - a\*d)^3\*(b\*c\*(-1 + n) + a\*d\*(1 + 3\*n)))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/(a^2\*n))/b^4

**fricas [F]** time = 1.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^4x^{4n} + 4cd^3x^{3n} + 6c^2d^2x^{2n} + 4c^3dx^n + c^4}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^4/(a+b\*x^n)^2,x, algorithm="fricas")

[Out] integral((d^4\*x^(4\*n) + 4\*c\*d^3\*x^(3\*n) + 6\*c^2\*d^2\*x^(2\*n) + 4\*c^3\*d\*x^n + c^4)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^4}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^4/(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^4/(b\*x^n + a)^2, x)

**maple** [F] time = 0.62, size = 0, normalized size = 0.00

$$\int \frac{(d x^n + c)^4}{(b x^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^4/(b\*x^n+a)^2,x)

[Out] int((d\*x^n+c)^4/(b\*x^n+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-(a^4 d^4 (3n + 1) - 4 a^3 b c d^3 (2n + 1) + 6 a^2 b^2 c^2 d^2 (n + 1) - b^4 c^4 (n - 1) - 4 a b^3 c^3 d) \int \frac{1}{a b^5 n x^n + a^2 b^4 n} dx + \frac{(n^2 + n)}{ab^5 n x^n + a^2 b^4 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^4/(a+b\*x^n)^2,x, algorithm="maxima")

[Out]  $-(a^4 d^4 (3n + 1) - 4 a^3 b c d^3 (2n + 1) + 6 a^2 b^2 c^2 d^2 (n + 1) - b^4 c^4 (n - 1) - 4 a b^3 c^3 d) \int \frac{1}{a b^5 n x^n + a^2 b^4 n} dx + ((n^2 + n) a b^3 d^4 x x^{(3n)} + (4 * (2n^2 + n) a b^3 c d^3 - (3n^2 + n) a^2 b^2 d^4) x x^{(2n)} + (6 * (2n^3 + 3n^2 + n) a b^3 c^2 d^2 - 4 * (4n^3 + 4n^2 + n) a^2 b^2 c d^3 + (6n^3 + 5n^2 + n) a^3 b d^4) x x^n + ((2n^2 + 3n + 1) b^4 c^4 - 4 * (2n^2 + 3n + 1) a b^3 c^3 d + 6 * (2n^3 + 5n^2 + 4n + 1) a^2 b^2 c^2 d^2 - 4 * (4n^3 + 8n^2 + 5n + 1) a^3 b c d^3 + (6n^3 + 11n^2 + 6n + 1) a^4 d^4) x) / ((2n^3 + 3n^2 + n) a b^5 x^n + (2n^3 + 3n^2 + n) a^2 b^4)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + d x^n)^4}{(a + b x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^4/(a + b\*x^n)^2,x)

[Out] int((c + d\*x^n)^4/(a + b\*x^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d x^n)^4}{(a + b x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*4/(a+b\*x\*\*n)\*\*2,x)

[Out] Integral((c + d\*x\*\*n)\*\*4/(a + b\*x\*\*n)\*\*2, x)



$$3.306 \quad \int \frac{(c+dx^n)^3}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=200

$$\frac{x(bc-ad)^2(bc(1-n)-ad(2n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^3n} dx \frac{(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2))}{ab^3n(n+1)}$$

[Out]  $-d*(b^2*c^2*(1+n)+a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(3*n^2+4*n+2))*x/a/b^3/n/(1+n)-d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n)/a/b^2/n/(1+n)+(-a*d+b*c))*x*(c+d*x^n)^2/a/b/n/(a+b*x^n)-(-a*d+b*c)^2*(b*c*(1-n)-a*d*(1+2*n))*x*hypergeom([1, 1/n], [1+1/n], -b*x^n/a)/a^2/b^3/n$

**Rubi [A]** time = 0.26, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {413, 528, 388, 245}

$$\frac{dx(a^2d^2(2n^2+3n+1)-abcd(3n^2+4n+2)+b^2c^2(n+1))}{ab^3n(n+1)} \frac{x(bc-ad)^2(bc(1-n)-ad(2n+1)) {}_2F_1\left(1, \frac{1}{n}; 1+\frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^3n}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^3/(a + b\*x^n)^2, x]

[Out]  $-((d*(b^2*c^2*(1+n)+a^2*d^2*(1+3*n+2*n^2)-a*b*c*d*(2+4*n+3*n^2))*x)/(a*b^3*n*(1+n)) - (d*(b*c*(1+n)-a*d*(1+2*n))*x*(c+d*x^n))/(a*b^2*n*(1+n)) + ((b*c-a*d)*x*(c+d*x^n)^2)/(a*b*n*(a+b*x^n)) - ((b*c-a*d)^2*(b*c*(1-n)-a*d*(1+2*n))*x*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -((b*x^n)/a)])/(a^2*b^3*n)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(a\*b\*n\*(p+1)), x] - Dist[1/(a\*b\*n\*(p+1)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-2)\*Simp[c\*(a\*d - c\*b\*(n\*(p+1)+1)) + d\*(a\*d\*(n\*(q-1)+1) - b\*c\*(n\*(p+q)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 528

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(f\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q)/(b\*(n\*(p+q+1)+1)), x] + Dist[1/(b\*(n\*(p+q+1)+1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-1)\*Simp[c\*(b\*e - a\*f + b\*e\*n\*(p+q+1)) + (d\*(b\*e -

$a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /;$  FreeQ[{  
 $a, b, c, d, e, f, n, p\}$ ,  $x]$  && GtQ[ $q, 0]$  && NeQ[ $n*(p + q + 1) + 1, 0]$

Rubi steps

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2} dx = \frac{(bc - ad)x(c + dx^n)^2}{abn(a + bx^n)} + \frac{\int \frac{(c+dx^n)(c(ad-bc(1-n))-d(bc(1+n)-ad(1+2n))x^n)}{a+bx^n} dx}{abn}$$

$$= -\frac{d(bc(1+n) - ad(1+2n))x(c + dx^n)}{ab^2n(1+n)} + \frac{(bc - ad)x(c + dx^n)^2}{abn(a + bx^n)} + \frac{\int \frac{c(2abcd(1+n)-a^2d^2(1+2n)-b^2c^2(1+n))}{a+bx^n} dx}{abn}$$

$$= -\frac{d(b^2c^2(1+n) + a^2d^2(1+3n+2n^2) - abcd(2+4n+3n^2))x}{ab^3n(1+n)} - \frac{d(bc(1+n) - ad(1+2n))x}{ab^2n(1+n)}$$

$$= -\frac{d(b^2c^2(1+n) + a^2d^2(1+3n+2n^2) - abcd(2+4n+3n^2))x}{ab^3n(1+n)} - \frac{d(bc(1+n) - ad(1+2n))x}{ab^2n(1+n)}$$

**Mathematica [A]** time = 5.38, size = 167, normalized size = 0.84

$$x \left( \frac{a(-a^3d^3(2n^2+3n+1)+a^2bd^2(3c(n+1)^2-dn(2n+1)x^n)+ab^2d(-3c^2(n+1)+3cdn(n+1)x^n+d^2nx^{2n})+b^3c^3(n+1))}{(n+1)(a+bx^n)} + (bc - ad)^2(ad(2n + 1) + bc) \right) / a^2b^3n$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^3/(a + b\*x^n)^2,x]

[Out] (x\*((a\*(b^3\*c^3\*(1+n) - a^3\*d^3\*(1+3\*n+2\*n^2) + a^2\*b\*d^2\*(3\*c\*(1+n)^2 - d\*n\*(1+2\*n)\*x^n) + a\*b^2\*d\*(-3\*c^2\*(1+n) + 3\*c\*d\*n\*(1+n)\*x^n + d^2\*n\*x^(2\*n))))/((1+n)\*(a+b\*x^n)) + (b\*c - a\*d)^2\*(b\*c\*(-1+n) + a\*d\*(1+2\*n))\*Hypergeometric2F1[1, n^(-1), 1+n^(-1), -((b\*x^n)/a)])))/(a^2\*b^3\*n)

**fricas [F]** time = 0.86, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}{b^2x^{2n} + 2abx^n + a^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^3/(a+b\*x^n)^2,x, algorithm="fricas")

[Out] integral((d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^3}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^3/(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^3/(b\*x^n + a)^2, x)

**maple [F]** time = 0.68, size = 0, normalized size = 0.00

$$\int \frac{(d x^n + c)^3}{(b x^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^3/(b\*x^n+a)^2,x)

[Out] int((d\*x^n+c)^3/(b\*x^n+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(a^3d^3(2n+1) - 3a^2bcd^2(n+1) + b^3c^3(n-1) + 3ab^2c^2d) \int \frac{1}{ab^4nx^n + a^2b^3n} dx + \frac{ab^2d^3nxx^{2n} + (3(n^2+n)ab^2d^3n)}{ab^4nx^n + a^2b^3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^3/(a+b\*x^n)^2,x, algorithm="maxima")

[Out] (a^3\*d^3\*(2\*n + 1) - 3\*a^2\*b\*c\*d^2\*(n + 1) + b^3\*c^3\*(n - 1) + 3\*a\*b^2\*c^2\*d)\*integrate(1/(a\*b^4\*n\*x^n + a^2\*b^3\*n), x) + (a\*b^2\*d^3\*n\*x\*x^(2\*n) + (3\*(n^2 + n)\*a\*b^2\*c\*d^2 - (2\*n^2 + n)\*a^2\*b\*d^3)\*x\*x^n + (3\*(n^2 + 2\*n + 1)\*a^2\*b\*c\*d^2 - (2\*n^2 + 3\*n + 1)\*a^3\*d^3 + b^3\*c^3\*(n + 1) - 3\*a\*b^2\*c^2\*d\*(n + 1))\*x)/((n^2 + n)\*a\*b^4\*x^n + (n^2 + n)\*a^2\*b^3)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^3/(a + b\*x^n)^2,x)

[Out] int((c + d\*x^n)^3/(a + b\*x^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx^n)^3}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*3/(a+b\*x\*\*n)\*\*2,x)

[Out] Integral((c + d\*x\*\*n)\*\*3/(a + b\*x\*\*n)\*\*2, x)

$$3.307 \quad \int \frac{(c+dx^n)^2}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{x(bc-ad)(bc(1-n)-ad(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n} - \frac{dx(bc-ad(n+1))}{ab^2n} + \frac{x(bc-ad)(c+dx^n)}{abn(a+bx^n)}$$

[Out] -d\*(b\*c-a\*d\*(1+n))\*x/a/b^2/n+(-a\*d+b\*c)\*x\*(c+d\*x^n)/a/b/n/(a+b\*x^n)-(-a\*d+b\*c)\*(b\*c\*(1-n)-a\*d\*(1+n))\*x\*hypergeom([1, 1/n],[1+1/n],-b\*x^n/a)/a^2/b^2/n

**Rubi [A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {413, 388, 245}

$$\frac{x(bc-ad)(bc(1-n)-ad(n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n} - \frac{dx(bc-ad(n+1))}{ab^2n} + \frac{x(bc-ad)(c+dx^n)}{abn(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^2/(a + b\*x^n)^2, x]

[Out] -((d\*(b\*c - a\*d\*(1 + n))\*x)/(a\*b^2\*n)) + ((b\*c - a\*d)\*x\*(c + d\*x^n))/(a\*b\*n\*(a + b\*x^n)) - ((b\*c - a\*d)\*(b\*c\*(1 - n) - a\*d\*(1 + n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a^2\*b^2\*n)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rule 413

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[((a\*d - c\*b)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1))/(a\*b\*n\*(p + 1)), x] - Dist[1/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 2)\*Simp[c\*(a\*d - c\*b\*(n\*(p + 1) + 1)) + d\*(a\*d\*(n\*(q - 1) + 1) - b\*c\*(n\*(p + q) + 1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rubi steps

$$\begin{aligned} \int \frac{(c + dx^n)^2}{(a + bx^n)^2} dx &= \frac{(bc - ad)x(c + dx^n)}{abn(a + bx^n)} + \frac{\int \frac{c(ad - bc(1-n)) - d(bc - ad(1+n))x^n}{a + bx^n} dx}{abn} \\ &= -\frac{d(bc - ad(1+n))x}{ab^2n} + \frac{(bc - ad)x(c + dx^n)}{abn(a + bx^n)} - \frac{((bc - ad)(bc(1-n) - ad(1+n))) \int \frac{1}{a + bx^n} dx}{ab^2n} \\ &= -\frac{d(bc - ad(1+n))x}{ab^2n} + \frac{(bc - ad)x(c + dx^n)}{abn(a + bx^n)} - \frac{(bc - ad)(bc(1-n) - ad(1+n))x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2b^2n} \end{aligned}$$

**Mathematica [C]** time = 2.58, size = 666, normalized size = 5.79

$$x \left( -2bc^2n^6x^n {}_4F_3\left(2, 2, 2, 1 + \frac{1}{n}; 1, 1, 4 + \frac{1}{n}; -\frac{bx^n}{a}\right) - 4bcdn^6x^{2n} {}_4F_3\left(2, 2, 2, 1 + \frac{1}{n}; 1, 1, 4 + \frac{1}{n}; -\frac{bx^n}{a}\right) - 2bd^2n^6x^{3n} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^n)^2/(a + b\*x^n)^2, x]

[Out] (x\*(-2\*a\*(1 + 6\*n + 11\*n^2 + 6\*n^3)\*(c^2\*(1 + n)^3 + 2\*c\*d\*(1 + 3\*n + 4\*n^2 + n^3)\*x^n + d^2\*(1 + n)^3\*x^(2\*n))\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 1 + n^(-1)] + a\*(1 + 6\*n + 11\*n^2 + 6\*n^3)\*(c^2\*(1 + 2\*n)^3 + 2\*c\*d\*(1 + 2\*n)^3\*x^n + d^2\*(1 + 6\*n + 10\*n^2 + 6\*n^3)\*x^(2\*n))\*HurwitzLerchPhi[-((b\*x^n)/a), 1, 2 + n^(-1)] + a\*c^2\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 6\*a\*c^2\*n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 9\*a\*c^2\*n^2\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] - 4\*a\*c^2\*n^3\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] - 10\*a\*c^2\*n^4\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 10\*a\*c^2\*n^5\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 12\*a\*c^2\*n^6\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 2\*a\*c\*d\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 12\*a\*c\*d\*n\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 22\*a\*c\*d\*n^2\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 12\*a\*c\*d\*n^3\*x^n\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + a\*d^2\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 6\*a\*d^2\*n\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 11\*a\*d^2\*n^2\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] + 6\*a\*d^2\*n^3\*x^(2\*n)\*HurwitzLerchPhi[-((b\*x^n)/a), 1, n^(-1)] - 2\*b\*c^2\*n^6\*x^n\*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -(b\*x^n)/a] - 4\*b\*c\*d\*n^6\*x^(2\*n)\*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -(b\*x^n)/a] - 2\*b\*d^2\*n^6\*x^(3\*n)\*HypergeometricPFQ[{2, 2, 2, 1 + n^(-1)}, {1, 1, 4 + n^(-1)}, -(b\*x^n)/a]))/(2\*a^3\*n^4\*(1 + 6\*n + 11\*n^2 + 6\*n^3))

**fricas [F]** time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{d^2x^{2n} + 2cdx^n + c^2}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n)^2,x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^2}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^2/(b\*x^n + a)^2, x)

**maple** [F] time = 0.61, size = 0, normalized size = 0.00

$$\int \frac{(d x^n + c)^2}{(b x^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^2/(b\*x^n+a)^2,x)

[Out] int((d\*x^n+c)^2/(b\*x^n+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$-(a^2 d^2 (n+1) - b^2 c^2 (n-1) - 2abcd) \int \frac{1}{ab^3 n x^n + a^2 b^2 n} dx + \frac{abd^2 n x x^n + (a^2 d^2 (n+1) + b^2 c^2 - 2abcd)x}{ab^3 n x^n + a^2 b^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^2/(a+b\*x^n)^2,x, algorithm="maxima")

[Out] -(a^2\*d^2\*(n + 1) - b^2\*c^2\*(n - 1) - 2\*a\*b\*c\*d)\*integrate(1/(a\*b^3\*n\*x^n + a^2\*b^2\*n), x) + (a\*b\*d^2\*n\*x\*x^n + (a^2\*d^2\*(n + 1) + b^2\*c^2 - 2\*a\*b\*c\*d)\*x)/(a\*b^3\*n\*x^n + a^2\*b^2\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d x^n)^2}{(a + b x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^2/(a + b\*x^n)^2,x)

[Out] int((c + d\*x^n)^2/(a + b\*x^n)^2, x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + d x^n)^2}{(a + b x^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*2/(a+b\*x\*\*n)\*\*2,x)

[Out] Integral((c + d\*x\*\*n)\*\*2/(a + b\*x\*\*n)\*\*2, x)

$$3.308 \quad \int \frac{c+dx^n}{(a+bx^n)^2} dx$$

Optimal. Leaf size=72

$$\frac{x(ad - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} + \frac{x(bc - ad)}{abn(a + bx^n)}$$

[Out]  $(-a*d+b*c)*x/a/b/n/(a+b*x^n)+(a*d-b*c*(1-n))*x*\text{hypergeom}([1, 1/n], [1+1/n], -b*x^n/a)/a^2/b/n$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$ , Rules used = {385, 245}

$$\frac{x(ad - bc(1 - n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} + \frac{x(bc - ad)}{abn(a + bx^n)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)/(a + b\*x^n)^2, x]

[Out]  $((b*c - a*d)*x)/(a*b*n*(a + b*x^n)) + ((a*d - b*c*(1 - n))*x*\text{Hypergeometric}2F1[1, n^{(-1)}, 1 + n^{(-1)}, -((b*x^n)/a)])/(a^2*b*n)$

Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rubi steps

$$\begin{aligned} \int \frac{c + dx^n}{(a + bx^n)^2} dx &= \frac{(bc - ad)x}{abn(a + bx^n)} + \frac{(ad - bc(1 - n)) \int \frac{1}{a + bx^n} dx}{abn} \\ &= \frac{(bc - ad)x}{abn(a + bx^n)} + \frac{(ad - bc(1 - n))x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2bn} \end{aligned}$$

Mathematica [A] time = 0.04, size = 56, normalized size = 0.78

$$\frac{x \left( \frac{d}{a + bx^n} - \frac{(ad + bc(n-1)) {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} \right)}{b - bn}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)/(a + b\*x^n)^2,x]

[Out] (x\*(d/(a + b\*x^n) - ((a\*d + b\*c\*(-1 + n))\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b\*x^n)/a)]/a^2))/(b - b\*n)

**fricas** [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{dx^n + c}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n)^2,x, algorithm="fricas")

[Out] integral((d\*x^n + c)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^n + c}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x^n + c)/(b\*x^n + a)^2, x)

**maple** [F] time = 0.59, size = 0, normalized size = 0.00

$$\int \frac{dx^n + c}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)/(b\*x^n+a)^2,x)

[Out] int((d\*x^n+c)/(b\*x^n+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$(bc(n-1) + ad) \int \frac{1}{ab^2nx^n + a^2bn} dx + \frac{(bc - ad)x}{ab^2nx^n + a^2bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)/(a+b\*x^n)^2,x, algorithm="maxima")

[Out] (b\*c\*(n - 1) + a\*d)\*integrate(1/(a\*b^2\*n\*x^n + a^2\*b\*n), x) + (b\*c - a\*d)\*x/(a\*b^2\*n\*x^n + a^2\*b\*n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + dx^n}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)/(a + b\*x^n)^2,x)

[Out] int((c + d\*x^n)/(a + b\*x^n)^2, x)

**sympy** [C] time = 8.77, size = 592, normalized size = 8.22

$$c \left( \frac{nx\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} + \frac{nx\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} - \frac{x\Phi\left(\frac{bx^ne^{i\pi}}{a}, 1, \frac{1}{n}\right)\Gamma\left(\frac{1}{n}\right)}{a\left(an^3\Gamma\left(1 + \frac{1}{n}\right) + bn^3x^n\Gamma\left(1 + \frac{1}{n}\right)\right)} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)/(a+b\*x\*\*n)\*\*2,x)

[Out]  $c*(n*x*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1/n)*\text{gamma}(1/n)/(a*(a*n**3*\text{gamma}(1 + 1/n) + b*n**3*x**n*\text{gamma}(1 + 1/n))) + n*x*\text{gamma}(1/n)/(a*(a*n**3*\text{gamma}(1 + 1/n) + b*n**3*x**n*\text{gamma}(1 + 1/n))) - x*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1/n)*\text{gamma}(1/n)/(a*(a*n**3*\text{gamma}(1 + 1/n) + b*n**3*x**n*\text{gamma}(1 + 1/n))) + b*n*x*x**n*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1/n)*\text{gamma}(1/n)/(a**2*(a*n**3*\text{gamma}(1 + 1/n) + b*n**3*x**n*\text{gamma}(1 + 1/n))) - b*x*x**n*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1/n)*\text{gamma}(1/n)/(a**2*(a*n**3*\text{gamma}(1 + 1/n) + b*n**3*x**n*\text{gamma}(1 + 1/n)))) + d*(n**2*x*x**n*\text{gamma}(1 + 1/n)/(a*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n))) - n*x*x**n*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1 + 1/n)*\text{gamma}(1 + 1/n)/(a*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n))) + n*x*x**n*\text{gamma}(1 + 1/n)/(a*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n))) - x*x**n*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1 + 1/n)*\text{gamma}(1 + 1/n)/(a*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n))) - b*n*x*x**(2*n)*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1 + 1/n)*\text{gamma}(1 + 1/n)/(a**2*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n)))) - b*x*x**(2*n)*\text{lerchphi}(b*x**n*\exp\_polar(I*\pi)/a, 1, 1 + 1/n)*\text{gamma}(1 + 1/n)/(a**2*(a*n**3*\text{gamma}(2 + 1/n) + b*n**3*x**n*\text{gamma}(2 + 1/n))))$

$$3.309 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)} dx$$

**Optimal.** Leaf size=122

$$\frac{bx(ad(1-2n) - bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

[Out] b\*x/a/(-a\*d+b\*c)/n/(a+b\*x^n)+b\*(a\*d\*(1-2\*n)-b\*c\*(1-n))\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a^2/(-a\*d+b\*c)^2/n+d^2\*x\*hypergeom([1, 1/n], [1+1/n], -d\*x^n/c)/c/(-a\*d+b\*c)^2

**Rubi [A]** time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$ , Rules used = {414, 522, 245}

$$\frac{bx(ad(1-2n) - bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^2} + \frac{d^2x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c(bc-ad)^2} + \frac{bx}{an(bc-ad)(a+bx^n)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)), x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)) + (b\*(a\*d\*(1 - 2\*n) - b\*c\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/ (a^2\*(b\*c - a\*d)^2\*n) + (d^2\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/ (c\*(b\*c - a\*d)^2)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^n)^2 (c + dx^n)} dx &= \frac{bx}{a(bc - ad)n(a + bx^n)} - \frac{\int \frac{adn + b(c - cn) + bd(1 - n)x^n}{(a + bx^n)(c + dx^n)} dx}{a(bc - ad)n} \\ &= \frac{bx}{a(bc - ad)n(a + bx^n)} + \frac{d^2 \int \frac{1}{c + dx^n} dx}{(bc - ad)^2} + \frac{(b(ad(1 - 2n) - bc(1 - n))) \int \frac{1}{a + bx^n} dx}{a(bc - ad)^2 n} \\ &= \frac{bx}{a(bc - ad)n(a + bx^n)} + \frac{b(ad(1 - 2n) - bc(1 - n))x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2(bc - ad)^2 n} + \frac{d^2 x {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.20, size = 108, normalized size = 0.89

$$\frac{x \left( \frac{b^2 c - abd}{a^2 n + abn x^n} + \frac{b(ad(1 - 2n) + bc(n - 1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2 n} + \frac{d^2 {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c} \right)}{(bc - ad)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)), x]

[Out] (x\*((b^2\*c - a\*b\*d)/(a^2\*n + a\*b\*n\*x^n) + (b\*(a\*d\*(1 - 2\*n) + b\*c\*(-1 + n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a^2\*n) + (d^2\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)]/c))/(b\*c - a\*d)^2

**fricas [F]** time = 0.99, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{b^2 dx^{3n} + a^2 c + (b^2 c + 2abd)x^{2n} + (2abc + a^2 d)x^n}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n), x, algorithm="fricas")

[Out] integral(1/(b^2\*d\*x^(3\*n) + a^2\*c + (b^2\*c + 2\*a\*b\*d)\*x^(2\*n) + (2\*a\*b\*c + a^2\*d)\*x^n), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n), x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)), x)

**maple [F]** time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)^2/(d\*x^n+c), x)

[Out] int(1/(b\*x^n+a)^2/(d\*x^n+c), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$d^2 \int \frac{1}{b^2c^3 - 2abc^2d + a^2cd^2 + (b^2c^2d - 2abcd^2 + a^2d^3)x^n} dx - (abd(2n-1) - b^2c(n-1)) \int \frac{1}{a^2b^2c^2n - 2a^3bcdn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n),x, algorithm="maxima")

[Out] d^2\*integrate(1/(b^2\*c^3 - 2\*a\*b\*c^2\*d + a^2\*c\*d^2 + (b^2\*c^2\*d - 2\*a\*b\*c\*d^2 + a^2\*d^3)\*x^n), x) - (a\*b\*d\*(2\*n - 1) - b^2\*c\*(n - 1))\*integrate(1/(a^2\*b^2\*c^2\*n - 2\*a^3\*b\*c\*d\*n + a^4\*d^2\*n + (a\*b^3\*c^2\*n - 2\*a^2\*b^2\*c\*d\*n + a^3\*b\*d^2\*n)\*x^n), x) + b\*x/(a^2\*b\*c\*n - a^3\*d\*n + (a\*b^2\*c\*n - a^2\*b\*d\*n)\*x^n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + b x^n)^2 (c + d x^n)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)^2\*(c + d\*x^n)),x)

[Out] int(1/((a + b\*x^n)^2\*(c + d\*x^n)), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n),x)

[Out] Exception raised: HeuristicGCDFailed

$$3.310 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)^2} dx$$

**Optimal.** Leaf size=193

$$\frac{b^2x(ad(1-3n)-b(c-cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3} - \frac{d^2x(bc(1-3n)-ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3} + \frac{1}{an(bc-ad)}$$

[Out] d\*(a\*d+b\*c)\*x/a/c/(-a\*d+b\*c)^2/n/(c+d\*x^n)+b\*x/a/(-a\*d+b\*c)/n/(a+b\*x^n)/(c+d\*x^n)+b^2\*(a\*d\*(1-3\*n)-b\*(-c\*n+c))\*x\*hypergeom([1, 1/n], [1+1/n], -b\*x^n/a)/a^2/(-a\*d+b\*c)^3/n-d^2\*(b\*c\*(1-3\*n)-a\*d\*(1-n))\*x\*hypergeom([1, 1/n], [1+1/n], -d\*x^n/c)/c^2/(-a\*d+b\*c)^3/n

**Rubi [A]** time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {414, 527, 522, 245}

$$\frac{b^2x(ad(1-3n)-b(c-cn)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^3} - \frac{d^2x(bc(1-3n)-ad(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2n(bc-ad)^3} + \frac{1}{an(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^2), x]

[Out] (d\*(b\*c + a\*d)\*x)/(a\*c\*(b\*c - a\*d)^2\*n\*(c + d\*x^n)) + (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)) + (b^2\*(a\*d\*(1 - 3\*n) - b\*(c - c\*n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*x^n)/a)])/(a^2\*(b\*c - a\*d)^3\*n) - (d^2\*(b\*c\*(1 - 3\*n) - a\*d\*(1 - n))\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)])/(c^2\*(b\*c - a\*d)^3\*n)

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p + 1)\*(b\*c - a\*d) + d\*b\*(n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !(IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/(((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_))), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rule 527

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.)\*((e\_) + (f\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*e - a\*f)\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(b\*c - a\*d)\*(p + 1)), x] + Dist[1/(a\*n\*(b\*c - a\*d)\*(p

+ 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q\*Simp[c\*(b\*e - a\*f) + e\*n\*(b\*c - a\*d)\*(p + 1) + d\*(b\*e - a\*f)\*(n\*(p + q + 2) + 1)\*x^n, x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^n)^2 (c + dx^n)^2} dx &= \frac{bx}{a(bc - ad)n (a + bx^n) (c + dx^n)} - \frac{\int \frac{adn + b(c - cn) + bd(1 - 2n)x^n}{(a + bx^n)(c + dx^n)^2} dx}{a(bc - ad)n} \\ &= \frac{d(bc + ad)x}{ac(bc - ad)^2 n (c + dx^n)} + \frac{bx}{a(bc - ad)n (a + bx^n) (c + dx^n)} - \frac{\int \frac{n(b^2 c^2 (1 - n) + a^2 d^2 (1 - n) + 2}{(a + bx^n)} dx}{ac(bc - ad)^2 n} \\ &= \frac{d(bc + ad)x}{ac(bc - ad)^2 n (c + dx^n)} + \frac{bx}{a(bc - ad)n (a + bx^n) (c + dx^n)} + \frac{(d^2(ad(1 - n) - b(c - 3n)) - b^2c^2)}{c(bc - ad)^2 n} \\ &= \frac{d(bc + ad)x}{ac(bc - ad)^2 n (c + dx^n)} + \frac{bx}{a(bc - ad)n (a + bx^n) (c + dx^n)} + \frac{b^2(ad(1 - 3n) - b(c - 3n))}{a^2(bc - ad)^2 n} \end{aligned}$$

**Mathematica** [A] time = 0.28, size = 147, normalized size = 0.76

$$\frac{x \left( \frac{b^2(ad(1-3n)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} + \frac{b^2(bc-ad)}{a(a+bx^n)} + \frac{d^2(bc(3n-1)-ad(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2} + \frac{d^2(bc-ad)}{c(c+dx^n)} \right)}{n(bc-ad)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^2), x]

[Out] (x\*((b^2\*(b\*c - a\*d))/(a\*(a + b\*x^n)) + (d^2\*(b\*c - a\*d))/(c\*(c + d\*x^n)) + (b^2\*(a\*d\*(1 - 3\*n) + b\*c\*(-1 + n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1)], -((b\*x^n)/a)]/a^2 + (d^2\*(-(a\*d\*(-1 + n)) + b\*c\*(-1 + 3\*n))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d\*x^n)/c)]/c^2))/((b\*c - a\*d)^3\*n)

**fricas** [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2 d^2 x^{4n} + a^2 c^2 + 2(b^2 c d + a b d^2) x^{3n} + (b^2 c^2 + 4 a b c d + a^2 d^2) x^{2n} + 2(a b c^2 + a^2 c d) x^n}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="fricas")

[Out] integral(1/(b^2\*d^2\*x^(4\*n) + a^2\*c^2 + 2\*(b^2\*c\*d + a\*b\*d^2)\*x^(3\*n) + (b^2\*c^2 + 4\*a\*b\*c\*d + a^2\*d^2)\*x^(2\*n) + 2\*(a\*b\*c^2 + a^2\*c\*d)\*x^n), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^2), x)

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2 (dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)^2/(d\*x^n+c)^2,x)

[Out] int(1/(b\*x^n+a)^2/(d\*x^n+c)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$(ab^2d(3n-1) - b^3c(n-1)) \int -\frac{1}{a^2b^3c^3n - 3a^3b^2c^2dn + 3a^4bcd^2n - a^5d^3n + (ab^4c^3n - 3a^2b^3c^2dn + 3a^3b^2cd^2n)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^2,x, algorithm="maxima")

[Out] (a\*b^2\*d\*(3\*n - 1) - b^3\*c\*(n - 1))\*integrate(-1/(a^2\*b^3\*c^3\*n - 3\*a^3\*b^2\*c^2\*d\*n + 3\*a^4\*b\*c\*d^2\*n - a^5\*d^3\*n + (a\*b^4\*c^3\*n - 3\*a^2\*b^3\*c^2\*d\*n + 3\*a^3\*b^2\*c\*d^2\*n - a^4\*b\*d^3\*n)\*x^n), x) - (b\*c\*d^2\*(3\*n - 1) - a\*d^3\*(n - 1))\*integrate(-1/(b^3\*c^5\*n - 3\*a\*b^2\*c^4\*d\*n + 3\*a^2\*b\*c^3\*d^2\*n - a^3\*c^2\*d^3\*n + (b^3\*c^4\*d\*n - 3\*a\*b^2\*c^3\*d^2\*n + 3\*a^2\*b\*c^2\*d^3\*n - a^3\*c\*d^4\*n)\*x^n), x) + ((b^2\*c\*d + a\*b\*d^2)\*x\*x^n + (b^2\*c^2 + a^2\*d^2)\*x)/(a^2\*b^2\*c^4\*n - 2\*a^3\*b\*c^3\*d\*n + a^4\*c^2\*d^2\*n + (a\*b^3\*c^3\*d\*n - 2\*a^2\*b^2\*c^2\*d^2\*n + a^3\*b\*c\*d^3\*n)\*x^(2\*n) + (a\*b^3\*c^4\*n - a^2\*b^2\*c^3\*d\*n - a^3\*b\*c^2\*d^2\*n + a^4\*c\*d^3\*n)\*x^n)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + bx^n)^2 (c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)^2\*(c + d\*x^n)^2),x)

[Out] int(1/((a + b\*x^n)^2\*(c + d\*x^n)^2), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)\*\*2/(c+d\*x\*\*n)\*\*2,x)

[Out] Exception raised: HeuristicGCDFailed

$$3.311 \quad \int \frac{1}{(a+bx^n)^2(c+dx^n)^3} dx$$

**Optimal.** Leaf size=299

$$\frac{b^3x(ad(1-4n)-bc(1-n)){}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2n(bc-ad)^4} - \frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2(bc-ad)^3(c+dx^n)} + \frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)){}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4} - \frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2}$$

[Out]  $\frac{1}{2}d*(a*d+2*b*c)*x/a/c/(-a*d+b*c)^2/n/(c+d*x^n)^2+b*x/a/(-a*d+b*c)/n/(a+b*x^n)/(c+d*x^n)^2-1/2*d*(a*b*c*d*(1-6*n)-a^2*d^2*(1-2*n)-2*b^2*c^2*n)*x/a/c^2/(-a*d+b*c)^3/n^2/(c+d*x^n)+b^3*(a*d*(1-4*n)-b*c*(1-n))*x*hypergeom([1, 1/n], [1+1/n], -b*x^n/a)/a^2/(-a*d+b*c)^4/n+1/2*d^2*(a^2*d^2*(2*n^2-3*n+1)-2*a*b*c*d*(4*n^2-5*n+1)+b^2*c^2*(12*n^2-7*n+1))*x*hypergeom([1, 1/n], [1+1/n], -d*x^n/c)/c^3/(-a*d+b*c)^4/n^2$

**Rubi [A]** time = 0.55, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {414, 527, 522, 245}

$$\frac{d^2x(a^2d^2(2n^2-3n+1)-2abcd(4n^2-5n+1)+b^2c^2(12n^2-7n+1)){}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{2c^3n^2(bc-ad)^4} - \frac{dx(-a^2d^2(1-2n)+abcd(1-6n)-2b^2c^2n)}{2ac^2n^2}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^3), x]

[Out]  $\frac{d*(2*b*c + a*d)*x}{(2*a*c*(b*c - a*d)^2*n*(c + d*x^n)^2} + \frac{(b*x)}{a*(b*c - a*d)*n*(a + b*x^n)*(c + d*x^n)^2} - \frac{d*(a*b*c*d*(1 - 6*n) - a^2*d^2*(1 - 2*n) - 2*b^2*c^2*n)*x}{(2*a*c^2*(b*c - a*d)^3*n^2*(c + d*x^n)} + \frac{b^3*(a*d*(1 - 4*n) - b*c*(1 - n))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*x^n)/a)]}{(a^2*(b*c - a*d)^4*n} + \frac{(d^2*(a^2*d^2*(1 - 3*n + 2*n^2) - 2*a*b*c*d*(1 - 5*n + 4*n^2) + b^2*c^2*(1 - 7*n + 12*n^2))*x*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((d*x^n)/c)]}{(2*c^3*(b*c - a*d)^4*n^2}$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b\*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !LtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 414

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[1/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q\*Simp[b\*c + n\*(p+1)\*(b\*c - a\*d) + d\*b\*(n\*(p+q+2)+1)\*x^n, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && LtQ[p, -1] && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, n, p, q, x]

#### Rule 522

Int[((e\_) + (f\_.)\*(x\_)^(n\_))/((a\_) + (b\_.)\*(x\_)^(n\_))\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Dist[(b\*e - a\*f)/(b\*c - a\*d), Int[1/(a + b\*x^n), x], x] - Dist[(d\*e - c\*f)/(b\*c - a\*d), Int[1/(c + d\*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

#### Rule 527



```
Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_)*((e_) + (f_)*(x_)^(n_)), x_Symbol] := -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{1}{(a + bx^n)^2 (c + dx^n)^3} dx &= \frac{bx}{a(bc - ad)n(a + bx^n)(c + dx^n)^2} - \frac{\int \frac{adn + b(c - cn) + bd(1 - 3n)x^n}{(a + bx^n)(c + dx^n)^3} dx}{a(bc - ad)n} \\ &= \frac{d(2bc + ad)x}{2ac(bc - ad)^2n(c + dx^n)^2} + \frac{bx}{a(bc - ad)n(a + bx^n)(c + dx^n)^2} - \frac{\int \frac{n(a^2d^2(1 - 2n) + 2b^2c^2)}{(a + bx^n)(c + dx^n)^3} dx}{a(bc - ad)n} \\ &= \frac{d(2bc + ad)x}{2ac(bc - ad)^2n(c + dx^n)^2} + \frac{bx}{a(bc - ad)n(a + bx^n)(c + dx^n)^2} - \frac{d(abcd(1 - 6n) - 2ac^2(bc - ad))}{2ac^2(bc - ad)} \\ &= \frac{d(2bc + ad)x}{2ac(bc - ad)^2n(c + dx^n)^2} + \frac{bx}{a(bc - ad)n(a + bx^n)(c + dx^n)^2} - \frac{d(abcd(1 - 6n) - 2ac^2(bc - ad))}{2ac^2(bc - ad)} \\ &= \frac{d(2bc + ad)x}{2ac(bc - ad)^2n(c + dx^n)^2} + \frac{bx}{a(bc - ad)n(a + bx^n)(c + dx^n)^2} - \frac{d(abcd(1 - 6n) - 2ac^2(bc - ad))}{2ac^2(bc - ad)} \end{aligned}$$

**Mathematica [A]** time = 0.44, size = 233, normalized size = 0.78

$$x \left( \frac{2b^3n(ad(1-4n)+bc(n-1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{a^2} + \frac{d^2(a^2d^2(2n^2-3n+1) - 2abcd(4n^2-5n+1) + b^2c^2(12n^2-7n+1)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^3} + \frac{2b^3n(bc - ad)}{a(a + bx^n)(c + dx^n)^3} \right) \frac{1}{2n^2(bc - ad)^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^3), x]

[Out] (x\*((2\*b^3\*(b\*c - a\*d)\*n)/(a\*(a + b\*x^n)) + (d^2\*(b\*c - a\*d)^2\*n)/(c\*(c + d\*x^n)^2) + (d^2\*(-(b\*c) + a\*d)\*(a\*d\*(-1 + 2\*n) + b\*(c - 6\*c\*n)))/(c^2\*(c + d\*x^n)) + (2\*b^3\*(a\*d\*(1 - 4\*n) + b\*c\*(-1 + n))\*n\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(b\*x^n)/a])/a^2 + (d^2\*(a^2\*d^2\*(1 - 3\*n + 2\*n^2) - 2\*a\*b\*c\*d\*(1 - 5\*n + 4\*n^2) + b^2\*c^2\*(1 - 7\*n + 12\*n^2))\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(d\*x^n)/c])/c^3)/(2\*(b\*c - a\*d)^4\*n^2)

**fricas [F]** time = 0.99, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{b^2d^3x^{5n} + a^2c^3 + (3b^2cd^2 + 2abd^3)x^{4n} + (3b^2c^2d + 6abcd^2 + a^2d^3)x^{3n} + (b^2c^3 + 6abc^2d + 3a^2cd^2)x^{2n} + (2a^2c^2d + 3a^2cd^2)x^n + a^2c^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="fricas")

[Out] integral(1/(b^2\*d^3\*x^(5\*n) + a^2\*c^3 + (3\*b^2\*c\*d^2 + 2\*a\*b\*d^3)\*x^(4\*n) + (3\*b^2\*c^2\*d + 6\*a\*b\*c\*d^2 + a^2\*d^3)\*x^(3\*n) + (b^2\*c^3 + 6\*a\*b\*c^2\*d + 3\*a^2\*c\*d^2)\*x^(2\*n) + (2\*a\*b\*c^3 + 3\*a^2\*c^2\*d)\*x^n), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^3), x)

**maple** [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)^2/(d\*x^n+c)^3,x)

[Out] int(1/(b\*x^n+a)^2/(d\*x^n+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\left( (12n^2 - 7n + 1)b^2c^2d^2 - 2(4n^2 - 5n + 1)abcd^3 + (2n^2 - 3n + 1)a^2d^4 \right) \int \frac{1}{2(b^4c^7n^2 - 4ab^3c^6dn^2 + 6a^2b^2c^5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/(c+d\*x^n)^3,x, algorithm="maxima")

[Out] ((12\*n^2 - 7\*n + 1)\*b^2\*c^2\*d^2 - 2\*(4\*n^2 - 5\*n + 1)\*a\*b\*c\*d^3 + (2\*n^2 - 3\*n + 1)\*a^2\*d^4)\*integrate(1/2/(b^4\*c^7\*n^2 - 4\*a\*b^3\*c^6\*d\*n^2 + 6\*a^2\*b^2\*c^5\*d^2\*n^2 - 4\*a^3\*b\*c^4\*d^3\*n^2 + a^4\*c^3\*d^4\*n^2 + (b^4\*c^6\*d\*n^2 - 4\*a\*b^3\*c^5\*d^2\*n^2 + 6\*a^2\*b^2\*c^4\*d^3\*n^2 - 4\*a^3\*b\*c^3\*d^4\*n^2 + a^4\*c^2\*d^5\*n^2)\*x^n), x) - (a\*b^3\*d\*(4\*n - 1) - b^4\*c\*(n - 1))\*integrate(1/(a^2\*b^4\*c^4\*n - 4\*a^3\*b^3\*c^3\*d\*n + 6\*a^4\*b^2\*c^2\*d^2\*n - 4\*a^5\*b\*c\*d^3\*n + a^6\*d^4\*4\*n + (a\*b^5\*c^4\*n - 4\*a^2\*b^4\*c^3\*d\*n + 6\*a^3\*b^3\*c^2\*d^2\*n - 4\*a^4\*b^2\*c\*d^3\*n + a^5\*b\*d^4\*n)\*x^n), x) + 1/2\*((a\*b^2\*c\*d^3\*(6\*n - 1) - a^2\*b\*d^4\*(2\*n - 1) + 2\*b^3\*c^2\*d^2\*n)\*x\*x^(2\*n) + (a\*b^2\*c^2\*d^2\*(7\*n - 1) - a^3\*d^4\*(2\*n - 1) + 4\*b^3\*c^3\*d\*n + 3\*a^2\*b\*c\*d^3\*n)\*x\*x^n + (a^2\*b\*c^2\*d^2\*(7\*n - 1) - a^3\*c\*d^3\*(3\*n - 1) + 2\*b^3\*c^4\*n)\*x)/(a^2\*b^3\*c^7\*n^2 - 3\*a^3\*b^2\*c^6\*d\*n^2 + 3\*a^4\*b\*c^5\*d^2\*n^2 - a^5\*c^4\*d^3\*n^2 + (a\*b^4\*c^5\*d^2\*n^2 - 3\*a^2\*b^3\*c^4\*d^3\*n^2 + 3\*a^3\*b^2\*c^3\*d^4\*n^2 - a^4\*b\*c^2\*d^5\*n^2)\*x^(3\*n) + (2\*a\*b^4\*c^6\*d\*n^2 - 5\*a^2\*b^3\*c^5\*d^2\*n^2 + 3\*a^3\*b^2\*c^4\*d^3\*n^2 + a^4\*b\*c^3\*d^4\*n^2 - a^5\*c^2\*d^5\*n^2)\*x^(2\*n) + (a\*b^4\*c^7\*n^2 - a^2\*b^3\*c^6\*d\*n^2 - 3\*a^3\*b^2\*c^5\*d^2\*n^2 + 5\*a^4\*b\*c^4\*d^3\*n^2 - 2\*a^5\*c^3\*d^4\*n^2)\*x^n)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(a + bx^n)^2(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)^2\*(c + d\*x^n)^3),x)

[Out] int(1/((a + b\*x^n)^2\*(c + d\*x^n)^3), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^n)^2(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*x**n)**2/(c+d*x**n)**3,x)
```

```
[Out] Integral(1/((a + b*x**n)**2*(c + d*x**n)**3), x)
```

### 3.312 $\int (a + bx^n)^p (c + dx^n)^q dx$

**Optimal.** Leaf size=81

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

[Out]  $x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1(1/n, -p, -q, 1+1/n, -b*x^n/a, -d*x^n/c)/((1+b*x^n/a)^p)/((1+d*x^n/c)^q)$

**Rubi [A]** time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (c + dx^n)^q \left(\frac{dx^n}{c} + 1\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^q,x]

[Out]  $(x*(a + b*x^n)^p*(c + d*x^n)^q*AppellF1[n^(-1), -p, -q, 1 + n^(-1), -((b*x^n)/a), -((d*x^n)/c)])/((1 + (b*x^n)/a)^p*(1 + (d*x^n)/c)^q)$

#### Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

#### Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/((1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)^p (c + dx^n)^q dx &= \left( (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \right) \int \left(1 + \frac{bx^n}{a}\right)^p (c + dx^n)^q dx \\ &= \left( (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} (c + dx^n)^q \left(1 + \frac{dx^n}{c}\right)^{-q} \right) \int \left(1 + \frac{bx^n}{a}\right)^p \left(1 + \frac{dx^n}{c}\right)^q dx \\ &= x(a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} (c + dx^n)^q \left(1 + \frac{dx^n}{c}\right)^{-q} F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) \end{aligned}$$

**Mathematica [B]** time = 0.57, size = 190, normalized size = 2.35

$$\frac{ac(n+1)x(a + bx^n)^p (c + dx^n)^q F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{bcnpx^n F_1\left(1 + \frac{1}{n}; 1 - p, -q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + adnqx^n F_1\left(1 + \frac{1}{n}; -p, 1 - q; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; -p, -q; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^q,x]

[Out]  $(a*c*(1+n)*x*(a+b*x^n)^p*(c+d*x^n)^q*AppellF1[n^{(-1)}, -p, -q, 1+n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)])/(b*c*n*p*x^n*AppellF1[1+n^{(-1)}, 1-p, -q, 2+n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] + a*d*n*q*x^n*AppellF1[1+n^{(-1)}, -p, 1-q, 2+n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1+n)*AppellF1[n^{(-1)}, -p, -q, 1+n^{(-1)}, -((b*x^n)/a), -((d*x^n)/c)])$

**fricas** [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}((bx^n + a)^p(dx^n + c)^q, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="fricas")`

[Out] `integral((b*x^n + a)^p*(d*x^n + c)^q, x)`

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="giac")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^q, x)`

**maple** [F] time = 0.89, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^n+a)^p*(d*x^n+c)^q,x)`

[Out] `int((b*x^n+a)^p*(d*x^n+c)^q,x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x^n)^p*(c+d*x^n)^q,x, algorithm="maxima")`

[Out] `integrate((b*x^n + a)^p*(d*x^n + c)^q, x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + bx^n)^p(c + dx^n)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^n)^p*(c + d*x^n)^q,x)`

[Out] `int((a + b*x^n)^p*(c + d*x^n)^q, x)`

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n)**q,x)`

[Out] Exception raised: HeuristicGCDFailed

### 3.313 $\int (a + bx^n)^p (c + dx^n)^3 dx$

Optimal. Leaf size=402

$$\frac{dx (a + bx^n)^{p+1} (a^2 d^2 (2n^2 + 3n + 1) - abcd (n^2(p + 7) + n(2p + 9) + 2) + b^2 c^2 (n^2 (p^2 + 6p + 11) + 2n(p + 3) + 1))}{b^3 (np + n + 1)(n(p + 2) + 1)(n(p + 3) + 1)}$$

[Out]  $d*(a^2*d^2*(2*n^2+3*n+1)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3+p)+n^2*(p^2+6*p+11))*x*(a+b*x^n)^(1+p)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p))-d*(a*d*(1+2*n)-b*c*(1+n*(5+p)))*x*(a+b*x^n)^(1+p)*(c+d*x^n)/b^2/(1+n*(2+p))/(1+n*(3+p))+d*x*(a+b*x^n)^(1+p)*(c+d*x^n)^2/b/(n*p+3*n+1)-(a^3*d^3*(2*n^2+3*n+1)-3*a^2*b*c*d^2*(1+n*(1+n*(3+p)))+3*a*b^2*c^2*d*(1+n*(5+2*p))+n^2*(p^2+5*p+6))-b^3*c^3*(1+3*n*(2+p)+n^2*(3*p^2+12*p+11))+n^3*(p^3+6*p^2+11*p+6))*x*(a+b*x^n)^p*hypergeom([-p, 1/n], [1+1/n], -b*x^n/a)/b^3/(n*p+n+1)/(1+n*(2+p))/(1+n*(3+p))/((1+b*x^n/a)^p)$

**Rubi [A]** time = 0.58, antiderivative size = 401, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$ , Rules used = {416, 528, 388, 246, 245}

$$\frac{x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (-3a^2bcd^2(n+1)(n(p+3)+1) + a^3d^3(2n^2+3n+1) + 3ab^2c^2d(n^2(p^2+5p+6) + n(p+3) + 1))}{b^3(np+n+1)(n(p+2)+1)(n(p+3)+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^3,x]

[Out]  $(d*(a^2*d^2*(1+3*n+2*n^2)-a*b*c*d*(2+n^2*(7+p)+n*(9+2*p))+b^2*c^2*(1+2*n*(3+p)+n^2*(11+6*p+p^2)))*x*(a+b*x^n)^(1+p))/(b^3*(1+n+n*p)*(1+n*(2+p))*(1+n*(3+p)))-(d*(a*d*(1+2*n)-b*(c+c*n*(5+p)))*x*(a+b*x^n)^(1+p)*(c+d*x^n))/(b^2*(1+n*(2+p))*(1+n*(3+p)))+(d*x*(a+b*x^n)^(1+p)*(c+d*x^n)^2)/(b*(1+n*(3+p)))-((a^3*d^3*(1+3*n+2*n^2)-3*a^2*b*c*d^2*(1+n*(1+n*(3+p)))+3*a*b^2*c^2*d*(1+n*(5+2*p))+n^2*(6+5*p+p^2))-b^3*c^3*(1+3*n*(2+p)+n^2*(11+12*p+3*p^2))+n^3*(6+11*p+6*p^2+p^3))*x*(a+b*x^n)^p*Hypergeometric2F1[n^(-1), -p, 1+n^(-1), -(b*x^n/a)]/(b^3*(1+n+n*p)*(1+n*(2+p))*(1+n*(3+p))*(1+(b*x^n/a)^p)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] && !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] := Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q) + 1)), x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)
^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q) + 1, 0]
```

Rubi steps

$$\begin{aligned} \int (a + bx^n)^p (c + dx^n)^3 dx &= \frac{dx (a + bx^n)^{1+p} (c + dx^n)^2}{b(1 + n(3 + p))} + \frac{\int (a + bx^n)^p (c + dx^n) (-c(ad - b(c + cn(3 + p))) -}{b(1 + n(3 + p))} \\ &= -\frac{d(ad(1 + 2n) - b(c + cn(5 + p)))x (a + bx^n)^{1+p} (c + dx^n)}{b^2(1 + n(2 + p))(1 + n(3 + p))} + \frac{dx (a + bx^n)^{1+p} (c +}{b(1 + n(3 + p))} \\ &= \frac{d(a^2d^2(1 + 3n + 2n^2) - abcd(2 + n^2(7 + p) + n(9 + 2p)) + b^2c^2(1 + 2n(3 + p))}{b^3(1 + n + np)(1 + n(2 + p))(1 + n(3 + p))} \\ &= \frac{d(a^2d^2(1 + 3n + 2n^2) - abcd(2 + n^2(7 + p) + n(9 + 2p)) + b^2c^2(1 + 2n(3 + p))}{b^3(1 + n + np)(1 + n(2 + p))(1 + n(3 + p))} \\ &= \frac{d(a^2d^2(1 + 3n + 2n^2) - abcd(2 + n^2(7 + p) + n(9 + 2p)) + b^2c^2(1 + 2n(3 + p))}{b^3(1 + n + np)(1 + n(2 + p))(1 + n(3 + p))} \end{aligned}$$

**Mathematica [A]** time = 5.32, size = 168, normalized size = 0.42

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left( c^3 {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + \frac{3c^2 dx^n {}_2F_1\left(1 + \frac{1}{n}, -p; 2 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{n + 1} + \frac{3cd^2 x^{2n} {}_2F_1\left(2 + \frac{1}{n}, -p; 3 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{2(n + 1)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^3,x]

[Out] (x\*(a + b\*x^n)^p\*((3\*c^2\*d\*x^n\*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -((b\*x^n)/a)])/(1 + n) + (3\*c\*d^2\*x^(2\*n)\*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1), -((b\*x^n)/a)])/(1 + 2\*n) + (d^3\*x^(3\*n)\*Hypergeometric2F1[3 + n^(-1), -p, 4 + n^(-1), -((b\*x^n)/a)])/(1 + 3\*n) + c^3\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b\*x^n)/a)]))/(1 + (b\*x^n)/a)^p

**fricas [F]** time = 1.11, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3\right)(bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^3,x, algorithm="fricas")

[Out] integral((d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3)\*(b\*x^n + a)^p, x)

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error%%{1, [2,0,6,4,2,4,4,3,0]%%}+%%{4, [2,0,6,4,2,3,4,3,0]%%}+%%{6, [2,0,6,4,2,2,4,3,0]%%}+%%{4, [2,0,6,4,2,1,4,3,0]%%}+%%{1, [2,0,6,4,2,0,4,3,0]%%}+%%{3, [1,0,6,4,2,4,4,2,1]%%}+%%{12, [1,0,6,4,2,3,4,2,1]%%}+%%{18, [1,0,6,4,2,2,4,2,1]%%}+%%{12, [1,0,6,4,2,1,4,2,1]%%}+%%{3, [1,0,6,4,2,0,4,2,1]%%}+%%{-1, [1,0,6,4,1,4,5,3,0]%%}+%%{-4, [1,0,6,4,1,3,5,3,0]%%}+%%{-6, [1,0,6,4,1,2,5,3,0]%%}+%%{-4, [1,0,6,4,1,1,5,3,0]%%}+%%{-1, [1,0,6,4,1,0,5,3,0]%%}+%%{1, [0,0,6,4,3,4,3,0,3]%%}+%%{4, [0,0,6,4,3,3,3,0,3]%%}+%%{6, [0,0,6,4,3,2,3,0,3]%%}+%%{4, [0,0,6,4,3,1,3,0,3]%%}+%%{1, [0,0,6,4,3,0,3,0,3]%%}+%%{1, [0,0,6,3,3,3,3,0,3]%%}+%%{3, [0,0,6,3,3,2,3,0,3]%%}+%%{3, [0,0,6,3,3,1,3,0,3]%%}+%%{1, [0,0,6,3,3,0,3,0,3]%%}+%%{-3, [0,0,6,3,2,3,4,1,2]%%}+%%{-9, [0,0,6,3,2,2,4,1,2]%%}+%%{-9, [0,0,6,3,2,1,4,1,2]%%}+%%{-3, [0,0,6,3,2,0,4,1,2]%%}+%%{3, [0,0,6,3,1,3,5,2,1]%%}+%%{9, [0,0,6,3,1,2,5,2,1]%%}+%%{9, [0,0,6,3,1,1,5,2,1]%%}+%%{3, [0,0,6,3,1,0,5,2,1]%%}+%%{-1, [0,0,6,3,0,3,6,3,0]%%}+%%{-3, [0,0,6,3,0,2,6,3,0]%%}+%%{-3, [0,0,6,3,0,1,6,3,0]%%}+%%{-1, [0,0,6,3,0,0,6,3,0]%%} / %%{1, [0,0,7,4,3,4,4,0,0]%%}+%%{4, [0,0,7,4,3,3,4,0,0]%%}+%%{6, [0,0,7,4,3,2,4,0,0]%%}+%%{4, [0,0,7,4,3,1,4,0,0]%%}+%%{1, [0,0,7,4,3,0,4,0,0]%%} Error: Bad Argument Value

**maple** [F] time = 0.74, size = 0, normalized size = 0.00

$$\int (dx^n + c)^3 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p\*(d\*x^n+c)^3,x)

[Out] int((b\*x^n+a)^p\*(d\*x^n+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^3 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^3,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^3\*(b\*x^n + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + bx^n)^p (c + dx^n)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p\*(c + d\*x^n)^3,x)



[Out]  $\int (a + b*x^n)^p*(c + d*x^n)^3, x$

**sympy [C]** time = 93.23, size = 199, normalized size = 0.50

$$\frac{a^p c^3 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\frac{1}{n}, -p \mid \frac{bx^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)} + \frac{3a^p c^2 dx x^n \Gamma\left(1 + \frac{1}{n}\right) {}_2F_1\left(-p, 1 + \frac{1}{n} \mid \frac{bx^n e^{i\pi}}{a}\right)}{n \Gamma\left(2 + \frac{1}{n}\right)} + \frac{3a^p cd^2 xx^{2n} \Gamma\left(2 + \frac{1}{n}\right) {}_2F_1\left(-p, 2 + \frac{1}{n} \mid \frac{bx^n e^{i\pi}}{a}\right)}{n \Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n)**3,x)`

[Out] `a**p*c**3*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n, ), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + 3*a**p*c**2*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n, ), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n)) + 3*a**p*c*d**2*x*x**(2*n)*gamma(2 + 1/n)*hyper((-p, 2 + 1/n), (3 + 1/n, ), b*x**n*exp_polar(I*pi)/a)/(n*gamma(3 + 1/n)) + a**p*d**3*x*x**(3*n)*gamma(3 + 1/n)*hyper((-p, 3 + 1/n), (4 + 1/n, ), b*x**n*exp_polar(I*pi)/a)/(n*gamma(4 + 1/n))`

### 3.314 $\int (a + bx^n)^p (c + dx^n)^2 dx$

**Optimal.** Leaf size=202

$$\frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (bc(np + n + 1)(ad - bc(n(p + 2) + 1)) - ad(ad(n + 1) - bc(n(p + 3) + 1))) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b^2(np + n + 1)(n(p + 2) + 1)}$$

[Out]  $-d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^{(1+p)}/b^2/(n*p+n+1)/(1+n*(2+p))+d*x*(a+b*x^n)^{(1+p)}*(c+d*x^n)/b/(n*p+2*n+1)-(b*c*(n*p+n+1)*(a*d-b*c*(1+n*(2+p)))-a*d*(a*d*(1+n)-b*c*(1+n*(3+p)))*x*(a+b*x^n)^p*\text{hypergeom}([-p, 1/n], [1+1/n], -b*x^n/a)/b^2/(n*p+n+1)/(1+n*(2+p))/((1+b*x^n/a)^p)$

**Rubi [A]** time = 0.27, antiderivative size = 197, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$ , Rules used = {416, 388, 246, 245}

$$\frac{dx(a + bx^n)^{p+1} (ad(n + 1) - b(cn(p + 3) + c))}{b^2(np + n + 1)(n(p + 2) + 1)} - \frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(c(ad - b(cn(p + 2) + c)) - \frac{ad(ad(n+1)-b(cn(p+3)+c))}{b(np+n+1)}\right)}{b(n(p + 2) + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^2,x]

[Out]  $-((d*(a*d*(1+n) - b*(c + c*n*(3+p)))*x*(a + b*x^n)^{(1+p)})/(b^2*(1+n+n*p)*(1+n*(2+p))) + (d*x*(a + b*x^n)^{(1+p)}*(c + d*x^n))/(b*(1+n*(2+p))) - ((c*(a*d - b*(c + c*n*(2+p))) - (a*d*(a*d*(1+n) - b*(c + c*n*(3+p))))/(b*(1+n+n*p)))*x*(a + b*x^n)^p*\text{Hypergeometric2F1}[n^{-1}, -p, 1 + n^{-1}, -(b*x^n/a)]/(b*(1+n*(2+p))*(1 + (b*x^n/a)^p)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n/a)^FracPart[p], Int[(1 + (b\*x^n/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1))/(b\*(n\*(p+1)+1)), x] - Dist[(a\*d - b\*c\*(n\*(p+1)+1))/(b\*(n\*(p+1)+1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p+1)+1, 0]

#### Rule 416

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(d\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q-1))/(b\*(n\*(p+q)+1)), x] + Dist[1/(b\*(n\*(p+q)+1)), Int[(a + b\*x^n)^p\*(c + d\*x^n)^(q-2)\*Simp[c\*(b\*c\*(n\*(p+q)+1) - a\*d] + d\*(b\*c\*(n\*(p+2\*q-1)+1) - a\*d\*(n\*(q-1)+1))\*x^n, x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && GtQ[q, 1] && NeQ[n\*(p+q)+1, 0] && !IGtQ[p, 1] && IntBinomialQ[a

, b, c, d, n, p, q, x]

### Rubi steps

$$\begin{aligned} \int (a + bx^n)^p (c + dx^n)^2 dx &= \frac{dx (a + bx^n)^{1+p} (c + dx^n)}{b(1 + n(2 + p))} + \frac{\int (a + bx^n)^p (-c(ad - b(c + cn(2 + p))) - d(ad(1 + n(2 + p)))) dx}{b(1 + n(2 + p))} \\ &= -\frac{d(ad(1 + n) - b(c + cn(3 + p)))x (a + bx^n)^{1+p}}{b^2(1 + n + np)(1 + n(2 + p))} + \frac{dx (a + bx^n)^{1+p} (c + dx^n)}{b(1 + n(2 + p))} - \frac{c \int (a + bx^n)^p dx}{b(1 + n(2 + p))} \\ &= -\frac{d(ad(1 + n) - b(c + cn(3 + p)))x (a + bx^n)^{1+p}}{b^2(1 + n + np)(1 + n(2 + p))} + \frac{dx (a + bx^n)^{1+p} (c + dx^n)}{b(1 + n(2 + p))} - \frac{c \int (a + bx^n)^p dx}{b(1 + n(2 + p))} \\ &= -\frac{d(ad(1 + n) - b(c + cn(3 + p)))x (a + bx^n)^{1+p}}{b^2(1 + n + np)(1 + n(2 + p))} + \frac{dx (a + bx^n)^{1+p} (c + dx^n)}{b(1 + n(2 + p))} - \frac{c \int (a + bx^n)^p dx}{b(1 + n(2 + p))} \end{aligned}$$

**Mathematica [A]** time = 5.20, size = 140, normalized size = 0.69

$$\frac{x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left( (n+1) \left( c^2(2n+1) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + d^2 x^{2n} {}_2F_1\left(2 + \frac{1}{n}, -p; 3 + \frac{1}{n}; -\frac{bx^n}{a}\right) \right) + 2c \int (a + bx^n)^p dx}{(n+1)(2n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^2,x]

[Out] (x\*(a + b\*x^n)^p\*(2\*c\*d\*(1 + 2\*n)\*x^n\*Hypergeometric2F1[1 + n^(-1), -p, 2 + n^(-1), -(b\*x^n)/a]) + (1 + n)\*(d^2\*x^(2\*n)\*Hypergeometric2F1[2 + n^(-1), -p, 3 + n^(-1), -(b\*x^n)/a]) + c^2\*(1 + 2\*n)\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a]))/((1 + n)\*(1 + 2\*n)\*(1 + (b\*x^n)/a)^p)

**fricas [F]** time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(d^2 x^{2n} + 2cdx^n + c^2\right)(bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^2,x, algorithm="fricas")

[Out] integral((d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2)\*(b\*x^n + a)^p, x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^2,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{-1, [1,0,4,3,1,3,3,2,0]%%}+%%{-3, [1,0,4,3,1,2,3,2,0]%%}+%%{-3, [1,0,4,3,1,1,3,2,0]%%}+%%{-1, [1,0,4,3,1,0,3,2,0]%%}+%%{-1, [0,0,4,3,2,3,2,0,2]%%}+%%{-3, [0,0,4,3,2,2,2,0,2]%%}+%%{-3, [0,0,4,3,2,1,2,0,2]%%}+%%{-1, [0,0,4,3,2,0,2,0,2]%%}+%%{-1, [0,0,4,2,2,2,0,2]%%}+%%{-2, [0,0,4,2,2,1,2,0,2]%%}+%%{-1, [0,0,4,2,2,0,2,0,2]%%}+%%{2, [0,0,4,2,1,2,3,1,1]%%}+%%{4, [0,0,4,2,1,1,3,1,1]%%}+%%{2, [0,0,4,2,1,0,3,1,1]%%}+%%{-1, [0,0,4,2,

0,2,4,2,0]%%}+%%{-2,[0,0,4,2,0,1,4,2,0]%%}+%%{-1,[0,0,4,2,0,0,4,2,0]%%} / %%{-1,[0,0,5,3,2,3,3,0,0]%%}+%%{-3,[0,0,5,3,2,2,3,0,0]%%}+%%{-3,[0,0,5,3,2,1,3,0,0]%%}+%%{-1,[0,0,5,3,2,0,3,0,0]%%} Error: Bad Argument Value

**maple** [F] time = 0.72, size = 0, normalized size = 0.00

$$\int (dx^n + c)^2 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p\*(d\*x^n+c)^2,x)

[Out] int((b\*x^n+a)^p\*(d\*x^n+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^2 (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^2,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^2\*(b\*x^n + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (a + bx^n)^p (c + dx^n)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p\*(c + d\*x^n)^2,x)

[Out] int((a + b\*x^n)^p\*(c + d\*x^n)^2, x)

**sympy** [C] time = 42.75, size = 143, normalized size = 0.71

$$\frac{a^p c^2 x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\frac{1}{n}, -p \left| \frac{bx^n e^{i\pi}}{a} \right. \right)}{n \Gamma\left(1 + \frac{1}{n}\right)} + \frac{2a^p c dx x^n \Gamma\left(1 + \frac{1}{n}\right) {}_2F_1\left(-p, 1 + \frac{1}{n} \left| \frac{bx^n e^{i\pi}}{a} \right. \right)}{n \Gamma\left(2 + \frac{1}{n}\right)} + \frac{a^p d^2 x x^{2n} \Gamma\left(2 + \frac{1}{n}\right) {}_2F_1\left(-p, 2 + \frac{1}{n} \left| \frac{bx^n e^{i\pi}}{a} \right. \right)}{n \Gamma\left(3 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*2,x)

[Out] a\*\*p\*c\*\*2\*x\*gamma(1/n)\*hyper((1/n, -p), (1 + 1/n,), b\*x\*\*n\*exp\_polar(I\*pi)/a)/(n\*gamma(1 + 1/n)) + 2\*a\*\*p\*c\*d\*x\*x\*\*n\*gamma(1 + 1/n)\*hyper((-p, 1 + 1/n), (2 + 1/n,), b\*x\*\*n\*exp\_polar(I\*pi)/a)/(n\*gamma(2 + 1/n)) + a\*\*p\*d\*\*2\*x\*x\*\*(2\*n)\*gamma(2 + 1/n)\*hyper((-p, 2 + 1/n), (3 + 1/n,), b\*x\*\*n\*exp\_polar(I\*pi)/a)/(n\*gamma(3 + 1/n))

### 3.315 $\int (a + bx^n)^p (c + dx^n) dx$

**Optimal.** Leaf size=98

$$\frac{dx (a + bx^n)^{p+1}}{b(np + n + 1)} - \frac{x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} (ad - bc(np + n + 1)) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)}{b(np + n + 1)}$$

[Out] d\*x\*(a+b\*x^n)^(1+p)/b/(n\*p+n+1)-(a\*d-b\*c\*(n\*p+n+1))\*x\*(a+b\*x^n)^p\*hypergeom([-p, 1/n], [1+1/n], -b\*x^n/a)/b/(n\*p+n+1)/((1+b\*x^n/a)^p)

**Rubi [A]** time = 0.05, antiderivative size = 89, normalized size of antiderivative = 0.91, number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$ , Rules used = {388, 246, 245}

$$x (a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left(c - \frac{ad}{bnp + bn + b}\right) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + \frac{dx (a + bx^n)^{p+1}}{b(np + n + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n), x]

[Out] (d\*x\*(a + b\*x^n)^(1 + p))/(b\*(1 + n + n\*p)) + ((c - (a\*d)/(b + b\*n + b\*n\*p))\*x\*(a + b\*x^n)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -(b\*x^n)/a])/ (1 + (b\*x^n)/a)^p

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rule 388

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a + b\*x^n)^(p + 1))/(b\*(n\*(p + 1) + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(b\*(n\*(p + 1) + 1)), Int[(a + b\*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)^p (c + dx^n) dx &= \frac{dx (a + bx^n)^{1+p}}{b(1 + n + np)} - \left(-c + \frac{ad}{b + bn + bnp}\right) \int (a + bx^n)^p dx \\ &= \frac{dx (a + bx^n)^{1+p}}{b(1 + n + np)} - \left(\left(-c + \frac{ad}{b + bn + bnp}\right) (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p}\right) \int \left(1 + \frac{bx^n}{a}\right)^p dx \\ &= \frac{dx (a + bx^n)^{1+p}}{b(1 + n + np)} + \left(c - \frac{ad}{b + bn + bnp}\right) x (a + bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 94, normalized size = 0.96

$$\frac{x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} \left( (bc(np + n + 1) - ad) {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) + d(a + bx^n) \left(\frac{bx^n}{a} + 1\right)^p \right)}{b(np + n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n), x]

[Out] (x\*(a + b\*x^n)^p\*(d\*(a + b\*x^n)\*(1 + (b\*x^n)/a)^p + (-a\*d) + b\*c\*(1 + n + n\*p))\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b\*x^n)/a)])/(b\*(1 + n + n\*p)\*(1 + (b\*x^n)/a)^p)

**fricas [F]** time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}((dx^n + c)(bx^n + a)^p, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n), x, algorithm="fricas")

[Out] integral((d\*x^n + c)\*(b\*x^n + a)^p, x)

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{1, [0,0,2,2,1,2,1,0,1]%%}+%%{2, [0,0,2,2,1,1,1,0,1]%%}+%%{1, [0,0,2,2,1,0,1,0,1]%%}+%%{1, [0,0,2,1,1,1,1,0,1]%%}+%%{1, [0,0,2,1,1,0,1,0,1]%%}+%%{-1, [0,0,2,1,0,1,2,1,0]%%}+%%{-1, [0,0,2,1,0,0,2,1,0]%%} / %%{1, [0,0,3,2,1,2,2,0,0]%%}+%%{2, [0,0,3,2,1,1,2,0,0]%%}+%%{1, [0,0,3,2,1,0,2,0,0]%%} Error: Bad Argument Value

**maple [F]** time = 0.60, size = 0, normalized size = 0.00

$$\int (dx^n + c)(bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p\*(d\*x^n+c), x)

[Out] int((b\*x^n+a)^p\*(d\*x^n+c), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)(bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n), x, algorithm="maxima")

[Out] integrate((d\*x^n + c)\*(b\*x^n + a)^p, x)

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int (a + bx^n)^p (c + dx^n) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^n)^p*(c + d*x^n), x)`

[Out] `int((a + b*x^n)^p*(c + d*x^n), x)`

**sympy [C]** time = 6.15, size = 87, normalized size = 0.89

$$\frac{a^p c x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\frac{1}{n}, -p \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)} + \frac{a^p d x x^n \Gamma\left(1 + \frac{1}{n}\right) {}_2F_1\left(-p, 1 + \frac{1}{n} \middle| \frac{b x^n e^{i\pi}}{a}\right)}{n \Gamma\left(2 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**n)**p*(c+d*x**n), x)`

[Out] `a**p*c*x*gamma(1/n)*hyper((1/n, -p), (1 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(1 + 1/n)) + a**p*d*x*x**n*gamma(1 + 1/n)*hyper((-p, 1 + 1/n), (2 + 1/n,), b*x**n*exp_polar(I*pi)/a)/(n*gamma(2 + 1/n))`

### 3.316 $\int (a + bx^n)^p dx$

**Optimal.** Leaf size=46

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)$$

[Out]  $x*(a+b*x^n)^p*\text{hypergeom}([-p, 1/n], [1+1/n], -b*x^n/a)/((1+b*x^n/a)^p)$

**Rubi [A]** time = 0.01, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$ , Rules used = {246, 245}

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p, x]

[Out]  $(x*(a + b*x^n)^p*\text{Hypergeometric2F1}[n^{(-1)}, -p, 1 + n^{(-1)}, -(b*x^n)/a])/((1 + (b*x^n)/a)^p)$

#### Rule 245

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[a^p\*x\*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b\*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

#### Rule 246

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p, x], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)^p dx &= \left( (a + bx^n)^p \left( 1 + \frac{bx^n}{a} \right)^{-p} \right) \int \left( 1 + \frac{bx^n}{a} \right)^p dx \\ &= x(a + bx^n)^p \left( 1 + \frac{bx^n}{a} \right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right) \end{aligned}$$

**Mathematica [A]** time = 0.01, size = 46, normalized size = 1.00

$$x(a + bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{bx^n}{a}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^p, x]

[Out]  $(x*(a + b*x^n)^p*\text{Hypergeometric2F1}[n^{(-1)}, -p, 1 + n^{(-1)}, -(b*x^n)/a])/((1 + (b*x^n)/a)^p)$

**fricas [F]** time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}((bx^n + a)^p, x)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p, x)

**maple** [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p,x)

[Out] int((b\*x^n+a)^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p, x)

**mupad** [B] time = 2.30, size = 47, normalized size = 1.02

$$\frac{x(a + bx^n)^p {}_2F_1\left(\frac{1}{n}, -p; \frac{1}{n} + 1; -\frac{bx^n}{a}\right)}{\left(\frac{bx^n}{a} + 1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p,x)

[Out] (x\*(a + b\*x^n)^p\*hypergeom([1/n, -p], 1/n + 1, -(b\*x^n)/a))/((b\*x^n)/a + 1)^p

**sympy** [C] time = 1.90, size = 37, normalized size = 0.80

$$\frac{a^p x \Gamma\left(\frac{1}{n}\right) {}_2F_1\left(\frac{1}{n}, -p \middle| \frac{bx^n e^{i\pi}}{a}\right)}{n \Gamma\left(1 + \frac{1}{n}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p,x)

[Out] a\*\*p\*x\*gamma(1/n)\*hyper((1/n, -p), (1 + 1/n, ), b\*x\*\*n\*exp\_polar(I\*pi)/a)/(n\*gamma(1 + 1/n))

$$3.317 \quad \int \frac{(a+bx^n)^p}{c+dx^n} dx$$

**Optimal.** Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c}$$

[Out] x\*(a+b\*x^n)^p\*AppellF1(1/n,-p,1,1+1/n,-b\*x^n/a,-d\*x^n/c)/c/((1+b\*x^n/a)^p)

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n), x]

[Out] (x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 1, 1 + n^(-1), -(b\*x^n)/a, -(d\*x^n)/c])/((c\*(1 + (b\*x^n)/a)^p)

**Rule 429**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -(b\*x^n)/a, -(d\*x^n)/c], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

**Rule 430**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

**Rubi steps**

$$\begin{aligned} \int \frac{(a+bx^n)^p}{c+dx^n} dx &= \left( (a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \right) \int \frac{\left(1 + \frac{bx^n}{a}\right)^p}{c+dx^n} dx \\ &= \frac{x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c} \end{aligned}$$

**Mathematica [B]** time = 0.31, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n) \left( bcnp x^n F_1\left(1 + \frac{1}{n}; 1-p, 1; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - adnx^n F_1\left(1 + \frac{1}{n}; -p, 2; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1)F_1\left(\frac{1}{n}; -p, 1; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n), x]

```
[Out] (a*c*(1+n)*x*(a+b*x^n)^p*AppellF1[n^(-1), -p, 1, 1+n^(-1), -((b*x^n)/a), -((d*x^n)/c)]/((c+d*x^n)*(b*c*n*p*x^n*AppellF1[1+n^(-1), 1-p, 1, 2+n^(-1), -((b*x^n)/a), -((d*x^n)/c)] - a*d*n*x^n*AppellF1[1+n^(-1), -p, 2, 2+n^(-1), -((b*x^n)/a), -((d*x^n)/c)] + a*c*(1+n)*AppellF1[n^(-1), -p, 1, 1+n^(-1), -((b*x^n)/a), -((d*x^n)/c)]))
```

**fricas** [F] time = 1.07, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n + a)^p}{dx^n + c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n), x, algorithm="fricas")
```

```
[Out] integral((b*x^n + a)^p/(d*x^n + c), x)
```

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n), x, algorithm="giac")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

**maple** [F] time = 0.91, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^n+a)^p/(d*x^n+c), x)
```

```
[Out] int((b*x^n+a)^p/(d*x^n+c), x)
```

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{dx^n + c} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x^n)^p/(c+d*x^n), x, algorithm="maxima")
```

```
[Out] integrate((b*x^n + a)^p/(d*x^n + c), x)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + bx^n)^p}{c + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^p/(c + d*x^n), x)
```

```
[Out] int((a + b*x^n)^p/(c + d*x^n), x)
```

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n), x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.318 \quad \int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx$$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^2}$$

[Out] x\*(a+b\*x^n)^p\*AppellF1(1/n,-p,2,1+1/n,-b\*x^n/a,-d\*x^n/c)/c^2/((1+b\*x^n/a)^p)

**Rubi [A]** time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n)^2,x]

[Out] (x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)])/(c^2\*(1 + (b\*x^n)/a)^p)

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^n)^p}{(c+dx^n)^2} dx &= \left( (a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \right) \int \frac{\left(1 + \frac{bx^n}{a}\right)^p}{(c+dx^n)^2} dx \\ &= \frac{x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^2} \end{aligned}$$

**Mathematica [B]** time = 0.34, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 2; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n)^2 \left( bcnp x^n F_1\left(1 + \frac{1}{n}; 1-p, 2; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - 2adnx^n F_1\left(1 + \frac{1}{n}; -p, 3; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) + ac(n+1) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n)^2,x]

[Out] (a\*c\*(1 + n)\*x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)]/((c + d\*x^n)^2\*(b\*c\*n\*p\*x^n\*AppellF1[1 + n^(-1), 1 - p, 2, 2 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)] - 2\*a\*d\*n\*x^n\*AppellF1[1 + n^(-1), -p, 3, 2 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)] + a\*c\*(1 + n)\*AppellF1[n^(-1), -p, 2, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)]))

**fricas** [F] time = 0.90, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^2x^{2n} + 2cdx^n + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^2,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d^2\*x^(2\*n) + 2\*c\*d\*x^n + c^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^2,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^2, x)

**maple** [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p/(d\*x^n+c)^2,x)

[Out] int((b\*x^n+a)^p/(d\*x^n+c)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^2,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p/(c + d\*x^n)^2,x)

[Out] int((a + b\*x^n)^p/(c + d\*x^n)^2, x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p/(c+d\*x\*\*n)\*\*2,x)

[Out] Exception raised: HeuristicGCDFailed

$$3.319 \quad \int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx$$

Optimal. Leaf size=59

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^3}$$

[Out] x\*(a+b\*x^n)^p\*AppellF1(1/n,-p,3,1+1/n,-b\*x^n/a,-d\*x^n/c)/c^3/((1+b\*x^n/a)^p)

Rubi [A] time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$ , Rules used = {430, 429}

$$\frac{x(a+bx^n)^p \left(\frac{bx^n}{a} + 1\right)^{-p} F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p/(c + d\*x^n)^3,x]

[Out] (x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)]/(c^3\*(1 + (b\*x^n)/a)^p)

Rule 429

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[a^p\*c^q\*x\*AppellF1[1/n, -p, -q, 1 + 1/n, -((b\*x^n)/a), -((d\*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])

Rule 430

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Dist[(a^IntPart[p]\*(a + b\*x^n)^FracPart[p])/(1 + (b\*x^n)/a)^FracPart[p], Int[(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \frac{(a+bx^n)^p}{(c+dx^n)^3} dx &= \left( (a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} \right) \int \frac{\left(1 + \frac{bx^n}{a}\right)^p}{(c+dx^n)^3} dx \\ &= \frac{x(a+bx^n)^p \left(1 + \frac{bx^n}{a}\right)^{-p} F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{c^3} \end{aligned}$$

Mathematica [B] time = 0.65, size = 180, normalized size = 3.05

$$\frac{ac(n+1)x(a+bx^n)^p F_1\left(\frac{1}{n}; -p, 3; 1 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right)}{(c+dx^n)^3 \left( bcnp x^n F_1\left(1 + \frac{1}{n}; 1-p, 3; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) - 3adn x^n F_1\left(1 + \frac{1}{n}; -p, 4; 2 + \frac{1}{n}; -\frac{bx^n}{a}, -\frac{dx^n}{c}\right) \right) + ac(n+1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p/(c + d\*x^n)^3,x]

[Out] (a\*c\*(1 + n)\*x\*(a + b\*x^n)^p\*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)]/((c + d\*x^n)^3\*(b\*c\*n\*p\*x^n\*AppellF1[1 + n^(-1), 1 - p, 3, 2 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)] - 3\*a\*d\*n\*x^n\*AppellF1[1 + n^(-1), -p, 4, 2 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)] + a\*c\*(1 + n)\*AppellF1[n^(-1), -p, 3, 1 + n^(-1), -((b\*x^n)/a), -((d\*x^n)/c)]))

**fricas** [F] time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n + a)^p}{d^3x^{3n} + 3cd^2x^{2n} + 3c^2dx^n + c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^3,x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d^3\*x^(3\*n) + 3\*c\*d^2\*x^(2\*n) + 3\*c^2\*d\*x^n + c^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^3,x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^3, x)

**maple** [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p/(d\*x^n+c)^3,x)

[Out] int((b\*x^n+a)^p/(d\*x^n+c)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(bx^n + a)^p}{(dx^n + c)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p/(c+d\*x^n)^3,x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p/(d\*x^n + c)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p/(c + d\*x^n)^3,x)

[Out] int((a + b\*x^n)^p/(c + d\*x^n)^3, x)



sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**p/(c+d*x**n)**3,x)
```

```
[Out] Integral((a + b*x**n)**p/(c + d*x**n)**3, x)
```

$$3.320 \quad \int (a + bx^n)^p (c + dx^n)^{-1-\frac{1}{n}-p} dx$$

**Optimal.** Leaf size=93

$$\frac{x (a + bx^n)^p (c + dx^n)^{-\frac{1}{n}-p} \left( \frac{c(ax^n)}{a(c+dx^n)} \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)} \right)}{c}$$

[Out] x\*(a+b\*x^n)^p\*(c+d\*x^n)^(-1/n-p)\*hypergeom([-p, 1/n], [1+1/n], -(a\*d+b\*c)\*x^n/a/(c+d\*x^n))/c/((c\*(a+b\*x^n)/a/(c+d\*x^n))^p)

**Rubi [A]** time = 0.02, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$ , Rules used = {380}

$$\frac{x (a + bx^n)^p (c + dx^n)^{-\frac{1}{n}-p} \left( \frac{c(ax^n)}{a(c+dx^n)} \right)^{-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)} \right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^(-1 - n^(-1) - p), x]

[Out] (x\*(a + b\*x^n)^p\*(c + d\*x^n)^(-n^(-1) - p)\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p)

**Rule 380**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

**Rubi steps**

$$\int (a + bx^n)^p (c + dx^n)^{-1-\frac{1}{n}-p} dx = \frac{x (a + bx^n)^p \left( \frac{c(ax^n)}{a(c+dx^n)} \right)^{-p} (c + dx^n)^{-\frac{1}{n}-p} {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)} \right)}{c}$$

**Mathematica [A]** time = 0.09, size = 94, normalized size = 1.01

$$\frac{x (a + bx^n)^p \left( \frac{bx^n}{a} + 1 \right)^{-p} (c + dx^n)^{-\frac{np+1}{n}} \left( \frac{dx^n}{c} + 1 \right)^p {}_2F_1 \left( \frac{1}{n}, -p; 1 + \frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)} \right)}{c}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^(-1 - n^(-1) - p), x]

[Out] (x\*(a + b\*x^n)^p\*(1 + (d\*x^n)/c)^p\*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), ((-b\*c) + a\*d)\*x^n/(a\*(c + d\*x^n))])/(c\*(1 + (b\*x^n)/a)^p\*(c + d\*x^n)^(1 + n\*p)/n)

**fricas [F]** time = 1.10, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+n+1}{n}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-1-1/n-p),x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d\*x^n + c)^((n\*p + n + 1)/n), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^{-p-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-1-1/n-p),x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x)

maple [F] time = 0.91, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^{-p-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p\*(d\*x^n+c)^(-1-1/n-p),x)

[Out] int((b\*x^n+a)^p\*(d\*x^n+c)^(-1-1/n-p),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p(dx^n + c)^{-p-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-1-1/n-p),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + bx^n)^p}{(c + dx^n)^{p+\frac{1}{n}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p/(c + d\*x^n)^(p + 1/n + 1),x)

[Out] int((a + b\*x^n)^p/(c + d\*x^n)^(p + 1/n + 1), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*(-1-1/n-p),x)

[Out] Timed out

$$3.321 \quad \int (a + bx^n)^3 (c + dx^n)^{-4-\frac{1}{n}} dx$$

**Optimal.** Leaf size=178

$$\frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)} + \frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)} + \frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)} + \frac{x(a+bx^n)^3(c+dx^n)^{-4-\frac{1}{n}}}{c(3n+1)}$$

[Out]  $x*(a+b*x^n)^3*(c+d*x^n)^{-3-1/n}/c/(1+3*n)+3*a*n*x*(a+b*x^n)^2*(c+d*x^n)^{-2-1/n}/c^2/(6*n^2+5*n+1)+6*a^2*n^2*x*(a+b*x^n)*(c+d*x^n)^{-1-1/n}/c^3/(6*n^2+11*n^2+6*n+1)+6*a^3*n^3*x/c^4/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^{(1/n)})$

**Rubi [A]** time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {378, 191}

$$\frac{6a^2n^2x(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^3(n+1)(2n+1)(3n+1)} + \frac{6a^3n^3x(c+dx^n)^{-1/n}}{c^4(n+1)(2n+1)(3n+1)} + \frac{3anx(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c^2(6n^2+5n+1)} + \frac{x(a+bx^n)^3(c+dx^n)^{-4-\frac{1}{n}}}{c(3n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^3\*(c + d\*x^n)^(-4 - n^(-1)), x]

[Out]  $(x*(a + b*x^n)^3*(c + d*x^n)^{-3 - n^(-1)})/(c*(1 + 3*n)) + (3*a*n*x*(a + b*x^n)^2*(c + d*x^n)^{-2 - n^(-1)})/(c^2*(1 + 5*n + 6*n^2)) + (6*a^2*n^2*x*(a + b*x^n)*(c + d*x^n)^{-1 - n^(-1)})/(c^3*(1 + n)*(1 + 2*n)*(1 + 3*n)) + (6*a^3*n^3*x)/(c^4*(1 + n)*(1 + 2*n)*(1 + 3*n)*(c + d*x^n)^{n^(-1)})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_.), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int (a + bx^n)^3 (c + dx^n)^{-4-\frac{1}{n}} dx &= \frac{x(a + bx^n)^3 (c + dx^n)^{-3-\frac{1}{n}}}{c(1 + 3n)} + \frac{(3an) \int (a + bx^n)^2 (c + dx^n)^{-3-\frac{1}{n}} dx}{c(1 + 3n)} \\ &= \frac{x(a + bx^n)^3 (c + dx^n)^{-3-\frac{1}{n}}}{c(1 + 3n)} + \frac{3anx(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c^2(1 + 5n + 6n^2)} + \frac{(6a^2n^2) \int (a + bx^n) (c + dx^n)^{-2-\frac{1}{n}} dx}{c^2(1 + 5n + 6n^2)} \\ &= \frac{x(a + bx^n)^3 (c + dx^n)^{-3-\frac{1}{n}}}{c(1 + 3n)} + \frac{3anx(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c^2(1 + 5n + 6n^2)} + \frac{6a^2n^2x(a + bx^n) (c + dx^n)^{-1-\frac{1}{n}}}{c^3(1 + n)(1 + 2n)(1 + 3n)} \\ &= \frac{x(a + bx^n)^3 (c + dx^n)^{-3-\frac{1}{n}}}{c(1 + 3n)} + \frac{3anx(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c^2(1 + 5n + 6n^2)} + \frac{6a^2n^2x(a + bx^n) (c + dx^n)^{-1-\frac{1}{n}}}{c^3(1 + n)(1 + 2n)(1 + 3n)} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 218, normalized size = 1.22

$$\frac{x(c + dx^n)^{-\frac{1}{n}-3} \left( a^3 \left( c^3 (6n^3 + 11n^2 + 6n + 1) + 3c^2 dn (6n^2 + 5n + 1) x^n + 6cd^2 n^2 (3n + 1) x^{2n} + 6d^3 n^3 x^{3n} \right) + 3c^4(n + 1) \right)}{c^4(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^3\*(c + d\*x^n)^(-4 - n^(-1)), x]

[Out] (x\*(c + d\*x^n)^(-3 - n^(-1))\*(b^3\*c^3\*(1 + 3\*n + 2\*n^2)\*x^(3\*n) + 3\*a\*b^2\*c^2\*(1 + n)\*x^(2\*n)\*(c + 3\*c\*n + d\*n\*x^n) + 3\*a^2\*b\*c\*x^n\*(c^2\*(1 + 5\*n + 6\*n^2) + 2\*c\*d\*n\*(1 + 3\*n)\*x^n + 2\*d^2\*n^2\*x^(2\*n)) + a^3\*(c^3\*(1 + 6\*n + 11\*n^2 + 6\*n^3) + 3\*c^2\*d\*n\*(1 + 5\*n + 6\*n^2)\*x^n + 6\*c\*d^2\*n^2\*(1 + 3\*n)\*x^(2\*n) + 6\*d^3\*n^3\*x^(3\*n)))/(c^4\*(1 + n)\*(1 + 2\*n)\*(1 + 3\*n))

**fricas [B]** time = 0.80, size = 478, normalized size = 2.69

$$\frac{(6a^3d^4n^3 + b^3c^3d + (2b^3c^3d + 3ab^2c^2d^2 + 6a^2bcd^3)n^2 + 3(b^3c^3d + ab^2c^2d^2)n)xx^{4n} + (24a^3cd^3n^3 + b^3c^4 + 3c^4(n + 1))}{c^4(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^3\*(c+d\*x^n)^(-4-1/n), x, algorithm="fricas")

[Out] ((6\*a^3\*d^4\*n^3 + b^3\*c^3\*d + (2\*b^3\*c^3\*d + 3\*a\*b^2\*c^2\*d^2 + 6\*a^2\*b\*c\*d^3)\*n^2 + 3\*(b^3\*c^3\*d + a\*b^2\*c^2\*d^2)\*n)\*x\*x^(4\*n) + (24\*a^3\*c\*d^3\*n^3 + b^3\*c^4 + 3\*a\*b^2\*c^3\*d + 2\*(b^3\*c^4 + 6\*a\*b^2\*c^3\*d + 12\*a^2\*b\*c^2\*d^2 + 3\*a^3\*c\*d^3)\*n^2 + 3\*(b^3\*c^4 + 5\*a\*b^2\*c^3\*d + 2\*a^2\*b\*c^2\*d^2)\*n)\*x\*x^(3\*n) + 3\*(12\*a^3\*c^2\*d^2\*n^3 + a\*b^2\*c^4 + a^2\*b\*c^3\*d + (3\*a\*b^2\*c^4 + 12\*a^2\*b\*c^3\*d + 7\*a^3\*c^2\*d^2)\*n^2 + (4\*a\*b^2\*c^4 + 7\*a^2\*b\*c^3\*d + a^3\*c^2\*d^2)\*n)\*x\*x^(2\*n) + (24\*a^3\*c^3\*d\*n^3 + 3\*a^2\*b\*c^4 + a^3\*c^3\*d + 2\*(9\*a^2\*b\*c^4 + 13\*a^3\*c^3\*d)\*n^2 + 3\*(5\*a^2\*b\*c^4 + 3\*a^3\*c^3\*d)\*n)\*x\*x^n + (6\*a^3\*c^4\*n^3 + 11\*a^3\*c^4\*n^2 + 6\*a^3\*c^4\*n + a^3\*c^4)\*x)/((6\*c^4\*n^3 + 11\*c^4\*n^2 + 6\*c^4\*n + c^4)\*(d\*x^n + c)^((4\*n + 1)/n))

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^3\*(c+d\*x^n)^(-4-1/n), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error%%{81, [2,0,6,4,2,4,3,0]%%}+%%{108, [2,0,6,3,2,4,3,0]%%}+%%{54, [2,0,6,2,2,4,3,0]%%}+%%{12, [2,0,6,1,2,4,3,0]%%}+%%{1, [2,0,6,0,2,4,3,0]%%}+%%{243, [1,0,6,4,2,4,2,1]%%}+%%{-81, [1,0,6,4,1,5,3,0]%%}+%%{324, [1,0,6,3,2,4,2,1]%%}+%%{-108, [1,0,6,3,1,5,3,0]%%}+%%{162, [1,0,6,2,2,4,2,1]%%}+%%{-54, [1,0,6,2,1,5,3,0]%%}+%%{36, [1,0,6,1,2,4,2,1]%%}+%%{-12, [1,0,6,1,1,5,3,0]%%}+%%{3, [1,0,6,0,2,4,2,1]%%}+%%{-1, [1,0,6,0,1,5,3,0]%%}+%%{81, [0,0,6,4,3,3,0,3]%%}+%%{81, [0,0,6,3,3,3,0,3]%%}+%%{81, [0,0,6,3,2,4,1,2]%%}+%%{-81, [0,0,6,3,1,5,2,1]%%}+%%{27, [0,0,6,3,0,6,3,0]%%}+%%{27, [0,0,6,2,3,3,0,3]%%}+%%{81, [0,0,6,2,2,4,1,2]%%}+%%{-81, [0,0,6,2,1,5,2,1]%%}+%%{27, [0,0,6,2,0,6,3,0]%%}+%%{3, [0,0,6,1,3,3,0,3]%%}+%%{27, [0,0,6,1,2,4,1,2]%%}+%%{-27, [0,0,6,1,1,5,2,1]%%}+%%{9, [0,0,6,1,0,6,3,0]%%}+%%{3, [0,0,6,0,2,4,1,2]%%}+%%{-3, [0,0,6,0,1,5,2,1]%%}+%%{1, [0,0,6,0,0,6,3,0]%%} / %%{81, [0,0,7,4,3,4,0,0]%%}+%%{108, [0,0,7,3,3,4,0,0]%%}+%%{54, [0,0,7,2,3,4,0,0]%%}+%%{12, [0,0,7,1,3,4,0,0]%%}+%%{1, [0,0,7,0,3,4,0,0]%%} Error: Bad Argument Value

**maple** [F] time = 0.83, size = 0, normalized size = 0.00

$$\int (bx^n + a)^3 (dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^3\*(d\*x^n+c)^(-1/n-4),x)

[Out] int((b\*x^n+a)^3\*(d\*x^n+c)^(-1/n-4),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^3 (dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^3\*(c+d\*x^n)^(-4-1/n),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^3\*(d\*x^n + c)^(-1/n - 4), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + bx^n)^3}{(c + dx^n)^{\frac{1}{n}+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^3/(c + d\*x^n)^(1/n + 4),x)

[Out] int((a + b\*x^n)^3/(c + d\*x^n)^(1/n + 4), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*3\*(c+d\*x\*\*n)\*\*(-4-1/n),x)

[Out] Timed out

### 3.322 $\int (a + bx^n)^2 (c + dx^n)^{-3-\frac{1}{n}} dx$

**Optimal.** Leaf size=116

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

[Out]  $x*(a+b*x^n)^2*(c+d*x^n)^{(-2-1/n)/c/(1+2*n)+2*a*n*x*(a+b*x^n)*(c+d*x^n)^{(-1-1/n)/c^2/(2*n^2+3*n+1)+2*a^2*n^2*x/c^3/(2*n^2+3*n+1)/((c+d*x^n)^{(1/n))}$

**Rubi [A]** time = 0.04, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {378, 191}

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}}{c^3(n+1)(2n+1)} + \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}}{c^2(n+1)(2n+1)} + \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}}{c(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out]  $(x*(a + b*x^n)^2*(c + d*x^n)^{(-2 - n^(-1))}/(c*(1 + 2*n)) + (2*a*n*x*(a + b*x^n)*(c + d*x^n)^{(-1 - n^(-1))}/(c^2*(1 + n)*(1 + 2*n)) + (2*a^2*n^2*x)/(c^3*(1 + n)*(1 + 2*n)*(c + d*x^n)^{n^(-1)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)^2 (c + dx^n)^{-3-\frac{1}{n}} dx &= \frac{x(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c(1 + 2n)} + \frac{(2an) \int (a + bx^n) (c + dx^n)^{-2-\frac{1}{n}} dx}{c(1 + 2n)} \\ &= \frac{x(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c(1 + 2n)} + \frac{2anx(a + bx^n)(c + dx^n)^{-1-\frac{1}{n}}}{c^2(1 + n)(1 + 2n)} + \frac{(2a^2n^2) \int (c + dx^n)^{-1-\frac{1}{n}} dx}{c^2(1 + n)} \\ &= \frac{x(a + bx^n)^2 (c + dx^n)^{-2-\frac{1}{n}}}{c(1 + 2n)} + \frac{2anx(a + bx^n)(c + dx^n)^{-1-\frac{1}{n}}}{c^2(1 + n)(1 + 2n)} + \frac{2a^2n^2x(c + dx^n)^{-\frac{1}{n}-2}}{c^3(1 + n)(1 + 2n)} \end{aligned}$$

**Mathematica [A]** time = 0.08, size = 113, normalized size = 0.97

$$\frac{x(c+dx^n)^{-\frac{1}{n}-2} \left( a^2 \left( c^2 (2n^2 + 3n + 1) + 2cdn(2n + 1)x^n + 2d^2n^2x^{2n} \right) + 2abcx^n (2cn + c + dnx^n) + b^2c^2(n + 1) \right)}{c^3(n + 1)(2n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^2\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out] (x\*(c + d\*x^n)^(-2 - n^(-1))\*(b^2\*c^2\*(1 + n)\*x^(2\*n) + 2\*a\*b\*c\*x^n\*(c + 2\*c\*n + d\*n\*x^n) + a^2\*(c^2\*(1 + 3\*n + 2\*n^2) + 2\*c\*d\*n\*(1 + 2\*n)\*x^n + 2\*d^2\*n^2\*x^(2\*n))))/(c^3\*(1 + n)\*(1 + 2\*n))

**fricas** [A] time = 1.23, size = 231, normalized size = 1.99

$$\frac{(2a^2d^3n^2 + b^2c^2d + (b^2c^2d + 2abcd^2)n)xx^{3n} + (6a^2cd^2n^2 + b^2c^3 + 2abc^2d + (b^2c^3 + 6abc^2d + 2a^2cd^2)n)xx^{2n}}{(2c^3n^2 + 3c^3n + c^3)(dx^n + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n)^(-3-1/n), x, algorithm="fricas")

[Out] ((2\*a^2\*d^3\*n^2 + b^2\*c^2\*d + (b^2\*c^2\*d + 2\*a\*b\*c\*d^2)\*n)\*x\*x^(3\*n) + (6\*a^2\*c\*d^2\*n^2 + b^2\*c^3 + 2\*a\*b\*c^2\*d + (b^2\*c^3 + 6\*a\*b\*c^2\*d + 2\*a^2\*c\*d^2)\*n)\*x\*x^(2\*n) + (6\*a^2\*c^2\*d\*n^2 + 2\*a\*b\*c^3 + a^2\*c^2\*d + (4\*a\*b\*c^3 + 5\*a^2\*c^2\*d)\*n)\*x\*x^n + (2\*a^2\*c^3\*n^2 + 3\*a^2\*c^3\*n + a^2\*c^3)\*x)/((2\*c^3\*n^2 + 3\*c^3\*n + c^3)\*(d\*x^n + c)^((3\*n + 1)/n))

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n)^(-3-1/n), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{8, [1,0,4,3,1,3,2,0]%%}+%%{12, [1,0,4,2,1,3,2,0]%%}+%%{6, [1,0,4,1,1,3,2,0]%%}+%%{1, [1,0,4,0,1,3,2,0]%%}+%%{8, [0,0,4,3,2,2,0,2]%%}+%%{8, [0,0,4,2,2,2,0,2]%%}+%%{8, [0,0,4,2,1,3,1,1]%%}+%%{-4, [0,0,4,2,0,4,2,0]%%}+%%{2, [0,0,4,1,2,2,0,2]%%}+%%{8, [0,0,4,1,1,3,1,1]%%}+%%{-4, [0,0,4,1,0,4,2,0]%%}+%%{2, [0,0,4,0,1,3,1,1]%%}+%%{-1, [0,0,4,0,0,4,2,0]%%} / %%{8, [0,0,5,3,2,3,0,0]%%}+%%{12, [0,0,5,2,2,3,0,0]%%}+%%{6, [0,0,5,1,2,3,0,0]%%}+%%{1, [0,0,5,0,2,3,0,0]%%} Error: Bad Argument Value

**maple** [F] time = 0.80, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2\*(d\*x^n+c)^(-1/n-3), x)

[Out] int((b\*x^n+a)^2\*(d\*x^n+c)^(-1/n-3), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n)^(-3-1/n), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^{\frac{1}{n}+3}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

```
[Out] int((a + b*x^n)^2/(c + d*x^n)^(1/n + 3), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**2*(c+d*x**n)**(-3-1/n), x)
```

```
[Out] Timed out
```

$$3.323 \quad \int (a + bx^n)(c + dx^n)^{-2-\frac{1}{n}} dx$$

**Optimal.** Leaf size=58

$$\frac{x(a + bx^n)(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)} + \frac{anx(c + dx^n)^{-1/n}}{c^2(n+1)}$$

[Out]  $x*(a+b*x^n)*(c+d*x^n)^{-1-1/n}/c/(1+n)+a*n*x/c^2/(1+n)/((c+d*x^n)^{1/n})$

**Rubi [A]** time = 0.01, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {378, 191}

$$\frac{x(a + bx^n)(c + dx^n)^{-\frac{1}{n}-1}}{c(n+1)} + \frac{anx(c + dx^n)^{-1/n}}{c^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(a + b*x^n)*(c + d*x^n)^{-1 - n^{-1}})/(c*(1 + n)) + (a*n*x)/(c^2*(1 + n)*(c + d*x^n)^{n^{-1}})$

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rule 378**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int (a + bx^n)(c + dx^n)^{-2-\frac{1}{n}} dx &= \frac{x(a + bx^n)(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{(an) \int (c + dx^n)^{-1-\frac{1}{n}} dx}{c(1+n)} \\ &= \frac{x(a + bx^n)(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{anx(c + dx^n)^{-1/n}}{c^2(1+n)} \end{aligned}$$

**Mathematica [C]** time = 0.14, size = 82, normalized size = 1.41

$$\frac{x(c + dx^n)^{-\frac{n+1}{n}} \left( a(n+1)(c + dx^n) \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} {}_2F_1 \left( 2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + bcx^n \right)}{c^2(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(b*c*x^n + a*(1 + n)*(c + d*x^n)*(1 + (d*x^n)/c)^{n^{-1}}*Hypergeometric2F1[2 + n^{-1}, n^{-1}, 1 + n^{-1}, -((d*x^n)/c)]))/(c^2*(1 + n)*(c + d*x^n)^{((1 + n)/n)})$

**fricas** [A] time = 0.83, size = 85, normalized size = 1.47

$$\frac{(ad^2n + bcd)xx^{2n} + (2acd + bc^2 + acd)xx^n + (ac^2n + ac^2)x}{(c^2n + c^2)(dx^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-2-1/n),x, algorithm="fricas")

[Out] ((a\*d^2\*n + b\*c\*d)\*x\*x^(2\*n) + (2\*a\*c\*d\*n + b\*c^2 + a\*c\*d)\*x\*x^n + (a\*c^2\*n + a\*c^2)\*x)/((c^2\*n + c^2)\*(d\*x^n + c)^((2\*n + 1)/n))

**giac** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-2-1/n),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{1, [0,0,2,2,1,1,0,1]%%}+%%{1, [0,0,2,1,1,1,0,1]%%}+%%{1, [0,0,2,1,0,2,1,0]%%}+%%{1, [0,0,2,0,0,2,1,0]%%} / %%{1, [0,0,3,2,1,2,0,0]%%}+%%{2, [0,0,3,1,1,2,0,0]%%}+%%{1, [0,0,3,0,1,2,0,0]%%} Error: Bad Argument Value

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(d\*x^n+c)^(-1/n-2),x)

[Out] int((b\*x^n+a)\*(d\*x^n+c)^(-1/n-2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-2-1/n),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)\*(d\*x^n + c)^(-1/n - 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + bx^n}{(c + dx^n)^{\frac{1}{n}+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n)^(1/n + 2),x)

[Out] int((a + b\*x^n)/(c + d\*x^n)^(1/n + 2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)*(c+d*x**n)**(-2-1/n),x)
```

```
[Out] Timed out
```

$$3.324 \quad \int (c + dx^n)^{-1-\frac{1}{n}} dx$$

**Optimal.** Leaf size=18

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

[Out] x/c/((c+d\*x^n)^(1/n))

**Rubi [A]** time = 0.00, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$ , Rules used = {191}

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(-1 - n^(-1)), x]

[Out] x/(c\*(c + d\*x^n)^n^(-1))

**Rule 191**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

**Rubi steps**

$$\int (c + dx^n)^{-1-\frac{1}{n}} dx = \frac{x(c + dx^n)^{-1/n}}{c}$$

**Mathematica [A]** time = 0.04, size = 18, normalized size = 1.00

$$\frac{x(c + dx^n)^{-1/n}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-1 - n^(-1)), x]

[Out] x/(c\*(c + d\*x^n)^n^(-1))

**fricas [A]** time = 1.00, size = 31, normalized size = 1.72

$$\frac{dx^n + cx}{(dx^n + c)^{\frac{n+1}{n}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n), x, algorithm="fricas")

[Out] (d\*x\*x^n + c\*x)/((d\*x^n + c)^((n + 1)/n)\*c)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n),x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n - 1), x)

**maple [B]** time = 0.07, size = 53, normalized size = 2.94

$$\frac{dx e^{n \ln(x)} e^{\left(-\frac{1}{n}-1\right) \ln(d e^{n \ln(x)+c})}}{c} + x e^{\left(-\frac{1}{n}-1\right) \ln(d e^{n \ln(x)+c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n-1),x)

[Out] x\*exp((-1/n-1)\*ln(c+d\*exp(n\*ln(x))))+1/c\*d\*x\*exp(n\*ln(x))\*exp((-1/n-1)\*ln(c+d\*exp(n\*ln(x))))

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^n + c)^{-\frac{1}{n}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n),x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n - 1), x)

**mupad [B]** time = 1.76, size = 75, normalized size = 4.17

$$\frac{dx^{n+1} \left( \frac{c}{dx^n} - \left( \frac{c}{dx^n} + 1 \right)^{\frac{n+1}{n}} + 1 \right)}{cn \left( \frac{n+1}{n} - 1 \right) (c + dx^n)^{\frac{n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + d\*x^n)^(1/n + 1),x)

[Out] (d\*x^(n + 1)\*(c/(d\*x^n) - (c/(d\*x^n) + 1)^((n + 1)/n) + 1))/(c\*n\*((n + 1)/n - 1)\*(c + d\*x^n)^((n + 1)/n))

**sympy [A]** time = 33.05, size = 211, normalized size = 11.72

$$\left\{ \begin{array}{l} -\frac{d^{-\frac{1}{n}} x x^{-n} (x^n)^{-\frac{1}{n}}}{dn} \\ 0^{-1-\frac{1}{n}} x \\ x (0^n)^{-1-\frac{1}{n}} \end{array} \right. \begin{array}{l} \text{for } c = 0 \\ \text{for } c = - \\ \text{for } c = 0 \\ \text{otherwise} \end{array}$$

$$\frac{c^2 x}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{cdx^n}{c^3 (c+dx^n)^{\frac{1}{n}} + 2c^2 dx^n (c+dx^n)^{\frac{1}{n}} + cd^2 x^{2n} (c+dx^n)^{\frac{1}{n}}} + \frac{dx^n}{c^2 (c+dx^n)^{\frac{1}{n}} + cdx^n (c+dx^n)^{\frac{1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(-1-1/n),x)

[Out] Piecewise((-d\*\*(-1/n)\*x\*x\*\*(-n)\*(x\*\*n)\*\*(-1/n)/(d\*n), Eq(c, 0)), (0\*\*(-1 - 1/n)\*x, Eq(c, -d\*x\*\*n)), (x\*(0\*\*n)\*\*(-1 - 1/n), Eq(c, 0\*\*n - d\*x\*\*n)), (c\*\*2\*x/(c\*\*3\*(c + d\*x\*\*n)\*\*(1/n) + 2\*c\*\*2\*d\*x\*\*n\*(c + d\*x\*\*n)\*\*(1/n) + c\*d\*\*2\*x\*\*(2\*n)\*(c + d\*x\*\*n)\*\*(1/n)) + c\*d\*x\*x\*\*n/(c\*\*3\*(c + d\*x\*\*n)\*\*(1/n) + 2\*c\*\*2\*d\*x\*\*n\*(c + d\*x\*\*n)\*\*(1/n) + c\*d\*\*2\*x\*\*(2\*n)\*(c + d\*x\*\*n)\*\*(1/n)) + d\*x\*x\*\*n/(c\*\*2\*(c + d\*x\*\*n)\*\*(1/n) + c\*d\*x\*\*n\*(c + d\*x\*\*n)\*\*(1/n)), True))

$$3.325 \quad \int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx$$

Optimal. Leaf size=53

$$\frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

[Out] x\*hypergeom([1, 1/n], [1+1/n], -(-a\*d+b\*c)\*x^n/a/(c+d\*x^n))/a/((c+d\*x^n)^(1/n))

**Rubi [A]** time = 0.01, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$ , Rules used = {379}

$$\frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)\*(c + d\*x^n)^n^(-1)), x]

[Out] (x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(a\*(c + d\*x^n)^n^(-1))

Rule 379

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

Rubi steps

$$\int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx = \frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a}$$

**Mathematica [A]** time = 0.01, size = 52, normalized size = 0.98

$$\frac{x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b\*x^n)\*(c + d\*x^n)^n^(-1)), x]

[Out] (x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), ((-b\*c) + a\*d)\*x^n/(a\*(c + d\*x^n))])/(a\*(c + d\*x^n)^n^(-1))

**fricas [F]** time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/((c+d\*x^n)^(1/n)),x, algorithm="fricas")

[Out] integral(1/((b\*x^n + a)\*(d\*x^n + c)^(1/n)), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/((c+d\*x^n)^(1/n)),x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^(1/n)), x)

**maple** [F] time = 0.98, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b\*x^n+a)/((d\*x^n+c)^(1/n)),x)

[Out] int(1/(b\*x^n+a)/((d\*x^n+c)^(1/n)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)(dx^n + c)^{\left(\frac{1}{n}\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)/((c+d\*x^n)^(1/n)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^n + a)\*(d\*x^n + c)^(1/n)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + bx^n)(c + dx^n)^{1/n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)\*(c + d\*x^n)^(1/n)),x)

[Out] int(1/((a + b\*x^n)\*(c + d\*x^n)^(1/n)), x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x\*\*n)/((c+d\*x\*\*n)\*\*(1/n)),x)

[Out] Exception raised: HeuristicGCDFailed



$$3.326 \quad \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx$$

Optimal. Leaf size=54

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

[Out] c\*x\*hypergeom([2, 1/n], [1+1/n], -(-a\*d+b\*c)\*x^n/a/(c+d\*x^n))/a^2/((c+d\*x^n)^(1/n))

**Rubi [A]** time = 0.01, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {379}

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^2, x]

[Out] (c\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])/(a^2\*(c + d\*x^n)^n^(-1))

Rule 379

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

Rubi steps

$$\int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx = \frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2}$$

**Mathematica [A]** time = 0.01, size = 53, normalized size = 0.98

$$\frac{cx(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1 + \frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^2, x]

[Out] (c\*x\*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), ((-b\*c) + a\*d)\*x^n/(a\*(c + d\*x^n))])/(a^2\*(c + d\*x^n)^n^(-1))

**fricas [F]** time = 1.16, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{n-1}{n}}}{b^2x^{2n} + 2abx^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^2,x, algorithm="fricas")

[Out] integral((d\*x^n + c)^((n - 1)/n)/(b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^2,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^2, x)

**maple** [F] time = 0.92, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n+1)/(b\*x^n+a)^2,x)

[Out] int((d\*x^n+c)^(-1/n+1)/(b\*x^n+a)^2,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^2,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^2, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c + dx^n)^{1-\frac{1}{n}}}{(a + bx^n)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^(1 - 1/n)/(a + b\*x^n)^2,x)

[Out] int((c + d\*x^n)^(1 - 1/n)/(a + b\*x^n)^2, x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(1-1/n)/(a+b\*x\*\*n)\*\*2,x)

[Out] Exception raised: HeuristicGCDFailed

$$3.327 \quad \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx$$

Optimal. Leaf size=56

$$\frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

[Out]  $c^2 x \text{hypergeom}([3, 1/n], [1+1/n], -(-a*d+b*c)*x^n/a/(c+d*x^n))/a^3/((c+d*x^n)^{(1/n)})$

**Rubi [A]** time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$ , Rules used = {379}

$$\frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^3, x]

[Out]  $(c^2 x \text{Hypergeometric2F1}[3, n^{(-1)}, 1 + n^{(-1)}, -((b*c - a*d)*x^n)/(a*(c + d*x^n))])/(a^3*(c + d*x^n)^{n^{(-1)}})$

Rule 379

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

Rubi steps

$$\int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx = \frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^3}$$

**Mathematica [A]** time = 0.01, size = 55, normalized size = 0.98

$$\frac{c^2 x (c + dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1 + \frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^3, x]

[Out]  $(c^2 x \text{Hypergeometric2F1}[3, n^{(-1)}, 1 + n^{(-1)}, ((-b*c) + a*d)*x^n/(a*(c + d*x^n))])/(a^3*(c + d*x^n)^{n^{(-1)}})$

**fricas [F]** time = 1.29, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{2n-1}{n}}}{b^3 x^{3n} + 3ab^2 x^{2n} + 3a^2 b x^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^3,x, algorithm="fricas")

[Out] integral((d\*x^n + c)^((2\*n - 1)/n)/(b^3\*x^(3\*n) + 3\*a\*b^2\*x^(2\*n) + 3\*a^2\*b\*x^n + a^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^3,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^3, x)

**maple** [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(d x^n + c)^{-\frac{1}{n}+2}}{(b x^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n+2)/(b\*x^n+a)^3,x)

[Out] int((d\*x^n+c)^(-1/n+2)/(b\*x^n+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^3,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(c + d x^n)^{2-\frac{1}{n}}}{(a + b x^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^(2 - 1/n)/(a + b\*x^n)^3,x)

[Out] int((c + d\*x^n)^(2 - 1/n)/(a + b\*x^n)^3, x)

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(2-1/n)/(a+b\*x\*\*n)\*\*3,x)

[Out] Exception raised: HeuristicGCDFailed

### 3.328 $\int (a + bx^n)^p (c + dx^n)^{-2-\frac{1}{n}-p} dx$

**Optimal.** Leaf size=193

$$\frac{x(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n}-p-1} (n(p+1)(bc - ad) + bc) \left(\frac{c+bx^n}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n}, -p-1; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{acn(p+1)(bc - ad)} bx(a + bx^n)^p$$

[Out]  $-b*x*(a+b*x^n)^{(1+p)}*(c+d*x^n)^{(-1-1/n-p)}/a/(-a*d+b*c)/n/(1+p)+(b*c+(-a*d+b*c)*n*(1+p))*x*(a+b*x^n)^{(1+p)}*(c*(a+b*x^n)/a/(c+d*x^n))^{(-1-p)}*(c+d*x^n)^{(-1-1/n-p)}*hypergeom([1/n, -1-p], [1+1/n], -(a*d+b*c)*x^n/a/(c+d*x^n))/a/c/(-a*d+b*c)/n/(1+p)$

**Rubi [A]** time = 0.08, antiderivative size = 179, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$ , Rules used = {382, 380}

$$\frac{x(a + bx^n)^{p+1} (c + dx^n)^{-\frac{1}{n}-p-1} \left(\frac{b}{n(p+1)(bc-ad)} + \frac{1}{c}\right) \left(\frac{c+bx^n}{a(c+dx^n)}\right)^{-p-1} {}_2F_1\left(\frac{1}{n}, -p-1; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a} bx(a + bx^n)^p$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^p\*(c + d\*x^n)^(-2 - n^(-1) - p), x]

[Out]  $-((b*x*(a + b*x^n)^{(1+p)}*(c + d*x^n)^{(-1 - n^{(-1)} - p)})/(a*(b*c - a*d)*n*(1+p))) + ((c^{(-1)} + b/((b*c - a*d)*n*(1+p)))*x*(a + b*x^n)^{(1+p)}*((c*(a + b*x^n))/(a*(c + d*x^n)))^{(-1-p)}*(c + d*x^n)^{(-1 - n^{(-1)} - p)}*Hypergeometric2F1[n^{(-1)}, -1 - p, 1 + n^{(-1)}, -((b*c - a*d)*x^n)/(a*(c + d*x^n))])/a$

#### Rule 380

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(x\*(a + b\*x^n)^p\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c\*((c\*(a + b\*x^n))/(a\*(c + d\*x^n)))^p\*(c + d\*x^n)^(1/n + p)), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(b\*x\*(a + b\*x^n)^(p+1)\*(c + d\*x^n)^(q+1))/(a\*n\*(p+1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p+1)\*(b\*c - a\*d))/(a\*n\*(p+1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p+1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)^p (c + dx^n)^{-2-\frac{1}{n}-p} dx &= -\frac{bx(a + bx^n)^{1+p} (c + dx^n)^{-1-\frac{1}{n}-p}}{a(bc - ad)n(1+p)} + \frac{\left(1 + \frac{bc}{(bc-ad)n(1+p)}\right) \int (a + bx^n)^{1+p} (c + dx^n)^{-2-\frac{1}{n}-p} dx}{a} \\ &= -\frac{bx(a + bx^n)^{1+p} (c + dx^n)^{-1-\frac{1}{n}-p}}{a(bc - ad)n(1+p)} + \frac{\left(1 + \frac{bc}{(bc-ad)n(1+p)}\right) x(a + bx^n)^{1+p} \left(\frac{c+bx^n}{a(c+dx^n)}\right)^{-p-1}}{a(bc - ad)n(1+p)} \end{aligned}$$

**Mathematica** [B] time = 51.21, size = 1414, normalized size = 7.33

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^n)^p\*(c + d\*x^n)^(-2 - n^(-1) - p),x]

[Out] (c^4\*(1 + n)\*(1 + 2\*n)\*(1 + 3\*n)\*x\*(a + b\*x^n)^(3 + p)\*(c + d\*x^n)^(-2 - n^(-1) - p)\*(1 + (d\*x^n)/c)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*(Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (d\*n\*x^n\*((c\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]))/(1 + n) + ((b\*c - a\*d)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/((1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]))/c^2)/(-(c\*d\*(1 + 3\*n)\*(1 + n + n\*p)\*x^n\*(a + b\*x^n)^2\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + b\*c\*n\*(1 + 3\*n)\*p\*x^n\*(a + b\*x^n)\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + c\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) + n^2\*x^n\*(c + d\*x^n)\*(a\*c^2\*(-(b\*c) + a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*p\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[2, 1 - p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + c\*d\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[1, -p, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])) - d\*(b\*c - a\*d)\*x^n\*(b\*c\*(1 + n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] - c\*(1 + n)\*(1 + 3\*n)\*(a + b\*x^n)^2\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + a\*c\*n\*(1 + 3\*n)\*p\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Gamma[-p]\*Hypergeometric2F1[2, 1 - p, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] - 2\*a\*(-(b\*c) + a\*d)\*n\*(1 + n)\*(-1 + p)\*x^n\*Gamma[1 + n^(-1)]\*Gamma[1 - p]\*Hypergeometric2F1[3, 2 - p, 4 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]))))

**fricas** [F] time = 1.14, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(bx^n + a)^p}{(dx^n + c)^{\frac{np+2n+1}{n}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-2-1/n-p),x, algorithm="fricas")

[Out] integral((b\*x^n + a)^p/(d\*x^n + c)^((n\*p + 2\*n + 1)/n), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^p (dx^n + c)^{-p - \frac{1}{n} - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-2-1/n-p),x, algorithm="giac")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 2), x)

**maple** [F] time = 0.92, size = 0, normalized size = 0.00

$$\int (b x^n + a)^p (d x^n + c)^{-p - \frac{1}{n} - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^p\*(d\*x^n+c)^(-2-1/n-p),x)

[Out] int((b\*x^n+a)^p\*(d\*x^n+c)^(-2-1/n-p),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b x^n + a)^p (d x^n + c)^{-p - \frac{1}{n} - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^p\*(c+d\*x^n)^(-2-1/n-p),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^p\*(d\*x^n + c)^(-p - 1/n - 2), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b x^n)^p}{(c + d x^n)^{p + \frac{1}{n} + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p/(c + d\*x^n)^(p + 1/n + 2),x)

[Out] int((a + b\*x^n)^p/(c + d\*x^n)^(p + 1/n + 2), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*n)\*\*(-2-1/n-p),x)

[Out] Timed out

$$3.329 \quad \int (a + bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c + dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx$$

Optimal. Leaf size=57

$$\frac{x (a + bx^n)^{-\frac{bc}{n(bc-ad)}} (c + dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

[Out]  $x*(c+d*x^n)^{(a*d/(-a*d+b*c)/n)}/a/c/((a+b*x^n)^{(b*c/(-a*d+b*c)/n))$

Rubi [A] time = 0.03, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 69,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.014$ , Rules used = {381}

$$\frac{x (a + bx^n)^{-\frac{bc}{n(bc-ad)}} (c + dx^n)^{\frac{ad}{n(bc-ad)}}}{ac}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^((a\*d\*n - b\*c\*(1 + n))/((b\*c - a\*d)\*n))\*(c + d\*x^n)^((a\*d - b\*c\*n + a\*d\*n)/(b\*c\*n - a\*d\*n)), x]

[Out] (x\*(c + d\*x^n)^((a\*d)/((b\*c - a\*d)\*n)))/(a\*c\*(a + b\*x^n)^((b\*c)/((b\*c - a\*d)\*n)))

Rule 381

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*c), x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && EqQ[a\*d\*(p + 1) + b\*c\*(q + 1), 0]

Rubi steps

$$\int (a + bx^n)^{\frac{adn-bc(1+n)}{(bc-ad)n}} (c + dx^n)^{\frac{ad-bcn+adn}{bcn-adn}} dx = \frac{x (a + bx^n)^{-\frac{bc}{(bc-ad)n}} (c + dx^n)^{\frac{ad}{(bc-ad)n}}}{ac}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 0.96

$$\frac{x (a + bx^n)^{-\frac{bc}{bcn-adn}} (c + dx^n)^{\frac{ad}{bcn-adn}}}{ac}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)^((a\*d\*n - b\*c\*(1 + n))/((b\*c - a\*d)\*n))\*(c + d\*x^n)^((a\*d - b\*c\*n + a\*d\*n)/(b\*c\*n - a\*d\*n)), x]

[Out] (x\*(c + d\*x^n)^((a\*d)/(b\*c\*n - a\*d\*n)))/(a\*c\*(a + b\*x^n)^((b\*c)/(b\*c\*n - a\*d\*n)))

fricas [A] time = 1.27, size = 108, normalized size = 1.89

$$\frac{(bdxx^{2n} + acx + (bc + ad)xx^n)(dx^n + c)^{\frac{ad-(bc-ad)n}{(bc-ad)n}}}{(bx^n + a)^{\frac{bc+(bc-ad)n}{(bc-ad)n}} ac}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((a+b\*x^n)^((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x^n)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x, algorithm="fricas")

[Out] (b\*d\*x\*x^(2\*n) + a\*c\*x + (b\*c + a\*d)\*x\*x^n)\*(d\*x^n + c)^((a\*d - (b\*c - a\*d)\*n)/((b\*c - a\*d)\*n))/((b\*x^n + a)^((b\*c + (b\*c - a\*d)\*n)/((b\*c - a\*d)\*n))\*a\*c)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}} (dx^n + c)^{\frac{bcn-adn-ad}{bcn-adn}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x^n)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^((b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)\*n))\*(d\*x^n + c)^((b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n))), x)

**maple** [F] time = 0.90, size = 0, normalized size = 0.00

$$\int (bx^n + a)^{\frac{adn-(n+1)bc}{(-ad+bc)n}} (dx^n + c)^{\frac{adn-bcn+ad}{-adn+bcn}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^((a\*d\*n-b\*c\*(n+1))/(-a\*d+b\*c)/n)\*(d\*x^n+c)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x)

[Out] int((b\*x^n+a)^((a\*d\*n-b\*c\*(n+1))/(-a\*d+b\*c)/n)\*(d\*x^n+c)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^{\frac{bc(n+1)-adn}{(bc-ad)n}} (dx^n + c)^{\frac{bcn-adn-ad}{bcn-adn}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^((a\*d\*n-b\*c\*(1+n))/(-a\*d+b\*c)/n)\*(c+d\*x^n)^((a\*d\*n-b\*c\*n+a\*d)/(-a\*d\*n+b\*c\*n)),x, algorithm="maxima")

[Out] integrate(1/((b\*x^n + a)^((b\*c\*(n + 1) - a\*d\*n)/((b\*c - a\*d)\*n))\*(d\*x^n + c)^((b\*c\*n - a\*d\*n - a\*d)/(b\*c\*n - a\*d\*n))), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(a + bx^n)^{\frac{adn-bc(n+1)}{n(ad-bc)}} (c + dx^n)^{\frac{ad+adn-bcn}{adn-bcn}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)^((a\*d\*n - b\*c\*(n + 1))/(n\*(a\*d - b\*c)))\*(c + d\*x^n)^((a\*d + a\*d\*n - b\*c\*n)/(a\*d\*n - b\*c\*n))),x)

[Out] int(1/((a + b\*x^n)^((a\*d\*n - b\*c\*(n + 1))/(n\*(a\*d - b\*c)))\*(c + d\*x^n)^((a\*d + a\*d\*n - b\*c\*n)/(a\*d\*n - b\*c\*n))), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*x**n)**((a*d*n-b*c*(1+n))/(-a*d+b*c)/n)*(c+d*x**n)**((a*d*n-b*c*n+a*d)/(-a*d*n+b*c*n)),x)
```

```
[Out] Timed out
```

### 3.330 $\int (a + bx^n)^2 (c + dx^n)^{-4-\frac{1}{n}} dx$

**Optimal.** Leaf size=327

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}$$

[Out]  $-1/3*b*x*(a+b*x^n)^3*(c+d*x^n)^{-3-1/n}/a/(-a*d+b*c)/n-1/3*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)^3*(c+d*x^n)^{-3-1/n}/a/c/(-a*d+b*c)/n/(1+3*n)-(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)^2*(c+d*x^n)^{-2-1/n}/c^2/(-a*d+b*c)/(6*n^2+5*n+1)-2*a*n*(3*a*d*n-b*(3*c*n+c))*x*(a+b*x^n)*(c+d*x^n)^{-1-1/n}/c^3/(-a*d+b*c)/(6*n^3+11*n^2+6*n+1)-2*a^2*n^2*(3*a*d*n-b*(3*c*n+c))*x/c^4/(-a*d+b*c)/(6*n^3+11*n^2+6*n+1)/((c+d*x^n)^{1/n})$

**Rubi [A]** time = 0.18, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$ , Rules used = {382, 378, 191}

$$\frac{2a^2n^2x(c+dx^n)^{-1/n}(3adn-b(3cn+c))}{c^4(n+1)(2n+1)(3n+1)(bc-ad)} - \frac{x(a+bx^n)^2(c+dx^n)^{-\frac{1}{n}-2}(3adn-b(3cn+c))}{c^2(6n^2+5n+1)(bc-ad)} - \frac{2anx(a+bx^n)(c+dx^n)^{-\frac{1}{n}-1}(3adn-b(3cn+c))}{c^3(n+1)(2n+1)(3n+1)(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)^2\*(c + d\*x^n)^(-4 - n^(-1)), x]

[Out]  $-(b*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n^{-1}})/(3*a*(b*c-a*d)*n)-((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^3*(c+d*x^n)^{-3-n^{-1}})/(3*a*c*(b*c-a*d)*n*(1+3*n))-((3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)^2*(c+d*x^n)^{-2-n^{-1}})/(c^2*(b*c-a*d)*(1+5*n+6*n^2))-2*a*n*(3*a*d*n-b*(c+3*c*n))*x*(a+b*x^n)*(c+d*x^n)^{-1-n^{-1}}/(c^3*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n))-2*a^2*n^2*(3*a*d*n-b*(c+3*c*n))*x/(c^4*(b*c-a*d)*(1+n)*(1+2*n)*(1+3*n)*(c+d*x^n)^{n^{-1}})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 378

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q)/(a\*n\*(p + 1)), x] - Dist[(c\*q)/(a\*(p + 1)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q - 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && GtQ[q, 0] && NeQ[p, -1]

#### Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)), Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

#### Rubi steps



[In] integrate((a+b\*x^n)^2\*(c+d\*x^n)^(-4-1/n),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP  
 UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding er  
 ror%%{27, [1,0,4,3,1,3,2,0]%%}+%%{27, [1,0,4,2,1,3,2,0]%%}+%%{9, [1,0,4,1  
 ,1,3,2,0]%%}+%%{1, [1,0,4,0,1,3,2,0]%%}+%%{27, [0,0,4,3,2,2,0,2]%%}+%%{  
 18, [0,0,4,2,2,2,0,2]%%}+%%{18, [0,0,4,2,1,3,1,1]%%}+%%{-9, [0,0,4,2,0,4,2  
 ,0]%%}+%%{3, [0,0,4,1,2,2,0,2]%%}+%%{12, [0,0,4,1,1,3,1,1]%%}+%%{-6, [0,  
 0,4,1,0,4,2,0]%%}+%%{2, [0,0,4,0,1,3,1,1]%%}+%%{-1, [0,0,4,0,0,4,2,0]%%}  
 / %%{27, [0,0,5,3,2,3,0,0]%%}+%%{27, [0,0,5,2,2,3,0,0]%%}+%%{9, [0,0,5,1  
 ,2,3,0,0]%%}+%%{1, [0,0,5,0,2,3,0,0]%%} Error: Bad Argument Value

maple [F] time = 0.77, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2 (dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)^2\*(d\*x^n+c)^(-1/n-4),x)

[Out] int((b\*x^n+a)^2\*(d\*x^n+c)^(-1/n-4),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)^2(dx^n + c)^{-\frac{1}{n}-4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)^2\*(c+d\*x^n)^(-4-1/n),x, algorithm="maxima")

[Out] integrate((b\*x^n + a)^2\*(d\*x^n + c)^(-1/n - 4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a + bx^n)^2}{(c + dx^n)^{\frac{1}{n}+4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^2/(c + d\*x^n)^(1/n + 4),x)

[Out] int((a + b\*x^n)^2/(c + d\*x^n)^(1/n + 4), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*\*2\*(c+d\*x\*\*n)\*\*(-4-1/n),x)

[Out] Timed out

### 3.331 $\int (a + bx^n)(c + dx^n)^{-3-\frac{1}{n}} dx$

**Optimal.** Leaf size=127

$$\frac{nx(c + dx^n)^{-1/n}(2adn + bc)}{c^3d(n+1)(2n+1)} + \frac{x(c + dx^n)^{-\frac{1}{n}-1}(2adn + bc)}{c^2d(n+1)(2n+1)} - \frac{x(bc - ad)(c + dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

[Out]  $-(-a*d+b*c)*x*(c+d*x^n)^{(-2-1/n)}/c/d/(1+2*n)+(2*a*d*n+b*c)*x*(c+d*x^n)^{(-1-1/n)}/c^2/d/(1+n)/(1+2*n)+n*(2*a*d*n+b*c)*x/c^3/d/(1+n)/(1+2*n)/((c+d*x^n)^{(1/n))}$

**Rubi [A]** time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$ , Rules used = {385, 192, 191}

$$\frac{x(c + dx^n)^{-\frac{1}{n}-1}(2adn + bc)}{c^2d(n+1)(2n+1)} + \frac{nx(c + dx^n)^{-1/n}(2adn + bc)}{c^3d(n+1)(2n+1)} - \frac{x(bc - ad)(c + dx^n)^{-\frac{1}{n}-2}}{cd(2n+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^n)\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out]  $-(((b*c - a*d)*x*(c + d*x^n)^{(-2 - n^(-1))})/(c*d*(1 + 2*n))) + ((b*c + 2*a*d*n)*x*(c + d*x^n)^{(-1 - n^(-1))})/(c^2*d*(1 + n)*(1 + 2*n)) + (n*(b*c + 2*a*d*n)*x)/(c^3*d*(1 + n)*(1 + 2*n)*(c + d*x^n)^{n^(-1)})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] := -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rule 385

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := -Simp[((b\*c - a\*d)\*x\*(a + b\*x^n)^(p + 1))/(a\*b\*n\*(p + 1)), x] - Dist[(a\*d - b\*c\*(n\*(p + 1) + 1))/(a\*b\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b\*c - a\*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

#### Rubi steps

$$\begin{aligned} \int (a + bx^n)(c + dx^n)^{-3-\frac{1}{n}} dx &= -\frac{(bc - ad)x(c + dx^n)^{-2-\frac{1}{n}}}{cd(1 + 2n)} + \frac{(bc + 2adn) \int (c + dx^n)^{-2-\frac{1}{n}} dx}{cd(1 + 2n)} \\ &= -\frac{(bc - ad)x(c + dx^n)^{-2-\frac{1}{n}}}{cd(1 + 2n)} + \frac{(bc + 2adn)x(c + dx^n)^{-1-\frac{1}{n}}}{c^2d(1 + n)(1 + 2n)} + \frac{(n(bc + 2adn)) \int (c + dx^n)^{-1-\frac{1}{n}} dx}{c^2d(1 + n)(1 + 2n)} \\ &= -\frac{(bc - ad)x(c + dx^n)^{-2-\frac{1}{n}}}{cd(1 + 2n)} + \frac{(bc + 2adn)x(c + dx^n)^{-1-\frac{1}{n}}}{c^2d(1 + n)(1 + 2n)} + \frac{n(bc + 2adn)x(c + dx^n)^{-\frac{1}{n}}}{c^3d(1 + n)(1 + 2n)} \end{aligned}$$

**Mathematica [C]** time = 0.17, size = 94, normalized size = 0.74

$$\frac{x(c + dx^n)^{-1/n} \left( \frac{dx^n}{c} + 1 \right)^{\frac{1}{n}} \left( (ad - bc) {}_2F_1 \left( 3 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) + bc {}_2F_1 \left( 2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c} \right) \right)}{c^3 d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^n)\*(c + d\*x^n)^(-3 - n^(-1)), x]

[Out] (x\*(1 + (d\*x^n)/c)^n^(-1)\*(b\*c\*Hypergeometric2F1[2 + n^(-1), n^(-1), 1 + n^(-1), -(d\*x^n)/c]) + (-b\*c) + a\*d)\*Hypergeometric2F1[3 + n^(-1), n^(-1), 1 + n^(-1), -(d\*x^n)/c])/(c^3\*d\*(c + d\*x^n)^n^(-1))

**fricas [A]** time = 1.35, size = 173, normalized size = 1.36

$$\frac{(2ad^3n^2 + bcd^2n)xx^{3n} + (6acd^2n^2 + bc^2d + (3bc^2d + 2acd^2)n)xx^{2n} + (6ac^2dn^2 + bc^3 + ac^2d + (2bc^3 + 5acd^2n))xx^{n} + (2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}{(2c^3n^2 + 3c^3n + c^3)(dx^n + c)^{\frac{3n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-3-1/n), x, algorithm="fricas")

[Out] ((2\*a\*d^3\*n^2 + b\*c\*d^2\*n)\*x\*x^(3\*n) + (6\*a\*c\*d^2\*n^2 + b\*c^2\*d + (3\*b\*c^2\*d + 2\*a\*c\*d^2)\*n)\*x\*x^(2\*n) + (6\*a\*c^2\*d\*n^2 + b\*c^3 + a\*c^2\*d + (2\*b\*c^3 + 5\*a\*c^2\*d)\*n)\*x\*x^n + (2\*a\*c^3\*n^2 + 3\*a\*c^3\*n + a\*c^3)\*x)/((2\*c^3\*n^2 + 3\*c^3\*n + c^3)\*(d\*x^n + c)^((3\*n + 1)/n))

**giac [F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-3-1/n), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%{4, [0,0,2,2,1,1,0,1]}%{2, [0,0,2,1,1,1,0,1]}%{2, [0,0,2,1,0,2,1,0]}%{1, [0,0,2,0,0,2,1,0]} / %%{4, [0,0,3,2,1,2,0,0]}%{4, [0,0,3,1,1,2,0,0]}%{1, [0,0,3,0,1,2,0,0]} Error: Bad Argument Value

**maple [F]** time = 0.63, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^n+a)\*(d\*x^n+c)^(-1/n-3), x)

[Out] int((b\*x^n+a)\*(d\*x^n+c)^(-1/n-3), x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (bx^n + a)(dx^n + c)^{-\frac{1}{n}-3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x^n)\*(c+d\*x^n)^(-3-1/n), x, algorithm="maxima")

[Out] integrate((b\*x^n + a)\*(d\*x^n + c)^(-1/n - 3), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a + b x^n}{(c + d x^n)^{\frac{1}{n} + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)/(c + d\*x^n)^(1/n + 3), x)

[Out] int((a + b\*x^n)/(c + d\*x^n)^(1/n + 3), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b\*x\*\*n)\*(c+d\*x\*\*n)\*\*(-3-1/n), x)

[Out] Timed out



### 3.332 $\int (c + dx^n)^{-2-\frac{1}{n}} dx$

**Optimal.** Leaf size=50

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)} + \frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

[Out]  $x*(c+d*x^n)^{(-1-1/n)}/c/(1+n)+n*x/c^2/(1+n)/((c+d*x^n)^{(1/n)})$

**Rubi [A]** time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$ , Rules used = {192, 191}

$$\frac{nx(c+dx^n)^{-1/n}}{c^2(n+1)} + \frac{x(c+dx^n)^{-\frac{1}{n}-1}}{c(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(c + d*x^n)^{(-1 - n^{(-1)})})/(c*(1 + n)) + (n*x)/(c^2*(1 + n)*(c + d*x^n)^{n^{(-1)}})$

#### Rule 191

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> Simp[(x\*(a + b\*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

#### Rule 192

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_), x\_Symbol] :> -Simp[(x\*(a + b\*x^n)^(p + 1))/(a\*n\*(p + 1)), x] + Dist[(n\*(p + 1) + 1)/(a\*n\*(p + 1)), Int[(a + b\*x^n)^(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0] && NeQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int (c + dx^n)^{-2-\frac{1}{n}} dx &= \frac{x(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{n \int (c + dx^n)^{-1-\frac{1}{n}} dx}{c(1+n)} \\ &= \frac{x(c + dx^n)^{-1-\frac{1}{n}}}{c(1+n)} + \frac{nx(c + dx^n)^{-1/n}}{c^2(1+n)} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 55, normalized size = 1.10

$$\frac{x(c + dx^n)^{-1/n} \left(\frac{dx^n}{c} + 1\right)^{\frac{1}{n}} {}_2F_1\left(2 + \frac{1}{n}, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{dx^n}{c}\right)}{c^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-2 - n^(-1)), x]

[Out]  $(x*(1 + (d*x^n)/c)^{n^{(-1)}}*Hypergeometric2F1[2 + n^{(-1)}, n^{(-1)}, 1 + n^{(-1)}, -((d*x^n)/c)])/(c^2*(c + d*x^n)^{n^{(-1)}})$

**fricas** [A] time = 1.41, size = 68, normalized size = 1.36

$$\frac{d^2 n x x^{2n} + (2 c d n + c d) x x^n + (c^2 n + c^2) x}{(c^2 n + c^2) (d x^n + c)^{\frac{2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-2-1/n),x, algorithm="fricas")

[Out] (d^2\*n\*x\*x^(2\*n) + (2\*c\*d\*n + c\*d)\*x\*x^n + (c^2\*n + c^2)\*x)/((c^2\*n + c^2)\*(d\*x^n + c)^((2\*n + 1)/n))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d x^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-2-1/n),x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n - 2), x)

**maple** [F] time = 0.60, size = 0, normalized size = 0.00

$$\int (d x^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n-2),x)

[Out] int((d\*x^n+c)^(-1/n-2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (d x^n + c)^{-\frac{1}{n}-2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-2-1/n),x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n - 2), x)

**mupad** [B] time = 1.76, size = 64, normalized size = 1.28

$$\frac{x^{1-2n} \left(\frac{c}{d x^n} + 1\right)^{1/n} {}_2F_1\left(2, \frac{1}{n} + 2; 3; -\frac{c}{d x^n}\right)}{2 d^2 n (c + d x^n)^{1/n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c + d\*x^n)^(1/n + 2),x)

[Out] -(x^(1 - 2\*n)\*(c/(d\*x^n) + 1)^(1/n)\*hypergeom([2, 1/n + 2], 3, -c/(d\*x^n)))/(2\*d^2\*n\*(c + d\*x^n)^(1/n))

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(-2-1/n),x)

[Out] Timed out

$$3.333 \quad \int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx$$

**Optimal.** Leaf size=95

$$\frac{bx(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

[Out]  $-dx/c/(-a*d+b*c)/((c+d*x^n)^{(1/n)}+b*x*hypergeom([1, 1/n], [1+1/n], -(a*d+b*c)*x^n/a/(c+d*x^n))/a/(-a*d+b*c)/((c+d*x^n)^{(1/n)})$

**Rubi [A]** time = 0.03, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a(bc-ad)} - \frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x^n)^{-1 - n^{-1}}/(a + b*x^n), x]$

[Out]  $-((d*x)/(c*(b*c - a*d)*(c + d*x^n)^{n^{-1}})) + (b*x*Hypergeometric2F1[1, n^{-1}(-1), 1 + n^{-1}, -((b*c - a*d)*x^n)/(a*(c + d*x^n))])/(a*(b*c - a*d)*(c + d*x^n)^{n^{-1}})$

**Rule 379**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:\> \text{Simp}[(a^p*x*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b*c - a*d)*x^n)/(a*(c + d*x^n))])]/(c^{(p + 1)}*(c + d*x^n)^{(1/n)}, x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*(p + q + 1) + 1, 0] \&\& \text{ILtQ}[p, 0]$

**Rule 382**

$\text{Int}[(a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x\_Symbol]$   
 $:\> -\text{Simp}[(b*x*(a + b*x^n)^{(p + 1)}*(c + d*x^n)^{(q + 1)})/(a*n*(p + 1)*(b*c - a*d)), x] + \text{Dist}[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), \text{Int}[(a + b*x^n)^{(p + 1)}*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a, b, c, d, n, q\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{EqQ}[n*(p + q + 2) + 1, 0] \&\& (\text{LtQ}[p, -1] || !\text{LtQ}[q, -1]) \&\& \text{NeQ}[p, -1]$

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^n)^{-1-\frac{1}{n}}}{a+bx^n} dx &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{b \int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx}{bc-ad} \\ &= -\frac{dx(c+dx^n)^{-1/n}}{c(bc-ad)} + \frac{bx(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a(bc-ad)} \end{aligned}$$

**Mathematica [C]** time = 6.94, size = 153, normalized size = 1.61

$$\frac{x(c+dx^n)^{-\frac{n+1}{n}} \left( \frac{bnx^{2n}(ad-bc) {}_2F_1\left(2, 2+\frac{1}{n}; 3+\frac{1}{n}; \frac{(ad-bc)x^n}{a(dx^n+c)}\right)}{a^2(2n+1)(c+dx^n)} + \frac{bx^n \Phi\left(\frac{(ad-bc)x^n}{a(dx^n+c)}, 1, 1+\frac{1}{n}\right)}{a} + \frac{a(c+dx^n)}{c(a+bx^n)} \right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^n)^(-1 - n^(-1))/(a + b\*x^n), x]

[Out] (x\*((a\*(c + d\*x^n))/(c\*(a + b\*x^n)) + (b\*x^n\*HurwitzLerchPhi[(-(b\*c) + a\*d)\*x^n]/(a\*(c + d\*x^n)), 1, 1 + n^(-1)))/a + (b\*(-(b\*c) + a\*d)\*n\*x^(2\*n)\*Hypergeometric2F1[2, 2 + n^(-1), 3 + n^(-1), ((-b\*c) + a\*d)\*x^n]/(a\*(c + d\*x^n)))/((a^2\*(1 + 2\*n)\*(c + d\*x^n)))/((a\*(c + d\*x^n)^((1 + n)/n))

**fricas** [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{(bx^n + a)(dx^n + c)^{\frac{n+1}{n}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n)/(a+b\*x^n), x, algorithm="fricas")

[Out] integral(1/((b\*x^n + a)\*(d\*x^n + c)^((n + 1)/n)), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{\frac{1}{n}-1}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n)/(a+b\*x^n), x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n - 1)/(b\*x^n + a), x)

**maple** [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{\frac{1}{n}-1}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n-1)/(b\*x^n+a), x)

[Out] int((d\*x^n+c)^(-1/n-1)/(b\*x^n+a), x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{\frac{1}{n}-1}}{bx^n + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(-1-1/n)/(a+b\*x^n), x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n - 1)/(b\*x^n + a), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + bx^n)(c + dx^n)^{\frac{1}{n}+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^n)\*(c + d\*x^n)^(1/n + 1)), x)

```
[Out] int(1/((a + b*x^n)*(c + d*x^n)^(1/n + 1)), x)
```

```
sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c+d*x**n)**(-1-1/n)/(a+b*x**n),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

$$3.334 \quad \int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx$$

**Optimal.** Leaf size=127

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)} - \frac{x(c+dx^n)^{-1/n}(adn+bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

[Out] b\*x/a/(-a\*d+b\*c)/n/(a+b\*x^n)/((c+d\*x^n)^((1-n)/n))-((b\*c\*(1-n)+a\*d\*n)\*x\*Hypergeometric2F1[1, 1/n], [1+1/n], -(-a\*d+b\*c)\*x^n/a/(c+d\*x^n))/a^2/(-a\*d+b\*c)/n/((c+d\*x^n)^(1/n))

**Rubi [A]** time = 0.05, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$ , Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{an(bc-ad)(a+bx^n)} - \frac{x(c+dx^n)^{-1/n}(adn+bc(1-n)) {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{a^2n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b\*x^n)^2\*(c + d\*x^n)^n^(-1)), x]

[Out] (b\*x)/(a\*(b\*c - a\*d)\*n\*(a + b\*x^n)\*(c + d\*x^n)^((1 - n)/n)) - ((b\*c\*(1 - n) + a\*d\*n)\*x\*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(a^2\*(b\*c - a\*d)\*n\*(c + d\*x^n)^n^(-1))

**Rule 379**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

**Rule 382**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol] :> -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)], Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^n)^{-1/n}}{(a+bx^n)^2} dx &= \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n(a+bx^n)} - \frac{(bc-(bc-ad)n) \int \frac{(c+dx^n)^{-1/n}}{a+bx^n} dx}{a(bc-ad)n} \\ &= \frac{bx(c+dx^n)^{-\frac{1-n}{n}}}{a(bc-ad)n(a+bx^n)} - \frac{(bc(1-n)+adn)x(c+dx^n)^{-1/n} {}_2F_1\left(1, \frac{1}{n}; 1 + \frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{a^2(bc-ad)n} \end{aligned}$$

**Mathematica [B]** time = 63.26, size = 1070, normalized size = 8.43

$$-cd(1-n)(2n+1)(3n+1)(bx^n+a)^2 \left(2(bc-ad)n(dx^n+c)\Gamma\left(2+\frac{1}{n}\right) {}_2F_1\left(2, 3; 3+\frac{1}{n}; \frac{(bc-ad)x^n}{c(bx^n+a)}\right) x^n + c(bx^n+a)\right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((a + b\*x^n)^2\*(c + d\*x^n)^n^(-1)),x]

[Out] (c^2\*(1 + 2\*n)\*(1 + 3\*n)\*x\*(a + b\*x^n)\*(1 + (d\*x^n)/c)\*Gamma[2 + n^(-1)]\*Gamma[3 + n^(-1)]\*((c\*(c + c\*n + d\*n\*x^n)\*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/Gamma[2 + n^(-1)] + (2\*(b\*c - a\*d)\*n\*x^n\*(c + d\*x^n)\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])/((a + b\*x^n)\*Gamma[3 + n^(-1)]))/((c + d\*x^n)^n^(-1)\*(-(c\*d\*(1 - n)\*(1 + 2\*n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)^2\*(c\*(a + b\*x^n)\*(c + c\*n + d\*n\*x^n)\*Gamma[3 + n^(-1)]\*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 2\*(b\*c - a\*d)\*n\*x^n\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - 2\*b\*c\*n\*(1 + 2\*n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*(c + d\*x^n)\*(c\*(a + b\*x^n)\*(c + c\*n + d\*n\*x^n)\*Gamma[3 + n^(-1)]\*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 2\*(b\*c - a\*d)\*n\*x^n\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + c\*(1 + 2\*n)\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c + d\*x^n)\*(c\*(a + b\*x^n)\*(c + c\*n + d\*n\*x^n)\*Gamma[3 + n^(-1)]\*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 2\*(b\*c - a\*d)\*n\*x^n\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + n^2\*x^n\*(c + d\*x^n)\*(c^2\*d\*(1 + 2\*n)\*(1 + 3\*n)\*(a + b\*x^n)^3\*Gamma[3 + n^(-1)]\*Hypergeometric2F1[1, 2, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 2\*c\*d\*(b\*c - a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)^2\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - 2\*b\*c\*(b\*c - a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + 2\*c\*(b\*c - a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + 2\*a\*c\*(b\*c - a\*d)\*(1 + 3\*n)\*(a + b\*x^n)\*(c + c\*n + d\*n\*x^n)\*Gamma[3 + n^(-1)]\*Hypergeometric2F1[2, 3, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + 12\*a\*(b\*c - a\*d)^2\*n\*(1 + 2\*n)\*x^n\*(c + d\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[3, 4, 4 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])

**fricas** [F] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral} \left( \frac{1}{(b^2x^{2n} + 2abx^n + a^2)(dx^n + c)^{\left(\frac{1}{n}\right)}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/((c+d\*x^n)^(1/n)),x, algorithm="fricas")

[Out] integral(1/((b^2\*x^(2\*n) + 2\*a\*b\*x^n + a^2)\*(d\*x^n + c)^(1/n)), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\left(\frac{1}{n}\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b\*x^n)^2/((c+d\*x^n)^(1/n)),x, algorithm="giac")

[Out] integrate(1/((b\*x^n + a)^2\*(d\*x^n + c)^(1/n)), x)

**maple** [F] time = 1.04, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}}}{(bx^n + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(b*x^n+a)^2/((d*x^n+c)^(1/n)),x)`

[Out] `int(1/(b*x^n+a)^2/((d*x^n+c)^(1/n)),x)`

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^n + a)^2(dx^n + c)^{\frac{1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x^n)^2/((c+d*x^n)^(1/n)),x, algorithm="maxima")`

[Out] `integrate(1/((b*x^n + a)^2*(d*x^n + c)^(1/n)), x)`

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(a + bx^n)^2(c + dx^n)^{1/n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)),x)`

[Out] `int(1/((a + b*x^n)^2*(c + d*x^n)^(1/n)), x)`

**sympy** [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*x**n)**2/((c+d*x**n)**(1/n)),x)`

[Out] Exception raised: HeuristicGCDFailed



$$3.335 \quad \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx$$

**Optimal.** Leaf size=131

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2} - \frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n)) {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

[Out]  $1/2*b*x*(c+d*x^n)^(2-1/n)/a/(-a*d+b*c)/n/(a+b*x^n)^(2-1/2*c*(b*c*(1-2*n)+2*a*d*n)*x*hypergeom([2, 1/n], [1+1/n], -(a*d+b*c)*x^n/a/(c+d*x^n))/a^3/(-a*d+b*c)/n/((c+d*x^n)^(1/n))$

**Rubi [A]** time = 0.06, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2an(bc-ad)(a+bx^n)^2} - \frac{cx(c+dx^n)^{-1/n}(2adn+bc(1-2n)) {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{2a^3n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^3, x]

[Out]  $(b*x*(c+d*x^n)^(2-n^(-1)))/(2*a*(b*c-a*d)*n*(a+b*x^n)^2) - (c*(b*c*(1-2*n)+2*a*d*n)*x*Hypergeometric2F1[2, n^(-1), 1+n^(-1), -((b*c-a*d)*x^n)/(a*(c+d*x^n))])/(2*a^3*(b*c-a*d)*n*(c+d*x^n)^n^(-1))$

**Rule 379**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n))])]/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

**Rule 382**

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 :> -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)], Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^3} dx &= \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n(a+bx^n)^2} - \frac{(bc-2(bc-ad)n) \int \frac{(c+dx^n)^{1-\frac{1}{n}}}{(a+bx^n)^2} dx}{2a(bc-ad)n} \\ &= \frac{bx(c+dx^n)^{2-\frac{1}{n}}}{2a(bc-ad)n(a+bx^n)^2} - \frac{c(bc(1-2n)+2adn)x(c+dx^n)^{-1/n} {}_2F_1\left(2, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{2a^3(bc-ad)n} \end{aligned}$$

**Mathematica [B]** time = 44.86, size = 1241, normalized size = 9.47

result too large to display

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^n)^(1 - n^(-1))/(a + b\*x^n)^3,x]

[Out] -((c^3\*(1 + n)\*(1 + 2\*n)\*(1 + 3\*n)\*x\*(c + d\*x^n)^(2 - n^(-1))\*Gamma[2 + n^(-1)]\*(Hypergeometric2F1[1, 3, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + (d\*n\*x^n\*((c\*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n)))]/(1 + n) + (3\*(b\*c - a\*d)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n)))]/((1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)])))/c^2)/(c\*d\*(1 - 2\*n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)^2\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 3\*(b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + 3\*b\*c\*n\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 3\*(b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) - c\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c + d\*x^n)\*(c^2\*(1 + n)\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 1 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + d\*n\*x^n\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 3\*(b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + n^2\*x^n\*(c + d\*x^n)\*(3\*a\*c^2\*(-(b\*c) + a\*d)\*(1 + 2\*n)\*(1 + 3\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 4, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] - c\*d\*(1 + 3\*n)\*(a + b\*x^n)^2\*(c\*(1 + 2\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[1, 3, 2 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 3\*(b\*c - a\*d)\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))]) + 3\*d\*(b\*c - a\*d)\*x^n\*(b\*c\*(1 + n)\*(1 + 3\*n)\*x^n\*(a + b\*x^n)\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] - c\*(1 + n)\*(1 + 3\*n)\*(a + b\*x^n)^2\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] - a\*c\*n\*(1 + 3\*n)\*(a + b\*x^n)\*Gamma[2 + n^(-1)]\*Hypergeometric2F1[2, 4, 3 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))] + 8\*a\*(-(b\*c) + a\*d)\*n\*(1 + n)\*x^n\*Gamma[1 + n^(-1)]\*Hypergeometric2F1[3, 5, 4 + n^(-1), ((b\*c - a\*d)\*x^n)/(c\*(a + b\*x^n))])

**fricas** [F] time = 1.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{n-1}{n}}}{b^3x^{3n} + 3ab^2x^{2n} + 3a^2bx^n + a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^3,x, algorithm="fricas")

[Out] integral((d\*x^n + c)^((n - 1)/n)/(b^3\*x^(3\*n) + 3\*a\*b^2\*x^(2\*n) + 3\*a^2\*b\*x^n + a^3), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^3,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^3, x)

**maple** [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n+1)/(b\*x^n+a)^3,x)

[Out] int((d\*x^n+c)^(-1/n+1)/(b\*x^n+a)^3,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+1}}{(bx^n + a)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(1-1/n)/(a+b\*x^n)^3,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 1)/(b\*x^n + a)^3, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + dx^n)^{1-\frac{1}{n}}}{(a + bx^n)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^(1 - 1/n)/(a + b\*x^n)^3,x)

[Out] int((c + d\*x^n)^(1 - 1/n)/(a + b\*x^n)^3, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(1-1/n)/(a+b\*x\*\*n)\*\*3,x)

[Out] Timed out

$$3.336 \quad \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx$$

Optimal. Leaf size=133

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3} - \frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n)){}_2F_1\left(3, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

[Out] 1/3\*b\*x\*(c+d\*x^n)^(3-1/n)/a/(-a\*d+b\*c)/n/(a+b\*x^n)^3-1/3\*c^2\*(b\*c\*(1-3\*n)+3\*a\*d\*n)\*x\*hypergeom([3, 1/n], [1+1/n], -(-a\*d+b\*c)\*x^n/a/(c+d\*x^n))/a^4/(-a\*d+b\*c)/n/((c+d\*x^n)^(1/n))

**Rubi [A]** time = 0.05, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$ , Rules used = {382, 379}

$$\frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3an(bc-ad)(a+bx^n)^3} - \frac{c^2x(c+dx^n)^{-1/n}(3adn+bc(1-3n)){}_2F_1\left(3, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(dx^n+c)}\right)}{3a^4n(bc-ad)}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^4, x]

[Out] (b\*x\*(c + d\*x^n)^(3 - n^(-1)))/(3\*a\*(b\*c - a\*d)\*n\*(a + b\*x^n)^3) - (c^2\*(b\*c\*(1 - 3\*n) + 3\*a\*d\*n)\*x\*Hypergeometric2F1[3, n^(-1), 1 + n^(-1), -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(3\*a^4\*(b\*c - a\*d)\*n\*(c + d\*x^n)^n^(-1))

Rule 379

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := Simp[(a^p\*x\*Hypergeometric2F1[1/n, -p, 1 + 1/n, -(((b\*c - a\*d)\*x^n)/(a\*(c + d\*x^n)))])/(c^(p + 1)\*(c + d\*x^n)^(1/n)), x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 1) + 1, 0] && ILtQ[p, 0]

Rule 382

Int[((a\_) + (b\_.)\*(x\_)^(n\_))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_))^(q\_), x\_Symbol]  
 := -Simp[(b\*x\*(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^(q + 1))/(a\*n\*(p + 1)\*(b\*c - a\*d)), x] + Dist[(b\*c + n\*(p + 1)\*(b\*c - a\*d))/(a\*n\*(p + 1)\*(b\*c - a\*d)], Int[(a + b\*x^n)^(p + 1)\*(c + d\*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ[q, -1]) && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^4} dx &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3} - \frac{(bc-3(bc-ad)n) \int \frac{(c+dx^n)^{2-\frac{1}{n}}}{(a+bx^n)^3} dx}{3a(bc-ad)n} \\ &= \frac{bx(c+dx^n)^{3-\frac{1}{n}}}{3a(bc-ad)n(a+bx^n)^3} - \frac{c^2(bc(1-3n)+3adn)x(c+dx^n)^{-1/n} {}_2F_1\left(3, \frac{1}{n}; 1+\frac{1}{n}; -\frac{(bc-ad)x^n}{a(c+dx^n)}\right)}{3a^4(bc-ad)n} \end{aligned}$$

**Mathematica [F]** time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is Not applicable to the result.

[In] Integrate[(c + d\*x^n)^(2 - n^(-1))/(a + b\*x^n)^4, x]

[Out] \$Aborted

**fricas** [F] time = 1.37, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(dx^n + c)^{\frac{2n-1}{n}}}{b^4x^{4n} + 4ab^3x^{3n} + 6a^2b^2x^{2n} + 4a^3bx^n + a^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^4,x, algorithm="fricas")

[Out] integral((d\*x^n + c)^((2\*n - 1)/n)/(b^4\*x^(4\*n) + 4\*a\*b^3\*x^(3\*n) + 6\*a^2\*b^2\*x^(2\*n) + 4\*a^3\*b\*x^n + a^4), x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^4,x, algorithm="giac")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4, x)

**maple** [F] time = 0.95, size = 0, normalized size = 0.00

$$\int \frac{(d x^n + c)^{-\frac{1}{n}+2}}{(b x^n + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^n+c)^(-1/n+2)/(b\*x^n+a)^4, x)

[Out] int((d\*x^n+c)^(-1/n+2)/(b\*x^n+a)^4, x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(dx^n + c)^{-\frac{1}{n}+2}}{(bx^n + a)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^n)^(2-1/n)/(a+b\*x^n)^4,x, algorithm="maxima")

[Out] integrate((d\*x^n + c)^(-1/n + 2)/(b\*x^n + a)^4, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d x^n)^{2-\frac{1}{n}}}{(a + b x^n)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^n)^(2 - 1/n)/(a + b\*x^n)^4, x)

[Out] int((c + d\*x^n)^(2 - 1/n)/(a + b\*x^n)^4, x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*n)\*\*(2-1/n)/(a+b\*x\*\*n)\*\*4,x)

[Out] Exception raised: HeuristicGCDFailed

### 3.337 $\int x^5 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=152

$$\frac{(dx - c)^{7/2}(c + dx)^{7/2}(ad^2 + 3bc^2)}{7d^8} + \frac{c^2(dx - c)^{5/2}(c + dx)^{5/2}(2ad^2 + 3bc^2)}{5d^8} + \frac{c^4(dx - c)^{3/2}(c + dx)^{3/2}(ad^2 + bc^2)}{3d^8}$$

[Out]  $1/3*c^4*(a*d^2+b*c^2)*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^8+1/5*c^2*(2*a*d^2+3*b*c^2)*(d*x-c)^{(5/2)}*(d*x+c)^{(5/2)}/d^8+1/7*(a*d^2+3*b*c^2)*(d*x-c)^{(7/2)}*(d*x+c)^{(7/2)}/d^8+1/9*b*(d*x-c)^{(9/2)}*(d*x+c)^{(9/2)}/d^8$

**Rubi [A]** time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {460, 100, 12, 74}

$$\frac{x^4(dx - c)^{3/2}(c + dx)^{3/2}(3ad^2 + 2bc^2)}{21d^4} + \frac{4c^2x^2(dx - c)^{3/2}(c + dx)^{3/2}(3ad^2 + 2bc^2)}{105d^6} + \frac{8c^4(dx - c)^{3/2}(c + dx)^{3/2}(3ad^2 + 2bc^2)}{315d^8}$$

Antiderivative was successfully verified.

[In] Int[x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $(8*c^4*(2*b*c^2 + 3*a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(315*d^8) + (4*c^2*(2*b*c^2 + 3*a*d^2)*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(105*d^6) + ((2*b*c^2 + 3*a*d^2)*x^4*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(21*d^4) + (b*x^6*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(9*d^2)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 100

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^(p)\*(a2 + b2\*x^(n/2))^(p), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{-c+dx} \sqrt{c+dx} (a+bx^2) dx &= \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{1}{3} \left( 3a + \frac{2bc^2}{d^2} \right) \int x^5 \sqrt{-c+dx} \sqrt{c+dx} dx \\
&= \frac{(2bc^2+3ad^2)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{1}{3} \left( 3a + \frac{2bc^2}{d^2} \right) \int x^5 \sqrt{-c+dx} \sqrt{c+dx} dx \\
&= \frac{(2bc^2+3ad^2)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} + \frac{bx^6(-c+dx)^{3/2}(c+dx)^{3/2}}{9d^2} + \frac{1}{3} \left( 3a + \frac{2bc^2}{d^2} \right) \int x^5 \sqrt{-c+dx} \sqrt{c+dx} dx \\
&= \frac{4c^2(2bc^2+3ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(2bc^2+3ad^2)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} \\
&= \frac{4c^2(2bc^2+3ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(2bc^2+3ad^2)x^4(-c+dx)^{3/2}(c+dx)^{3/2}}{21d^4} \\
&= \frac{8c^4(2bc^2+3ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{315d^8} + \frac{4c^2(2bc^2+3ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6}
\end{aligned}$$

**Mathematica [A]** time = 0.10, size = 110, normalized size = 0.72

$$\frac{\sqrt{dx-c} \sqrt{c+dx} (d^2x^2 - c^2) (3ad^2 (8c^4 + 12c^2d^2x^2 + 15d^4x^4) + b (16c^6 + 24c^4d^2x^2 + 30c^2d^4x^4 + 35d^6x^6))}{315d^8}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*(a+b\*x^2),x]

[Out] (Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*(-c^2+d^2\*x^2)\*(3\*a\*d^2\*(8\*c^4+12\*c^2\*d^2\*x^2+15\*d^4\*x^4)+b\*(16\*c^6+24\*c^4\*d^2\*x^2+30\*c^2\*d^4\*x^4+35\*d^6\*x^6)))/(315\*d^8)

**fricas [A]** time = 1.34, size = 114, normalized size = 0.75

$$\frac{(35bd^8x^8 - 16bc^8 - 24ac^6d^2 - 5(bc^2d^6 - 9ad^8)x^6 - 3(2bc^4d^4 + 3ac^2d^6)x^4 - 4(2bc^6d^2 + 3ac^4d^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{315d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/315\*(35\*b\*d^8\*x^8 - 16\*b\*c^8 - 24\*a\*c^6\*d^2 - 5\*(b\*c^2\*d^6 - 9\*a\*d^8)\*x^6 - 3\*(2\*b\*c^4\*d^4 + 3\*a\*c^2\*d^6)\*x^4 - 4\*(2\*b\*c^6\*d^2 + 3\*a\*c^4\*d^4)\*x^2)\*sqrt(d\*x+c)\*sqrt(d\*x-c)/d^8

**giac [B]** time = 0.76, size = 621, normalized size = 4.09

$$168 \left( \left( \left( 2 \left( (dx+c) \left( 4(dx+c) \left( \frac{5(dx+c)}{d^5} - \frac{31c}{d^5} \right) + \frac{321c^2}{d^5} \right) - \frac{451c^3}{d^5} \right) (dx+c) + \frac{745c^4}{d^5} \right) (dx+c) - \frac{405c^5}{d^5} \right) \sqrt{dx+c} \sqrt{dx-c} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/40320\*(168\*(((2\*((d\*x+c))\*(4\*(d\*x+c))\*(5\*(d\*x+c)/d^5 - 31\*c/d^5) + 321\*c^2/d^5) - 451\*c^3/d^5)\*(d\*x+c) + 745\*c^4/d^5)\*(d\*x+c) - 405\*c^5/d^5)\*sqrt(d\*x+c)\*sqrt(d\*x-c) - 150\*c^6\*log(abs(-sqrt(d\*x+c) + sqrt(d\*x-c))))/d^5)\*a\*c + 3\*(((2\*((4\*(5\*(d\*x+c))\*(6\*(d\*x+c))\*(7\*(d\*x+c)/d^7 - 57\*c/d^7) + 1219\*c^2/d^7) - 12463\*c^3/d^7)\*(d\*x+c) + 64233\*c^4/d^7)\*(d\*x+c) - 53963\*c^5/d^7)\*(d\*x+c) + 59465\*c^6/d^7)\*(d\*x+c) - 23205\*c^7/d^7)\*sqrt(d\*x+c)\*sqrt(d\*x-c) - 7350\*c^8\*log(abs(-sqrt(d\*x+c) + sqrt(d\*x-c))))/d^8



$$\left. \right) / d^7) * b * c + 24 * (((2 * ((4 * (d * x + c) * (5 * (d * x + c) * (6 * (d * x + c) / d^6 - 43 * c / d^6) + 661 * c^2 / d^6) - 4551 * c^3 / d^6) * (d * x + c) + 4781 * c^4 / d^6) * (d * x + c) - 6335 * c^5 / d^6) * (d * x + c) + 2835 * c^6 / d^6) * \sqrt{d * x + c} * \sqrt{d * x - c} + 1050 * c^7 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^6) * a * d + (((2 * ((4 * (5 * (2 * (d * x + c) * (7 * (d * x + c) * (8 * (d * x + c) / d^8 - 73 * c / d^8) + 2073 * c^2 / d^8) - 9833 * c^3 / d^8) * (d * x + c) + 75293 * c^4 / d^8) * (d * x + c) - 310203 * c^5 / d^8) * (d * x + c) + 216993 * c^6 / d^8) * (d * x + c) - 205275 * c^7 / d^8) * (d * x + c) + 69615 * c^8 / d^8) * \sqrt{d * x + c} * \sqrt{d * x - c} + 22050 * c^9 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^8) * b * d) / d$$

**maple [A]** time = 0.04, size = 92, normalized size = 0.61

$$\frac{(dx + c)^{\frac{3}{2}} (35b x^6 d^6 + 45a d^6 x^4 + 30b c^2 d^4 x^4 + 36a c^2 d^4 x^2 + 24b c^4 d^2 x^2 + 24a c^4 d^2 + 16b c^6) (dx - c)^{\frac{3}{2}}}{315d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x)

[Out] 1/315\*(d\*x+c)^(3/2)\*(35\*b\*d^6\*x^6+45\*a\*d^6\*x^4+30\*b\*c^2\*d^4\*x^4+36\*a\*c^2\*d^4\*x^2+24\*b\*c^4\*d^2\*x^2+24\*a\*c^4\*d^2+16\*b\*c^6)\*(d\*x-c)^(3/2)/d^8

**maxima [A]** time = 0.50, size = 178, normalized size = 1.17

$$\frac{(d^2x^2 - c^2)^{\frac{3}{2}} bx^6}{9d^2} + \frac{2(d^2x^2 - c^2)^{\frac{3}{2}} bc^2x^4}{21d^4} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}} ax^4}{7d^2} + \frac{8(d^2x^2 - c^2)^{\frac{3}{2}} bc^4x^2}{105d^6} + \frac{4(d^2x^2 - c^2)^{\frac{3}{2}} ac^2x^2}{35d^4} + \frac{16(d^2x^2 - c^2)^{\frac{3}{2}} b^2c^6}{315d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="maxima")

[Out] 1/9\*(d^2\*x^2 - c^2)^(3/2)\*b\*x^6/d^2 + 2/21\*(d^2\*x^2 - c^2)^(3/2)\*b\*c^2\*x^4/d^4 + 1/7\*(d^2\*x^2 - c^2)^(3/2)\*a\*x^4/d^2 + 8/105\*(d^2\*x^2 - c^2)^(3/2)\*b\*c^4\*x^2/d^6 + 4/35\*(d^2\*x^2 - c^2)^(3/2)\*a\*c^2\*x^2/d^4 + 16/315\*(d^2\*x^2 - c^2)^(3/2)\*b\*c^6/d^8 + 8/105\*(d^2\*x^2 - c^2)^(3/2)\*a\*c^4/d^6

**mupad [B]** time = 1.76, size = 152, normalized size = 1.00

$$-\sqrt{dx - c} \left( \frac{(16bc^8 + 24ac^6d^2) \sqrt{c + dx}}{315d^8} - \frac{bx^8 \sqrt{c + dx}}{9} + \frac{x^4 (6bc^4d^4 + 9ac^2d^6) \sqrt{c + dx}}{315d^8} + \frac{x^2 (8bc^6d^6 + 16ac^4d^4) \sqrt{c + dx}}{315d^8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(a + b\*x^2)\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2), x)

[Out] -(d\*x - c)^(1/2)\*(((16\*b\*c^8 + 24\*a\*c^6\*d^2)\*(c + d\*x)^(1/2))/(315\*d^8) - (b\*x^8\*(c + d\*x)^(1/2))/9 + (x^4\*(9\*a\*c^2\*d^6 + 6\*b\*c^4\*d^4)\*(c + d\*x)^(1/2))/(315\*d^8) + (x^2\*(12\*a\*c^4\*d^4 + 8\*b\*c^6\*d^2)\*(c + d\*x)^(1/2))/(315\*d^8) - (x^6\*(45\*a\*d^8 - 5\*b\*c^2\*d^6)\*(c + d\*x)^(1/2))/(315\*d^8))

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2), x)

[Out] Integral(x\*\*5\*(a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

### 3.338 $\int x^3 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=109

$$\frac{(dx - c)^{5/2}(c + dx)^{5/2} (ad^2 + 2bc^2)}{5d^6} + \frac{c^2(dx - c)^{3/2}(c + dx)^{3/2} (ad^2 + bc^2)}{3d^6} + \frac{b(dx - c)^{7/2}(c + dx)^{7/2}}{7d^6}$$

[Out]  $1/3*c^2*(a*d^2+b*c^2)*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^6+1/5*(a*d^2+2*b*c^2)*(d*x-c)^{(5/2)}*(d*x+c)^{(5/2)}/d^6+1/7*b*(d*x-c)^{(7/2)}*(d*x+c)^{(7/2)}/d^6$

**Rubi [A]** time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.08, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {460, 100, 12, 74}

$$\frac{x^2(dx - c)^{3/2}(c + dx)^{3/2} (7ad^2 + 4bc^2)}{35d^4} + \frac{2c^2(dx - c)^{3/2}(c + dx)^{3/2} (7ad^2 + 4bc^2)}{105d^6} + \frac{bx^4(dx - c)^{3/2}(c + dx)^{3/2}}{7d^2}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $(2*c^2*(4*b*c^2 + 7*a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(105*d^6) + ((4*b*c^2 + 7*a*d^2)*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(35*d^4) + (b*x^4*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(7*d^2)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 100

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^(p\*(a2 + b2\*x^(n/2)))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{-c+dx} \sqrt{c+dx} (a+bx^2) dx &= \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} - \frac{1}{7} \left( -7a - \frac{4bc^2}{d^2} \right) \int x^3 \sqrt{-c+dx} \sqrt{c+dx} dx \\
&= \frac{(4bc^2+7ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} \\
&= \frac{(4bc^2+7ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4} + \frac{bx^4(-c+dx)^{3/2}(c+dx)^{3/2}}{7d^2} \\
&= \frac{2c^2(4bc^2+7ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{105d^6} + \frac{(4bc^2+7ad^2)x^2(-c+dx)^{3/2}(c+dx)^{3/2}}{35d^4}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 88, normalized size = 0.81

$$\frac{\sqrt{dx-c} \sqrt{c+dx} (d^2x^2 - c^2) (7ad^2 (2c^2 + 3d^2x^2) + b (8c^4 + 12c^2d^2x^2 + 15d^4x^4))}{105d^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(-c^2 + d^2\*x^2)\*(7\*a\*d^2\*(2\*c^2 + 3\*d^2\*x^2) + b\*(8\*c^4 + 12\*c^2\*d^2\*x^2 + 15\*d^4\*x^4)))/(105\*d^6)

**fricas [A]** time = 0.99, size = 90, normalized size = 0.83

$$\frac{(15bd^6x^6 - 8bc^6 - 14ac^4d^2 - 3(bc^2d^4 - 7ad^6)x^4 - (4bc^4d^2 + 7ac^2d^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{105d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/105\*(15\*b\*d^6\*x^6 - 8\*b\*c^6 - 14\*a\*c^4\*d^2 - 3\*(b\*c^2\*d^4 - 7\*a\*d^6)\*x^4 - (4\*b\*c^4\*d^2 + 7\*a\*c^2\*d^4)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d^6

**giac [B]** time = 0.75, size = 495, normalized size = 4.54

$$\frac{70 \left( (dx+c) \left( 2(dx+c) \left( \frac{3(dx+c)}{d^3} - \frac{13c}{d^3} \right) + \frac{43c^2}{d^3} \right) - \frac{39c^3}{d^3} \right) \sqrt{dx+c} \sqrt{dx-c} - \frac{18c^4 \log(|-\sqrt{dx+c} + \sqrt{dx-c}|)}{d^3}}{ac} + 7 \left( \left( 2 \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="giac")

[Out] 1/1680\*(70\*((((d\*x + c)\*(2\*(d\*x + c)\*(3\*(d\*x + c)/d^3 - 13\*c/d^3) + 43\*c^2/d^3) - 39\*c^3/d^3)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 18\*c^4\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^3)\*a\*c + 7\*((((2\*((d\*x + c)\*(4\*(d\*x + c)\*(5\*(d\*x + c)/d^5 - 31\*c/d^5) + 321\*c^2/d^5) - 451\*c^3/d^5)\*(d\*x + c) + 745\*c^4/d^5)\*(d\*x + c) - 405\*c^5/d^5)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 150\*c^6\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^5)\*b\*c + 14\*((((2\*(d\*x + c)\*(3\*(d\*x + c)\*(4\*(d\*x + c)/d^4 - 21\*c/d^4) + 133\*c^2/d^4) - 295\*c^3/d^4)\*(d\*x + c) + 195\*c^4/d^4)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 90\*c^5\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^4)\*a\*d + (((2\*((4\*(d\*x + c)\*(5\*(d\*x + c)\*(6\*(d\*x + c)/d^6 - 43\*c/d^6) + 661\*c^2/d^6) - 4551\*c^3/d^6)\*(d\*x + c) + 4781\*c^4/d^6)\*(d\*x + c) - 6335\*c^5/d^6)\*(d\*x + c) + 2835\*c^6/d^6)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 1050\*c^7\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^6)\*b\*d)/d

**maple** [A] time = 0.05, size = 68, normalized size = 0.62

$$\frac{(dx+c)^{\frac{3}{2}}(15bd^4x^4+21ad^4x^2+12bc^2d^2x^2+14ac^2d^2+8bc^4)(dx-c)^{\frac{3}{2}}}{105d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $1/105*(d*x+c)^{(3/2)}*(15*b*d^4*x^4+21*a*d^4*x^2+12*b*c^2*d^2*x^2+14*a*c^2*d^2+8*b*c^4)*(d*x-c)^{(3/2)}/d^6$

**maxima** [A] time = 0.50, size = 124, normalized size = 1.14

$$\frac{(d^2x^2-c^2)^{\frac{3}{2}}bx^4}{7d^2} + \frac{4(d^2x^2-c^2)^{\frac{3}{2}}bc^2x^2}{35d^4} + \frac{(d^2x^2-c^2)^{\frac{3}{2}}ax^2}{5d^2} + \frac{8(d^2x^2-c^2)^{\frac{3}{2}}bc^4}{105d^6} + \frac{2(d^2x^2-c^2)^{\frac{3}{2}}ac^2}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $1/7*(d^2*x^2-c^2)^{(3/2)}*b*x^4/d^2 + 4/35*(d^2*x^2-c^2)^{(3/2)}*b*c^2*x^2/d^4 + 1/5*(d^2*x^2-c^2)^{(3/2)}*a*x^2/d^2 + 8/105*(d^2*x^2-c^2)^{(3/2)}*b*c^4/d^6 + 2/15*(d^2*x^2-c^2)^{(3/2)}*a*c^2/d^4$

**mupad** [B] time = 1.74, size = 118, normalized size = 1.08

$$-\sqrt{dx-c} \left( \frac{(8bc^6+14ac^4d^2)\sqrt{c+dx}}{105d^6} - \frac{bx^6\sqrt{c+dx}}{7} + \frac{x^2(4bc^4d^2+7ac^2d^4)\sqrt{c+dx}}{105d^6} - \frac{x^4(21ad^6-3b^2c^4d^2)}{105d^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a+b*x^2)*(c+d*x)^(1/2)*(d*x-c)^(1/2),x)`

[Out]  $-(d*x-c)^{(1/2)}*(((8*b*c^6+14*a*c^4*d^2)*(c+d*x)^{(1/2)})/(105*d^6) - (b*x^6*(c+d*x)^{(1/2)})/7 + (x^2*(7*a*c^2*d^4+4*b*c^4*d^2)*(c+d*x)^{(1/2)})/(105*d^6) - (x^4*(21*a*d^6-3*b*c^2*d^4)*(c+d*x)^{(1/2)})/(105*d^6))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)`

[Out] `Integral(x**3*(a+b*x**2)*sqrt(-c+d*x)*sqrt(c+d*x),x)`

### 3.339 $\int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)dx$

**Optimal.** Leaf size=67

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}(ad^2+bc^2)}{3d^4} + \frac{b(dx-c)^{5/2}(c+dx)^{5/2}}{5d^4}$$

[Out]  $1/3*(a*d^2+b*c^2)*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^4+1/5*b*(d*x-c)^{(5/2)}*(d*x+c)^{(5/2)}/d^4$

**Rubi [A]** time = 0.04, antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {460, 74}

$$\frac{(dx-c)^{3/2}(c+dx)^{3/2}(5ad^2+2bc^2)}{15d^4} + \frac{bx^2(dx-c)^{3/2}(c+dx)^{3/2}}{5d^2}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $((2*b*c^2 + 5*a*d^2)*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(15*d^4) + (b*x^2*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(5*d^2)$

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_.))\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 460

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_.)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(q + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int x\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)dx &= \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2} - \frac{1}{5}\left(-5a - \frac{2bc^2}{d^2}\right) \int x\sqrt{-c+dx}\sqrt{c+dx}dx \\ &= \frac{(2bc^2 + 5ad^2)(-c+dx)^{3/2}(c+dx)^{3/2}}{15d^4} + \frac{bx^2(-c+dx)^{3/2}(c+dx)^{3/2}}{5d^2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 62, normalized size = 0.93

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(d^2x^2-c^2)(5ad^2+2bc^2+3bd^2x^2)}{15d^4}$$

Antiderivative was successfully verified.

[In] Integrate[x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $(\sqrt{-c + dx} \sqrt{c + dx} (-c^2 + d^2 x^2) (2bc^2 + 5ad^2 + 3bd^2 x^2)) / (15d^4)$

**fricas** [A] time = 1.14, size = 66, normalized size = 0.99

$$\frac{(3bd^4x^4 - 2bc^4 - 5ac^2d^2 - (bc^2d^2 - 5ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="fricas")`

[Out]  $1/15*(3*b*d^4*x^4 - 2*b*c^4 - 5*a*c^2*d^2 - (b*c^2*d^2 - 5*a*d^4)*x^2)*\sqrt{d*x + c}*\sqrt{d*x - c}/d^4$

**giac** [B] time = 0.37, size = 361, normalized size = 5.39

$$\frac{5 \left( (dx+c) \left( 2(dx+c) \left( \frac{3(dx+c)}{d^3} - \frac{13c}{d^3} \right) + \frac{43c^2}{d^3} \right) - \frac{39c^3}{d^3} \right) \sqrt{dx+c} \sqrt{dx-c} - \frac{18c^4 \log(|-\sqrt{dx+c} + \sqrt{dx-c}|)}{d^3}}{d^3} bc + 20 \left( \sqrt{dx+c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="giac")`

[Out]  $1/120*(5*((d*x + c)*(2*(d*x + c)*(3*(d*x + c)/d^3 - 13*c/d^3) + 43*c^2/d^3) - 39*c^3/d^3)*\sqrt{d*x + c}*\sqrt{d*x - c} - 18*c^4*\log(\text{abs}(-\sqrt{d*x + c} + \sqrt{d*x - c}))/d^3)*b*c + 20*(\sqrt{d*x + c}*\sqrt{d*x - c}*((d*x + c)*(2*(d*x + c)/d^2 - 7*c/d^2) + 9*c^2/d^2) + 6*c^3*\log(\text{abs}(-\sqrt{d*x + c} + \sqrt{d*x - c}))/d^2)*a*d + (((2*(d*x + c)*(3*(d*x + c)*(4*(d*x + c)/d^4 - 21*c/d^4) + 133*c^2/d^4) - 295*c^3/d^4)*(d*x + c) + 195*c^4/d^4)*\sqrt{d*x + c}*\sqrt{d*x - c} + 90*c^5*\log(\text{abs}(-\sqrt{d*x + c} + \sqrt{d*x - c}))/d^4)*b*d - 60*(2*c^2*\log(\text{abs}(-\sqrt{d*x + c} + \sqrt{d*x - c}))) - \sqrt{d*x + c}*\sqrt{d*x - c})*(d*x - 2*c))*a*c/d/d$

**maple** [A] time = 0.05, size = 44, normalized size = 0.66

$$\frac{(dx+c)^{\frac{3}{2}}(3bd^2x^2+5ad^2+2bc^2)(dx-c)^{\frac{3}{2}}}{15d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $1/15*(d*x+c)^{(3/2)}*(3*b*d^2*x^2+5*a*d^2+2*b*c^2)*(d*x-c)^{(3/2)}/d^4$

**maxima** [A] time = 0.62, size = 70, normalized size = 1.04

$$\frac{(d^2x^2 - c^2)^{\frac{3}{2}}bx^2}{5d^2} + \frac{2(d^2x^2 - c^2)^{\frac{3}{2}}bc^2}{15d^4} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}}a}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $1/5*(d^2*x^2 - c^2)^{(3/2)}*b*x^2/d^2 + 2/15*(d^2*x^2 - c^2)^{(3/2)}*b*c^2/d^4 + 1/3*(d^2*x^2 - c^2)^{(3/2)}*a/d^2$

**mupad** [B] time = 1.64, size = 83, normalized size = 1.24

$$\sqrt{dx-c} \left( \frac{bx^4 \sqrt{c+dx}}{5} - \frac{(2bc^4 + 5ac^2d^2) \sqrt{c+dx}}{15d^4} + \frac{x^2 (5ad^4 - bc^2d^2) \sqrt{c+dx}}{15d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2), x)
```

```
[Out] (d*x - c)^(1/2)*((b*x^4*(c + d*x)^(1/2))/5 - ((2*b*c^4 + 5*a*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4) + (x^2*(5*a*d^4 - b*c^2*d^2)*(c + d*x)^(1/2))/(15*d^4))
```

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x(a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2), x)
```

```
[Out] Integral(x*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)
```

$$3.340 \quad \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x} dx$$

Optimal. Leaf size=80

$$a\sqrt{dx-c} \sqrt{c+dx} - ac \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right) + \frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

[Out] 1/3\*b\*(d\*x-c)^(3/2)\*(d\*x+c)^(3/2)/d^2-a\*c\*arctan((d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c)+a\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)

**Rubi [A]** time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {460, 101, 12, 92, 205}

$$a\sqrt{dx-c} \sqrt{c+dx} - ac \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right) + \frac{b(dx-c)^{3/2}(c+dx)^{3/2}}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x,x]

[Out] a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] + (b\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(3\*d^2) - a\*c\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_)]\*Sqrt[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 101

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[((a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/(f\*(m + n + p + 1)), x] - Dist[1/(f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[c\*m\*(b\*e - a\*f) + a\*n\*(d\*e - c\*f) + (d\*m\*(b\*e - a\*f) + b\*n\*(d\*e - c\*f))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,



n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x} dx &= \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} + a \int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x} dx \\
 &= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - a \int \frac{c^2}{x\sqrt{-c+dx}\sqrt{c+dx}} dx \\
 &= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - (ac^2) \int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}} dx \\
 &= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - (ac^2d) \operatorname{Subst}\left(\int \frac{1}{c^2d + \dots} dx\right) \\
 &= a\sqrt{-c+dx}\sqrt{c+dx} + \frac{b(-c+dx)^{3/2}(c+dx)^{3/2}}{3d^2} - ac \tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)
 \end{aligned}$$

**Mathematica [A]** time = 0.24, size = 85, normalized size = 1.06

$$\frac{1}{3}\sqrt{dx-c}\sqrt{c+dx} \left( -\frac{3ac \tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right)}{\sqrt{d^2x^2-c^2}} + 3a + b\left(x^2 - \frac{c^2}{d^2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x,x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(3\*a + b\*(-(c^2/d^2) + x^2) - (3\*a\*c\*ArcTan[Sqrt[-c^2 + d^2\*x^2]/c]))/Sqrt[-c^2 + d^2\*x^2])/3

**fricas [A]** time = 0.94, size = 80, normalized size = 1.00

$$\frac{6acd^2 \arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right) - (bd^2x^2 - bc^2 + 3ad^2)\sqrt{dx+c}\sqrt{dx-c}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x,x, algorithm="fricas")

[Out] -1/3\*(6\*a\*c\*d^2\*arctan(-(d\*x - sqrt(d\*x + c))\*sqrt(d\*x - c))/c) - (b\*d^2\*x^2 - b\*c^2 + 3\*a\*d^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d^2

**giac [A]** time = 0.32, size = 78, normalized size = 0.98

$$2ac \arctan\left(\frac{(\sqrt{dx+c} - \sqrt{dx-c})^2}{2c}\right) + \frac{1}{3}\sqrt{dx+c}\sqrt{dx-c} \left( (dx+c) \left( \frac{(dx+c)b}{d^2} - \frac{2bc}{d^2} \right) + 3a \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x,x, algorithm="giac")

[Out] 2\*a\*c\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c) + 1/3\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*((d\*x + c)\*((d\*x + c)\*b/d^2 - 2\*b\*c/d^2) + 3\*a)

**maple [B]** time = 0.10, size = 174, normalized size = 2.18

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( 3ac^2d^2 \ln \left( -\frac{2(c^2-\sqrt{-c^2} \sqrt{d^2x^2-c^2})}{x} \right) + \sqrt{-c^2} \sqrt{d^2x^2-c^2} b d^2x^2 + 3\sqrt{-c^2} \sqrt{d^2x^2-c^2} a d^2 - \sqrt{-c^2} \right)}{3\sqrt{d^2x^2-c^2} \sqrt{-c^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x,x)

[Out] 1/3\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(x^2\*b\*d^2\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)+3\*ln(-2\*(c^2-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*a\*c^2\*d^2+3\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)\*a\*d^2-b\*c^2\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/(d^2\*x^2-c^2)^(1/2)/d^2/(-c^2)^(1/2)

**maxima [A]** time = 1.30, size = 52, normalized size = 0.65

$$ac \arcsin\left(\frac{c}{d|x|}\right) + \sqrt{d^2x^2-c^2} a + \frac{(d^2x^2-c^2)^{\frac{3}{2}} b}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x,x, algorithm="maxima")

[Out] a\*c\*arcsin(c/(d\*abs(x))) + sqrt(d^2\*x^2 - c^2)\*a + 1/3\*(d^2\*x^2 - c^2)^(3/2)\*b/d^2

**mupad [B]** time = 3.60, size = 248, normalized size = 3.10

$$a\sqrt{-c} \sqrt{c} \ln \left( \frac{(\sqrt{c+dx} - \sqrt{c})^2}{(\sqrt{-c} - \sqrt{dx-c})^2} + 1 \right) - a\sqrt{-c} \sqrt{c} \ln \left( \frac{\sqrt{c+dx} - \sqrt{c}}{\sqrt{-c} - \sqrt{dx-c}} \right) - \frac{b(c^2 - d^2x^2) \sqrt{c+dx} \sqrt{dx-c}}{3d^2} - \frac{(\sqrt{c+dx} - \sqrt{c})^2}{(\sqrt{-c} - \sqrt{dx-c})^2} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x^2)\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2))/x,x)

[Out] a\*(-c)^(1/2)\*c^(1/2)\*log(((c + d\*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + 1) - a\*(-c)^(1/2)\*c^(1/2)\*log(((c + d\*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d\*x - c)^(1/2))) - (b\*(c^2 - d^2\*x^2)\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2))/(3\*d^2) - (8\*a\*(-c)^(1/2)\*c^(1/2)\*((c + d\*x)^(1/2) - c^(1/2))^2)/(((-c)^(1/2) - (d\*x - c)^(1/2))^2\*((c + d\*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d\*x - c)^(1/2))^4 - (2\*((c + d\*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + 1)

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \sqrt{-c + dx} \sqrt{c + dx}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x,x)

[Out] Integral((a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/x, x)

$$3.341 \quad \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^3} dx$$

Optimal. Leaf size=96

$$-\frac{(2bc^2 - ad^2) \tan^{-1}\left(\frac{\sqrt{dx-c} \sqrt{c+dx}}{c}\right)}{2c} - \frac{a\sqrt{dx-c} \sqrt{c+dx}}{2x^2} + b\sqrt{dx-c} \sqrt{c+dx}$$

[Out]  $-1/2*(-a*d^2+2*b*c^2)*\arctan((d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c)/c+b*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}-1/2*a*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/x^2$

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.19, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {454, 101, 12, 92, 205}

$$\frac{1}{2}\sqrt{dx-c} \sqrt{c+dx} \left(2b - \frac{ad^2}{c^2}\right) - \frac{(2bc^2 - ad^2) \tan^{-1}\left(\frac{\sqrt{dx-c} \sqrt{c+dx}}{c}\right)}{2c} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{2c^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^3,x]

[Out]  $((2*b - (a*d^2)/c^2)*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/2 + (a*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(2*c^2*x^2) - ((2*b*c^2 - a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(2*c)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 101

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] := Simp[((a + b\*x)^m\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/(f\*(m + n + p + 1)), x] - Dist[1/(f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p\*Simp[c\*m\*(b\*e - a\*f) + a\*n\*(d\*e - c\*f) + (d\*m\*(b\*e - a\*f) + b\*n\*(d\*e - c\*f))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2\*m, 2\*n, 2\*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_.^(non2\_.)))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_.^(non2\_.)))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.^(n\_.))), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(

$m + 1))$ , `Int[(e*x)^(m+n)*(a1+b1*x^(n/2))^p*(a2+b2*x^(n/2))^p, x]`  
`/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]`

### Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^3} dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2} \left(2b - \frac{ad^2}{c^2}\right) \int \frac{\sqrt{-c+dx} \sqrt{c+dx}}{x} dx \\ &= \frac{1}{2} \left(2b - \frac{ad^2}{c^2}\right) \sqrt{-c+dx} \sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2} \left(-2b + \frac{ad^2}{c^2}\right) \int \frac{1}{x} dx \\ &= \frac{1}{2} \left(2b - \frac{ad^2}{c^2}\right) \sqrt{-c+dx} \sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} + \frac{1}{2} \left(-2bc^2 + ad^2\right) \ln|x| \\ &= \frac{1}{2} \left(2b - \frac{ad^2}{c^2}\right) \sqrt{-c+dx} \sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{1}{2} \left(d(2bc^2 - ad^2)\right) \ln|x| \\ &= \frac{1}{2} \left(2b - \frac{ad^2}{c^2}\right) \sqrt{-c+dx} \sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{2c^2x^2} - \frac{(2bc^2 - ad^2)}{2} \ln|x| \end{aligned}$$

**Mathematica [A]** time = 0.06, size = 114, normalized size = 1.19

$$\frac{\sqrt{dx-c} \sqrt{c+dx} \left( c(a-2bx^2) \sqrt{d^2x^2-c^2} + x^2(2bc^2-ad^2) \tan^{-1}\left(\frac{\sqrt{d^2x^2-c^2}}{c}\right) \right)}{2cx^2\sqrt{d^2x^2-c^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[-c+d*x]*Sqrt[c+d*x]*(a+b*x^2))/x^3,x]`

[Out] `-1/2*(Sqrt[-c+d*x]*Sqrt[c+d*x]*(c*(a-2*b*x^2)*Sqrt[-c^2+d^2*x^2] + (2*b*c^2-a*d^2)*x^2*ArcTan[Sqrt[-c^2+d^2*x^2]/c]))/(c*x^2*Sqrt[-c^2+d^2*x^2])`

**fricas [A]** time = 1.07, size = 85, normalized size = 0.89

$$\frac{2(2bc^2-ad^2)x^2 \arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right) - (2bcx^2-ac)\sqrt{dx+c}\sqrt{dx-c}}{2cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="fricas")`

[Out] `-1/2*(2*(2*b*c^2-a*d^2)*x^2*arctan(-(d*x-sqrt(d*x+c)*sqrt(d*x-c))/c) - (2*b*c*x^2-a*c)*sqrt(d*x+c)*sqrt(d*x-c))/(c*x^2)`

**giac [A]** time = 0.42, size = 157, normalized size = 1.64

$$\frac{\sqrt{dx+c} \sqrt{dx-c} bd + \frac{(2bc^2d-ad^3) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c} + \frac{2(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6 - 4ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2)}{((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^3,x, algorithm="giac")`

[Out]  $(\sqrt{d*x + c}*\sqrt{d*x - c})*b*d + (2*b*c^2*d - a*d^3)*\arctan(1/2*(\sqrt{d*x + c} - \sqrt{d*x - c})^2/c)/c + 2*(a*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^6 - 4*a*c^2*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^2)/((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2)^2/d$

**maple [B]** time = 0.07, size = 182, normalized size = 1.90

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( a d^2 x^2 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) - 2b c^2 x^2 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) - 2\sqrt{-c^2} \sqrt{d^2 x^2 - c^2} b \right)}{2\sqrt{d^2 x^2 - c^2} \sqrt{-c^2} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((b*x^2+a)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/x^3, x)$

[Out]  $-1/2*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}*(\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*a*d^2-2*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*b*c^2-2*x^2*b*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}+(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*a)/(d^2*x^2-c^2)^{(1/2)}/x^2/(-c^2)^{(1/2)}$

**maxima [A]** time = 1.46, size = 98, normalized size = 1.02

$$bc \arcsin\left(\frac{c}{d|x|}\right) - \frac{ad^2 \arcsin\left(\frac{c}{d|x|}\right)}{2c} + \sqrt{d^2x^2 - c^2} b - \frac{\sqrt{d^2x^2 - c^2} ad^2}{2c^2} + \frac{(d^2x^2 - c^2)^{3/2} a}{2c^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((b*x^2+a)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/x^3, x, \text{algorithm}="maxima")$

[Out]  $b*c*\arcsin(c/(d*\text{abs}(x))) - 1/2*a*d^2*\arcsin(c/(d*\text{abs}(x)))/c + \sqrt{d^2*x^2 - c^2}*b - 1/2*\sqrt{d^2*x^2 - c^2}*a*d^2/c^2 + 1/2*(d^2*x^2 - c^2)^{(3/2)}*a/(c^2*x^2)$

**mupad [B]** time = 6.89, size = 584, normalized size = 6.08

$$b \sqrt{-c} \sqrt{c} \ln \left( \frac{(\sqrt{c+dx} - \sqrt{c})^2}{(\sqrt{-c} - \sqrt{dx-c})^2} + 1 \right) - \frac{\frac{a \sqrt{-c} d^2}{32 c^{3/2}} + \frac{a \sqrt{-c} d^2 (\sqrt{c+dx} - \sqrt{c})^2}{16 c^{3/2} (\sqrt{-c} - \sqrt{dx-c})^2} - \frac{15 a \sqrt{-c} d^2 (\sqrt{c+dx} - \sqrt{c})^4}{32 c^{3/2} (\sqrt{-c} - \sqrt{dx-c})^4}}{\frac{(\sqrt{c+dx} - \sqrt{c})^2}{(\sqrt{-c} - \sqrt{dx-c})^2} + \frac{2(\sqrt{c+dx} - \sqrt{c})^4}{(\sqrt{-c} - \sqrt{dx-c})^4} + \frac{(\sqrt{c+dx} - \sqrt{c})^6}{(\sqrt{-c} - \sqrt{dx-c})^6}} - b \sqrt{-c} \sqrt{c} \ln$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(((a + b*x^2)*(c + d*x)^{(1/2)}*(d*x - c)^{(1/2)})/x^3, x)$

[Out]  $b*(-c)^{(1/2)}*c^{(1/2)}*\log(((c + d*x)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1) - ((a*(-c)^{(1/2)}*d^2)/(32*c^{(3/2)}) + (a*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/(16*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) - (15*a*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/(32*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4))/(((c + d*x)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1) - (2*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 + ((c + d*x)^{(1/2)} - c^{(1/2)})^6/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6) - b*(-c)^{(1/2)}*c^{(1/2)}*\log(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})) + (a*(-c)^{(1/2)}*d^2*\log(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})))/(2*c^{(3/2)}) - (a*(-c)^{(1/2)}*d^2*\log(((c + d*x)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1))/(2*c^{(3/2)}) - (a*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/(32*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) - (8*b*(-c)^{(1/2)}*c^{(1/2)}*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/(((c + d*x)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1) - (2*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 - (2*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \sqrt{-c + dx} \sqrt{c + dx}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*3,x)

[Out] Integral((a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x)/x\*\*3, x)

$$3.342 \quad \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^5} dx$$

**Optimal.** Leaf size=121

$$-\frac{\sqrt{dx-c} \sqrt{c+dx} (ad^2 + 4bc^2)}{8c^2x^2} + \frac{d^2 (ad^2 + 4bc^2) \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right)}{8c^3} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

[Out] 1/4\*a\*(d\*x-c)^(3/2)\*(d\*x+c)^(3/2)/c^2/x^4+1/8\*d^2\*(a\*d^2+4\*b\*c^2)\*arctan((d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c)/c^3-1/8\*(a\*d^2+4\*b\*c^2)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^2/x^2

**Rubi [A]** time = 0.10, antiderivative size = 164, normalized size of antiderivative = 1.36, number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {454, 94, 92, 205}

$$-\frac{\sqrt{dx-c} (c+dx)^{3/2} (ad^2 + 4bc^2)}{8c^3x^2} + \frac{d\sqrt{dx-c} \sqrt{c+dx} (ad^2 + 4bc^2)}{8c^3x} + \frac{d^2 (ad^2 + 4bc^2) \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right)}{8c^3} + \frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{4c^2x^4}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^5,x]

[Out] (d\*(4\*b\*c^2 + a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*c^3\*x) - ((4\*b\*c^2 + a\*d^2)\*Sqrt[-c + d\*x]\*(c + d\*x)^(3/2))/(8\*c^3\*x^2) + (a\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(4\*c^2\*x^4) + (d^2\*(4\*b\*c^2 + a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(8\*c^3)

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] :> Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 94

Int[((a\_.) + (b\_.)\*(x\_.))^(m\_.)\*((c\_.) + (d\_.)\*(x\_.))^(n\_.)\*((e\_.) + (f\_.)\*(x\_.))^(p\_.), x\_Symbol] :> Simp[((a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*e - a\*f)), x] - Dist[(n\*(d\*e - c\*f))/((m + 1)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

#### Rule 205

Int[((a\_.) + (b\_.)\*(x\_.)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^5} dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{4} \left( 4b + \frac{ad^2}{c^2} \right) \int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^3} dx \\
&= -\frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} + \frac{1}{8} \left( d \left( 4b + \frac{ad^2}{c^2} \right) \right) \\
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} \\
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4} \\
&= \frac{d(4bc^2+ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{8c^3x} - \frac{(4bc^2+ad^2)\sqrt{-c+dx}(c+dx)^{3/2}}{8c^3x^2} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{4c^2x^4}
\end{aligned}$$

**Mathematica [A]** time = 0.09, size = 137, normalized size = 1.13

$$\frac{\sqrt{dx-c}\sqrt{c+dx} \left( (c^2-d^2x^2)(2ac^2-ad^2x^2+4bc^2x^2) - d^2x^4\sqrt{1-\frac{d^2x^2}{c^2}}(ad^2+4bc^2)\tanh^{-1}\left(\sqrt{1-\frac{d^2x^2}{c^2}}\right) \right)}{8c^2d^2x^6-8c^4x^4}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*(a+b\*x^2))/x^5,x]

[Out] (Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*((c^2-d^2\*x^2)\*(2\*a\*c^2+4\*b\*c^2\*x^2-a\*d^2\*x^2)-d^2\*(4\*b\*c^2+a\*d^2)\*x^4\*Sqrt[1-(d^2\*x^2)/c^2]\*ArcTanh[Sqrt[1-(d^2\*x^2)/c^2]]))/(-8\*c^4\*x^4+8\*c^2\*d^2\*x^6)

**fricas [A]** time = 1.09, size = 100, normalized size = 0.83

$$\frac{2(4bc^2d^2+ad^4)x^4 \arctan\left(-\frac{dx-\sqrt{dx+c}\sqrt{dx-c}}{c}\right) - (2ac^3+(4bc^3-acd^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{8c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^5,x, algorithm="fricas")

[Out] 1/8\*(2\*(4\*b\*c^2\*d^2+a\*d^4)\*x^4\*arctan(-(d\*x-sqrt(d\*x+c))\*sqrt(d\*x-c))/c)-(2\*a\*c^3+(4\*b\*c^3-a\*c\*d^2)\*x^2)\*sqrt(d\*x+c)\*sqrt(d\*x-c)/(c^3\*x^4)

**giac [B]** time = 0.45, size = 324, normalized size = 2.68

$$\frac{(4bc^2d^3+ad^5)\arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^3} - \frac{2(4bc^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^{14}-ad^5(\sqrt{dx+c}-\sqrt{dx-c})^{14}+16bc^4d^3(\sqrt{dx+c}-\sqrt{dx-c})^{10}+28ac^2d^5(\sqrt{dx+c}-\sqrt{dx-c})^6)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^5,x, algorithm="giac")

[Out] -1/4\*((4\*b\*c^2\*d^3+a\*d^5)\*arctan(1/2\*(sqrt(d\*x+c)-sqrt(d\*x-c))^2/c)/c^3-2\*(4\*b\*c^2\*d^3\*(sqrt(d\*x+c)-sqrt(d\*x-c))^14-a\*d^5\*(sqrt(d\*x+c)-sqrt(d\*x-c))^14+16\*b\*c^4\*d^3\*(sqrt(d\*x+c)-sqrt(d\*x-c))^10+28\*a\*c^2\*d^5\*(sqrt(d\*x+c)-sqrt(d\*x-c))^6))



$$\frac{+ 28*a*c^2*d^5*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^10 - 64*b*c^6*d^3*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^6 - 112*a*c^4*d^5*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^6 - 256*b*c^8*d^3*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + 64*a*c^6*d^5*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2}{((\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^4 + 4*c^2)^4*c^2}/d$$

**maple [B]** time = 0.07, size = 226, normalized size = 1.87

$$\frac{\sqrt{dx - c} \sqrt{dx + c} \left( a d^4 x^4 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) + 4b c^2 d^2 x^4 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2} \right)}{8\sqrt{d^2 x^2 - c^2} \sqrt{-c^2} c^2 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^5,x)

[Out] -1/8\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^2\*(ln(-2\*(c^2-(c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^4\*a\*d^4+4\*ln(-2\*(c^2-(c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2))/x)\*x^4\*b\*c^2\*d^2-(c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)\*x^2\*a\*d^2+4\*(c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)\*x^2\*b\*c^2+2\*(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)\*a\*c^2)/(d^2\*x^2-c^2)^(1/2)/x^4/(-c^2)^(1/2)

**maxima [A]** time = 1.35, size = 162, normalized size = 1.34

$$\frac{bd^2 \arcsin\left(\frac{c}{d|x|}\right) - ad^4 \arcsin\left(\frac{c}{d|x|}\right) - \sqrt{d^2x^2 - c^2} bd^2 - \sqrt{d^2x^2 - c^2} ad^4}{2c} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}} b}{2c^2x^2} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}} ad^2}{8c^4x^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^5,x, algorithm="maxima")

[Out] -1/2\*b\*d^2\*arcsin(c/(d\*abs(x)))/c - 1/8\*a\*d^4\*arcsin(c/(d\*abs(x)))/c^3 - 1/2\*sqrt(d^2\*x^2 - c^2)\*b\*d^2/c^2 - 1/8\*sqrt(d^2\*x^2 - c^2)\*a\*d^4/c^4 + 1/2\*(d^2\*x^2 - c^2)^(3/2)\*b/(c^2\*x^2) + 1/8\*(d^2\*x^2 - c^2)^(3/2)\*a\*d^2/(c^4\*x^2) + 1/4\*(d^2\*x^2 - c^2)^(3/2)\*a/(c^2\*x^4)

**mupad [B]** time = 15.56, size = 1004, normalized size = 8.30

$$\frac{\frac{a\sqrt{-c}d^4}{1024c^{7/2}} + \frac{a\sqrt{-c}d^4(\sqrt{c+dx}-\sqrt{c})^2}{128c^{7/2}(\sqrt{-c}-\sqrt{dx-c})^2} + \frac{11a\sqrt{-c}d^4(\sqrt{c+dx}-\sqrt{c})^4}{512c^{7/2}(\sqrt{-c}-\sqrt{dx-c})^4} + \frac{7a\sqrt{-c}d^4(\sqrt{c+dx}-\sqrt{c})^6}{256c^{7/2}(\sqrt{-c}-\sqrt{dx-c})^6} - \frac{239a\sqrt{-c}d^4(\sqrt{c+dx}-\sqrt{c})^8}{1024c^{7/2}(\sqrt{-c}-\sqrt{dx-c})^8} + \frac{a\sqrt{-c}d^4(\sqrt{c+dx}-\sqrt{c})^{10}}{256c^{7/2}(\sqrt{-c}-\sqrt{dx-c})^{10}}}{\frac{(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{-c}-\sqrt{dx-c})^4} + \frac{4(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{-c}-\sqrt{dx-c})^6} + \frac{6(\sqrt{c+dx}-\sqrt{c})^8}{(\sqrt{-c}-\sqrt{dx-c})^8} + \frac{4(\sqrt{c+dx}-\sqrt{c})^{10}}{(\sqrt{-c}-\sqrt{dx-c})^{10}} + \frac{(\sqrt{c+dx}-\sqrt{c})^{12}}{(\sqrt{-c}-\sqrt{dx-c})^{12}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x^2)\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2))/x^5,x)

[Out] ((a\*(-c)^(1/2)\*d^4)/(1024\*c^(7/2)) + (a\*(-c)^(1/2)\*d^4\*((c + d\*x)^(1/2) - c^(1/2))^2)/(128\*c^(7/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^2) + (11\*a\*(-c)^(1/2)\*d^4\*((c + d\*x)^(1/2) - c^(1/2))^4)/(512\*c^(7/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^4) + (7\*a\*(-c)^(1/2)\*d^4\*((c + d\*x)^(1/2) - c^(1/2))^6)/(256\*c^(7/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^6) - (239\*a\*(-c)^(1/2)\*d^4\*((c + d\*x)^(1/2) - c^(1/2))^8)/(1024\*c^(7/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^8) + (a\*(-c)^(1/2)\*d^4\*((c + d\*x)^(1/2) - c^(1/2))^10)/(256\*c^(7/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^10)/(((c + d\*x)^(1/2) - c^(1/2))^4/((-c)^(1/2) - (d\*x - c)^(1/2))^4 + 4\*((c + d\*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d\*x - c)^(1/2))^6 + 6\*((c + d\*x)^(1/2) - c^(1/2))^8/((-c)^(1/2) - (d\*x - c)^(1/2))^8 + 4\*((c + d\*x)^(1/2) - c^(1/2))^10/((-c)^(1/2) - (d\*x - c)^(1/2))^10 + ((c + d\*x)^(1/2) - c^(1/2))^12/((-c)^(1/2) - (d\*x - c)^(1/2))^12) - ((b\*(-c)^(1/2)\*d^2)

$$\begin{aligned} &/ (32*c^{(3/2)}) + (b*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^2) / (16*c^{(3/2)} \\ &)*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 - (15*b*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} \\ &- c^{(1/2)})^4) / (32*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4) / (((c + d*x)^{(1/2)} \\ &- c^{(1/2)})^2 / ((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (2*((c + d*x)^{(1/2)} - \\ &c^{(1/2)})^4) / ((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 + ((c + d*x)^{(1/2)} - c^{(1/2)})^6 \\ &/ ((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6) + (a*(-c)^{(1/2)}*d^4*\log(((c + d*x)^{(1/2)} \\ &- c^{(1/2)}) / ((-c)^{(1/2)} - (d*x - c)^{(1/2)}))) / (8*c^{(7/2)}) + (b*(-c)^{(1/2)}*d \\ &^2*\log(((c + d*x)^{(1/2)} - c^{(1/2)}) / ((-c)^{(1/2)} - (d*x - c)^{(1/2)}))) / (2*c^{(3/2)}) \\ &- (a*(-c)^{(1/2)}*d^4*\log(((c + d*x)^{(1/2)} - c^{(1/2)})^2 / ((-c)^{(1/2)} - (d \\ &*x - c)^{(1/2)})^2 + 1)) / (8*c^{(7/2)}) - (b*(-c)^{(1/2)}*d^2*\log(((c + d*x)^{(1/2)} \\ &- c^{(1/2)})^2 / ((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1)) / (2*c^{(3/2)}) + (a*(-c)^{(1/2)} \\ &*d^4*((c + d*x)^{(1/2)} - c^{(1/2)})^2) / (256*c^{(7/2)}*((-c)^{(1/2)} - (d*x - \\ &c)^{(1/2)})^2) + (a*(-c)^{(1/2)}*d^4*((c + d*x)^{(1/2)} - c^{(1/2)})^4) / (1024*c^{(7/2)} \\ &*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4) - (b*(-c)^{(1/2)}*d^2*((c + d*x)^{(1/2)} - \\ &c^{(1/2)})^2) / (32*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) \end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*5,x)

[Out] Timed out

### 3.343 $\int x^4 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=208

$$\frac{c^2 x(dx-c)^{3/2}(c+dx)^{3/2}(8ad^2+5bc^2)}{64d^6} + \frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}(8ad^2+5bc^2)}{48d^4} - \frac{c^6(8ad^2+5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{64d^7}$$

[Out]  $1/64*c^2*(8*a*d^2+5*b*c^2)*x*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^6+1/48*(8*a*d^2+5*b*c^2)*x^3*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^4+1/8*b*x^5*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^2-1/64*c^6*(8*a*d^2+5*b*c^2)*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2)})/d^7+1/128*c^4*(8*a*d^2+5*b*c^2)*x*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^6$

**Rubi [A]** time = 0.15, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {460, 100, 12, 90, 38, 63, 217, 206}

$$\frac{x^3(dx-c)^{3/2}(c+dx)^{3/2}(8ad^2+5bc^2)}{48d^4} + \frac{c^4 x \sqrt{dx-c} \sqrt{c+dx} (8ad^2+5bc^2)}{128d^6} + \frac{c^2 x(dx-c)^{3/2}(c+dx)^{3/2}(8ad^2+5bc^2)}{64d^6}$$

Antiderivative was successfully verified.

[In] Int[x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out]  $(c^4*(5*b*c^2 + 8*a*d^2)*x*\operatorname{Sqrt}[-c + d*x]*\operatorname{Sqrt}[c + d*x])/(128*d^6) + (c^2*(5*b*c^2 + 8*a*d^2)*x*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(64*d^6) + ((5*b*c^2 + 8*a*d^2)*x^3*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(48*d^4) + (b*x^5*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(8*d^2) - (c^6*(5*b*c^2 + 8*a*d^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[-c + d*x]/\operatorname{Sqrt}[c + d*x]])/(64*d^7)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 38

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := Simp[(x\*(a + b\*x)^m\*(c + d\*x)^n)/(2\*m + 1), x] + Dist[(2\*a\*c\*m)/(2\*m + 1), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 90

Int[((a\_.) + (b\_.)\*(x\_))^(2\*...)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 3)), x] + Dist[1/(d\*f\*(n + p + 3)), Int[(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(n + p + 3) - b\*(b\*c\*e + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(n + p + 4) - b\*(d\*e\*(n + 2) + c\*f\*(p + 2)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 460

```
Int[((e_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m + n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rubi steps

$$\begin{aligned}
 \int x^4 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx &= \frac{bx^5(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^2} - \frac{1}{8} \left( -8a - \frac{5bc^2}{d^2} \right) \int x^4 \sqrt{-c + dx} \sqrt{c + dx} dx \\
 &= \frac{(5bc^2 + 8ad^2) x^3(-c + dx)^{3/2}(c + dx)^{3/2}}{48d^4} + \frac{bx^5(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^2} + \frac{(5bc^2 + 8ad^2) x^3(-c + dx)^{3/2}(c + dx)^{3/2}}{48d^4} + \frac{bx^5(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^2} + \frac{(5bc^2 + 8ad^2) x^3(-c + dx)^{3/2}(c + dx)^{3/2}}{48d^4} \\
 &= \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6} + \frac{(5bc^2 + 8ad^2) x^3(-c + dx)^{3/2}(c + dx)^{3/2}}{48d^4} \\
 &= \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6} + \frac{(5bc^2 + 8ad^2) x^3(-c + dx)^{3/2}(c + dx)^{3/2}}{48d^4} \\
 &= \frac{c^4(5bc^2 + 8ad^2) x \sqrt{-c + dx} \sqrt{c + dx}}{128d^6} + \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6} \\
 &= \frac{c^4(5bc^2 + 8ad^2) x \sqrt{-c + dx} \sqrt{c + dx}}{128d^6} + \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6} \\
 &= \frac{c^4(5bc^2 + 8ad^2) x \sqrt{-c + dx} \sqrt{c + dx}}{128d^6} + \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6} \\
 &= \frac{c^4(5bc^2 + 8ad^2) x \sqrt{-c + dx} \sqrt{c + dx}}{128d^6} + \frac{c^2(5bc^2 + 8ad^2) x(-c + dx)^{3/2}(c + dx)^{3/2}}{64d^6}
 \end{aligned}$$

**Mathematica [A]** time = 0.31, size = 161, normalized size = 0.77

$$\frac{\sqrt{dx-c} \sqrt{c+dx} \left( 3(8ac^5d^2 + 5bc^7) \sin^{-1}\left(\frac{dx}{c}\right) - dx\sqrt{1 - \frac{d^2x^2}{c^2}} (8ad^2(3c^4 + 2c^2d^2x^2 - 8d^4x^4) + b(15c^6 + 10c^4) \right)}{384d^7\sqrt{1 - \frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(-(d\*x\*Sqrt[1 - (d^2\*x^2)/c^2])\*(8\*a\*d^2\*(3\*c^4 + 2\*c^2\*d^2\*x^2 - 8\*d^4\*x^4) + b\*(15\*c^6 + 10\*c^4\*d^2\*x^2 + 8\*c^2\*d^4\*x^4 - 48\*d^6\*x^6))) + 3\*(5\*b\*c^7 + 8\*a\*c^5\*d^2)\*ArcSin[(d\*x)/c])/(384\*d^7\*Sqrt[1 - (d^2\*x^2)/c^2])

**fricas [A]** time = 1.32, size = 138, normalized size = 0.66

$$\frac{(48bd^7x^7 - 8(bc^2d^5 - 8ad^7)x^5 - 2(5bc^4d^3 + 8ac^2d^5)x^3 - 3(5bc^6d + 8ac^4d^3)x)\sqrt{dx+c}\sqrt{dx-c} + 3(5bc^8 - 8ad^7)}{384d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/384\*((48\*b\*d^7\*x^7 - 8\*(b\*c^2\*d^5 - 8\*a\*d^7)\*x^5 - 2\*(5\*b\*c^4\*d^3 + 8\*a\*c^2\*d^5)\*x^3 - 3\*(5\*b\*c^6\*d + 8\*a\*c^4\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 3\*(5\*b\*c^8 + 8\*a\*c^6\*d^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/d^7

**giac [B]** time = 0.63, size = 558, normalized size = 2.68

$$112 \left( \left( \left( 2(dx+c) \left( 3(dx+c) \left( \frac{4(dx+c)}{d^4} - \frac{21c}{d^4} \right) + \frac{133c^2}{d^4} \right) - \frac{295c^3}{d^4} \right) (dx+c) + \frac{195c^4}{d^4} \right) \sqrt{dx+c} \sqrt{dx-c} + \frac{90c^5 \log\left(\left| -\sqrt{dx-c} \right|\right)}{d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="giac")

[Out] 1/13440\*(112\*((2\*(d\*x + c)\*(3\*(d\*x + c)\*(4\*(d\*x + c)/d^4 - 21\*c/d^4) + 133\*c^2/d^4) - 295\*c^3/d^4)\*(d\*x + c) + 195\*c^4/d^4)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 90\*c^5\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^4)\*a\*c + 8\*((2\*((4\*(d\*x + c)\*(5\*(d\*x + c)\*(6\*(d\*x + c)/d^6 - 43\*c/d^6) + 661\*c^2/d^6) - 4551\*c^3/d^6)\*(d\*x + c) + 4781\*c^4/d^6)\*(d\*x + c) - 6335\*c^5/d^6)\*(d\*x + c) + 2835\*c^6/d^6)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 1050\*c^7\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^6)\*b\*c + 56\*((2\*((2\*(d\*x + c)\*(4\*(d\*x + c)\*(5\*(d\*x + c)/d^5 - 31\*c/d^5) + 321\*c^2/d^5) - 451\*c^3/d^5)\*(d\*x + c) + 745\*c^4/d^5)\*(d\*x + c) - 405\*c^5/d^5)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 150\*c^6\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^5)\*a\*d + (((2\*((4\*(5\*(d\*x + c)\*(6\*(d\*x + c)\*(7\*(d\*x + c)/d^7 - 57\*c/d^7) + 1219\*c^2/d^7) - 12463\*c^3/d^7)\*(d\*x + c) + 64233\*c^4/d^7)\*(d\*x + c) - 53963\*c^5/d^7)\*(d\*x + c) + 59465\*c^6/d^7)\*(d\*x + c) - 23205\*c^7/d^7)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 7350\*c^8\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^7)\*b\*d)/d

**maple [C]** time = 0.10, size = 298, normalized size = 1.43

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( 48\sqrt{d^2x^2 - c^2} b d^7 x^7 \operatorname{csgn}(d) + 64\sqrt{d^2x^2 - c^2} a d^7 x^5 \operatorname{csgn}(d) - 8\sqrt{d^2x^2 - c^2} b c^2 d^5 x^5 \operatorname{csgn}(d) \right)}{384d^7\sqrt{1 - \frac{d^2x^2}{c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{384}(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}*(48*c*sgn(d)*x^7*b*d^7*(d^2*x^2-c^2)^{(1/2)}+64*c*sgn(d)*x^5*a*d^7*(d^2*x^2-c^2)^{(1/2)}-8*c*sgn(d)*x^5*b*c^2*d^5*(d^2*x^2-c^2)^{(1/2)}-16*c*sgn(d)*x^3*a*c^2*d^5*(d^2*x^2-c^2)^{(1/2)}-10*c*sgn(d)*x^3*b*c^4*d^3*(d^2*x^2-c^2)^{(1/2)}-24*c*sgn(d)*d^3*(d^2*x^2-c^2)^{(1/2)}*x*a*c^4-15*c*sgn(d)*d*(d^2*x^2-c^2)^{(1/2)}*x*b*c^6-24*\ln((c*sgn(d)*(d^2*x^2-c^2)^{(1/2)}+d*x)*c*sgn(d))*a*c^6*d^2-15*\ln((c*sgn(d)*(d^2*x^2-c^2)^{(1/2)}+d*x)*c*sgn(d))*b*c^8)*c*sgn(d)/(d^2*x^2-c^2)^{(1/2)}/d^7$

**maxima [A]** time = 0.55, size = 246, normalized size = 1.18

$$\frac{(d^2x^2 - c^2)^{\frac{3}{2}}bx^5}{8d^2} + \frac{5(d^2x^2 - c^2)^{\frac{3}{2}}bc^2x^3}{48d^4} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}}ax^3}{6d^2} - \frac{5bc^8 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right)}{128d^7} - \frac{ac^6 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right)}{16d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{8}(d^2*x^2 - c^2)^{(3/2)}*b*x^5/d^2 + \frac{5}{48}(d^2*x^2 - c^2)^{(3/2)}*b*c^2*x^3/d^4 + \frac{1}{6}(d^2*x^2 - c^2)^{(3/2)}*a*x^3/d^2 - \frac{5}{128}*b*c^8*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d^7 - \frac{1}{16}*a*c^6*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d^5 + \frac{5}{128}*\sqrt{d^2*x^2 - c^2}*b*c^6*x/d^6 + \frac{1}{16}*\sqrt{d^2*x^2 - c^2}*a*c^4*x/d^4 + \frac{5}{64}*(d^2*x^2 - c^2)^{(3/2)}*b*c^4*x/d^6 + \frac{1}{8}*(d^2*x^2 - c^2)^{(3/2)}*a*c^2*x/d^4$

**mupad [B]** time = 39.15, size = 2314, normalized size = 11.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)`

[Out]  $((35*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^3)/(12*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^3) - (a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)}))/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})) + (757*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^5)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^5) + (7339*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^7)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^7) + (41929*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^9)/(6*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^9) + (25661*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^11)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^11) + (25661*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^13)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^13) + (41929*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^15)/(6*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^15) + (7339*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^17)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^17) + (757*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^19)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^19) + (35*a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^21)/(12*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^21) - (a*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^23)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^23))/((d^5 - (12*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (66*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 - (220*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^6)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6 + (495*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^8)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^8 - (792*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^10)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^10 + (924*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^12)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^12 - (792*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^14)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^14 + (495*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^16)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^16 - (220*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^18)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^18 + (66*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^20)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^20 - (12*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^22)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^22 + (d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^24)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^24) - ((5*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)}))/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})) - (235*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^3)/(96*((-c)^{(1/2)} - (d*x - c)^{(1/2)}))$

$$\begin{aligned}
& -c)^{(1/2)} - (d*x - c)^{(1/2)})^3 + (1723*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^5 \\
& )/(96*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^5) + (72283*b*c^8*((c + d*x)^{(1/2)} - c \\
& ^{(1/2)})^7)/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^7) + (848801*b*c^8*((c + d*x) \\
& ^{(1/2)} - c^{(1/2)})^9)/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^9) + (4181067*b*c^8 \\
& *((c + d*x)^{(1/2)} - c^{(1/2)})^{11})/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{11}) + ( \\
& 10994181*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{13})/(32*((-c)^{(1/2)} - (d*x - c) \\
& ^{(1/2)})^{13}) + (17457599*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{15})/(32*((-c)^{(1/2)} \\
& ) - (d*x - c)^{(1/2)})^{15}) + (17457599*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{17})/ \\
& (32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{17}) + (10994181*b*c^8*((c + d*x)^{(1/2)} - \\
& c^{(1/2)})^{19})/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{19}) + (4181067*b*c^8*((c + \\
& d*x)^{(1/2)} - c^{(1/2)})^{21})/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{21}) + (848801 \\
& *b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{23})/(32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{23}) \\
& + (72283*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{25})/(32*((-c)^{(1/2)} - (d*x - \\
& c)^{(1/2)})^{25}) + (1723*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{27})/(96*((-c)^{(1/2)} \\
& - (d*x - c)^{(1/2)})^{27}) - (235*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{29})/(96*(( \\
& -c)^{(1/2)} - (d*x - c)^{(1/2)})^{29}) + (5*b*c^8*((c + d*x)^{(1/2)} - c^{(1/2)})^{31}) \\
& /((32*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{31}))/((d^7 - (16*d^7*((c + d*x)^{(1/2)} - \\
& c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (120*d^7*((c + d*x)^{(1/2)} - \\
& c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 - (560*d^7*((c + d*x)^{(1/2)} - \\
& c^{(1/2)})^6)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6 + (1820*d^7*((c + d*x)^{(1/2)} - \\
& c^{(1/2)})^8)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^8 - (4368*d^7*((c + d*x)^{(1/2)} \\
& - c^{(1/2)})^{10})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{10} + (8008*d^7*((c + d*x)^{(1/2)} \\
& - c^{(1/2)})^{12})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{12} - (11440*d^7*((c + d*x) \\
& ^{(1/2)} - c^{(1/2)})^{14})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{14} + (12870*d^7*((c + \\
& d*x)^{(1/2)} - c^{(1/2)})^{16})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{16} - (11440*d^7*(( \\
& c + d*x)^{(1/2)} - c^{(1/2)})^{18})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{18} + (8008*d^7 \\
& *((c + d*x)^{(1/2)} - c^{(1/2)})^{20})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{20} - (4368* \\
& d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{22})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{22} + (18 \\
& 20*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{24})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{24} - \\
& (560*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{26})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{26} \\
& + (120*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{28})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{28} \\
& - (16*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{30})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{30} \\
& + (d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{32})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{32} \\
& ) + (a*c^6*atanh(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})) \\
& ))/(4*d^5) + (5*b*c^8*atanh(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x \\
& - c)^{(1/2)})))/(32*d^7)
\end{aligned}$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2), x)

[Out] Integral(x\*\*4\*(a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

### 3.344 $\int x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=159

$$\frac{c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)}{16d^4} + \frac{x(dx - c)^{3/2} (c + dx)^{3/2} (2ad^2 + bc^2)}{8d^4} - \frac{c^4 (2ad^2 + bc^2) \tanh^{-1} \left( \frac{\sqrt{dx - c}}{\sqrt{c + dx}} \right)}{8d^5} + \frac{bx^3 (dx - c)^{3/2} (c + dx)^{3/2}}{8d^5}$$

[Out]  $1/8*(2*a*d^2+b*c^2)*x*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^4+1/6*b*x^3*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/d^2-1/8*c^4*(2*a*d^2+b*c^2)*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2)})/d^5+1/16*c^2*(2*a*d^2+b*c^2)*x*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^4$

**Rubi [A]** time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {460, 90, 12, 38, 63, 217, 206}

$$\frac{c^2 x \sqrt{dx - c} \sqrt{c + dx} (2ad^2 + bc^2)}{16d^4} + \frac{x(dx - c)^{3/2} (c + dx)^{3/2} (2ad^2 + bc^2)}{8d^4} - \frac{c^4 (2ad^2 + bc^2) \tanh^{-1} \left( \frac{\sqrt{dx - c}}{\sqrt{c + dx}} \right)}{8d^5} + \frac{bx^3 (dx - c)^{3/2} (c + dx)^{3/2}}{8d^5}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^2 \operatorname{Sqrt}[-c + d*x] \operatorname{Sqrt}[c + d*x] (a + b*x^2), x]$

[Out]  $(c^2*(b*c^2 + 2*a*d^2)*x*\operatorname{Sqrt}[-c + d*x]*\operatorname{Sqrt}[c + d*x])/(16*d^4) + ((b*c^2 + 2*a*d^2)*x*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(8*d^4) + (b*x^3*(-c + d*x)^{(3/2)}*(c + d*x)^{(3/2)})/(6*d^2) - (c^4*(b*c^2 + 2*a*d^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[-c + d*x]/\operatorname{Sqrt}[c + d*x]])/(8*d^5)$

#### Rule 12

$\operatorname{Int}[(a_*)*(u_), x\_Symbol] \rightarrow \operatorname{Dist}[a, \operatorname{Int}[u, x], x] /;$  FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 38

$\operatorname{Int}[(a_*) + (b_*)*(x_)^m * ((c_*) + (d_*)*(x_)^n), x\_Symbol] \rightarrow \operatorname{Simp}[(x*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + \operatorname{Dist}[(2*a*c*m)/(2*m + 1), \operatorname{Int}[(a + b*x)^{m-1}*(c + d*x)^{m-1}, x], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

#### Rule 63

$\operatorname{Int}[(a_*) + (b_*)*(x_)^m * ((c_*) + (d_*)*(x_)^n), x\_Symbol] \rightarrow \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 90

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2 * ((c_*) + (d_*)*(x_)^n) * ((e_*) + (f_*)*(x_)^p), x\_Symbol] \rightarrow \operatorname{Simp}[(b*(a + b*x)*(c + d*x)^{(n+1)}*(e + f*x)^{(p+1)})/(d*f*(n + p + 3)), x] + \operatorname{Dist}[1/(d*f*(n + p + 3)), \operatorname{Int}[(c + d*x)^n*(e + f*x)^p * \operatorname{Simp}[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))]*x, x], x] /;$  FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 206

$\operatorname{Int}[(a_*) + (b_*)*(x_)^2 * (-1), x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && Gt



Q[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 460

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rubi steps

$$\begin{aligned} \int x^2 \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx &= \frac{bx^3(-c + dx)^{3/2}(c + dx)^{3/2}}{6d^2} + \frac{1}{2} \left( 2a + \frac{bc^2}{d^2} \right) \int x^2 \sqrt{-c + dx} \sqrt{c + dx} dx \\ &= \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} + \frac{bx^3(-c + dx)^{3/2}(c + dx)^{3/2}}{6d^2} + \dots \\ &= \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} + \frac{bx^3(-c + dx)^{3/2}(c + dx)^{3/2}}{6d^2} + \dots \\ &= \frac{c^2(bc^2 + 2ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{16d^4} + \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} \\ &= \frac{c^2(bc^2 + 2ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{16d^4} + \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} \\ &= \frac{c^2(bc^2 + 2ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{16d^4} + \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} \\ &= \frac{c^2(bc^2 + 2ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{16d^4} + \frac{(bc^2 + 2ad^2)x(-c + dx)^{3/2}(c + dx)^{3/2}}{8d^4} \end{aligned}$$

**Mathematica [A]** time = 0.19, size = 135, normalized size = 0.85

$$\frac{\sqrt{dx - c} \sqrt{c + dx} \left( 3(2ac^3d^2 + bc^5) \sin^{-1}\left(\frac{dx}{c}\right) + dx \sqrt{1 - \frac{d^2x^2}{c^2}} (b(-3c^4 - 2c^2d^2x^2 + 8d^4x^4) - 6ad^2(c^2 - 2d^2x^2)) \right)}{48d^5 \sqrt{1 - \frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(d\*x\*Sqrt[1 - (d^2\*x^2)/c^2]\*(-6\*a\*d^2\*(c^2 - 2\*d^2\*x^2) + b\*(-3\*c^4 - 2\*c^2\*d^2\*x^2 + 8\*d^4\*x^4)) + 3\*(b\*c^5 + 2\*a\*c^3\*d^2)\*ArcSin[(d\*x)/c])/(48\*d^5\*Sqrt[1 - (d^2\*x^2)/c^2])

**fricas [A]** time = 1.50, size = 112, normalized size = 0.70

$$\frac{(8bd^5x^5 - 2(bc^2d^3 - 6ad^5)x^3 - 3(bc^4d + 2ac^2d^3)x)\sqrt{dx + c}\sqrt{dx - c} + 3(bc^6 + 2ac^4d^2)\log(-dx + \sqrt{dx + c})}{48d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $\frac{1}{48} * ((8 * b * d^5 * x^5 - 2 * (b * c^2 * d^3 - 6 * a * d^5) * x^3 - 3 * (b * c^4 * d + 2 * a * c^2 * d^3) * x) * \sqrt{d * x + c} * \sqrt{d * x - c} + 3 * (b * c^6 + 2 * a * c^4 * d^2) * \log(-d * x + \sqrt{d * x + c} * \sqrt{d * x - c})) / d^5$

**giac** [B] time = 0.46, size = 432, normalized size = 2.72

$$40 \left( \sqrt{dx+c} \sqrt{dx-c} \left( (dx+c) \left( \frac{2(dx+c)}{d^2} - \frac{7c}{d^2} \right) + \frac{9c^2}{d^2} \right) + \frac{6c^3 \log(|-\sqrt{dx+c} + \sqrt{dx-c}|)}{d^2} \right) ac + 2 \left( \left( \left( 2(dx+c) \left( 3(dx+c) \left( \frac{4(dx+c)}{d^4} \right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x, algorithm="giac")

[Out]  $\frac{1}{240} * (40 * (\sqrt{d * x + c} * \sqrt{d * x - c} * ((d * x + c) * (2 * (d * x + c) / d^2 - 7 * c / d^2) + 9 * c^2 / d^2) + 6 * c^3 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^2) * a * c + 2 * (((2 * (d * x + c) * (3 * (d * x + c) * (4 * (d * x + c) / d^4 - 21 * c / d^4) + 133 * c^2 / d^4) - 295 * c^3 / d^4) * (d * x + c) + 195 * c^4 / d^4) * \sqrt{d * x + c} * \sqrt{d * x - c} + 90 * c^5 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^4) * b * c + 10 * (((d * x + c) * (2 * (d * x + c) * (3 * (d * x + c) / d^3 - 13 * c / d^3) + 43 * c^2 / d^3) - 39 * c^3 / d^3) * \sqrt{d * x + c} * \sqrt{d * x - c} - 18 * c^4 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^3) * a * d + (((2 * ((d * x + c) * (4 * (d * x + c) * (5 * (d * x + c) / d^5 - 31 * c / d^5) + 321 * c^2 / d^5) - 451 * c^3 / d^5) * (d * x + c) + 745 * c^4 / d^5) * (d * x + c) - 405 * c^5 / d^5) * \sqrt{d * x + c} * \sqrt{d * x - c} - 150 * c^6 * \log(\text{abs}(-\sqrt{d * x + c} + \sqrt{d * x - c}))) / d^5) * b * d) / d$

**maple** [C] time = 0.07, size = 240, normalized size = 1.51

$$\sqrt{dx-c} \sqrt{dx+c} \left( 8 \sqrt{d^2 x^2 - c^2} b d^5 x^5 \text{csgn}(d) + 12 \sqrt{d^2 x^2 - c^2} a d^5 x^3 \text{csgn}(d) - 2 \sqrt{d^2 x^2 - c^2} b c^2 d^3 x^3 \text{csgn}(d) - \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x)

[Out]  $\frac{1}{48} * (d * x - c)^{(1/2)} * (d * x + c)^{(1/2)} * (8 * \text{csgn}(d) * x^5 * b * d^5 * (d^2 * x^2 - c^2)^{(1/2)} + 12 * \text{csgn}(d) * x^3 * a * d^5 * (d^2 * x^2 - c^2)^{(1/2)} - 2 * \text{csgn}(d) * x^3 * b * c^2 * d^3 * (d^2 * x^2 - c^2)^{(1/2)} - 6 * \text{csgn}(d) * d^3 * (d^2 * x^2 - c^2)^{(1/2)} * x * a * c^2 - 3 * \text{csgn}(d) * d * (d^2 * x^2 - c^2)^{(1/2)} * x * b * c^4 - 6 * \ln((d * x + (d^2 * x^2 - c^2)^{(1/2)} * \text{csgn}(d)) * \text{csgn}(d)) * a * c^4 * d^2 - 3 * \ln((d * x + (d^2 * x^2 - c^2)^{(1/2)} * \text{csgn}(d)) * \text{csgn}(d)) * b * c^6) * \text{csgn}(d) / (d^2 * x^2 - c^2)^{(1/2)}) / d^5$

**maxima** [A] time = 0.67, size = 192, normalized size = 1.21

$$\frac{(d^2 x^2 - c^2)^{\frac{3}{2}} b x^3}{6 d^2} - \frac{b c^6 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d\right)}{16 d^5} - \frac{a c^4 \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d\right)}{8 d^3} + \frac{\sqrt{d^2 x^2 - c^2} b c^4 x}{16 d^4} + \frac{\sqrt{d^2 x^2 - c^2} a c^2 x}{8 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{6} * (d^2 * x^2 - c^2)^{(3/2)} * b * x^3 / d^2 - \frac{1}{16} * b * c^6 * \log(2 * d^2 * x + 2 * \sqrt{d^2 * x^2 - c^2} * d) / d^5 - \frac{1}{8} * a * c^4 * \log(2 * d^2 * x + 2 * \sqrt{d^2 * x^2 - c^2} * d) / d^3 + \frac{1}{16} * \sqrt{d^2 * x^2 - c^2} * b * c^4 * x / d^4 + \frac{1}{8} * \sqrt{d^2 * x^2 - c^2} * a * c^2 * x / d^2 + \frac{1}{8} * (d^2 * x^2 - c^2)^{(3/2)} * b * c^2 * x / d^4 + \frac{1}{4} * (d^2 * x^2 - c^2)^{(3/2)} * a * x / d^2$

**mupad** [B] time = 42.57, size = 1681, normalized size = 10.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)`

[Out] 
$$\begin{aligned} & ((35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^3)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (b*c^6*((c + d*x)^(1/2) - c^(1/2)))/(4*((-c)^(1/2) - (d*x - c)^(1/2))) \\ & + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^5)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^7)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^7) \\ & + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^9)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^9) + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) \\ & + (25661*b*c^6*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) + (41929*b*c^6*((c + d*x)^(1/2) - c^(1/2))^15)/(6*((-c)^(1/2) - (d*x - c)^(1/2))^15) \\ & + (7339*b*c^6*((c + d*x)^(1/2) - c^(1/2))^17)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^17) + (757*b*c^6*((c + d*x)^(1/2) - c^(1/2))^19)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^19) \\ & + (35*b*c^6*((c + d*x)^(1/2) - c^(1/2))^21)/(12*((-c)^(1/2) - (d*x - c)^(1/2))^21) - (b*c^6*((c + d*x)^(1/2) - c^(1/2))^23)/(4*((-c)^(1/2) - (d*x - c)^(1/2))^23) \\ & / (d^5 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 \\ & + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 \\ & + (924*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (792*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 \\ & + (495*d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 - (220*d^5*((c + d*x)^(1/2) - c^(1/2))^18)/((-c)^(1/2) - (d*x - c)^(1/2))^18 \\ & + (66*d^5*((c + d*x)^(1/2) - c^(1/2))^20)/((-c)^(1/2) - (d*x - c)^(1/2))^20 - (12*d^5*((c + d*x)^(1/2) - c^(1/2))^22)/((-c)^(1/2) - (d*x - c)^(1/2))^22 \\ & + (d^5*((c + d*x)^(1/2) - c^(1/2))^24)/((-c)^(1/2) - (d*x - c)^(1/2))^24 - ((a*c^4*((c + d*x)^(1/2) - c^(1/2)))/(2*((-c)^(1/2) - (d*x - c)^(1/2))) \\ & + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) \\ & + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (715*a*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9) \\ & + (273*a*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (35*a*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) \\ & + (a*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15) / (d^3 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 \\ & + (70*d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^3*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 \\ & + (28*d^3*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^3*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 \\ & + (d^3*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16 + (a*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^3) \\ & + (b*c^6*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(4*d^5) \end{aligned}$$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2),x)`

[Out] `Integral(x**2*(a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x), x)`

### 3.345 $\int \sqrt{-c + dx} \sqrt{c + dx} (a + bx^2) dx$

**Optimal.** Leaf size=114

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+bc^2)}{8d^2} - \frac{c^2(4ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3} + \frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

[Out] 1/4\*b\*x\*(d\*x-c)^(3/2)\*(d\*x+c)^(3/2)/d^2-1/4\*c^2\*(4\*a\*d^2+b\*c^2)\*arctanh((d\*x-c)^(1/2)/(d\*x+c)^(1/2))/d^3+1/8\*(4\*a\*d^2+b\*c^2)\*x\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

**Rubi [A]** time = 0.05, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$ , Rules used = {389, 38, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2+bc^2)}{8d^2} - \frac{c^2(4ad^2+bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^3} + \frac{bx(dx-c)^{3/2}(c+dx)^{3/2}}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] ((b\*c^2 + 4\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*d^2) + (b\*x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2))/(4\*d^2) - (c^2\*(b\*c^2 + 4\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(4\*d^3)

#### Rule 38

Int[((a\_) + (b\_.)\*(x\_))^(m\_)\*((c\_) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> Simp[(x\*(a + b\*x)^m\*(c + d\*x)^n)/(2\*m + 1), x] + Dist[(2\*a\*c\*m)/(2\*m + 1), Int[(a + b\*x)^(m - 1)\*(c + d\*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 389

Int[((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*(n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d - b1\*b2\*c\*(n\*(p + 1) + 1))/(b1\*b2\*(n\*(p + 1) + 1)), Int[(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && Eq

$Q[\text{non2}, n/2] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ \text{NeQ}[n*(p + 1) + 1, 0]$

### Rubi steps

$$\begin{aligned} \int \sqrt{-c+dx} \sqrt{c+dx} (a+bx^2) dx &= \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{(-bc^2-4ad^2) \int \sqrt{-c+dx} \sqrt{c+dx} dx}{4d^2} \\ &= \frac{(bc^2+4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} + \frac{c^2(-bc^2-4ad^2)}{4d^2} \\ &= \frac{(bc^2+4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2(bc^2+4ad^2)}{4d^2} \\ &= \frac{(bc^2+4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2(bc^2+4ad^2)}{4d^2} \\ &= \frac{(bc^2+4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^2} + \frac{bx(-c+dx)^{3/2}(c+dx)^{3/2}}{4d^2} - \frac{c^2(bc^2+4ad^2)}{4d^2} \end{aligned}$$

**Mathematica [A]** time = 0.28, size = 129, normalized size = 1.13

$$\frac{dx(c^2-d^2x^2)(b(c^2-2d^2x^2)-4ad^2)-2c^{5/2}\sqrt{dx-c}\sqrt{\frac{dx}{c}+1}(4ad^2+bc^2)\sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right)}{8d^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2), x]

[Out] (d\*x\*(c^2 - d^2\*x^2)\*(-4\*a\*d^2 + b\*(c^2 - 2\*d^2\*x^2)) - 2\*c^(5/2)\*(b\*c^2 + 4\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[1 + (d\*x)/c]\*ArcSinh[Sqrt[-c + d\*x]/(Sqrt[2]\*Sqrt[c])])/(8\*d^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.13, size = 88, normalized size = 0.77

$$\frac{(2bd^3x^3 - (bc^2d - 4ad^3)x)\sqrt{dx+c}\sqrt{dx-c} + (bc^4 + 4ac^2d^2)\log(-dx + \sqrt{dx+c}\sqrt{dx-c})}{8d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/8\*((2\*b\*d^3\*x^3 - (b\*c^2\*d - 4\*a\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + (b\*c^4 + 4\*a\*c^2\*d^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/d^3

**giac [B]** time = 0.35, size = 288, normalized size = 2.53

$$24 \left( 2c \log \left( \left| -\sqrt{dx+c} + \sqrt{dx-c} \right| \right) + \sqrt{dx+c}\sqrt{dx-c} \right) ac + 4 \left( \sqrt{dx+c}\sqrt{dx-c} \left( (dx+c) \left( \frac{2(dx+c)}{d^2} - \frac{7c}{d^2} \right) + \frac{9c^2}{d^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2), x, algorithm="giac")

[Out] 1/24\*(24\*(2\*c\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c))) + sqrt(d\*x + c)\*sqrt(d\*x - c))\*a\*c + 4\*(sqrt(d\*x + c)\*sqrt(d\*x - c)\*((d\*x + c)\*(2\*(d\*x + c)/d^2 - 7\*c/d^2) + 9\*c^2/d^2) + 6\*c^3\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c))))/d

$\wedge 2) * b * c + (((d * x + c) * (2 * (d * x + c) * (3 * (d * x + c) / d^3 - 13 * c / d^3) + 43 * c^2 / d^3) - 39 * c^3 / d^3) * \text{sqrt}(d * x + c) * \text{sqrt}(d * x - c) - 18 * c^4 * \log(\text{abs}(-\text{sqrt}(d * x + c) + \text{sqrt}(d * x - c))) / d^3) * b * d - 12 * (2 * c^2 * \log(\text{abs}(-\text{sqrt}(d * x + c) + \text{sqrt}(d * x - c))) - \text{sqrt}(d * x + c) * \text{sqrt}(d * x - c) * (d * x - 2 * c)) * a) / d$

**maple** [C] time = 0.06, size = 182, normalized size = 1.60

$$\frac{\sqrt{dx - c} \sqrt{dx + c} \left( 2\sqrt{d^2x^2 - c^2} b d^3 x^3 \text{csgn}(d) - 4a c^2 d^2 \ln \left( \left( dx + \sqrt{d^2x^2 - c^2} \text{csgn}(d) \right) \text{csgn}(d) \right) + 4\sqrt{d^2x^2 - c^2} \right)}{8\sqrt{d^2x^2 - c^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x)`

[Out]  $\frac{1}{8} * (d * x - c)^{(1/2)} * (d * x + c)^{(1/2)} * (2 * c * \text{sgn}(d) * x^3 * b * d^3 * (d^2 * x^2 - c^2)^{(1/2)} + 4 * c * \text{sgn}(d) * d^3 * (d^2 * x^2 - c^2)^{(1/2)} * x * a - c * \text{sgn}(d) * d * (d^2 * x^2 - c^2)^{(1/2)} * x * b * c^2 - 4 * \ln((d * x + (d^2 * x^2 - c^2)^{(1/2)} * \text{sgn}(d)) * \text{sgn}(d)) * a * c^2 * d^2 - \ln((d * x + (d^2 * x^2 - c^2)^{(1/2)} * \text{sgn}(d)) * \text{sgn}(d)) * b * c^4) * \text{sgn}(d) / (d^2 * x^2 - c^2)^{(1/2)} / d^3$

**maxima** [A] time = 0.54, size = 137, normalized size = 1.20

$$\frac{bc^4 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d \right)}{8 d^3} - \frac{ac^2 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d \right)}{2 d} + \frac{1}{2} \sqrt{d^2 x^2 - c^2} a x + \frac{\sqrt{d^2 x^2 - c^2} b c^2 x}{8 d^2} + \frac{(d^2 x^2 - c^2)^{3/2}}{4 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $-1/8 * b * c^4 * \log(2 * d^2 * x + 2 * \text{sqrt}(d^2 * x^2 - c^2) * d) / d^3 - 1/2 * a * c^2 * \log(2 * d^2 * x + 2 * \text{sqrt}(d^2 * x^2 - c^2) * d) / d + 1/2 * \text{sqrt}(d^2 * x^2 - c^2) * a * x + 1/8 * \text{sqrt}(d^2 * x^2 - c^2) * b * c^2 * x / d^2 + 1/4 * (d^2 * x^2 - c^2)^{(3/2)} * b * x / d^2$

**mupad** [B] time = 17.43, size = 734, normalized size = 6.44

$$\frac{a x \sqrt{c + d x} \sqrt{d x - c}}{2} - \frac{bc^4 (\sqrt{c+dx} - \sqrt{c})}{2(\sqrt{-c} - \sqrt{dx-c})} + \frac{35bc^4 (\sqrt{c+dx} - \sqrt{c})^3}{2(\sqrt{-c} - \sqrt{dx-c})^3} + \frac{273bc^4 (\sqrt{c+dx} - \sqrt{c})^5}{2(\sqrt{-c} - \sqrt{dx-c})^5} + \frac{715bc^4 (\sqrt{c+dx} - \sqrt{c})^7}{2(\sqrt{-c} - \sqrt{dx-c})^7} + \frac{715bc^4 (\sqrt{c+dx} - \sqrt{c})^9}{2(\sqrt{-c} - \sqrt{dx-c})^9} + \frac{273bc^4 (\sqrt{c+dx} - \sqrt{c})^{11}}{2(\sqrt{-c} - \sqrt{dx-c})^{11}} + \frac{35bc^4 (\sqrt{c+dx} - \sqrt{c})^{13}}{2(\sqrt{-c} - \sqrt{dx-c})^{13}} + \frac{bc^4 (\sqrt{c+dx} - \sqrt{c})^{15}}{2(\sqrt{-c} - \sqrt{dx-c})^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2),x)`

[Out]  $(a * x * (c + d * x)^{(1/2)} * (d * x - c)^{(1/2)}) / 2 - ((b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})) + (35 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^3) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^3) + (273 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^5) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^5) + (715 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^7) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^7) + (715 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^9) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^9) + (273 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{11}) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{11}) + (35 * b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{13}) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{13}) + (b * c^4 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{15}) / (2 * ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{15})) / (d^3 - (8 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^2) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^2 + (28 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^4) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^4 - (56 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^6) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^6 + (70 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^8) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^8 - (56 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{10}) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{10} + (28 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{12}) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{12} - (8 * d^3 * ((c + d * x)^{(1/2)} - c^{(1/2)})^{14}) / ((-c)^{(1/2)} - (d * x - c)^{(1/2)})^{14}$

$$\frac{1}{2})^{14} + (d^3((c + dx)^{1/2} - c^{1/2})^{16} / ((-c)^{1/2} - (dx - c)^{1/2})^{16}) - (a*c^2*\log(dx + (c + dx)^{1/2}*(dx - c)^{1/2})) / (2*d) + (b*c^4 * \operatorname{atanh}(((c + dx)^{1/2} - c^{1/2}) / ((-c)^{1/2} - (dx - c)^{1/2}))) / (2*d^3)$$

**sympy [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (a + bx^2) \sqrt{-c + dx} \sqrt{c + dx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2), x)

[Out] Integral((a + b\*x\*\*2)\*sqrt(-c + d\*x)\*sqrt(c + d\*x), x)

$$3.346 \quad \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^2} dx$$

**Optimal.** Leaf size=104

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

[Out]  $a*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/c^2/x-(-2*a*d^2+b*c^2)*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2)})/d+1/2*(b-2*a*d^2/c^2)*x*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {454, 38, 63, 217, 206}

$$\frac{1}{2}x\sqrt{dx-c}\sqrt{c+dx}\left(b-\frac{2ad^2}{c^2}\right)-\frac{(bc^2-2ad^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}+\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{c^2x}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(\operatorname{Sqrt}[-c+d*x]*\operatorname{Sqrt}[c+d*x]*(a+b*x^2))/x^2,x]$

[Out]  $((b-(2*a*d^2)/c^2)*x*\operatorname{Sqrt}[-c+d*x]*\operatorname{Sqrt}[c+d*x])/2+(a*(-c+d*x)^{(3/2)}*(c+d*x)^{(3/2)})/(c^2*x)-((b*c^2-2*a*d^2)*\operatorname{ArcTanh}[\operatorname{Sqrt}[-c+d*x]/\operatorname{Sqrt}[c+d*x]])/d$

### Rule 38

$\operatorname{Int}[(a_+ + (b_+)(x_+))^{m_+}((c_+ + (d_+)(x_+))^{n_+}), x\_Symbol] \rightarrow \operatorname{Simp}[(x_+^{m_+}(a_+ + b_+x_+)^m(c_+ + d_+x_+)^m)/(2m + 1), x] + \operatorname{Dist}[(2a_+c_+m)/(2m + 1), \operatorname{Int}[(a_+ + b_+x_+)^{m-1}(c_+ + d_+x_+)^{m-1}], x] /;$  FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0] && IGtQ[m + 1/2, 0]

### Rule 63

$\operatorname{Int}[(a_+ + (b_+)(x_+))^{m_+}((c_+ + (d_+)(x_+))^{n_+}), x\_Symbol] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p(m+1)-1)}(c - (a*d)/b + (d*x^p)/b)^n], x, (a + b*x)^{(1/p)}], x] /;$  FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 206

$\operatorname{Int}[(a_+ + (b_+)(x_+)^2)^{-1}], x\_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x]/\operatorname{Rt}[a, 2])]/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$  FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_+ + (b_+)(x_+)^2)], x\_Symbol] \rightarrow \operatorname{Subst}[\operatorname{Int}[1/(1 - b*x^2)], x], x, x/\operatorname{Sqrt}[a + b*x^2]] /;$  FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 454

$\operatorname{Int}[(e_+)(x_+)^{m_+}((a1_+ + (b1_+)(x_+)^{\operatorname{non}2_+})^{p_+})((a2_+ + (b2_+)(x_+)^{\operatorname{non}2_+})^{p_+})((c_+ + (d_+)(x_+)^{n_+}), x\_Symbol] \rightarrow \operatorname{Simp}[(c_+)(e_+)^{m_+}(m_+ + 1)(a1_+ + b1_+x_+^{n_+/2})^{p_+ + 1}(a2_+ + b2_+x_+^{n_+/2})^{p_+ + 1}]/(a1_+a2_+e_+^{m_+ + 1}), x] + \operatorname{Dist}[(a1_+a2_+d_+(m_+ + 1) - b1_+b2_+c_+(m_+ + n_+(p_+ + 1) + 1))/(a1_+a2_+e_+^{n_+}(m_+ + 1)), \operatorname{Int}[(e_+)(x_+)^{m_+ + n_+}(a1_+ + b1_+x_+^{n_+/2})^{p_+}(a2_+ + b2_+x_+^{n_+/2})^{p_+}], x]$



/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^2} dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \left(b - \frac{2ad^2}{c^2}\right) \int \sqrt{-c+dx}\sqrt{c+dx} dx \\ &= \frac{1}{2} \left(b - \frac{2ad^2}{c^2}\right) x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{1}{2}(-bc^2 + \\ &= \frac{1}{2} \left(b - \frac{2ad^2}{c^2}\right) x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{(-bc^2 + \\ &= \frac{1}{2} \left(b - \frac{2ad^2}{c^2}\right) x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} + \frac{(-bc^2 + \\ &= \frac{1}{2} \left(b - \frac{2ad^2}{c^2}\right) x\sqrt{-c+dx}\sqrt{c+dx} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{c^2x} - \frac{(bc^2 - 2ad^2)}{2c} \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 101, normalized size = 0.97

$$\frac{\sqrt{dx-c}\sqrt{c+dx} \left( cd (bx^2 - 2a) \sqrt{1 - \frac{d^2x^2}{c^2}} + x (bc^2 - 2ad^2) \sin^{-1} \left( \frac{dx}{c} \right) \right)}{2cdx\sqrt{1 - \frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(a + b\*x^2))/x^2,x]

[Out] (Sqrt[-c + d\*x]\*Sqrt[c + d\*x]\*(c\*d\*(-2\*a + b\*x^2)\*Sqrt[1 - (d^2\*x^2)/c^2] + (b\*c^2 - 2\*a\*d^2)\*x\*ArcSin[(d\*x)/c]))/(2\*c\*d\*x\*Sqrt[1 - (d^2\*x^2)/c^2])

**fricas [A]** time = 1.22, size = 83, normalized size = 0.80

$$\frac{2ad^2x - (bc^2 - 2ad^2)x \log(-dx + \sqrt{dx+c}\sqrt{dx-c}) - (bdx^2 - 2ad)\sqrt{dx+c}\sqrt{dx-c}}{2dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^2,x, algorithm="fricas")

[Out] -1/2\*(2\*a\*d^2\*x - (b\*c^2 - 2\*a\*d^2)\*x\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)) - (b\*d\*x^2 - 2\*a\*d)\*sqrt(d\*x + c)\*sqrt(d\*x - c))/(d\*x)

**giac [A]** time = 0.40, size = 110, normalized size = 1.06

$$\frac{\frac{32ac^2d^2}{(\sqrt{dx+c}-\sqrt{dx-c})^4+4c^2} - 2((dx+c)b-bc)\sqrt{dx+c}\sqrt{dx-c} - (bc^2-2ad^2)\log\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^2,x, algorithm="giac")

[Out]  $-1/4*(32*a*c^2*d^2/((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2) - 2*((d*x + c)*b - b*c)*\sqrt{d*x + c}*\sqrt{d*x - c} - (b*c^2 - 2*a*d^2)*\log((\sqrt{d*x + c} - \sqrt{d*x - c})^4))/d$

**maple** [C] time = 0.06, size = 153, normalized size = 1.47

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( 2a d^2 x \ln \left( \left( dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) \right) \operatorname{csgn}(d) \right) - b c^2 x \ln \left( \left( dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) \right) \operatorname{csgn}(d) \right) \right)}{2\sqrt{d^2 x^2 - c^2} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x)`

[Out]  $1/2*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}*(\operatorname{csgn}(d)*x^2*b*d*(d^2*x^2-c^2)^{(1/2)}+2*\ln((d*x+(d^2*x^2-c^2)^{(1/2)}*\operatorname{csgn}(d))*\operatorname{csgn}(d))*x*a*d^2-\ln((d*x+(d^2*x^2-c^2)^{(1/2)}*\operatorname{csgn}(d))*\operatorname{csgn}(d))*x*b*c^2-2*\operatorname{csgn}(d)*d*(d^2*x^2-c^2)^{(1/2)}*a)*\operatorname{csgn}(d)/(d^2*x^2-c^2)^{(1/2)}/x/d$

**maxima** [A] time = 1.50, size = 105, normalized size = 1.01

$$-\frac{bc^2 \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d \right)}{2 d} + ad \log \left( 2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d \right) + \frac{1}{2} \sqrt{d^2 x^2 - c^2} bx - \frac{\sqrt{d^2 x^2 - c^2} a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)*(d*x-c)^(1/2)*(d*x+c)^(1/2)/x^2,x, algorithm="maxima")`

[Out]  $-1/2*b*c^2*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d + a*d*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d) + 1/2*\sqrt{d^2*x^2 - c^2}*b*x - \sqrt{d^2*x^2 - c^2}*a/x$

**mapad** [B] time = 3.49, size = 243, normalized size = 2.34

$$\frac{ad + \frac{5ad(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2}}{\frac{4(\sqrt{c+dx}-\sqrt{c})}{\sqrt{-c}-\sqrt{dx-c}} + \frac{4(\sqrt{c+dx}-\sqrt{c})^3}{(\sqrt{-c}-\sqrt{dx-c})^3}} - 4ad \operatorname{atanh} \left( \frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}} \right) + \frac{bx\sqrt{c+dx}\sqrt{dx-c}}{2} - \frac{bc^2 \ln(dx + \sqrt{c+dx}\sqrt{c})}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x^2)*(c + d*x)^(1/2)*(d*x - c)^(1/2))/x^2,x)`

[Out]  $(a*d + (5*a*d*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2)/((4*((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)}) + (4*((c + d*x)^{(1/2)} - c^{(1/2)})^3)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^3) - 4*a*d*\operatorname{atanh}(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})) + (b*x*(c + d*x)^{(1/2)}*(d*x - c)^{(1/2)})/2 - (b*c^2*\log(d*x + (c + d*x)^{(1/2)}*(d*x - c)^{(1/2)}))/(2*d) + (a*d*((c + d*x)^{(1/2)} - c^{(1/2)})/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})))$

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2) \sqrt{-c + dx} \sqrt{c + dx}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)*(d*x-c)**(1/2)*(d*x+c)**(1/2)/x**2,x)`

[Out] `Integral((a + b*x**2)*sqrt(-c + d*x)*sqrt(c + d*x)/x**2, x)`

$$3.347 \quad \int \frac{\sqrt{-c+dx} \sqrt{c+dx} (a+bx^2)}{x^4} dx$$

**Optimal.** Leaf size=84

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3} - \frac{b\sqrt{dx-c}\sqrt{c+dx}}{x} + 2bd \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

[Out]  $1/3*a*(d*x-c)^{(3/2)}*(d*x+c)^{(3/2)}/c^2/x^3+2*b*d*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2)})-b*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/x$

**Rubi [A]** time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {454, 97, 12, 63, 217, 206}

$$\frac{a(dx-c)^{3/2}(c+dx)^{3/2}}{3c^2x^3} - \frac{b\sqrt{dx-c}\sqrt{c+dx}}{x} + 2bd \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)$$

Antiderivative was successfully verified.

[In] `Int[(Sqrt[-c + d*x]*Sqrt[c + d*x]*(a + b*x^2))/x^4, x]`

[Out] `-((b*Sqrt[-c + d*x]*Sqrt[c + d*x])/x) + (a*(-c + d*x)^(3/2)*(c + d*x)^(3/2))/(3*c^2*x^3) + 2*b*d*ArcTanh[Sqrt[-c + d*x]/Sqrt[c + d*x]]`

#### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

#### Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

#### Rule 97

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])`

#### Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

#### Rule 217

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]`

#### Rule 454

```
Int[((e_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_.) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(a1*a2*e^(m +
1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(a1*a2*e^(n*(
m + 1))], Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^(p*(a2 + b2*x^(n/2))^(p, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-c+dx}\sqrt{c+dx}(a+bx^2)}{x^4} dx &= \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b \int \frac{\sqrt{-c+dx}\sqrt{c+dx}}{x^2} dx \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + b \int \frac{d^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (bd^2) \int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd) \operatorname{Subst} \left( \int \frac{1}{\sqrt{2c+dx}} dx \right) \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + (2bd) \operatorname{Subst} \left( \int \frac{1}{1-x^2} dx \right) \\ &= -\frac{b\sqrt{-c+dx}\sqrt{c+dx}}{x} + \frac{a(-c+dx)^{3/2}(c+dx)^{3/2}}{3c^2x^3} + 2bd \tanh^{-1} \left( \frac{\sqrt{-c+dx}}{\sqrt{c+dx}} \right) \end{aligned}$$

**Mathematica [A]** time = 0.09, size = 105, normalized size = 1.25

$$\frac{\sqrt{dx-c}\sqrt{c+dx} \left( \sqrt{1-\frac{d^2x^2}{c^2}} \left( a(c^2-d^2x^2) + 3bc^2x^2 \right) + 3bcdx^3 \sin^{-1} \left( \frac{dx}{c} \right) \right)}{3c^2x^3 \sqrt{1-\frac{d^2x^2}{c^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*(a+b\*x^2))/x^4,x]

[Out] -1/3\*(Sqrt[-c+d\*x]\*Sqrt[c+d\*x]\*(Sqrt[1-(d^2\*x^2)/c^2]\*(3\*b\*c^2\*x^2+a\*(c^2-d^2\*x^2))+3\*b\*c\*d\*x^3\*ArcSin[(d\*x)/c]))/(c^2\*x^3\*Sqrt[1-(d^2\*x^2)/c^2])

**fricas [A]** time = 1.34, size = 100, normalized size = 1.19

$$\frac{3bc^2dx^3 \log(-dx + \sqrt{dx+c}\sqrt{dx-c}) + (3bc^2d - ad^3)x^3 + (ac^2 + (3bc^2 - ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^4,x, algorithm="fricas")

[Out] -1/3\*(3\*b\*c^2\*d\*x^3\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)) + (3\*b\*c^2\*d - a\*d^3)\*x^3 + (a\*c^2 + (3\*b\*c^2 - a\*d^2)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c))/(c^2\*x^3)

**giac [B]** time = 0.38, size = 171, normalized size = 2.04

$$\frac{3bd^2 \log\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4\right) + \frac{16\left(3bc^2d^2(\sqrt{dx+c}-\sqrt{dx-c})^8 - 3ad^4(\sqrt{dx+c}-\sqrt{dx-c})^8 + 24bc^4d^2(\sqrt{dx+c}-\sqrt{dx-c})^4 + 48bc^6d\right)}{\left(\left(\sqrt{dx+c}-\sqrt{dx-c}\right)^4 + 4c^2\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^4,x, algorithm="giac")

[Out] -1/6\*(3\*b\*d^2\*log((sqrt(d\*x + c) - sqrt(d\*x - c))^4) + 16\*(3\*b\*c^2\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^8 - 3\*a\*d^4\*(sqrt(d\*x + c) - sqrt(d\*x - c))^8 + 24\*b\*c^4\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 48\*b\*c^6\*d^2 - 16\*a\*c^4\*d^4)/((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^3)/d

**maple [C]** time = 0.07, size = 153, normalized size = 1.82

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left(3bc^2dx^3 \ln\left(\left(dx + \sqrt{d^2x^2 - c^2} \operatorname{csgn}(d)\right) \operatorname{csgn}(d)\right) + \sqrt{d^2x^2 - c^2} a d^2x^2 \operatorname{csgn}(d) - 3\sqrt{d^2x^2 - c^2} \right)}{3\sqrt{d^2x^2 - c^2} c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^4,x)

[Out] 1/3\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(3\*ln((d\*x+(d^2\*x^2-c^2)^(1/2))\*csgn(d))\*csgn(d)\*x^3\*b\*c^2\*d+csgn(d)\*x^2\*a\*d^2\*(d^2\*x^2-c^2)^(1/2)-3\*csgn(d)\*x^2\*b\*c^2\*(d^2\*x^2-c^2)^(1/2)-csgn(d)\*a\*c^2\*(d^2\*x^2-c^2)^(1/2))\*csgn(d)/(d^2\*x^2-c^2)^(1/2)/c^2/x^3

**maxima [A]** time = 1.47, size = 75, normalized size = 0.89

$$bd \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right) - \frac{\sqrt{d^2x^2 - c^2}b}{x} + \frac{(d^2x^2 - c^2)^{\frac{3}{2}}a}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/x^4,x, algorithm="maxima")

[Out] b\*d\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*d) - sqrt(d^2\*x^2 - c^2)\*b/x + 1/3\*(d^2\*x^2 - c^2)^(3/2)\*a/(c^2\*x^3)

**mupad [B]** time = 3.44, size = 236, normalized size = 2.81

$$\frac{bd + \frac{5bd(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2}}{\frac{4(\sqrt{c+dx}-\sqrt{c})}{\sqrt{-c}-\sqrt{dx-c}} + \frac{4(\sqrt{c+dx}-\sqrt{c})^3}{(\sqrt{-c}-\sqrt{dx-c})^3}} - 4bd \operatorname{atanh}\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right) - \frac{\left(\frac{a\sqrt{c+dx}}{3} - \frac{ad^2x^2\sqrt{c+dx}}{3c^2}\right)\sqrt{dx-c}}{x^3} + \frac{bd(\sqrt{c+dx}-\sqrt{c})}{4(\sqrt{-c}-\sqrt{dx-c})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x^2)\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2))/x^4,x)

[Out] (b\*d + (5\*b\*d\*((c + d\*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d\*x - c)^(1/2))^2)/((4\*((c + d\*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d\*x - c)^(1/2)) + (4\*((c + d\*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d\*x - c)^(1/2))^3) - 4\*b\*d\*atanh(((c + d\*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d\*x - c)^(1/2))) - (((a\*(c + d\*x)^(1/2))/3 - (a\*d^2\*x^2\*(c + d\*x)^(1/2))/(3\*c^2))\*(d\*x - c)^(1/2))/x^3 + (b\*d\*((c + d\*x)^(1/2) - c^(1/2)))/(4\*((-c)^(1/2) - (d\*x - c)^(1/2)))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*(d\*x-c)\*\*(1/2)\*(d\*x+c)\*\*(1/2)/x\*\*4,x)

[Out] Timed out

$$3.348 \quad \int \frac{x^4(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$$

**Optimal.** Leaf size=125

$$\frac{(6ac^2 + 5b) \cosh^{-1}(cx)}{16c^7} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{16c^6} + \frac{x^3\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{24c^4} + \frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

[Out] 1/16\*(6\*a\*c^2+5\*b)\*arccosh(c\*x)/c^7+1/16\*(6\*a\*c^2+5\*b)\*x\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^6+1/24\*(6\*a\*c^2+5\*b)\*x^3\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^4+1/6\*b\*x^5\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

**Rubi [A]** time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {460, 100, 12, 90, 52}

$$\frac{x^3\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{24c^4} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(6ac^2 + 5b)}{16c^6} + \frac{(6ac^2 + 5b) \cosh^{-1}(cx)}{16c^7} + \frac{bx^5\sqrt{cx-1}\sqrt{cx+1}}{6c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((5\*b + 6\*a\*c^2)\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(16\*c^6) + ((5\*b + 6\*a\*c^2)\*x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(24\*c^4) + (b\*x^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(6\*c^2) + ((5\*b + 6\*a\*c^2)\*ArcCosh[c\*x])/(16\*c^7)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 52

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)]), x\_Symbol] := Simp[ArcCosh[(b\*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

#### Rule 90

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 3)), x] + Dist[1/(d\*f\*(n + p + 3)), Int[(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(n + p + 3) - b\*(b\*c\*e + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(n + p + 4) - b\*(d\*e\*(n + 2) + c\*f\*(p + 2)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 100

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

#### Rule 460

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_))^(non2\_)^(p\_)\*((a2\_) + (b2\_)\*(x\_))^(non2\_)^(p\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Simp[(d\*(e\*x)^(m+1) \* ((a1 + b1\*x)^(non2) \* (a2 + b2\*x)^(non2) \* (c + d\*x)^n) / (d\*(e\*x)^(m+1) \* ((a1 + b1\*x)^(non2) \* (a2 + b2\*x)^(non2) \* (c + d\*x)^n) \* (c + d\*x)^n) / (d\*(e\*x)^(m+1) \* ((a1 + b1\*x)^(non2) \* (a2 + b2\*x)^(non2) \* (c + d\*x)^n) \* (c + d\*x)^n)

$(m + 1) \cdot (a_1 + b_1 x^{n/2})^{p+1} \cdot (a_2 + b_2 x^{n/2})^{p+1} / (b_1 b_2 e^{(m+n)(p+1)+1})$ ,  $x] - \text{Dist}[(a_1 a_2 d^{(m+1)} - b_1 b_2 c^{(m+n)(p+1)+1}) / (b_1 b_2 (m+n)(p+1)+1)]$ ,  $\text{Int}[(e x)^m (a_1 + b_1 x^{n/2})^p (a_2 + b_2 x^{n/2})^p, x]$ ,  $x] /;$   $\text{FreeQ}\{a_1, b_1, a_2, b_2, c, d, e, m, n, p\}, x] \ \&\& \ \text{EqQ}[\text{non2}, n/2] \ \&\& \ \text{EqQ}[a_2 b_1 + a_1 b_2, 0] \ \&\& \ \text{NeQ}[m + n(p+1) + 1, 0]$

### Rubi steps

$$\begin{aligned} \int \frac{x^4 (a + bx^2)}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx &= \frac{bx^5 \sqrt{-1 + cx} \sqrt{1 + cx}}{6c^2} - \frac{1}{6} \left( -6a - \frac{5b}{c^2} \right) \int \frac{x^4}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx \\ &= \frac{(5b + 6ac^2) x^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{24c^4} + \frac{bx^5 \sqrt{-1 + cx} \sqrt{1 + cx}}{6c^2} + \frac{(5b + 6ac^2) \int \frac{3x^2}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx}{24c^4} \\ &= \frac{(5b + 6ac^2) x^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{24c^4} + \frac{bx^5 \sqrt{-1 + cx} \sqrt{1 + cx}}{6c^2} + \frac{(5b + 6ac^2) \int \frac{x^2}{\sqrt{-1 + cx} \sqrt{1 + cx}} dx}{8c^4} \\ &= \frac{(5b + 6ac^2) x \sqrt{-1 + cx} \sqrt{1 + cx}}{16c^6} + \frac{(5b + 6ac^2) x^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{24c^4} + \frac{bx^5 \sqrt{-1 + cx} \sqrt{1 + cx}}{6c^2} \\ &= \frac{(5b + 6ac^2) x \sqrt{-1 + cx} \sqrt{1 + cx}}{16c^6} + \frac{(5b + 6ac^2) x^3 \sqrt{-1 + cx} \sqrt{1 + cx}}{24c^4} + \frac{bx^5 \sqrt{-1 + cx} \sqrt{1 + cx}}{6c^2} \end{aligned}$$

**Mathematica [A]** time = 0.14, size = 117, normalized size = 0.94

$$\frac{3\sqrt{c^2 x^2 - 1} (6ac^2 + 5b) \tanh^{-1}\left(\frac{cx}{\sqrt{c^2 x^2 - 1}}\right) + cx(c^2 x^2 - 1)(6ac^2(2c^2 x^2 + 3) + b(8c^4 x^4 + 10c^2 x^2 + 15))}{48c^7 \sqrt{cx - 1} \sqrt{cx + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (c\*x\*(-1 + c^2\*x^2)\*(6\*a\*c^2\*(3 + 2\*c^2\*x^2) + b\*(15 + 10\*c^2\*x^2 + 8\*c^4\*x^4)) + 3\*(5\*b + 6\*a\*c^2)\*Sqrt[-1 + c^2\*x^2]\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(48\*c^7\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas [A]** time = 1.25, size = 96, normalized size = 0.77

$$\frac{(8bc^5x^5 + 2(6ac^5 + 5bc^3)x^3 + 3(6ac^3 + 5bc)x)\sqrt{cx+1}\sqrt{cx-1} - 3(6ac^2 + 5b)\log(-cx + \sqrt{cx+1}\sqrt{cx-1})}{48c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/48\*((8\*b\*c^5\*x^5 + 2\*(6\*a\*c^5 + 5\*b\*c^3)\*x^3 + 3\*(6\*a\*c^3 + 5\*b\*c)\*x)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - 3\*(6\*a\*c^2 + 5\*b)\*log(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1))/c^7

**giac [A]** time = 0.23, size = 172, normalized size = 1.38

$$\frac{\left( \left( 2 \left( (cx + 1) \left( 4 (cx + 1) \left( \frac{(cx+1)b}{c^6} - \frac{5b}{c^6} \right) + \frac{3(2ac^{38} + 15bc^{36})}{c^{42}} \right) - \frac{18ac^{38} + 55bc^{36}}{c^{42}} \right) (cx + 1) + \frac{54ac^{38} + 85bc^{36}}{c^{42}} \right) (cx + 1) - \frac{3(10ac^{38} + 15bc^{36})}{c^{42}} \right)}{48c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")



[Out]  $\frac{1}{48} * (((2 * ((c * x + 1) * (4 * (c * x + 1) * ((c * x + 1) * b / c^6 - 5 * b / c^6) + 3 * (2 * a * c^{38} + 15 * b * c^{36}) / c^{42}) - (18 * a * c^{38} + 55 * b * c^{36}) / c^{42}) * (c * x + 1) + (54 * a * c^{38} + 85 * b * c^{36}) / c^{42}) * (c * x + 1) - 3 * (10 * a * c^{38} + 11 * b * c^{36}) / c^{42}) * \sqrt{c * x + 1}) * \sqrt{c * x - 1} - 6 * (6 * a * c^2 + 5 * b) * \log(\sqrt{c * x + 1} - \sqrt{c * x - 1}) / c^6) / c$

**maple [C]** time = 0.11, size = 191, normalized size = 1.53

$$\frac{\sqrt{cx-1} \sqrt{cx+1} \left( 8\sqrt{c^2x^2-1} b c^5 x^5 \operatorname{csgn}(c) + 12\sqrt{c^2x^2-1} a c^5 x^3 \operatorname{csgn}(c) + 10\sqrt{c^2x^2-1} b c^3 x^3 \operatorname{csgn}(c) + 18\sqrt{c^2x^2-1} a c^3 x^3 \operatorname{csgn}(c) \right)}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $\frac{1}{48} * (c * x - 1)^{(1/2)} * (c * x + 1)^{(1/2)} * (8 * \operatorname{csgn}(c) * x^5 * b * c^5 * (c^2 * x^2 - 1)^{(1/2)} + 12 * \operatorname{csgn}(c) * x^3 * a * c^5 * (c^2 * x^2 - 1)^{(1/2)} + 10 * (c^2 * x^2 - 1)^{(1/2)} * \operatorname{csgn}(c) * c^3 * x^3 * b + 18 * (c^2 * x^2 - 1)^{(1/2)} * \operatorname{csgn}(c) * c^3 * x * a + 15 * (c^2 * x^2 - 1)^{(1/2)} * \operatorname{csgn}(c) * c * x * b + 18 * \ln(((c^2 * x^2 - 1)^{(1/2)} * \operatorname{csgn}(c) + c * x) * \operatorname{csgn}(c)) * a * c^2 + 15 * \ln(((c^2 * x^2 - 1)^{(1/2)} * \operatorname{csgn}(c) + c * x) * \operatorname{csgn}(c)) * b) * \operatorname{csgn}(c) / c^7 / (c^2 * x^2 - 1)^{(1/2)}$

**maxima [A]** time = 0.55, size = 153, normalized size = 1.22

$$\frac{\sqrt{c^2x^2-1} b x^5}{6c^2} + \frac{\sqrt{c^2x^2-1} a x^3}{4c^2} + \frac{5\sqrt{c^2x^2-1} b x^3}{24c^4} + \frac{3\sqrt{c^2x^2-1} a x}{8c^4} + \frac{3a \log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{8c^5} + \frac{5\sqrt{c^2x^2-1}}{16c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")`

[Out]  $\frac{1}{6} * \sqrt{c^2 * x^2 - 1} * b * x^5 / c^2 + \frac{1}{4} * \sqrt{c^2 * x^2 - 1} * a * x^3 / c^2 + \frac{5}{24} * \sqrt{c^2 * x^2 - 1} * b * x^3 / c^4 + \frac{3}{8} * \sqrt{c^2 * x^2 - 1} * a * x / c^4 + \frac{3}{8} * a * \log(2 * c^2 * x + 2 * \sqrt{c^2 * x^2 - 1} * c) / c^5 + \frac{5}{16} * \sqrt{c^2 * x^2 - 1} * b * x / c^6 + \frac{5}{16} * b * \log(2 * c^2 * x + 2 * \sqrt{c^2 * x^2 - 1} * c) / c^7$

**mupad [B]** time = 32.63, size = 1154, normalized size = 9.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^4*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)`

[Out]  $((23 * a * ((c * x - 1)^{(1/2)} - 1i)^3) / (2 * ((c * x + 1)^{(1/2)} - 1)^3) + (333 * a * ((c * x - 1)^{(1/2)} - 1i)^5) / (2 * ((c * x + 1)^{(1/2)} - 1)^5) + (671 * a * ((c * x - 1)^{(1/2)} - 1i)^7) / (2 * ((c * x + 1)^{(1/2)} - 1)^7) + (671 * a * ((c * x - 1)^{(1/2)} - 1i)^9) / (2 * ((c * x + 1)^{(1/2)} - 1)^9) + (333 * a * ((c * x - 1)^{(1/2)} - 1i)^11) / (2 * ((c * x + 1)^{(1/2)} - 1)^11) + (23 * a * ((c * x - 1)^{(1/2)} - 1i)^13) / (2 * ((c * x + 1)^{(1/2)} - 1)^13) - (3 * a * ((c * x - 1)^{(1/2)} - 1i)^15) / (2 * ((c * x + 1)^{(1/2)} - 1)^15) - (3 * a * ((c * x - 1)^{(1/2)} - 1i)) / (2 * ((c * x + 1)^{(1/2)} - 1))) / (c^5 - (8 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^2) / ((c * x + 1)^{(1/2)} - 1)^2 + (28 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^4) / ((c * x + 1)^{(1/2)} - 1)^4 - (56 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^6) / ((c * x + 1)^{(1/2)} - 1)^6 + (70 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^8) / ((c * x + 1)^{(1/2)} - 1)^8 - (56 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^10) / ((c * x + 1)^{(1/2)} - 1)^10 + (28 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^12) / ((c * x + 1)^{(1/2)} - 1)^12 - (8 * c^5 * ((c * x - 1)^{(1/2)} - 1i)^14) / ((c * x + 1)^{(1/2)} - 1)^14 + (c^5 * ((c * x - 1)^{(1/2)} - 1i)^16) / ((c * x + 1)^{(1/2)} - 1)^16 - ((311 * b * ((c * x - 1)^{(1/2)} - 1i)^5) / (4 * ((c * x + 1)^{(1/2)} - 1)^5) - (175 * b * ((c * x - 1)^{(1/2)} - 1i)^3) / (12 * ((c * x + 1)^{(1/2)} - 1)^3) + (8361 * b * ((c * x - 1)^{(1/2)} - 1i)^7) / (4 * ((c * x + 1)^{(1/2)} - 1)^7) + (42259 * b * ((c * x - 1)^{(1/2)} - 1i)^9) / (6 * ((c * x + 1)^{(1/2)} - 1)^9) + (25295 * b * ((c * x - 1)^{(1/2)} - 1i)^11) / (2 * ((c * x + 1)^{(1/2)} - 1)^11) + (25295 * b * ((c * x - 1)^{(1/2)} - 1i)^13) / (2 * ((c * x + 1)^{(1/2)} - 1)^13)$

$$\begin{aligned} & / (2*((c*x + 1)^{(1/2)} - 1)^{13}) + (42259*b*((c*x - 1)^{(1/2)} - 1i)^{15}) / (6*((c*x + 1)^{(1/2)} - 1)^{15}) + (8361*b*((c*x - 1)^{(1/2)} - 1i)^{17}) / (4*((c*x + 1)^{(1/2)} - 1)^{17}) + (311*b*((c*x - 1)^{(1/2)} - 1i)^{19}) / (4*((c*x + 1)^{(1/2)} - 1)^{19}) - (175*b*((c*x - 1)^{(1/2)} - 1i)^{21}) / (12*((c*x + 1)^{(1/2)} - 1)^{21}) + (5*b*((c*x - 1)^{(1/2)} - 1i)^{23}) / (4*((c*x + 1)^{(1/2)} - 1)^{23}) + (5*b*((c*x - 1)^{(1/2)} - 1i)) / (4*((c*x + 1)^{(1/2)} - 1)) / (c^7 - (12*c^7*((c*x - 1)^{(1/2)} - 1i)^2) / ((c*x + 1)^{(1/2)} - 1)^2 + (66*c^7*((c*x - 1)^{(1/2)} - 1i)^4) / ((c*x + 1)^{(1/2)} - 1)^4 - (220*c^7*((c*x - 1)^{(1/2)} - 1i)^6) / ((c*x + 1)^{(1/2)} - 1)^6 + (495*c^7*((c*x - 1)^{(1/2)} - 1i)^8) / ((c*x + 1)^{(1/2)} - 1)^8 - (792*c^7*((c*x - 1)^{(1/2)} - 1i)^10) / ((c*x + 1)^{(1/2)} - 1)^10 + (924*c^7*((c*x - 1)^{(1/2)} - 1i)^12) / ((c*x + 1)^{(1/2)} - 1)^12 - (792*c^7*((c*x - 1)^{(1/2)} - 1i)^14) / ((c*x + 1)^{(1/2)} - 1)^14 + (495*c^7*((c*x - 1)^{(1/2)} - 1i)^16) / ((c*x + 1)^{(1/2)} - 1)^16 - (220*c^7*((c*x - 1)^{(1/2)} - 1i)^18) / ((c*x + 1)^{(1/2)} - 1)^18 + (66*c^7*((c*x - 1)^{(1/2)} - 1i)^20) / ((c*x + 1)^{(1/2)} - 1)^20 - (12*c^7*((c*x - 1)^{(1/2)} - 1i)^22) / ((c*x + 1)^{(1/2)} - 1)^22 + (c^7*((c*x - 1)^{(1/2)} - 1i)^24) / ((c*x + 1)^{(1/2)} - 1)^24 + (3*a*atanh(((c*x - 1)^{(1/2)} - 1i) / ((c*x + 1)^{(1/2)} - 1))) / (2*c^5) + (5*b*atanh(((c*x - 1)^{(1/2)} - 1i) / ((c*x + 1)^{(1/2)} - 1))) / (4*c^7) \end{aligned}$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] Timed out

$$3.349 \quad \int \frac{x^3(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^6} + \frac{x^2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^4} + \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

[Out] 2/15\*(5\*a\*c^2+4\*b)\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^6+1/15\*(5\*a\*c^2+4\*b)\*x^2\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^4+1/5\*b\*x^4\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

**Rubi [A]** time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$ , Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^4} + \frac{2\sqrt{cx-1}\sqrt{cx+1}(5ac^2+4b)}{15c^6} + \frac{bx^4\sqrt{cx-1}\sqrt{cx+1}}{5c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (2\*(4\*b + 5\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^6) + ((4\*b + 5\*a\*c^2)\*x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(15\*c^4) + (b\*x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(5\*c^2)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 100

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx &= \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} - \frac{1}{5} \left(-5a - \frac{4b}{c^2}\right) \int \frac{x^3}{\sqrt{-1+cx}\sqrt{1+cx}} dx \\
&= \frac{(4b+5ac^2)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} + \frac{(4b+5ac^2) \int \frac{2x}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{15c^4} \\
&= \frac{(4b+5ac^2)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2} + \frac{(2(4b+5ac^2)) \int \frac{2x}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{15c^4} \\
&= \frac{2(4b+5ac^2)\sqrt{-1+cx}\sqrt{1+cx}}{15c^6} + \frac{(4b+5ac^2)x^2\sqrt{-1+cx}\sqrt{1+cx}}{15c^4} + \frac{bx^4\sqrt{-1+cx}\sqrt{1+cx}}{5c^2}
\end{aligned}$$

**Mathematica** [A] time = 0.04, size = 70, normalized size = 0.68

$$\frac{(c^2x^2-1)(5ac^2(c^2x^2+2)+b(3c^4x^4+4c^2x^2+8))}{15c^6\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a+b\*x^2))/(Sqrt[-1+c\*x]\*Sqrt[1+c\*x]),x]

[Out] ((-1+c^2\*x^2)\*(5\*a\*c^2\*(2+c^2\*x^2)+b\*(8+4\*c^2\*x^2+3\*c^4\*x^4)))/(15\*c^6\*Sqrt[-1+c\*x]\*Sqrt[1+c\*x])

**fricas** [A] time = 1.13, size = 55, normalized size = 0.53

$$\frac{(3bc^4x^4+10ac^2+(5ac^4+4bc^2)x^2+8b)\sqrt{cx+1}\sqrt{cx-1}}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/15\*(3\*b\*c^4\*x^4+10\*a\*c^2+(5\*a\*c^4+4\*b\*c^2)\*x^2+8\*b)\*sqrt(c\*x+1)\*sqrt(c\*x-1)/c^6

**giac** [A] time = 0.22, size = 108, normalized size = 1.05

$$\frac{\left(\left((cx+1)\left(3(cx+1)\left(\frac{(cx+1)b}{c^5}-\frac{4b}{c^5}\right)+\frac{5ac^{27}+22bc^{25}}{c^{30}}\right)-\frac{10(ac^{27}+2bc^{25})}{c^{30}}\right)(cx+1)+\frac{15(ac^{27}+bc^{25})}{c^{30}}\right)\sqrt{cx+1}\sqrt{cx-1}}{15c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] 1/15\*(((c\*x+1)\*(3\*(c\*x+1)\*((c\*x+1)\*b/c^5-4\*b/c^5)+(5\*a\*c^27+22\*b\*c^25)/c^30)-10\*(a\*c^27+2\*b\*c^25)/c^30)\*(c\*x+1)+15\*(a\*c^27+b\*c^25)/c^30)\*sqrt(c\*x+1)\*sqrt(c\*x-1)/c

**maple** [A] time = 0.05, size = 57, normalized size = 0.55

$$\frac{\sqrt{cx+1}\sqrt{cx-1}(3bx^4c^4+5ac^4x^2+4bc^2x^2+10ac^2+8b)}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out]  $1/15*(c*x+1)^{(1/2)}*(c*x-1)^{(1/2)}*(3*b*c^4*x^4+5*a*c^4*x^2+4*b*c^2*x^2+10*a*c^2+8*b)/c^6$

**maxima** [A] time = 0.63, size = 95, normalized size = 0.92

$$\frac{\sqrt{c^2x^2-1}bx^4}{5c^2} + \frac{\sqrt{c^2x^2-1}ax^2}{3c^2} + \frac{4\sqrt{c^2x^2-1}bx^2}{15c^4} + \frac{2\sqrt{c^2x^2-1}a}{3c^4} + \frac{8\sqrt{c^2x^2-1}b}{15c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")`

[Out]  $1/5*\text{sqrt}(c^2*x^2-1)*b*x^4/c^2 + 1/3*\text{sqrt}(c^2*x^2-1)*a*x^2/c^2 + 4/15*\text{sqrt}(c^2*x^2-1)*b*x^2/c^4 + 2/3*\text{sqrt}(c^2*x^2-1)*a/c^4 + 8/15*\text{sqrt}(c^2*x^2-1)*b/c^6$

**mupad** [B] time = 2.44, size = 108, normalized size = 1.05

$$\frac{\sqrt{cx-1} \left( \frac{10ac^2+8b}{15c^6} + \frac{bx^5}{5c} + \frac{bx^4}{5c^2} + \frac{x^2(5ac^4+4bc^2)}{15c^6} + \frac{x^3(5ac^5+4bc^3)}{15c^6} + \frac{x(10ac^3+8bc)}{15c^6} \right)}{\sqrt{cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a+b*x^2))/((c*x-1)^(1/2)*(c*x+1)^(1/2)),x)`

[Out]  $((c*x-1)^{(1/2)}*((8*b+10*a*c^2)/(15*c^6) + (b*x^5)/(5*c) + (b*x^4)/(5*c^2) + (x^2*(5*a*c^4+4*b*c^2))/(15*c^6) + (x^3*(5*a*c^5+4*b*c^3))/(15*c^6) + (x*(8*b*c+10*a*c^3))/(15*c^6)))/(c*x+1)^{(1/2)}$

**sympy** [C] time = 62.84, size = 216, normalized size = 2.10

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{c^2x^2} \right) + iaG_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2} \right) + bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{matrix} \right)}{4\pi^{\frac{3}{2}}c^4 + 4\pi^{\frac{3}{2}}c^4 + 4\pi^{\frac{3}{2}}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out]  $a*\text{meijerg}((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c**2*x**2))/(4*\text{pi}**(3/2)*c**4) + I*a*\text{meijerg}((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), \text{exp\_polar}(2*I*\text{pi})/(c**2*x**2))/(4*\text{pi}**(3/2)*c**4) + b*\text{meijerg}((( -9/4, -7/4), (-2, -2, -3/2, 1)), (( -5/2, -9/4, -2, -7/4, -3/2, 0), ()), 1/(c**2*x**2))/(4*\text{pi}**(3/2)*c**6) + I*b*\text{meijerg}((( -3, -11/4, -5/2, -9/4, -2, 1), ()), (( -11/4, -9/4), (-3, -5/2, -5/2, 0)), \text{exp\_polar}(2*I*\text{pi})/(c**2*x**2))/(4*\text{pi}**(3/2)*c**6)$

$$3.350 \quad \int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx$$

**Optimal.** Leaf size=87

$$\frac{(4ac^2 + 3b) \cosh^{-1}(cx)}{8c^5} + \frac{x\sqrt{cx-1}\sqrt{cx+1}(4ac^2 + 3b)}{8c^4} + \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

[Out] 1/8\*(4\*a\*c^2+3\*b)\*arccosh(c\*x)/c^5+1/8\*(4\*a\*c^2+3\*b)\*x\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^4+1/4\*b\*x^3\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

**Rubi [A]** time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {460, 90, 52}

$$\frac{x\sqrt{cx-1}\sqrt{cx+1}(4ac^2 + 3b)}{8c^4} + \frac{(4ac^2 + 3b) \cosh^{-1}(cx)}{8c^5} + \frac{bx^3\sqrt{cx-1}\sqrt{cx+1}}{4c^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((3\*b + 4\*a\*c^2)\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(8\*c^4) + (b\*x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(4\*c^2) + ((3\*b + 4\*a\*c^2)\*ArcCosh[c\*x])/(8\*c^5)

#### Rule 52

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)]), x\_Symbol] :> Simp[ArcCosh[(b\*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

#### Rule 90

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] :> Simp[(b\*(a + b\*x)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 3)), x] + Dist[1/(d\*f\*(n + p + 3)), Int[(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(n + p + 3) - b\*(b\*c\*e + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(n + p + 4) - b\*(d\*e\*(n + 2) + c\*f\*(p + 2)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 460

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_))^(non2\_)^(p\_)\*((a2\_) + (b2\_)\*(x\_))^(non2\_)^(p\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{x^2(a+bx^2)}{\sqrt{-1+cx}\sqrt{1+cx}} dx &= \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} - \frac{1}{4} \left( -4a - \frac{3b}{c^2} \right) \int \frac{x^2}{\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= \frac{(3b+4ac^2)x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2) \int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}}}{8c^4} \\ &= \frac{(3b+4ac^2)x\sqrt{-1+cx}\sqrt{1+cx}}{8c^4} + \frac{bx^3\sqrt{-1+cx}\sqrt{1+cx}}{4c^2} + \frac{(3b+4ac^2) \cosh^{-1}(cx)}{8c^5} \end{aligned}$$

**Mathematica [A]** time = 0.11, size = 98, normalized size = 1.13

$$\frac{cx(c^2x^2 - 1)(4ac^2 + b(2c^2x^2 + 3)) + \sqrt{c^2x^2 - 1}(4ac^2 + 3b)\tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2 - 1}}\right)}{8c^5\sqrt{cx - 1}\sqrt{cx + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (c\*x\*(-1 + c^2\*x^2)\*(4\*a\*c^2 + b\*(3 + 2\*c^2\*x^2)) + (3\*b + 4\*a\*c^2)\*Sqrt[-1 + c^2\*x^2]\*ArcTanh[(c\*x)/Sqrt[-1 + c^2\*x^2]])/(8\*c^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas [A]** time = 1.01, size = 77, normalized size = 0.89

$$\frac{(2bc^3x^3 + (4ac^3 + 3bc)x)\sqrt{cx + 1}\sqrt{cx - 1} - (4ac^2 + 3b)\log(-cx + \sqrt{cx + 1}\sqrt{cx - 1})}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/8\*((2\*b\*c^3\*x^3 + (4\*a\*c^3 + 3\*b\*c)\*x)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - (4\*a\*c^2 + 3\*b)\*log(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1)))/c^5

**giac [A]** time = 0.37, size = 121, normalized size = 1.39

$$\frac{(cx + 1)\left(2(cx + 1)\left(\frac{(cx+1)b}{c^4} - \frac{3b}{c^4}\right) + \frac{4ac^{18} + 9bc^{16}}{c^{20}}\right) - \frac{4ac^{18} + 5bc^{16}}{c^{20}}\sqrt{cx + 1}\sqrt{cx - 1} - \frac{2(4ac^2 + 3b)\log(\sqrt{cx+1} - \sqrt{cx-1})}{c^4}}{8c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] 1/8\*(((c\*x + 1)\*(2\*(c\*x + 1)\*((c\*x + 1)\*b/c^4 - 3\*b/c^4) + (4\*a\*c^18 + 9\*b\*c^16)/c^20) - (4\*a\*c^18 + 5\*b\*c^16)/c^20)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1) - 2\*(4\*a\*c^2 + 3\*b)\*log(sqrt(c\*x + 1) - sqrt(c\*x - 1))/c^4)/c

**maple [C]** time = 0.08, size = 147, normalized size = 1.69

$$\frac{\sqrt{cx - 1}\sqrt{cx + 1}\left(2\sqrt{c^2x^2 - 1}bc^3x^3\text{csgn}(c) + 4\sqrt{c^2x^2 - 1}ac^3x\text{csgn}(c) + 4ac^2\ln\left(\left(cx + \sqrt{c^2x^2 - 1}\text{csgn}(c)\right)\right)\right)}{8\sqrt{c^2x^2 - 1}c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out] 1/8\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)\*(2\*(c^2\*x^2-1)^(1/2)\*b\*c^3\*x^3\*csgn(c)+4\*(c^2\*x^2-1)^(1/2)\*a\*c^3\*x\*csgn(c)+3\*(c^2\*x^2-1)^(1/2)\*b\*c\*x\*csgn(c)+4\*a\*c^2\*ln((c\*x+(c^2\*x^2-1)^(1/2)\*csgn(c))\*csgn(c))+3\*b\*ln((c\*x+(c^2\*x^2-1)^(1/2)\*csgn(c))\*csgn(c)))\*csgn(c)/c^5/(c^2\*x^2-1)^(1/2)

**maxima [A]** time = 0.54, size = 113, normalized size = 1.30

$$\frac{\sqrt{c^2x^2 - 1}bx^3}{4c^2} + \frac{\sqrt{c^2x^2 - 1}ax}{2c^2} + \frac{a\log\left(2c^2x + 2\sqrt{c^2x^2 - 1}c\right)}{2c^3} + \frac{3\sqrt{c^2x^2 - 1}bx}{8c^4} + \frac{3b\log\left(2c^2x + 2\sqrt{c^2x^2 - 1}c\right)}{8c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="maxima")

[Out]  $\frac{1}{4}\sqrt{c^2x^2 - 1}bx^3/c^2 + \frac{1}{2}\sqrt{c^2x^2 - 1}ax/c^2 + \frac{1}{2}a\log(2c^2x + 2\sqrt{c^2x^2 - 1}c)/c^3 + \frac{3}{8}\sqrt{c^2x^2 - 1}bx/c^4 + \frac{3}{8}b\log(2c^2x + 2\sqrt{c^2x^2 - 1}c)/c^5$

**mupad [B]** time = 22.50, size = 720, normalized size = 8.28

$$\frac{\frac{23b(\sqrt{cx-1}-i)^3}{2(\sqrt{cx+1}-1)^3} + \frac{333b(\sqrt{cx-1}-i)^5}{2(\sqrt{cx+1}-1)^5} + \frac{671b(\sqrt{cx-1}-i)^7}{2(\sqrt{cx+1}-1)^7} + \frac{671b(\sqrt{cx-1}-i)^9}{2(\sqrt{cx+1}-1)^9} + \frac{333b(\sqrt{cx-1}-i)^{11}}{2(\sqrt{cx+1}-1)^{11}} + \frac{23b(\sqrt{cx-1}-i)^{13}}{2(\sqrt{cx+1}-1)^{13}} - \frac{3b(\sqrt{cx-1}-i)^{15}}{2(\sqrt{cx+1}-1)^{15}}}{c^5 - \frac{8c^5(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + \frac{28c^5(\sqrt{cx-1}-i)^4}{(\sqrt{cx+1}-1)^4} - \frac{56c^5(\sqrt{cx-1}-i)^6}{(\sqrt{cx+1}-1)^6} + \frac{70c^5(\sqrt{cx-1}-i)^8}{(\sqrt{cx+1}-1)^8} - \frac{56c^5(\sqrt{cx-1}-i)^{10}}{(\sqrt{cx+1}-1)^{10}} + \frac{28c^5(\sqrt{cx-1}-i)^{12}}{(\sqrt{cx+1}-1)^{12}} - \frac{8c^5(\sqrt{cx-1}-i)^{14}}{(\sqrt{cx+1}-1)^{14}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2*(a + b*x^2))/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)`

[Out]  $\frac{(23*b*((c*x - 1)^{(1/2)} - 1i)^3)/(2*((c*x + 1)^{(1/2)} - 1)^3) + (333*b*((c*x - 1)^{(1/2)} - 1i)^5)/(2*((c*x + 1)^{(1/2)} - 1)^5) + (671*b*((c*x - 1)^{(1/2)} - 1i)^7)/(2*((c*x + 1)^{(1/2)} - 1)^7) + (671*b*((c*x - 1)^{(1/2)} - 1i)^9)/(2*((c*x + 1)^{(1/2)} - 1)^9) + (333*b*((c*x - 1)^{(1/2)} - 1i)^{11})/(2*((c*x + 1)^{(1/2)} - 1)^{11}) + (23*b*((c*x - 1)^{(1/2)} - 1i)^{13})/(2*((c*x + 1)^{(1/2)} - 1)^{13}) - (3*b*((c*x - 1)^{(1/2)} - 1i)^{15})/(2*((c*x + 1)^{(1/2)} - 1)^{15}) - (3*b*((c*x - 1)^{(1/2)} - 1i))/(2*((c*x + 1)^{(1/2)} - 1)))/(c^5 - (8*c^5*((c*x - 1)^{(1/2)} - 1i)^2)/((c*x + 1)^{(1/2)} - 1)^2 + (28*c^5*((c*x - 1)^{(1/2)} - 1i)^4)/((c*x + 1)^{(1/2)} - 1)^4 - (56*c^5*((c*x - 1)^{(1/2)} - 1i)^6)/((c*x + 1)^{(1/2)} - 1)^6 + (70*c^5*((c*x - 1)^{(1/2)} - 1i)^8)/((c*x + 1)^{(1/2)} - 1)^8 - (56*c^5*((c*x - 1)^{(1/2)} - 1i)^{10})/((c*x + 1)^{(1/2)} - 1)^{10} + (28*c^5*((c*x - 1)^{(1/2)} - 1i)^{12})/((c*x + 1)^{(1/2)} - 1)^{12} - (8*c^5*((c*x - 1)^{(1/2)} - 1i)^{14})/((c*x + 1)^{(1/2)} - 1)^{14} + (c^5*((c*x - 1)^{(1/2)} - 1i)^{16})/((c*x + 1)^{(1/2)} - 1)^{16} - ((14*a*((c*x - 1)^{(1/2)} - 1i)^3)/((c*x + 1)^{(1/2)} - 1)^3 + (14*a*((c*x - 1)^{(1/2)} - 1i)^5)/((c*x + 1)^{(1/2)} - 1)^5 + (2*a*((c*x - 1)^{(1/2)} - 1i)^7)/((c*x + 1)^{(1/2)} - 1)^7 + (2*a*((c*x - 1)^{(1/2)} - 1i))/((c*x + 1)^{(1/2)} - 1))/(c^3 - (4*c^3*((c*x - 1)^{(1/2)} - 1i)^2)/((c*x + 1)^{(1/2)} - 1)^2 + (6*c^3*((c*x - 1)^{(1/2)} - 1i)^4)/((c*x + 1)^{(1/2)} - 1)^4 - (4*c^3*((c*x - 1)^{(1/2)} - 1i)^6)/((c*x + 1)^{(1/2)} - 1)^6 + (c^3*((c*x - 1)^{(1/2)} - 1i)^8)/((c*x + 1)^{(1/2)} - 1)^8 + (2*a*atanh(((c*x - 1)^{(1/2)} - 1i)/((c*x + 1)^{(1/2)} - 1)))/c^3 + (3*b*atanh(((c*x - 1)^{(1/2)} - 1i)/((c*x + 1)^{(1/2)} - 1)))/(2*c^5)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out] Timed out



$$3.351 \quad \int \frac{x(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (3ac^2 + 2b)}{3c^4} + \frac{bx^2 \sqrt{cx-1} \sqrt{cx+1}}{3c^2}$$

[Out] 1/3\*(3\*a\*c^2+2\*b)\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^4+1/3\*b\*x^2\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

**Rubi [A]** time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$ , Rules used = {460, 74}

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (3ac^2 + 2b)}{3c^4} + \frac{bx^2 \sqrt{cx-1} \sqrt{cx+1}}{3c^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((2\*b + 3\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*c^4) + (b\*x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*c^2)

Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx^2)}{\sqrt{-1+cx} \sqrt{1+cx}} dx &= \frac{bx^2 \sqrt{-1+cx} \sqrt{1+cx}}{3c^2} - \frac{1}{3} \left( -3a - \frac{2b}{c^2} \right) \int \frac{x}{\sqrt{-1+cx} \sqrt{1+cx}} dx \\ &= \frac{(2b + 3ac^2) \sqrt{-1+cx} \sqrt{1+cx}}{3c^4} + \frac{bx^2 \sqrt{-1+cx} \sqrt{1+cx}}{3c^2} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 52, normalized size = 0.80

$$\frac{(c^2x^2 - 1)(3ac^2 + b(c^2x^2 + 2))}{3c^4 \sqrt{cx-1} \sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*x^2))/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((-1 + c^2\*x^2)\*(3\*a\*c^2 + b\*(2 + c^2\*x^2)))/(3\*c^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas** [A] time = 1.04, size = 37, normalized size = 0.57

$$\frac{(bc^2x^2 + 3ac^2 + 2b)\sqrt{cx+1}\sqrt{cx-1}}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/3\*(b\*c^2\*x^2 + 3\*a\*c^2 + 2\*b)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)/c^4

**giac** [A] time = 0.22, size = 59, normalized size = 0.91

$$\frac{\sqrt{cx+1}\sqrt{cx-1}\left((cx+1)\left(\frac{(cx+1)b}{c^3} - \frac{2b}{c^3}\right) + \frac{3(ac^{11}+bc^9)}{c^{12}}\right)}{3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] 1/3\*sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*((c\*x + 1)\*((c\*x + 1)\*b/c^3 - 2\*b/c^3) + 3\*(a\*c^11 + b\*c^9)/c^12)/c

**maple** [A] time = 0.04, size = 38, normalized size = 0.58

$$\frac{\sqrt{cx+1}\sqrt{cx-1}(bc^2x^2 + 3ac^2 + 2b)}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out] 1/3\*(c\*x+1)^(1/2)\*(c\*x-1)^(1/2)\*(b\*c^2\*x^2+3\*a\*c^2+2\*b)/c^4

**maxima** [A] time = 0.47, size = 54, normalized size = 0.83

$$\frac{\sqrt{c^2x^2-1}bx^2}{3c^2} + \frac{\sqrt{c^2x^2-1}a}{c^2} + \frac{2\sqrt{c^2x^2-1}b}{3c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="maxima")

[Out] 1/3\*sqrt(c^2\*x^2 - 1)\*b\*x^2/c^2 + sqrt(c^2\*x^2 - 1)\*a/c^2 + 2/3\*sqrt(c^2\*x^2 - 1)\*b/c^4

**mupad** [B] time = 2.36, size = 66, normalized size = 1.02

$$\frac{\sqrt{cx-1}\left(\frac{3ac^2+2b}{3c^4} + \frac{bx^3}{3c} + \frac{bx^2}{3c^2} + \frac{x(3ac^3+2bc)}{3c^4}\right)}{\sqrt{cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*x^2))/((c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2)),x)

[Out] ((c\*x - 1)^(1/2)\*((2\*b + 3\*a\*c^2)/(3\*c^4) + (b\*x^3)/(3\*c) + (b\*x^2)/(3\*c^2) + (x\*(2\*b\*c + 3\*a\*c^3))/(3\*c^4)))/(c\*x + 1)^(1/2)

sympy [C] time = 41.91, size = 202, normalized size = 3.11

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) + iaG_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right) + bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, - \\ -\frac{3}{2}, -\frac{5}{4}, -1, - \end{matrix} \right)}{4\pi^{\frac{3}{2}} c^2} + \frac{4\pi^{\frac{3}{2}} c^2} + \frac{4\pi^{\frac{3}{2}} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] a\*meijerg((( -1/4, 1/4), (0, 0, 1/2, 1)), (( -1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*2) + I\*a\*meijerg((( -1, -3/4, -1/2, -1/4, 0, 1), ()), (( -3/4, -1/4), (-1, -1/2, -1/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*2) + b\*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*4) + I\*b\*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*4)

$$3.352 \quad \int \frac{a+bx^2}{\sqrt{-1+cx} \sqrt{1+cx}} dx$$

Optimal. Leaf size=47

$$\frac{(2ac^2 + b) \cosh^{-1}(cx)}{2c^3} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

[Out] 1/2\*(2\*a\*c^2+b)\*arccosh(c\*x)/c^3+1/2\*b\*x\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

**Rubi [A]** time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$ , Rules used = {389, 52}

$$\frac{(2ac^2 + b) \cosh^{-1}(cx)}{2c^3} + \frac{bx\sqrt{cx-1}\sqrt{cx+1}}{2c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (b\*x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*c^2) + ((b + 2\*a\*c^2)\*ArcCosh[c\*x])/(2\*c^3)

#### Rule 52

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)]\*Sqrt[(c\_) + (d\_.)\*(x\_)]), x\_Symbol] :> Simp[ArcCosh[(b\*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

#### Rule 389

Int[((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*(n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d - b1\*b2\*c\*(n\*(p + 1) + 1))/(b1\*b2\*(n\*(p + 1) + 1)), Int[(a1 + b1\*x^(n/2))^(p\*(a2 + b2\*x^(n/2))^(p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{\sqrt{-1+cx} \sqrt{1+cx}} dx &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2} - \frac{(-b-2ac^2) \int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}} dx}{2c^2} \\ &= \frac{bx\sqrt{-1+cx}\sqrt{1+cx}}{2c^2} + \frac{(b+2ac^2) \cosh^{-1}(cx)}{2c^3} \end{aligned}$$

**Mathematica [B]** time = 0.21, size = 101, normalized size = 2.15

$$\frac{4(ac^2 + b) \tanh^{-1}\left(\sqrt{\frac{cx-1}{cx+1}}\right) + \frac{b\left(cx\sqrt{-(cx-1)^2}\sqrt{cx+1} - 2\sqrt{cx-1} \sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\right)}{\sqrt{1-cx}}}{2c^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out]  $((b*(c*x*\text{Sqrt}[-(-1 + c*x)^2]*\text{Sqrt}[1 + c*x] - 2*\text{Sqrt}[-1 + c*x]*\text{ArcSin}[\text{Sqrt}[1 - c*x]/\text{Sqrt}[2]]))/\text{Sqrt}[1 - c*x] + 4*(b + a*c^2)*\text{ArcTanh}[\text{Sqrt}[(-1 + c*x)/(1 + c*x)]])/ (2*c^3)$

**fricas** [A] time = 1.10, size = 55, normalized size = 1.17

$$\frac{\sqrt{cx+1}\sqrt{cx-1}bcx - (2ac^2 + b)\log(-cx + \sqrt{cx+1}\sqrt{cx-1})}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")`

[Out]  $1/2*(\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)*b*c*x - (2*a*c^2 + b)*\log(-c*x + \text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)))/c^3$

**giac** [A] time = 0.18, size = 69, normalized size = 1.47

$$\frac{\sqrt{cx+1}\sqrt{cx-1}\left(\frac{(cx+1)b}{c^2} - \frac{b}{c^2}\right) - \frac{2(2ac^2+b)\log(\sqrt{cx+1}-\sqrt{cx-1})}{c^2}}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")`

[Out]  $1/2*(\text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)*((c*x + 1)*b/c^2 - b/c^2) - 2*(2*a*c^2 + b)*\log(\text{sqrt}(c*x + 1) - \text{sqrt}(c*x - 1)))/c^2/c$

**maple** [C] time = 0.07, size = 103, normalized size = 2.19

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(2ac^2\ln\left(\left(cx + \sqrt{c^2x^2-1}\text{csgn}(c)\right)\text{csgn}(c)\right) + \sqrt{c^2x^2-1}bcx\text{csgn}(c) + b\ln\left(\left(cx + \sqrt{c^2x^2-1}\right)\text{csgn}(c)\right)\right)}{2\sqrt{c^2x^2-1}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $1/2*(c*x-1)^(1/2)*(c*x+1)^(1/2)*((c^2*x^2-1)^(1/2)*b*c*x*\text{csgn}(c) + 2*a*c^2*\ln((c*x+(c^2*x^2-1)^(1/2)*\text{csgn}(c))*\text{csgn}(c)) + b*\ln((c*x+(c^2*x^2-1)^(1/2)*\text{csgn}(c))*\text{csgn}(c)))/c^3/(c^2*x^2-1)^(1/2)$

**maxima** [A] time = 0.50, size = 74, normalized size = 1.57

$$\frac{a\log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{c} + \frac{\sqrt{c^2x^2-1}bx}{2c^2} + \frac{b\log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")`

[Out]  $a*\log(2*c^2*x + 2*\text{sqrt}(c^2*x^2 - 1)*c)/c + 1/2*\text{sqrt}(c^2*x^2 - 1)*b*x/c^2 + 1/2*b*\log(2*c^2*x + 2*\text{sqrt}(c^2*x^2 - 1)*c)/c^3$

**mupad** [B] time = 12.69, size = 293, normalized size = 6.23

$$\frac{\frac{14b(\sqrt{cx-1}-i)^3}{(\sqrt{cx+1}-1)^3} + \frac{14b(\sqrt{cx-1}-i)^5}{(\sqrt{cx+1}-1)^5} + \frac{2b(\sqrt{cx-1}-i)^7}{(\sqrt{cx+1}-1)^7} + \frac{2b(\sqrt{cx-1}-i)}{\sqrt{cx+1}-1}}{c^3 - \frac{4c^3(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + \frac{6c^3(\sqrt{cx-1}-i)^4}{(\sqrt{cx+1}-1)^4} - \frac{4c^3(\sqrt{cx-1}-i)^6}{(\sqrt{cx+1}-1)^6} + \frac{c^3(\sqrt{cx-1}-i)^8}{(\sqrt{cx+1}-1)^8}} + \frac{2b\text{atanh}\left(\frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1}\right)}{c^3} - \frac{4a\text{atan}\left(\frac{c(\sqrt{cx-1}-i)}{(\sqrt{cx+1}-1)}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
```

```
[Out] (2*b*atanh(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1)))/c^3 - ((14*b*((c*x - 1)^(1/2) - 1i)^3)/((c*x + 1)^(1/2) - 1)^3 + (14*b*((c*x - 1)^(1/2) - 1i)^5)/((c*x + 1)^(1/2) - 1)^5 + (2*b*((c*x - 1)^(1/2) - 1i)^7)/((c*x + 1)^(1/2) - 1)^7 + (2*b*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1))/c^3 - (4*c^3*((c*x - 1)^(1/2) - 1i)^2)/((c*x + 1)^(1/2) - 1)^2 + (6*c^3*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 - (4*c^3*((c*x - 1)^(1/2) - 1i)^6)/((c*x + 1)^(1/2) - 1)^6 + (c^3*((c*x - 1)^(1/2) - 1i)^8)/((c*x + 1)^(1/2) - 1)^8 - (4*a*atan((c*((c*x - 1)^(1/2) - 1i))/((c*x + 1)^(1/2) - 1)*(-c^2)^(1/2))))/(-c^2)^(1/2)
```

```
sympy [C] time = 45.51, size = 182, normalized size = 3.87
```

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) - iaG_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right) + bG_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \right)}{4\pi^{\frac{3}{2}}c} + \frac{4\pi^{\frac{3}{2}}c^3}{4\pi^{\frac{3}{2}}c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
```

```
[Out] a*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*a*meijerg((-1/2, -1/4, 0, 1/4, 1/2, 1), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c) + b*meijerg((-3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c**3) - I*b*meijerg((-3/2, -5/4, -1, -3/4, -1/2, 1), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c**3)
```

$$3.353 \quad \int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx$$

Optimal. Leaf size=46

$$a \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

[Out] a\*arctan((c\*x-1)^(1/2)\*(c\*x+1)^(1/2))+b\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/c^2

Rubi [A] time = 0.06, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {460, 92, 205}

$$a \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right) + \frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (b\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/c^2 + a\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 460

Int[((e\_.)\*(x\_)^(m\_.))\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m+1)\*(a1 + b1\*x^(n/2))^(p+1)\*(a2 + b2\*x^(n/2))^(p+1))/(b1\*b2\*e\*(m+n\*(p+1)+1)), x] - Dist[(a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1))/(b1\*b2\*(m+n\*(p+1)+1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p+1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + a \int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + (ac) \text{Subst}\left(\int \frac{1}{c+cx^2} dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right) \\ &= \frac{b\sqrt{-1+cx}\sqrt{1+cx}}{c^2} + a \tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 1.43

$$\frac{ac^2\sqrt{c^2x^2-1}\tan^{-1}\left(\sqrt{c^2x^2-1}\right)+b(c^2x^2-1)}{c^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (b\*(-1 + c^2\*x^2) + a\*c^2\*Sqrt[-1 + c^2\*x^2]\*ArcTan[Sqrt[-1 + c^2\*x^2]])/(c^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas [A]** time = 0.98, size = 48, normalized size = 1.04

$$\frac{2ac^2\arctan\left(-cx+\sqrt{cx+1}\sqrt{cx-1}\right)+\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x, algorithm="fricas")

[Out] (2\*a\*c^2\*arctan(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*b)/c^2

**giac [A]** time = 0.22, size = 45, normalized size = 0.98

$$-2a\arctan\left(\frac{1}{2}\left(\sqrt{cx+1}-\sqrt{cx-1}\right)^2\right)+\frac{\sqrt{cx+1}\sqrt{cx-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x, algorithm="giac")

[Out] -2\*a\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2) + sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*b/c^2

**maple [A]** time = 0.07, size = 62, normalized size = 1.35

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-ac^2\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right)+\sqrt{c^2x^2-1}b\right)}{\sqrt{c^2x^2-1}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x)

[Out] (c\*x-1)^(1/2)\*(c\*x+1)^(1/2)\*(-arctan(1/(c^2\*x^2-1)^(1/2))\*a\*c^2+(c^2\*x^2-1)^(1/2)\*b)/(c^2\*x^2-1)^(1/2)/c^2

**maxima [A]** time = 1.15, size = 29, normalized size = 0.63

$$-a\arcsin\left(\frac{1}{c|x|}\right)+\frac{\sqrt{c^2x^2-1}b}{c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x, algorithm="maxima")

[Out] -a\*arcsin(1/(c\*abs(x))) + sqrt(c^2\*x^2 - 1)\*b/c^2



**mupad [B]** time = 3.86, size = 77, normalized size = 1.67

$$\frac{b\sqrt{cx-1}\sqrt{cx+1}}{c^2} - a \left( \ln \left( \frac{(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + 1 \right) - \ln \left( \frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1} \right) \right) 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x\*(c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2)),x)

[Out] (b\*(c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2))/c^2 - a\*(log(((c\*x - 1)^(1/2) - 1i)^2/((c\*x + 1)^(1/2) - 1)^2 + 1) - log(((c\*x - 1)^(1/2) - 1i)/((c\*x + 1)^(1/2) - 1)))\*1i

**sympy [C]** time = 40.14, size = 162, normalized size = 3.52

$$\frac{{}_aG_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) + i {}_aG_{6,6}^{2,6} \left( \begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right) + b {}_aG_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{c^2} \right)}{4\pi^{\frac{3}{2}} + 4\pi^{\frac{3}{2}} + 4\pi^{\frac{3}{2}} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] -a\*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)) + I\*a\*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)) + b\*meijerg((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*2) + I\*b\*meijerg((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp\_polar(2\*I\*pi)/(c\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*2)

$$3.354 \quad \int \frac{a+bx^2}{x^2 \sqrt{-1+cx} \sqrt{1+cx}} dx$$

Optimal. Leaf size=33

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \cosh^{-1}(cx)}{c}$$

[Out] b\*arccosh(c\*x)/c+a\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {454, 52}

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} + \frac{b \cosh^{-1}(cx)}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/x + (b\*ArcCosh[c\*x])/c

Rule 52

Int[1/(Sqrt[(a\_) + (b\_.)\*(x\_)]\*Sqrt[(c\_) + (d\_.)\*(x\_)]), x\_Symbol] :> Simp[ArcCosh[(b\*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(c\*(e\*x)^(m+1)\*(a1 + b1\*x^(n/2))^(p+1)\*(a2 + b2\*x^(n/2))^(p+1))/(a1\*a2\*e^(m+1)), x] + Dist[(a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1))/(a1\*a2\*e^(m+1)), Int[(e\*x)^(m+n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^2 \sqrt{-1+cx} \sqrt{1+cx}} dx &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x} + b \int \frac{1}{\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= \frac{a\sqrt{-1+cx}\sqrt{1+cx}}{x} + \frac{b \cosh^{-1}(cx)}{c} \end{aligned}$$

Mathematica [B] time = 0.03, size = 73, normalized size = 2.21

$$\frac{\sqrt{c^2x^2-1} \left( \frac{a\sqrt{c^2x^2-1}}{x} + \frac{b \tanh^{-1}\left(\frac{cx}{\sqrt{c^2x^2-1}}\right)}{c} \right)}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out]  $(\text{Sqrt}[-1 + c^2*x^2]*((a*\text{Sqrt}[-1 + c^2*x^2])/x + (b*\text{ArcTanh}[(c*x)/\text{Sqrt}[-1 + c^2*x^2]])/c))/(\text{Sqrt}[-1 + c*x]*\text{Sqrt}[1 + c*x])$

**fricas** [A] time = 1.18, size = 56, normalized size = 1.70

$$\frac{ac^2x + \sqrt{cx+1}\sqrt{cx-1}ac - bx \log(-cx + \sqrt{cx+1}\sqrt{cx-1})}{cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="fricas")`

[Out]  $(a*c^2*x + \text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)*a*c - b*x*\log(-c*x + \text{sqrt}(c*x + 1)*\text{sqrt}(c*x - 1)))/(c*x)$

**giac** [A] time = 0.22, size = 58, normalized size = 1.76

$$\frac{\frac{16ac^2}{(\sqrt{cx+1}-\sqrt{cx-1})^4+4} - b \log\left(\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^4\right)}{2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="giac")`

[Out]  $1/2*(16*a*c^2/((\text{sqrt}(c*x + 1) - \text{sqrt}(c*x - 1))^4 + 4) - b*\log((\text{sqrt}(c*x + 1) - \text{sqrt}(c*x - 1))^4))/c$

**maple** [C] time = 0.07, size = 77, normalized size = 2.33

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(\sqrt{c^2x^2-1}ac \text{csgn}(c) + bx \ln\left(\left(cx + \sqrt{c^2x^2-1} \text{csgn}(c)\right) \text{csgn}(c)\right)\right) \text{csgn}(c)}{\sqrt{c^2x^2-1}cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x)`

[Out]  $(c*x-1)^{(1/2)}*(c*x+1)^{(1/2)}*(\text{csgn}(c)*c*(c^2*x^2-1)^{(1/2)}*a+\ln((c*x+(c^2*x^2-1)^{(1/2)}*\text{csgn}(c))*\text{csgn}(c))*x*b)*\text{csgn}(c)/(c^2*x^2-1)^{(1/2)}/c/x$

**maxima** [A] time = 1.18, size = 44, normalized size = 1.33

$$\frac{b \log\left(2c^2x + 2\sqrt{c^2x^2-1}c\right)}{c} + \frac{\sqrt{c^2x^2-1}a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^2/(c*x-1)^(1/2)/(c*x+1)^(1/2),x, algorithm="maxima")`

[Out]  $b*\log(2*c^2*x + 2*\text{sqrt}(c^2*x^2 - 1)*c)/c + \text{sqrt}(c^2*x^2 - 1)*a/x$

**mupad** [B] time = 2.59, size = 61, normalized size = 1.85

$$\frac{a\sqrt{cx-1}\sqrt{cx+1}}{x} - \frac{4b \operatorname{atan}\left(\frac{c(\sqrt{cx-1}-i)}{(\sqrt{cx+1}-1)\sqrt{-c^2}}\right)}{\sqrt{-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x^2*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)`

[Out]  $(a*(c*x - 1)^{(1/2)}*(c*x + 1)^{(1/2)})/x - (4*b*atan((c*((c*x - 1)^{(1/2)} - 1i)))/(((c*x + 1)^{(1/2)} - 1)*(-c^2)^{(1/2)})))/(-c^2)^{(1/2)}$

**sympy** [C] time = 35.17, size = 148, normalized size = 4.48

$$\frac{acG_{6,6}^{5,3} \left( \begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| \frac{1}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{iacG_{6,6}^{2,6} \left( \begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{5}{4} & \frac{1}{2}, 1, 1, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}} + \frac{bG_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} & \frac{1}{2}, \frac{1}{2}, 1, 1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{1}{c^2x^2} \right)}{4\pi^{\frac{3}{2}}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*2/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out]  $-a*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + b*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)*c) - I*b*meijerg(((1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)*c)$

$$3.355 \quad \int \frac{a+bx^2}{x^3 \sqrt{-1+cx} \sqrt{1+cx}} dx$$

Optimal. Leaf size=60

$$\frac{1}{2} (ac^2 + 2b) \tan^{-1} \left( \sqrt{cx-1} \sqrt{cx+1} \right) + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{2x^2}$$

[Out] 1/2\*(a\*c^2+2\*b)\*arctan((c\*x-1)^(1/2)\*(c\*x+1)^(1/2))+1/2\*a\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x^2

**Rubi [A]** time = 0.06, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$ , Rules used = {454, 92, 205}

$$\frac{1}{2} (ac^2 + 2b) \tan^{-1} \left( \sqrt{cx-1} \sqrt{cx+1} \right) + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(2\*x^2) + ((2\*b + a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/2

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a1 + b1\*x^(n/2))^(p+1)\*(a2 + b2\*x^(n/2))^(q+1))/(a1\*a2\*e^(m+1)), x] + Dist[(a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1))/(a1\*a2\*e^(m+1)), Int[(e\*x)^(m+n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

#### Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^3 \sqrt{-1+cx} \sqrt{1+cx}} dx &= \frac{a\sqrt{-1+cx} \sqrt{1+cx}}{2x^2} + \frac{1}{2} (2b+ac^2) \int \frac{1}{x\sqrt{-1+cx} \sqrt{1+cx}} dx \\ &= \frac{a\sqrt{-1+cx} \sqrt{1+cx}}{2x^2} + \frac{1}{2} (c(2b+ac^2)) \text{Subst} \left( \int \frac{1}{c+cx^2} dx, x, \sqrt{-1+cx} \sqrt{1+cx} \right) \\ &= \frac{a\sqrt{-1+cx} \sqrt{1+cx}}{2x^2} + \frac{1}{2} (2b+ac^2) \tan^{-1} \left( \sqrt{-1+cx} \sqrt{1+cx} \right) \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 77, normalized size = 1.28

$$\frac{x^2\sqrt{c^2x^2-1} (ac^2 + 2b) \tan^{-1}\left(\sqrt{c^2x^2-1}\right) + a(c^2x^2-1)}{2x^2\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (a\*(-1 + c^2\*x^2) + (2\*b + a\*c^2)\*x^2\*Sqrt[-1 + c^2\*x^2]\*ArcTan[Sqrt[-1 + c^2\*x^2]])/(2\*x^2\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas [A]** time = 1.07, size = 57, normalized size = 0.95

$$\frac{2(ac^2 + 2b)x^2 \arctan(-cx + \sqrt{cx+1}\sqrt{cx-1}) + \sqrt{cx+1}\sqrt{cx-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/2\*(2\*(a\*c^2 + 2\*b)\*x^2\*arctan(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + sqrt(c\*x + 1)\*sqrt(c\*x - 1)\*a)/x^2

**giac [B]** time = 0.21, size = 114, normalized size = 1.90

$$\frac{(ac^3 + 2bc) \arctan\left(\frac{1}{2}(\sqrt{cx+1} - \sqrt{cx-1})^2\right) + \frac{2(ac^3(\sqrt{cx+1}-\sqrt{cx-1})^6 - 4ac^3(\sqrt{cx+1}-\sqrt{cx-1})^2)}{((\sqrt{cx+1}-\sqrt{cx-1})^4 + 4)^2}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] -((a\*c^3 + 2\*b\*c)\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2) + 2\*(a\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^6 - 4\*a\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2)/((sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 4)^2)/c

**maple [A]** time = 0.08, size = 84, normalized size = 1.40

$$\frac{\sqrt{cx-1}\sqrt{cx+1} \left( a c^2 x^2 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) + 2b x^2 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) - \sqrt{c^2x^2-1} a \right)}{2\sqrt{c^2x^2-1} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^3/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out] -1/2\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)\*(arctan(1/(c^2\*x^2-1)^(1/2))\*x^2\*a\*c^2+2\*a\*rctan(1/(c^2\*x^2-1)^(1/2))\*x^2\*b-(c^2\*x^2-1)^(1/2)\*a)/(c^2\*x^2-1)^(1/2)/x^2

**maxima [A]** time = 1.23, size = 45, normalized size = 0.75

$$-\frac{1}{2}ac^2 \arcsin\left(\frac{1}{c|x|}\right) - b \arcsin\left(\frac{1}{c|x|}\right) + \frac{\sqrt{c^2x^2-1}a}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="maxima")

[Out] -1/2\*a\*c^2\*arcsin(1/(c\*abs(x))) - b\*arcsin(1/(c\*abs(x))) + 1/2\*sqrt(c^2\*x^2 - 1)\*a/x^2

**mupad [B]** time = 8.67, size = 297, normalized size = 4.95

$$\frac{\frac{ac^2 1i}{32} + \frac{ac^2(\sqrt{cx-1}-i)^2 1i}{16(\sqrt{cx+1}-1)^2} - \frac{ac^2(\sqrt{cx-1}-i)^4 15i}{32(\sqrt{cx+1}-1)^4}}{\frac{(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + \frac{2(\sqrt{cx-1}-i)^4}{(\sqrt{cx+1}-1)^4} + \frac{(\sqrt{cx-1}-i)^6}{(\sqrt{cx+1}-1)^6}} - b \left( \ln \left( \frac{(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + 1 \right) - \ln \left( \frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1} \right) \right) 1i - \frac{ac^2 \ln \left( \frac{(\sqrt{cx-1}-i)}{(\sqrt{cx+1}-1)} \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^3*(c*x - 1)^(1/2)*(c*x + 1)^(1/2)),x)
[Out] ((a*c^2*1i)/32 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(16*((c*x + 1)^(1/2) - 1)^2) - (a*c^2*((c*x - 1)^(1/2) - 1i)^4*15i)/(32*((c*x + 1)^(1/2) - 1)^4)) / (((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + (2*((c*x - 1)^(1/2) - 1i)^4)/((c*x + 1)^(1/2) - 1)^4 + ((c*x - 1)^(1/2) - 1i)^6/((c*x + 1)^(1/2) - 1)^6) - b*(log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1) - log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))) * 1i - (a*c^2*log(((c*x - 1)^(1/2) - 1i)^2/((c*x + 1)^(1/2) - 1)^2 + 1)*1i)/2 + (a*c^2*log(((c*x - 1)^(1/2) - 1i)/((c*x + 1)^(1/2) - 1))*1i)/2 + (a*c^2*((c*x - 1)^(1/2) - 1i)^2*1i)/(32*((c*x + 1)^(1/2) - 1)^2)
```

**sympy [C]** time = 63.62, size = 141, normalized size = 2.35

$$\frac{ac^2 G_{6,6}^{5,3} \left( \begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} \end{matrix} \middle| \frac{1}{c^2 x^2} \right) + iac^2 G_{6,6}^{2,6} \left( \begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 \\ \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right) - bG_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} + 4\pi^{\frac{3}{2}} - 4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**3/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)
[Out] -a*c**2*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*a*c**2*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*b*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))
```

$$3.356 \quad \int \frac{a+bx^2}{x^4 \sqrt{-1+cx} \sqrt{1+cx}} dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (2ac^2 + 3b)}{3x} + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{3x^3}$$

[Out] 1/3\*a\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x^3+1/3\*(2\*a\*c^2+3\*b)\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {454, 95}

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (2ac^2 + 3b)}{3x} + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x^3) + ((3\*b + 2\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(3\*x)

Rule 95

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a\*d\*f\*(m + 1) + b\*c\*f\*(n + 1) + b\*d\*e\*(p + 1), 0] && NeQ[m, -1]

Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^4 \sqrt{-1+cx} \sqrt{1+cx}} dx &= \frac{a\sqrt{-1+cx} \sqrt{1+cx}}{3x^3} + \frac{1}{3} (3b + 2ac^2) \int \frac{1}{x^2 \sqrt{-1+cx} \sqrt{1+cx}} dx \\ &= \frac{a\sqrt{-1+cx} \sqrt{1+cx}}{3x^3} + \frac{(3b + 2ac^2) \sqrt{-1+cx} \sqrt{1+cx}}{3x} \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.82

$$\frac{(c^2x^2 - 1)(2ac^2x^2 + a + 3bx^2)}{3x^3 \sqrt{cx-1} \sqrt{cx+1}}$$

Antiderivative was successfully verified.



[In] Integrate[(a + b\*x^2)/(x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((-1 + c^2\*x^2)\*(a + 3\*b\*x^2 + 2\*a\*c^2\*x^2))/(3\*x^3\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas** [A] time = 0.70, size = 52, normalized size = 0.84

$$\frac{(2ac^3 + 3bc)x^3 + ((2ac^2 + 3b)x^2 + a)\sqrt{cx+1}\sqrt{cx-1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/3\*((2\*a\*c^3 + 3\*b\*c)\*x^3 + ((2\*a\*c^2 + 3\*b)\*x^2 + a)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^3

**giac** [B] time = 0.21, size = 116, normalized size = 1.87

$$\frac{8\left(3bc^2(\sqrt{cx+1} - \sqrt{cx-1})^8 + 24ac^4(\sqrt{cx+1} - \sqrt{cx-1})^4 + 24bc^2(\sqrt{cx+1} - \sqrt{cx-1})^4 + 32ac^4 + 48bc^2\right)}{3\left((\sqrt{cx+1} - \sqrt{cx-1})^4 + 4\right)^3 c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] 8/3\*(3\*b\*c^2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^8 + 24\*a\*c^4\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 24\*b\*c^2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 32\*a\*c^4 + 48\*b\*c^2)/(((sqrt(c\*x + 1) - sqrt(c\*x - 1))^4 + 4)^3\*c)

**maple** [A] time = 0.05, size = 37, normalized size = 0.60

$$\frac{\sqrt{cx+1}\sqrt{cx-1}(2ac^2x^2 + 3bx^2 + a)}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^4/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out] 1/3\*(c\*x+1)^(1/2)\*(c\*x-1)^(1/2)\*(2\*a\*c^2\*x^2+3\*b\*x^2+a)/x^3

**maxima** [A] time = 1.23, size = 54, normalized size = 0.87

$$\frac{2\sqrt{c^2x^2-1}ac^2}{3x} + \frac{\sqrt{c^2x^2-1}b}{x} + \frac{\sqrt{c^2x^2-1}a}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="maxima")

[Out] 2/3\*sqrt(c^2\*x^2 - 1)\*a\*c^2/x + sqrt(c^2\*x^2 - 1)\*b/x + 1/3\*sqrt(c^2\*x^2 - 1)\*a/x^3

**mupad** [B] time = 2.44, size = 53, normalized size = 0.85

$$\frac{\sqrt{cx-1}\left(\left(\frac{2ac^3}{3} + bc\right)x^3 + \left(\frac{2ac^2}{3} + b\right)x^2 + \frac{acx}{3} + \frac{a}{3}\right)}{x^3\sqrt{cx+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^4\*(c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2)),x)

[Out]  $((c*x - 1)^{(1/2)}*(a/3 + x^3*(b*c + (2*a*c^3)/3) + x^2*(b + (2*a*c^2)/3) + (a*c*x)/3))/(x^3*(c*x + 1)^{(1/2)})$

**sympy** [C] time = 61.44, size = 146, normalized size = 2.35

$$\frac{ac^3 G_{6,6}^{5,3} \left( \begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 & \frac{5}{2}, \frac{5}{2}, 3 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 & 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) iac^3 G_{6,6}^{2,6} \left( \begin{matrix} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 \\ \frac{7}{4}, \frac{9}{4} & \frac{3}{2}, 2, 2, 0 \end{matrix} \middle| \frac{e^{2i\pi}}{c^2 x^2} \right) bc G_{6,6}^{5,3} \left( \begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 & \frac{3}{2}, \frac{3}{2}, 2 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 & 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right)}{4\pi^{\frac{3}{2}} \quad 4\pi^{\frac{3}{2}} \quad 4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*4/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out]  $-a*c**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*a*c**3*meijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) - b*c*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) - I*b*c*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))$

$$3.357 \quad \int \frac{a+bx^2}{x^5 \sqrt{-1+cx} \sqrt{1+cx}} dx$$

**Optimal.** Leaf size=99

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (3ac^2 + 4b)}{8x^2} + \frac{1}{8}c^2 (3ac^2 + 4b) \tan^{-1} \left( \sqrt{cx-1} \sqrt{cx+1} \right) + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{4x^4}$$

[Out] 1/8\*c^2\*(3\*a\*c^2+4\*b)\*arctan((c\*x-1)^(1/2)\*(c\*x+1)^(1/2))+1/4\*a\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x^4+1/8\*(3\*a\*c^2+4\*b)\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/x^2

**Rubi [A]** time = 0.08, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$ , Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{cx-1} \sqrt{cx+1} (3ac^2 + 4b)}{8x^2} + \frac{1}{8}c^2 (3ac^2 + 4b) \tan^{-1} \left( \sqrt{cx-1} \sqrt{cx+1} \right) + \frac{a\sqrt{cx-1} \sqrt{cx+1}}{4x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (a\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(4\*x^4) + ((4\*b + 3\*a\*c^2)\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])/(8\*x^2) + (c^2\*(4\*b + 3\*a\*c^2)\*ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]])/8

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_)]\*Sqrt[(c\_.) + (d\_.)\*(x\_)]\*((e\_.) + (f\_.)\*(x\_))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_.)\*(x\_))^(m\_)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_)\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e\*(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x]

/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

### Rubi steps

$$\begin{aligned}
 \int \frac{a + bx^2}{x^5 \sqrt{-1 + cx} \sqrt{1 + cx}} dx &= \frac{a\sqrt{-1 + cx} \sqrt{1 + cx}}{4x^4} + \frac{1}{4} (4b + 3ac^2) \int \frac{1}{x^3 \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{a\sqrt{-1 + cx} \sqrt{1 + cx}}{4x^4} + \frac{(4b + 3ac^2) \sqrt{-1 + cx} \sqrt{1 + cx}}{8x^2} + \frac{1}{8} (4b + 3ac^2) \int \frac{1}{x \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{a\sqrt{-1 + cx} \sqrt{1 + cx}}{4x^4} + \frac{(4b + 3ac^2) \sqrt{-1 + cx} \sqrt{1 + cx}}{8x^2} + \frac{1}{8} (c^2 (4b + 3ac^2)) \int \frac{1}{x \sqrt{-1 + cx} \sqrt{1 + cx}} dx \\
 &= \frac{a\sqrt{-1 + cx} \sqrt{1 + cx}}{4x^4} + \frac{(4b + 3ac^2) \sqrt{-1 + cx} \sqrt{1 + cx}}{8x^2} + \frac{1}{8} (c^3 (4b + 3ac^2)) \text{Subst} \left( \int \frac{1}{x \sqrt{-1 + cx} \sqrt{1 + cx}} dx, x, \frac{x^2}{c} \right) \\
 &= \frac{a\sqrt{-1 + cx} \sqrt{1 + cx}}{4x^4} + \frac{(4b + 3ac^2) \sqrt{-1 + cx} \sqrt{1 + cx}}{8x^2} + \frac{1}{8} c^2 (4b + 3ac^2) \tan^{-1} \left( \sqrt{\frac{-1 + cx}{1 + cx}} \right)
 \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 102, normalized size = 1.03

$$\frac{(c^2 x^2 - 1) (a (3c^2 x^2 + 2) + 4bx^2) - c^2 x^4 \sqrt{1 - c^2 x^2} (3ac^2 + 4b) \tanh^{-1} \left( \sqrt{1 - c^2 x^2} \right)}{8x^4 \sqrt{cx - 1} \sqrt{cx + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^5\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] ((-1 + c^2\*x^2)\*(4\*b\*x^2 + a\*(2 + 3\*c^2\*x^2)) - c^2\*(4\*b + 3\*a\*c^2)\*x^4\*Sqrt[1 - c^2\*x^2]\*ArcTanh[Sqrt[1 - c^2\*x^2]])/(8\*x^4\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas [A]** time = 1.31, size = 78, normalized size = 0.79

$$\frac{2(3ac^4 + 4bc^2)x^4 \arctan(-cx + \sqrt{cx + 1} \sqrt{cx - 1}) + ((3ac^2 + 4b)x^2 + 2a)\sqrt{cx + 1} \sqrt{cx - 1}}{8x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^5/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] 1/8\*(2\*(3\*a\*c^4 + 4\*b\*c^2)\*x^4\*arctan(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1)) + ((3\*a\*c^2 + 4\*b)\*x^2 + 2\*a)\*sqrt(c\*x + 1)\*sqrt(c\*x - 1))/x^4

**giac [B]** time = 0.22, size = 268, normalized size = 2.71

$$(3ac^5 + 4bc^3) \arctan \left( \frac{1}{2} (\sqrt{cx + 1} - \sqrt{cx - 1})^2 \right) + \frac{2(3ac^5(\sqrt{cx+1}-\sqrt{cx-1})^{14} + 4bc^3(\sqrt{cx+1}-\sqrt{cx-1})^{14} + 44ac^5(\sqrt{cx+1}-\sqrt{cx-1})^{14})}{8x^4}$$

4c

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^5/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] -1/4\*((3\*a\*c^5 + 4\*b\*c^3)\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2) + 2\*(3\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^14 + 4\*b\*c^3\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^14 + 44\*a\*c^5\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^14))/x^4

$t(cx - 1)^{14} + 44ac^5(\sqrt{cx + 1} - \sqrt{cx - 1})^{10} + 16b^3c^3(\sqrt{cx + 1} - \sqrt{cx - 1})^{10} - 176a^5c^5(\sqrt{cx + 1} - \sqrt{cx - 1})^6 - 64b^3c^3(\sqrt{cx + 1} - \sqrt{cx - 1})^6 - 192a^5c^5(\sqrt{cx + 1} - \sqrt{cx - 1})^2 - 256b^3c^3(\sqrt{cx + 1} - \sqrt{cx - 1})^2 / ((\sqrt{cx + 1} - \sqrt{cx - 1})^4 + 4)^4 / c$

**maple [A]** time = 0.07, size = 125, normalized size = 1.26

$$\frac{\sqrt{cx-1} \sqrt{cx+1} \left(3ac^4x^4 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) + 4bc^2x^4 \arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) - 3\sqrt{c^2x^2-1} ac^2x^2 - 4\sqrt{c^2x^2-1} b\right)}{8\sqrt{c^2x^2-1} x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^5/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x)

[Out] -1/8\*(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)\*(3\*arctan(1/(c^2\*x^2-1)^(1/2))\*x^4\*a\*c^4+4\*arctan(1/(c^2\*x^2-1)^(1/2))\*x^4\*b\*c^2-3\*(c^2\*x^2-1)^(1/2)\*x^2\*a\*c^2-4\*(c^2\*x^2-1)^(1/2)\*x^2\*b-2\*(c^2\*x^2-1)^(1/2)\*a)/(c^2\*x^2-1)^(1/2)/x^4

**maxima [A]** time = 1.11, size = 85, normalized size = 0.86

$$-\frac{3}{8}ac^4 \arcsin\left(\frac{1}{|cx|}\right) - \frac{1}{2}bc^2 \arcsin\left(\frac{1}{|cx|}\right) + \frac{3\sqrt{c^2x^2-1}ac^2}{8x^2} + \frac{\sqrt{c^2x^2-1}b}{2x^2} + \frac{\sqrt{c^2x^2-1}a}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^5/(c\*x-1)^(1/2)/(c\*x+1)^(1/2), x, algorithm="maxima")

[Out] -3/8\*a\*c^4\*arcsin(1/(c\*abs(x))) - 1/2\*b\*c^2\*arcsin(1/(c\*abs(x))) + 3/8\*sqrt(c^2\*x^2 - 1)\*a\*c^2/x^2 + 1/2\*sqrt(c^2\*x^2 - 1)\*b/x^2 + 1/4\*sqrt(c^2\*x^2 - 1)\*a/x^4

**mupad [B]** time = 21.45, size = 650, normalized size = 6.57

$$\frac{\frac{bc^2 1i}{32} + \frac{bc^2(\sqrt{cx-1}-i)^2 1i}{16(\sqrt{cx+1}-1)^2} - \frac{bc^2(\sqrt{cx-1}-i)^4 15i}{32(\sqrt{cx+1}-1)^4} - \frac{ac^4 1i}{1024} - \frac{ac^4(\sqrt{cx-1}-i)^2 3i}{128(\sqrt{cx+1}-1)^2} - \frac{ac^4(\sqrt{cx-1}-i)^4 53i}{512(\sqrt{cx+1}-1)^4} + \frac{ac^4(\sqrt{cx-1}-i)^6 87i}{256(\sqrt{cx+1}-1)^6} + \frac{ac^4(\sqrt{cx-1}-i)^8 121i}{1024(\sqrt{cx+1}-1)^8} + \frac{ac^4(\sqrt{cx-1}-i)^{10} 657i}{512(\sqrt{cx+1}-1)^{10}} + \frac{ac^4(\sqrt{cx-1}-i)^{12} 121i}{256(\sqrt{cx+1}-1)^{12}}}{\frac{(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + \frac{2(\sqrt{cx-1}-i)^4}{(\sqrt{cx+1}-1)^4} + \frac{(\sqrt{cx-1}-i)^6}{(\sqrt{cx+1}-1)^6} + \frac{(\sqrt{cx-1}-i)^4}{(\sqrt{cx+1}-1)^4} + \frac{4(\sqrt{cx-1}-i)^6}{(\sqrt{cx+1}-1)^6} + \frac{6(\sqrt{cx-1}-i)^8}{(\sqrt{cx+1}-1)^8} + \frac{4(\sqrt{cx-1}-i)^{10}}{(\sqrt{cx+1}-1)^{10}} + \frac{6(\sqrt{cx-1}-i)^{12}}{(\sqrt{cx+1}-1)^{12}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^5\*(c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2)), x)

[Out] ((b\*c^2\*1i)/32 + (b\*c^2\*((c\*x - 1)^(1/2) - 1i)^2\*1i)/(16\*((c\*x + 1)^(1/2) - 1)^2) - (b\*c^2\*((c\*x - 1)^(1/2) - 1i)^4\*15i)/(32\*((c\*x + 1)^(1/2) - 1)^4) / (((c\*x - 1)^(1/2) - 1i)^2/((c\*x + 1)^(1/2) - 1)^2 + (2\*((c\*x - 1)^(1/2) - 1i)^4)/((c\*x + 1)^(1/2) - 1)^4 + ((c\*x - 1)^(1/2) - 1i)^6/((c\*x + 1)^(1/2) - 1)^6) - ((a\*c^4\*1i)/1024 - (a\*c^4\*((c\*x - 1)^(1/2) - 1i)^2\*3i)/(128\*((c\*x + 1)^(1/2) - 1)^2) - (a\*c^4\*((c\*x - 1)^(1/2) - 1i)^4\*53i)/(512\*((c\*x + 1)^(1/2) - 1)^4) + (a\*c^4\*((c\*x - 1)^(1/2) - 1i)^6\*87i)/(256\*((c\*x + 1)^(1/2) - 1)^6) + (a\*c^4\*((c\*x - 1)^(1/2) - 1i)^8\*657i)/(1024\*((c\*x + 1)^(1/2) - 1)^8) + (a\*c^4\*((c\*x - 1)^(1/2) - 1i)^10\*121i)/(256\*((c\*x + 1)^(1/2) - 1)^10) ) / (((c\*x - 1)^(1/2) - 1i)^4/((c\*x + 1)^(1/2) - 1)^4 + (4\*((c\*x - 1)^(1/2) - 1i)^6)/((c\*x + 1)^(1/2) - 1)^6 + (6\*((c\*x - 1)^(1/2) - 1i)^8)/((c\*x + 1)^(1/2) - 1)^8 + (4\*((c\*x - 1)^(1/2) - 1i)^10)/((c\*x + 1)^(1/2) - 1)^10 + ((c\*x - 1)^(1/2) - 1i)^12/((c\*x + 1)^(1/2) - 1)^12) - (a\*c^4\*log(((c\*x - 1)^(1/2) - 1i)^2/((c\*x + 1)^(1/2) - 1)^2 + 1)\*3i)/8 - (b\*c^2\*log(((c\*x - 1)^(1/2) - 1i)^2/((c\*x + 1)^(1/2) - 1)^2 + 1)\*1i)/2 + (a\*c^4\*log(((c\*x - 1)^(1/2) - 1i)/((c\*x + 1)^(1/2) - 1))\*3i)/8 + (b\*c^2\*log(((c\*x - 1)^(1/2) - 1i)/((c\*x + 1)^(1/2) - 1))\*1i)/2 + (a\*c^4\*log(((c\*x - 1)^(1/2) - 1i)^2\*7i)/(256\*((c\*x + 1)^(1/2) - 1)^2)

$$1)^{(1/2)} - 1)^2) - (a*c^4*((c*x - 1)^{(1/2)} - 1i)^4*1i)/(1024*((c*x + 1)^{(1/2)} - 1)^4) + (b*c^2*((c*x - 1)^{(1/2)} - 1i)^2*1i)/(32*((c*x + 1)^{(1/2)} - 1)^2)$$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*5/(c\*x-1)\*\*(1/2)/(c\*x+1)\*\*(1/2),x)

[Out] Timed out

$$3.358 \quad \int \frac{x^4(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$$

Optimal. Leaf size=164

$$\frac{c^2 x \sqrt{dx-c} \sqrt{c+dx} (6ad^2 + 5bc^2)}{16d^6} + \frac{x^3 \sqrt{dx-c} \sqrt{c+dx} (6ad^2 + 5bc^2)}{24d^4} + \frac{c^4 (6ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7} + \frac{bx^5}{d^2}$$

[Out] 1/8\*c^4\*(6\*a\*d^2+5\*b\*c^2)\*arctanh((d\*x-c)^(1/2)/(d\*x+c)^(1/2))/d^7+1/16\*c^2\*(6\*a\*d^2+5\*b\*c^2)\*x\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^6+1/24\*(6\*a\*d^2+5\*b\*c^2)\*x^3\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^4+1/6\*b\*x^5\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

**Rubi [A]** time = 0.12, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {460, 100, 12, 90, 63, 217, 206}

$$\frac{x^3 \sqrt{dx-c} \sqrt{c+dx} (6ad^2 + 5bc^2)}{24d^4} + \frac{c^2 x \sqrt{dx-c} \sqrt{c+dx} (6ad^2 + 5bc^2)}{16d^6} + \frac{c^4 (6ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{8d^7} + \frac{bx^5}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (c^2\*(5\*b\*c^2 + 6\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(16\*d^6) + ((5\*b\*c^2 + 6\*a\*d^2)\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(24\*d^4) + (b\*x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(6\*d^2) + (c^4\*(5\*b\*c^2 + 6\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(8\*d^7)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 90

Int[((a\_.) + (b\_.)\*(x\_))^(2\*((c\_.) + (d\_.)\*(x\_))^(n\_.))\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 3)), x] + Dist[1/(d\*f\*(n + p + 3)), Int[(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(n + p + 3) - b\*(b\*c\*e + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(n + p + 4) - b\*(d\*e\*(n + 2) + c\*f\*(p + 2)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 100

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}

}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

### Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

### Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

### Rule 460

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

### Rubi steps

$$\begin{aligned} \int \frac{x^4(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx &= \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} - \frac{1}{6} \left( -6a - \frac{5bc^2}{d^2} \right) \int \frac{x^4}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{(5bc^2+6ad^2) \int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}} dx}{24d^4} \\ &= \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} + \frac{(c^2(5bc^2+6ad^2)) \int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}} dx}{8d^4} \\ &= \frac{c^2(5bc^2+6ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} \\ &= \frac{c^2(5bc^2+6ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} \\ &= \frac{c^2(5bc^2+6ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} \\ &= \frac{c^2(5bc^2+6ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} \\ &= \frac{c^2(5bc^2+6ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{16d^6} + \frac{(5bc^2+6ad^2)x^3\sqrt{-c+dx}\sqrt{c+dx}}{24d^4} + \frac{bx^5\sqrt{-c+dx}\sqrt{c+dx}}{6d^2} \end{aligned}$$

**Mathematica** [A] time = 0.12, size = 148, normalized size = 0.90

$$\frac{3c^4\sqrt{d^2x^2-c^2}(6ad^2+5bc^2)\tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2-c^2}}\right)+dx(d^2x^2-c^2)(6ad^2(3c^2+2d^2x^2)+b(15c^4+10c^2d^2x^2+8d^4))}{48d^7\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.



[In] Integrate[(x^4\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (d\*x\*(-c^2 + d^2\*x^2)\*(6\*a\*d^2\*(3\*c^2 + 2\*d^2\*x^2) + b\*(15\*c^4 + 10\*c^2\*d^2\*x^2 + 8\*d^4\*x^4)) + 3\*c^4\*(5\*b\*c^2 + 6\*a\*d^2)\*Sqrt[-c^2 + d^2\*x^2]\*ArcTanh[(d\*x)/Sqrt[-c^2 + d^2\*x^2]])/(48\*d^7\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas** [A] time = 1.24, size = 115, normalized size = 0.70

$$\frac{(8bd^5x^5 + 2(5bc^2d^3 + 6ad^5)x^3 + 3(5bc^4d + 6ac^2d^3)x)\sqrt{dx+c}\sqrt{dx-c} - 3(5bc^6 + 6ac^4d^2)\log(-dx + \sqrt{dx+c})}{48d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/48\*((8\*b\*d^5\*x^5 + 2\*(5\*b\*c^2\*d^3 + 6\*a\*d^5)\*x^3 + 3\*(5\*b\*c^4\*d + 6\*a\*c^2\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 3\*(5\*b\*c^6 + 6\*a\*c^4\*d^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/d^7

**giac** [A] time = 0.28, size = 203, normalized size = 1.24

$$\frac{\left(2\left(dx+c\right)\left(4\left(dx+c\right)\left(\frac{(dx+c)b}{d^6}-\frac{5bc}{d^6}\right)+\frac{3(15bc^2d^{36}+2ad^{38})}{d^{42}}\right)-\frac{55bc^3d^{36}+18acd^{38}}{d^{42}}\right)(dx+c)+\frac{85bc^4d^{36}+54ac^2d^{38}}{d^{42}}}{48d}(dx + \sqrt{dx+c})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/48\*(((2\*((d\*x + c)\*(4\*(d\*x + c)\*((d\*x + c)\*b/d^6 - 5\*b\*c/d^6) + 3\*(15\*b\*c^2\*d^36 + 2\*a\*d^5\*x^3)/d^42) - (55\*b\*c^3\*d^36 + 18\*a\*c\*d^38)/d^42)\*(d\*x + c) + (85\*b\*c^4\*d^36 + 54\*a\*c^2\*d^38)/d^42)\*(d\*x + c) - 3\*(11\*b\*c^5\*d^36 + 10\*a\*c^3\*d^38)/d^42)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 6\*(5\*b\*c^6 + 6\*a\*c^4\*d^2)\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^6)/d

**maple** [C] time = 0.10, size = 240, normalized size = 1.46

$$\frac{\sqrt{dx-c}\sqrt{dx+c}\left(8\sqrt{d^2x^2-c^2}bd^5x^5\text{csgn}(d)+12\sqrt{d^2x^2-c^2}ad^5x^3\text{csgn}(d)+10\sqrt{d^2x^2-c^2}bc^2d^3x^3\text{csgn}(d)\right)}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 1/48\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)\*(8\*(d^2\*x^2-c^2)^(1/2)\*b\*d^5\*x^5\*csgn(d)+12\*(d^2\*x^2-c^2)^(1/2)\*a\*d^5\*x^3\*csgn(d)+10\*(d^2\*x^2-c^2)^(1/2)\*b\*c^2\*d^3\*x^3\*csgn(d)+18\*(d^2\*x^2-c^2)^(1/2)\*a\*c^2\*d^3\*x\*csgn(d)+15\*(d^2\*x^2-c^2)^(1/2)\*b\*c^4\*d\*x\*csgn(d)+18\*a\*c^4\*d^2\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))+15\*b\*c^6\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))\*csgn(d)/d^7/(d^2\*x^2-c^2)^(1/2)

**maxima** [A] time = 0.57, size = 196, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2}bx^5}{6d^2} + \frac{5\sqrt{d^2x^2-c^2}bc^2x^3}{24d^4} + \frac{\sqrt{d^2x^2-c^2}ax^3}{4d^2} + \frac{5bc^6\log\left(2d^2x+2\sqrt{d^2x^2-c^2}d\right)}{16d^7} + \frac{3ac^4\log\left(2d^2x+2\sqrt{d^2x^2-c^2}d\right)}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/6\*sqrt(d^2\*x^2 - c^2)\*b\*x^5/d^2 + 5/24\*sqrt(d^2\*x^2 - c^2)\*b\*c^2\*x^3/d^4 + 1/4\*sqrt(d^2\*x^2 - c^2)\*a\*x^3/d^2 + 5/16\*b\*c^6\*log(2\*d^2\*x + 2\*sqrt(d^2\*x

$$\frac{d^2 - c^2}{d^7} + \frac{3}{8} a c^4 \log(2d^2 x + 2\sqrt{d^2 x^2 - c^2} d) / d^5 + \frac{5}{16} \sqrt{d^2 x^2 - c^2} b c^4 x / d^6 + \frac{3}{8} \sqrt{d^2 x^2 - c^2} a c^2 x / d^4$$

**mupad [B]** time = 42.66, size = 1682, normalized size = 10.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((x^4(a + b*x^2))/((c + d*x)^{(1/2)}*(d*x - c)^{(1/2)}), x)$

[Out]  $((5*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)}))/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})) - (175*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^3)/(12*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^3) + (311*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^5)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^5) + (8361*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^7)/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^7) + (42259*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^9)/(6*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^9) + (25295*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{11})/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{11}) + (25295*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{13})/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{13}) + (42259*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{15})/(6*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{15}) + (8361*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{17})/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{17}) + (311*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{19})/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{19}) - (175*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{21})/(12*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{21}) + (5*b*c^6*((c + d*x)^{(1/2)} - c^{(1/2)})^{23})/(4*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{23}))/((d^7 - (12*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (66*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 - (220*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^6)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6 + (495*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^8)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^8 - (792*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{10})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{10} + (924*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{12})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{12} - (792*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{14})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{14} + (495*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{16})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{16} - (220*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{18})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{18} + (66*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{20})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{20} - (12*d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{22})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{22} + (d^7*((c + d*x)^{(1/2)} - c^{(1/2)})^{24})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{24} - ((23*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^3)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^3) - (3*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)}))/((2*((-c)^{(1/2)} - (d*x - c)^{(1/2)}))) + (333*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^5)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^5) + (671*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^7)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^7) + (671*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^9)/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^9) + (333*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^{11})/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{11}) + (23*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^{13})/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{13}) - (3*a*c^4*((c + d*x)^{(1/2)} - c^{(1/2)})^{15})/(2*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{15}))/((d^5 - (8*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (28*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 - (56*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^6)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6 + (70*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^8)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^8 - (56*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^{10})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{10} + (28*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^{12})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{12} - (8*d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^{14})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{14} + (d^5*((c + d*x)^{(1/2)} - c^{(1/2)})^{16})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{16} - (3*a*c^4*atanh(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})))/((2*d^5) - (5*b*c^6*atanh(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})))/((4*d^7)$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

$$3.359 \quad \int \frac{x^3(a+bx^2)}{\sqrt{-c+dx} \sqrt{c+dx}} dx$$

**Optimal.** Leaf size=118

$$\frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4} + \frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

[Out]  $2/15*c^2*(5*a*d^2+4*b*c^2)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^6+1/15*(5*a*d^2+4*b*c^2)*x^2*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^4+1/5*b*x^4*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^2$

**Rubi [A]** time = 0.09, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {460, 100, 12, 74}

$$\frac{x^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^4} + \frac{2c^2\sqrt{dx-c}\sqrt{c+dx}(5ad^2+4bc^2)}{15d^6} + \frac{bx^4\sqrt{dx-c}\sqrt{c+dx}}{5d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out]  $(2*c^2*(4*b*c^2 + 5*a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^6) + ((4*b*c^2 + 5*a*d^2)*x^2*Sqrt[-c + d*x]*Sqrt[c + d*x])/(15*d^4) + (b*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x])/(5*d^2)$

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 100

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)^(m - 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(m + n + p + 1)), x] + Dist[1/(d\*f\*(m + n + p + 1)), Int[(a + b\*x)^(m - 2)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(m + n + p + 1) - b\*(b\*c\*e\*(m - 1) + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(2\*m + n + p) - b\*(d\*e\*(m + n) + c\*f\*(m + p)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{x^3 (a + bx^2)}{\sqrt{-c + dx} \sqrt{c + dx}} dx &= \frac{bx^4 \sqrt{-c + dx} \sqrt{c + dx}}{5d^2} - \frac{1}{5} \left( -5a - \frac{4bc^2}{d^2} \right) \int \frac{x^3}{\sqrt{-c + dx} \sqrt{c + dx}} dx \\
&= \frac{(4bc^2 + 5ad^2) x^2 \sqrt{-c + dx} \sqrt{c + dx}}{15d^4} + \frac{bx^4 \sqrt{-c + dx} \sqrt{c + dx}}{5d^2} + \frac{(4bc^2 + 5ad^2) \int \frac{x^3}{\sqrt{-c + dx} \sqrt{c + dx}} dx}{15d^4} \\
&= \frac{(4bc^2 + 5ad^2) x^2 \sqrt{-c + dx} \sqrt{c + dx}}{15d^4} + \frac{bx^4 \sqrt{-c + dx} \sqrt{c + dx}}{5d^2} + \frac{(2c^2 (4bc^2 + 5ad^2) \int \frac{x^3}{\sqrt{-c + dx} \sqrt{c + dx}} dx)}{15d^4} \\
&= \frac{2c^2 (4bc^2 + 5ad^2) \sqrt{-c + dx} \sqrt{c + dx}}{15d^6} + \frac{(4bc^2 + 5ad^2) x^2 \sqrt{-c + dx} \sqrt{c + dx}}{15d^4} + \frac{bx^4 \sqrt{-c + dx} \sqrt{c + dx}}{5d^2}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 87, normalized size = 0.74

$$\frac{(d^2 x^2 - c^2) (5ad^2 (2c^2 + d^2 x^2) + b (8c^4 + 4c^2 d^2 x^2 + 3d^4 x^4))}{15d^6 \sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] ((-c^2 + d^2\*x^2)\*(5\*a\*d^2\*(2\*c^2 + d^2\*x^2) + b\*(8\*c^4 + 4\*c^2\*d^2\*x^2 + 3\*d^4\*x^4)))/(15\*d^6\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.10, size = 66, normalized size = 0.56

$$\frac{(3bd^4x^4 + 8bc^4 + 10ac^2d^2 + (4bc^2d^2 + 5ad^4)x^2)\sqrt{dx + c}\sqrt{dx - c}}{15d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/15\*(3\*b\*d^4\*x^4 + 8\*b\*c^4 + 10\*a\*c^2\*d^2 + (4\*b\*c^2\*d^2 + 5\*a\*d^4)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d^6

**giac [A]** time = 0.25, size = 124, normalized size = 1.05

$$\frac{\left( (dx + c) \left( 3(dx + c) \left( \frac{(dx+c)b}{d^5} - \frac{4bc}{d^5} \right) + \frac{22bc^2d^{25} + 5ad^{27}}{d^{30}} \right) - \frac{10(2bc^3d^{25} + acd^{27})}{d^{30}} \right) (dx + c) + \frac{15(bc^4d^{25} + ac^2d^{27})}{d^{30}} \right) \sqrt{dx + c} \sqrt{dx - c}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/15\*(((d\*x + c)\*(3\*(d\*x + c)\*((d\*x + c)\*b/d^5 - 4\*b\*c/d^5) + (22\*b\*c^2\*d^25 + 5\*a\*d^27)/d^30) - 10\*(2\*b\*c^3\*d^25 + a\*c\*d^27)/d^30)\*(d\*x + c) + 15\*(b\*c^4\*d^25 + a\*c^2\*d^27)/d^30)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d

**maple [A]** time = 0.05, size = 68, normalized size = 0.58

$$\frac{\sqrt{dx + c} (3bd^4x^4 + 5ad^4x^2 + 4bc^2d^2x^2 + 10ac^2d^2 + 8bc^4) \sqrt{dx - c}}{15d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out]  $1/15*(d*x+c)^{(1/2)}*(3*b*d^4*x^4+5*a*d^4*x^2+4*b*c^2*d^2*x^2+10*a*c^2*d^2+8*b*c^4)/d^6*(d*x-c)^{(1/2)}$

**maxima** [A] time = 0.55, size = 124, normalized size = 1.05

$$\frac{\sqrt{d^2x^2 - c^2} bx^4}{5d^2} + \frac{4\sqrt{d^2x^2 - c^2} bc^2x^2}{15d^4} + \frac{\sqrt{d^2x^2 - c^2} ax^2}{3d^2} + \frac{8\sqrt{d^2x^2 - c^2} bc^4}{15d^6} + \frac{2\sqrt{d^2x^2 - c^2} ac^2}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $1/5*\sqrt{d^2*x^2 - c^2}*b*x^4/d^2 + 4/15*\sqrt{d^2*x^2 - c^2}*b*c^2*x^2/d^4 + 1/3*\sqrt{d^2*x^2 - c^2}*a*x^2/d^2 + 8/15*\sqrt{d^2*x^2 - c^2}*b*c^4/d^6 + 2/3*\sqrt{d^2*x^2 - c^2}*a*c^2/d^4$

**mupad** [B] time = 2.70, size = 130, normalized size = 1.10

$$\frac{\sqrt{dx-c} \left( \frac{8bc^5+10ac^3d^2}{15d^6} + \frac{x^3(4bc^2d^3+5ad^5)}{15d^6} + \frac{x(8bc^4d+10ac^2d^3)}{15d^6} + \frac{bx^5}{5d} + \frac{x^2(4bc^3d^2+5acd^4)}{15d^6} + \frac{bcx^4}{5d^2} \right)}{\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)`

[Out]  $((d*x - c)^{(1/2)}*((8*b*c^5 + 10*a*c^3*d^2)/(15*d^6) + (x^3*(5*a*d^5 + 4*b*c^2*d^3))/(15*d^6) + (x*(10*a*c^2*d^3 + 8*b*c^4*d))/(15*d^6) + (b*x^5)/(5*d) + (x^2*(4*b*c^3*d^2 + 5*a*c*d^4))/(15*d^6) + (b*c*x^4)/(5*d^2)))/(c + d*x)^{(1/2)}$

**sympy** [C] time = 70.84, size = 240, normalized size = 2.03

$$\frac{ac^3 G_{6,6}^{6,2} \left( \begin{matrix} -\frac{5}{4}, -\frac{3}{4} & -1, -1, -\frac{1}{2}, 1 \\ -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) + iac^3 G_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}, -1, 1 \\ -\frac{7}{4}, -\frac{5}{4} & -2, -\frac{3}{2}, -\frac{3}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^4} + \frac{bc^5 G_{6,6}^{6,2}}{4\pi^{\frac{3}{2}} d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $a*c**3*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**4) + I*a*c**3*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**4) + b*c**5*meijerg((( -9/4, -7/4), (-2, -2, -3/2, 1)), (( -5/2, -9/4, -2, -7/4, -3/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**6) + I*b*c**5*meijerg((( -3, -11/4, -5/2, -9/4, -2, 1), ()), (( -11/4, -9/4), (-3, -5/2, -5/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**6)$

$$3.360 \quad \int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

**Optimal.** Leaf size=118

$$\frac{c^2(4ad^2 + 3bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5} + \frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 3bc^2)}{8d^4} + \frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

[Out] 1/4\*c^2\*(4\*a\*d^2+3\*b\*c^2)\*arctanh((d\*x-c)^(1/2)/(d\*x+c)^(1/2))/d^5+1/8\*(4\*a\*d^2+3\*b\*c^2)\*x\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^4+1/4\*b\*x^3\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

**Rubi [A]** time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$ , Rules used = {460, 90, 12, 63, 217, 206}

$$\frac{x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 3bc^2)}{8d^4} + \frac{c^2(4ad^2 + 3bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^5} + \frac{bx^3\sqrt{dx-c}\sqrt{c+dx}}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] ((3\*b\*c^2 + 4\*a\*d^2)\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*d^4) + (b\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(4\*d^2) + (c^2\*(3\*b\*c^2 + 4\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/(4\*d^5)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 90

Int[((a\_.) + (b\_.)\*(x\_))^(2)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(a + b\*x)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 3)), x] + Dist[1/(d\*f\*(n + p + 3)), Int[(c + d\*x)^n\*(e + f\*x)^p\*Simp[a^2\*d\*f\*(n + p + 3) - b\*(b\*c\*e + a\*(d\*e\*(n + 1) + c\*f\*(p + 1))) + b\*(a\*d\*f\*(n + p + 4) - b\*(d\*e\*(n + 2) + c\*f\*(p + 2)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

## Rule 460

```
Int[((e_.)*(x_))^(m_.)*((a1_.) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_.) + (b2_.)
*(x_)^(non2_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(d*(e*x)^(
m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m +
n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/
(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^(m*(a1 + b1*x^(n/2))^(p*(a2 + b2*x^(n/
2))^(p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,
n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

## Rubi steps

$$\begin{aligned} \int \frac{x^2(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx &= \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} - \frac{1}{4} \left( -4a - \frac{3bc^2}{d^2} \right) \int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= \frac{(3bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{(3bc^2 + 4ad^2) \int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx}{8d^4} \\ &= \frac{(3bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2 + 4ad^2)) \int \frac{x^2}{\sqrt{-c+dx}\sqrt{c+dx}} dx}{8d^4} \\ &= \frac{(3bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2 + 4ad^2)) \operatorname{Su}}{8d^4} \\ &= \frac{(3bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{(c^2(3bc^2 + 4ad^2)) \operatorname{Su}}{8d^4} \\ &= \frac{(3bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^4} + \frac{bx^3\sqrt{-c+dx}\sqrt{c+dx}}{4d^2} + \frac{c^2(3bc^2 + 4ad^2) \operatorname{tanh}^{-1}\left(\frac{dx}{\sqrt{d^2x^2 - c^2}}\right)}{4d^5} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 121, normalized size = 1.03

$$\frac{dx(d^2x^2 - c^2)(4ad^2 + 3bc^2 + 2bd^2x^2) + c^2\sqrt{d^2x^2 - c^2}(4ad^2 + 3bc^2) \tanh^{-1}\left(\frac{dx}{\sqrt{d^2x^2 - c^2}}\right)}{8d^5\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (d\*x\*(-c^2 + d^2\*x^2)\*(3\*b\*c^2 + 4\*a\*d^2 + 2\*b\*d^2\*x^2) + c^2\*(3\*b\*c^2 + 4\*a\*d^2)\*Sqrt[-c^2 + d^2\*x^2]\*ArcTanh[(d\*x)/Sqrt[-c^2 + d^2\*x^2]])/(8\*d^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 0.91, size = 90, normalized size = 0.76

$$\frac{(2bd^3x^3 + (3bc^2d + 4ad^3)x)\sqrt{dx+c}\sqrt{dx-c} - (3bc^4 + 4ac^2d^2) \log(-dx + \sqrt{dx+c}\sqrt{dx-c})}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2), x, algorithm="fricas")

[Out] 1/8\*((2\*b\*d^3\*x^3 + (3\*b\*c^2\*d + 4\*a\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - (3\*b\*c^4 + 4\*a\*c^2\*d^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/d^5

**giac [A]** time = 0.23, size = 140, normalized size = 1.19

$$\frac{\left((dx+c)\left(2(dx+c)\left(\frac{(dx+c)b}{d^4} - \frac{3bc}{d^4}\right) + \frac{9bc^2d^{16}+4ad^{18}}{d^{20}}\right) - \frac{5bc^3d^{16}+4acd^{18}}{d^{20}}\right)\sqrt{dx+c}\sqrt{dx-c} - \frac{2(3bc^4+4ac^2d^2) \log\left(-\sqrt{dx+c} + \sqrt{dx-c}\right)}{d^4}}{8d}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] 1/8*(((d*x + c)*(2*(d*x + c)*((d*x + c)*b/d^4 - 3*b*c/d^4) + (9*b*c^2*d^16 + 4*a*d^18)/d^20) - (5*b*c^3*d^16 + 4*a*c*d^18)/d^20)*sqrt(d*x + c)*sqrt(d*x - c) - 2*(3*b*c^4 + 4*a*c^2*d^2)*log(abs(-sqrt(d*x + c) + sqrt(d*x - c)))/d^4)/d
```

**maple [C]** time = 0.08, size = 182, normalized size = 1.54

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( 2\sqrt{d^2x^2-c^2} b d^3 x^3 \operatorname{csgn}(d) + 4a c^2 d^2 \ln \left( \left( dx + \sqrt{d^2x^2-c^2} \operatorname{csgn}(d) \right) \operatorname{csgn}(d) \right) + 4\sqrt{d^2x^2-c^2} \right)}{8\sqrt{d^2x^2-c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)
[Out] 1/8*(d*x-c)^(1/2)*(d*x+c)^(1/2)*(2*(d^2*x^2-c^2)^(1/2)*b*d^3*x^3*csgn(d)+4*(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)+3*(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)+4*a*c^2*d^2*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d))+3*b*c^4*ln((d*x+(d^2*x^2-c^2)^(1/2)*csgn(d))*csgn(d)))*csgn(d)/d^5/(d^2*x^2-c^2)^(1/2)
```

**maxima [A]** time = 0.64, size = 142, normalized size = 1.20

$$\frac{\sqrt{d^2x^2-c^2} b x^3}{4d^2} + \frac{3bc^4 \log\left(2d^2x + 2\sqrt{d^2x^2-c^2}d\right)}{8d^5} + \frac{ac^2 \log\left(2d^2x + 2\sqrt{d^2x^2-c^2}d\right)}{2d^3} + \frac{3\sqrt{d^2x^2-c^2}bc^2x}{8d^4} + \frac{\sqrt{d^2x^2-c^2}bc^2x}{8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x^2+a)/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] 1/4*sqrt(d^2*x^2 - c^2)*b*x^3/d^2 + 3/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5 + 1/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3 + 3/8*sqrt(d^2*x^2 - c^2)*b*c^2*x/d^4 + 1/2*sqrt(d^2*x^2 - c^2)*a*x/d^2
```

**mupad [B]** time = 25.51, size = 1048, normalized size = 8.88

$$\frac{2ac^2(\sqrt{c+dx}-\sqrt{c})}{\sqrt{c}-\sqrt{dx-c}} + \frac{14ac^2(\sqrt{c+dx}-\sqrt{c})^3}{(\sqrt{c}-\sqrt{dx-c})^3} + \frac{14ac^2(\sqrt{c+dx}-\sqrt{c})^5}{(\sqrt{c}-\sqrt{dx-c})^5} + \frac{2ac^2(\sqrt{c+dx}-\sqrt{c})^7}{(\sqrt{c}-\sqrt{dx-c})^7} - \frac{23bc^4(\sqrt{c+dx}-\sqrt{c})^3}{2(\sqrt{c}-\sqrt{dx-c})^3} - \frac{3bc^4(\sqrt{c+dx}-\sqrt{c})^5}{2(\sqrt{c}-\sqrt{dx-c})^5} - \frac{3bc^4(\sqrt{c+dx}-\sqrt{c})^7}{2(\sqrt{c}-\sqrt{dx-c})^7} - \frac{d^3 - \frac{4d^3(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{c}-\sqrt{dx-c})^2} + \frac{6d^3(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{c}-\sqrt{dx-c})^4} - \frac{4d^3(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{c}-\sqrt{dx-c})^6} + \frac{d^3(\sqrt{c+dx}-\sqrt{c})^8}{(\sqrt{c}-\sqrt{dx-c})^8}}{d^5 - \frac{8d^5(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{c}-\sqrt{dx-c})^2} + \frac{28d^5(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{c}-\sqrt{dx-c})^4} - \frac{28d^5(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{c}-\sqrt{dx-c})^6} + \frac{8d^5(\sqrt{c+dx}-\sqrt{c})^8}{(\sqrt{c}-\sqrt{dx-c})^8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2))/((c + d*x)^(1/2)*(d*x - c)^(1/2)),x)
[Out] ((2*a*c^2*((c + d*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d*x - c)^(1/2)) + (14*a*c^2*((c + d*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d*x - c)^(1/2))^3 + (14*a*c^2*((c + d*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d*x - c)^(1/2))^5 + (2*a*c^2*((c + d*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d*x - c)^(1/2))^7)/(d^3 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (6*d^3*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (4*d^3*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (d^3*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - ((23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^3)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^3) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2)))/((2*((-c)^(1/2) - (d*x - c)^(1/2)))) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^5)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^5) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^7)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^7) + (671*b*c^4*((c + d*x)^(1/2) - c^(1/2))^9)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^9))
```

```

*x - c)^(1/2))^9) + (333*b*c^4*((c + d*x)^(1/2) - c^(1/2))^11)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^11) + (23*b*c^4*((c + d*x)^(1/2) - c^(1/2))^13)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^13) - (3*b*c^4*((c + d*x)^(1/2) - c^(1/2))^15)/(2*((-c)^(1/2) - (d*x - c)^(1/2))^15))/(d^5 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d*x - c)^(1/2))^2 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d*x - c)^(1/2))^4 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d*x - c)^(1/2))^6 + (70*d^5*((c + d*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d*x - c)^(1/2))^8 - (56*d^5*((c + d*x)^(1/2) - c^(1/2))^10)/((-c)^(1/2) - (d*x - c)^(1/2))^10 + (28*d^5*((c + d*x)^(1/2) - c^(1/2))^12)/((-c)^(1/2) - (d*x - c)^(1/2))^12 - (8*d^5*((c + d*x)^(1/2) - c^(1/2))^14)/((-c)^(1/2) - (d*x - c)^(1/2))^14 + (d^5*((c + d*x)^(1/2) - c^(1/2))^16)/((-c)^(1/2) - (d*x - c)^(1/2))^16) - (2*a*c^2*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/d^3 - (3*b*c^4*atanh(((c + d*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d*x - c)^(1/2))))/(2*d^5)

```

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] Timed out

$$3.361 \quad \int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx$$

Optimal. Leaf size=72

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4} + \frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

[Out] 1/3\*(3\*a\*d^2+2\*b\*c^2)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^4+1/3\*b\*x^2\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {460, 74}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(3ad^2+2bc^2)}{3d^4} + \frac{bx^2\sqrt{dx-c}\sqrt{c+dx}}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] ((2\*b\*c^2 + 3\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*d^4) + (b\*x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*d^2)

Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^(m\*(a1 + b1\*x^(n/2)))^(p\*(a2 + b2\*x^(n/2)))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{x(a+bx^2)}{\sqrt{-c+dx}\sqrt{c+dx}} dx &= \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2} - \frac{1}{3} \left( -3a - \frac{2bc^2}{d^2} \right) \int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= \frac{(2bc^2 + 3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^4} + \frac{bx^2\sqrt{-c+dx}\sqrt{c+dx}}{3d^2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.85

$$\frac{(d^2x^2 - c^2)(3ad^2 + 2bc^2 + bd^2x^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*x^2))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] ((-c^2 + d^2\*x^2)\*(2\*b\*c^2 + 3\*a\*d^2 + b\*d^2\*x^2))/(3\*d^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas** [A] time = 1.14, size = 42, normalized size = 0.58

$$\frac{(bd^2x^2 + 2bc^2 + 3ad^2)\sqrt{dx + c}\sqrt{dx - c}}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/3\*(b\*d^2\*x^2 + 2\*b\*c^2 + 3\*a\*d^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/d^4

**giac** [A] time = 0.20, size = 65, normalized size = 0.90

$$\frac{\sqrt{dx + c}\sqrt{dx - c}\left((dx + c)\left(\frac{(dx+c)b}{d^3} - \frac{2bc}{d^3}\right) + \frac{3(bc^2d^9 + ad^{11})}{d^{12}}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/3\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*((d\*x + c)\*((d\*x + c)\*b/d^3 - 2\*b\*c/d^3) + 3\*(b\*c^2\*d^9 + a\*d^11)/d^12)/d

**maple** [A] time = 0.04, size = 43, normalized size = 0.60

$$\frac{\sqrt{dx + c} (bd^2x^2 + 3ad^2 + 2bc^2)\sqrt{dx - c}}{3d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 1/3\*(d\*x+c)^(1/2)\*(b\*d^2\*x^2+3\*a\*d^2+2\*b\*c^2)/d^4\*(d\*x-c)^(1/2)

**maxima** [A] time = 0.64, size = 69, normalized size = 0.96

$$\frac{\sqrt{d^2x^2 - c^2}bx^2}{3d^2} + \frac{2\sqrt{d^2x^2 - c^2}bc^2}{3d^4} + \frac{\sqrt{d^2x^2 - c^2}a}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/3\*sqrt(d^2\*x^2 - c^2)\*b\*x^2/d^2 + 2/3\*sqrt(d^2\*x^2 - c^2)\*b\*c^2/d^4 + sqrt(d^2\*x^2 - c^2)\*a/d^2

**mupad** [B] time = 2.66, size = 76, normalized size = 1.06

$$\frac{\sqrt{dx - c} \left( \frac{2bc^3 + 3acd^2}{3d^4} + \frac{bx^3}{3d} + \frac{x(2bc^2d + 3ad^3)}{3d^4} + \frac{bcx^2}{3d^2} \right)}{\sqrt{c + dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x\*(a + b\*x^2))/((c + d\*x)^(1/2)\*(d\*x - c)^(1/2)),x)

[Out] ((d\*x - c)^(1/2)\*((2\*b\*c^3 + 3\*a\*c\*d^2)/(3\*d^4) + (b\*x^3)/(3\*d) + (x\*(3\*a\*d^3 + 2\*b\*c^2\*d))/(3\*d^4) + (b\*c\*x^2)/(3\*d^2)))/(c + d\*x)^(1/2)

sympy [C] time = 44.70, size = 223, normalized size = 3.10

$$\frac{{}_6G_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) + i {}_6G_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right) + bc^3 {}_6G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4} \end{matrix} \right)}{4\pi^{\frac{3}{2}} d^2} + \frac{\dots}{4\pi^{\frac{3}{2}} d^2} + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)
```

```
[Out] a*c*meijerg((( -1/4, 1/4), (0, 0, 1/2, 1)), (( -1/2, -1/4, 0, 1/4, 1/2, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d**2) + I*a*c*meijerg((( -1, -3/4, -1/2, -1/4, 0, 1), ()), (( -3/4, -1/4), (-1, -1/2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d**2) + b*c**3*meijerg((( -5/4, -3/4), (-1, -1, -1/2, 1)), (( -3/2, -5/4, -1, -3/4, -1/2, 0), ()), c**2/(d**2*x**2))/(4*pi***(3/2)*d**4) + I*b*c**3*meijerg((( -2, -7/4, -3/2, -5/4, -1, 1), ()), (( -7/4, -5/4), (-2, -3/2, -3/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi***(3/2)*d**4)
```

$$3.362 \quad \int \frac{a+bx^2}{\sqrt{-c+dx} \sqrt{c+dx}} dx$$

Optimal. Leaf size=68

$$\frac{(2ad^2 + bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

[Out] (2\*a\*d^2+b\*c^2)\*arctanh((d\*x-c)^(1/2)/(d\*x+c)^(1/2))/d^3+1/2\*b\*x\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

**Rubi [A]** time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {389, 63, 217, 206}

$$\frac{(2ad^2 + bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} + \frac{bx\sqrt{dx-c}\sqrt{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (b\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(2\*d^2) + ((b\*c^2 + 2\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^3

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_)^(m\_))\*((c\_.) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 389

Int[((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] :> Simp[(d\*x\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*(n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d - b1\*b2\*c\*(n\*(p + 1) + 1))/(b1\*b2\*(n\*(p + 1) + 1)), Int[(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[n\*(p + 1) + 1, 0]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2}{\sqrt{-c + dx} \sqrt{c + dx}} dx &= \frac{bx\sqrt{-c + dx} \sqrt{c + dx}}{2d^2} - \frac{(-bc^2 - 2ad^2) \int \frac{1}{\sqrt{-c + dx} \sqrt{c + dx}} dx}{2d^2} \\
&= \frac{bx\sqrt{-c + dx} \sqrt{c + dx}}{2d^2} + \frac{(bc^2 + 2ad^2) \text{Subst}\left(\int \frac{1}{\sqrt{2c + x^2}} dx, x, \sqrt{-c + dx}\right)}{d^3} \\
&= \frac{bx\sqrt{-c + dx} \sqrt{c + dx}}{2d^2} + \frac{(bc^2 + 2ad^2) \text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, \frac{\sqrt{-c + dx}}{\sqrt{c + dx}}\right)}{d^3} \\
&= \frac{bx\sqrt{-c + dx} \sqrt{c + dx}}{2d^2} + \frac{(bc^2 + 2ad^2) \tanh^{-1}\left(\frac{\sqrt{-c + dx}}{\sqrt{c + dx}}\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.35, size = 119, normalized size = 1.75

$$\frac{4(ad^2 + bc^2) \tanh^{-1}\left(\frac{\sqrt{dx - c}}{\sqrt{c + dx}}\right) - \frac{2bc^{5/2} \sqrt{\frac{dx}{c} + 1} \sinh^{-1}\left(\frac{\sqrt{dx - c}}{\sqrt{2} \sqrt{c}}\right)}{\sqrt{c + dx}} + bdx\sqrt{dx - c} \sqrt{c + dx}}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (b\*d\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x] - (2\*b\*c^(5/2)\*Sqrt[1 + (d\*x)/c]\*ArcSinh[Sqrt[-c + d\*x]/(Sqrt[2]\*Sqrt[c])])/Sqrt[c + d\*x] + 4\*(b\*c^2 + a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]]/(2\*d^3)

**fricas [A]** time = 1.17, size = 63, normalized size = 0.93

$$\frac{\sqrt{dx + c} \sqrt{dx - c} bdx - (bc^2 + 2ad^2) \log(-dx + \sqrt{dx + c} \sqrt{dx - c})}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/2\*(sqrt(d\*x + c)\*sqrt(d\*x - c)\*b\*d\*x - (b\*c^2 + 2\*a\*d^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/d^3

**giac [A]** time = 0.26, size = 79, normalized size = 1.16

$$\frac{\sqrt{dx + c} \sqrt{dx - c} \left(\frac{(dx+c)b}{d^2} - \frac{bc}{d^2}\right) - \frac{2(bc^2+2ad^2) \log(|-\sqrt{dx+c} + \sqrt{dx-c}|)}{d^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/2\*(sqrt(d\*x + c)\*sqrt(d\*x - c)\*((d\*x + c)\*b/d^2 - b\*c/d^2) - 2\*(b\*c^2 + 2\*a\*d^2)\*log(abs(-sqrt(d\*x + c) + sqrt(d\*x - c)))/d^2)/d

**maple [C]** time = 0.07, size = 124, normalized size = 1.82

$$\frac{\sqrt{dx - c} \sqrt{dx + c} \left(2a d^2 \ln\left(\left(dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d)\right) \operatorname{csgn}(d)\right) + b c^2 \ln\left(\left(dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d)\right) \operatorname{csgn}(d)\right)\right)}{2\sqrt{d^2 x^2 - c^2} d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 1/2\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^3\*((d^2\*x^2-c^2)^(1/2)\*csgn(d)\*d\*x\*b+ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))\*b\*c^2+2\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))\*a\*d^2)/(d^2\*x^2-c^2)^(1/2)\*csgn(d)

**maxima** [A] time = 0.46, size = 89, normalized size = 1.31

$$\frac{bc^2 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right)}{2d^3} + \frac{a \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right)}{d} + \frac{\sqrt{d^2x^2 - c^2}bx}{2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/2\*b\*c^2\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*d)/d^3 + a\*log(2\*d^2\*x + 2\*sqrt(d^2\*x^2 - c^2)\*d)/d + 1/2\*sqrt(d^2\*x^2 - c^2)\*b\*x/d^2

**mupad** [B] time = 10.80, size = 417, normalized size = 6.13

$$\frac{\frac{2bc^2(\sqrt{c+dx}-\sqrt{c})}{\sqrt{c}-\sqrt{dx-c}} + \frac{14bc^2(\sqrt{c+dx}-\sqrt{c})^3}{(\sqrt{c}-\sqrt{dx-c})^3} + \frac{14bc^2(\sqrt{c+dx}-\sqrt{c})^5}{(\sqrt{c}-\sqrt{dx-c})^5} + \frac{2bc^2(\sqrt{c+dx}-\sqrt{c})^7}{(\sqrt{c}-\sqrt{dx-c})^7}}{d^3 - \frac{4d^3(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{c}-\sqrt{dx-c})^2} + \frac{6d^3(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{c}-\sqrt{dx-c})^4} - \frac{4d^3(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{c}-\sqrt{dx-c})^6} + \frac{d^3(\sqrt{c+dx}-\sqrt{c})^8}{(\sqrt{c}-\sqrt{dx-c})^8}} + \frac{4a \operatorname{atan}\left(\frac{d(\sqrt{c}-\sqrt{dx-c})}{\sqrt{-d^2}(\sqrt{c+dx}-\sqrt{c})}\right)}{\sqrt{-d^2}} - \frac{2bc^2}{\sqrt{-d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/((c + d\*x)^(1/2)\*(d\*x - c)^(1/2)),x)

[Out] ((2\*b\*c^2\*((c + d\*x)^(1/2) - c^(1/2)))/((-c)^(1/2) - (d\*x - c)^(1/2)) + (14\*b\*c^2\*((c + d\*x)^(1/2) - c^(1/2))^3)/((-c)^(1/2) - (d\*x - c)^(1/2))^3 + (14\*b\*c^2\*((c + d\*x)^(1/2) - c^(1/2))^5)/((-c)^(1/2) - (d\*x - c)^(1/2))^5 + (2\*b\*c^2\*((c + d\*x)^(1/2) - c^(1/2))^7)/((-c)^(1/2) - (d\*x - c)^(1/2))^7)/(d^3 - (4\*d^3\*((c + d\*x)^(1/2) - c^(1/2))^2)/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + (6\*d^3\*((c + d\*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d\*x - c)^(1/2))^4 - (4\*d^3\*((c + d\*x)^(1/2) - c^(1/2))^6)/((-c)^(1/2) - (d\*x - c)^(1/2))^6 + (d^3\*((c + d\*x)^(1/2) - c^(1/2))^8)/((-c)^(1/2) - (d\*x - c)^(1/2))^8) + (4\*a\*atan((d\*((-c)^(1/2) - (d\*x - c)^(1/2)))/((-d^2)^(1/2)\*((c + d\*x)^(1/2) - c^(1/2)))))/((-d^2)^(1/2) - (2\*b\*c^2\*atanh(((c + d\*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d\*x - c)^(1/2)))))/d^3

**sympy** [C] time = 41.78, size = 199, normalized size = 2.93

$$\frac{aG_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{4\pi^{\frac{3}{2}}d} - \frac{iaG_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2} \right)}{4\pi^{\frac{3}{2}}d} + \frac{bc^2G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \right)}{4\pi^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] a\*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d) - I\*a\*meijerg(((1/4, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d) + b\*c\*\*2\*meijerg(((3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3) - I\*b\*c\*\*2\*meijerg(((3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3)



$$3.363 \quad \int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}} dx$$

Optimal. Leaf size=56

$$\frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c} + \frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

[Out] a\*arctan((d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c)/c+b\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/d^2

Rubi [A] time = 0.07, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {460, 92, 205}

$$\frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c} + \frac{b\sqrt{dx-c}\sqrt{c+dx}}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (b\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/d^2 + (a\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/c

Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 460

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^(p\*(a2 + b2\*x^(n/2)))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x\sqrt{-c+dx}\sqrt{c+dx}} dx &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + a \int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + (ad) \text{Subst}\left(\int \frac{1}{c^2d+dx^2} dx, x, \sqrt{-c+dx}\sqrt{c+dx}\right) \\ &= \frac{b\sqrt{-c+dx}\sqrt{c+dx}}{d^2} + \frac{a \tan^{-1}\left(\frac{\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{c} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 87, normalized size = 1.55

$$\frac{ad^2\sqrt{d^2x^2 - c^2} \tan^{-1}\left(\frac{\sqrt{d^2x^2 - c^2}}{c}\right) - bc^3 + bcd^2x^2}{cd^2\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out]  $(-(b*c^3) + b*c*d^2*x^2 + a*d^2*Sqrt[-c^2 + d^2*x^2]*ArcTan[Sqrt[-c^2 + d^2*x^2]/c])/(c*d^2*Sqrt[-c + d*x]*Sqrt[c + d*x])$

**fricas [A]** time = 0.97, size = 61, normalized size = 1.09

$$\frac{2ad^2 \arctan\left(-\frac{dx - \sqrt{dx+c}\sqrt{dx-c}}{c}\right) + \sqrt{dx+c}\sqrt{dx-c}bc}{cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2), x, algorithm="fricas")

[Out]  $(2*a*d^2*\arctan(-(d*x - \text{sqrt}(d*x + c))*\text{sqrt}(d*x - c))/c) + \text{sqrt}(d*x + c)*\text{sqrt}(d*x - c)*b*c)/(c*d^2)$

**giac [A]** time = 0.21, size = 55, normalized size = 0.98

$$-\frac{2a \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c} + \frac{\sqrt{dx+c}\sqrt{dx-c}b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2), x, algorithm="giac")

[Out]  $-2*a*\arctan(1/2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2/c)/c + \text{sqrt}(d*x + c)*\text{sqrt}(d*x - c)*b/d^2$

**maple [B]** time = 0.08, size = 108, normalized size = 1.93

$$\frac{\left(-ad^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2x^2 - c^2})}{x}\right) + \sqrt{-c^2} \sqrt{d^2x^2 - c^2} b\right) \sqrt{dx - c} \sqrt{dx + c}}{\sqrt{d^2x^2 - c^2} \sqrt{-c^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2), x)

[Out]  $(-\ln(-2*(c^2 - (-c^2)^{1/2})*(d^2*x^2 - c^2)^{1/2})/x)*a*d^2 + b*(-c^2)^{1/2}*(d^2*x^2 - c^2)^{1/2}*(d*x - c)^{1/2}*(d*x + c)^{1/2}/(d^2*x^2 - c^2)^{1/2}/d^2/(-c^2)^{1/2}$

**maxima [A]** time = 1.34, size = 37, normalized size = 0.66

$$-\frac{a \arcsin\left(\frac{c}{d|x|}\right)}{c} + \frac{\sqrt{d^2x^2 - c^2} b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2), x, algorithm="maxima")

[Out]  $-a \cdot \arcsin(c/(d \cdot \text{abs}(x)))/c + \sqrt{d^2 \cdot x^2 - c^2} \cdot b/d^2$

**mupad [B]** time = 3.97, size = 108, normalized size = 1.93

$$\frac{b \sqrt{c+dx} \sqrt{dx-c}}{d^2} - \frac{a \sqrt{-c} \left( \ln \left( \frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2} + 1 \right) - \ln \left( \frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}} \right) \right)}{c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)`

[Out]  $(b \cdot (c + d \cdot x)^{1/2} \cdot (d \cdot x - c)^{1/2})/d^2 - (a \cdot (-c)^{1/2} \cdot (\log(((c + d \cdot x)^{1/2} \cdot (2 - c^{1/2})^2/((-c)^{1/2} - (d \cdot x - c)^{1/2})^2 + 1) - \log(((c + d \cdot x)^{1/2} \cdot (2 - c^{1/2})/((-c)^{1/2} - (d \cdot x - c)^{1/2})))))/c^{3/2}$

**sympy [C]** time = 39.16, size = 178, normalized size = 3.18

$$\frac{{}_aG_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c} + \frac{{}_i a G_{6,6}^{2,6} \left( \begin{matrix} 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1 \\ \frac{1}{4}, \frac{3}{4} & 0, \frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c} + \frac{{}_b c G_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \right)}{4\pi^{\frac{3}{2}} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a \cdot \text{meijerg}(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c^{**2}/(d^{**2} \cdot x^{**2}))/ (4 \cdot \pi^{**}(3/2) \cdot c) + I \cdot a \cdot \text{meijerg}(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c^{**2} \cdot \exp\_polar(2 \cdot I \cdot \pi)/(d^{**2} \cdot x^{**2}))/ (4 \cdot \pi^{**}(3/2) \cdot c) + b \cdot c \cdot \text{meijerg}((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), c^{**2}/(d^{**2} \cdot x^{**2}))/ (4 \cdot \pi^{**}(3/2) \cdot d^{**2}) + I \cdot b \cdot c \cdot \text{meijerg}((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), c^{**2} \cdot \exp\_polar(2 \cdot I \cdot \pi)/(d^{**2} \cdot x^{**2}))/ (4 \cdot \pi^{**}(3/2) \cdot d^{**2})$

$$3.364 \quad \int \frac{a+bx^2}{x^2 \sqrt{-c+dx} \sqrt{c+dx}} dx$$

Optimal. Leaf size=57

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

[Out]  $2*b*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2}))/d+a*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2/x$

**Rubi [A]** time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {454, 63, 217, 206}

$$\frac{a\sqrt{dx-c}\sqrt{c+dx}}{c^2x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]), x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(c^2\*x) + (2\*b\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 206

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(1\*ArcTanh[Rt[-b, 2]\*x]/Rt[a, 2])]/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] :> Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(q + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

#### Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2}{x^2 \sqrt{-c + dx} \sqrt{c + dx}} dx &= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{c^2 x} + b \int \frac{1}{\sqrt{-c + dx} \sqrt{c + dx}} dx \\
&= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{c^2 x} + \frac{(2b) \operatorname{Subst}\left(\int \frac{1}{\sqrt{2c + x^2}} dx, x, \sqrt{-c + dx}\right)}{d} \\
&= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{c^2 x} + \frac{(2b) \operatorname{Subst}\left(\int \frac{1}{1 - x^2} dx, x, \frac{\sqrt{-c + dx}}{\sqrt{c + dx}}\right)}{d} \\
&= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{c^2 x} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{-c + dx}}{\sqrt{c + dx}}\right)}{d}
\end{aligned}$$

**Mathematica [A]** time = 0.04, size = 90, normalized size = 1.58

$$\frac{\sqrt{d^2 x^2 - c^2} \left( \frac{a\sqrt{d^2 x^2 - c^2}}{c^2 x} + \frac{b \tanh^{-1}\left(\frac{dx}{\sqrt{d^2 x^2 - c^2}}\right)}{d} \right)}{\sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (Sqrt[-c^2 + d^2\*x^2]\*((a\*Sqrt[-c^2 + d^2\*x^2])/(c^2\*x) + (b\*ArcTanh[(d\*x)/Sqrt[-c^2 + d^2\*x^2]])/d))/(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.29, size = 68, normalized size = 1.19

$$\frac{bc^2 x \log(-dx + \sqrt{dx + c} \sqrt{dx - c}) - ad^2 x - \sqrt{dx + c} \sqrt{dx - c} ad}{c^2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^2/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] -(b\*c^2\*x\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)) - a\*d^2\*x - sqrt(d\*x + c)\*sqrt(d\*x - c)\*a\*d)/(c^2\*d\*x)

**giac [A]** time = 0.22, size = 66, normalized size = 1.16

$$\frac{\frac{16ad^2}{(\sqrt{dx+c} - \sqrt{dx-c})^4 + 4c^2} - b \log\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^4\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^2/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 1/2\*(16\*a\*d^2/((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2) - b\*log((sqrt(d\*x + c) - sqrt(d\*x - c))^4))/d

**maple [C]** time = 0.07, size = 97, normalized size = 1.70

$$\frac{\sqrt{dx - c} \sqrt{dx + c} \left( b c^2 x \ln\left(\left(dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d)\right) \operatorname{csgn}(d)\right) + \sqrt{d^2 x^2 - c^2} ad \operatorname{csgn}(d)\right) \operatorname{csgn}(d)}{\sqrt{d^2 x^2 - c^2} c^2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x)`

[Out]  $(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2*(b*c^2*x*\ln((d*x+(d^2*x^2-c^2)^{(1/2)}*c\operatorname{sgn}(d)))*c\operatorname{sgn}(d))+(d^2*x^2-c^2)^{(1/2)}*a*d*c\operatorname{sgn}(d)*c\operatorname{sgn}(d)/(d^2*x^2-c^2)^{(1/2)}/d/x$

**maxima** [A] time = 1.26, size = 55, normalized size = 0.96

$$\frac{b \log\left(2 d^2 x + 2 \sqrt{d^2 x^2 - c^2} d\right)}{d} + \frac{\sqrt{d^2 x^2 - c^2} a}{c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^2/(d*x-c)^(1/2)/(d*x+c)^(1/2),x, algorithm="maxima")`

[Out]  $b*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d + \sqrt{d^2*x^2 - c^2}*a/(c^2*x)$

**mupad** [B] time = 2.94, size = 77, normalized size = 1.35

$$\frac{4 b \operatorname{atan}\left(\frac{d(\sqrt{-c}-\sqrt{d x-c})}{\sqrt{-d^2}(\sqrt{c+d x}-\sqrt{c})}\right)}{\sqrt{-d^2}} + \frac{a \sqrt{c+d x} \sqrt{d x-c}}{c^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x^2*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)`

[Out]  $(4*b*\operatorname{atan}((d*((-c)^{(1/2)} - (d*x - c)^{(1/2)}))/((-d^2)^{(1/2)}*((c + d*x)^{(1/2)} - c^{(1/2)})))/(-d^2)^{(1/2)} + (a*(c + d*x)^{(1/2)}*(d*x - c)^{(1/2)})/(c^2*x)$

**sympy** [C] time = 36.82, size = 165, normalized size = 2.89

$$\frac{{}_2F_1\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ \frac{3}{2}, \frac{3}{2}, 2 \end{matrix} \middle| \frac{c^2}{d^2 x^2}\right)}{4\pi^{\frac{3}{2}} c^2} - \frac{{}_2F_1\left(\begin{matrix} \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2}\right)}{4\pi^{\frac{3}{2}} c^2} + \frac{{}_2F_1\left(\begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1}{2}, 1, 1 \end{matrix} \middle| \frac{c^2}{d^2 x^2}\right)}{4\pi^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**2/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a*d*\operatorname{meijerg}(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*a*d*\operatorname{meijerg}(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*\exp\_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2) + b*\operatorname{meijerg}(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), c**2/(d**2*x**2))/(4*pi**(3/2)*d) - I*b*\operatorname{meijerg}((-1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), c**2*\exp\_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*d)$

$$3.365 \quad \int \frac{a+bx^2}{x^3 \sqrt{-c+dx} \sqrt{c+dx}} dx$$

Optimal. Leaf size=76

$$\frac{(ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c} \sqrt{c+dx}}{c}\right)}{2c^3} + \frac{a\sqrt{dx-c} \sqrt{c+dx}}{2c^2x^2}$$

[Out] 1/2\*(a\*d^2+2\*b\*c^2)\*arctan((d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c)/c^3+1/2\*a\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^2/x^2

**Rubi [A]** time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {454, 92, 205}

$$\frac{(ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c} \sqrt{c+dx}}{c}\right)}{2c^3} + \frac{a\sqrt{dx-c} \sqrt{c+dx}}{2c^2x^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/((2\*c^2\*x^2) + ((2\*b\*c^2 + a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c]))/(2\*c^3)

Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

Rule 205

Int[((a\_.) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 454

Int[((e\_.)\*(x\_)^(m\_.))\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] := Simp[(c\*(e\*x)^(m+1)\*(a1 + b1\*x^(n/2))^(p+1)\*(a2 + b2\*x^(n/2))^(q+1))/(a1\*a2\*e^(m+1)), x] + Dist[(a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1))/(a1\*a2\*e^n\*(m+1)), Int[(e\*x)^(m+n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^3 \sqrt{-c+dx} \sqrt{c+dx}} dx &= \frac{a\sqrt{-c+dx} \sqrt{c+dx}}{2c^2x^2} + \frac{1}{2} \left(2b + \frac{ad^2}{c^2}\right) \int \frac{1}{x\sqrt{-c+dx} \sqrt{c+dx}} dx \\ &= \frac{a\sqrt{-c+dx} \sqrt{c+dx}}{2c^2x^2} + \frac{1}{2} \left(d \left(2b + \frac{ad^2}{c^2}\right)\right) \text{Subst}\left(\int \frac{1}{c^2d+dx^2} dx, x, \sqrt{-c+dx} \sqrt{c+dx}\right) \\ &= \frac{a\sqrt{-c+dx} \sqrt{c+dx}}{2c^2x^2} + \frac{(2bc^2 + ad^2) \tan^{-1}\left(\frac{\sqrt{-c+dx} \sqrt{c+dx}}{c}\right)}{2c^3} \end{aligned}$$

**Mathematica [A]** time = 0.10, size = 102, normalized size = 1.34

$$\frac{x^2 \sqrt{d^2 x^2 - c^2} (ad^2 + 2bc^2) \tan^{-1} \left( \frac{\sqrt{d^2 x^2 - c^2}}{c} \right) + a (cd^2 x^2 - c^3)}{2c^3 x^2 \sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (a\*(-c^3 + c\*d^2\*x^2) + (2\*b\*c^2 + a\*d^2)\*x^2\*Sqrt[-c^2 + d^2\*x^2]\*ArcTan[Sqrt[-c^2 + d^2\*x^2]/c])/(2\*c^3\*x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.27, size = 73, normalized size = 0.96

$$\frac{2(2bc^2 + ad^2)x^2 \arctan\left(-\frac{dx - \sqrt{dx+c} \sqrt{dx-c}}{c}\right) + \sqrt{dx+c} \sqrt{dx-c} ac}{2c^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/2\*(2\*(2\*b\*c^2 + a\*d^2)\*x^2\*arctan(-(d\*x - sqrt(d\*x + c)\*sqrt(d\*x - c))/c) + sqrt(d\*x + c)\*sqrt(d\*x - c)\*a\*c)/(c^3\*x^2)

**giac [B]** time = 0.49, size = 141, normalized size = 1.86

$$\frac{(2bc^2d+ad^3) \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right) + \frac{2(ad^3(\sqrt{dx+c}-\sqrt{dx-c})^6 - 4ac^2d^3(\sqrt{dx+c}-\sqrt{dx-c})^2)}{((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2)^2 c^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] -((2\*b\*c^2\*d + a\*d^3)\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c^3 + 2\*(a\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^6 - 4\*a\*c^2\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^2\*c^2)/d

**maple [B]** time = 0.07, size = 158, normalized size = 2.08

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( a d^2 x^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) + 2b c^2 x^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2} a \right)}{2\sqrt{d^2 x^2 - c^2} \sqrt{-c^2} c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^3/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] -1/2\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^2\*(a\*d^2\*x^2\*ln(-2\*(c^2-(-c^2)^(1/2))\*(d^2\*x^2-c^2)^(1/2))/x)+2\*b\*c^2\*x^2\*ln(-2\*(c^2-(-c^2)^(1/2))\*(d^2\*x^2-c^2)^(1/2))/x)-(-c^2)^(1/2)\*(d^2\*x^2-c^2)^(1/2)\*a/(d^2\*x^2-c^2)^(1/2)/x^2/(-c^2)^(1/2)

**maxima [A]** time = 1.29, size = 60, normalized size = 0.79

$$-\frac{b \arcsin\left(\frac{c}{d|x|}\right)}{c} - \frac{ad^2 \arcsin\left(\frac{c}{d|x|}\right)}{2c^3} + \frac{\sqrt{d^2 x^2 - c^2} a}{2c^2 x^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] -b\*arcsin(c/(d\*abs(x)))/c - 1/2\*a\*d^2\*arcsin(c/(d\*abs(x)))/c^3 + 1/2\*sqrt(d^2\*x^2 - c^2)\*a/(c^2\*x^2)

**mupad [B]** time = 7.50, size = 457, normalized size = 6.01

$$\frac{a(-c)^{3/2} d^2 \ln\left(\frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2} + 1\right)}{2c^{9/2}} - \frac{b\sqrt{-c} \left( \ln\left(\frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2} + 1\right) - \ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right) \right)}{c^{3/2}} - \frac{a(-c)^{3/2} d^2 \ln\left(\frac{\sqrt{c+dx}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{2c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^3\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2)),x)

[Out] (a\*(-c)^(3/2)\*d^2\*log(((c + d\*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + 1))/(2\*c^(9/2)) - (b\*(-c)^(1/2)\*(log(((c + d\*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + 1) - log(((c + d\*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d\*x - c)^(1/2))))/c^(3/2) - (a\*(-c)^(3/2)\*d^2\*log(((c + d\*x)^(1/2) - c^(1/2))/((-c)^(1/2) - (d\*x - c)^(1/2))))/(2\*c^(9/2)) - ((a\*(-c)^(3/2)\*d^2)/(32\*c^(9/2)) + (a\*(-c)^(3/2)\*d^2\*((c + d\*x)^(1/2) - c^(1/2))^2)/(16\*c^(9/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^2 - (15\*a\*(-c)^(3/2)\*d^2\*((c + d\*x)^(1/2) - c^(1/2))^4)/(32\*c^(9/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^4)/((c + d\*x)^(1/2) - c^(1/2))^2/((-c)^(1/2) - (d\*x - c)^(1/2))^2 + (2\*((c + d\*x)^(1/2) - c^(1/2))^4)/((-c)^(1/2) - (d\*x - c)^(1/2))^4 + ((c + d\*x)^(1/2) - c^(1/2))^6/((-c)^(1/2) - (d\*x - c)^(1/2))^6 + (a\*d^2\*((c + d\*x)^(1/2) - c^(1/2))^2)/(32\*(-c)^(3/2)\*c^(3/2)\*((-c)^(1/2) - (d\*x - c)^(1/2))^2)

**sympy [C]** time = 69.67, size = 162, normalized size = 2.13

$$\frac{ad^2 G_{6,6}^{5,3} \left( \begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} + \frac{id^2 G_{6,6}^{2,6} \left( \begin{matrix} 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, 1 \\ \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^3} - \frac{b G_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*3/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] -a\*d\*\*2\*meijerg(((7/4, 9/4, 1), (2, 2, 5/2)), ((3/2, 7/4, 2, 9/4, 5/2), (0,)), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3) + I\*a\*d\*\*2\*meijerg(((1, 5/4, 3/2, 7/4, 2, 1), ()), ((5/4, 7/4), (1, 3/2, 3/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c\*\*3) - b\*meijerg(((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), c\*\*2/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c) + I\*b\*meijerg(((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*c)

$$3.366 \quad \int \frac{a+bx^2}{x^4 \sqrt{-c+dx} \sqrt{c+dx}} dx$$

Optimal. Leaf size=75

$$\frac{\sqrt{dx-c} \sqrt{c+dx} (2ad^2 + 3bc^2)}{3c^4x} + \frac{a\sqrt{dx-c} \sqrt{c+dx}}{3c^2x^3}$$

[Out]  $1/3*a*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2/x^3+1/3*(2*a*d^2+3*b*c^2)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^4/x$

**Rubi [A]** time = 0.07, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {454, 95}

$$\frac{\sqrt{dx-c} \sqrt{c+dx} (2ad^2 + 3bc^2)}{3c^4x} + \frac{a\sqrt{dx-c} \sqrt{c+dx}}{3c^2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^2\*x^3) + ((3\*b\*c^2 + 2\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(3\*c^4\*x)

Rule 95

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && EqQ[a\*d\*f\*(m + 1) + b\*c\*f\*(n + 1) + b\*d\*e\*(p + 1), 0] && NeQ[m, -1]

Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^4 \sqrt{-c+dx} \sqrt{c+dx}} dx &= \frac{a\sqrt{-c+dx} \sqrt{c+dx}}{3c^2x^3} + \frac{1}{3} \left( 3b + \frac{2ad^2}{c^2} \right) \int \frac{1}{x^2 \sqrt{-c+dx} \sqrt{c+dx}} dx \\ &= \frac{a\sqrt{-c+dx} \sqrt{c+dx}}{3c^2x^3} + \frac{(3bc^2 + 2ad^2) \sqrt{-c+dx} \sqrt{c+dx}}{3c^4x} \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 66, normalized size = 0.88

$$-\frac{(c^2 - d^2x^2)(a(c^2 + 2d^2x^2) + 3bc^2x^2)}{3c^4x^3 \sqrt{dx-c} \sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out]  $-1/3*((c^2 - d^2*x^2)*(3*b*c^2*x^2 + a*(c^2 + 2*d^2*x^2)))/(c^4*x^3*Sqrt[-c + d*x]*Sqrt[c + d*x])$

**fricas** [A] time = 0.67, size = 67, normalized size = 0.89

$$\frac{(3bc^2d + 2ad^3)x^3 + (ac^2 + (3bc^2 + 2ad^2)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="fricas")

[Out]  $1/3*((3*b*c^2*d + 2*a*d^3)*x^3 + (a*c^2 + (3*b*c^2 + 2*a*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c))/(c^4*x^3)$

**giac** [B] time = 0.24, size = 137, normalized size = 1.83

$$\frac{8\left(3bd^2(\sqrt{dx+c}-\sqrt{dx-c})^8 + 24bc^2d^2(\sqrt{dx+c}-\sqrt{dx-c})^4 + 24ad^4(\sqrt{dx+c}-\sqrt{dx-c})^4 + 48bc^4d^2 + 48b^2c^4\right)}{3\left((\sqrt{dx+c}-\sqrt{dx-c})^4 + 4c^2\right)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out]  $8/3*(3*b*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^8 + 24*b*c^2*d^2*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 24*a*d^4*(sqrt(d*x + c) - sqrt(d*x - c))^4 + 48*b*c^4*d^2 + 32*a*c^2*d^4)/(((sqrt(d*x + c) - sqrt(d*x - c))^4 + 4*c^2)^3*d)$

**maple** [A] time = 0.04, size = 49, normalized size = 0.65

$$\frac{\sqrt{dx+c} (2ad^2x^2 + 3bc^2x^2 + ac^2)\sqrt{dx-c}}{3c^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^4/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out]  $1/3*(d*x+c)^(1/2)*(2*a*d^2*x^2+3*b*c^2*x^2+a*c^2)/x^3/c^4*(d*x-c)^(1/2)$

**maxima** [A] time = 1.32, size = 75, normalized size = 1.00

$$\frac{\sqrt{d^2x^2 - c^2} b}{c^2x} + \frac{2\sqrt{d^2x^2 - c^2} ad^2}{3c^4x} + \frac{\sqrt{d^2x^2 - c^2} a}{3c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out]  $sqrt(d^2*x^2 - c^2)*b/(c^2*x) + 2/3*sqrt(d^2*x^2 - c^2)*a*d^2/(c^4*x) + 1/3*sqrt(d^2*x^2 - c^2)*a/(c^2*x^3)$

**mupad** [B] time = 2.77, size = 79, normalized size = 1.05

$$\frac{\sqrt{dx-c} \left( \frac{a}{3c} + \frac{x^2(3bc^3+2acd^2)}{3c^4} + \frac{x^3(3bc^2d+2ad^3)}{3c^4} + \frac{adx}{3c^2} \right)}{x^3\sqrt{c+dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x^4*(c + d*x)^(1/2)*(d*x - c)^(1/2)),x)`

[Out]  $((d*x - c)^{(1/2)}*(a/(3*c) + (x^2*(3*b*c^3 + 2*a*c*d^2))/(3*c^4) + (x^3*(2*a*d^3 + 3*b*c^2*d))/(3*c^4) + (a*d*x)/(3*c^2)))/(x^3*(c + d*x)^{(1/2)})$

**sympy** [C] time = 70.80, size = 170, normalized size = 2.27

$$\frac{ad^3 G_{6,6}^{5,3} \left( \begin{matrix} \frac{9}{4}, \frac{11}{4}, 1 \\ 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}, 3 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^4} - \frac{iad^3 G_{6,6}^{2,6} \left( \begin{matrix} \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 1 \\ \frac{7}{4}, \frac{9}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^4} - \frac{bd G_{6,6}^{5,3} \left( \begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**4/(d*x-c)**(1/2)/(d*x+c)**(1/2),x)`

[Out]  $-a*d**3*meijerg(((9/4, 11/4, 1), (5/2, 5/2, 3)), ((2, 9/4, 5/2, 11/4, 3), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**4) - I*a*d**3*meijerg(((3/2, 7/4, 2, 9/4, 5/2, 1), ()), ((7/4, 9/4), (3/2, 2, 2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**4) - b*d*meijerg(((5/4, 7/4, 1), (3/2, 3/2, 2)), ((1, 5/4, 3/2, 7/4, 2), (0,)), c**2/(d**2*x**2))/(4*pi**(3/2)*c**2) - I*b*d*meijerg(((1/2, 3/4, 1, 5/4, 3/2, 1), ()), ((3/4, 5/4), (1/2, 1, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(4*pi**(3/2)*c**2)$

$$3.367 \quad \int \frac{a+bx^2}{x^5 \sqrt{-c+dx} \sqrt{c+dx}} dx$$

**Optimal.** Leaf size=123

$$\frac{d^2 (3ad^2 + 4bc^2) \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right)}{8c^5} + \frac{\sqrt{dx-c} \sqrt{c+dx} (3ad^2 + 4bc^2)}{8c^4 x^2} + \frac{a \sqrt{dx-c} \sqrt{c+dx}}{4c^2 x^4}$$

[Out] 1/8\*d^2\*(3\*a\*d^2+4\*b\*c^2)\*arctan((d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c)/c^5+1/4\*a\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^2/x^4+1/8\*(3\*a\*d^2+4\*b\*c^2)\*(d\*x-c)^(1/2)\*(d\*x+c)^(1/2)/c^4/x^2

**Rubi [A]** time = 0.10, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {454, 103, 12, 92, 205}

$$\frac{\sqrt{dx-c} \sqrt{c+dx} (3ad^2 + 4bc^2)}{8c^4 x^2} + \frac{d^2 (3ad^2 + 4bc^2) \tan^{-1} \left( \frac{\sqrt{dx-c} \sqrt{c+dx}}{c} \right)}{8c^5} + \frac{a \sqrt{dx-c} \sqrt{c+dx}}{4c^2 x^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]),x]

[Out] (a\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(4\*c^2\*x^4) + ((4\*b\*c^2 + 3\*a\*d^2)\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/(8\*c^4\*x^2) + (d^2\*(4\*b\*c^2 + 3\*a\*d^2)\*ArcTan[(Sqrt[-c + d\*x]\*Sqrt[c + d\*x])/c])/(8\*c^5)

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

#### Rule 92

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)]\*((e\_) + (f\_)\*(x\_))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 103

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(

```
m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^q, x], x]
/; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 +
a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (L
tQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rubi steps

$$\int \frac{a + bx^2}{x^5 \sqrt{-c + dx} \sqrt{c + dx}} dx = \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{4c^2 x^4} + \frac{1}{4} \left( 4b + \frac{3ad^2}{c^2} \right) \int \frac{1}{x^3 \sqrt{-c + dx} \sqrt{c + dx}} dx$$

$$= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{4c^2 x^4} + \frac{(4bc^2 + 3ad^2) \sqrt{-c + dx} \sqrt{c + dx}}{8c^4 x^2} + \frac{(4bc^2 + 3ad^2) \int \frac{1}{x \sqrt{-c + dx} \sqrt{c + dx}} dx}{8c^4}$$

$$= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{4c^2 x^4} + \frac{(4bc^2 + 3ad^2) \sqrt{-c + dx} \sqrt{c + dx}}{8c^4 x^2} + \frac{(d^2 (4bc^2 + 3ad^2)) \int \frac{1}{x \sqrt{-c + dx} \sqrt{c + dx}} dx}{8c^4}$$

$$= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{4c^2 x^4} + \frac{(4bc^2 + 3ad^2) \sqrt{-c + dx} \sqrt{c + dx}}{8c^4 x^2} + \frac{(d^3 (4bc^2 + 3ad^2)) \operatorname{Subst}\left(\int \frac{1}{u \sqrt{-c + du} \sqrt{c + du}} du\right)}{8c^4}$$

$$= \frac{a\sqrt{-c + dx} \sqrt{c + dx}}{4c^2 x^4} + \frac{(4bc^2 + 3ad^2) \sqrt{-c + dx} \sqrt{c + dx}}{8c^4 x^2} + \frac{d^2 (4bc^2 + 3ad^2) \tan^{-1}\left(\frac{\sqrt{-c + dx} \sqrt{c + dx}}{x}\right)}{8c^5}$$

**Mathematica [A]** time = 0.11, size = 144, normalized size = 1.17

$$\frac{(c^2 - d^2 x^2) \left( c^2 \sqrt{1 - \frac{d^2 x^2}{c^2}} (2ac^2 + 3ad^2 x^2 + 4bc^2 x^2) + d^2 x^4 (3ad^2 + 4bc^2) \tanh^{-1} \left( \sqrt{1 - \frac{d^2 x^2}{c^2}} \right) \right)}{8c^6 x^4 \sqrt{dx - c} \sqrt{c + dx} \sqrt{1 - \frac{d^2 x^2}{c^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*Sqrt[-c + d*x]*Sqrt[c + d*x]), x]
[Out] -1/8*((c^2 - d^2*x^2)*(c^2*(2*a*c^2 + 4*b*c^2*x^2 + 3*a*d^2*x^2)*Sqrt[1 - (d^2*x^2)/c^2] + d^2*(4*b*c^2 + 3*a*d^2)*x^4*ArcTanh[Sqrt[1 - (d^2*x^2)/c^2]])/(c^6*x^4*Sqrt[-c + d*x]*Sqrt[c + d*x]*Sqrt[1 - (d^2*x^2)/c^2])
```

**fricas [A]** time = 1.08, size = 100, normalized size = 0.81

$$\frac{2(4bc^2d^2 + 3ad^4)x^4 \arctan\left(-\frac{dx - \sqrt{dx+c} \sqrt{dx-c}}{c}\right) + (2ac^3 + (4bc^3 + 3acd^2)x^2)\sqrt{dx+c} \sqrt{dx-c}}{8c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(1/2)/(d*x+c)^(1/2), x, algorithm="fricas")
[Out] 1/8*(2*(4*b*c^2*d^2 + 3*a*d^4)*x^4*arctan(-(d*x - sqrt(d*x + c))*sqrt(d*x - c))/c) + (2*a*c^3 + (4*b*c^3 + 3*a*c*d^2)*x^2)*sqrt(d*x + c)*sqrt(d*x - c)/(c^5*x^4)
```

**giac [B]** time = 0.27, size = 325, normalized size = 2.64

$$\frac{(4bc^2d^3 + 3ad^5) \arctan\left(\frac{(\sqrt{dx+c} - \sqrt{dx-c})^2}{2c}\right)}{c^5} + \frac{2(4bc^2d^3(\sqrt{dx+c} - \sqrt{dx-c})^{14} + 3ad^5(\sqrt{dx+c} - \sqrt{dx-c})^{14} + 16bc^4d^3(\sqrt{dx+c} - \sqrt{dx-c})^{10} + 44ac^2d^5(\sqrt{dx+c} - \sqrt{dx-c})^6)}{8c^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^5/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="giac")

[Out] 
$$-1/4*((4*b*c^2*d^3 + 3*a*d^5)*\arctan(1/2*(\sqrt{d*x + c} - \sqrt{d*x - c}))^2/c^5 + 2*(4*b*c^2*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^{14} + 3*a*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^{14} + 16*b*c^4*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^{10} + 44*a*c^2*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^{10} - 64*b*c^6*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^6 - 176*a*c^4*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^6 - 256*b*c^8*d^3*(\sqrt{d*x + c} - \sqrt{d*x - c})^2 - 192*a*c^6*d^5*(\sqrt{d*x + c} - \sqrt{d*x - c})^2)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2)^4*c^4)/d$$

**maple [B]** time = 0.07, size = 227, normalized size = 1.85

$$\frac{\sqrt{dx-c} \sqrt{dx+c} \left( 3a d^4 x^4 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) + 4b c^2 d^2 x^4 \ln \left( -\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x} \right) - 3\sqrt{-c^2} \sqrt{d^2 x^2 - c^2} \right)}{8\sqrt{d^2 x^2 - c^2} \sqrt{-c^2} c^4 x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^5/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x)

[Out] 
$$-1/8*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^4*(3*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}))/x)*x^4*a*d^4+4*b*c^2*d^2*x^4*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}))/x)-3*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*x^2*a*d^2-4*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*x^2*b*c^2-2*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*a*c^2)/(d^2*x^2-c^2)^{(1/2)}/x^4/(-c^2)^{(1/2)}$$

**maxima [A]** time = 1.21, size = 114, normalized size = 0.93

$$-\frac{bd^2 \arcsin\left(\frac{c}{d|x|}\right)}{2c^3} - \frac{3ad^4 \arcsin\left(\frac{c}{d|x|}\right)}{8c^5} + \frac{\sqrt{d^2x^2 - c^2} b}{2c^2x^2} + \frac{3\sqrt{d^2x^2 - c^2} ad^2}{8c^4x^2} + \frac{\sqrt{d^2x^2 - c^2} a}{4c^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^5/(d\*x-c)^(1/2)/(d\*x+c)^(1/2),x, algorithm="maxima")

[Out] 
$$-1/2*b*d^2*\arcsin(c/(d*\text{abs}(x)))/c^3 - 3/8*a*d^4*\arcsin(c/(d*\text{abs}(x)))/c^5 + 1/2*\sqrt{d^2*x^2 - c^2}*b/(c^2*x^2) + 3/8*\sqrt{d^2*x^2 - c^2}*a*d^2/(c^4*x^2) + 1/4*\sqrt{d^2*x^2 - c^2}*a/(c^2*x^4)$$

**mupad [B]** time = 19.13, size = 1005, normalized size = 8.17

$$\frac{3a\sqrt{-c}d^4 \ln\left(\frac{\sqrt{c+dx}-\sqrt{c}}{\sqrt{-c}-\sqrt{dx-c}}\right)}{8c^{11/2}} - \frac{b(-c)^{3/2}d^2}{32c^{9/2}} + \frac{b(-c)^{3/2}d^2(\sqrt{c+dx}-\sqrt{c})^2}{16c^{9/2}(\sqrt{-c}-\sqrt{dx-c})^2} - \frac{15b(-c)^{3/2}d^2(\sqrt{c+dx}-\sqrt{c})^4}{32c^{9/2}(\sqrt{-c}-\sqrt{dx-c})^4} - \frac{a\sqrt{-c}d^4}{1024c^{11/2}} - \frac{3a\sqrt{-c}d^4}{128c^{11/2}} \left( \frac{(\sqrt{c+dx}-\sqrt{c})^2}{(\sqrt{-c}-\sqrt{dx-c})^2} + \frac{2(\sqrt{c+dx}-\sqrt{c})^4}{(\sqrt{-c}-\sqrt{dx-c})^4} + \frac{(\sqrt{c+dx}-\sqrt{c})^6}{(\sqrt{-c}-\sqrt{dx-c})^6} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^5\*(c + d\*x)^(1/2)\*(d\*x - c)^(1/2)),x)

[Out] 
$$(3*a*(-c)^{(1/2)}*d^4*\log(((c + d*x)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)}))/((8*c^{(11/2)}) - ((b*(-c)^{(3/2)}*d^2)/(32*c^{(9/2)}) + (b*(-c)^{(3/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/(16*c^{(9/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) - (15*b*(-c)^{(3/2)}*d^2*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/(32*c^{(9/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4)/(((c + d*x)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + (2*((c + d*x)^{(1/2)} - c^{(1/2)})^4)/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4 + ((c + d*x)^{(1/2)} - c^{(1/2)})^6/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^6) - ((a*(-c)^{(1/2)}*d^4)/(1024*c^{(11/2)}) - (3*a*(-c)^{(1/2)}*d^4*((c + d*x)^{(1/2)} - c^{(1/2)})^2)/(128*c^{(11/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) - (53*$$

$$\begin{aligned}
& a(-c)^{(1/2)}d^4((c + dx)^{(1/2)} - c^{(1/2)})^4/(512c^{(11/2)}*((-c)^{(1/2)} - \\
& (dx - c)^{(1/2)})^4) + (87*a*(-c)^{(1/2)}*d^4*((c + dx)^{(1/2)} - c^{(1/2)})^6/ \\
& (256*c^{(11/2)}*((-c)^{(1/2)} - (dx - c)^{(1/2)})^6) + (657*a*(-c)^{(1/2)}*d^4*((c \\
& + dx)^{(1/2)} - c^{(1/2)})^8)/(1024*c^{(11/2)}*((-c)^{(1/2)} - (dx - c)^{(1/2)})^8 \\
& ) + (121*a*(-c)^{(1/2)}*d^4*((c + dx)^{(1/2)} - c^{(1/2)})^{10})/(256*c^{(11/2)}*((- \\
& c)^{(1/2)} - (dx - c)^{(1/2)})^{10})/(((c + dx)^{(1/2)} - c^{(1/2)})^4/((-c)^{(1/2)} \\
& - (dx - c)^{(1/2)})^4 + (4*((c + dx)^{(1/2)} - c^{(1/2)})^6)/((-c)^{(1/2)} - (d* \\
& x - c)^{(1/2)})^6 + (6*((c + dx)^{(1/2)} - c^{(1/2)})^8)/((-c)^{(1/2)} - (d*x - c) \\
& ^{(1/2)})^8 + (4*((c + dx)^{(1/2)} - c^{(1/2)})^{10})/((-c)^{(1/2)} - (d*x - c)^{(1/2)} \\
& ))^{10} + ((c + dx)^{(1/2)} - c^{(1/2)})^{12}/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^{12} - \\
& (b*(-c)^{(3/2)}*d^2*\log(((c + dx)^{(1/2)} - c^{(1/2)})/((-c)^{(1/2)} - (d*x - c)^{(1/2)})))/ \\
& (2*c^{(9/2)}) - (3*a*(-c)^{(1/2)}*d^4*\log(((c + dx)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1))/ \\
& (8*c^{(11/2)}) + (b*(-c)^{(3/2)}*d^2*\log(((c + dx)^{(1/2)} - c^{(1/2)})^2/((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2 + 1))/ \\
& (2*c^{(9/2)}) - (7*a*d^4*((c + dx)^{(1/2)} - c^{(1/2)})^2)/(256*(-c)^{(1/2)}*c^{(9/2)}* \\
& ((-c)^{(1/2)} - (d*x - c)^{(1/2)})^2) + (a*d^4*((c + dx)^{(1/2)} - c^{(1/2)})^4)/( \\
& 1024*(-c)^{(1/2)}*c^{(9/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)})^4) + (b*d^2*((c + d* \\
& x)^{(1/2)} - c^{(1/2)})^2)/(32*(-c)^{(3/2)}*c^{(3/2)}*((-c)^{(1/2)} - (d*x - c)^{(1/2)} \\
& )^2)
\end{aligned}$$

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*5/(d\*x-c)\*\*(1/2)/(d\*x+c)\*\*(1/2),x)

[Out] Timed out



$$3.368 \quad \int \frac{x^4(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=161

$$\frac{3c^2(4ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7} + \frac{3x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 5bc^2)}{8d^6} - \frac{x^3(4ad^2 + 5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $\frac{3}{4}c^2(4ad^2+5bc^2)\operatorname{arctanh}\left(\frac{(dx-c)^{1/2}}{(dx+c)^{1/2}}\right)/d^7-1/4(4ad^2+5bc^2)x^3/d^4/(dx-c)^{1/2}/(dx+c)^{1/2}+1/4bx^5/d^2/(dx-c)^{1/2}/(dx+c)^{1/2}+3/8(4ad^2+5bc^2)xx(dx-c)^{1/2}(dx+c)^{1/2}/d^6$

**Rubi [A]** time = 0.12, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$ , Rules used = {460, 98, 21, 90, 12, 63, 217, 206}

$$-\frac{x^3(4ad^2 + 5bc^2)}{4d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{3x\sqrt{dx-c}\sqrt{c+dx}(4ad^2 + 5bc^2)}{8d^6} + \frac{3c^2(4ad^2 + 5bc^2) \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{4d^7} + \frac{bx^5}{4d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] `Int[(x^4*(a + b*x^2))/((-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]`

[Out]  $-\frac{(5bc^2 + 4ad^2)x^3}{4d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^5}{4d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c+dx}\sqrt{c+dx}}{8d^6} + \frac{3c^2(5bc^2 + 4ad^2)\operatorname{ArcTanh}\left[\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right]}{4d^7}$

### Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]`

### Rule 21

`Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m+n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])`

### Rule 63

`Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m+1)-1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

### Rule 90

`Int[((a_) + (b_)*(x_))^(2)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n+1)*(e + f*x)^(p+1))/(d*f*(n+p+3)), x] + Dist[1/(d*f*(n+p+3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n+p+3) - b*(b*c*e + a*(d*e*(n+1) + c*f*(p+1))) + b*(a*d*f*(n+p+4) - b*(d*e*(n+2) + c*f*(p+2))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n+p+3, 0]`

### Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

### Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

### Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

### Rule 460

```
Int[((e_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(non2_))^(p_)*((a2_) + (b2_.)*(x_)^(non2_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/(b1*b2*e*(m + n*(p + 1) + 1)), x] - Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/(b1*b2*(m + n*(p + 1) + 1)), Int[(e*x)^m*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

### Rubi steps

$$\begin{aligned}
 \int \frac{x^4 (a + bx^2)}{(-c + dx)^{3/2} (c + dx)^{3/2}} dx &= \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} - \frac{1}{4} \left( -4a - \frac{5bc^2}{d^2} \right) \int \frac{x^4}{(-c + dx)^{3/2} (c + dx)^{3/2}} dx \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} - \frac{\left(4a + \frac{5bc^2}{d^2}\right) \int \frac{x^2(-3c^2 - 3cdx)}{\sqrt{-c + dx} (c + dx)^{3/2}}}{4cd^2} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2) \int \frac{x^2}{\sqrt{-c + dx} \sqrt{c + dx}}}{4d^4} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{8d^6} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{8d^6} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{8d^6} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{8d^6} \\
 &= -\frac{(5bc^2 + 4ad^2)x^3}{4d^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{bx^5}{4d^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{3(5bc^2 + 4ad^2)x\sqrt{-c + dx}\sqrt{c + dx}}{8d^6}
 \end{aligned}$$

**Mathematica [A]** time = 0.16, size = 119, normalized size = 0.74

$$\frac{3c^3 \sqrt{1 - \frac{d^2 x^2}{c^2}} (4ad^2 + 5bc^2) \sin^{-1}\left(\frac{dx}{c}\right) + 4ad^3 x (d^2 x^2 - 3c^2) + bdx (-15c^4 + 5c^2 d^2 x^2 + 2d^4 x^4)}{8d^7 \sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] (4\*a\*d^3\*x\*(-3\*c^2 + d^2\*x^2) + b\*d\*x\*(-15\*c^4 + 5\*c^2\*d^2\*x^2 + 2\*d^4\*x^4) + 3\*c^3\*(5\*b\*c^2 + 4\*a\*d^2)\*Sqrt[1 - (d^2\*x^2)/c^2]\*ArcSin[(d\*x)/c])/(8\*d^7\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.08, size = 190, normalized size = 1.18

$$\frac{8bc^6 + 8ac^4d^2 - 8(bc^4d^2 + ac^2d^4)x^2 + (2bd^5x^5 + (5bc^2d^3 + 4ad^5)x^3 - 3(5bc^4d + 4ac^2d^3)x)\sqrt{dx + c}\sqrt{dx - c}}{8(d^9x^2 - c^2d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/8\*(8\*b\*c^6 + 8\*a\*c^4\*d^2 - 8\*(b\*c^4\*d^2 + a\*c^2\*d^4)\*x^2 + (2\*b\*d^5\*x^5 + (5\*b\*c^2\*d^3 + 4\*a\*d^5)\*x^3 - 3\*(5\*b\*c^4\*d + 4\*a\*c^2\*d^3)\*x)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 3\*(5\*b\*c^6 + 4\*a\*c^4\*d^2 - (5\*b\*c^4\*d^2 + 4\*a\*c^2\*d^4)\*x^2)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/(d^9\*x^2 - c^2\*d^7)

**giac [A]** time = 0.40, size = 214, normalized size = 1.33

$$\frac{\left(\left((dx + c)\left(2(dx + c)\left(\frac{(dx+c)b}{d^7} - \frac{5bc}{d^7}\right) + \frac{25bc^2d^{35} + 4ad^{37}}{d^{42}}\right) - \frac{35bc^3d^{35} + 12acd^{37}}{d^{42}}\right)(dx + c) + \frac{2(7bc^4d^{35} + 2ac^2d^{37})}{d^{42}}\right)\sqrt{dx + c}}{8\sqrt{dx - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] 1/8\*(((d\*x + c)\*(2\*(d\*x + c)\*((d\*x + c)\*b/d^7 - 5\*b\*c/d^7) + (25\*b\*c^2\*d^35 + 4\*a\*d^37)/d^42) - (35\*b\*c^3\*d^35 + 12\*a\*c\*d^37)/d^42)\*(d\*x + c) + 2\*(7\*b\*c^4\*d^35 + 2\*a\*c^2\*d^37)/d^42)\*sqrt(d\*x + c)/sqrt(d\*x - c) - 3/8\*(5\*b\*c^4 + 4\*a\*c^2\*d^2)\*log((sqrt(d\*x + c) - sqrt(d\*x - c))^2/d^7 - 2\*(b\*c^5 + a\*c^3\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*d^7))

**maple [C]** time = 0.09, size = 316, normalized size = 1.96

$$\frac{\left(2\sqrt{d^2x^2 - c^2} b d^5 x^5 \operatorname{csgn}(d) + 12a c^2 d^4 x^2 \ln\left(\left(dx + \sqrt{d^2x^2 - c^2} \operatorname{csgn}(d)\right) \operatorname{csgn}(d)\right) + 4\sqrt{d^2x^2 - c^2} a d^5 x^3 \operatorname{csgn}(d)\right)}{8\sqrt{dx - c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out] 1/8\*(2\*(d^2\*x^2-c^2)^(1/2)\*b\*d^5\*x^5\*csgn(d)+4\*(d^2\*x^2-c^2)^(1/2)\*a\*d^5\*x^3\*csgn(d)+5\*(d^2\*x^2-c^2)^(1/2)\*b\*c^2\*d^3\*x^3\*csgn(d)+12\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))\*x^2\*a\*c^2\*d^4+15\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))\*x^2\*b\*c^4\*d^2-12\*(d^2\*x^2-c^2)^(1/2)\*a\*c^2\*d^3\*x\*csgn(d)-15\*(d^2\*x^2-c^2)^(1/2)\*b\*c^4\*d\*x\*csgn(d)-12\*a\*c^4\*d^2\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d))-15\*b\*c^6\*ln((d\*x+(d^2\*x^2-c^2)^(1/2)\*csgn(d))\*csgn(d)))csgn(d)/(d^2\*x^2-c^2)^(1/2)/d^7/(d\*x+c)^(1/2)/(d\*x-c)^(1/2)

**maxima** [A] time = 0.49, size = 196, normalized size = 1.22

$$\frac{bx^5}{4\sqrt{d^2x^2 - c^2}d^2} + \frac{5bc^2x^3}{8\sqrt{d^2x^2 - c^2}d^4} + \frac{ax^3}{2\sqrt{d^2x^2 - c^2}d^2} - \frac{15bc^4x}{8\sqrt{d^2x^2 - c^2}d^6} - \frac{3ac^2x}{2\sqrt{d^2x^2 - c^2}d^4} + \frac{15bc^4 \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}\right)}{8d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
[Out] 1/4*b*x^5/(sqrt(d^2*x^2 - c^2)*d^2) + 5/8*b*c^2*x^3/(sqrt(d^2*x^2 - c^2)*d^4) + 1/2*a*x^3/(sqrt(d^2*x^2 - c^2)*d^2) - 15/8*b*c^4*x/(sqrt(d^2*x^2 - c^2)*d^6) - 3/2*a*c^2*x/(sqrt(d^2*x^2 - c^2)*d^4) + 15/8*b*c^4*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^7 + 3/2*a*c^2*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^5
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4 (bx^2 + a)}{(c + dx)^{3/2} (dx - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
[Out] int((x^4*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
[Out] Timed out
```

$$3.369 \quad \int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=115

$$\frac{2\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6} - \frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-1/3*(3*a*d^2+4*b*c^2)*x^2/d^4/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+1/3*b*x^4/d^2/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+2/3*(3*a*d^2+4*b*c^2)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/d^6$

**Rubi [A]** time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {460, 98, 21, 74}

$$-\frac{x^2(3ad^2+4bc^2)}{3d^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{2\sqrt{dx-c}\sqrt{c+dx}(3ad^2+4bc^2)}{3d^6} + \frac{bx^4}{3d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out]  $-((4*b*c^2 + 3*a*d^2)*x^2)/(3*d^4*sqrt[-c + d*x]*sqrt[c + d*x]) + (b*x^4)/(3*d^2*sqrt[-c + d*x]*sqrt[c + d*x]) + (2*(4*b*c^2 + 3*a*d^2)*sqrt[-c + d*x]*sqrt[c + d*x])/(3*d^6)$

#### Rule 21

Int[(u\_.)\*((a\_) + (b\_.)\*(v\_))^(m\_.)\*((c\_) + (d\_.)\*(v\_))^(n\_.), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 74

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

#### Rule 98

Int[((a\_.) + (b\_.)\*(x\_))^(m\_.)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[((b\*c - a\*d)\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 1)\*(e + f\*x)^(p + 1))/(b\*(b\*e - a\*f)\*(m + 1)), x] + Dist[1/(b\*(b\*e - a\*f)\*(m + 1)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^(n - 2)\*(e + f\*x)^p\*Simp[a\*d\*(d\*e\*(n - 1) + c\*f\*(p + 1)) + b\*c\*(d\*e\*(m - n + 2) - c\*f\*(m + p + 2)) + d\*(a\*d\*f\*(n + p) + b\*(d\*e\*(m + 1) - c\*f\*(m + n + p + 1)))\*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2\*m, 2\*n, 2\*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

#### Rule 460

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2,

$n/2$  && EqQ[ $a_2*b_1 + a_1*b_2, 0$ ] && NeQ[ $m + n*(p + 1) + 1, 0$ ]

Rubi steps

$$\begin{aligned} \int \frac{x^3(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx &= \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{1}{3} \left(-3a - \frac{4bc^2}{d^2}\right) \int \frac{x^3}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx \\ &= -\frac{(4bc^2+3ad^2)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} - \frac{\left(3a + \frac{4bc^2}{d^2}\right) \int \frac{x(-2c^2-2cdx)}{\sqrt{-c+dx}(c+dx)^{3/2}}}{3cd^2} \\ &= -\frac{(4bc^2+3ad^2)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(2\left(3a + \frac{4bc^2}{d^2}\right)\right) \int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}}}{3d^2} \\ &= -\frac{(4bc^2+3ad^2)x^2}{3d^4\sqrt{-c+dx}\sqrt{c+dx}} + \frac{bx^4}{3d^2\sqrt{-c+dx}\sqrt{c+dx}} + \frac{2(4bc^2+3ad^2)\sqrt{-c+dx}\sqrt{c+dx}}{3d^6} \end{aligned}$$

**Mathematica [A]** time = 0.05, size = 72, normalized size = 0.63

$$\frac{-6ac^2d^2 + 3ad^4x^2 - 8bc^4 + 4bc^2d^2x^2 + bd^4x^4}{3d^6\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out] (-8\*b\*c^4 - 6\*a\*c^2\*d^2 + 4\*b\*c^2\*d^2\*x^2 + 3\*a\*d^4\*x^2 + b\*d^4\*x^4)/(3\*d^6\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.18, size = 80, normalized size = 0.70

$$\frac{(bd^4x^4 - 8bc^4 - 6ac^2d^2 + (4bc^2d^2 + 3ad^4)x^2)\sqrt{dx+c}\sqrt{dx-c}}{3(d^8x^2 - c^2d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="fricas")

[Out] 1/3\*(b\*d^4\*x^4 - 8\*b\*c^4 - 6\*a\*c^2\*d^2 + (4\*b\*c^2\*d^2 + 3\*a\*d^4)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/(d^8\*x^2 - c^2\*d^6)

**giac [B]** time = 0.43, size = 200, normalized size = 1.74

$$\frac{\left(2(dx+c)\left((dx+c)\left(\frac{(dx+c)b}{d^6} - \frac{4bc}{d^6}\right) + \frac{10bc^2d^2+3ad^26}{d^30}\right) - \frac{3(9bc^3d^24+5acd^26)}{d^30}\right)\sqrt{dx+c}}{6\sqrt{dx-c}} + \frac{2(b^2c}{(bc^4(\sqrt{dx+c} - \sqrt{dx-c})^2 + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="giac")

[Out] 1/6\*(2\*(d\*x + c)\*((d\*x + c)\*((d\*x + c)\*b/d^6 - 4\*b\*c/d^6) + (10\*b\*c^2\*d^24 + 3\*a\*d^26)/d^30) - 3\*(9\*b\*c^3\*d^24 + 5\*a\*c\*d^26)/d^30)\*sqrt(d\*x + c)/sqrt(d\*x - c) + 2\*(b^2\*c^8 + 2\*a\*b\*c^6\*d^2 + a^2\*c^4\*d^4)/((b\*c^4\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2 + a\*c^2\*d^2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*b\*c^5 + 2\*a\*c^3\*d^2)\*d^6)

**maple [A]** time = 0.05, size = 68, normalized size = 0.59

$$\frac{-b d^4 x^4 - 3a d^4 x^2 - 4b c^2 d^2 x^2 + 6a c^2 d^2 + 8b c^4}{3\sqrt{dx+c} \sqrt{dx-c} d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out] -1/3\*(-b\*d^4\*x^4-3\*a\*d^4\*x^2-4\*b\*c^2\*d^2\*x^2+6\*a\*c^2\*d^2+8\*b\*c^4)/(d\*x+c)^(1/2)/d^6/(d\*x-c)^(1/2)

**maxima [A]** time = 0.46, size = 123, normalized size = 1.07

$$\frac{bx^4}{3\sqrt{d^2x^2-c^2}d^2} + \frac{4bc^2x^2}{3\sqrt{d^2x^2-c^2}d^4} + \frac{ax^2}{\sqrt{d^2x^2-c^2}d^2} - \frac{8bc^4}{3\sqrt{d^2x^2-c^2}d^6} - \frac{2ac^2}{\sqrt{d^2x^2-c^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/3\*b\*x^4/(sqrt(d^2\*x^2 - c^2)\*d^2) + 4/3\*b\*c^2\*x^2/(sqrt(d^2\*x^2 - c^2)\*d^4) + a\*x^2/(sqrt(d^2\*x^2 - c^2)\*d^2) - 8/3\*b\*c^4/(sqrt(d^2\*x^2 - c^2)\*d^6) - 2\*a\*c^2/(sqrt(d^2\*x^2 - c^2)\*d^4)

**mupad [B]** time = 2.80, size = 90, normalized size = 0.78

$$\frac{\sqrt{dx-c} \left( \frac{x^2(4bc^2d^2+3ad^4)}{3d^7} - \frac{8bc^4+6ac^2d^2}{3d^7} + \frac{bx^4}{3d^3} \right)}{x\sqrt{c+dx} - \frac{c\sqrt{c+dx}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3\*(a + b\*x^2))/((c + d\*x)^(3/2)\*(d\*x - c)^(3/2)),x)

[Out] ((d\*x - c)^(1/2)\*((x^2\*(3\*a\*d^4 + 4\*b\*c^2\*d^2))/(3\*d^7) - (8\*b\*c^4 + 6\*a\*c^2\*d^2)/(3\*d^7) + (b\*x^4)/(3\*d^3)))/(x\*(c + d\*x)^(1/2) - (c\*(c + d\*x)^(1/2))/d)

**sympy [C]** time = 177.26, size = 226, normalized size = 1.97

$$a \left( \frac{c G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} & -1, 0, \frac{1}{2}, 1 \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i c G_{6,6}^{2,6} \left( \begin{matrix} -2, -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, 1 \\ -\frac{5}{4}, -\frac{3}{4} & -2, -\frac{3}{2}, -\frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} d^4} \right) + b \left( \frac{c^3 G_{6,6}^{6,2}}{2\pi^{\frac{3}{2}} d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] a\*(c\*meijerg((( -3/4, -1/4), (-1, 0, 1/2, 1)), (( -3/4, -1/2, -1/4, 0, 1/2, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\* (3/2)\*d\*\*4) - I\*c\*meijerg((( -2, -3/2, -5/4, -1, -3/4, 1), ()), (( -5/4, -3/4), (-2, -3/2, -1/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\* (3/2)\*d\*\*4) + b\*(c\*\*3\*meijerg((( -7/4, -5/4), (-2, -1, -1/2, 1)), (( -7/4, -3/2, -5/4, -1, -1/2, 0), ()), c\*\*2/(d\*\*2\*x\*\*2))/(2\*pi\*\* (3/2)\*d\*\*6) - I\*c\*\*3\*meijerg((( -3, -5/2, -9/4, -2, -7/4, 1), ()), (( -9/4, -7/4), (-3, -5/2, -3/2, 0)), c\*\*2\*exp\_polar(2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\* (3/2)\*d\*\*6)

$$3.370 \quad \int \frac{x^2(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=152

$$\frac{\sqrt{dx-c}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}} - \frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\sqrt{c+dx}} + \frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5} + \frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out] (2\*a\*d^2+3\*b\*c^2)\*arctanh((d\*x-c)^(1/2)/(d\*x+c)^(1/2))/d^5-1/2\*c\*(2\*a\*d^2+3\*b\*c^2)/d^5/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)+1/2\*b\*x^3/d^2/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)-1/2\*(2\*a\*d^2+3\*b\*c^2)\*(d\*x-c)^(1/2)/d^5/(d\*x+c)^(1/2)

**Rubi [A]** time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {460, 89, 12, 78, 63, 217, 206}

$$\frac{\sqrt{dx-c}(2ad^2+3bc^2)}{2d^5\sqrt{c+dx}} - \frac{c(2ad^2+3bc^2)}{2d^5\sqrt{dx-c}\sqrt{c+dx}} + \frac{(2ad^2+3bc^2)\tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^5} + \frac{bx^3}{2d^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] -(c\*(3\*b\*c^2 + 2\*a\*d^2))/(2\*d^5\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) + (b\*x^3)/(2\*d^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) - ((3\*b\*c^2 + 2\*a\*d^2)\*Sqrt[-c + d\*x])/(2\*d^5\*Sqrt[c + d\*x]) + ((3\*b\*c^2 + 2\*a\*d^2)\*ArcTanh[Sqrt[-c + d\*x]/Sqrt[c + d\*x]])/d^5

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

### Rule 78

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := -Simp[((b\*e - a\*f)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(f\*(p + 1)\*(c\*f - d\*e)), x] - Dist[(a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)))/(f\*(p + 1)\*(c\*f - d\*e)), Int[(c + d\*x)^n\*(e + f\*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

### Rule 89

Int[((a\_.) + (b\_.)\*(x\_))^(2)\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] := Simp[((b\*c - a\*d)^2\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d^2\*(d\*e - c\*f)\*(n + 1)), x] - Dist[1/(d^2\*(d\*e - c\*f)\*(n + 1)), Int[(c + d\*x)^(n + 1)\*(e + f\*x)^p\*Simp[a^2\*d^2\*f\*(n + p + 2) + b^2\*c\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - 2\*a\*b\*d\*(d\*e\*(n + 1) + c\*f\*(p + 1)) - b^2\*d\*(d\*e - c\*f)\*(n + 1)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] || (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,



1]))))

Rule 206

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTanh[(Rt[-b, 2]\*x)/Rt[a, 2]])/(Rt[a, 2]\*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 460

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(d\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(b1\*b2\*e\*(m + n\*(p + 1) + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(b1\*b2\*(m + n\*(p + 1) + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p + 1) + 1, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{x^2(a + bx^2)}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{1}{2} \left( -2a - \frac{3bc^2}{d^2} \right) \int \frac{x^2}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{\left( -2a - \frac{3bc^2}{d^2} \right) \int \frac{cd^2x}{\sqrt{-c + dx}(c + dx)}}{2cd^3} \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} + \frac{(3bc^2 + 2ad^2) \int \frac{x}{\sqrt{-c + dx}(c + dx)}}{2d^3} \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{(3bc^2 + 2ad^2)\sqrt{-c + dx}}{2d^5\sqrt{c + dx}} \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{(3bc^2 + 2ad^2)\sqrt{-c + dx}}{2d^5\sqrt{c + dx}} \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{(3bc^2 + 2ad^2)\sqrt{-c + dx}}{2d^5\sqrt{c + dx}} \\
 &= -\frac{c(3bc^2 + 2ad^2)}{2d^5\sqrt{-c + dx}\sqrt{c + dx}} + \frac{bx^3}{2d^2\sqrt{-c + dx}\sqrt{c + dx}} - \frac{(3bc^2 + 2ad^2)\sqrt{-c + dx}}{2d^5\sqrt{c + dx}}
 \end{aligned}$$

**Mathematica [A]** time = 0.12, size = 90, normalized size = 0.59

$$\frac{c\sqrt{1 - \frac{d^2x^2}{c^2}} (2ad^2 + 3bc^2) \sin^{-1}\left(\frac{dx}{c}\right) - 2ad^3x - 3bc^2dx + bd^3x^3}{2d^5\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $(-3bc^2dx - 2ad^3x + bd^3x^3 + c(3bc^2 + 2ad^2)\sqrt{1 - (d^2x^2)/c^2})\text{ArcSin}[(dx)/c]/(2d^5\sqrt{-c + dx}\sqrt{c + dx})$

**fricas** [A] time = 0.98, size = 159, normalized size = 1.05

$$\frac{2bc^4 + 2ac^2d^2 - 2(bc^2d^2 + ad^4)x^2 + (bd^3x^3 - (3bc^2d + 2ad^3)x)\sqrt{dx + c}\sqrt{dx - c} + (3bc^4 + 2ac^2d^2 - (3bc^2d^2 - 2ad^4)x^2)}{2(d^7x^2 - c^2d^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")`

[Out]  $1/2*(2bc^4 + 2ac^2d^2 - 2(bc^2d^2 + ad^4)x^2 + (bd^3x^3 - (3bc^2d + 2ad^3)x)\sqrt{dx + c}\sqrt{dx - c} + (3bc^4 + 2ac^2d^2 - (3bc^2d^2 + 2ad^4)x^2)\log(-dx + \sqrt{dx + c}\sqrt{dx - c}))/d^7x^2 - c^2d^5)$

**giac** [A] time = 0.32, size = 147, normalized size = 0.97

$$\frac{\sqrt{dx + c} \left( (dx + c) \left( \frac{(dx+c)b}{d^5} - \frac{3bc}{d^5} \right) + \frac{bc^2d^{15}-ad^{17}}{d^{20}} \right) (3bc^2 + 2ad^2) \log \left( \left( \sqrt{dx + c} - \sqrt{dx - c} \right)^2 \right)}{2\sqrt{dx - c}} - \frac{(3bc^2 + 2ad^2) \log \left( \left( \sqrt{dx + c} - \sqrt{dx - c} \right)^2 \right)}{2d^5} - \frac{2(bc^3 + ad^3)}{\left( \left( \sqrt{dx + c} - \sqrt{dx - c} \right)^2 + 2c \right) d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")`

[Out]  $1/2*\sqrt{d*x + c}*((d*x + c)*((d*x + c)*b/d^5 - 3*b*c/d^5) + (b*c^2*d^{15} - a*d^{17})/d^{20})/\sqrt{d*x - c} - 1/2*(3*b*c^2 + 2*a*d^2)*\log((\sqrt{d*x + c} - \sqrt{d*x - c})^2)/d^5 - 2*(b*c^3 + a*c*d^2)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 2*c)*d^5)$

**maple** [C] time = 0.08, size = 254, normalized size = 1.67

$$\frac{\left( 2a d^4 x^2 \ln \left( \left( dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) \right) \operatorname{csgn}(d) \right) + 3b c^2 d^2 x^2 \ln \left( \left( dx + \sqrt{d^2 x^2 - c^2} \operatorname{csgn}(d) \right) \operatorname{csgn}(d) \right) + \sqrt{d^2 x^2 - c^2} \right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $1/2*((d^2x^2-c^2)^(1/2)*bd^3x^3\operatorname{csgn}(d)+2*\ln((d*x+(d^2x^2-c^2)^(1/2))*\operatorname{csgn}(d))*\operatorname{csgn}(d))*x^2ad^4+3*\ln((d*x+(d^2x^2-c^2)^(1/2))*\operatorname{csgn}(d))*\operatorname{csgn}(d))*x^2*b*c^2*d^2-2*(d^2x^2-c^2)^(1/2)*ad^3*x*\operatorname{csgn}(d)-3*(d^2x^2-c^2)^(1/2)*b*c^2*d*x*\operatorname{csgn}(d)-2*a*c^2*d^2*\ln((d*x+(d^2x^2-c^2)^(1/2))*\operatorname{csgn}(d))*\operatorname{csgn}(d))-3*b*c^4*\ln((d*x+(d^2x^2-c^2)^(1/2))*\operatorname{csgn}(d))*\operatorname{csgn}(d))*\operatorname{csgn}(d)/(d^2x^2-c^2)^(1/2)/d^5/(d*x+c)^(1/2)/(d*x-c)^(1/2)$

**maxima** [A] time = 0.50, size = 138, normalized size = 0.91

$$\frac{bx^3}{2\sqrt{d^2x^2 - c^2}d^2} - \frac{3bc^2x}{2\sqrt{d^2x^2 - c^2}d^4} - \frac{ax}{\sqrt{d^2x^2 - c^2}d^2} + \frac{3bc^2 \log \left( 2d^2x + 2\sqrt{d^2x^2 - c^2}d \right)}{2d^5} + \frac{a \log \left( 2d^2x + 2\sqrt{d^2x^2 - c^2}d \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")`

[Out]  $1/2*b*x^3/(\sqrt{d^2*x^2 - c^2}*d^2) - 3/2*b*c^2*x/(\sqrt{d^2*x^2 - c^2}*d^4) - a*x/(\sqrt{d^2*x^2 - c^2}*d^2) + 3/2*b*c^2*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d^5 + a*\log(2*d^2*x + 2*\sqrt{d^2*x^2 - c^2}*d)/d^3$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2 (b x^2 + a)}{(c + d x)^{3/2} (d x - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2\*(a + b\*x^2))/((c + d\*x)^(3/2)\*(d\*x - c)^(3/2)),x)

[Out] int((x^2\*(a + b\*x^2))/((c + d\*x)^(3/2)\*(d\*x - c)^(3/2)), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x\*\*2+a)/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] Timed out

$$3.371 \quad \int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=76

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+2bc^2)}{c^2d^4} - \frac{x^2\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-(a/c^2+b/d^2)*x^2/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+(a*d^2+2*b*c^2)*(d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c^2/d^4$

**Rubi [A]** time = 0.05, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$ , Rules used = {458, 74}

$$\frac{\sqrt{dx-c}\sqrt{c+dx}(ad^2+2bc^2)}{c^2d^4} - \frac{x^2\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $-(((a/c^2 + b/d^2)*x^2)/(Sqrt[-c + d*x]*Sqrt[c + d*x])) + ((2*b*c^2 + a*d^2)*Sqrt[-c + d*x]*Sqrt[c + d*x])/(c^2*d^4)$

**Rule 74**

Int[((a\_.) + (b\_.)\*(x\_))\*((c\_.) + (d\_.)\*(x\_))^(n\_.)\*((e\_.) + (f\_.)\*(x\_))^(p\_.), x\_Symbol] :> Simp[(b\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/(d\*f\*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a\*d\*f\*(n + p + 2) - b\*(d\*e\*(n + 1) + c\*f\*(p + 1)), 0]

**Rule 458**

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> -Simp[((b1\*b2\*c - a1\*a2\*d)\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*b1\*b2\*e\*n\*(p + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*b1\*b2\*n\*(p + 1)), Int[(e\*x)^(m\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && LtQ[p, -1] && (!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n\*(p + 1))])

**Rubi steps**

$$\begin{aligned} \int \frac{x(a+bx^2)}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx &= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}} - \left(-\frac{a}{c^2} - \frac{2b}{d^2}\right) \int \frac{x}{\sqrt{-c+dx}\sqrt{c+dx}} dx \\ &= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x^2}{\sqrt{-c+dx}\sqrt{c+dx}} + \frac{\left(\frac{a}{c^2} + \frac{2b}{d^2}\right)\sqrt{-c+dx}\sqrt{c+dx}}{d^2} \end{aligned}$$

**Mathematica [A]** time = 0.07, size = 45, normalized size = 0.59

$$\frac{-ad^2 - 2bc^2 + bd^2x^2}{d^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a + b\*x^2))/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out]  $(-2*b*c^2 - a*d^2 + b*d^2*x^2)/(d^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])$

**fricas** [A] time = 1.14, size = 56, normalized size = 0.74

$$\frac{(bd^2x^2 - 2bc^2 - ad^2)\sqrt{dx + c}\sqrt{dx - c}}{d^6x^2 - c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $(b*d^2*x^2 - 2*b*c^2 - a*d^2)*\text{sqrt}(d*x + c)*\text{sqrt}(d*x - c)/(d^6*x^2 - c^2*d^4)$

**giac** [B] time = 0.28, size = 152, normalized size = 2.00

$$\frac{\sqrt{dx + c} \left( \frac{2(dx+c)b}{d^4} - \frac{5bc^2d^8 + ad^{10}}{cd^{12}} \right)}{2\sqrt{dx - c}} + \frac{2(b^2c^4 + 2abc^2d^2 + a^2d^4)}{(bc^2(\sqrt{dx + c} - \sqrt{dx - c})^2 + ad^2(\sqrt{dx + c} - \sqrt{dx - c})^2 + 2bc^3 + 2acd^2)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out]  $1/2*\text{sqrt}(d*x + c)*(2*(d*x + c)*b/d^4 - (5*b*c^2*d^8 + a*d^{10})/(c*d^{12}))/\text{sqrt}(d*x - c) + 2*(b^2*c^4 + 2*a*b*c^2*d^2 + a^2*d^4)/((b*c^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + a*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + 2*b*c^3 + 2*a*c*d^2)*d^4)$

**maple** [A] time = 0.04, size = 43, normalized size = 0.57

$$-\frac{-b d^2 x^2 + a d^2 + 2 b c^2}{\sqrt{d x + c} \sqrt{d x - c} d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out]  $-(-b*d^2*x^2+a*d^2+2*b*c^2)/(d*x+c)^(1/2)/d^4/(d*x-c)^(1/2)$

**maxima** [A] time = 0.58, size = 69, normalized size = 0.91

$$\frac{bx^2}{\sqrt{d^2x^2 - c^2}d^2} - \frac{2bc^2}{\sqrt{d^2x^2 - c^2}d^4} - \frac{a}{\sqrt{d^2x^2 - c^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out]  $b*x^2/(\text{sqrt}(d^2*x^2 - c^2)*d^2) - 2*b*c^2/(\text{sqrt}(d^2*x^2 - c^2)*d^4) - a/(\text{sqrt}(d^2*x^2 - c^2)*d^2)$

**mupad** [B] time = 2.75, size = 67, normalized size = 0.88

$$\frac{a d^2 \sqrt{d x - c} + 2 b c^2 \sqrt{d x - c} - b d^2 x^2 \sqrt{d x - c}}{d^4 \sqrt{c + d x} (c - d x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(a + b*x^2))/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)`

[Out] `(a*d^2*(d*x - c)^(1/2) + 2*b*c^2*(d*x - c)^(1/2) - b*d^2*x^2*(d*x - c)^(1/2))/((d^4*(c + d*x)^(1/2)*(c - d*x))`

**sympy** [C] time = 136.30, size = 201, normalized size = 2.64

$$a \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 & 0, 1, \frac{3}{2} \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} & 0 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i G_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 \\ -\frac{1}{4}, \frac{1}{4} & -1, -\frac{1}{2}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} \right) + b \left( \frac{c G_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{2}, 0 \end{matrix} \right)}{2\pi^{\frac{3}{2}} d^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out] `a*(-meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I*meijerg((-1, -1/2, -1/4, 0, 1/4, 1), (), (-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2) + b*(c*meijerg((-3/4, -1/4), (-1, 0, 1/2, 1), (-3/4, -1/2, -1/4, 0, 1/2, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**4) - I*c*meijerg((-2, -3/2, -5/4, -1, -3/4, 1), ()), (-5/4, -3/4), (-2, -3/2, -1/2, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**4)`

$$3.372 \quad \int \frac{a+bx^2}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

Optimal. Leaf size=63

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} - \frac{x\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $2*b*\operatorname{arctanh}((d*x-c)^{(1/2)}/(d*x+c)^{(1/2)})/d^3-(a/c^2+b/d^2)*x/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$ , Rules used = {386, 63, 217, 206}

$$\frac{2b \tanh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{c+dx}}\right)}{d^3} - \frac{x\left(\frac{a}{c^2} + \frac{b}{d^2}\right)}{\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a + b*x^2)/((-c + d*x)^{(3/2)}*(c + d*x)^{(3/2))}, x]$

[Out]  $-(((a/c^2 + b/d^2)*x)/(\operatorname{Sqrt}[-c + d*x]*\operatorname{Sqrt}[c + d*x])) + (2*b*\operatorname{ArcTanh}[\operatorname{Sqrt}[-c + d*x]/\operatorname{Sqrt}[c + d*x]])/d^3$

Rule 63

$\operatorname{Int}[(a_. + (b_.)*(x_.))^{(m_.)*((c_.) + (d_.)*(x_.))^{(n_.)}, x\_Symbol] := \operatorname{With}[\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{(p*(m+1)-1)}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \operatorname{FreeQ}[\{a, b, c, d\}, x] \&\& \operatorname{NeQ}[b*c - a*d, 0] \&\& \operatorname{LtQ}[-1, m, 0] \&\& \operatorname{LeQ}[-1, n, 0] \&\& \operatorname{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \&\& \operatorname{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 206

$\operatorname{Int}[(a_. + (b_.)*(x_.)^2)^{-1}, x\_Symbol] := \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{NegQ}[a/b] \&\& (\operatorname{GtQ}[a, 0] \mid\mid \operatorname{LtQ}[b, 0])$

Rule 217

$\operatorname{Int}[1/\operatorname{Sqrt}[(a_. + (b_.)*(x_.)^2], x\_Symbol] := \operatorname{Subst}[\operatorname{Int}[1/(1 - b*x^2), x], x, x/\operatorname{Sqrt}[a + b*x^2]] /; \operatorname{FreeQ}[\{a, b\}, x] \&\& \operatorname{!GtQ}[a, 0]$

Rule 386

$\operatorname{Int}[(a1_. + (b1_.)*(x_.)^{(non2_.)})^{(p_.)*((a2_. + (b2_.)*(x_.)^{(non2_.)})^{(p_.)*((c_.) + (d_.)*(x_.)^{(n_.))}, x\_Symbol] := -\operatorname{Simp}[(b1*b2*c - a1*a2*d)*x*(a1 + b1*x^{(n/2)})^{(p+1)}*(a2 + b2*x^{(n/2)})^{(p+1)}]/(a1*a2*b1*b2*n*(p+1)), x] - \operatorname{Dist}[(a1*a2*d - b1*b2*c*(n*(p+1) + 1))/(a1*a2*b1*b2*n*(p+1)), \operatorname{Int}[(a1 + b1*x^{(n/2)})^{(p+1)}*(a2 + b2*x^{(n/2)})^{(p+1)}, x], x] /; \operatorname{FreeQ}[\{a1, b1, a2, b2, c, d, n\}, x] \&\& \operatorname{EqQ}[non2, n/2] \&\& \operatorname{EqQ}[a2*b1 + a1*b2, 0] \&\& (\operatorname{LtQ}[p, -1] \mid\mid \operatorname{ILtQ}[1/n + p, 0])$

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2}{(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x}{\sqrt{-c + dx}\sqrt{c + dx}} + \frac{b \int \frac{1}{\sqrt{-c+dx}\sqrt{c+dx}} dx}{d^2} \\
&= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x}{\sqrt{-c + dx}\sqrt{c + dx}} + \frac{(2b) \text{Subst}\left(\int \frac{1}{\sqrt{2c+x^2}} dx, x, \sqrt{-c + dx}\right)}{d^3} \\
&= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x}{\sqrt{-c + dx}\sqrt{c + dx}} + \frac{(2b) \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3} \\
&= -\frac{\left(\frac{a}{c^2} + \frac{b}{d^2}\right)x}{\sqrt{-c + dx}\sqrt{c + dx}} + \frac{2b \tanh^{-1}\left(\frac{\sqrt{-c+dx}}{\sqrt{c+dx}}\right)}{d^3}
\end{aligned}$$

**Mathematica [A]** time = 0.24, size = 86, normalized size = 1.37

$$\frac{2bc^{5/2}\sqrt{\frac{dx}{c} + 1} \sinh^{-1}\left(\frac{\sqrt{dx-c}}{\sqrt{2}\sqrt{c}}\right) - \frac{dx(ad^2+bc^2)}{\sqrt{dx-c}}}{c^2d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/((-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out] (-((d\*(b\*c^2 + a\*d^2)\*x)/Sqrt[-c + d\*x]) + 2\*b\*c^(5/2)\*Sqrt[1 + (d\*x)/c]\*ArcSinh[Sqrt[-c + d\*x]/(Sqrt[2]\*Sqrt[c])])/(c^2\*d^3\*Sqrt[c + d\*x])

**fricas [B]** time = 1.41, size = 129, normalized size = 2.05

$$\frac{bc^4 + ac^2d^2 - (bc^2d + ad^3)\sqrt{dx+c}\sqrt{dx-c}x - (bc^2d^2 + ad^4)x^2 - (bc^2d^2x^2 - bc^4)\log(-dx + \sqrt{dx+c}\sqrt{dx-c})}{c^2d^5x^2 - c^4d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="fricas")

[Out] (b\*c^4 + a\*c^2\*d^2 - (b\*c^2\*d + a\*d^3)\*sqrt(d\*x + c)\*sqrt(d\*x - c)\*x - (b\*c^2\*d^2 + a\*d^4)\*x^2 - (b\*c^2\*d^2\*x^2 - b\*c^4)\*log(-d\*x + sqrt(d\*x + c)\*sqrt(d\*x - c)))/(c^2\*d^5\*x^2 - c^4\*d^3)

**giac [B]** time = 0.28, size = 113, normalized size = 1.79

$$-\frac{b \log\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2\right)}{d^3} - \frac{2(bc^2 + ad^2)}{\left(\left(\sqrt{dx+c} - \sqrt{dx-c}\right)^2 + 2c\right)cd^3} - \frac{(bc^2d^3 + ad^5)\sqrt{dx+c}}{2\sqrt{dx-c}c^2d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="giac")

[Out] -b\*log((sqrt(d\*x + c) - sqrt(d\*x - c))^2)/d^3 - 2\*(b\*c^2 + a\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*c\*d^3) - 1/2\*(b\*c^2\*d^3 + a\*d^5)\*sqrt(d\*x + c)/(sqrt(d\*x - c)\*c^2\*d^6)

**maple [C]** time = 0.07, size = 160, normalized size = 2.54

$$\frac{\left(b c^2 d^2 x^2 \ln\left(\left(dx + \sqrt{(dx-c)(dx+c)}\right) \text{csgn}(d)\right) - \sqrt{d^2 x^2 - c^2} a d^3 x \text{csgn}(d) - b c^4 \ln\left(\left(dx + \sqrt{(dx-c)(dx+c)}\right) \text{csgn}(d)\right)\right)}{\sqrt{d^2 x^2 - c^2} \sqrt{dx+c} \sqrt{dx-c} c^2 d^3}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)
```

```
[Out] (ln((csgn(d)*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*x^2*b*c^2*d^2-(d^2*x^2-c^2)^(1/2)*a*d^3*x*csgn(d)-(d^2*x^2-c^2)^(1/2)*b*c^2*d*x*csgn(d)-ln((csgn(d)*((d*x-c)*(d*x+c))^(1/2)+d*x)*csgn(d))*b*c^4)*csgn(d)/(d^2*x^2-c^2)^(1/2)/c^2/d^3/(d*x+c)^(1/2)/(d*x-c)^(1/2)
```

**maxima** [A] time = 0.55, size = 76, normalized size = 1.21

$$-\frac{ax}{\sqrt{d^2x^2 - c^2}c^2} - \frac{bx}{\sqrt{d^2x^2 - c^2}d^2} + \frac{b \log\left(2d^2x + 2\sqrt{d^2x^2 - c^2}d\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -a*x/(sqrt(d^2*x^2 - c^2)*c^2) - b*x/(sqrt(d^2*x^2 - c^2)*d^2) + b*log(2*d^2*x + 2*sqrt(d^2*x^2 - c^2)*d)/d^3
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{bx^2 + a}{(c + dx)^{3/2}(dx - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/((c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

**sympy** [C] time = 112.36, size = 182, normalized size = 2.89

$$a \left( \frac{{}_G_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 \end{matrix} \middle| \frac{c^2}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^2d} + \frac{{}_iG_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 \\ \frac{1}{4}, \frac{3}{4} \end{matrix} \middle| \frac{c^2e^{2i\pi}}{d^2x^2} \right)}{2\pi^{\frac{3}{2}}c^2d} \right) + b \left( \frac{{}_G_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1, 0 \end{matrix} \middle| -\frac{1}{2} \right)}{2\pi^{\frac{3}{2}}d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] a*(-meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + I*meijerg((-1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**2*d) + b*(meijerg((-1/4, 1/4), (-1/2, 1/2, 1, 1)), ((-1/4, 0, 1/4, 1/2, 1, 0), ()), c**2/(d**2*x**2))/(2*pi**(3/2)*d**3) + I*meijerg((-3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((-3/4, -1/4), (-3/2, -1, 0, 0)), c**2*exp_polar(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*d**3))
```

$$3.373 \quad \int \frac{a+bx^2}{x(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

Optimal. Leaf size=65

$$-\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}} - \frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

[Out]  $-a \arctan((d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c)/c^3 + (-a/c^2 - b/d^2)/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {458, 92, 205}

$$-\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{dx-c}\sqrt{c+dx}} - \frac{a \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{c^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $-\left(\frac{a/c^2 + b/d^2}{\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]}\right) - \left(\frac{a*\text{ArcTan}[\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]]}{c}\right)/c^3$

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] :> Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] :> Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 458

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.)), x\_Symbol] :> -Simp[((b1\*b2\*c - a1\*a2\*d)\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*b1\*b2\*e\*n\*(p + 1)), x] - Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*b1\*b2\*n\*(p + 1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1), x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && LtQ[p, -1] && ((!IntegerQ[p + 1/2] && NeQ[p, -5/4]) || !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m, -(n\*(p + 1))]))

#### Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{x(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= -\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{-c + dx} \sqrt{c + dx}} - \frac{a \int \frac{1}{x\sqrt{-c+dx}\sqrt{c+dx}} dx}{c^2} \\ &= -\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{-c + dx} \sqrt{c + dx}} - \frac{(ad) \text{Subst} \left( \int \frac{1}{c^2 d + dx^2} dx, x, \sqrt{-c + dx} \sqrt{c + dx} \right)}{c^2} \\ &= -\frac{\frac{a}{c^2} + \frac{b}{d^2}}{\sqrt{-c + dx} \sqrt{c + dx}} - \frac{a \tan^{-1} \left( \frac{\sqrt{-c+dx}\sqrt{c+dx}}{c} \right)}{c^3} \end{aligned}$$

**Mathematica [A]** time = 0.04, size = 84, normalized size = 1.29

$$\frac{ad^2 \sqrt{d^2 x^2 - c^2} \tan^{-1} \left( \frac{\sqrt{d^2 x^2 - c^2}}{c} \right) + acd^2 + bc^3}{c^3 d^2 \sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] -((b\*c^3 + a\*c\*d^2 + a\*d^2\*Sqrt[-c^2 + d^2\*x^2]\*ArcTan[Sqrt[-c^2 + d^2\*x^2]/c])/(c^3\*d^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]))

**fricas [A]** time = 1.10, size = 101, normalized size = 1.55

$$\frac{(bc^3 + acd^2)\sqrt{dx + c}\sqrt{dx - c} + 2(ad^4x^2 - ac^2d^2) \arctan\left(-\frac{dx - \sqrt{dx+c}\sqrt{dx-c}}{c}\right)}{c^3 d^4 x^2 - c^5 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] -((b\*c^3 + a\*c\*d^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c) + 2\*(a\*d^4\*x^2 - a\*c^2\*d^2)\*arctan(-(d\*x - sqrt(d\*x + c)\*sqrt(d\*x - c))/c))/(c^3\*d^4\*x^2 - c^5\*d^2)

**giac [B]** time = 0.36, size = 115, normalized size = 1.77

$$\frac{2a \arctan\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{c^3} - \frac{(bc^2 + ad^2)\sqrt{dx + c}}{2\sqrt{dx - c}c^3d^2} + \frac{2(bc^2 + ad^2)}{\left((\sqrt{dx + c} - \sqrt{dx - c})^2 + 2c\right)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out] 2\*a\*arctan(1/2\*(sqrt(d\*x + c) - sqrt(d\*x - c))^2/c)/c^3 - 1/2\*(b\*c^2 + a\*d^2)\*sqrt(d\*x + c)/(sqrt(d\*x - c)\*c^3\*d^2) + 2\*(b\*c^2 + a\*d^2)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*c^2\*d^2)

**maple [B]** time = 0.08, size = 188, normalized size = 2.89

$$\frac{a d^4 x^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - a c^2 d^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2} a d^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2}}{\sqrt{-c^2} \sqrt{d^2 x^2 - c^2} \sqrt{dx + c} \sqrt{dx - c} c^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $\frac{1}{c^2} \left( \ln(-2*(c^2 - (-c^2)^{1/2})*(d^2*x^2 - c^2)^{1/2})/x \right) * x^2 * a * d^4 - \ln(-2*(c^2 - (-c^2)^{1/2})*(d^2*x^2 - c^2)^{1/2})/x * a * c^2 * d^2 - (-c^2)^{1/2} * (d^2*x^2 - c^2)^{1/2} * a * d^2 - b * c^2 * (-c^2)^{1/2} * (d^2*x^2 - c^2)^{1/2} / (-c^2)^{1/2} / (d^2*x^2 - c^2)^{1/2} / d^2 / (d*x+c)^{1/2} / (d*x-c)^{1/2}$

**maxima** [A] time = 1.46, size = 58, normalized size = 0.89

$$\frac{a \arcsin\left(\frac{c}{d|x|}\right)}{c^3} - \frac{a}{\sqrt{d^2x^2 - c^2} c^2} - \frac{b}{\sqrt{d^2x^2 - c^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")`

[Out]  $a * \arcsin(c/(d * \text{abs}(x))) / c^3 - a / (\text{sqrt}(d^2 * x^2 - c^2) * c^2) - b / (\text{sqrt}(d^2 * x^2 - c^2) * d^2)$

**mupad** [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{b x^2 + a}{x (c + d x)^{3/2} (d x - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)`

[Out] `int((a + b*x^2)/(x*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)`

**sympy** [C] time = 136.44, size = 172, normalized size = 2.65

$$a \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{5}{4}, \frac{7}{4}, 1 \\ \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2, \frac{5}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) - i G_{6,6}^{2,6} \left( \begin{matrix} 0, \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, 1 \\ \frac{3}{4}, \frac{5}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^3} \right) + b \left( \frac{G_{6,6}^{5,3} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 \\ \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{2} \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out]  $a * (-\text{meijerg}(((5/4, 7/4, 1), (1, 2, 5/2)), ((5/4, 3/2, 7/4, 2, 5/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c**3) - I * \text{meijerg}(((0, 1/2, 3/4, 1, 5/4, 1), ()), ((3/4, 5/4), (0, 1/2, 3/2, 0)), c**2 * \text{exp\_polar}(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c**3)) + b * (-\text{meijerg}(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), c**2/(d**2*x**2))/(2*pi**(3/2)*c*d**2) - I * \text{meijerg}((-1, -1/2, -1/4, 0, 1/4, 1), ()), ((-1/4, 1/4), (-1, -1/2, 1/2, 0)), c**2 * \text{exp\_polar}(2*I*pi)/(d**2*x**2))/(2*pi**(3/2)*c*d**2))$

$$3.374 \quad \int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

Optimal. Leaf size=67

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

[Out] a/c^2/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)-(2\*a\*d^2+b\*c^2)\*x/c^4/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)

Rubi [A] time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$ , Rules used = {454, 39}

$$\frac{a}{c^2x\sqrt{dx-c}\sqrt{c+dx}} - \frac{x(2ad^2+bc^2)}{c^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^2\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] a/(c^2\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) - ((b\*c^2 + 2\*a\*d^2)\*x)/(c^4\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

Rule 39

Int[1/(((a\_) + (b\_.)\*(x\_))^(3/2)\*((c\_) + (d\_.)\*(x\_))^(3/2)), x\_Symbol] :> Simp[x/(a\*c\*Sqrt[a + b\*x]\*Sqrt[c + d\*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0]

Rule 454

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{a+bx^2}{x^2(-c+dx)^{3/2}(c+dx)^{3/2}} dx &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} + \left(b + \frac{2ad^2}{c^2}\right) \int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx \\ &= \frac{a}{c^2x\sqrt{-c+dx}\sqrt{c+dx}} - \frac{(bc^2+2ad^2)x}{c^4\sqrt{-c+dx}\sqrt{c+dx}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.76

$$\frac{a(c^2 - 2d^2x^2) - bc^2x^2}{c^4x\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^2\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out]  $-(b*c^2*x^2) + a*(c^2 - 2*d^2*x^2)/(c^4*x*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])$

**fricas** [A] time = 1.00, size = 103, normalized size = 1.54

$$\frac{(bc^2d^2 + 2ad^4)x^3 - (ac^2d - (bc^2d + 2ad^3)x^2)\sqrt{dx+c}\sqrt{dx-c} - (bc^4 + 2ac^2d^2)x}{c^4d^3x^3 - c^6dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^2/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out]  $-((b*c^2*d^2 + 2*a*d^4)*x^3 - (a*c^2*d - (b*c^2*d + 2*a*d^3)*x^2)*\text{sqrt}(d*x + c)*\text{sqrt}(d*x - c) - (b*c^4 + 2*a*c^2*d^2)*x)/(c^4*d^3*x^3 - c^6*d*x)$

**giac** [B] time = 0.46, size = 219, normalized size = 3.27

$$\frac{(bc^2 + ad^2)\sqrt{dx+c} - 2\left(bc^2(\sqrt{dx+c} - \sqrt{dx-c})^4 + ad^2(\sqrt{dx+c} - \sqrt{dx-c})^4 + 4acd^2(\sqrt{dx+c} - \sqrt{dx-c})^2\right)}{2\sqrt{dx-c}c^4d \left((\sqrt{dx+c} - \sqrt{dx-c})^6 + 2c(\sqrt{dx+c} - \sqrt{dx-c})^4 + 4c^2(\sqrt{dx+c} - \sqrt{dx-c})^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^2/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out]  $-1/2*(b*c^2 + a*d^2)*\text{sqrt}(d*x + c)/(\text{sqrt}(d*x - c)*c^4*d) - 2*(b*c^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^4 + a*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^4 + 4*a*c*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + 4*b*c^4 + 12*a*c^2*d^2)/(((\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^6 + 2*c*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^4 + 4*c^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + 8*c^3)*c^3*d)$

**maple** [A] time = 0.05, size = 48, normalized size = 0.72

$$\frac{-2ad^2x^2 - bc^2x^2 + ac^2}{\sqrt{dx+c}\sqrt{dx-c}c^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^2/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out]  $(-2*a*d^2*x^2 - b*c^2*x^2 + a*c^2)/(d*x+c)^(1/2)/x/c^4/(d*x-c)^(1/2)$

**maxima** [A] time = 1.36, size = 71, normalized size = 1.06

$$-\frac{bx}{\sqrt{d^2x^2 - c^2}c^2} - \frac{2ad^2x}{\sqrt{d^2x^2 - c^2}c^4} + \frac{a}{\sqrt{d^2x^2 - c^2}c^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^2/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out]  $-b*x/(\text{sqrt}(d^2*x^2 - c^2)*c^2) - 2*a*d^2*x/(\text{sqrt}(d^2*x^2 - c^2)*c^4) + a/(\text{sqrt}(d^2*x^2 - c^2)*c^2*x)$

**mupad** [B] time = 2.87, size = 73, normalized size = 1.09

$$\frac{2ad^2x^2\sqrt{dx-c} - ac^2\sqrt{dx-c} + bc^2x^2\sqrt{dx-c}}{c^4x\sqrt{c+dx}(c-dx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^2\*(c + d\*x)^(3/2)\*(d\*x - c)^(3/2)),x)

[Out]  $(2*a*d^2*x^2*(d*x - c)^{(1/2)} - a*c^2*(d*x - c)^{(1/2)} + b*c^2*x^2*(d*x - c)^{(1/2)})/(c^4*x*(c + d*x)^{(1/2)}*(c - d*x))$

**sympy [C]** time = 136.13, size = 165, normalized size = 2.46

$$a \left( \frac{{}_dG_{6,6}^{5,3} \left( \begin{matrix} \frac{7}{4}, \frac{9}{4}, 1 \\ \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 3 \end{matrix} \middle| \frac{c^2}{d^2 x^2} \right) + {}_{id}G_{6,6}^{2,6} \left( \begin{matrix} \frac{1}{2}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 1 \\ \frac{5}{4}, \frac{7}{4} \end{matrix} \middle| \frac{c^2 e^{2i\pi}}{d^2 x^2} \right)}{2\pi^{\frac{3}{2}} c^4} \right) + b \left( \frac{{}_G_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 \\ \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2 \end{matrix} \middle| \frac{1}{2}, \frac{3}{2}, 2 \right)}{2\pi^{\frac{3}{2}} c^2 d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*2/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out]  $a*(-d*\text{meijerg}((\frac{7}{4}, \frac{9}{4}, 1), (\frac{3}{2}, \frac{5}{2}, 3)), ((\frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}, 3), (0,)), c**2/(d**2*x**2))/(2*\pi**(\frac{3}{2})*c**4) + I*d*\text{meijerg}((\frac{1}{2}, 1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 1), ()), ((\frac{5}{4}, \frac{7}{4}), (\frac{1}{2}, 1, 2, 0)), c**2*\exp\_polar(2*I*\pi)/(d**2*x**2))/(2*\pi**(\frac{3}{2})*c**4) + b*(-\text{meijerg}((\frac{3}{4}, \frac{5}{4}, 1), (\frac{1}{2}, \frac{3}{2}, 2)), ((\frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2}, 2), (0,)), c**2/(d**2*x**2))/(2*\pi**(\frac{3}{2})*c**2*d) + I*\text{meijerg}((-\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1), ()), ((\frac{1}{4}, \frac{3}{4}), (-\frac{1}{2}, 0, 1, 0)), c**2*\exp\_polar(2*I*\pi)/(d**2*x**2))/(2*\pi**(\frac{3}{2})*c**2*d)$

$$3.375 \quad \int \frac{a+bx^2}{x^3(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

Optimal. Leaf size=117

$$-\frac{(3ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5} - \frac{3ad^2 + 2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-1/2*(3*a*d^2+2*b*c^2)*\arctan((d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c)/c^5+1/2*(-3*a*d^2-2*b*c^2)/c^4/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+1/2*a/c^2/x^2/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$ , Rules used = {454, 104, 21, 92, 205}

$$-\frac{3ad^2 + 2bc^2}{2c^4\sqrt{dx-c}\sqrt{c+dx}} - \frac{(3ad^2 + 2bc^2) \tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{2c^5} + \frac{a}{2c^2x^2\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $-(2*b*c^2 + 3*a*d^2)/(2*c^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(2*c^2*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - ((2*b*c^2 + 3*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(2*c^5)$

#### Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

#### Rule 92

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)]\*((e\_) + (f\_)\*(x\_))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 104

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[2\*m, 2\*n, 2\*p]

#### Rule 205

Int[((a\_) + (b\_)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 454

Int[((e\_)\*(x\_))^(m\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_)\*((c\_) + (d\_)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^



$(m + 1) \cdot (a_1 + b_1 x^{n/2})^{p+1} \cdot (a_2 + b_2 x^{n/2})^{p+1} / (a_1 a_2 e^{(m+1)x}) + \text{Dist}[(a_1 a_2 d(m+1) - b_1 b_2 c(m+n(p+1)+1)) / (a_1 a_2 e^{n(m+1)x}), x] + \text{Int}[(e^x)^{m+n} \cdot (a_1 + b_1 x^{n/2})^p \cdot (a_2 + b_2 x^{n/2})^p, x] /;$   
 $\text{FreeQ}\{a_1, b_1, a_2, b_2, c, d, e, p\}, x\} \ \&\& \ \text{EqQ}[\text{non2}, n/2] \ \&\& \ \text{EqQ}[a_2 b_1 + a_1 b_2, 0] \ \&\& \ (\text{IntegerQ}[n] \ || \ \text{GtQ}[e, 0]) \ \&\& \ ((\text{GtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1]) \ || \ (\text{LtQ}[n, 0] \ \&\& \ \text{GtQ}[m+n, -1])) \ \&\& \ !\text{LtQ}[p, -1]$

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{x^3(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= \frac{a}{2c^2 x^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{1}{2} \left( 2b + \frac{3ad^2}{c^2} \right) \int \frac{1}{x(-c + dx)^{3/2}(c + dx)^{3/2}} dx \\ &= -\frac{2bc^2 + 3ad^2}{2c^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{a}{2c^2 x^2 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{\left(-2b - \frac{3ad^2}{c^2}\right) \int \frac{ca}{x\sqrt{-c+dx}\sqrt{c+dx}} dx}{2c^2 d} \\ &= -\frac{2bc^2 + 3ad^2}{2c^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{a}{2c^2 x^2 \sqrt{-c + dx} \sqrt{c + dx}} - \frac{(2bc^2 + 3ad^2) \int \frac{ca}{x\sqrt{-c+dx}\sqrt{c+dx}} dx}{2c^4} \\ &= -\frac{2bc^2 + 3ad^2}{2c^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{a}{2c^2 x^2 \sqrt{-c + dx} \sqrt{c + dx}} - \frac{(d(2bc^2 + 3ad^2)) \text{Subst}\left(\int \frac{ca}{x\sqrt{-c+dx}\sqrt{c+dx}} dx\right)}{2c^4} \\ &= -\frac{2bc^2 + 3ad^2}{2c^4 \sqrt{-c + dx} \sqrt{c + dx}} + \frac{a}{2c^2 x^2 \sqrt{-c + dx} \sqrt{c + dx}} - \frac{(2bc^2 + 3ad^2) \tan^{-1}\left(\frac{d\sqrt{-c+dx}\sqrt{c+dx}}{c}\right)}{2c^5} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 75, normalized size = 0.64

$$\frac{ac^2 - x^2(3ad^2 + 2bc^2) {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1 - \frac{d^2 x^2}{c^2}\right)}{2c^4 x^2 \sqrt{dx - c} \sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^3\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] (a\*c^2 - (2\*b\*c^2 + 3\*a\*d^2)\*x^2\*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (d^2\*x^2)/c^2])/(2\*c^4\*x^2\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 1.07, size = 138, normalized size = 1.18

$$\frac{(ac^3 - (2bc^3 + 3acd^2)x^2)\sqrt{dx + c}\sqrt{dx - c} - 2((2bc^2d^2 + 3ad^4)x^4 - (2bc^4 + 3ac^2d^2)x^2) \arctan\left(-\frac{dx - \sqrt{dx+c}}{c}\right)}{2(c^5d^2x^4 - c^7x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="fricas")

[Out] 1/2\*((a\*c^3 - (2\*b\*c^3 + 3\*a\*c\*d^2)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c) - 2\*((2\*b\*c^2\*d^2 + 3\*a\*d^4)\*x^4 - (2\*b\*c^4 + 3\*a\*c^2\*d^2)\*x^2)\*arctan(-(d\*x - sqrt(d\*x + c)\*sqrt(d\*x - c))/c))/(c^5\*d^2\*x^4 - c^7\*x^2)

**giac [B]** time = 0.54, size = 211, normalized size = 1.80

$$\frac{(2bc^2 + 3ad^2) \arctan\left(\frac{(\sqrt{dx+c} - \sqrt{dx-c})^2}{2c}\right)}{c^5} - \frac{(bc^2 + ad^2)\sqrt{dx + c}}{2\sqrt{dx - c}c^5} + \frac{2(bc^2 + ad^2)}{\left((\sqrt{dx + c} - \sqrt{dx - c})^2 + 2c\right)c^4} + \frac{2(ad^2(\sqrt{dx + c} - \sqrt{dx - c}))}{c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="giac")

[Out]  $(2*b*c^2 + 3*a*d^2)*\arctan(1/2*(\sqrt{d*x + c} - \sqrt{d*x - c})^2/c)/c^5 - 1/2*(b*c^2 + a*d^2)*\sqrt{d*x + c}/(\sqrt{d*x - c}*c^5) + 2*(b*c^2 + a*d^2)/((\sqrt{d*x + c} - \sqrt{d*x - c})^2 + 2*c)*c^4 + 2*(a*d^2*(\sqrt{d*x + c} - \sqrt{d*x - c}))^6 - 4*a*c^2*d^2*(\sqrt{d*x + c} - \sqrt{d*x - c})^2)/(((\sqrt{d*x + c} - \sqrt{d*x - c})^4 + 4*c^2)^2*c^4)$

**maple [B]** time = 0.09, size = 315, normalized size = 2.69

$$\frac{3a d^4 x^4 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) + 2b c^2 d^2 x^4 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - 3a c^2 d^2 x^2 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - 2b c^2}{2\sqrt{-c^2} \sqrt{d^2 x^2 - c^2} \sqrt{d^2 x^2 - c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)/x^3/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x)

[Out]  $1/2/c^4*(3*a*d^4*x^4*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)+2*b*c^2*d^2*x^4*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)-3*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*a*c^2*d^2-2*\ln(-2*(c^2-(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)})/x)*x^2*b*c^4-3*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*a*d^2*x^2-2*(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*b*c^2*x^2+(-c^2)^{(1/2)}*(d^2*x^2-c^2)^{(1/2)}*a*c^2)/(-c^2)^{(1/2)}/x^2/(d^2*x^2-c^2)^{(1/2)}/(d*x+c)^{(1/2)}/(d*x-c)^{(1/2)}$

**maxima [A]** time = 1.26, size = 104, normalized size = 0.89

$$\frac{b \arcsin\left(\frac{c}{d|x|}\right)}{c^3} + \frac{3 a d^2 \arcsin\left(\frac{c}{d|x|}\right)}{2 c^5} - \frac{b}{\sqrt{d^2 x^2 - c^2} c^2} - \frac{3 a d^2}{2 \sqrt{d^2 x^2 - c^2} c^4} + \frac{a}{2 \sqrt{d^2 x^2 - c^2} c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^3/(d\*x-c)^(3/2)/(d\*x+c)^(3/2),x, algorithm="maxima")

[Out]  $b*\arcsin(c/(d*\text{abs}(x)))/c^3 + 3/2*a*d^2*\arcsin(c/(d*\text{abs}(x)))/c^5 - b/(\sqrt{d^2*x^2 - c^2})*c^2 - 3/2*a*d^2/(\sqrt{d^2*x^2 - c^2})*c^4 + 1/2*a/(\sqrt{d^2*x^2 - c^2})*c^2*x^2)$

**mupad [F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b x^2 + a}{x^3 (c + d x)^{3/2} (d x - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^2)/(x^3\*(c + d\*x)^(3/2)\*(d\*x - c)^(3/2)),x)

[Out] int((a + b\*x^2)/(x^3\*(c + d\*x)^(3/2)\*(d\*x - c)^(3/2)), x)

**sympy [F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)/x\*\*3/(d\*x-c)\*\*(3/2)/(d\*x+c)\*\*(3/2),x)

[Out] Timed out

$$3.376 \quad \int \frac{a+bx^2}{x^4(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

Optimal. Leaf size=119

$$-\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

[Out] 1/3\*a/c^2/x^3/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)+1/3\*(4\*a\*d^2+3\*b\*c^2)/c^4/x/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)-2/3\*d^2\*(4\*a\*d^2+3\*b\*c^2)\*x/c^6/(d\*x-c)^(1/2)/(d\*x+c)^(1/2)

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$ , Rules used = {454, 103, 12, 39}

$$-\frac{2d^2x(4ad^2+3bc^2)}{3c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{4ad^2+3bc^2}{3c^4x\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{3c^2x^3\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^4\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)),x]

[Out] a/(3\*c^2\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) + (3\*b\*c^2 + 4\*a\*d^2)/(3\*c^4\*x\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x]) - (2\*d^2\*(3\*b\*c^2 + 4\*a\*d^2)\*x)/(3\*c^6\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

#### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_) /; FreeQ[b, x]]

#### Rule 39

Int[1/(((a\_) + (b\_.)\*(x\_))^(3/2)\*((c\_) + (d\_.)\*(x\_))^(3/2)), x\_Symbol] := Simp[x/(a\*c\*Sqrt[a + b\*x]\*Sqrt[c + d\*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b\*c + a\*d, 0]

#### Rule 103

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_)\*((e\_.) + (f\_.)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

#### Rule 454

Int[((e\_.)\*(x\_))^(m\_)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_)\*((c\_) + (d\_.)\*(x\_)^(n\_)), x\_Symbol] := Simp[(c\*(e\*x)^(m + 1)\*(a1 + b1\*x^(n/2))^(p + 1)\*(a2 + b2\*x^(n/2))^(p + 1))/(a1\*a2\*e^(m + 1)), x] + Dist[(a1\*a2\*d\*(m + 1) - b1\*b2\*c\*(m + n\*(p + 1) + 1))/(a1\*a2\*e^n\*(m + 1)), Int[(e\*x)^(m + n)\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^2}{x^4(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= \frac{a}{3c^2x^3\sqrt{-c + dx}\sqrt{c + dx}} + \frac{1}{3} \left( 3b + \frac{4ad^2}{c^2} \right) \int \frac{1}{x^2(-c + dx)^{3/2}(c + dx)^{3/2}} dx \\
&= \frac{a}{3c^2x^3\sqrt{-c + dx}\sqrt{c + dx}} + \frac{3bc^2 + 4ad^2}{3c^4x\sqrt{-c + dx}\sqrt{c + dx}} + \frac{\left( 3b + \frac{4ad^2}{c^2} \right) \int \frac{2d^2}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c + dx}\sqrt{c + dx}} + \frac{3bc^2 + 4ad^2}{3c^4x\sqrt{-c + dx}\sqrt{c + dx}} + \frac{\left( 2d^2 \left( 3b + \frac{4ad^2}{c^2} \right) \right) \int \frac{1}{(-c+dx)^{3/2}(c+dx)^{3/2}} dx}{3c^2} \\
&= \frac{a}{3c^2x^3\sqrt{-c + dx}\sqrt{c + dx}} + \frac{3bc^2 + 4ad^2}{3c^4x\sqrt{-c + dx}\sqrt{c + dx}} - \frac{2d^2(3bc^2 + 4ad^2)x}{3c^6\sqrt{-c + dx}\sqrt{c + dx}}
\end{aligned}$$

**Mathematica [A]** time = 0.03, size = 77, normalized size = 0.65

$$\frac{a(c^4 + 4c^2d^2x^2 - 8d^4x^4) + 3bc^2x^2(c^2 - 2d^2x^2)}{3c^6x^3\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x^2)/(x^4\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out] (3\*b\*c^2\*x^2\*(c^2 - 2\*d^2\*x^2) + a\*(c^4 + 4\*c^2\*d^2\*x^2 - 8\*d^4\*x^4))/(3\*c^6\*x^3\*Sqrt[-c + d\*x]\*Sqrt[c + d\*x])

**fricas [A]** time = 0.88, size = 132, normalized size = 1.11

$$\frac{2(3bc^2d^3 + 4ad^5)x^5 - 2(3bc^4d + 4ac^2d^3)x^3 - (ac^4 - 2(3bc^2d^2 + 4ad^4)x^4 + (3bc^4 + 4ac^2d^2)x^2)\sqrt{dx + c}\sqrt{dx - c}}{3(c^6d^2x^5 - c^8x^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="fricas")

[Out] -1/3\*(2\*(3\*b\*c^2\*d^3 + 4\*a\*d^5)\*x^5 - 2\*(3\*b\*c^4\*d + 4\*a\*c^2\*d^3)\*x^3 - (a\*c^4 - 2\*(3\*b\*c^2\*d^2 + 4\*a\*d^4)\*x^4 + (3\*b\*c^4 + 4\*a\*c^2\*d^2)\*x^2)\*sqrt(d\*x + c)\*sqrt(d\*x - c)/(c^6\*d^2\*x^5 - c^8\*x^3)

**giac [B]** time = 0.73, size = 242, normalized size = 2.03

$$\frac{\frac{(bc^2d + ad^3)\sqrt{dx + c}}{2\sqrt{dx - c}c^6} - \frac{2(bc^2d + ad^3)}{\left(\left(\sqrt{dx + c} - \sqrt{dx - c}\right)^2 + 2c\right)c^5} + \frac{8\left(3bc^2d(\sqrt{dx + c} - \sqrt{dx - c})^8 + 3ad^3(\sqrt{dx + c} - \sqrt{dx - c})^8\right)}{3c^6}}{3c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2+a)/x^4/(d\*x-c)^(3/2)/(d\*x+c)^(3/2), x, algorithm="giac")

[Out] -1/2\*(b\*c^2\*d + a\*d^3)\*sqrt(d\*x + c)/(sqrt(d\*x - c)\*c^6) - 2\*(b\*c^2\*d + a\*d^3)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^2 + 2\*c)\*c^5) - 8/3\*(3\*b\*c^2\*d\*(sqrt(d\*x + c) - sqrt(d\*x - c))^8 + 3\*a\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^8 + 2\*4\*b\*c^4\*d\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 48\*a\*c^2\*d^3\*(sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 48\*b\*c^6\*d + 80\*a\*c^4\*d^3)/(((sqrt(d\*x + c) - sqrt(d\*x - c))^4 + 4\*c^2)^3\*c^4)

**maple [A]** time = 0.05, size = 73, normalized size = 0.61

$$\frac{-8ad^4x^4 - 6bc^2d^2x^4 + 4ac^2d^2x^2 + 3bc^4x^2 + ac^4}{3\sqrt{dx + c}\sqrt{dx - c}c^6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $1/3*(-8*a*d^4*x^4-6*b*c^2*d^2*x^4+4*a*c^2*d^2*x^2+3*b*c^4*x^2+a*c^4)/(d*x+c)^{(1/2)}/x^3/c^6/(d*x-c)^{(1/2)}$

**maxima** [A] time = 1.34, size = 125, normalized size = 1.05

$$-\frac{2bd^2x}{\sqrt{d^2x^2-c^2}c^4} - \frac{8ad^4x}{3\sqrt{d^2x^2-c^2}c^6} + \frac{b}{\sqrt{d^2x^2-c^2}c^2x} + \frac{4ad^2}{3\sqrt{d^2x^2-c^2}c^4x} + \frac{a}{3\sqrt{d^2x^2-c^2}c^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^4/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")`

[Out]  $-2*b*d^2*x/(\sqrt{d^2*x^2-c^2}*c^4) - 8/3*a*d^4*x/(\sqrt{d^2*x^2-c^2}*c^6) + b/(\sqrt{d^2*x^2-c^2}*c^2*x) + 4/3*a*d^2/(\sqrt{d^2*x^2-c^2}*c^4*x) + 1/3*a/(\sqrt{d^2*x^2-c^2}*c^2*x^3)$

**mupad** [B] time = 2.90, size = 104, normalized size = 0.87

$$\frac{\sqrt{dx-c} \left( \frac{a}{3c^2d} + \frac{x^2(3bc^4+4ac^2d^2)}{3c^6d} - \frac{x^4(6bc^2d^2+8ad^4)}{3c^6d} \right)}{x^4\sqrt{c+dx} - \frac{cx^3\sqrt{c+dx}}{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^2)/(x^4*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)`

[Out]  $((d*x - c)^{(1/2)}*(a/(3*c^2*d) + (x^2*(3*b*c^4 + 4*a*c^2*d^2))/(3*c^6*d) - (x^4*(8*a*d^4 + 6*b*c^2*d^2))/(3*c^6*d)))/(x^4*(c + d*x)^{(1/2)} - (c*x^3*(c + d*x)^{(1/2)})/d)$

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)/x**4/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)`

[Out] Timed out

$$3.377 \quad \int \frac{a+bx^2}{x^5(-c+dx)^{3/2}(c+dx)^{3/2}} dx$$

**Optimal.** Leaf size=166

$$-\frac{3d^2(5ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7} - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\sqrt{c+dx}} + \frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}} + \frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

[Out]  $-3/8*d^2*(5*a*d^2+4*b*c^2)*\arctan((d*x-c)^{(1/2)}*(d*x+c)^{(1/2)}/c)/c^7-3/8*d^2*(5*a*d^2+4*b*c^2)/c^6/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+1/4*a/c^2/x^4/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}+1/8*(5*a*d^2+4*b*c^2)/c^4/x^2/(d*x-c)^{(1/2)}/(d*x+c)^{(1/2)}$

**Rubi [A]** time = 0.12, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$ , Rules used = {454, 103, 12, 104, 21, 92, 205}

$$\frac{5ad^2+4bc^2}{8c^4x^2\sqrt{dx-c}\sqrt{c+dx}} - \frac{3d^2(5ad^2+4bc^2)}{8c^6\sqrt{dx-c}\sqrt{c+dx}} - \frac{3d^2(5ad^2+4bc^2)\tan^{-1}\left(\frac{\sqrt{dx-c}\sqrt{c+dx}}{c}\right)}{8c^7} + \frac{a}{4c^2x^4\sqrt{dx-c}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)/(x^5\*(-c + d\*x)^(3/2)\*(c + d\*x)^(3/2)), x]

[Out]  $(-3*d^2*(4*b*c^2 + 5*a*d^2))/(8*c^6*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + a/(4*c^2*x^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) + (4*b*c^2 + 5*a*d^2)/(8*c^4*x^2*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x]) - (3*d^2*(4*b*c^2 + 5*a*d^2)*\text{ArcTan}[(\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])/c])/(8*c^7)$

### Rule 12

Int[(a\_)\*(u\_), x\_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b\_)\*(v\_)] /; FreeQ[b, x]

### Rule 21

Int[(u\_)\*((a\_) + (b\_)\*(v\_))^(m\_)\*((c\_) + (d\_)\*(v\_))^(n\_), x\_Symbol] := Dist[(b/d)^m, Int[u\*(c + d\*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b\*c - a\*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d\*x, a + b\*x])

### Rule 92

Int[1/(Sqrt[(a\_) + (b\_)\*(x\_)]\*Sqrt[(c\_) + (d\_)\*(x\_)]\*((e\_) + (f\_)\*(x\_))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

### Rule 103

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_)\*((e\_) + (f\_)\*(x\_))^(p\_), x\_Symbol] := Simp[(b\*(a + b\*x)^(m + 1)\*(c + d\*x)^(n + 1)\*(e + f\*x)^(p + 1))/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), x] + Dist[1/((m + 1)\*(b\*c - a\*d)\*(b\*e - a\*f)), Int[(a + b\*x)^(m + 1)\*(c + d\*x)^n\*(e + f\*x)^p\*Simp[a\*d\*f\*(m + 1) - b\*(d\*e\*(m + n + 2) + c\*f\*(m + p + 2)) - b\*d\*f\*(m + n + p + 3)\*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegerQ[m] && (IntegerQ[n] || IntegersQ[2\*n, 2\*p])

### Rule 104

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && LtQ[m, -1] && IntegersQ[2*m, 2*n, 2*p]
```

### Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

### Rule 454

```
Int[((e_.)*(x_))^(m_)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a1 + b1*x^(n/2))^(p + 1)*(a2 + b2*x^(n/2))^(p + 1))/((a1*a2*e^(m + 1)), x] + Dist[(a1*a2*d*(m + 1) - b1*b2*c*(m + n*(p + 1) + 1))/((a1*a2*e^n*(m + 1)), Int[(e*x)^(m + n)*(a1 + b1*x^(n/2))^p*(a2 + b2*x^(n/2))^p, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, p}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

### Rubi steps

$$\begin{aligned} \int \frac{a + bx^2}{x^5(-c + dx)^{3/2}(c + dx)^{3/2}} dx &= \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{1}{4} \left( 4b + \frac{5ad^2}{c^2} \right) \int \frac{1}{x^3(-c + dx)^{3/2}(c + dx)^{3/2}} dx \\ &= \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{4bc^2 + 5ad^2}{8c^4x^2\sqrt{-c + dx}\sqrt{c + dx}} + \frac{(4bc^2 + 5ad^2) \int \frac{1}{x(-c + dx)^{3/2}(c + dx)^{3/2}} dx}{8c^4} \\ &= \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{4bc^2 + 5ad^2}{8c^4x^2\sqrt{-c + dx}\sqrt{c + dx}} + \frac{(3d^2(4bc^2 + 5ad^2)) \int \frac{1}{x(-c + dx)^{3/2}(c + dx)^{3/2}} dx}{8c^4} \\ &= -\frac{3d^2(4bc^2 + 5ad^2)}{8c^6\sqrt{-c + dx}\sqrt{c + dx}} + \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{4bc^2 + 5ad^2}{8c^4x^2\sqrt{-c + dx}\sqrt{c + dx}} \\ &= -\frac{3d^2(4bc^2 + 5ad^2)}{8c^6\sqrt{-c + dx}\sqrt{c + dx}} + \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{4bc^2 + 5ad^2}{8c^4x^2\sqrt{-c + dx}\sqrt{c + dx}} \\ &= -\frac{3d^2(4bc^2 + 5ad^2)}{8c^6\sqrt{-c + dx}\sqrt{c + dx}} + \frac{a}{4c^2x^4\sqrt{-c + dx}\sqrt{c + dx}} + \frac{4bc^2 + 5ad^2}{8c^4x^2\sqrt{-c + dx}\sqrt{c + dx}} \end{aligned}$$

**Mathematica [C]** time = 0.03, size = 78, normalized size = 0.47

$$\frac{ac^4 - d^2x^4(5ad^2 + 4bc^2) {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; 1 - \frac{d^2x^2}{c^2}\right)}{4c^6x^4\sqrt{dx - c}\sqrt{c + dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2)/(x^5*(-c + d*x)^(3/2)*(c + d*x)^(3/2)),x]
```

[Out]  $(a*c^4 - d^2*(4*b*c^2 + 5*a*d^2))*x^4*\text{Hypergeometric2F1}[-1/2, 2, 1/2, 1 - (d^2*x^2)/c^2]/(4*c^6*x^4*\text{Sqrt}[-c + d*x]*\text{Sqrt}[c + d*x])$

**fricas** [A] time = 0.69, size = 165, normalized size = 0.99

$$\frac{(2ac^5 - 3(4bc^3d^2 + 5acd^4)x^4 + (4bc^5 + 5ac^3d^2)x^2)\sqrt{dx+c}\sqrt{dx-c} - 6((4bc^2d^4 + 5ad^6)x^6 - (4bc^4d^2 + 5ac^2d^2)x^4)}{8(c^7d^2x^6 - c^9x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="fricas")`

[Out]  $1/8*((2*a*c^5 - 3*(4*b*c^3*d^2 + 5*a*c*d^4))*x^4 + (4*b*c^5 + 5*a*c^3*d^2)*x^2)*\text{sqrt}(d*x + c)*\text{sqrt}(d*x - c) - 6*((4*b*c^2*d^4 + 5*a*d^6)*x^6 - (4*b*c^4*d^2 + 5*a*c^2*d^4)*x^4)*\text{arctan}(-(d*x - \text{sqrt}(d*x + c))*\text{sqrt}(d*x - c))/c)/c^7*d^2*x^6 - c^9*x^4$

**giac** [B] time = 0.79, size = 402, normalized size = 2.42

$$\frac{3(4bc^2d^2 + 5ad^4)\text{arctan}\left(\frac{(\sqrt{dx+c}-\sqrt{dx-c})^2}{2c}\right)}{4c^7} - \frac{(bc^2d^2 + ad^4)\sqrt{dx+c}}{2\sqrt{dx-c}c^7} + \frac{2(bc^2d^2 + ad^4)}{((\sqrt{dx+c}-\sqrt{dx-c})^2 + 2c)c^6} + \frac{4bc^2d^2}{c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="giac")`

[Out]  $3/4*(4*b*c^2*d^2 + 5*a*d^4)*\text{arctan}(1/2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2/c)/c^7 - 1/2*(b*c^2*d^2 + a*d^4)*\text{sqrt}(d*x + c)/(\text{sqrt}(d*x - c)*c^7) + 2*(b*c^2*d^2 + a*d^4)/(((\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 + 2*c)*c^6) + 1/2*(4*b*c^2*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^14 + 7*a*d^4*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^14 + 16*b*c^4*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^10 + 60*a*c^2*d^4*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^10 - 64*b*c^6*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^6 - 240*a*c^4*d^4*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^6 - 256*b*c^8*d^2*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2 - 448*a*c^6*d^4*(\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^2)/(((\text{sqrt}(d*x + c) - \text{sqrt}(d*x - c))^4 + 4*c^2)^4*c^6)$

**maple** [B] time = 0.08, size = 387, normalized size = 2.33

$$\frac{15a d^6 x^6 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) + 12b c^2 d^4 x^6 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right) - 15a c^2 d^4 x^4 \ln\left(-\frac{2(c^2 - \sqrt{-c^2} \sqrt{d^2 x^2 - c^2})}{x}\right)}{c^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x)`

[Out]  $1/8/c^6*(15*\ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*a*d^6+12*\ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^6*b*c^2*d^4-15*\ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*a*c^2*d^4-12*\ln(-2*(c^2-(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2))/x)*x^4*b*c^4*d^2-15*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*a*d^4-12*(-c^2)^(1/2)*(d^2*x^2-c^2)^(1/2)*x^4*b*c^2*d^2+5*x^2*a*c^2*d^2*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+4*x^2*b*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2)+2*a*c^4*(d^2*x^2-c^2)^(1/2)*(-c^2)^(1/2))/(-c^2)^(1/2)/x^4/(d^2*x^2-c^2)^(1/2)/(d*x+c)^(1/2)/(d*x-c)^(1/2)$

**maxima** [A] time = 1.48, size = 162, normalized size = 0.98

$$\frac{3bd^2 \arcsin\left(\frac{c}{d|x|}\right)}{2c^5} + \frac{15ad^4 \arcsin\left(\frac{c}{d|x|}\right)}{8c^7} - \frac{3bd^2}{2\sqrt{d^2x^2-c^2}c^4} - \frac{15ad^4}{8\sqrt{d^2x^2-c^2}c^6} + \frac{b}{2\sqrt{d^2x^2-c^2}c^2x^2} + \frac{5ad^2}{8\sqrt{d^2x^2-c^2}c^4x^2}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2+a)/x^5/(d*x-c)^(3/2)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*b*d^2*arcsin(c/(d*abs(x)))/c^5 + 15/8*a*d^4*arcsin(c/(d*abs(x)))/c^7 -
3/2*b*d^2/(sqrt(d^2*x^2 - c^2)*c^4) - 15/8*a*d^4/(sqrt(d^2*x^2 - c^2)*c^6)
+ 1/2*b/(sqrt(d^2*x^2 - c^2)*c^2*x^2) + 5/8*a*d^2/(sqrt(d^2*x^2 - c^2)*c^4*
x^2) + 1/4*a/(sqrt(d^2*x^2 - c^2)*c^2*x^4)
```

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{bx^2 + a}{x^5 (c + dx)^{3/2} (dx - c)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)),x)
```

```
[Out] int((a + b*x^2)/(x^5*(c + d*x)^(3/2)*(d*x - c)^(3/2)), x)
```

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x**2+a)/x**5/(d*x-c)**(3/2)/(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

$$3.378 \quad \int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx$$

Optimal. Leaf size=40

$$\sqrt{cx-1}\sqrt{cx+1} + \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

[Out] arctan((c\*x-1)^(1/2)\*(c\*x+1)^(1/2))+(c\*x-1)^(1/2)\*(c\*x+1)^(1/2)

**Rubi [A]** time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {460, 92, 205}

$$\sqrt{cx-1}\sqrt{cx+1} + \tan^{-1}\left(\sqrt{cx-1}\sqrt{cx+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + c^2\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]), x]

[Out] Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x] + ArcTan[Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]]

#### Rule 92

Int[1/(Sqrt[(a\_.) + (b\_.)\*(x\_.)]\*Sqrt[(c\_.) + (d\_.)\*(x\_.)]\*((e\_.) + (f\_.)\*(x\_.))), x\_Symbol] := Dist[b\*f, Subst[Int[1/(d\*(b\*e - a\*f)^2 + b\*f^2\*x^2), x], x, Sqrt[a + b\*x]\*Sqrt[c + d\*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2\*b\*d\*e - f\*(b\*c + a\*d), 0]

#### Rule 205

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(Rt[a/b, 2]\*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

#### Rule 460

Int[((e\_.)\*(x\_.))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_.)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_.)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_.)^(n\_.)), x\_Symbol] := Simp[(d\*(e\*x)^(m+1)\*(a1 + b1\*x^(n/2))^(p+1)\*(a2 + b2\*x^(n/2))^(q+1))/(b1\*b2\*e\*(m+n\*(p+1)+1)), x] - Dist[(a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1))/(b1\*b2\*(m+n\*(p+1)+1)), Int[(e\*x)^m\*(a1 + b1\*x^(n/2))^p\*(a2 + b2\*x^(n/2))^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && NeQ[m + n\*(p+1) + 1, 0]

#### Rubi steps

$$\begin{aligned} \int \frac{1+c^2x^2}{x\sqrt{-1+cx}\sqrt{1+cx}} dx &= \sqrt{-1+cx}\sqrt{1+cx} + \int \frac{1}{x\sqrt{-1+cx}\sqrt{1+cx}} dx \\ &= \sqrt{-1+cx}\sqrt{1+cx} + c \operatorname{Subst}\left(\int \frac{1}{c+cx^2} dx, x, \sqrt{-1+cx}\sqrt{1+cx}\right) \\ &= \sqrt{-1+cx}\sqrt{1+cx} + \tan^{-1}\left(\sqrt{-1+cx}\sqrt{1+cx}\right) \end{aligned}$$

**Mathematica [A]** time = 0.03, size = 56, normalized size = 1.40

$$\frac{c^2x^2 + \sqrt{c^2x^2 - 1} \tan^{-1}\left(\sqrt{c^2x^2 - 1}\right) - 1}{\sqrt{cx-1}\sqrt{cx+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + c^2\*x^2)/(x\*Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x]),x]

[Out] (-1 + c^2\*x^2 + Sqrt[-1 + c^2\*x^2]\*ArcTan[Sqrt[-1 + c^2\*x^2]])/(Sqrt[-1 + c\*x]\*Sqrt[1 + c\*x])

**fricas** [A] time = 1.24, size = 39, normalized size = 0.98

$$\sqrt{cx+1}\sqrt{cx-1} + 2 \arctan\left(-cx + \sqrt{cx+1}\sqrt{cx-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2\*x^2+1)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(c\*x + 1)\*sqrt(c\*x - 1) + 2\*arctan(-c\*x + sqrt(c\*x + 1)\*sqrt(c\*x - 1))

**giac** [A] time = 0.17, size = 40, normalized size = 1.00

$$\sqrt{cx+1}\sqrt{cx-1} - 2 \arctan\left(\frac{1}{2}\left(\sqrt{cx+1} - \sqrt{cx-1}\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2\*x^2+1)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="giac")

[Out] sqrt(c\*x + 1)\*sqrt(c\*x - 1) - 2\*arctan(1/2\*(sqrt(c\*x + 1) - sqrt(c\*x - 1))^2)

**maple** [A] time = 0.07, size = 53, normalized size = 1.32

$$\frac{\sqrt{cx-1}\sqrt{cx+1}\left(-\arctan\left(\frac{1}{\sqrt{c^2x^2-1}}\right) + \sqrt{c^2x^2-1}\right)}{\sqrt{c^2x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2\*x^2+1)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x)

[Out] (c\*x-1)^(1/2)\*(c\*x+1)^(1/2)/(c^2\*x^2-1)^(1/2)\*((c^2\*x^2-1)^(1/2)-arctan(1/(c^2\*x^2-1)^(1/2)))

**maxima** [A] time = 1.35, size = 23, normalized size = 0.58

$$\sqrt{c^2x^2-1} - \arcsin\left(\frac{1}{c|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2\*x^2+1)/x/(c\*x-1)^(1/2)/(c\*x+1)^(1/2),x, algorithm="maxima")

[Out] sqrt(c^2\*x^2 - 1) - arcsin(1/(c\*abs(x)))

**mupad** [B] time = 3.65, size = 72, normalized size = 1.80

$$\sqrt{cx-1}\sqrt{cx+1} - \ln\left(\frac{(\sqrt{cx-1}-i)^2}{(\sqrt{cx+1}-1)^2} + 1\right) \text{li} + \ln\left(\frac{\sqrt{cx-1}-i}{\sqrt{cx+1}-1}\right) \text{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c^2\*x^2 + 1)/(x\*(c\*x - 1)^(1/2)\*(c\*x + 1)^(1/2)),x)

[Out]  $\log(((c*x - 1)^{(1/2)} - 1i)/((c*x + 1)^{(1/2)} - 1))*1i - \log(((c*x - 1)^{(1/2)} - 1i)^2/((c*x + 1)^{(1/2)} - 1)^2 + 1)*1i + (c*x - 1)^{(1/2)}*(c*x + 1)^{(1/2)}$

**sympy [C]** time = 30.11, size = 148, normalized size = 3.70

$$\frac{G_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} & 0, 0, \frac{1}{2}, 1 \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) G_{6,6}^{5,3} \left( \begin{matrix} \frac{3}{4}, \frac{5}{4}, 1 & 1, 1, \frac{3}{2} \\ \frac{1}{2}, \frac{3}{4}, 1, \frac{5}{4}, \frac{3}{2} & 0 \end{matrix} \middle| \frac{1}{c^2 x^2} \right) + i G_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} & -1, -\frac{1}{2}, -\frac{1}{2}, 0 \end{matrix} \right)}{4\pi^{\frac{3}{2}} - 4\pi^{\frac{3}{2}} + 4\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c**2*x**2+1)/x/(c*x-1)**(1/2)/(c*x+1)**(1/2),x)`

[Out] `meijerg((-1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(c**2*x**2))/(4*pi**(3/2)) - meijerg((3/4, 5/4, 1), (1, 1, 3/2)), ((1/2, 3/4, 1, 5/4, 3/2), (0,)), 1/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg((-1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2)) + I*meijerg((0, 1/4, 1/2, 3/4, 1, 1), ()), ((1/4, 3/4), (0, 1/2, 1/2, 0)), exp_polar(2*I*pi)/(c**2*x**2))/(4*pi**(3/2))`

$$3.379 \quad \int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}} (c+dx^2)}{\sqrt{-a+bx} \sqrt{a+bx}} dx$$

**Optimal.** Leaf size=53

$$\sqrt{bx-a} \sqrt{a+bx} \left( \frac{c}{a^2} + \frac{d}{b^2} \right) x^{-\frac{b^2c}{a^2d+b^2c}}$$

[Out] (c/a^2+d/b^2)\*(b\*x-a)^(1/2)\*(b\*x+a)^(1/2)/(x^(b^2\*c/(a^2\*d+b^2\*c)))

**Rubi [A]** time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$ , Rules used = {450}

$$\sqrt{bx-a} \sqrt{a+bx} \left( \frac{c}{a^2} + \frac{d}{b^2} \right) x^{-\frac{b^2c}{a^2d+b^2c}}$$

Antiderivative was successfully verified.

[In] Int[(c + d\*x^2)/(x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x]), x]

[Out] ((c/a^2 + d/b^2)\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x])/x^((b^2\*c)/(b^2\*c + a^2\*d))

**Rule 450**

Int[((e\_.)\*(x\_))^(m\_.)\*((a1\_.) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_.) + (b2\_.)\*(x\_)^(non2\_.))^(q\_.)\*((c\_.) + (d\_.)\*(x\_)^(n\_.)), x\_Symbol] :> Simp[(c\*(e\*x)^(m+1)\*(a1+b1\*x^(n/2))^(p+1)\*(a2+b2\*x^(n/2))^(q+1))/(a1\*a2\*e\*(m+1)), x] /; FreeQ[{a1, b1, a2, b2, c, d, e, m, n, p}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && EqQ[a1\*a2\*d\*(m+1) - b1\*b2\*c\*(m+n\*(p+1)+1), 0] && NeQ[m, -1]

**Rubi steps**

$$\int \frac{x^{-\frac{2b^2c+a^2d}{b^2c+a^2d}} (c+dx^2)}{\sqrt{-a+bx} \sqrt{a+bx}} dx = \left( \frac{c}{a^2} + \frac{d}{b^2} \right) x^{-\frac{b^2c}{b^2c+a^2d}} \sqrt{-a+bx} \sqrt{a+bx}$$

**Mathematica [C]** time = 0.31, size = 244, normalized size = 4.60

$$\frac{\sqrt{1 - \frac{b^2x^2}{a^2}} (a^2d + b^2c) x^{-\frac{b^2c}{a^2d+b^2c}} \left( b^2dx^2 {}_2F_1\left(\frac{1}{2}, \frac{2da^2+b^2c}{2da^2+2b^2c}; \frac{4da^2+3b^2c}{2da^2+2b^2c}; \frac{b^2x^2}{a^2}\right) - (2a^2d + b^2c) {}_2F_1\left(\frac{1}{2}, -\frac{b^2c}{2(da^2+b^2c)}; \frac{2da^2+b^2c}{2da^2+b^2c}\right) \right)}{b^2 \sqrt{bx-a} \sqrt{a+bx} (2a^2d + b^2c)}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d\*x^2)/(x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x]), x]

[Out] ((b^2\*c + a^2\*d)\*Sqrt[1 - (b^2\*x^2)/a^2]\*(-(b^2\*c + 2\*a^2\*d)\*Hypergeometric2F1[1/2, -1/2\*(b^2\*c)/(b^2\*c + a^2\*d), (b^2\*c + 2\*a^2\*d)/(2\*b^2\*c + 2\*a^2\*d), (b^2\*x^2)/a^2]) + b^2\*d\*x^2\*Hypergeometric2F1[1/2, (b^2\*c + 2\*a^2\*d)/(2\*b^2\*c + 2\*a^2\*d), (3\*b^2\*c + 4\*a^2\*d)/(2\*b^2\*c + 2\*a^2\*d), (b^2\*x^2)/a^2])/(b^2\*(b^2\*c + 2\*a^2\*d)\*x^((b^2\*c)/(b^2\*c + a^2\*d))\*Sqrt[-a + b\*x]\*Sqrt[a + b\*x])

**fricas** [A] time = 1.01, size = 65, normalized size = 1.23

$$\frac{(b^2c + a^2d)\sqrt{bx + a}\sqrt{bx - a}x}{a^2b^2x^{\frac{2b^2c+a^2d}{b^2c+a^2d}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))/(b\*x-a)^(1/2)/(b\*x+a)^(1/2),x, algorithm="fricas")

[Out] (b^2\*c + a^2\*d)\*sqrt(b\*x + a)\*sqrt(b\*x - a)\*x/(a^2\*b^2\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d)))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{dx^2 + c}{\sqrt{bx + a}\sqrt{bx - a}x^{\frac{2b^2c+a^2d}{b^2c+a^2d}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))/(b\*x-a)^(1/2)/(b\*x+a)^(1/2),x, algorithm="giac")

[Out] integrate((d\*x^2 + c)/(sqrt(b\*x + a)\*sqrt(b\*x - a)\*x^((2\*b^2\*c + a^2\*d)/(b^2\*c + a^2\*d))), x)

**maple** [A] time = 0.05, size = 66, normalized size = 1.25

$$\frac{(a^2d + b^2c)\sqrt{bx + a}\sqrt{bx - a}x^{\frac{a^2d+2b^2c}{a^2d+b^2c}}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^2+c)/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))/(b\*x-a)^(1/2)/(b\*x+a)^(1/2),x)

[Out] x\*(a^2\*d+b^2\*c)\*(b\*x+a)^(1/2)/b^2/a^2/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))\*(b\*x-a)^(1/2)

**maxima** [A] time = 1.24, size = 79, normalized size = 1.49

$$\frac{(b^2c + a^2d)\sqrt{bx + a}\sqrt{bx - a}xe^{\left(\frac{2b^2c\log(x)}{b^2c+a^2d} - \frac{a^2d\log(x)}{b^2c+a^2d}\right)}}{a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2+c)/(x^((a^2\*d+2\*b^2\*c)/(a^2\*d+b^2\*c)))/(b\*x-a)^(1/2)/(b\*x+a)^(1/2),x, algorithm="maxima")

[Out] (b^2\*c + a^2\*d)\*sqrt(b\*x + a)\*sqrt(b\*x - a)\*x\*e^(-2\*b^2\*c\*log(x)/(b^2\*c + a^2\*d) - a^2\*d\*log(x)/(b^2\*c + a^2\*d))/(a^2\*b^2)

**mupad** [B] time = 3.27, size = 96, normalized size = 1.81

$$\frac{\frac{x(da^4+ca^2b^2)}{a^2b^2} - \frac{x^3(da^2b^2+cb^4)}{a^2b^2}}{x^{\frac{da^2+2cb^2}{da^2+cb^2}}\sqrt{a+bx}\sqrt{bx-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)/(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2)),x)
```

```
[Out] -((x*(a^4*d + a^2*b^2*c))/(a^2*b^2) - (x^3*(b^4*c + a^2*b^2*d))/(a^2*b^2))/
(x^((a^2*d + 2*b^2*c)/(a^2*d + b^2*c))*(a + b*x)^(1/2)*(b*x - a)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)/(x**((a**2*d+2*b**2*c)/(a**2*d+b**2*c)))/(b*x-a)**(1/2)
)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

$$3.380 \quad \int \frac{1}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}} \sqrt{1+x}} dx$$

**Optimal.** Leaf size=36

$$\frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1} \sqrt{\sqrt{x}-1}}$$

[Out] arcsin(x)\*(1-x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2)

**Rubi [A]** time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$ , Rules used = {519, 41, 216}

$$\frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1} \sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]]\*Sqrt[1 + x]),x]

[Out] (Sqrt[1 - x]\*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]])

#### Rule 41

Int[((a\_) + (b\_)\*(x\_))^(m\_)\*((c\_) + (d\_)\*(x\_))^(n\_), x\_Symbol] := Int[(a\*c + b\*d\*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b\*c + a\*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))

#### Rule 216

Int[1/Sqrt[(a\_) + (b\_)\*(x\_)^2], x\_Symbol] := Simp[ArcSin[Rt[-b, 2]\*x]/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

#### Rule 519

Int[(u\_)\*((c\_) + (d\_)\*(x\_)^(n\_))^(q\_)\*((a1\_) + (b1\_)\*(x\_)^(non2\_))^(p\_)\*((a2\_) + (b2\_)\*(x\_)^(non2\_))^(p\_), x\_Symbol] := Dist[((a1 + b1\*x^(n/2))^(FracPart[p])\*(a2 + b2\*x^(n/2))^(FracPart[p]))/(a1\*a2 + b1\*b2\*x^n)^(FracPart[p]), Int[u\*(a1\*a2 + b1\*b2\*x^n)^p\*(c + d\*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

#### Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}} \sqrt{1+x}} dx &= \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-x} \sqrt{1+x}} dx}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}}} \\ &= \frac{\sqrt{1-x} \int \frac{1}{\sqrt{1-x^2}} dx}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}}} \\ &= \frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-1-\sqrt{x}} \sqrt{-1+\sqrt{x}}} \end{aligned}$$



**Mathematica [A]** time = 0.02, size = 36, normalized size = 1.00

$$\frac{\sqrt{1-x} \sin^{-1}(x)}{\sqrt{-\sqrt{x}-1} \sqrt{\sqrt{x}-1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]]\*Sqrt[1 + x]),x]

[Out] (Sqrt[1 - x]\*ArcSin[x])/(Sqrt[-1 - Sqrt[x]]\*Sqrt[-1 + Sqrt[x]])

**fricas [C]** time = 0.92, size = 69, normalized size = 1.92

$$-i \log\left(\frac{\sqrt{x+1} \sqrt{\sqrt{x}-1} \sqrt{-\sqrt{x}-1} + ix - 1}{x}\right) + i \log\left(\frac{\sqrt{x+1} \sqrt{\sqrt{x}-1} \sqrt{-\sqrt{x}-1} - ix - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] -I\*log((sqrt(x + 1)\*sqrt(sqrt(x) - 1)\*sqrt(-sqrt(x) - 1) + I\*x - 1)/x) + I\*log((sqrt(x + 1)\*sqrt(sqrt(x) - 1)\*sqrt(-sqrt(x) - 1) - I\*x - 1)/x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} \sqrt{\sqrt{x}-1} \sqrt{-\sqrt{x}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(x + 1)\*sqrt(sqrt(x) - 1)\*sqrt(-sqrt(x) - 1)), x)

**maple [F]** time = 0.86, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} \sqrt{-\sqrt{x}-1} \sqrt{\sqrt{x}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)

[Out] int(1/(x+1)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} \sqrt{\sqrt{x}-1} \sqrt{-\sqrt{x}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2)/(-1-x^(1/2))^(1/2)/(-1+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x + 1)\*sqrt(sqrt(x) - 1)\*sqrt(-sqrt(x) - 1)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{\sqrt{x}-1} \sqrt{-\sqrt{x}-1} \sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^(1/2) - 1)^(1/2)\*(- x^(1/2) - 1)^(1/2)\*(x + 1)^(1/2)), x)

[Out] int(1/((x^(1/2) - 1)^(1/2)\*(- x^(1/2) - 1)^(1/2)\*(x + 1)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-\sqrt{x}-1} \sqrt{\sqrt{x}-1} \sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)\*\*(1/2)/(-1-x\*\*(1/2))\*\*(1/2)/(-1+x\*\*(1/2))\*\*(1/2), x)

[Out] Integral(1/(sqrt(-sqrt(x) - 1)\*sqrt(sqrt(x) - 1)\*sqrt(x + 1)), x)

$$3.381 \quad \int \frac{1}{\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}} \sqrt{a^2+b^2x}} dx$$

**Optimal.** Leaf size=75

$$\frac{2\sqrt{a^2-b^2x} \tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}}$$

[Out]  $-2*\arctan((-b^2*x+a^2)^{(1/2)}/(b^2*x+a^2)^{(1/2)})*(-b^2*x+a^2)^{(1/2)}/b^2/(a-b*x^{(1/2)})^{(1/2)}/(a+b*x^{(1/2)})^{(1/2)}$

**Rubi [A]** time = 0.05, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 41,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$ , Rules used = {519, 63, 217, 203}

$$\frac{2\sqrt{a^2-b^2x} \tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a - b\*Sqrt[x]]\*Sqrt[a + b\*Sqrt[x]]\*Sqrt[a^2 + b^2\*x]),x]

[Out]  $(-2*\text{Sqrt}[a^2 - b^2*x]*\text{ArcTan}[\text{Sqrt}[a^2 - b^2*x]/\text{Sqrt}[a^2 + b^2*x]])/(b^2*\text{Sqrt}[a - b*\text{Sqrt}[x]]*\text{Sqrt}[a + b*\text{Sqrt}[x]])$

#### Rule 63

Int[((a\_.) + (b\_.)\*(x\_))^(m\_)\*((c\_.) + (d\_.)\*(x\_))^(n\_), x\_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p\*(m + 1) - 1)\*(c - (a\*d)/b + (d\*x^p)/b)^n, x], x, (a + b\*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b\*c - a\*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

#### Rule 203

Int[((a\_) + (b\_.)\*(x\_)^2)^(-1), x\_Symbol] := Simp[(1\*ArcTan[(Rt[b, 2]\*x)/Rt[a, 2]]/(Rt[a, 2]\*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

#### Rule 217

Int[1/Sqrt[(a\_) + (b\_.)\*(x\_)^2], x\_Symbol] := Subst[Int[1/(1 - b\*x^2), x], x, x/Sqrt[a + b\*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

#### Rule 519

Int[(u\_.)\*((c\_) + (d\_.)\*(x\_)^(n\_.))^(q\_.)\*((a1\_) + (b1\_.)\*(x\_)^(non2\_.))^(p\_.)\*((a2\_) + (b2\_.)\*(x\_)^(non2\_.))^(p\_), x\_Symbol] := Dist[((a1 + b1\*x^(n/2))^FracPart[p]\*(a2 + b2\*x^(n/2))^FracPart[p])/(a1\*a2 + b1\*b2\*x^n)^FracPart[p], Int[u\*(a1\*a2 + b1\*b2\*x^n)^p\*(c + d\*x^n)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

#### Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}} \sqrt{a^2+b^2x}} dx &= \frac{\sqrt{a^2-b^2x} \int \frac{1}{\sqrt{a^2-b^2x} \sqrt{a^2+b^2x}} dx}{\sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}} \\
&= \frac{(2\sqrt{a^2-b^2x}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{2a^2-x^2}} dx, x, \sqrt{a^2-b^2x}\right)}{b^2 \sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}} \\
&= \frac{(2\sqrt{a^2-b^2x}) \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2 \sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}} \\
&= \frac{2\sqrt{a^2-b^2x} \tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2 \sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}}
\end{aligned}$$

**Mathematica [A]** time = 0.06, size = 75, normalized size = 1.00

$$\frac{2\sqrt{a^2-b^2x} \tan^{-1}\left(\frac{\sqrt{a^2-b^2x}}{\sqrt{a^2+b^2x}}\right)}{b^2 \sqrt{a-b\sqrt{x}} \sqrt{a+b\sqrt{x}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]]*Sqrt[a^2 + b^2*x]),x]
[Out] (-2*Sqrt[a^2 - b^2*x]*ArcTan[Sqrt[a^2 - b^2*x]/Sqrt[a^2 + b^2*x]])/(b^2*Sqrt[a - b*Sqrt[x]]*Sqrt[a + b*Sqrt[x]])
```

**fricas [A]** time = 1.32, size = 50, normalized size = 0.67

$$\frac{2 \arctan\left(-\frac{a^2 - \sqrt{b^2x+a^2} \sqrt{b\sqrt{x}+a} \sqrt{-b\sqrt{x}+a}}{b^2x}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b^2*x+a^2)^(1/2)/(a-b*x^(1/2))^(1/2)/(a+b*x^(1/2))^(1/2),x, algorithm="fricas")
[Out] -2*arctan(-(a^2 - sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a))/(b^2*x))/b^2
```

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x+a^2} \sqrt{b\sqrt{x}+a} \sqrt{-b\sqrt{x}+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x)
[Out] integrate(1/(sqrt(b^2*x + a^2)*sqrt(b*sqrt(x) + a)*sqrt(-b*sqrt(x) + a)), x)
```

**maple** [F] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x + a^2} \sqrt{-b\sqrt{x} + a} \sqrt{b\sqrt{x} + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b^2\*x+a^2)^(1/2)/(a-b\*x^(1/2))^(1/2)/(b\*x^(1/2)+a)^(1/2),x)

[Out] int(1/(b^2\*x+a^2)^(1/2)/(a-b\*x^(1/2))^(1/2)/(b\*x^(1/2)+a)^(1/2),x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b^2x + a^2} \sqrt{b\sqrt{x} + a} \sqrt{-b\sqrt{x} + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b^2\*x+a^2)^(1/2)/(a-b\*x^(1/2))^(1/2)/(a+b\*x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b^2\*x + a^2)\*sqrt(b\*sqrt(x) + a)\*sqrt(-b\*sqrt(x) + a)), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + b\sqrt{x}} \sqrt{a - b\sqrt{x}} \sqrt{a^2 + x b^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b\*x^(1/2))^(1/2)\*(a - b\*x^(1/2))^(1/2)\*(b^2\*x + a^2)^(1/2)),x)

[Out] int(1/((a + b\*x^(1/2))^(1/2)\*(a - b\*x^(1/2))^(1/2)\*(b^2\*x + a^2)^(1/2)), x)

**sympy** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a - b\sqrt{x}} \sqrt{a + b\sqrt{x}} \sqrt{a^2 + b^2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b\*\*2\*x+a\*\*2)\*\*(1/2)/(a-b\*x\*\*(1/2))\*\*(1/2)/(a+b\*x\*\*(1/2))\*\*(1/2),x)

[Out] Integral(1/(sqrt(a - b\*sqrt(x))\*sqrt(a + b\*sqrt(x))\*sqrt(a\*\*2 + b\*\*2\*x)), x)

### 3.382 $\int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx$

**Optimal.** Leaf size=113

$$x(a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

[Out] x\*(a-b\*x^n)^p\*(a+b\*x^n)^p\*(c+d\*x^(2\*n))^q\*AppellF1(1/2/n, -p, -q, 1+1/2/n, b^2\*x^(2\*n)/a^2, -d\*x^(2\*n)/c)/((1-b^2\*x^(2\*n)/a^2)^p)/((1+d\*x^(2\*n)/c)^q)

**Rubi [A]** time = 0.09, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {519, 430, 429}

$$x(a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(\frac{dx^{2n}}{c} + 1\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)$$

Antiderivative was successfully verified.

[In] Int[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q,x]

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q\*AppellF1[1/(2\*n), -p, -q, (2 + n^(-1))/2, (b^2\*x^(2\*n))/a^2, -((d\*x^(2\*n))/c)]/((1 - (b^2\*x^(2\*n))/a^2)^p\*(1 + (d\*x^(2\*n))/c)^q)

#### Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

#### Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

#### Rule 519

```
Int[(u_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.)*((a1_) + (b1_.)*(x_)^(non2_.))^(p_.)*((a2_) + (b2_.)*(x_)^(non2_.))^(p_), x_Symbol]
:> Dist[((a1 + b1*x^(n/2))^FracPart[p]*(a2 + b2*x^(n/2))^FracPart[p])/(a1*a2 + b1*b2*x^n)^FracPart[p], Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])
```

#### Rubi steps

$$\begin{aligned} \int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx &= \left( (a - bx^n)^p (a + bx^n)^p (a^2 - b^2 x^{2n})^{-p} \right) \int (a^2 - b^2 x^{2n})^p (c + dx^{2n})^q dx \\ &= \left( (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} \right) \int \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^p (c + dx^{2n})^q dx \\ &= \left( (a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(1 + \frac{dx^{2n}}{c}\right)^{-q} \right) \int \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^p \left(1 + \frac{dx^{2n}}{c}\right)^q dx \\ &= x(a - bx^n)^p (a + bx^n)^p \left(1 - \frac{b^2 x^{2n}}{a^2}\right)^{-p} (c + dx^{2n})^q \left(1 + \frac{dx^{2n}}{c}\right)^{-q} F_1\left(\frac{1}{2n}; -p, -q; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2 x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right) \end{aligned}$$

**Mathematica** [F] time = 0.44, size = 0, normalized size = 0.00

$$\int (a - bx^n)^p (a + bx^n)^p (c + dx^{2n})^q dx$$

Verification is Not applicable to the result.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q,x]

[Out] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(c + d\*x^(2\*n))^q, x]

**fricas** [F] time = 1.41, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx^{2n} + c\right)^q (bx^n + a)^p (-bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(c+d\*x^(2\*n))^q,x, algorithm="fricas")

[Out] integral((d\*x^(2\*n) + c)^q\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^{2n} + c)^q (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(c+d\*x^(2\*n))^q,x, algorithm="giac")

[Out] integrate((d\*x^(2\*n) + c)^q\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**maple** [F] time = 1.60, size = 0, normalized size = 0.00

$$\int (-bx^n + a)^p (bx^n + a)^p (dx^{2n} + c)^q dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^n+a)^p\*(b\*x^n+a)^p\*(c+d\*x^(2\*n))^q,x)

[Out] int((-b\*x^n+a)^p\*(b\*x^n+a)^p\*(c+d\*x^(2\*n))^q,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx^{2n} + c)^q (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(c+d\*x^(2\*n))^q,x, algorithm="maxima")

[Out] integrate((d\*x^(2\*n) + c)^q\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (c + dx^{2n})^q (a + bx^n)^p (a - bx^n)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^(2\*n))^q\*(a + b\*x^n)^p\*(a - b\*x^n)^p,x)

[Out] int((c + d\*x^(2\*n))^q\*(a + b\*x^n)^p\*(a - b\*x^n)^p, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p\*(c+d\*x\*\*(2\*n))\*\*q,x)

[Out] Timed out



### 3.383 $\int (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p dx$

Optimal. Leaf size=87

$$x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left(1 - \frac{b^4 x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n}, -p; \frac{1}{4}\left(4 + \frac{1}{n}\right); \frac{b^4 x^{4n}}{a^4}\right)$$

[Out]  $x*(a-b*x^n)^p*(a+b*x^n)^p*(a^2+b^2*x^(2*n))^p*\text{hypergeom}([-p, 1/4/n], [1+1/4/n], b^4*x^(4*n)/a^4)/((1-b^4*x^(4*n)/a^4)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$ , Rules used = {519, 253, 246, 245}

$$x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left(1 - \frac{b^4 x^{4n}}{a^4}\right)^{-p} {}_2F_1\left(\frac{1}{4n}, -p; \frac{1}{4}\left(4 + \frac{1}{n}\right); \frac{b^4 x^{4n}}{a^4}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p, x]$

[Out]  $(x*(a - b*x^n)^p*(a + b*x^n)^p*(a^2 + b^2*x^(2*n))^p*\text{Hypergeometric2F1}[1/(4*n), -p, (4 + n^(-1))/4, (b^4*x^(4*n))/a^4])/(1 - (b^4*x^(4*n))/a^4)^p$

#### Rule 245

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_), x\_Symbol] \rightarrow \text{Simp}[a^p*x*\text{Hypergeometric2F1}[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

#### Rule 246

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_), x\_Symbol] \rightarrow \text{Dist}[(a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]})/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p, x], x] /; \text{FreeQ}\{a, b, n, p\}, x \ \&\& \ !\text{IGtQ}[p, 0] \ \&\& \ !\text{IntegerQ}[1/n] \ \&\& \ !\text{ILtQ}[\text{Simplify}[1/n + p], 0] \ \&\& \ !(\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

#### Rule 253

$\text{Int}[(a1_ + (b1_)*(x_)^(n_))^(p_)*((a2_ + (b2_)*(x_)^(n_))^(p_)), x\_Symbol] \rightarrow \text{Dist}[(a1 + b1*x^n)^{\text{FracPart}[p]}*(a2 + b2*x^n)^{\text{FracPart}[p]}/(a1*a2 + b1*b2*x^(2*n))^{\text{FracPart}[p]}, \text{Int}[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; \text{FreeQ}\{a1, b1, a2, b2, n, p\}, x \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ !\text{IntegerQ}[p]$

#### Rule 519

$\text{Int}[(u_)*((c_ + (d_)*(x_)^(n_))^(q_))*((a1_ + (b1_)*(x_)^(non2_))^(p_))*((a2_ + (b2_)*(x_)^(non2_))^(p_)), x\_Symbol] \rightarrow \text{Dist}[(a1 + b1*x^(n/2))^{\text{FracPart}[p]}*(a2 + b2*x^(n/2))^{\text{FracPart}[p]}/(a1*a2 + b1*b2*x^n)^{\text{FracPart}[p]}, \text{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x] /; \text{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q\}, x \ \&\& \ \text{EqQ}[non2, n/2] \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ !(\text{EqQ}[n, 2] \ \&\& \ \text{IGtQ}[q, 0])$

#### Rubi steps

$$\begin{aligned}
\int (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p dx &= \left( (a - bx^n)^p (a + bx^n)^p (a^2 - b^2 x^{2n})^{-p} \right) \int (a^2 - b^2 x^{2n})^p (a^2 + b^2 x^{2n})^p dx \\
&= \left( (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p (a^4 - b^4 x^{4n})^{-p} \right) \int (a^4 - b^4 x^{4n})^p dx \\
&= \left( (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left( 1 - \frac{b^4 x^{4n}}{a^4} \right)^{-p} \right) \int \left( 1 - \frac{b^4 x^{4n}}{a^4} \right)^p dx \\
&= x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left( 1 - \frac{b^4 x^{4n}}{a^4} \right)^{-p} {}_2F_1 \left( \frac{1}{4n}, -p; \frac{1}{4} \left( 1 - \frac{b^4 x^{4n}}{a^4} \right) \right)
\end{aligned}$$

**Mathematica** [A] time = 0.05, size = 87, normalized size = 1.00

$$x (a - bx^n)^p (a + bx^n)^p (a^2 + b^2 x^{2n})^p \left( 1 - \frac{b^4 x^{4n}}{a^4} \right)^{-p} {}_2F_1 \left( \frac{1}{4n}, -p; 1 + \frac{1}{4n}; \frac{b^4 x^{4n}}{a^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p,x]

[Out] (x\*(a - b\*x^n)^p\*(a + b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p\*Hypergeometric2F1[1/(4\*n), -p, 1 + 1/(4\*n), (b^4\*x^(4\*n))/a^4])/(1 - (b^4\*x^(4\*n))/a^4)^p

**fricas** [F] time = 1.31, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 x^{2n} + a^2\right)^p (bx^n + a)^p (-bx^n + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(a^2+b^2\*x^(2\*n))^p,x, algorithm="fricas")

[Out] integral((b^2\*x^(2\*n) + a^2)^p\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( b^2 x^{2n} + a^2 \right)^p (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(a^2+b^2\*x^(2\*n))^p,x, algorithm="giac")

[Out] integrate((b^2\*x^(2\*n) + a^2)^p\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**maple** [F] time = 1.50, size = 0, normalized size = 0.00

$$\int (-b x^n + a)^p (b x^n + a)^p \left( b^2 x^{2n} + a^2 \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b\*x^n+a)^p\*(b\*x^n+a)^p\*(a^2+b^2\*x^(2\*n))^p,x)

[Out] int((-b\*x^n+a)^p\*(b\*x^n+a)^p\*(a^2+b^2\*x^(2\*n))^p,x)

**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left( b^2 x^{2n} + a^2 \right)^p (bx^n + a)^p (-bx^n + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^n)^p\*(a+b\*x^n)^p\*(a^2+b^2\*x^(2\*n))^p,x, algorithm="maxima")

[Out] integrate((b^2\*x^(2\*n) + a^2)^p\*(b\*x^n + a)^p\*(-b\*x^n + a)^p, x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (a + b x^n)^p (a - b x^n)^p (a^2 + b^2 x^{2n})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b\*x^n)^p\*(a - b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p,x)

[Out] int((a + b\*x^n)^p\*(a - b\*x^n)^p\*(a^2 + b^2\*x^(2\*n))^p, x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*n)\*\*p\*(a+b\*x\*\*n)\*\*p\*(a\*\*2+b\*\*2\*x\*\*(2\*n))\*\*p,x)

[Out] Timed out

$$3.384 \quad \int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)} dx$$

**Optimal.** Leaf size=76

$$\frac{x(c+dx^{2n})^p \left(\frac{dx^{2n}}{c} + 1\right)^{-p} F_1\left(\frac{1}{2n}; 1, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2}$$

[Out]  $x*(c+d*x^(2*n))^p*AppellF1(1/2/n, 1, -p, 1+1/2/n, b^2*x^(2*n)/a^2, -d*x^(2*n)/c)/a^2/((1+d*x^(2*n)/c)^p)$

**Rubi [A]** time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$ , Rules used = {517, 430, 429}

$$\frac{x(c+dx^{2n})^p \left(\frac{dx^{2n}}{c} + 1\right)^{-p} F_1\left(\frac{1}{2n}; 1, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(c + d*x^(2*n))^p/((a - b*x^n)*(a + b*x^n)), x]$

[Out]  $(x*(c + d*x^(2*n))^p*AppellF1[1/(2*n), 1, -p, (2 + n^(-1))/2, (b^2*x^(2*n))/a^2, -((d*x^(2*n))/c)])/(a^2*(1 + (d*x^(2*n))/c)^p)$

**Rule 429**

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_)*((c_ + (d_)*(x_)^(n_))^(q_)), x\_Symbol]$   
 $\rightarrow \text{Simp}[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ (\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0]) \ \&\& \ (\text{IntegerQ}[q] \ || \ \text{GtQ}[c, 0])$

**Rule 430**

$\text{Int}[(a_ + (b_)*(x_)^(n_))^(p_)*((c_ + (d_)*(x_)^(n_))^(q_)), x\_Symbol]$   
 $\rightarrow \text{Dist}[a^{\text{IntPart}[p]}*(a + b*x^n)^{\text{FracPart}[p]}/(1 + (b*x^n)/a)^{\text{FracPart}[p]}, \text{Int}[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;$   
 $\text{FreeQ}\{a, b, c, d, n, p, q, x\} \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{NeQ}[n, -1] \ \&\& \ !(\text{IntegerQ}[p] \ || \ \text{GtQ}[a, 0])$

**Rule 517**

$\text{Int}[(u_)*((c_ + (d_)*(x_)^(n_))^(q_))*((a1_ + (b1_)*(x_)^(non2_))^(p_))*((a2_ + (b2_)*(x_)^(non2_))^(p_)), x\_Symbol]$   
 $\rightarrow \text{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /;$   
 $\text{FreeQ}\{a1, b1, a2, b2, c, d, n, p, q, x\} \ \&\& \ \text{EqQ}[a2*b1 + a1*b2, 0] \ \&\& \ (\text{IntegerQ}[p] \ || \ (\text{GtQ}[a1, 0] \ \&\& \ \text{GtQ}[a2, 0]))$

**Rubi steps**

$$\begin{aligned} \int \frac{(c+dx^{2n})^p}{(a-bx^n)(a+bx^n)} dx &= \int \frac{(c+dx^{2n})^p}{a^2 - b^2x^{2n}} dx \\ &= \left( (c+dx^{2n})^p \left(1 + \frac{dx^{2n}}{c}\right)^{-p} \right) \int \frac{\left(1 + \frac{dx^{2n}}{c}\right)^p}{a^2 - b^2x^{2n}} dx \\ &= \frac{x(c+dx^{2n})^p \left(1 + \frac{dx^{2n}}{c}\right)^{-p} F_1\left(\frac{1}{2n}; 1, -p; \frac{1}{2}\left(2 + \frac{1}{n}\right); \frac{b^2x^{2n}}{a^2}, -\frac{dx^{2n}}{c}\right)}{a^2} \end{aligned}$$

**Mathematica [B]** time = 0.43, size = 258, normalized size = 3.39

$$\frac{a^2 c (2n+1) x (c + dx^{2n})^p F_1\left(\frac{1}{2n}; -p, 1; 1 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right)}{(a^2 - b^2 x^{2n}) \left(2a^2 d n p x^{2n} F_1\left(1 + \frac{1}{2n}; 1 - p, 1; 2 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right) + 2b^2 c n x^{2n} F_1\left(1 + \frac{1}{2n}; -p, 2; 2 + \frac{1}{2n}; -\frac{dx^{2n}}{c}, \frac{b^2 x^{2n}}{a^2}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d\*x^(2\*n))^p/((a - b\*x^n)\*(a + b\*x^n)),x]

[Out] (a^2\*c\*(1 + 2\*n)\*x\*(c + d\*x^(2\*n))^p\*AppellF1[1/(2\*n), -p, 1, 1 + 1/(2\*n), -((d\*x^(2\*n))/c), (b^2\*x^(2\*n))/a^2])/((a^2 - b^2\*x^(2\*n))\*(2\*a^2\*d\*n\*p\*x^(2\*n)\*AppellF1[1 + 1/(2\*n), 1 - p, 1, 2 + 1/(2\*n), -((d\*x^(2\*n))/c), (b^2\*x^(2\*n))/a^2] + 2\*b^2\*c\*n\*x^(2\*n)\*AppellF1[1 + 1/(2\*n), -p, 2, 2 + 1/(2\*n), -((d\*x^(2\*n))/c), (b^2\*x^(2\*n))/a^2] + a^2\*c\*(1 + 2\*n)\*AppellF1[1/(2\*n), -p, 1, 1 + 1/(2\*n), -((d\*x^(2\*n))/c), (b^2\*x^(2\*n))/a^2]))

**fricas [F]** time = 1.00, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(dx^{2n} + c)^p}{b^2x^{2n} - a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^(2\*n))^p/(a-b\*x^n)/(a+b\*x^n),x, algorithm="fricas")

[Out] integral(-(d\*x^(2\*n) + c)^p/(b^2\*x^(2\*n) - a^2), x)

**giac [F]** time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(dx^{2n} + c)^p}{(bx^n + a)(bx^n - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^(2\*n))^p/(a-b\*x^n)/(a+b\*x^n),x, algorithm="giac")

[Out] integrate(-(d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x)

**maple [F]** time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(dx^{2n} + c)^p}{(-bx^n + a)(bx^n + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d\*x^(2\*n)+c)^p/(-b\*x^n+a)/(b\*x^n+a),x)

[Out] int((d\*x^(2\*n)+c)^p/(-b\*x^n+a)/(b\*x^n+a),x)

**maxima [F]** time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(dx^{2n} + c)^p}{(bx^n + a)(bx^n - a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x^(2\*n))^p/(a-b\*x^n)/(a+b\*x^n),x, algorithm="maxima")

[Out] -integrate((d\*x^(2\*n) + c)^p/((b\*x^n + a)\*(b\*x^n - a)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(c + dx^{2n})^p}{a^2 - b^2 x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d\*x^(2\*n))^p/((a + b\*x^n)\*(a - b\*x^n)), x)

[Out] -int(-(c + d\*x^(2\*n))^p/(a^2 - b^2\*x^(2\*n)), x)

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c+d\*x\*\*(2\*n))\*\*p/(a-b\*x\*\*n)/(a+b\*x\*\*n), x)

[Out] Exception raised: HeuristicGCDFailed

$$3.385 \quad \int (a - bx^{n/2})^p (a + bx^{n/2})^p \left( \frac{a^2 d(1+p)}{b^2 \left(1 + \frac{-1-2n-np}{n}\right)} + dx^n \right)^{\frac{-1-2n-np}{n}}$$

Optimal. Leaf size=96

$$\frac{b^2 x(np+n+1) (a - bx^{n/2})^{p+1} (a + bx^{n/2})^{p+1} \left( dx^n - \frac{a^2 dn(p+1)}{b^2(np+n+1)} \right)^{-\frac{np+n+1}{n}}}{a^4 dn(p+1)}$$

[Out]  $-b^2*(n*p+n+1)*x*(a-b*x^{(1/2*n)})^{(1+p)}*(a+b*x^{(1/2*n)})^{(1+p)}/a^4/d/n/(1+p)/((-a^2*d*n*(1+p)/b^2/(n*p+n+1)+d*x^n)^{((n*p+n+1)/n)}$

**Rubi [A]** time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, integrand size = 76,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$ , Rules used = {519, 381}

$$\frac{b^2 x(np+n+1) (a^2 - b^2 x^n) (a - bx^{n/2})^p (a + bx^{n/2})^p \left( dx^n - \frac{a^2 dn(p+1)}{b^2(np+n+1)} \right)^{-\frac{np+n+1}{n}}}{a^4 dn(p+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a - b*x^{(n/2)})^p*(a + b*x^{(n/2)})^p*((a^2*d*(1 + p))/(b^2*(1 + (-1 - 2*n - n*p)/n)) + d*x^n)^{((-1 - 2*n - n*p)/n)}, x]$

[Out]  $-((b^2*(1 + n + n*p)*x*(a - b*x^{(n/2)})^p*(a + b*x^{(n/2)})^p*(a^2 - b^2*x^n))/(a^4*d*n*(1 + p)*(-(a^2*d*n*(1 + p))/(b^2*(1 + n + n*p))) + d*x^n)^{((1 + n + n*p)/n))}$

#### Rule 381

$\text{Int}[(a + b*x^n)^p*(c + d*x^n)^q, x]$   
 $\text{Simp}[(x*(a + b*x^n)^{p+1}*(c + d*x^n)^{q+1})/(a*c), x]$  /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b\*c - a\*d, 0] && EqQ[n\*(p + q + 2) + 1, 0] && EqQ[a\*d\*(p + 1) + b\*c\*(q + 1), 0]

#### Rule 519

$\text{Int}[(a + b*x^n)^p*(c + d*x^n)^q*(a1 + b1*x^{n/2})^p*(a2 + b2*x^{n/2})^p, x]$   
 $\text{Dist}[(a1 + b1*x^{n/2})^p*(a2 + b2*x^{n/2})^p/(a1*a2 + b1*b2*x^n)^p, \text{Int}[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x], x]$  /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2\*b1 + a1\*b2, 0] && !(EqQ[n, 2] && IGtQ[q, 0])

#### Rubi steps

$$\int (a - bx^{n/2})^p (a + bx^{n/2})^p \left( \frac{a^2 d(1+p)}{b^2 \left(1 + \frac{-1-2n-np}{n}\right)} + dx^n \right)^{\frac{-1-2n-np}{n}} dx = \left( (a - bx^{n/2})^p (a + bx^{n/2})^p (a^2 - b^2 x^n)^{-p} \right) \int \frac{b^2(1+n+np)x (a - bx^{n/2})^p (a + bx^{n/2})^p}{a^4 dn(1)}$$

**Mathematica** [A] time = 0.39, size = 103, normalized size = 1.07

$$\frac{b^2 x (np + n + 1) (a^2 - b^2 x^n) (a - b x^{n/2})^p (a + b x^{n/2})^p \left( d \left( x^n - \frac{a^2 n (p+1)}{b^2 (np+n+1)} \right) \right)^{-\frac{np+n+1}{n}}}{a^4 d n (p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a - b\*x^(n/2))^p\*(a + b\*x^(n/2))^p\*((a^2\*d\*(1 + p))/(b^2\*(1 + (-1 - 2\*n - n\*p)/n)) + d\*x^n)^(((-1 - 2\*n - n\*p)/n), x]

[Out] -((b^2\*(1 + n + n\*p)\*x\*(a - b\*x^(n/2))^p\*(a + b\*x^(n/2))^p\*(a^2 - b^2\*x^n))/(a^4\*d\*n\*(1 + p)\*(d\*(-((a^2\*n\*(1 + p))/(b^2\*(1 + n + n\*p)))) + x^n)^((1 + n + n\*p)/n))

**fricas** [A] time = 1.24, size = 180, normalized size = 1.88

$$\frac{\left( (b^4 np + b^4 n + b^4) x x^{2n} - (2 a^2 b^2 np + 2 a^2 b^2 n + a^2 b^2) x x^n + (a^4 np + a^4 n) x \right) \left( b x^{\frac{1}{2}n} + a \right)^p \left( -b x^{\frac{1}{2}n} + a \right)^p}{(a^4 np + a^4 n) \left( -\frac{a^2 d np + a^2 d n - (b^2 d np + b^2 d n + b^2 d) x^n}{b^2 np + b^2 n + b^2} \right)^{\frac{np+2n+1}{n}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(1+p)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x, algorithm="fricas")

[Out] ((b^4\*n\*p + b^4\*n + b^4)\*x\*x^(2\*n) - (2\*a^2\*b^2\*n\*p + 2\*a^2\*b^2\*n + a^2\*b^2)\*x\*x^n + (a^4\*n\*p + a^4\*n)\*x)\*(b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/((a^4\*n\*p + a^4\*n)\*(-(a^2\*d\*n\*p + a^2\*d\*n - (b^2\*d\*n\*p + b^2\*d\*n + b^2\*d)\*x^n)/(b^2\*n\*p + b^2\*n + b^2))^((n\*p + 2\*n + 1)/n))

**giac** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left( b x^{\frac{1}{2}n} + a \right)^p \left( -b x^{\frac{1}{2}n} + a \right)^p}{\left( d x^n - \frac{a^2 d (p+1)}{b^2 \left( \frac{np+2n+1}{n} - 1 \right)} \right)^{\frac{np+2n+1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(1+p)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x, algorithm="giac")

[Out] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/(d\*x^n - a^2\*d\*(p + 1)/(b^2\*((n\*p + 2\*n + 1)/n - 1)))^((n\*p + 2\*n + 1)/n), x)

**maple** [F] time = 2.17, size = 0, normalized size = 0.00

$$\int \left( -b x^{\frac{n}{2}} + a \right)^p \left( b x^{\frac{n}{2}} + a \right)^p \left( d x^n + \frac{(p+1) a^2 d}{\left( \frac{-np-2n-1}{n} + 1 \right) b^2} \right)^{\frac{-np-2n-1}{n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(p+1)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x)

[Out] int((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(p+1)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n), x)



**maxima** [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(bx^{\frac{1}{2}n} + a\right)^p \left(-bx^{\frac{1}{2}n} + a\right)^p}{\left(dx^n - \frac{a^2 d(p+1)}{b^2 \left(\frac{np+2n+1}{n} - 1\right)}\right)^{\frac{np+2n+1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x^(1/2\*n))^p\*(a+b\*x^(1/2\*n))^p\*(a^2\*d\*(1+p)/b^2/(1+(-n\*p-2\*n-1)/n)+d\*x^n)^((-n\*p-2\*n-1)/n),x, algorithm="maxima")

[Out] integrate((b\*x^(1/2\*n) + a)^p\*(-b\*x^(1/2\*n) + a)^p/(d\*x^n - a^2\*d\*(p + 1)/(b^2\*((n\*p + 2\*n + 1)/n - 1)))^((n\*p + 2\*n + 1)/n), x)

**mupad** [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + b x^{n/2}\right)^p \left(a - b x^{n/2}\right)^p}{\left(d x^n - \frac{a^2 d(p+1)}{b^2 \left(\frac{2n+np+1}{n} - 1\right)}\right)^{\frac{2n+np+1}{n}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b\*x^(n/2))^p\*(a - b\*x^(n/2))^p)/(d\*x^n - (a^2\*d\*(p + 1))/(b^2\*((2\*n + n\*p + 1)/n - 1))))^((2\*n + n\*p + 1)/n),x)

[Out] int(((a + b\*x^(n/2))^p\*(a - b\*x^(n/2))^p)/(d\*x^n - (a^2\*d\*(p + 1))/(b^2\*((2\*n + n\*p + 1)/n - 1))))^((2\*n + n\*p + 1)/n), x)

**sympy** [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a-b\*x\*\*(1/2\*n))\*\*p\*(a+b\*x\*\*(1/2\*n))\*\*p\*(a\*\*2\*d\*(1+p)/b\*\*2/(1+(-n\*p-2\*n-1)/n)+d\*x\*\*n)\*\*((-n\*p-2\*n-1)/n),x)

[Out] Timed out



# Chapter 4

## Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```

(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType, expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]], 2]],
      Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3, ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
    If[Head[expn]===RootSum,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
    If[Head[expn]===Integrate || Head[expn]===Int,
      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
    9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

## 4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
  debug:=false;

  leaf_count_result:=leafcount(result);
  #do NOT call ExpnType() if leaf size is too large. Recursion problem
  if leaf_count_result > 500000 then
    return "B";
  fi;

  leaf_count_optimal:=leafcount(optimal);

  ExpnType_result:=ExpnType(result);
  ExpnType_optimal:=ExpnType(optimal);

  if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
  fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
  return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
  if debug then
    print("ExpnType_result<=ExpnType_optimal");
  fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+' or type(expn,'*') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```

```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

### 4.0.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
    asinh,acosh,atanh,acoth,asech,acsch
  ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
    gamma,loggamma,digamma,zeta,polylog,LambertW,
    elliptic_f,elliptic_e,elliptic_pi,exp_polar
  ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

```



```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`' or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

```

```

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

## 4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True

```

```

        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):

```

```

        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(6,m1)      #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
expn.args)))
        return max(8,m1)      #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```